A Classical Diary In A Quantum Black Hole

Project Review

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Preliminary

QUBITS

A quantum bit or qubit, is the fundamental building block of a quantum computer, as a bit is of a classical computer. However, a qubit can not only be in one of two states but also in a superposition:

$$|\text{qubit}\rangle = \frac{1}{\sqrt{2}}\left(a|\text{state } \alpha\rangle + b|\text{state } \beta\rangle\right)$$

Here, *a* and *b* are complex coefficients, $|\text{state } \alpha\rangle$ and $|\text{state } \beta\rangle$ are the possible basis states, and the normalization factor $\frac{1}{\sqrt{2}}$ ensures that the total probability is conserved.

Bell states or EPR pair

The Bell states, or EPR pairs (after Einstein, Podolsky and Rosen), are chosen as the Bell basis states and defined as follows:

$$|\Psi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \tag{1}$$

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} - |11\rangle_{AB}) \tag{2}$$

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |10\rangle_{AB}) \tag{3}$$

$$|\Phi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} - |10\rangle_{AB}) \tag{4}$$

Here, $|ij\rangle_{AB}$ represents the joint state of qubit A being in state i and qubit B being in state j.

Super Dense Coding

Super dense coding is a protocol that enables two parties to transmit two classical bits of information by sending only one qubit.

Significance: Double the classical communication capacity of a noiseless qubit channel using noiseless entanglement.
Advantages and its Application: Enhanced Communication Efficiency, Resource Optimization, Quantum Error Correction

Super dense Coding Process

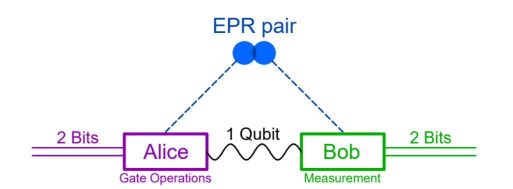

Figure: Illustration of the Superdense Coding Protocol.

Step 1: Preparation
Alice and Bob share an entangled pair of qubits which is created using a Hadamard gate on qubit A, and then a CNOT gate with qubit A as control and qubit B as target.

Step 2: Gate Operation
Alice encodes 2 classical bits using one of four quantum operations {I, X, Z, XZ} on her qubit, depending on the message she wants to send.

The state becomes one of the following four Bell states :

Table: Gate operations Alice must apply to encode her two-bit message for Bob

Message	Gate Operation	Resulting State
'00'	I	$\frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 11\rangle)_{AB} = \lvert\Phi^+\rangle_{AB}$
'01'	Z	$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle)_{AB} = \lvert\Phi^-\rangle_{AB}$
'10'	X	$\frac{1}{\sqrt{2}}(\lvert 10\rangle + \lvert 01\rangle)_{AB} = \lvert\Psi^+\rangle_{AB}$
'11'	XZ	$-\frac{1}{\sqrt{2}}(\lvert 10\rangle - \lvert 01\rangle)_{AB} = -\lvert\Psi^-\rangle_{AB}$

Step 3: Measurement

Bob finally performs a Bell measurement (a measurement in the basis $\{\phi^+{}_{AB}, \phi^-{}_{AB}, \psi^+{}_{AB}, \psi^-{}_{AB}\}$) to extract the 2 classical bits encoded in Alice's qubit.

Quantum Teleportation

Quantum Teleportation

Quantum teleportation is a protocol that enables the transmission of a quantum state from one location to another without directly transferring the physical qubit itself. Instead, the protocol involves using a shared entangled bell pair and classical communication to transfer only the information of the state.
Significance: It destroys the quantum state of a qubit in one location and recreates it on a qubit at a distant location, without violating the no-cloning theorem.
Advantages and its Application: Secure Communication, Quantum Cryptography, Quantum Key Distribution (QKD)

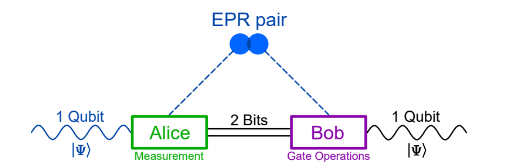

Figure: Illustration of the Quantum Teleportation Protocol

Step 1: Preparation
Alice and Bob share an entangled pair of qubits prepared by using a Hadamard gate on qubit A, and then a CNOT gate with qubit A as control and qubit B as target

Step 2: Measurement
Alice entangles her message qubit with her half of the Bell pair through a Bell basis measurement, resulting in random classical bits (b_1, b_2).

Step 3: Gate Operations
Alice sends her classical bits (b_1, b_2) to Bob. Bob applies a restoration operation on his qubit based on these bits, choosing from options like identity, Pauli X, Pauli Z, or the Pauli operator ZX.

Table: Gate operations Bob must apply to the state of his qubit to reconstruct $|\Psi\rangle$, depending on Alice's measurement outcome.

Alice's Result	Bob's State	Gate Operation
$\|00\rangle_{AS}$	$a\|0\rangle + b\|1\rangle$	$I\ \|\Psi\rangle_B$
$\|01\rangle_{AS}$	$a\|0\rangle - b\|1\rangle$	$Z\ \|\Psi\rangle_B$
$\|10\rangle_{AS}$	$a\|1\rangle + b\|0\rangle$	$X\ \|\Psi\rangle_B$
$\|11\rangle_{AS}$	$a\|1\rangle - b\|0\rangle$	$ZX\ \|\Psi\rangle_B$

Teleportation Through Wormhole Protocol

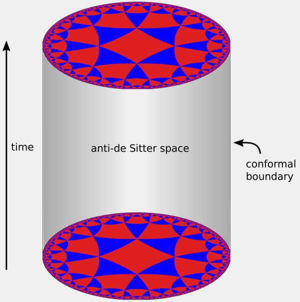

Figure: Ads-CFT

- Equivalent description of the same thing. One of the descriptions is strongly coupled, highly quantum mechanical system. The other description is quantum gravity. (Duality)
- It's a dictionary for translating calculation in one theory to the other.

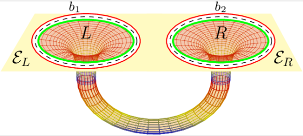

Figure: ER=EPR

- How to encode the information of the gravitational universe in much simpler quantum system?
- Through ER=EPR: Entanglement between different sub regions of boundaries/black hole horizons (the quantum mechanical description) is equivalent to the geometrical connection between those horizons (wormholes)

Teleportation Through Wormholes

- Can these ideas be tested?
- Classically Wormholes aren't traversable.
- But combining quantum teleportation with the idea that entangled black holes are connected by Einstein-Rosen bridges implies that ER=EPR could in-principle be tested by observers who themselves never cross the horizon.
- We create two horizons of N qubits that are entangled with each other. Teleportee is scrambled with the d.o.f of one of the horizons (A).
- Measurement of any two qubits of horizon A is made, and results are sent to B classically. Teleporte is found to be transferred to B by performing unitary operations on horizon B.
- Conformation of duality is achieved by applying the Stanford-Shenker shockwave that negative energy for the teleporte to appear at bob's side at $t = 0$.

The Paper

- Our paper explores the relationship between black holes and quantum information.
- We investigate the scenario of dropping a classical diary into a quantum black hole, and how the in-falling information contained within it can be recovered (As opposed to Qunatum Diary; as done by Hayden-Preskell [1]).

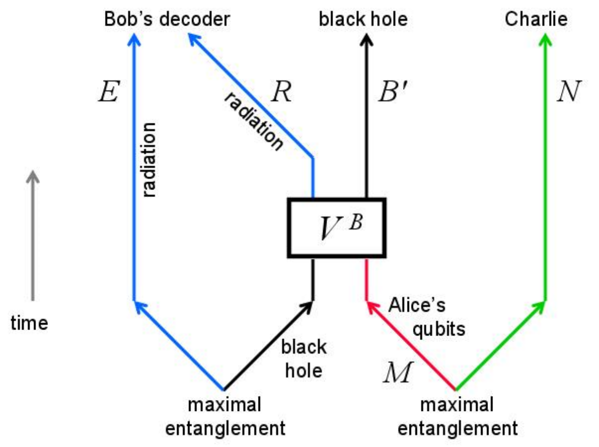

Figure: Hayden-Preskill Model

Modeling black hole in terms of secret sharing language

- Secret sharing is a cryptography technique that allows a secret message to be divided into multiple shares, which can then be distributed among different parties.
- We show that this technique can be used to model the behavior of a black hole, where the secret message is the information that falls into the black hole and the parties are the different regions of space-time.
- This approach provides a new perspective on the nature of black holes and their relationship to quantum information.
- It also has implications for quantum error correction, as it suggests that the behavior of black holes can be understood in terms of error-correcting codes.

Dropping a classical diary

- For a classical diary, a set of orthogonal basis is enough to reconstruct the secret. We are also not constrained by no-cloning theorem, which adds to the efficiency of the process.
- For the quantum diary, the entire superposition of the orthogonal basis is required.

Super-dense coding through the wormhole

- H-P model $\equiv$ Traversable Wormhole
- recovering information of infalling classical diary from quantum black hole $\equiv$ super-dense coding via wormhole
- By using super-dense coding, Alice can encode two classical bits of information into one qubit, and send it to Bob.
- Bob can then use a decoding algorithm to recover the original two bits of information.

- We investigate the scenario of dropping a classical diary into a black hole, and find that the information actually comes out faster than in the case of a quantum diary. $t_{CD} = \frac{1}{2} t_{QD}$
- Super-dense coding through the wormhole allows for more efficient communication than classical communication and can be used to extract information from a black hole.

Patrick Hayden and John Preskill.
Black holes as mirrors: Quantum information in random subsystems.
JHEP, 09:120, 2007.