Lecturer: Luna Lomonaco
Title: The Mandelbrot set and its copies

Abstract: We consider dynamical systems on the Riemann sphere whose evolution rule is given by a complex polynomial. Here infinity is a (super-)attracting fixed point, and the filled Julia set is the set of points with bounded orbit. The quadratic family, P_c(z)=z^2+c, gives rise to the Mandelbrot set M: the set of parameters c such that the filled Julia set of P_c is connected.

Computer experiments quickly reveal the existence of small homeomorphic copies of M inside itself; the existence of such copies is explained by the theory of polynomial-like maps, introduced by Douady and Hubbard in 1985.

Each little copy is either primitive (with a cusp on the boundary of its main cardioid region) or a satellite (without a cusp). Lyubich proved that the primitive copies of M satisfy a stronger regularity condition: they are quasiconformally homeomorphic to M. The satellite copies are not quasiconformally homeomorphic to M (as we cannot straighten a cusp quasiconformally), and we showed (together with Carsten Petersen) that they are mutually quasiconformally homeomorphic if and only if their rotation numbers have the same denominator.

This minicourse aims to introduce students without previous knowledge of complex dynamics and quasiconformal surgery to the Mandelbrots and its copies.

The program of the course includes:

1- Polynomials on the Riemann sphere and the Mandelbrot set
2- Quasiconformal maps and quasiconformal surgery
3- Polynomial-like maps
4- Primitive and satellite copies of the Mandelbrot set