SURFACE WAVE PRACTICAL

This practical deals with surface waves which are usually the largest amplitude arrivals on the seismogram for shallow earthquakes. The velocity at which surface waves propagate is a function of their frequency, that is they are **dispersed**, and the dispersion primarily depends on the shear wave velocity structure. In general, the lowest shear wave velocities are found for near-surface material in the Earth. The velocity increases with depth in the lithosphere, decreases in the asthenosphere, and then increases again below the asthenosphere. Fundamental mode Rayleigh waves (those dealt with in this practical) are most sensitive to the velocity structure within about 0.4 wavelengths of the surface. Therefore, intermediate period fundamental mode Rayleigh waves propagate with greater speed than both longer and shorter period waves. Average Rayleigh wave dispersion curves for the Earth are shown in the attached paper by Oliver – "A summary of observed seismic surface wave dispersion" (BSSA, **52**, 81-86, 1962) which you should read. Surface wave propagation is defined by its **group** and **phase velocities**. If we can measure the dispersion characteristics of surface waves in a region, we can use the dispersion curves to determine the Earth shear wave velocity structure by fitting the observed dispersion curves with theoretical dispersion curves calculated for proposed velocity models. This practical shows how this procedure is accomplished.

1 Surface wave dispersion

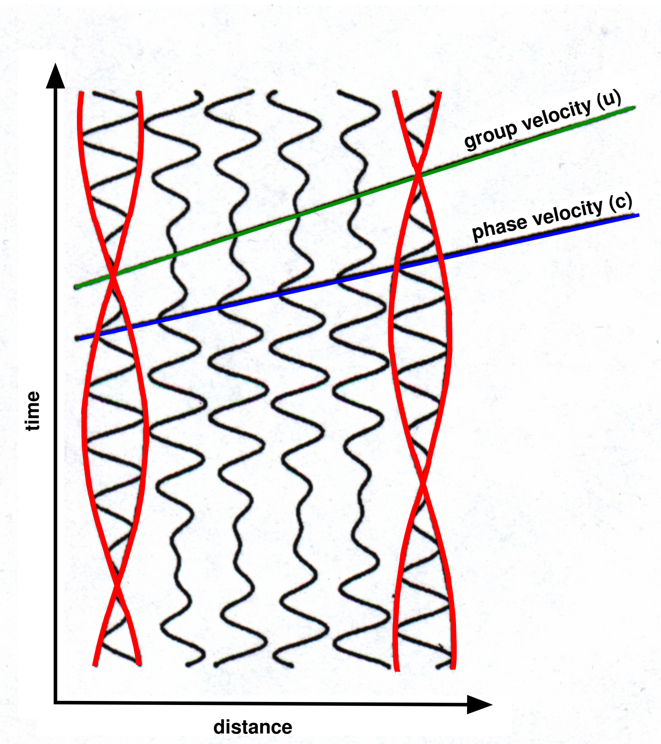

Figure 1: A sum of two sinusoids (black lines) with slightly different angular frequencies. The envelope (red lines) propagates at the **group velocity** u while the carrier propagates at the **phase velocity** c.

For

$$u(x,t) = \cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$$

where

$$\omega_1 = \omega + \delta\omega, \quad \omega_2 = \omega - \delta\omega, \quad \omega >> \delta\omega$$
$$k_1 = k + \delta k, \quad k_2 = k - \delta k, \quad k >> \delta k$$

then

$$\begin{aligned} u(x,t) &= \cos(\omega t + \delta\omega t - kx - \delta kx) + \\ &\quad \cos(\omega t - \delta\omega t - kx + \delta kx) \\ &= 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx) \end{aligned}$$

where $\cos(\omega t - kx)$ is the carrier propagating with $c = \omega/k$ and $\cos(\delta\omega t - \delta kx)$ is the envelope propagating with $u = \delta\omega/\delta k$.

In the lecture we examined briefly the Rayleigh wave period equation for a liquid layer over an elastic half-space. The period equation relates the phase velocity c to the period T or the frequency ω. By substituting values of T in the period equation we can solve for c and plot $c(T)$, the phase velocity curve. For each T along the curve there is a $c(T)$ which is a solution to the wave equation. The total motion (the seismogram) is the sum of the motion for each T (or ω) scaled by an appropriate amplitude $A(\omega)$ factor and shifted by an appropriate initial phase $\phi(\omega)$, that is

$$u(\mathbf{x}, t) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)\; \exp\{i\,[\omega t \;-\; k(\omega)x \;+\; \phi(\omega)]\}\, d\omega \;=\; \frac{1}{2\pi}\int_{\infty}^{\infty} A(\omega)\, e^{i\Phi}\, d\omega$$

This is the fundamental form of a dispersed wave train (for a single mode). We can solve an integral of this form by the method of stationary phase (Mathews & Walker, *Mathematical methods of physics, 2nd Ed.,1970, p90).* For large t and x, the only contribution to the integral is for the particular combinations of t and x (Fig. 2) where

$$\frac{d\Phi}{d\omega} \;=\; \frac{d}{d\omega}\{\omega t \;-\; k(\omega)x \;+\; \phi(\omega)\} \;=\; \left\{t \;-\; \frac{dk}{d\omega}\,x \;+\; \frac{d\phi}{d\omega}\right\} \;=\; 0$$

or

$$t \;=\; \frac{dk}{d\omega}x \;+\; \frac{d\phi}{d\omega} \qquad\Rightarrow\qquad t \;=\; x\,\frac{dk}{d\omega} \qquad\Rightarrow\qquad \frac{d\omega}{dk} \;=\; u \;=\; \frac{x}{t \;+\; d\phi/d\omega}$$

We can see this by plotting $\mathrm{Re}[e^{i\Phi}]$ *vs.* ω

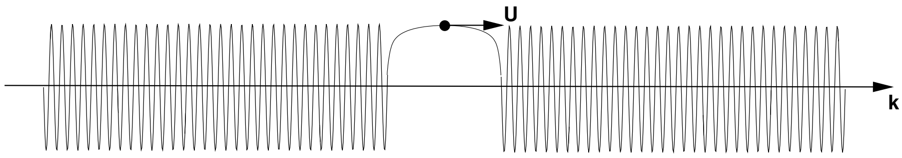

Figure 2: Point of stationary phase.

Thus, the only contribution to the integral occurs near where $d\Phi/d\omega \;=\; 0$ (Fig.2). This defines the group arrival time t at x and the group velocity u, i.e., where constructive interference leads to the presence of energy of frequency ω on the seismogram. The velocity of this point of constructive interference is

$$u(T_i) \;=\; \frac{x}{t_i}\;(\text{for}\;\frac{d\phi}{d\omega} \;=\; 0) \;=\; \text{group velocity} \tag{1}$$

The phase velocity c is given by

$$c(\omega) \;=\; \frac{\omega}{k}$$

and the phase velocity c and group velocity u are related by

$$u(\omega) \;=\; \frac{d\omega}{dk} \;=\; c \;+\; k\,\frac{dc}{dk} \qquad \text{or} \qquad u(T) \;=\; \frac{c}{1 \;-\; (T/c)\,(dc/dT)}$$

so $u(\omega)$ can be determined from the period equation.

1.1 Determining the Group and Phase Velocity Curves

In this practical you will measure the fundamental mode Rayleigh wave group and phase velocity dispersion curves for an earthquake. You will then use the observed group velocity curves and a group velocity curve calculated using the Rayleigh wave period equation to determine the average properties of the Earth structure along the propagation path between the earthquake and the seismographs. You will then use the observed phase velocity curve to determine the average Earth structure along the propagation path between the two stations. The long period seismograms (Plates 1 and 2) are for the magnitude 5.9 New Britain region (5.7°S,151.1°E) earthquake which occurred at 03:12:08.7 GMT on 28 May, 2006. The seismograms were recorded in western North America at two seismograph stations: WDC of the University of California seismic network (40.58°N, 122.54°W) and station DUG of the

US national network (40.1950°N, 112.8133°W). The epicentral distance is 10112 km for WDC and 10934 km for DUG. The azimuth to the source from each station is ∼ 50°. The onset of the Rayleigh waves is at about 2450 sec at WDC and 2750 sec at DUG.

1.2 Measuring the arrival time and period

The three-component seismographs are oriented vertical, north-south and east-west. For this event location, this is close to being naturally rotated (east-west is radial and north-south is transverse). However, you should use the vertical components (LHZ – Plates 2) to measure the Rayleigh wave dispersion since there should be no interference between the Love and Rayleigh waves on the vertical component. The dispersion can be seen as a progressive change in frequency of the waves with time. First, you must determine the arrival time of the energy of a given frequency or period. Using Plate 2 measure the arrival times of the peaks and troughs of the surface wave at each station. The grid shown on Plate 2 denotes the time in seconds after the earthquake origin time. Make a table of arrival times for each station by measuring t_1, t_2, t_3, ..., *etc.* as shown in Figure 3. You can estimate the period T_2, T_3, T_4, ..., *etc.* of the arrival as shown in Figure 3 where, for example, $T_2 = (t_3 - t_1)$.

You can measure this more accurately (with some smoothing) by plotting phase number *vs.* arrival time. The slope of the curve at a point then gives the period of the wave at that time.

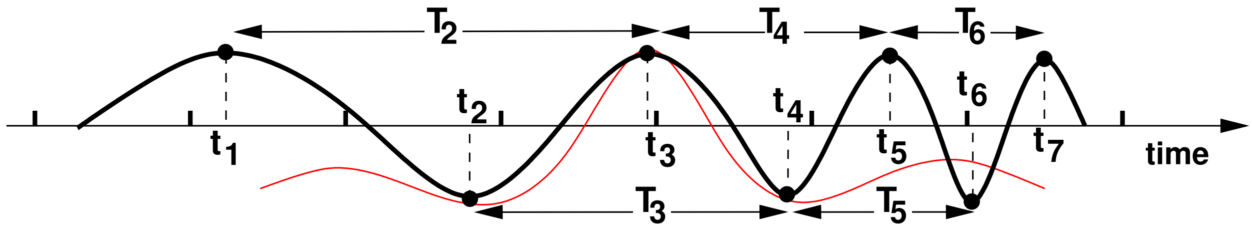

Figure 3: Determining the arrival time and period from the seismogram.

1.3 Group velocity determination

The group velocity is then given by EQ. (1) where we have assumed that the initial source phase $(d\phi/d\omega)$ is small and constant across the frequency band being considered. This is a valid assumption because the epicentral distance is large.

1.4 Phase velocity determination

The wave on the seismogram corresponds to values of equal phase to multiples of 2π (see lecture handout).

$$kx - \omega t + \phi \pm \pi/4 = 2N\pi$$

Dividing by k we obtain an expression for the phase velocity

$$c(T) = \frac{x}{t - (\phi + N \pm 1/8)T}$$

To determine $c(T)$ we need ϕ and N. ϕ can be determined if the source mechanism is known, but this is often not the case. However, ϕ can be eliminated by using seismograms from two stations on the same great circle path as the earthquake. In this case, the phase at the two stations is

$$kx_1 - \omega t + \phi \pm \pi/4 = 2N_1\pi \quad \text{and} \quad kx_2 - \omega t + \phi \pm \pi/4 = 2N_2\pi$$

Since the stations are nearly on the same great circle path as the earthquake, the seismograms from both stations, in this case WDC and DUG, sample the same part of the source mechanism radiation pattern (We have not talked about the source mechanism yet but will do so soon). Taking the difference of the two expressions eliminates ϕ and the phase velocity for the part of the Earth between the two stations is

$$c(T_i) = \frac{\Delta x}{\Delta t_i - NT_i} \qquad (2)$$

where $N = N_{DUG} - N_{WDC}$ is the integer number of cycles separating the phase at the two stations and Δx is the inter-station distance.

The times measured in determining the group velocity are points of equal phase (Fig. 3) at stations WDC and DUG. The wave period is the average for the two stations, $T_i = [T_i(\text{WDC}) + T_i(\text{DUG})]/2$ and $c(T)$ is computed from EQ.(2). For the phase velocity determination you must determine the integer N. N can be determined by examining the dispersion at long periods since for these, a wrong choice of N gives an unrealistic value of $c(T)$. Once N is determined for the long period part of the wave motion, the same value of N is applied for all periods.

This method of determining the group and phase velocities will not work if the wave is pulse-like or does not have a simple dispersed form. In these cases it is necessary to compute the dispersion from the Fourier phase spectra.

1.5 Determining the Velocity Structure

Plot your group velocity measurements from the WDC and DUG seismograms on the attached "oceanic" group velocity curves (Fig. 7). These curves were computed from the fundamental mode Rayleigh wave period equation for a liquid layer over an elastic half–space (see lecture handout). The different curves are all computed for the same elastic parameters but are for varying thickness of the water layer. Which theoretical group velocity curve gives the best fit to your observations? When you have access to an atlas, estimate the mean depth of the Pacific Ocean. How does it compare with your value determined from the group velocity measurements?

Plot your inter-station phase velocity measurements from the WDC and DUG seismograms on the attached continental phase velocity curves (Fig. 8). You will need to determine c for various values of N and decide which value of N is appropriate for your measurements. These phase velocity curves are also computed from the Rayleigh wave period equation for a layered elastic structure. Which theoretical phase velocity curve best fits your observations? What is a typical thickness for the continental crust? How does this compare with the thickness of "normal" continental crust? If it is different, why might this be?

1.6 Other observations from the seismograms

The epicenter of this earthquake was in New Britain and the great circle path to WDC and DUG is almost east-west (azimuth 264° as seen from WDC and 270° as seen from DUG). The east-west component (LHE – Plate 1) is nearly radial and should yield similar dispersion results as observed on the vertical above. Does it? What is the particle motion for the Rayleigh wave (plot the particle motion of the LHZ *vs.* the LHE components of Plate 2). Does this show retrograde motion typical of the Rayleigh wave?

The north-south component (LHN – Plate 1) is oriented nearly transverse. What is the nature of the pulse that arrives at about 03:49:15 UTC on the WDC LHN-component and at about 03:52:30 UTC sec on the DUG LHN-component?

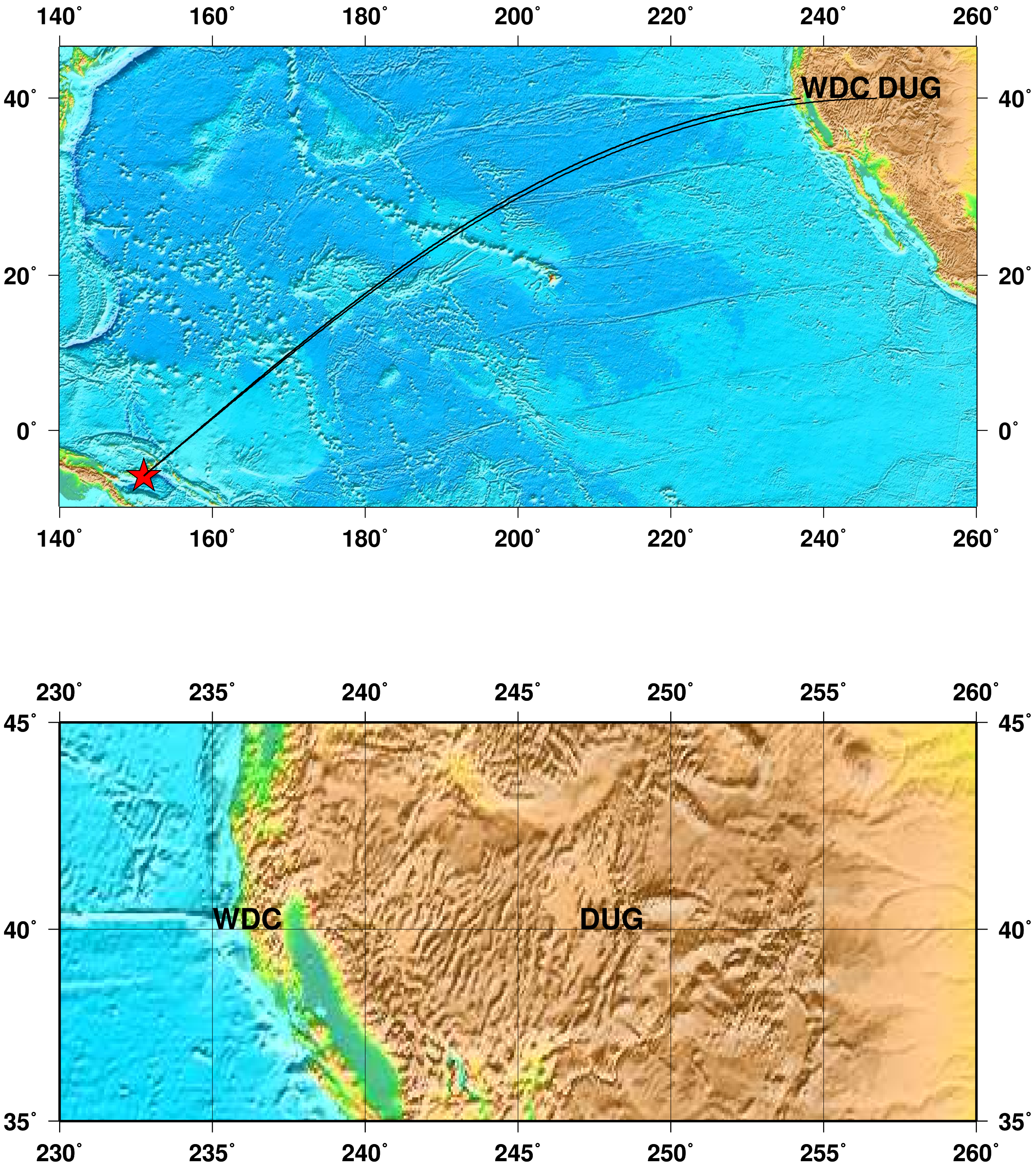

Figure 4: The upper panel shows the great-circle paths between the magnitude 6.1 New Britain earthquake of 28 May, 2006 (5.7°S, 151.1°E) and the seismographs WDC (40.1°N, 123.0°W) and DUG ((40.1°N, 113.0°W). The lower panel shows the relationship of the sseismographs sites WDC and DUG

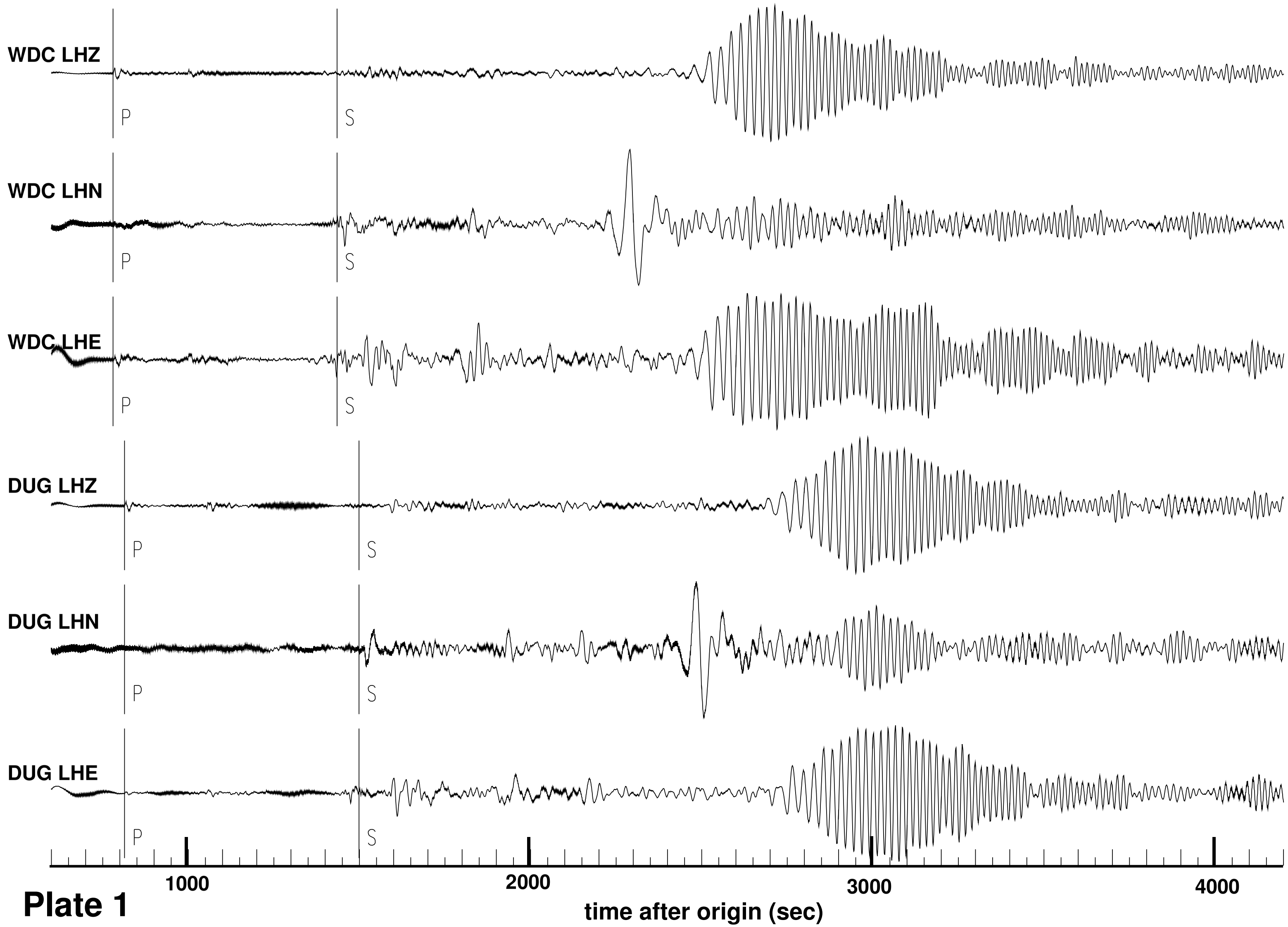

Figure 5: Plate 1 – Three-component seismograms for the magnitude 6.1 New Britain region (5.7°S,151.1°E) earthquake of 28 May, 2006. The vertical lines mark the P- and S-wave arrival times predicted for a standard earth model (IASPIE91).

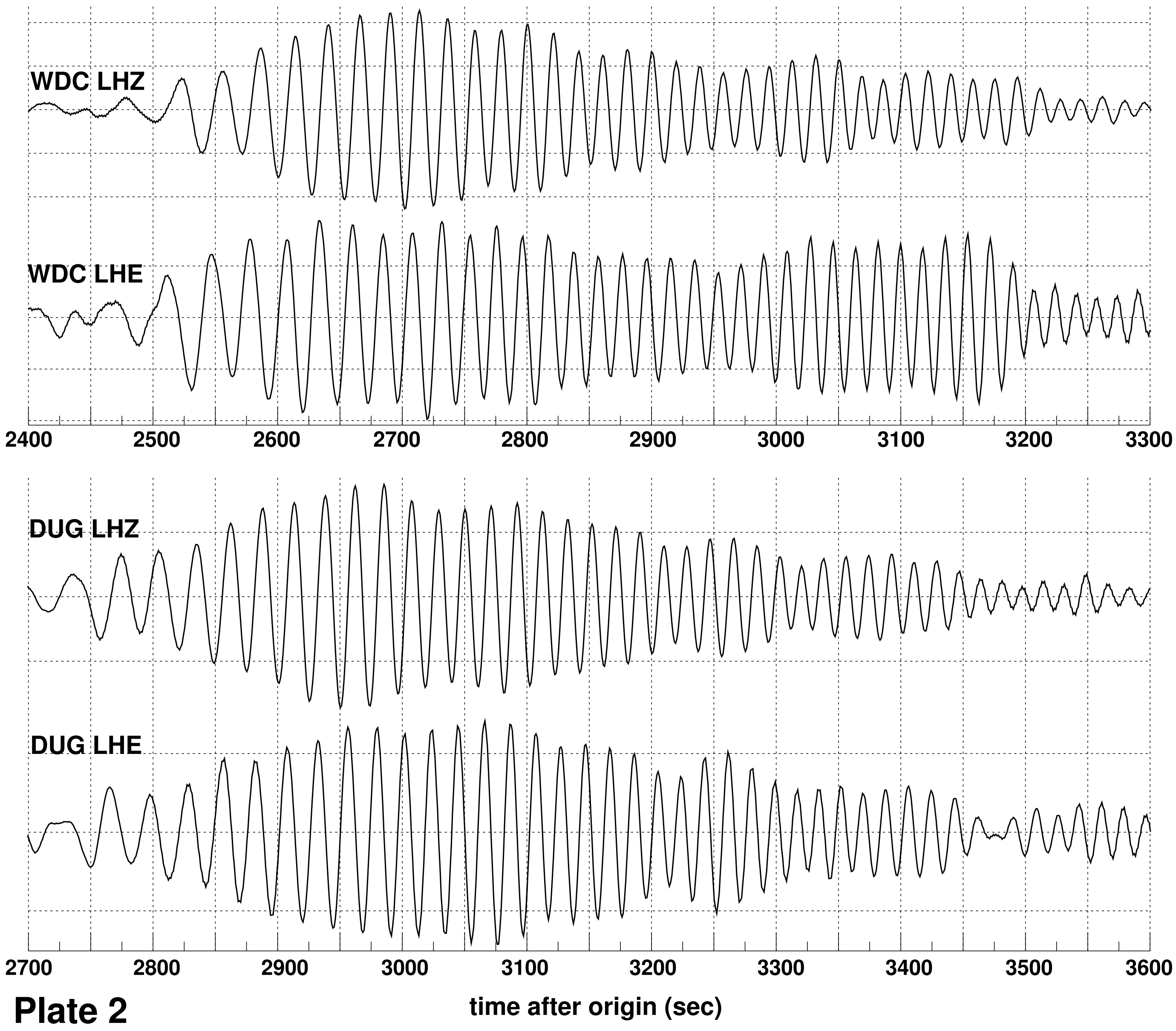

Figure 6: Plate 2 – Same as Plate 1 but an enlarged section of the LHZ and LHE components centered on the fundamental mode Rayleigh wave recorded at each station.

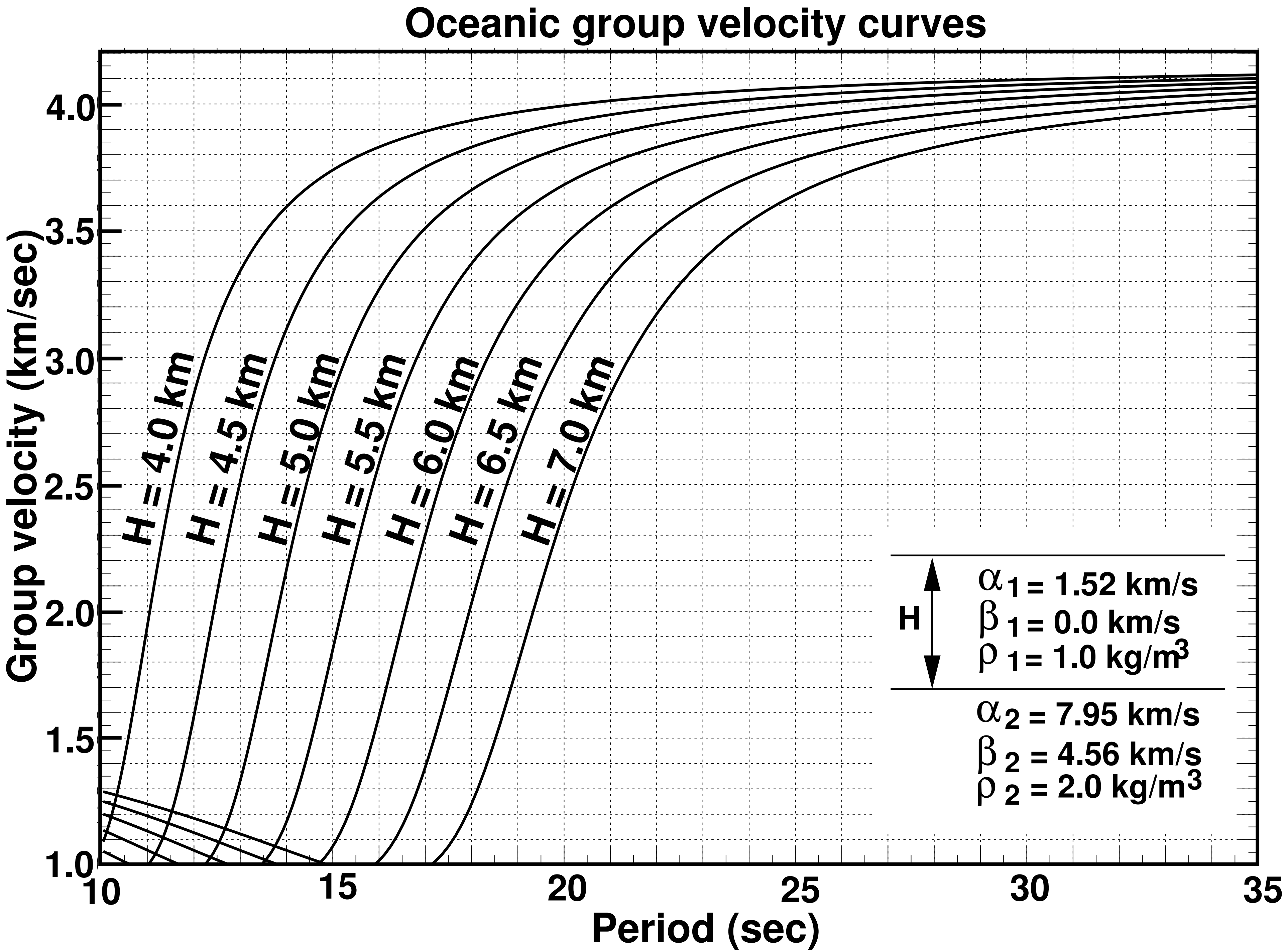

Figure 7: Group velocity curves for a fluid layer over an elastic half-space. This model simulates the oceanic structure. The step part of the group velocity curve is due to the fluid layer and the period at which the decrease occurs is controlled by the thickness of the fluid layer.

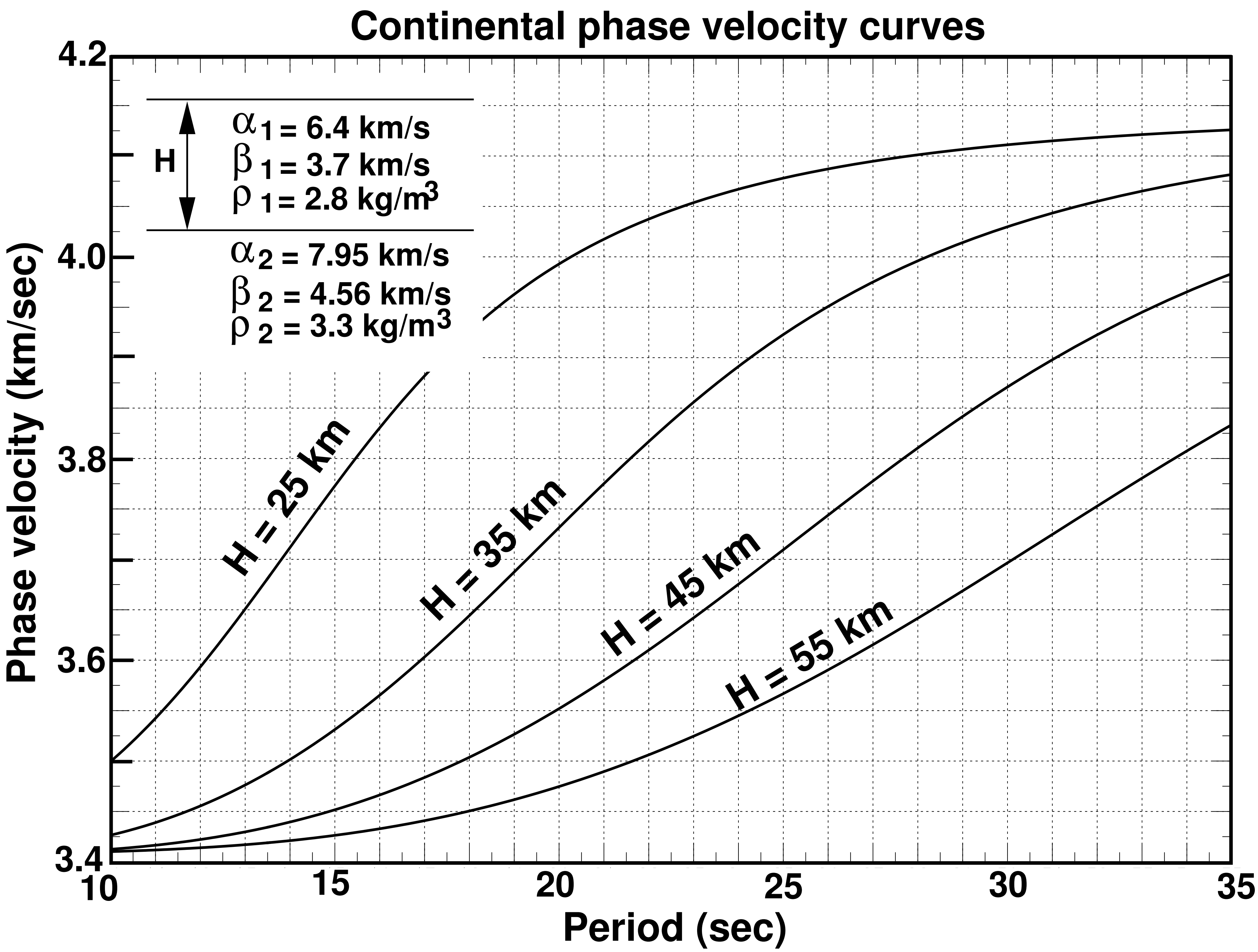

Figure 8: Phase velocity curve for a lower wavespeed elastic layer over a higher wavespeed elastic half-space. This simulates the continental crust.

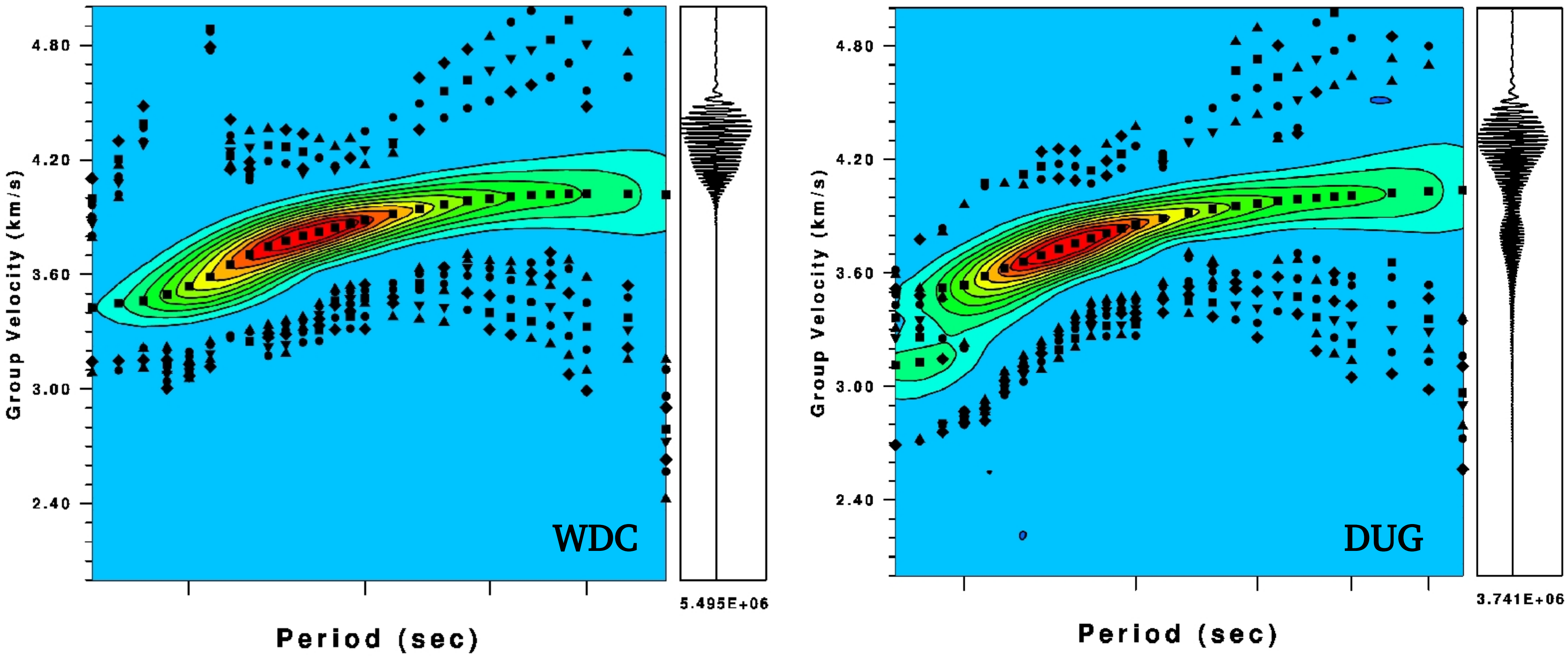

Figure 9: FTAN analysis shown in Figure 5

2 Surface wave tomography

This exercise consists of two parts. First, you will invert real ambient noise data recorded from Australia to develop tomographic velocity maps for the region, then you will invert a synthetic data set to gain insight on the resolution for the tomographic velocity model. The objective of this exercise is to provide experience into how seismic tomography is carried out, and gain insight into the various considerations that are needed to produce a robust tomographic model for a region that can be interpreted with a high degree of confidence. A minimal familiarity with the **LINUX** operating system is assumed. In this document the > symbol at the beginning of a line signifies the computer prompt followed by the command you enter. In some cases this is followed by # and a comment as to what the command is doing. Various PDF viewers are available; below **evince** is suggested but use whatever PDF viewer you are familiar with. For text editing **gedit** is suggested but use whatever text editor you are familiar with.

There are a number of computer codes making up this exercise, many evoked by the k-shell script **ttomoss** which calls the necessary executable in order to perform iterative non-linear tomography. These are:

fm2dss: Solves the forward problem of travel-time prediction by applying the so-called Fast Marching Method (FMM), a grid-based eikonal solver. **fm2dss** can also output ray paths and Fréchet derivatives, the latter of which are required by the inversion routine.

grid2dss: A program for constructing 2-D models in the format required by fm2dss. It can be used to generate checkerboard and other synthetic test models (e.g. random structure), and initial models for tomographic inversion.

misfitss: Simply prints to screen a measure of model roughness and smoothness. It can be useful when working out the most appropriate regularization parameters for "tuning" the solution.

residualss: Computes summary travel-time residuals (RMS and variance) associated with a given model.

resplotss: This program takes travel-time information generated by the tomographic inversion process and converts it into a format suitable for input into a GMT (Generic Mapping Tools) script that plots a frequency histogram of the initial or final data fit.

subinvss: Uses a subspace inversion method to perform a linearized inversion of the data. In order to address the non-linearity of the problem, this program is applied iteratively in sequence with fm2dss.

synthtss: A simple program for adding gaussian noise to a synthetic data set in order to simulate the effects of picking errors that is present in observational data.

tslicess: A translation code that converts output from the tomography software into an input format that can be used by GMT (Generic Mapping Tools). This allows the velocity field to be visualized as a color contour map, with ray paths, wavefronts and source/receiver locations superimposed. GMT scripts are provided (see the examples that come with the distribution) to generate these plots.

2.1 Inversion of the Australian ambient noise data set

Enter the sub-directory called **exercise1** and run the tomographic inversion:
> **cd exercise1**
> **ttomoss**

This is a shell script which iteratively solves the non-linear tomography problem. It iteratively applies a method for computing inter-station surface wave travel times and paths, and a subspace inversion method. It is set to run for six iterations, although this can be changed on line 1 of the file **ttomoss.in** (leave it at the default value for this practical).

1. Enter the subdirectory **gmtplot** and execute **tslicess**. Next, execute **plotgmt** (note: this is a script, and may need to be made executable).

> **cd gmtplot**
> **tslicess**
> **chmod u+x plotgmt**
> **plotgmt**
> **evince plotgmt.pdf**
The file **plotgmt.pdf** displays the 2D velocity model of the result.

2. Now open the script **plotgmt** in a text editor and uncomment the line beginning *gmt psxy rays.dat* (i.e., remove the "#" at the start of the line), save changes and execute *plotgmt*. The PDF figure will now be plotted with rays superimposed:
> **plotgmt**
> **evince plotgmt.pdf**

3. Now move back up to the sub-directory **exercise1**:
> **cd ..**
We can now look at some diagnostics to help see whether the solution makes sense. The file **residuals.dat** lists the travel-time misfit between the model and observations as a function of iteration. The misfit of the final model that you plotted above is given on the last line. The entry in the first column is RMS misfit in milliseconds. The second entry in column two is the variance in seconds (so take the square root to get standard deviation of the misfit).

4. To calculate the misfitss:
> **misfitss**
This provides information about the solution model, namely the variance reduction of the solution model with respect to the starting model, i.e., a measure of how much the initial model has been *perturbed*), and the model roughness, which is a measure of solution curvature (the higher the number, the *rougher* the model).

5. The inversion can be run in linear or non-linear mode. By default, it runs in non-linear mode, which is the mode that should be used to answer questions 6.i-iv below. Make a backup of your current results:
> **mv plotgmt.pdf plotgmt-nonlinear.pdf**
Then answer question 8.v by running the inversion in the linear mode.:
> **cd gmtplot**
> **cp plotgmt.pdf plotgmt_non_linear.pdf**
> **cd ..**
> **cp gridi.vtx gridc.vtx**
> **fm2dss**
> **mv raypath.out raypathref.in**
> **gedit fm2dss.in** # (edit line 25 so that it reads 1 instead of 0)
> **ttomoss**
Changing the value at line 25 command **ttomoss** in linear mode.

6. Given the above information, consider the following questions:

i. Inspect the PDF file created in step 4, and comment on the variations in ray path density. What two factors contribute to this overall pattern?

ii. Examine the effect on the solution of varying the damping factor (line 12 of the file **subinvss.in**). Each time you make a change, re-run **ttomoss** and take a note of the diagnostics (travel-time misfit, model variance) for the solution model. Try a range of values between 0 and 1000 and make a plot of travel-time misfit versus model variance. Travel time misfit of the final iteration is the bottom line of the second column of **residuals.dat** a **libreoffice** file is provided to help you with this – it is located in the **exercise 1** sub-directory and is called **practical_template.ods**. To open it, use:
> **libreoffice practical_template.ods**
Describe what happens to the solution model when the value of the damping factor is increased. Is there an obvious value of the damping factor which produces the best model (good data fit and small

model variance)?

iii. Now examine the effect on the solution of varying the smoothing factor (line 15 of the file **subinvss.in** Try a range of values between 0 and 10,000, and keep the damping factor fixed on the optimum value you found above. Use the same practical_template.ods file as in the previous question. Plot the travel-time misfit versus model roughness. Travel time misfit of the final iteration is the bottom line of the second column of **residuals.dat**. Describe what happens to the solution model when the value of the smoothing factor is increased. Is there an obvious value of the smoothing factor which produces the best model (good data fit and small model roughness)?

iv. Plot the solution model created using the optimum damping and smoothing values you found above. Does this procedure for finding the optimum model appear to be robust?

v. Now run **ttomoss** in linear mode and plot your output. How does the solution model compare with what you obtained in non-linear mode?

2.2 Exercise 2: Synthetic checkerboard test

1. Change to the sub-directory **exercise2**:
> **cd ..** # (if in exercise1)
> **cd exercise2**

2. Edit the file **subinvss.in** and change the damping and smoothing values so that they match what you determined in Exercise 1:
> **gedit subinvss.in**

3. Begin with the low resolution checkerboard:
> **cp gridi_lr.vtx gridi.vtx**
> **cp otimes_lr.dat otimes.dat**

4. Execute **ttomoss**
> **ttomoss**

5. Now plot the result (as in Exercise 1, **plotgmt** may need to be made executable):
> **cd gmtplot**
> **tslicess**
> **plotgmt**
This plot (**plotgmt.pdf** shows you the pattern of the **recovered** checkerboard, which can be compared to the input checkerboard shown in **checker_low.pdf**

6. Now try the high resolution checkerboard (but backup your **plotgmt.pdf] plot first):**
> **cp gridi_hr.vtx gridi.vtx**
> **cp otimes_hr.dat otimes.dat**

7. Repeat steps 4 and 5 above. In this case, the input checkerboard is in checker_high.pdf

8. Based on what you have learned from the above exercise, consider the following questions:

i. Comment on the accuracy of the checkerboard recovery. Given the ray path coverage (can be included with plotgmt as in exercise 1), is this to the expected? How do the low and high resolution checkerboard test results differ, and what does this tell us about the wavelength of structure we might expect to recover?

ii. Compared to the input checkerboard, do the amplitudes of the anomalies tend to be underestimated or overestimated?

iii. What are the strengths and weaknesses of the checkerboard test?