This is from the syllabus of a topics course I taught at Ohio State in spring. I am sharing it to give you the list of references. A combination of Bourdon, Ledrappier and Roblin is a good way to understand Busemann functions. See also 'Lectures on spaces of nonpositive curvature' by Ballmann, which is probably the best current English-language resource. I am planning to write a textbook called 'Ergodic Geometry' based partly on the material in this course. This will be available in a year or two (??) — Dan Thompson, July 2024.

Course title: Math 8210 Topics in Real Analysis - Ergodic Geometry, Spring 2024, Taught by Dan Thompson

Brief Description: The phrase Géométrie Ergodique has been used for several years in the French school to refer to the ergodic theory of geodesic flows, particularly via geometric methods. The phrase 'Ergodic Geometry' has not been much used in the English-language literature, and we use it to describe a subfield of dynamical systems and ergodic theory covering both the French meaning of this area, and also covering related areas of smooth dynamics where a geometric perspective is valuable, including the thermodynamic formalism and dimension theory of e.g. rational maps of the Riemann sphere and beyond.

The aim of this course is to give a comprehensive and accessible account of this theory, and will tie in with plans to write a book on this topic suitable for advanced graduate students. The focus will be the theory of Gibbs measures for geodesic flows, developed geometrically. I plan to develop this material methodically. The excellent reference sources for this material are:

1) Entropie et Principe Variatonnel pour le flot géodésique en courbure négative pincée by Francois Ledrappier, [Survey paper 2013].

2) Ergodicité et Equidistribution en courbure négative by Thomas Roblin [SMF research monograph 2003]

3) Principe variationnel et groupes Kleiniens by Otal and Peigné [Duke 2004]

4) `Equilibrium states in negative curvature' by Paulin, Pollicott and Schapira [Astérisque, 2013]. This book is essentially a very long (very good) research article.

5) Structure conforme au bord et flot géodésique d'un CAT(-1)-espace by Bourdon, 1995

My goal for the course (and the book) is to give an account of this theory which includes more background, and is accessible to graduate students, more in the spirit of the French-language resources.

Reference books include:

1) Ergodic Theory by Walters

2) Foundations of Ergodic Theory by Viana and Oliveira

3) Ergodic Theory and dynamical systems by Yves Coudene

4) Geodesic and Horocylic Trajectories by Dal'Bo

Topics will include:

1) Entropy background

2) Geometry background (including Busemann functions and some basics of CAT(-1) spaces)

3) Constructing Gibbs measures geometrically

4) Advanced entropy theory - the theory of measurable partitions.

5) The Otal-Peigné "Principe variationnel"

6) Equidistribution of closed geodesics

7) Recent generalizations - equilibrium states, coarse constructions

8) Student-led topics (e.g. connections with dimension theory, connections with complex dynamics)