Trieste Algebraic Geometry Summer School (TAGSS) 2024 - Tropical Geometry and Related Topics | (SMR 3965)

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T01

Assembling the Chirotopal Tropical Grassmannian

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The moduli space $\mathcal{M}_{0,n}$ plays an essential role in the CHY integral formulation of scattering amplitudes in theoretical particle physics. In particular, its tropicalization encodes the biadjoint cubic scalar amplitude at tree-level. In practice, such amplitudes are decomposed into partial amplitudes, corresponding to relabelings of the positive tropical Grassmannian $\mathrm{Trop}^+G(2,n)$ modulo lineality. In recent work in physics by Cachazo, Early and Zhang, this story was extended to the case where $\mathcal{M}_{0,n}$ is replaced by the moduli space $X(k,n)$ of n points in projective space $\mathbb{P}^{k-1}$. This required the introduction of the chirotopal Dressian $\mathrm{Dr}^\chi(k,n)$ and the chirotopal tropical Grassmannian $\mathrm{Trop}^\chi G(k,n)$, using the combinatorial data of oriented matroids χ. This gives rise to polyhedral fans which are not related by relabeling to $\mathrm{Trop}^+G(k,n)$. It remained to give a complete description of the fan structures.

In this talk, we explain our computation of $\mathrm{Dr}^\chi(3,n)$ for $n=6,7,8$ and any given realizable chirotope $\chi \in \{\pm 1\}^{\binom{n}{3}}$. In particular, they are pure fans of dimension $2(n-4)$, and have the following properties: (1) the maximal cones in $\mathrm{Dr}^\chi(3,n)$ are exactly the maximal cliques in the graph of compatible pairs of chirotropical Plucker vectors, and (2) any cone of $\mathrm{Dr}^\chi(3,n)$ of any dimension is equal to the pairwise intersection of exactly two maximal cones.

[1] Freddy Cachazo, Nick Early and Yong Zhang, *Color-Dressed Generalized Biadjoint Scalar Amplitudes: Local Planarity*, SIGMA **20** (2024) doi:10.3842/SIGMA.2024.016

T02

Toric degenerations of Grassmannian parametrizations

Viktoriia Borovik and Svala Sverrisdóttir

Abstract

In this paper, we present a generalization of the results in [1]. We consider a birational parametrization of the Plücker embedding and study the graph of this map. The question we pose is:

Can we lift some toric degeneration of the Grassmannian to its graph?

Specifically, it is natural for us to focus on the degeneration of the Grassmannian corresponding to the pointed diagonal matching field or, equivalently, to the FFLV degeneration. The resulting toric degeneration is related to the so-called FFLV (Feigin–Fourier–Littelmann–Vinberg) polytope and is defined by the Hibi-Li ideal of a distributive lattice, which is known as the PBW-poset. We extend this PBW-poset to the graph and prove the finite Khovanskii basis property for the parameterization.

In particular, this helps us to represent the degree of the graph as the normalized volume of a polytope. In the case of the Grassmannian of lines, we find this volume explicitly by finding the Stanley-Reisner ideal for the unimodular triangulation of this polytope.

The question is motivated by a nonlinear eigenvalue problem on the Grassmannian arising in quantum chemistry. The degree of the graph is equal to the number of complex solutions for this problem, and toric degenerations allow us to solve this problem by Homotopy continuation methods.

References

[1] Viktoriia Borovik, Bernd Sturmfels, and Svala Sverrisdóttir. Coupled cluster degree of the grassmannian. `arXiv:2310.15474`, 2023.

T03

Genus 0 logarithmic and tropical fixed-domain counts for Hirzebruch surfaces

Alessio Cela and Aitor Iribar López

Abstract

For a non-singular projective toric variety X, the virtual logarithmic Tevelev degrees are defined as the virtual degree of the morphism from the moduli stack of logarithmic stable maps $\overline{\mathcal{M}}_\Gamma(X)$ to the product $\overline{\mathcal{M}}_{g,n} \times X^n$. In this talk I will explain how to use tropical methods to provide closed formulas for the the virtual logarithmic Tevelev degrees of Hirzebruch surfaces in genus 0.

2020 MSC 14T90, 14N35 (primary).

T04

On generalization of tropical enumerative invariants of $\mathbb{F}_2$

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Maxim Lvovich Kontsevich found a recursive formula for counting the number of complex plane rational curves of degree d that pass through $3d-1$ general given points [2]. These numbers, which are called N_d, are Gromov-Witten (GW) invariants for $\mathbb{P}^2$. Hannah Markwig and Andreas Gathmann provided a combinatorial proof of Kontsevich's formula by using tropical geometric methods [1]. In this talk, we are going to see a further generalization of Kontsevich's formula to the case where the curves are in a Hirzebruch surface, $\mathbb{F}_2$, and they are required to satisfy not only point and line conditions but also multiple cross-ratio conditions.

[1] A. Gathmann and H. Markwig, "Kontsevich's formula and the WDVV equations in tropical geometry," Adv. Math., vol. 217, no. 2, pp. 537–560, (2008).

[2] M. Kontsevich and Y. Manin, "Gromov-Witten classes, quantum cohomology, and enumerative geometry" in Mirror symmetry, II, ser. AMS/IP Stud. Adv. Math. Amer. Math. Soc., Providence, RI, vol. 1, pp. 607–65, (1997)

T05

Limits of Real Tropical Linear Spaces

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In a previous work, we showed that the Goldman–Iwahori space of seminorms on a finite-dimensional vector space is the limit of all tropicalized linear embeddings $\iota\colon \mathbb{P}^n \hookrightarrow \mathbb{P}^m$. Now, we define for a real closed field K a signed analogue of the Goldman–Iwahori space consisting of signed seminorms on a finite-dimensional vector space over K. This space can be seen as a linear analogue of the real analytification of projective space over K which was introduced by Jell, Scheiderer, and Yu. We show that there is natural real tropicalization map from the signed Goldman–Iwahori space to a real tropicalized linear space and that this space is the limit of all real tropicalized linear embeddings $\iota\colon \mathbb{P}^n \hookrightarrow \mathbb{P}^m$. We observe that this space is in fact the real Bergman fan associated to the universal realizable oriented matroid.

T06

Buildings, valuated matroids, and tropical linear spaces

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Affine Bruhat–Tits buildings are geometric spaces extracting the combinatorics of algebraic groups. The building of PGL parametrizes flags of subspaces/lattices in or, equivalently, norms on a fixed finite-dimensional vector space, up to homothety. It has first been studied by Goldman and Iwahori as a piecewise-linear analogue of symmetric spaces. The space of seminorms compactifies the space of norms and admits a natural surjective restriction map from the Berkovich analytification of projective space. Inspired by Payne's result that analytification is the limit of all tropicalizations, we show that the space of seminorms is the limit of all tropicalized linear subspaces of rank r (as the embedding and the dimension of the ambient projective space vary), and prove a faithful tropicalization result for compactified linear spaces. The space of seminorms is in fact the tropical linear space associated to the universal realizable valuated matroid, extending a result of Dress and Terhalle. This is joint work with Luca Battistella, Kevin Kühn, Martin Ulirsch and Alejandro Vargas.

T07

Enumeration of plane tropical curves with first-order tangency to a tropical line.