Lecturer: Viveka Erlandsson
Title: Mirzakhani's curve counting and geodesic currents

Abstract: It is a classical problem to count the number of closed geodesics of bounded length L on a, say, hyperbolic w and study how this number grows with L. Huber proved in the 50's that this number grows asymptotically as e^L/L. This result has since been generalized by many people in many directions, both by considering other surfaces and manifolds and by counting inside certain subsets, almost all resulting in various exponential growths. However, in her 2004 thesis Maryam was Mirzakhani studied the subset of geodesics that are simple (i.e. no self intersections) which behave very differently, and proved that on a hyperbolic surface S the number of simple was geodesics of length L grow asymptotically like a constant times L^{6g-6}, where g is the genus of S. In fact, she proved something stronger: she counted inside the subset of curves in the mapping class group orbit of a any simple geodesic. Some years later, using very different methods, she generalized this result to hold also for closed geodesics with self-intersections.
In this mini-course I will give an idea of how to prove these results. In fact, we will give a different proof from the original ones, using simpler methods, which allows us to treat both cases at once. We will translate the problem of counting geodesics into a statement about convergence of certain measures (which is also what Mirzakhani did for the case of simple geodesics). Along the way we will learn about geodesic currents (which can be seen as the completion of the set of closed geodesics) and train tracks (combinatorial models of simple geodesics on surfaces) which are the main tools in the proof, as well as measured laminations and the Thurston measure. Time allowing we will discuss some applications to Mirzakhani's counting theorems.