Quantum optimization of coherent chaotic systems: A case for buses of Kathmandu

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Introduction

1. - physics without frontier Nepal Holographic Himalaya
2. - logo at place
3. -aman - remove repeated parts and make presentation concise
4. eva- rohit ko part lai continue garera prepare garnu paryo - work on the sequence of the presentation
5. format of bibliography- done
6. add citations if necessary.

Random Matrix

Consider a matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots & a_{2n} \\ a_{31} & \dots & & & \\ \vdots & & & & \\ \vdots & & & & \\ \vdots & & & & \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix}$$

where some or all the entries are drawn randomly from various probability distributions traditionally referred to as the random matrix ensembles. The main goal of the Random Matrix Theory is to provide understanding of the diverse properties of random matrix theory by studying statistics of matrix eigenvalues(spacing, discrete spectral density, gap probability, etc.).

Characterizing Quantum Systems using RMT

Random matrix theory is used to characterize complex quantum systems when there is limited knowledge about the Hamiltonian. The fundamental hypothesis is that the Hamiltonian can be treated as a random matrix drawn from an ensemble with appropriate symmetries. This approach is particularly useful for systems with many degrees of freedom and unknown interaction couplings among them.

Eigen Spectrum of Hamiltonian

Integrable Hamiltonian Models Let the eigenvalues be $\lambda_1 \leq \lambda_2 \leq ....$ and let $S_n = \lambda_{n+1} - \lambda_n$ be the consecutive splittings. In this case, the distribution $P(s)$ of the neighboring spacings $s = S/D$, where S is a particular spacing and D is the mean distance between neighboring intervals is given by:

$$P(s) = \frac{1}{D} e^{-s}$$

Non-Integrable Hamiltonian Models

In contrast, the level spacing distribution P(s) of chaotic models is closely approximated by the Wigner-Dyson (WD) distribution:

$$P(s) = b_\beta s^\beta e^{-\alpha_\beta s^2} \tag{1}$$

where β depends on which universality class of random matrices the chaotic Hamiltonian belongs to.

Broody Distribution

To compare the spectral statistics with regular and chaotic limits and also exhibit interpolation between them, different distribution functions have been proposed. For example, for the Gaussian Orthogonal Ensemble (GOE) statistics, one popular intermediate distribution is Broody distribution:

$$P(s) = b(1+q)s^q e^{-bs^{q+1}}, b = \left(\Gamma\frac{2+q}{1+q}\right)^{q+1}$$

where $q = 0$ corresponds to the Poisson limit while $q = 1$ corresponds to Wigner-Dyson limit.

Signature of Quantum Chaos

Bohigas-Giannoni-Schmit conjecture The BGS conjecture associates the quantum chaotic properties of a system with the correlations between its energy levels. Chaotic Hamiltonian exhibit level correlations in agreement with the predictions of random matrix theory (RMT): Adjacent eigenvalues show level repulsion and, at larger energy scales, signals of spectral rigidity. This understanding can be extended to the eigen spectrum of the Hamiltonian, allowing us to **determine the chaotic nature of a quantum state based on its level spacing distribution**.

Thank you

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Cuernavaca Transport

- The bus system is decentralized.
- The transport system of Cuernavaca is a chaotic coherent Quantum system so, it's statistical properties are described by Wigner Dyson random matrix ensembles.

Bus statistical data

- The bus statistical data is collected from line number four near city center in which 3500 arrivals were recorded within 27 days.
- Evaluation of bus spacing distribution and comparison of results with predictions of Gaussian Unitary Ensemble(GUE).
-

Cuernavaca Buses and Dyson(One dimensional interacting) gas

- Coulomb potential is given by:
  $V = -\sum_{i<j}(\log|x_i - x_j|$
- The acceleration of buses is :
  $\frac{dvi}{dt} \approx \frac{f((v_{i+1}, v_i)}{(x_{i+1}-x_i)^a}$
- For low velocities and a=1, the buses accelerate as similar as dyson gas particles.
- For given a in case of buses and for particular temperature in case of gases, the distribution of both gas particles and buses is given by GUE.

Interaction Potential

- No constraints to influence the transport which makes each bus a property of driver
- Tries to maximize income and passengers
- Leads to competition and mutual interaction
- Engage people to record bus arrivals at bus stops to avoid bus clustering so that distribution of buses is Wigner Dyson rather than Poisson
- With the information about position of preceding and following bus, drivers try to optimize distances between them
-

Relationship between GUE prediction and statistical results

- Assuming the random matrix of bus distribution in Cuernavaca is hermitian, the total information of the system is minimum and hence it conforms *Gaussian Unitary Ensemble(GUE) prediction*.
- The bus number variance is simply obtained as:
  N(T) $= \sum_{i=1}^{n}(n(T_i) - T)^2$
  where, $(n(T_i)$ is actual number of bus arrivals
  T is expected average bus arrivals
- The bus number variance from GUE prediction is:
  $N(T) \approx \frac{1}{\pi^2}(\ln(2\pi T) + \gamma + 1)$
- The number variance from bus data is consistent with GUE upto the time interval $T \approx 3$ which implies strong interaction between three subsequent buses whereas weaker long range correlation between more than three buses.

Thank you

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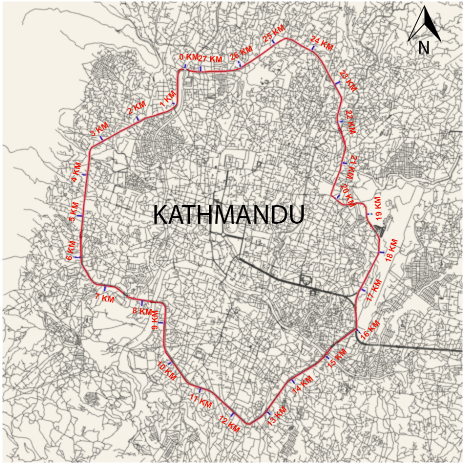
Motivation and Objective

- What we learned from RMT and Case of Cuernavaca
  - Chaotic system can give an optimum distribution
  - RMT can be used to model bus system of Cuernavaca
  - Wigner-Dyson Distribution-GUE gives optimum spacing distribution for Cuernavaca bus system
  - Similar with Ring Road Bus system of Kathmandu
- Objective of the project
  - Design theoretical protocol for optimization of bus spacing of Ring road bus system of Kathmandu

Kathmandu Ring Road

- Properties of Bus system in Ring Road
  - 40 buses in the system by Mahanagar Yatayat(Metropolitan Transport)
  - Leave bus system at a predefined time with no route-scheduling
  - Drivers drive buses independently of another(no mutual information shared)
  - Income is based on the number of passengers
  - Virtually no interaction with bus drivers *
- What to optimize
  - Optimal bus spacing
  - Reduce clustering and maximize passenger and profit



Optimization protocol

- Generate a distribution $G(x_1, x_2, x_3, \ldots, t)$ with x's as a real-time position of the bus system at time t,
- Shift bus position in $G(x_{i^s})$ by δx_i in time δx_i such that $G^{'}(x_1 + \delta x_1, x_1 + \delta x_1, x_2 + \delta x_2, ..., x_n + \delta x_n, t + \delta t)$ closely resembles Wigner Dyson distribution
$$|G^{'}(x_i, t) - P_{WD}| \leq \epsilon$$
- If $\delta x_i / \langle v \rangle \leq t_s$, the bus should stop at bus stop for $\delta x_i / \langle v \rangle$ time,
- If $\delta x_i / \langle v \rangle > t_s$, the bus should stop at bus stop for $\delta x_i - \langle v \rangle t_s$ time

Protocol using Quantum Algorithms

- Prepare a Hamiltonian using $G(x_{i^s})$
- Encode the Hamiltonian $G(x_{i^s}$ into a quantum circuit using parameterized variational quantum algorithms
- Check eigen spectrum of Hamiltonian using spectral analysis [1]
- Build a cost function using spectral analysis so as to get close to a Hamiltonian based on the Wigner-Dyson Distribution
- Use the evaluations to generate a new set of parameters to feed back into the variational algorithm

- The algorithm will run in a loop to minimize the cost function and results in a Hamiltonian that is required.
- Evaluate δx_i using initial and final Hamiltonian.
- Check eigen spectrum of Hamiltonian using spectral analysis
- Build a cost function using spectral analysis so as to get close to a Hamiltonian based on the Wigner-Dyson Distribution
- Use the evaluations to generate a new set of parameters to feed back into the variational algorithm
- If $\delta x_i / \langle v \rangle \leq t_s$, the bus should stop at bus stop for $\delta x_i / \langle v \rangle$ time,
- If $\delta x_i / \langle v \rangle > t_s$, the bus should stop at bus stop for $\delta x_i - \langle v \rangle t_s$ time
- Get an optimized bus spacing for the bus system

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Q&A

Thank You!

I will now be taking questions.

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