GNSS Multipath: Characterization, Modeling & Mitigation

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Dr. Gary A. McGraw, PNT Consultant
Genova Technologies
Phone: +1-319-210-9707
Email: mcgraw.pnt@gmail.com

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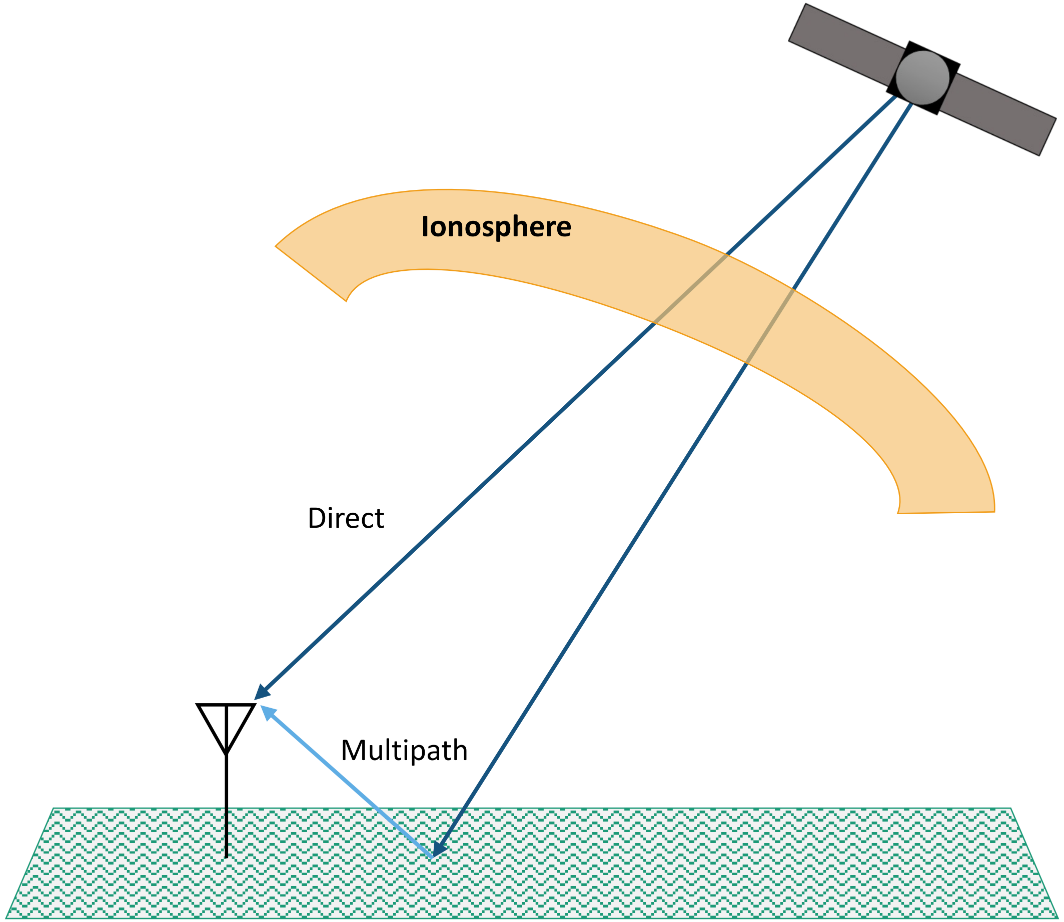
Why is there a talk on GNSS multipath at a space weather workshop?

- GNSS provides an important tool for observing the ionosphere
  (and tells you where you are and what time it is!)
- Multipath (short for "multiple signal paths") is often the dominant GNSS measurement error source
- Important to know how to:
  - Minimize multipath errors when siting a GNSS antenna (but sometimes you can't avoid it)
  - Process GNSS code and carrier measurements to mitigate errors



Overview

Part 1: GNSS Multipath Characterization & Modeling

- GNSS multipath introduction & definitions
- Different propagation environments
  - Specular vs. diffuse multipath
  - Multipath relative amplitude
  - Fading frequency
- Signal modeling
- Multipath effects on GNSS measurements
  - SNR
  - Pseudoranges (code)
  - Carrier Phase

Part 2: Multipath Mitigation & Measurement Processing

- Multipath mitigation techniques
  - Code type
  - Antenna design & siting
  - Receiver signal processing
  - Measurement processing
- GNSS measurement combinations
  - Wide and narrow-lane carrier phase
  - Ionospheric free
  - Ionospheric estimation
  - Divergence free
- Carrier smoothed code processing
  - Processing overview
  - Smoothing filter gain
  - Divergence-free smoothing

Part 1: GNSS Multipath Characterization & Modeling

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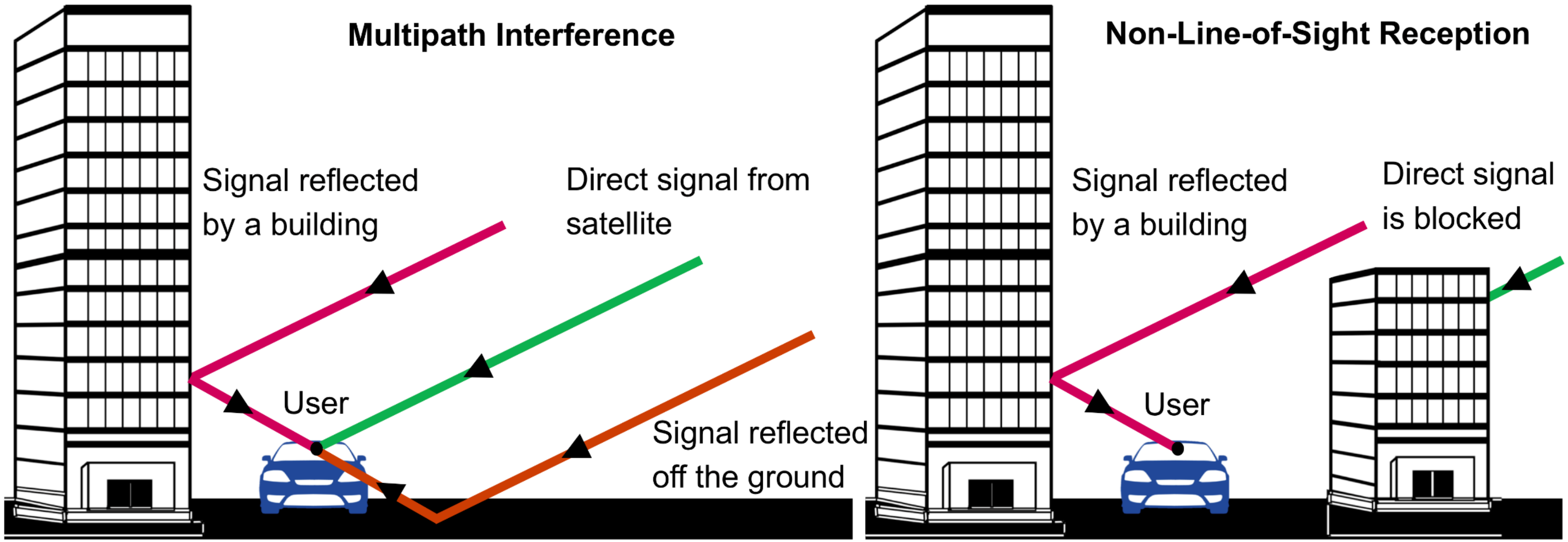
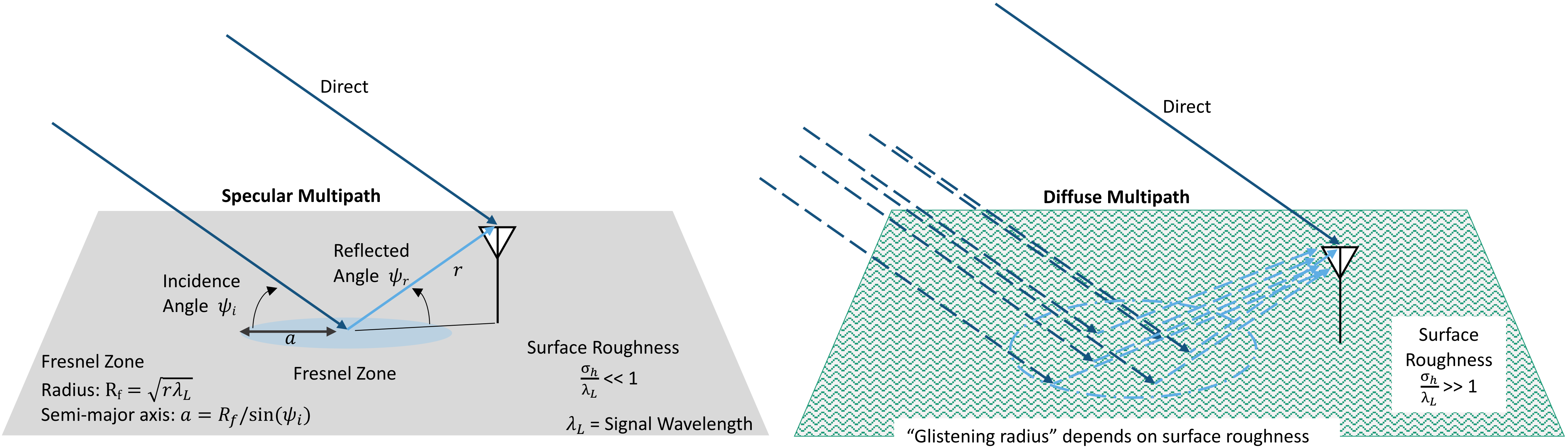
Multipath vs. Non-LOS Reception

- Multipath = Multiple signal propagation paths, including direct signal
- Non-LOS reception = Direct signal is blocked, but strong reflected signals are present



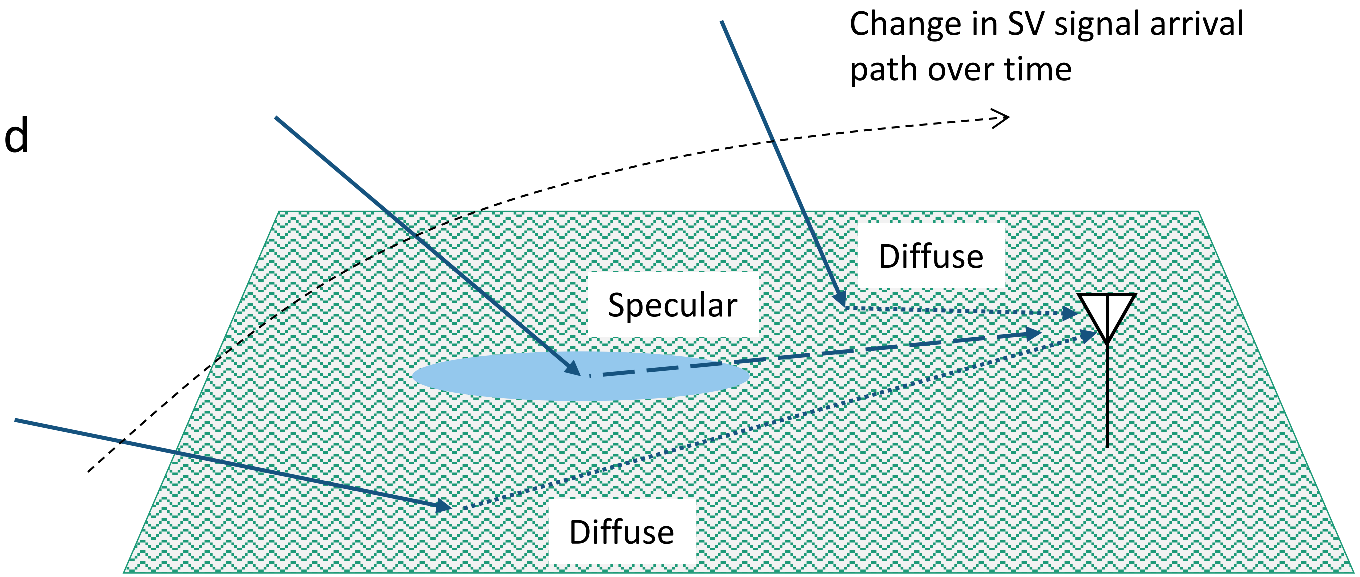
Specular vs. Diffuse Multipath

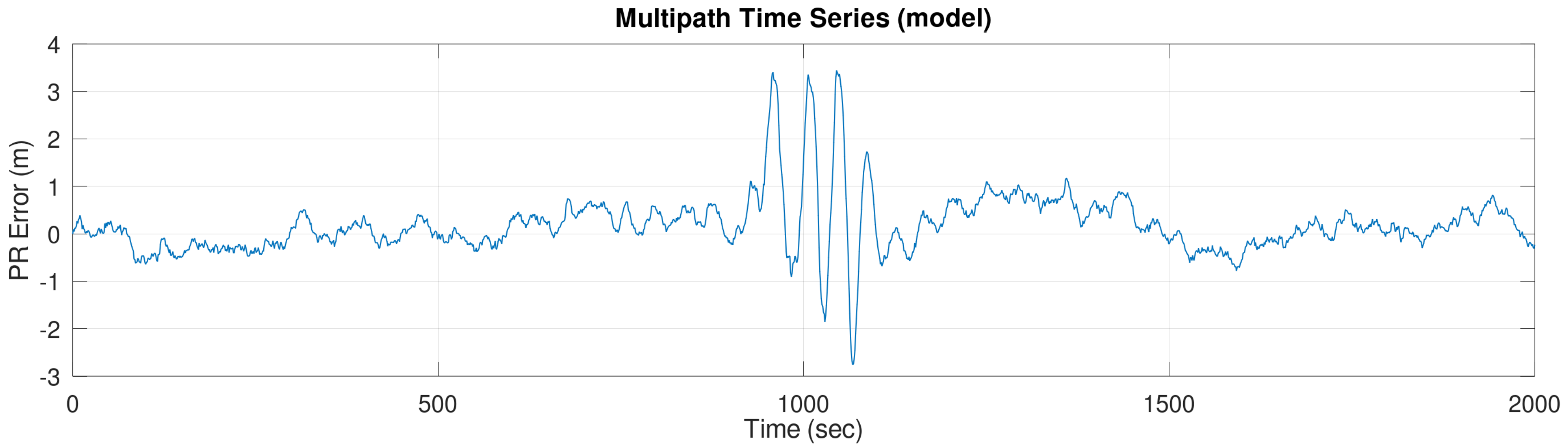


- For specular reflection $\psi_r = \psi_i$
- Amplitude of multipath dependent on surface composition
- Reflecting objects need to be larger than the Fresnel zone to create specular multipath
- GNSS signals are right-hand circularly-polarized (RHCP) signals; multipath usually dominated by left-hand circularly-polarized (LHCP) signals

Multipath Error Characteristics

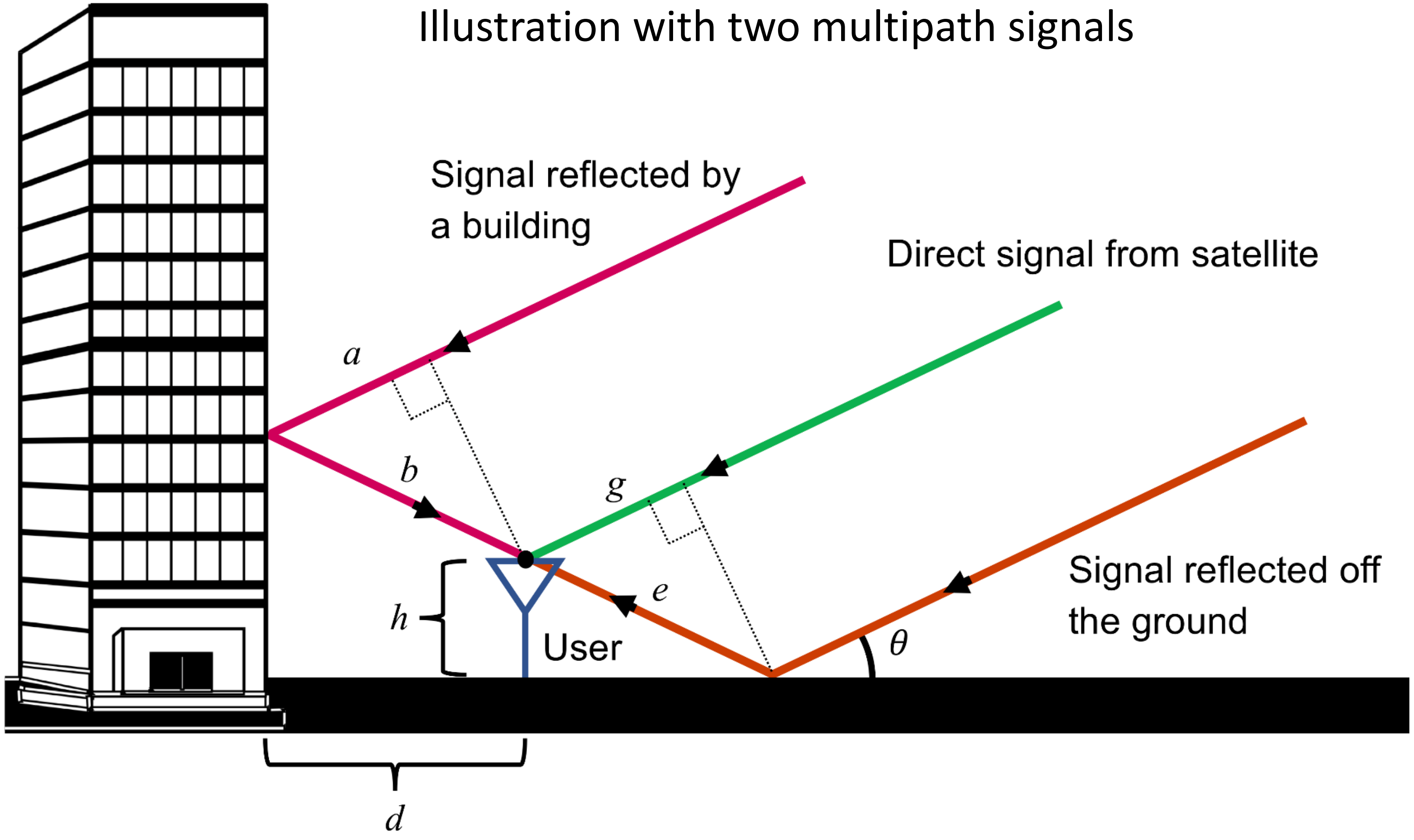
- Diffuse multipath appears like bandlimited noise
- Specular multipath has sinusoidal measurement error characteristics
- Often both types are present





Specular Multipath Characterization

Illustration with two multipath signals



Multipath Delay & Phase

<u>Ground Reflected Signal Delay</u>

$$\Delta_i = e - g = 2h\sin(\theta) \ \text{(m)}$$

<u>Building Reflected Signal Delay</u>

$$\Delta_i = a - b = 2d\cos(\theta) \ \text{(m)}$$

<u>Phase Shift</u>

$$\phi_i = \left(\frac{2\pi\Delta_i}{\lambda_L} + \phi_{Ri}\right)\mathrm{MOD}2\pi \ \text{(rad)}$$

ϕ_{Ri} = Phase shift at reflection = π rad when incidence angle is less than the Brewster angle

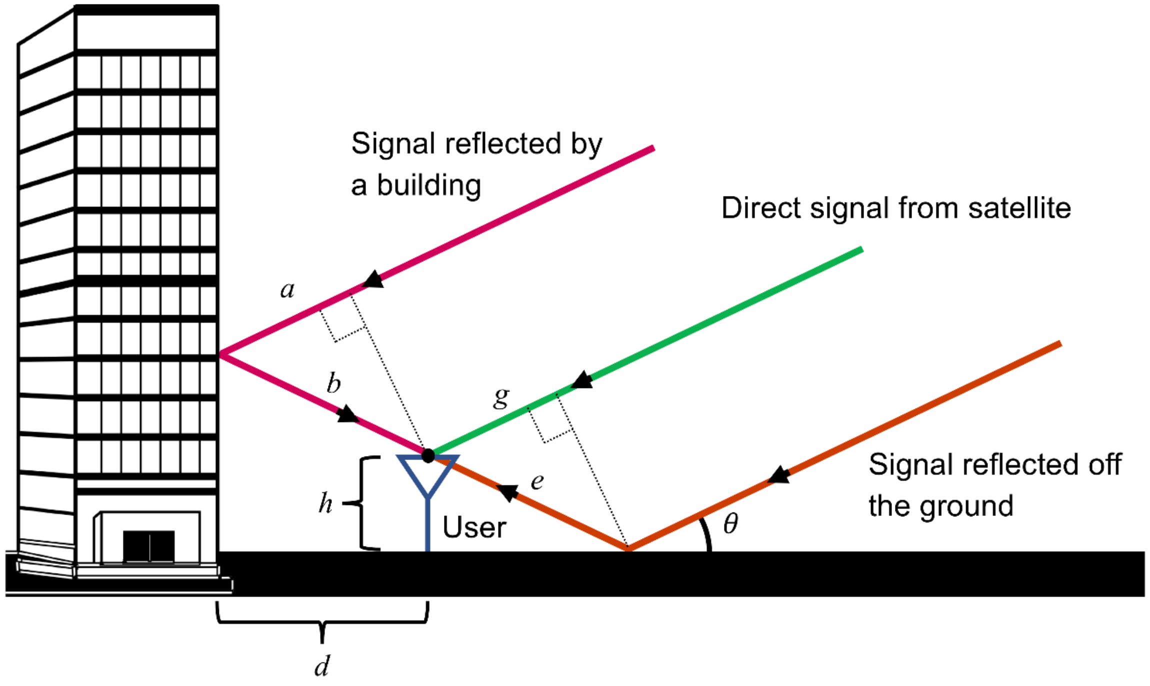
λ_L = wavelength (m)

<u>Multipath delay in code chips</u>

$$\delta_i = \Delta_i/\lambda_C = \Delta_i/cT_C$$

λ_C = PRN code chip length (m)

T_C = PRN code period (s)



- Delay increases with antenna height / distance
- Elevation angle greatly influences multipath characteristics

Multipath Fading Frequencies - Ground Reflection Relative Doppler

<u>Ground Reflected Signal</u>

$$\delta f_i = \left(\frac{2}{\lambda_L}\sin\theta\right)\frac{\partial h}{\partial t} - \left(\frac{2h}{\lambda_L}\cos\theta\right)\frac{\partial \theta}{\partial t}$$

Example:

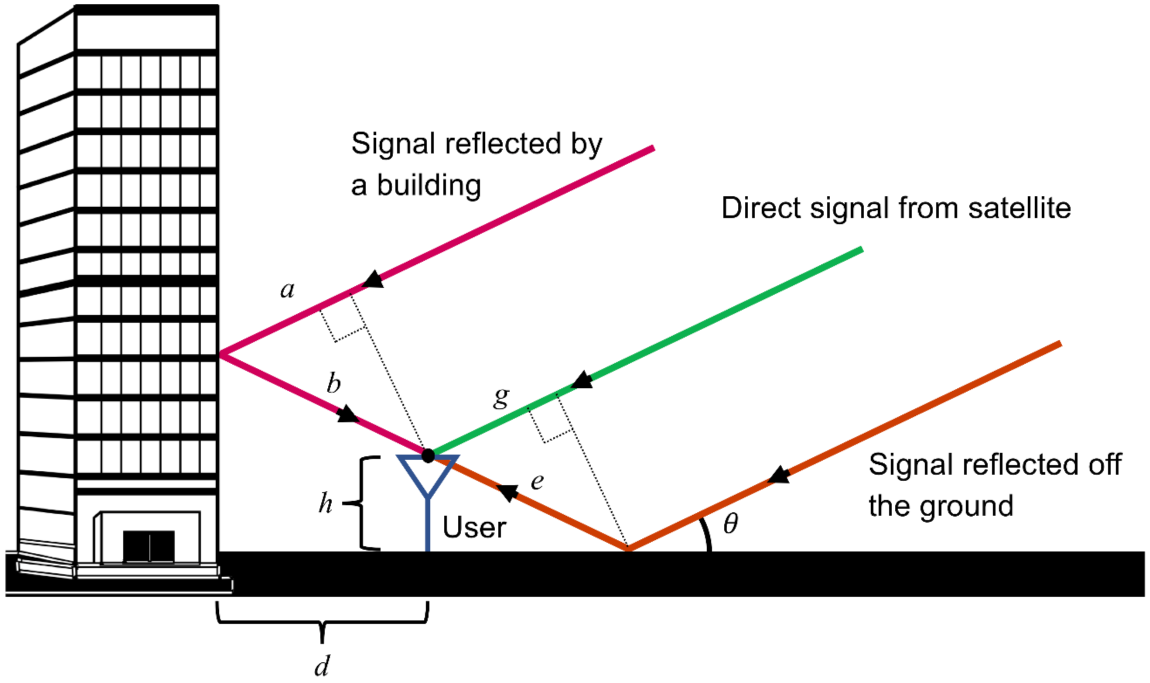
$h = 1$ m (fixed)

GNSS satellite angular rate:

$$\frac{\partial\theta}{\partial t} \approx 180\ \mathrm{deg}/\ 6\ \mathrm{hrs} \approx 0.15\ \mathrm{mrad/s}$$

L1 wavelength: λ_L=19 cm

=> $\delta f_i \approx 1.6$ mHz near the horizon; $\delta f_i \approx 0$ near zenith



- Frequencies dependent on relative satellite and antenna motion
- LEO satellite orbital angular rate ~10X faster than GNSS

Multipath Fading Frequencies – Building Reflected Relative Doppler

Building Reflected Signal

$$\delta f_i = \left(\frac{2}{\lambda_L}\cos\theta\right)\frac{\partial d}{\partial t} - \left(\frac{2d}{\lambda_L}\sin\theta\right)\frac{\partial \theta}{\partial t}$$

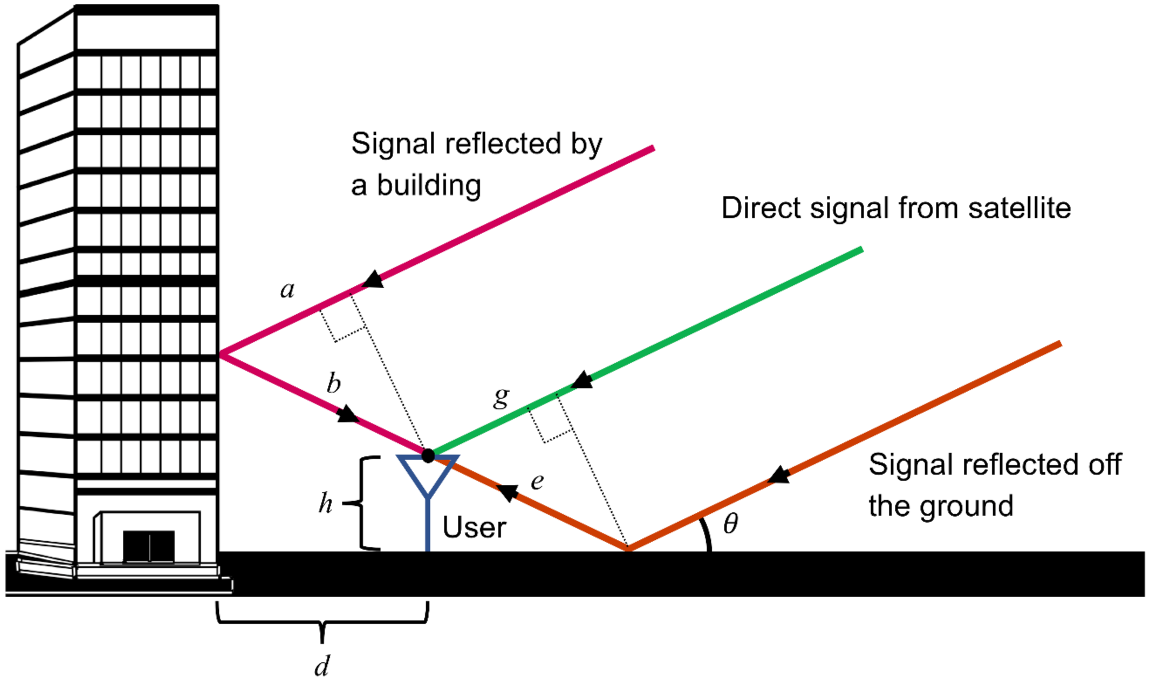
Example:

Antenna horizontal speed: $\frac{\partial d}{\partial t} = 1$ m/s

Satellite elevation angle: $\theta = 30°$

L1 wavelength: λ_L=19 cm

$\delta f_i \approx$ 5.3 Hz



- At higher speeds, fading frequency would exceed carrier tracking loop bandwidth and would appear as noise
- Effects due to satellite motion similar to ground bounce case

Received Signal Model

$$r(t) = \sqrt{2P}\sum_{i=0}^{n}\Big\{\alpha_i C(t - t_0 - \Delta_i/c) D(t - t_0 - \Delta_i/c) \times \exp\Big(j\Big[2\pi\big(f_L + f_D + \delta f_i\big)\big(t - t_0 - \Delta_i/c\big) + \phi_i\Big]\Big)\Big\} + w(t)$$

P = direct signal power

n = number of reflected signals (i=0 is the direct signal)

α_i = relative amplitude of reflected signals (α_0 = 1)

$C(\cdot)$ = pseudorandom noise (PRN) spreading code

$D(\cdot)$ = downlink data

t_0 = propagation delay for the direct signal (sec)

c = speed of light (m/s)

f_L = carrier frequency (Hz)

f_D = Doppler shift (Hz)

δf_i = relative multipath Doppler (Hz)

Δ_i = relative multipath delay (m)

ϕ_i = phase shift relative to direct (rad)

$w(\cdot)$ = bandlimited white Gaussian noise (WGN)

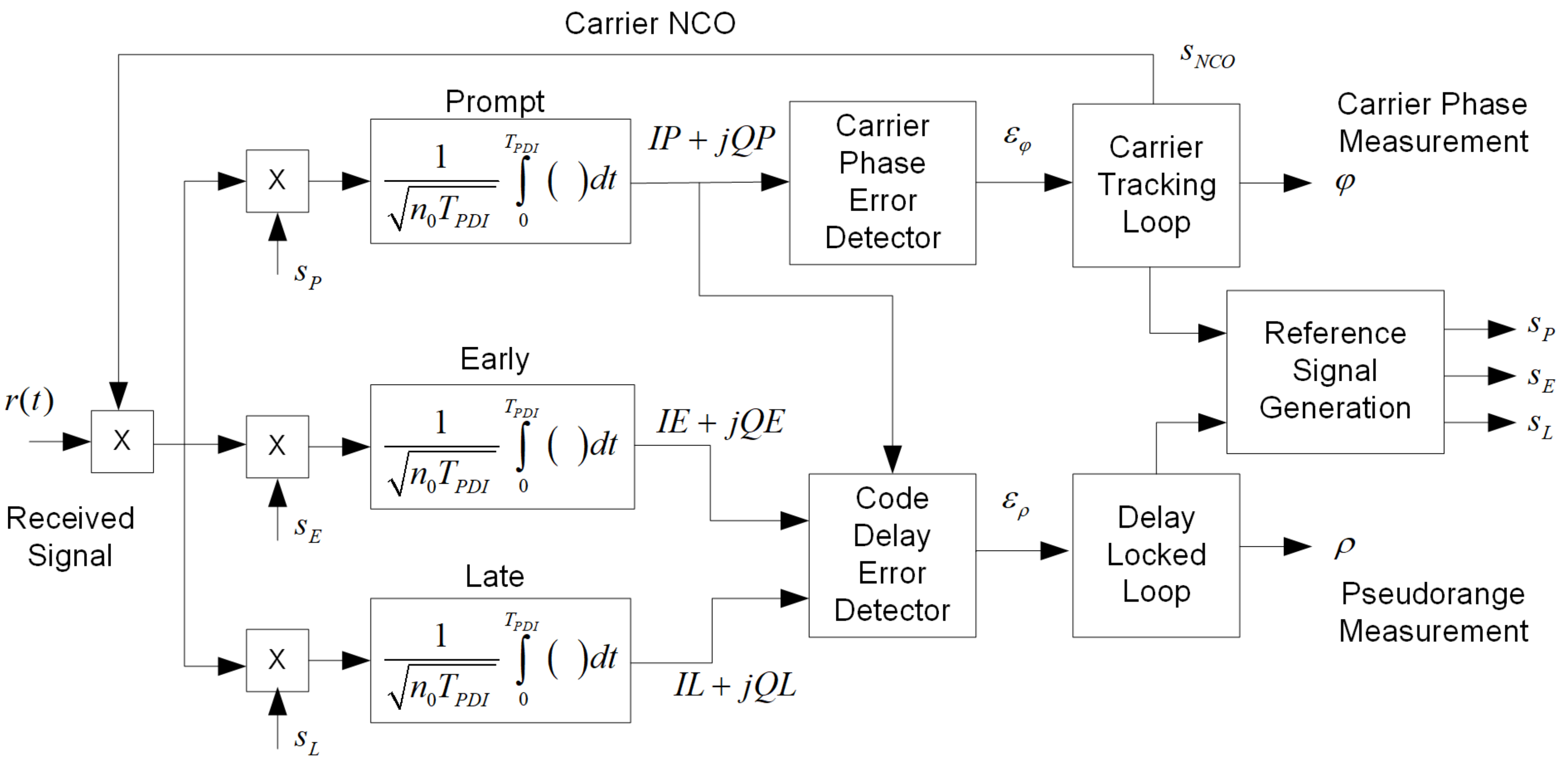
Relative Multipath Amplitude

$$\alpha_i = \sqrt{\frac{G_i R_i k_i}{G_0 R_0 k_0}}$$

G_0 = antenna gain for direct signal
G_i = antenna gain for the i^{th} multipath component
R_i and R_0 = reflection coefficients (R_0 = 1 in our case)
k_i and k_0 = signal attenuation coefficients (due to foliage, etc.)

- Antenna gain for direct signal typically ranges from -6 dB to +3 dB
- Multipath antenna gain typically smaller than direct – but not true for mobile devices!
- Reflection coefficients depend on the properties of the reflecting surface
  - Calm water, metal, glass can have reflection coefficients as large as 0.5-0.9
  - Other surfaces will have lower reflection coefficients
- Attenuation coefficients ~1, unless foliage or scattering is present

Receiver Signal Processing Block Diagram



Correlator Output Signals

$$IE = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C + dT_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_{PDI}\big)\cos(\delta\varphi+\phi_i)\Big]+w_{IE}$$

$$IP = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_{PDI}\big)\cos(\delta\varphi+\phi_i)\Big]+w_{IP}$$

$$IL = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C - dT_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_{PDI}\big)\cos(\delta\varphi+\phi_i)\Big]+w_{IL}$$

$$QE = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C + dT_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_{PDI}\big)\sin(\delta\varphi+\phi_i)\Big]+w_{QE}$$

$$QP = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_{PDI}\big)\sin(\delta\varphi+\phi_i)\Big]+w_{QP}$$

$$QL = \sqrt{2(c/n_0)T_{PDI}}\sum_{i=0}^{n}\Big[\alpha_i R\big(\tau-\delta_i T_C - dT_C\big)\operatorname{sinc}\big(\pi\big(\delta f+\delta f_i\big)T_c\big)\sin(\delta\varphi+\phi_i)\Big]+w_{QL}$$

d = Correlator spacing (chips)

c/n_0 = carrier-power-to-noise-density ratio (ratio-Hz),

$R(\cdot)$ = PRN code autocorrelation function

$\tau = \hat{t}_0 - t_0$ = code tracking error (s)

δf = carrier frequency tracking error (Hz)

$\delta\varphi$ = carrier phase tracking error (Hz)

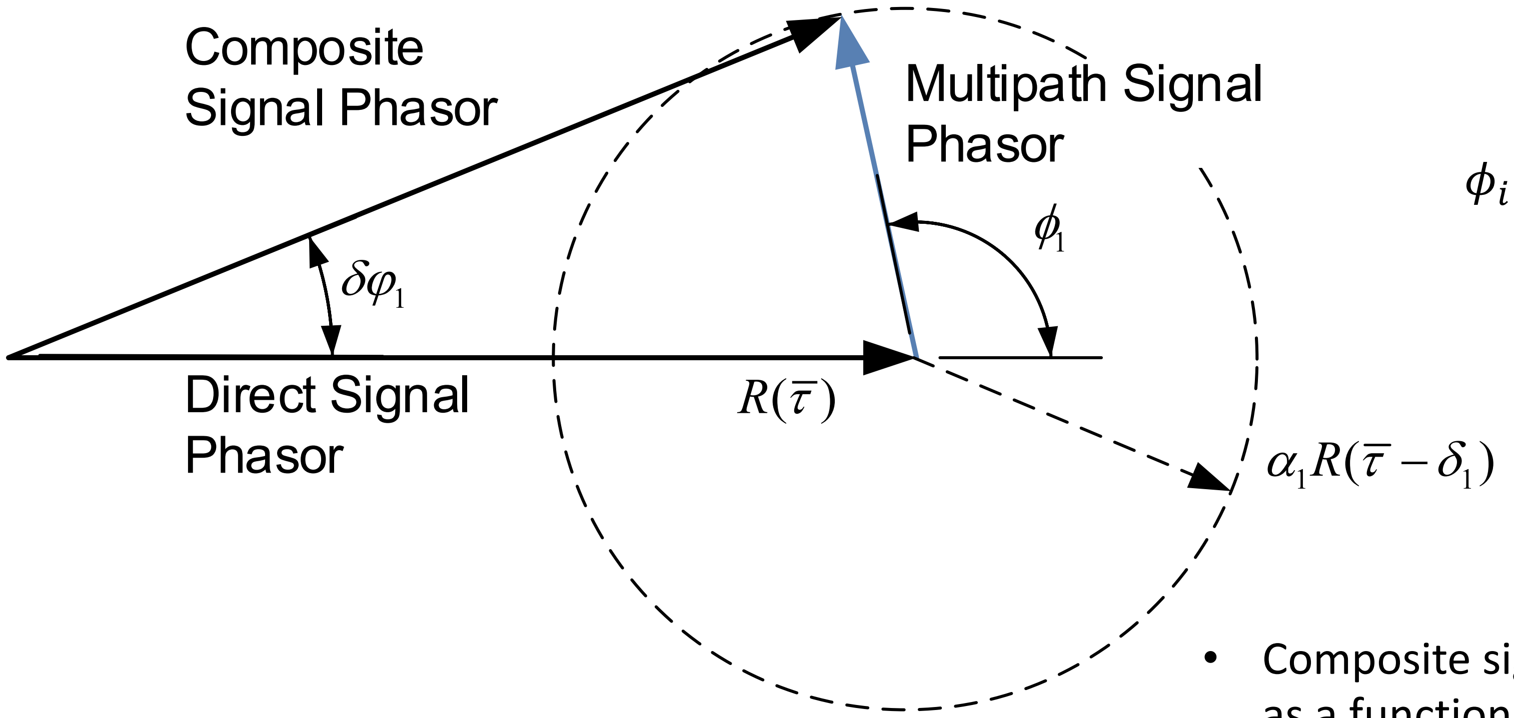
$$\operatorname{sinc}(\theta) = \begin{cases} \sin\theta/\theta, & \theta \neq 0 \\ 1, & \theta = 0 \end{cases}$$

I and *Q* denote in-phase and quadra-phase, *E*, *P* and *L* denote early, prompt and late

$w_{IE}, w_{IP}, w_{IL}, w_{QE}, w_{QP}, w_{QL}$ = I/Q WGN (zero mean and unit variance)

Ideal autocorrelation function for BPSK signals: $R(\tau) = E\{C(t)C(t-\tau)\} = \begin{cases} 1-|\tau/T_C|, & |\tau| < T_C \\ 0, & |\tau| \geq T_C \end{cases}$

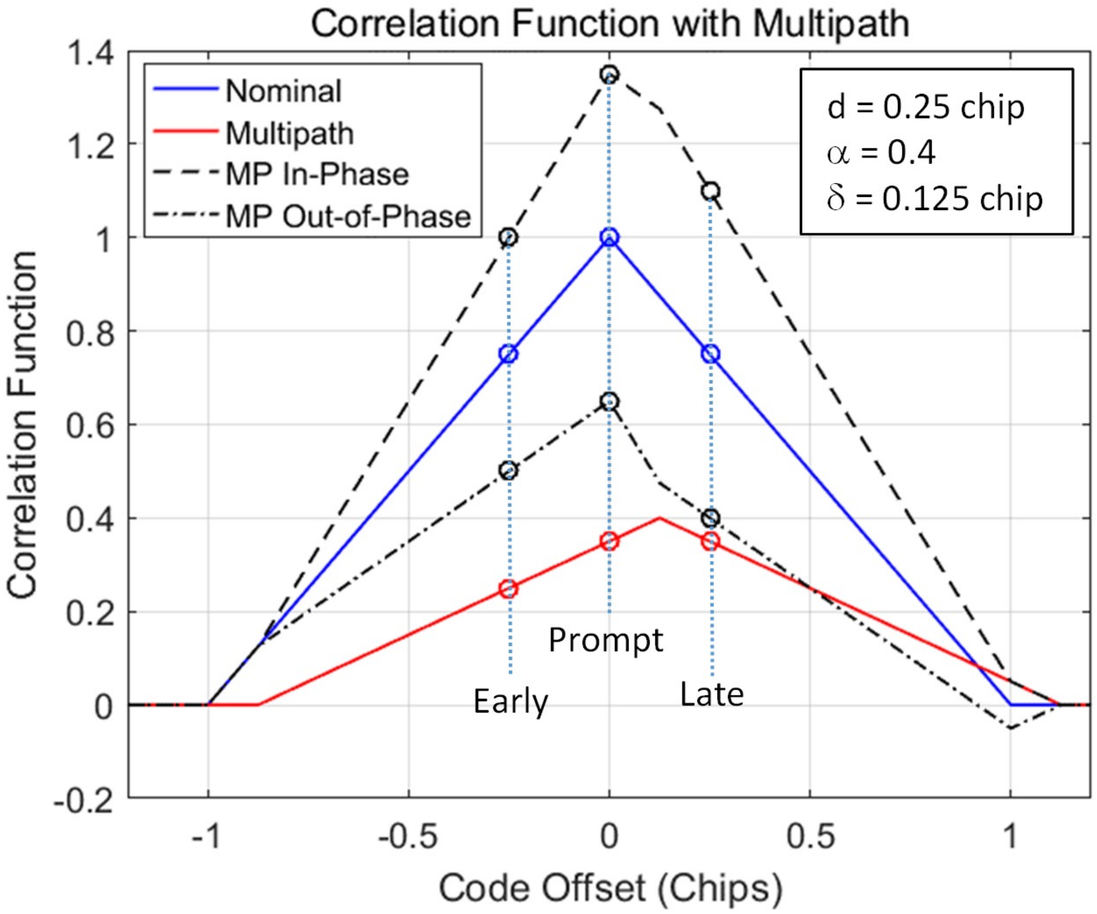
Composite Signal with Single Multipath



$$\phi_i = \left(\frac{2\pi\Delta_i}{\lambda_L} + \phi_{Ri}\right) \mathrm{MOD} 2\pi \quad \text{(rad)}$$

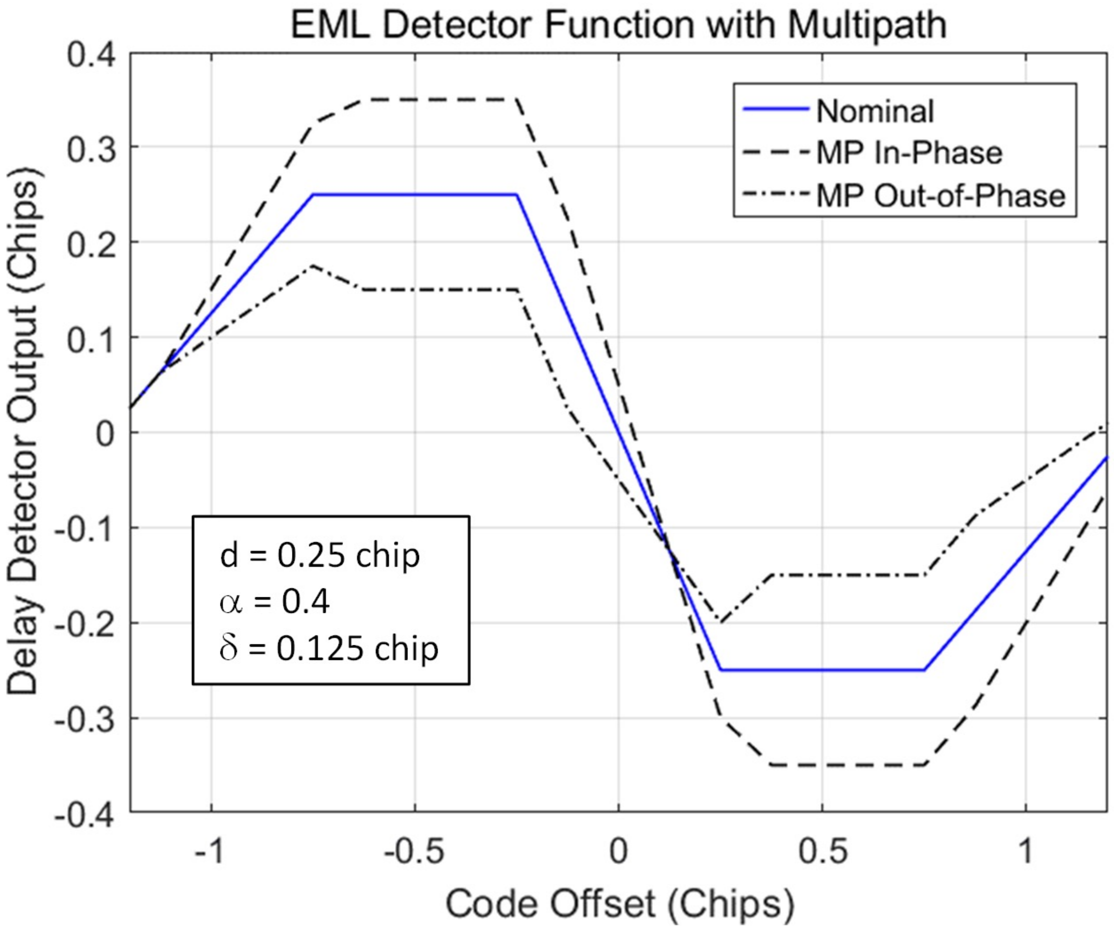
- Composite signal amplitude fluctuates as a function of multipath phase angle
- Deep fading can cause loss of lock

Ideal Code Correlation Functions for Single Multipath



- Binary Phase-Shift Key (BPSK) signal
- Infinite bandwidth
- Multipath distorts the shape of the correlation function

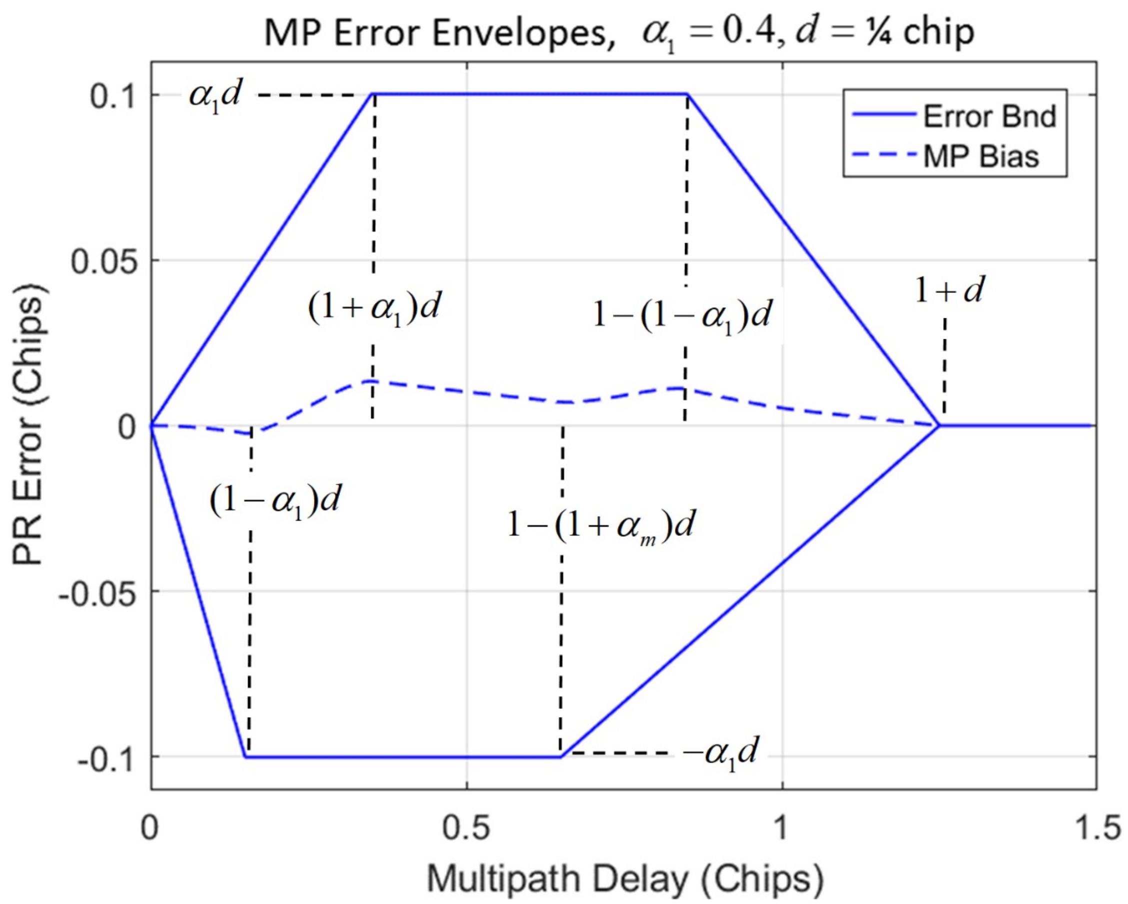
EML Code Tracking Error Discriminator



Early-Minus-Late (EML) Delay Lock Detector (DLD) function:

$$D_{EML}(\tau) = [R(\tau + dT_C) - R(\tau - dT_C)]/2$$

Ideal Code Tracking Error Envelopes



- Dot-product code tracking error detector:

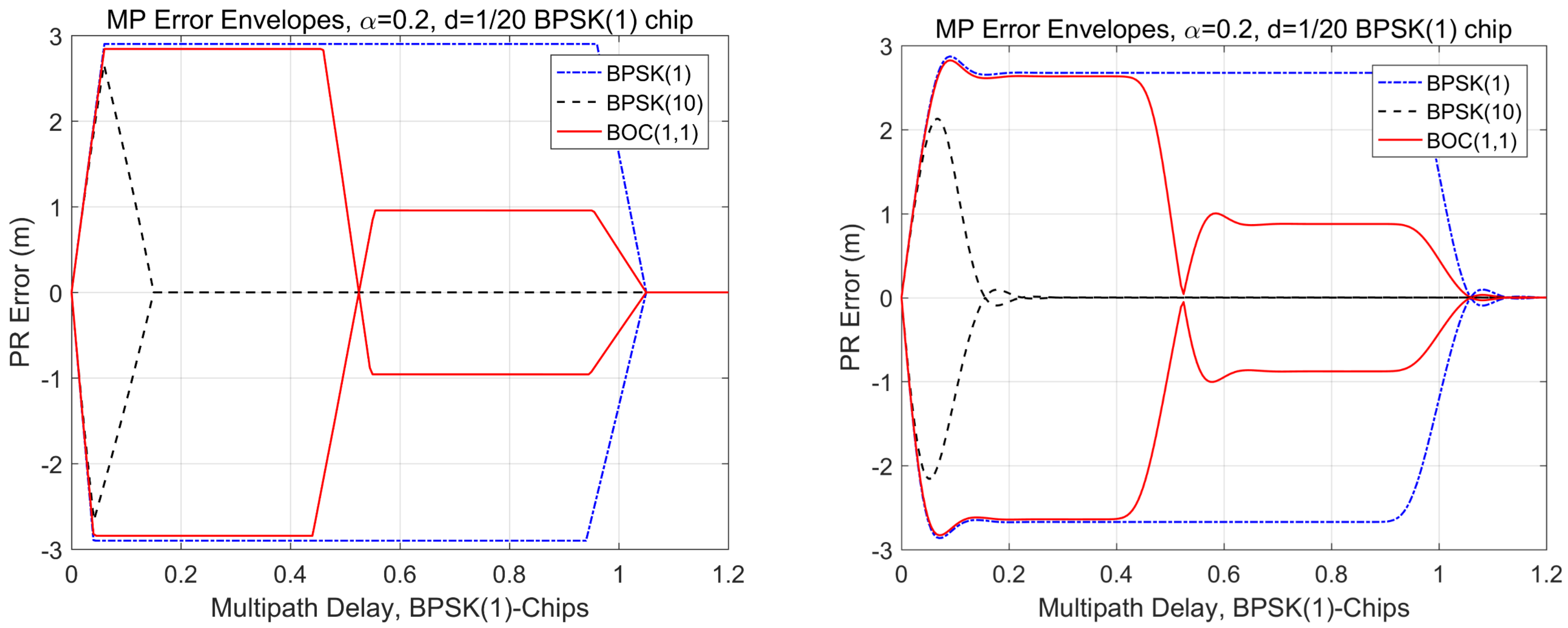
$$\varepsilon_D = \frac{ID_{EML} \cdot IP - QD_{EML} \cdot QP}{IP^2 + QP^2},$$

$$ID_{EML} = (IE - IL)/2, \quad QD_{EML} = (QE - QL)/2$$

- Ideal PRN code and infinite receiver bandwidth assumed
- Bounds represent perfect in-phase or out-of-phase multipath cases ($\theta = 0, \pi$)
- Other multipath phases will lie in between these two bounds
- MP bias represents average over full phase cycle at a given MP delay – MP is not zero mean
- Multipath with delay bigger than 1+d chip has little or no effect on PR measurements

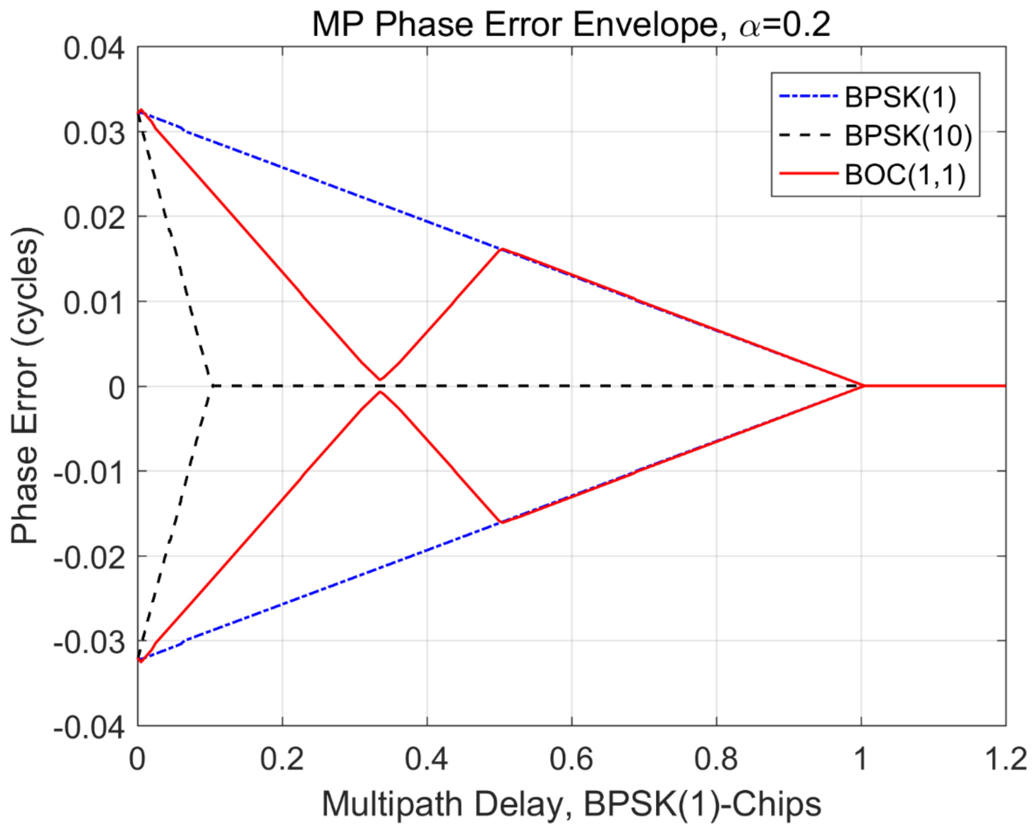
Multipath Code Tracking Error Envelopes for Different Code Types

Infinite bandwidth | **Band-limited with 10 MHz low-pass filter**

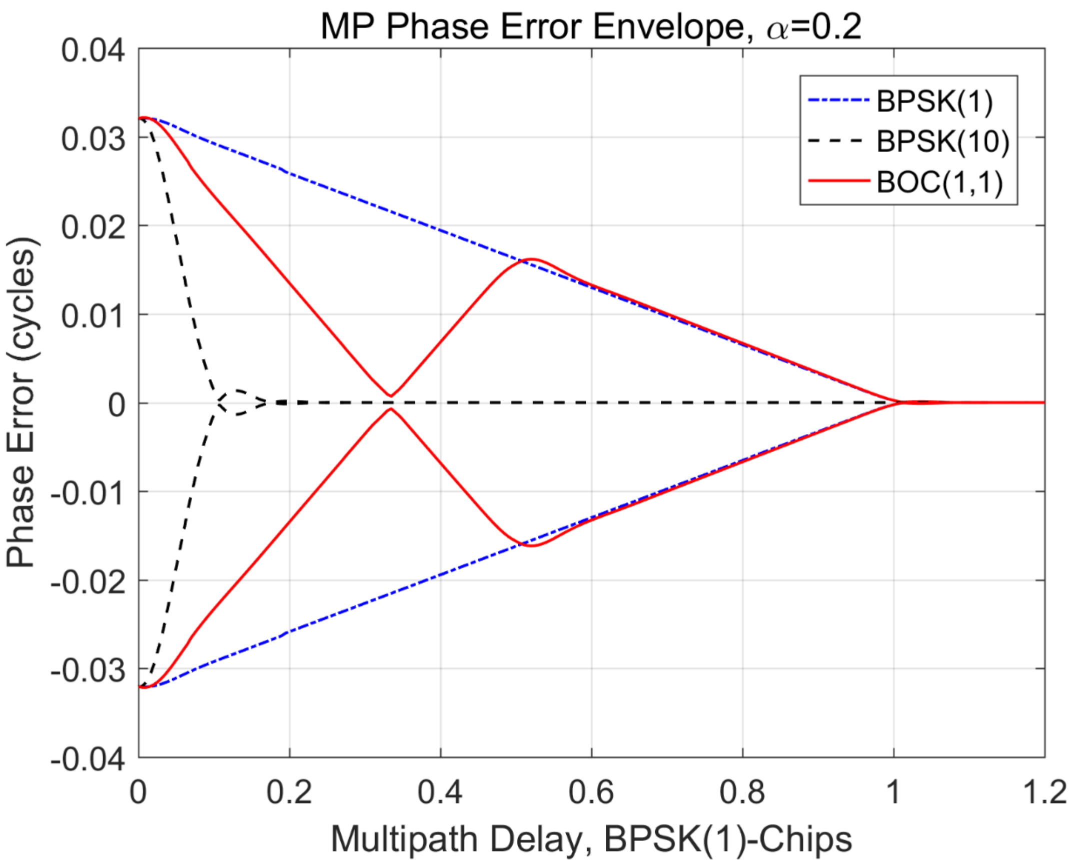


Multipath Phase Tracking Error Envelopes for Different Code Types

Infinite bandwidth



Band-limited with 10 MHz low-pass filter



GNSS Multipath Mitigation Techniques

- Code type
- Antenna design & siting
- Measurement processing
  - Code & carrier combinations
  - Carrier smoothing

} Discussed in Part 2

- Receiver design
  - Adaptive antenna array processing
  - Polarization processing
  - Correlator signal processing
  - Multipath estimation

} Not discussed here

Part 2: GNSS Multipath Mitigation, Measurement Processing & Carrier Smoothing

Part 2 Overview – GNSS Multipath Mitigation

- Effects of different code types
- Antenna design & siting
- GNSS measurement models
- Dual frequency code & carrier measurement combinations
  - Ionospheric-free
  - Wide-Lane (WL) / Narrow-Lane (NL)
  - Geometry-free
  - Divergence-free
- Carrier smoothing of code measurements
  - Single frequency
  - Dual frequency & divergence free

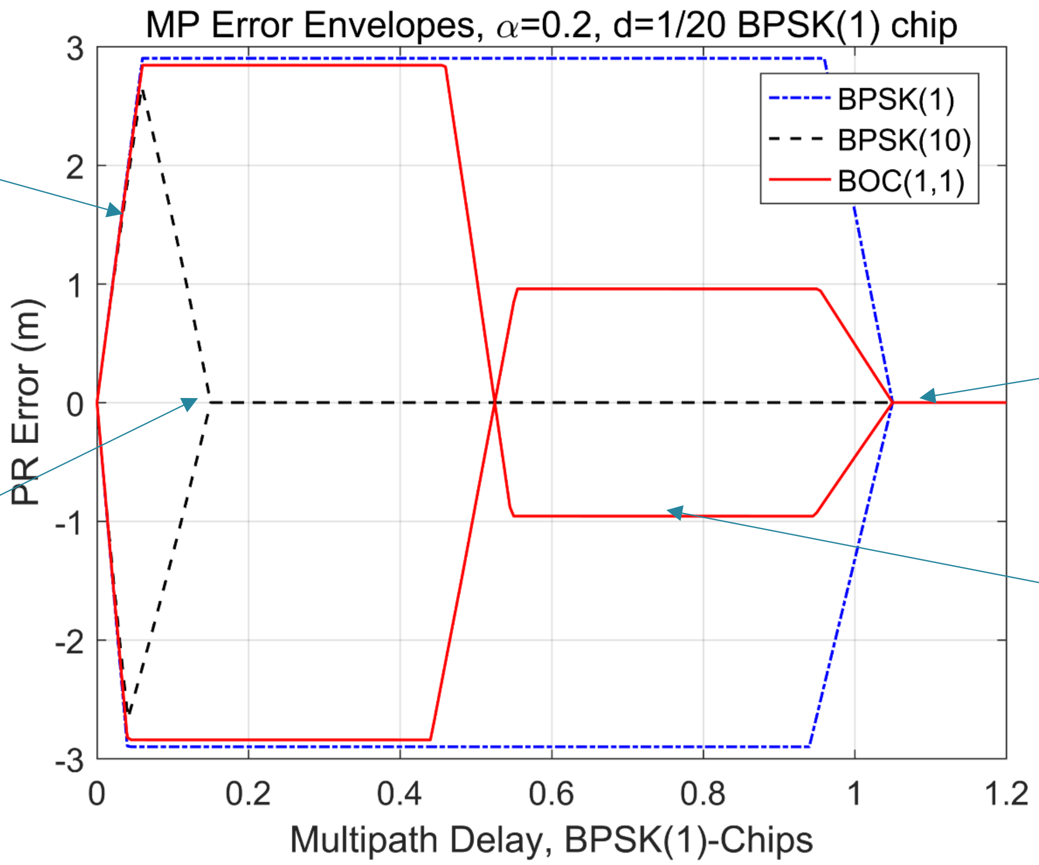
Code Type

Higher code chipping rates have improved multipath error characteristics

All code types have the same error for short multipath delays

BPSK(10) signals immune to multipath with delays >40 m

- GPS P(Y) code L1 & L2
- GPS L5
- Galileo E5a, E5b
- Beidou B2a



BPSK(1) signals immune to multipath with delays >300 m

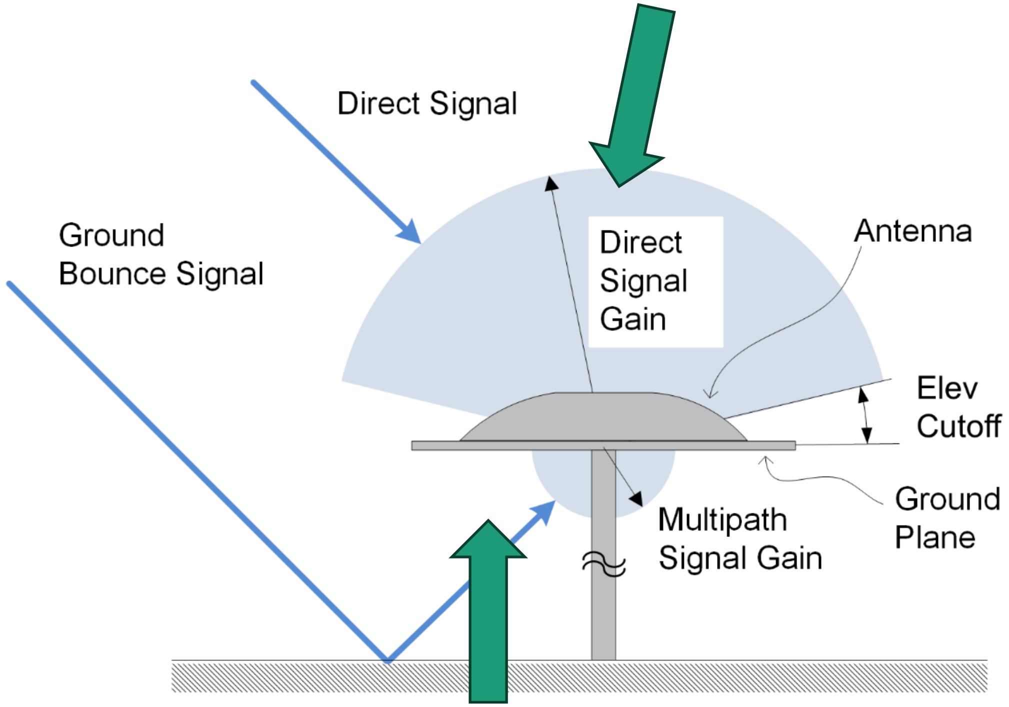
- GPS C/A code
- L1 SBAS

BOC(1,1) signals have reduced response to longer delay multipath

- GPS L1c
- Galileo E1 OS
- Beidou B1c

Antenna Design & Siting

Enhance gain to the direct signal



Minimize gain to the undesired signal

Siting

- Move antenna away from strong reflectors
- Raise antenna above reflecting objects in the vicinity

Try to increase Direct/Undesired (D/U) signal ratio

Measurement Model @ f_L

$$\rho_L = r + \delta_T + \delta_R + I_L + T + \delta\rho_{ML} + \varepsilon_{\rho L}$$
$$\varphi_L = r + \delta_T + \delta_R - I_L + T + \delta\varphi_{ML} + \varepsilon_{\varphi L} + N_L\lambda_L$$

ρ_L= Code pseudorange measurement (in meters)
φ_L= Carrier phase measurement (in meters)
r = Geometric Line-of-Sight (LOS) range
δ_T= Satellite clock and ephemeris errors projected along LOS
δ_R= Receiver clock bias
I_L= K_I/f_L^2 = Ionospheric refraction at f_L
T = Tropospheric delay
$\delta\rho_{ML}, \delta\varphi_{ML}$ = Code and carrier multipath at f_L
$\varepsilon_{\rho L}, \varepsilon_{\varphi L}$ = Code and carrier receiver noise and other errors
$N_L\lambda_L$= Carrier phase ambiguity for the carrier with wavelength λ_L, where N_L is an integer.

Simplified Measurement Model @ f_L

$$\rho_L = r + I_L + \varepsilon_{\rho L}$$
$$\varphi_L = r - I_L + \varepsilon_{\varphi L} + N_L \lambda_L$$

ρ_L= Code pseudorange measurement (in meters)
φ_L= Carrier phase measurement (in meters)
r = Geometric Line-of-Sight (LOS) range (including SV & rcvr clocks and tropo)
I_L= K_I/f_L^2 = Ionospheric refraction at f_L
$\varepsilon_{\rho L}, \varepsilon_{\varphi L}$ = Code and carrier receiver noise, multipath and other errors
$N_L \lambda_L$= Carrier phase ambiguity for the carrier with wavelength λ_L, where N_L is an integer.

Code - Carrier Combinations

Iono−Free:

$$\rho_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2,$$
$$\varphi_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_2^2}{f_1^2 - f_2^2}\varphi_2$$

Wide−Lane Carrier Phase/Narrow−Lane Code:

$$\rho_{NL} = \frac{f_1}{f_1 + f_2}\rho_1 + \frac{f_2}{f_1 + f_2}\rho_2,$$
$$\varphi_{WL} = \frac{f_1}{f_1 - f_2}\varphi_1 - \frac{f_2}{f_1 - f_2}\varphi_2$$

Narrow−Lane Carrier Phase/Wide−Lane Code:

$$\rho_{WL} = \frac{f_1}{f_1 - f_2}\rho_1 - \frac{f_2}{f_1 - f_2}\rho_2,$$
$$\varphi_{NL} = \frac{f_1}{f_1 + f_2}\varphi_1 + \frac{f_2}{f_1 + f_2}\varphi_2$$

Divergence Free Carrier Combinations:

$$f_1\text{:}\quad \rho = \rho_1, \quad \varphi = \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_1 - \frac{2f_2^2}{f_1^2 - f_2^2}\varphi_2$$
$$f_2\text{:}\quad \rho = \rho_2, \quad \varphi = \frac{2f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_2$$

Geometry-Free (*f_1* Iono-Estimation):

$$\rho = \frac{f_2^2}{f_1^2 - f_2^2}(\rho_2 - \rho_1),\ \varphi = \frac{f_2^2}{f_1^2 - f_2^2}(\varphi_2 - \varphi_1)$$

Iono-Free

PR noise amplification

$$\rho_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2 = r + \frac{f_1^2}{f_1^2 - f_2^2}\epsilon_{\rho 1} - \frac{f_2^2}{f_1^2 - f_2^2}\epsilon_{\rho 2}$$

$$\varphi_{IF} = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + \frac{f_1^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} - \frac{f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + N_{IF}\lambda_{IF}$$

Frequencies	PR Noise Amplification	λ_{IF} (cm)
L1, L2	2.98	0.31
L1, L5	2.59	0.28

- Iono canceled
- PR and CP noise amplified
- Short effective CP wavelength

Wide–Lane Carrier Phase/Narrow–Lane Code

PR noise & multipath attenuation

$$\rho_{NL} = \frac{f_1}{f_1+f_2}\rho_1 + \frac{f_2}{f_1+f_2}\rho_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1+f_2}\epsilon_{\rho 1} + \frac{f_2}{f_1+f_2}\epsilon_{\rho 2}$$

$$\varphi_{WL} = \frac{f_1}{f_1-f_2}\varphi_1 - \frac{f_2}{f_1-f_2}\varphi_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1-f_2}\epsilon_{\varphi 1} - \frac{f_2}{f_1-f_2}\epsilon_{\varphi 2} + N_{WL}\lambda_{WL}$$

Frequencies	PR Noise Amplification	λ_{WL} (cm)
L1, L2	0.713	86.3
L1, L5	0.714	75.2

- PR noise & multipath attenuated
- CP noise amplified
- Long effective CP wavelength aids ambiguity resolution
- PR & CP iono have same sign

Narrow–Lane Carrier Phase/ Wide–Lane Code

PR noise amplification

$$\rho_{WL} = \frac{f_1}{f_1 - f_2}\rho_1 - \frac{f_2}{f_1 - f_2}\rho_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 - f_2}\epsilon_{\rho 1} - \frac{f_2}{f_1 - f_2}\epsilon_{\rho 2}$$

$$\varphi_{NL} = \frac{f_1}{f_1 + f_2}\varphi_1 + \frac{f_2}{f_1 + f_2}\varphi_2 = r + \frac{k_I}{f_1 f_2} + \frac{f_1}{f_1 - f_2}\epsilon_{\varphi 1} - \frac{f_2}{f_1 - f_2}\epsilon_{\varphi 2} + N_{WL}\lambda_{WL}$$

Frequencies	PR Noise Amplification	λ_{WL} (cm)
L1, L2	5.74	10.7
L1, L5	4.93	10.9

- PR & CP iono have same sign
- PR and CP noise amplified
- Reduced effective CP wavelength

Geometry-Free (f_1 Iono Estimation)

PR noise amplification

$$\rho_{GF} = \frac{f_2^2}{f_1^2 - f_2^2}(\rho_2 - \rho_1) = I_1 + \frac{f_2^2}{f_1^2 - f_2^2}(\epsilon_{\rho 2} - \epsilon_{\rho 2})$$

$$\varphi_{GF} = \frac{f_2^2}{f_1^2 - f_2^2}(\varphi_1 - \varphi_2) = I_1 + \frac{f_2^2}{f_1^2 - f_2^2}(\epsilon_{\varphi 1} - \epsilon_{\varphi 2}) + N_{GF}\lambda_{GF}$$

Frequencies	PR Noise Amplification	λ_{GF} (cm)
L1, L2	2.19	0.25
L1, L5	1.78	0.21

- Iono delay measured
- PR and CP noise amplified
- Small effective CP wavelength

Divergence-Free Carrier Combinations for Single Frequency Code PR Measurements

$$f_1: \ \ \rho = \rho_1, \quad \varphi = \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_1 - \frac{2f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + I_1 + \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} - \frac{2f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + N_{D1}\lambda_{D1}$$

$$f_2: \ \ \rho = \rho_2, \quad \varphi = \frac{2f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\varphi_2 = r + I_2 - \frac{f_1^2 + f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 2} + \frac{2f_2^2}{f_1^2 - f_2^2}\epsilon_{\varphi 1} + N_{D2}\lambda_{D2}$$

Frequencies	PR Noise Amplification	λ_{DF}
L1, L2	1	<1 mm
L1, L5	1	<1 mm

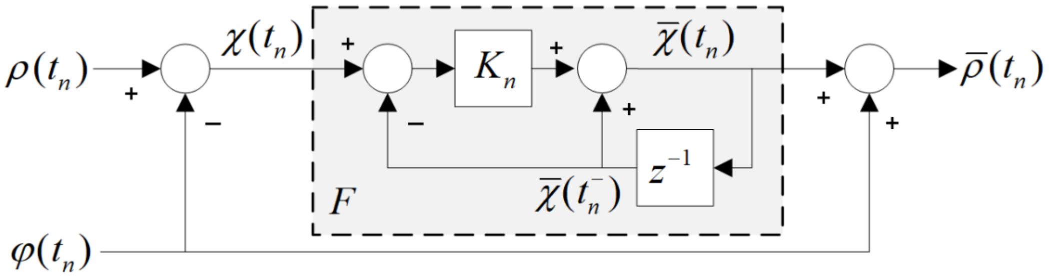
- PR & CP iono have same sign
- Tiny CP ambiguity wavelength

Code Carrier Smoothing

- Code PR have large noise+multipath errors (meter-level) but are unbiased
- Carrier phase measurements have small noise+multipath errors (cm-level) but have an integer cycle ambiguity
- Main idea: combine code and carrier measurements to yield a lower-noise, unbiased PR measurement
  - Low pass filter code and high pass filter carrier phase

Carrier Smoothing – Equivalent Formulations

Complementary Filter Formulation



Initialize: $\overline{\chi}(t_0^-) = 0$

For: $t_n, n = 1, 2, \ldots$

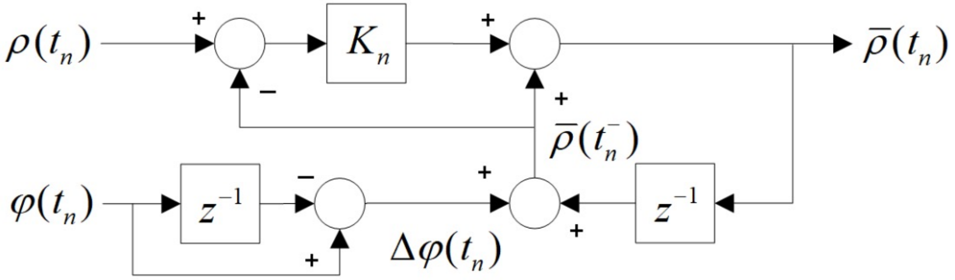
Input: $\chi(t_n) = \rho(t_n) - \varphi(t_n)$

Update: $\overline{\chi}(t_n) = \overline{\chi}(t_n^-) + K_n\left(\chi(t_n) - \overline{\chi}(t_n^-)\right)$

Output: $\overline{\rho}(t_n) = \overline{\chi}(t_n) + \varphi(t_n)$

Extrapolate: $\overline{\chi}(t_{n+1}^-) = \overline{\chi}(t_n)$

Hatch Filter Formulation



Initialize: $\overline{\rho}(t_0^-) = 0$

For: $t_n, n = 1, 2, \ldots$

Input: $\rho(t_n)$

Update/Output:

$$\overline{\rho}(t_n) = \overline{\rho}(t_n^-) + K_n\left(\rho(t_n) - \overline{\rho}(t_n^-)\right)$$

Extrapolate: $\Delta\varphi(t_n) = \varphi(t_n) - \varphi(t_{n-1})$

$$\overline{\rho}(t_n^-) = \overline{\rho}(t_{n-1}) + \Delta\varphi(t_n)$$

For time t_n, $n = 1, \ldots$, the gain is: $K_n = \begin{cases} 1/n, & n = 1, \ldots, N_{max} \\ 1/N_{max}, & n \geq N_{max} \end{cases}$

Qualitative Error Analysis

Complementary filter operation: $\bar{\rho} = F(\rho - \varphi) + \varphi = F\rho + (1 - F)\varphi$

For the steady-state gain, $K = 1/N_{max}$, and the complementary filter iterative equations for F can be written as:

$$\bar{\chi}(t_n) = (1 - K)\bar{\chi}(t_{n-1}) + K\chi(t_n)$$

This discrete time equation can be written in terms of a Z-transform as:

$$F(z) = \frac{K}{1 - (1 - K)z^{-1}} = \frac{Kz}{z - (1 - K)}$$

This is a low-pass filter => *1-F* is high-pass

A model for the smoothed single-frequency PR is:

$$\bar{\rho}_L = (r + \delta_T + \delta_R + T) + (2F - 1)I_L + F\left(\delta\rho_{ML} + \varepsilon_{\rho L}\right) + (1 - F)\left(\delta\varphi_{ML} + \varepsilon_{\varphi L} + N_L\lambda_L\right)$$

LOS range terms are unaffected

Iono delay is filtered

Code PR errors are low-pass filtered

Phase errors are high-pass filtered

Smoothing Filter Steady State Gain Calculation

The value for N_{max} can be determined by relating the CMC filter F to a first-order, continuous-time, low-pass filter:

$F(s) = \frac{1}{T_0 s+1}$, T_0 = time constant (s)

Discrete-time equivalent: $F(z) = \frac{\left(1-e^{-\Delta T/T_0}\right)z}{z-e^{-\Delta T/T_0}} = \frac{Kz}{z-(1-K)}$

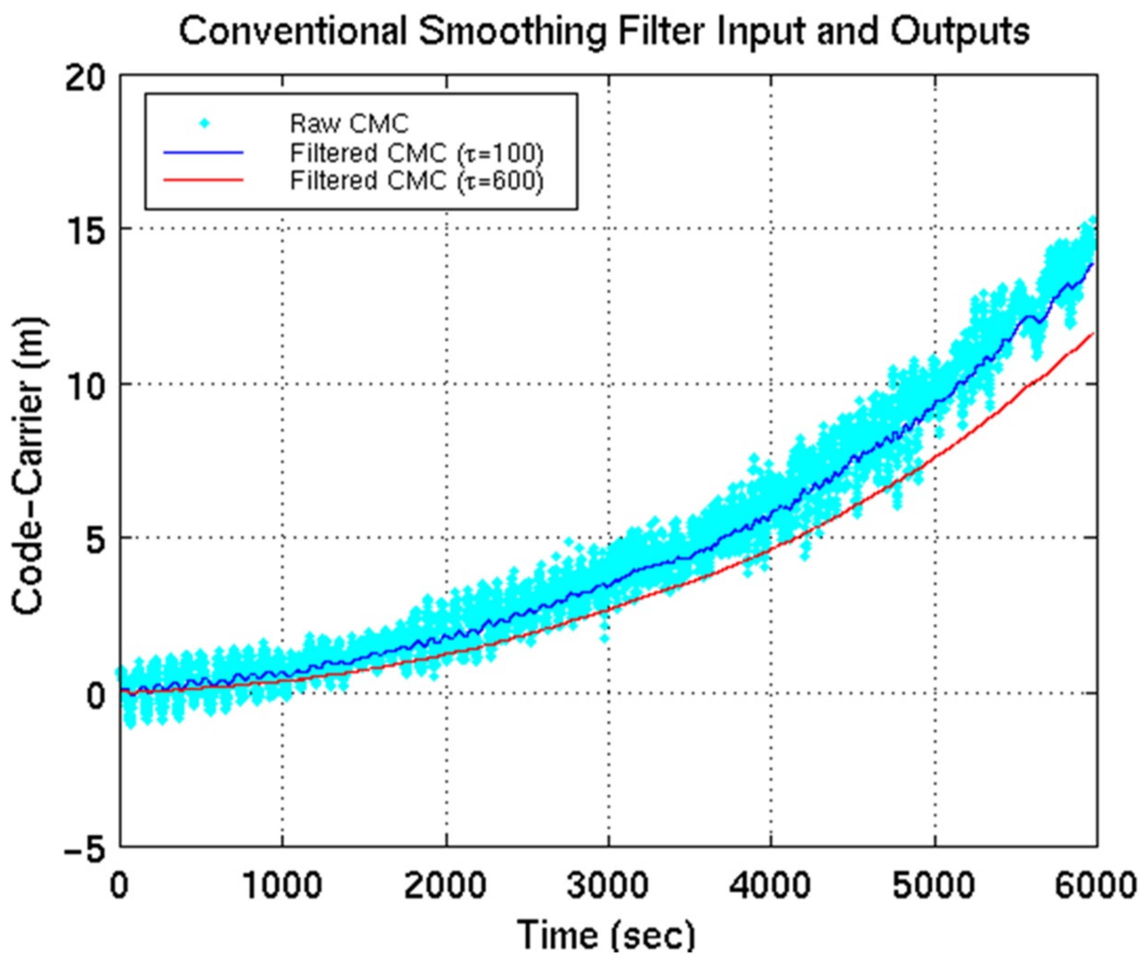
$$K = \frac{1}{N_{max}} = 1 - e^{-\Delta T/T_0} \approx \Delta T/T_0, \quad \Delta T \ll T_0$$

$$\Rightarrow N_{max} \approx T_0/\Delta T$$

For white noise, smoothed standard deviation given by:

$$\sigma_s = \sigma_\rho \sqrt{\frac{K}{2-K}} = \sigma_\rho \sqrt{\frac{1-e^{-\Delta T/T_0}}{1+e^{-\Delta T/T_0}}} \approx \sigma_\rho \sqrt{\frac{\Delta T/T_0}{2-\Delta T/T_0}} = \frac{\sigma_\rho}{\sqrt{2N_{max}-1}}$$

Single Frequency Smoothing Example Results

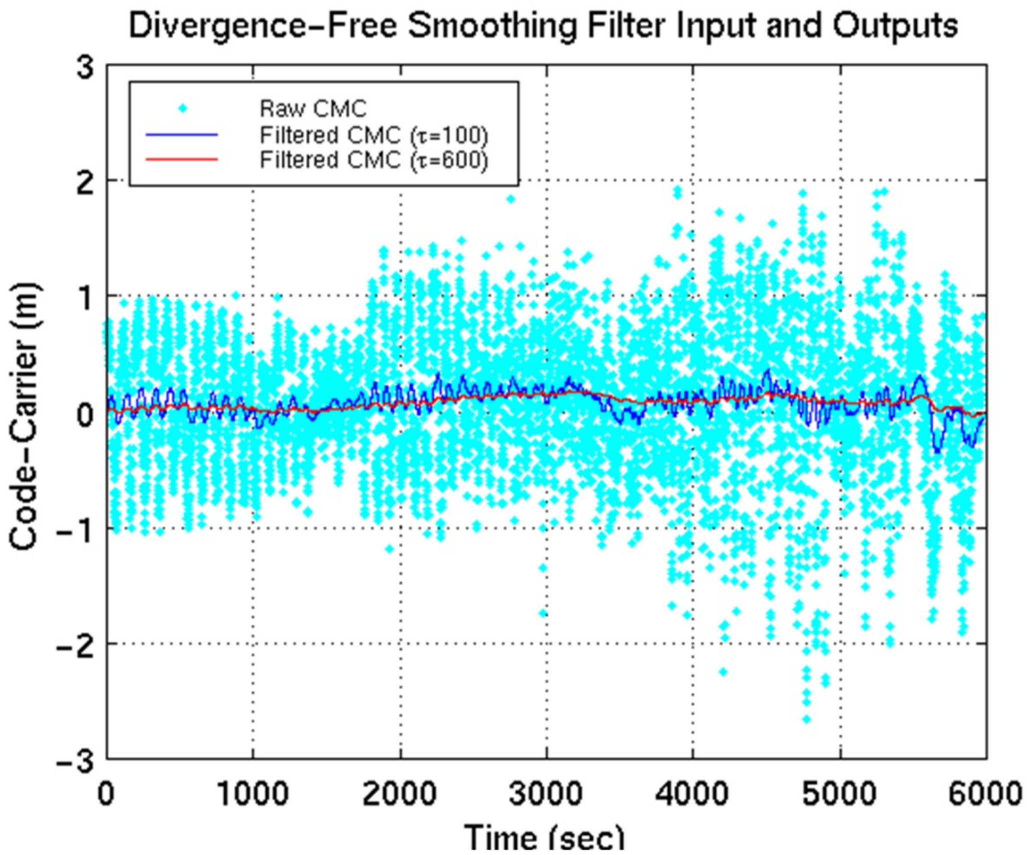


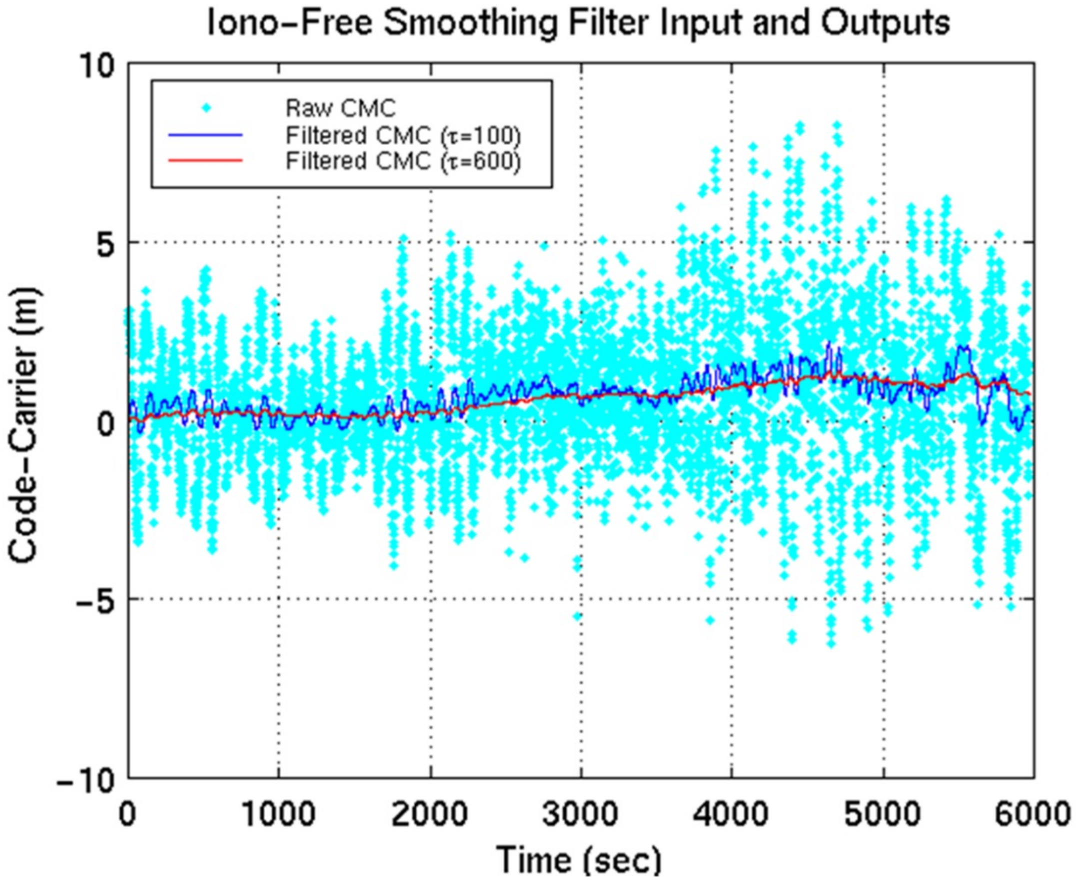
- Single frequency code and carrier phase
- Smoothing reduces meter-level noise to sub-decimeter level
- Longer smoothing time constant induces large bias due to iono divergence

Dual Frequency Smoothing

- Code-carrier iono divergence limits length of single frequency smoothing
  - Iono delays code and advances carrier phase
- Certain PR and CP combinations have equal iono delays (same sign)
  - Divergence free (single frequency code, dual frequency CP)
  - Iono Free
  - WL/NL
- Divergence free combinations enable extended carrier smoothing time constants

Dual Frequency Smoothing Example Results





- Examples use L1/L2 P(Y) code
- No iono divergence effects

- ~3X noise amplification due to iono-free combination is evident

Summary

- Multipath reception affects essentially all GNSS receiver applications
- For many applications it is the dominant error source
- Many techniques are available to mitigate multipath errors:
  - Antenna siting to avoid multipath
  - Antenna types that enhance direct signals and attenuate reflected signals, particularly for fixed sites
  - Adaptive antenna array processing
  - Correlation signal processing
  - Measurement processing techniques like carrier smoothing
  - Navigation processing to de-weight or exclude measurements impacted by multipath
  - Post-processing and modelling techniques that provide estimates to correct multipath errors
- Applicability of these techniques to different GNSS receiver types varies greatly, with mobile phones being especially constrained

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