Linear Analysis of Drift Alfven Waves in Dense Astrophysical Objects

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Introduction

What is Plasma?

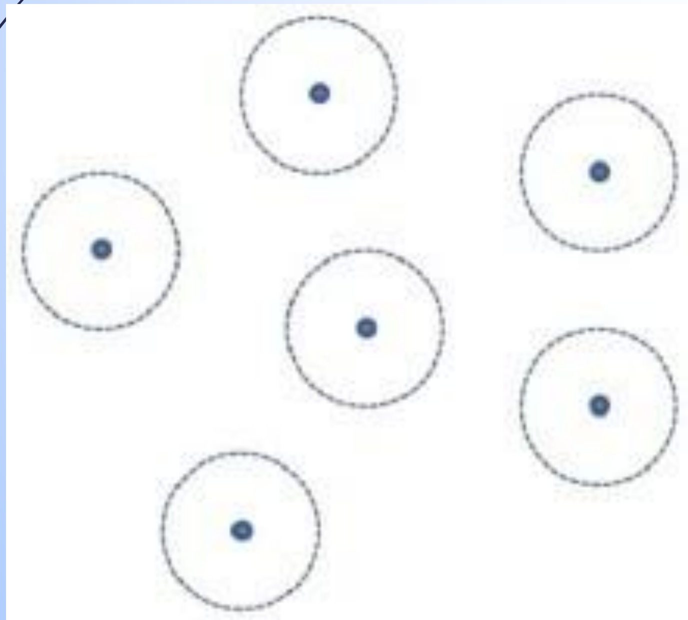
Plasma is an ionized gas.

Plasma in Nature

99 % of the photonic universe is plasma.

Classical Plasma

The classical regime focuses on high temperature and low densities.



$$\omega_p = \left(\frac{e^2 n}{m\varepsilon_0}\right)^{1/2}$$

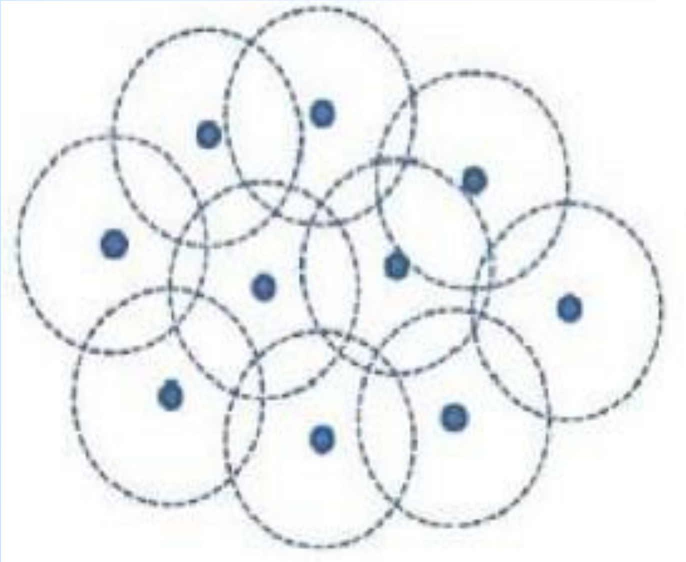
$$v_T = \left(\frac{k_B T}{m}\right)^{1/2}$$

$$\lambda_D = \frac{v_T}{\omega_p} = \left(\frac{\varepsilon_0 k_B T}{n e^2}\right)^{1/2}$$

$$\Gamma_c = \frac{q^2 n^{1/3}}{\epsilon_0 k_B T}$$

Quantum Plasma

The quantum regime focuses on low temperature and high number densities.



$$\omega_{pe} = \left(\frac{e^2 n}{m\varepsilon_0}\right)^{1/2}$$

$$V_{Fe} = \left(\frac{2E_F}{m_e}\right)^{\frac{1}{2}}$$

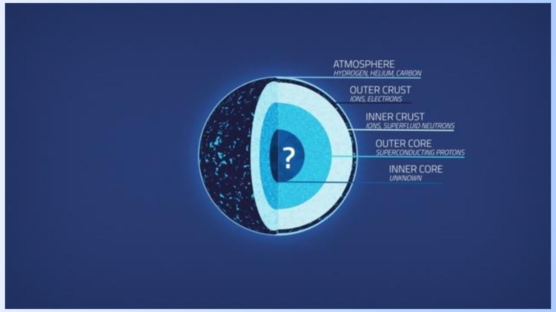
$$\lambda_{Fe} = \frac{V_{Fe}}{\omega_{Pe}}$$

Dense Astrophysical Objects

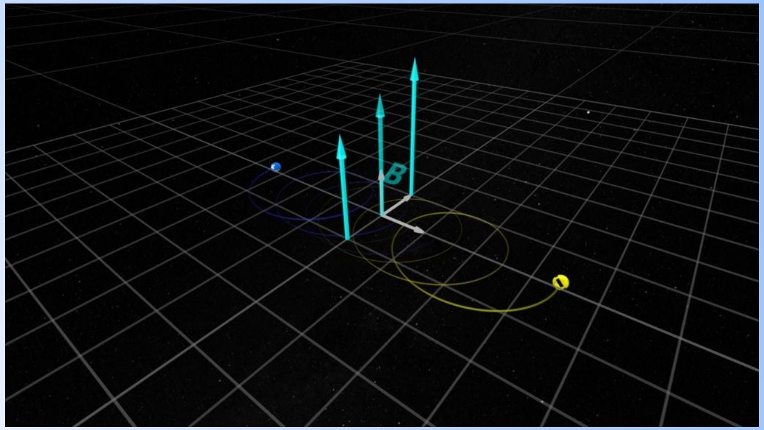
Exhaust most of their nuclear fuel.

High mass concentrations within relatively compact volumes.

- **White Dwarfs**
- **Neutron Stars**
- **Black Holes**



Drift



Charged particles in a magnetic field gyrate around the z-axis. When a drift source is introduced, it causes a drift, orthogonal to both the magnetic field and the drift source.

- $\boldsymbol{E} \times \boldsymbol{B}$ **Drift:** Introducing a uniform electric field.

$$v_{gc} = \frac{E \times B}{B^2} = \frac{E}{B}$$

- **Diamagnetic Drift:** Is collective due to pressure gradients, and absent in individual particles.

$$v_D = -\frac{k_B T}{q B_0 n_0} \frac{\partial n_0}{\partial x}$$

- **Polarization Drift:** Time-varying electric field is introduced.

$$v_p = \mp \frac{1}{\omega_C B} \frac{\partial E}{\partial t}$$

Waves in Plasma

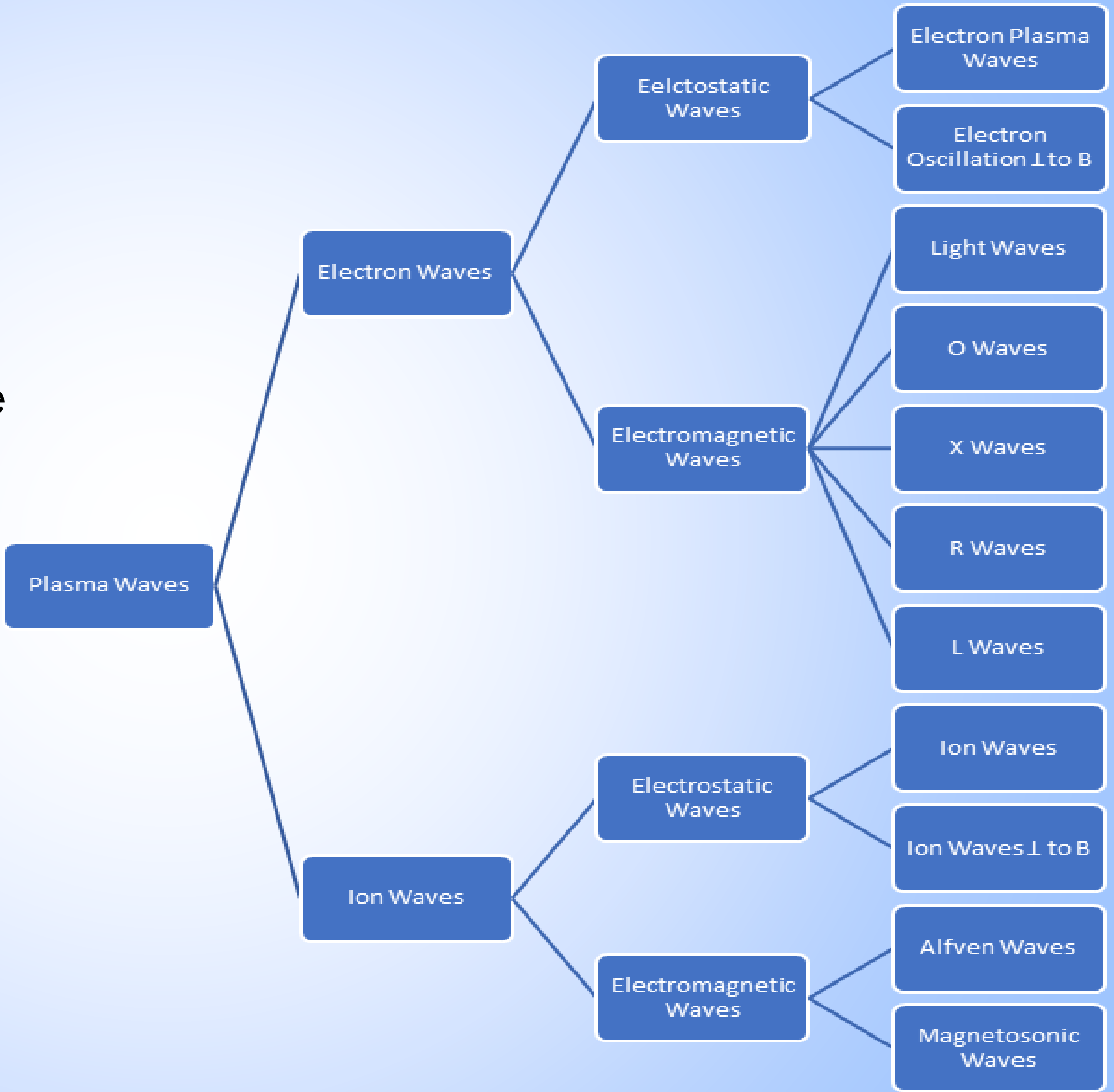
In plasma, due to oscillations of particles $\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0 m}\right)^{1/2}$, create waves, leading to electron and ion wave types, each with subcategories.

$$y = y_0 e^{i(kx-\omega t)}$$

$$n = n_0 e^{i(kx-\omega t)}$$

$$q = q_0 e^{i(kx-\omega t)}$$

$$E = E_0 e^{i(kx-\omega t)}$$



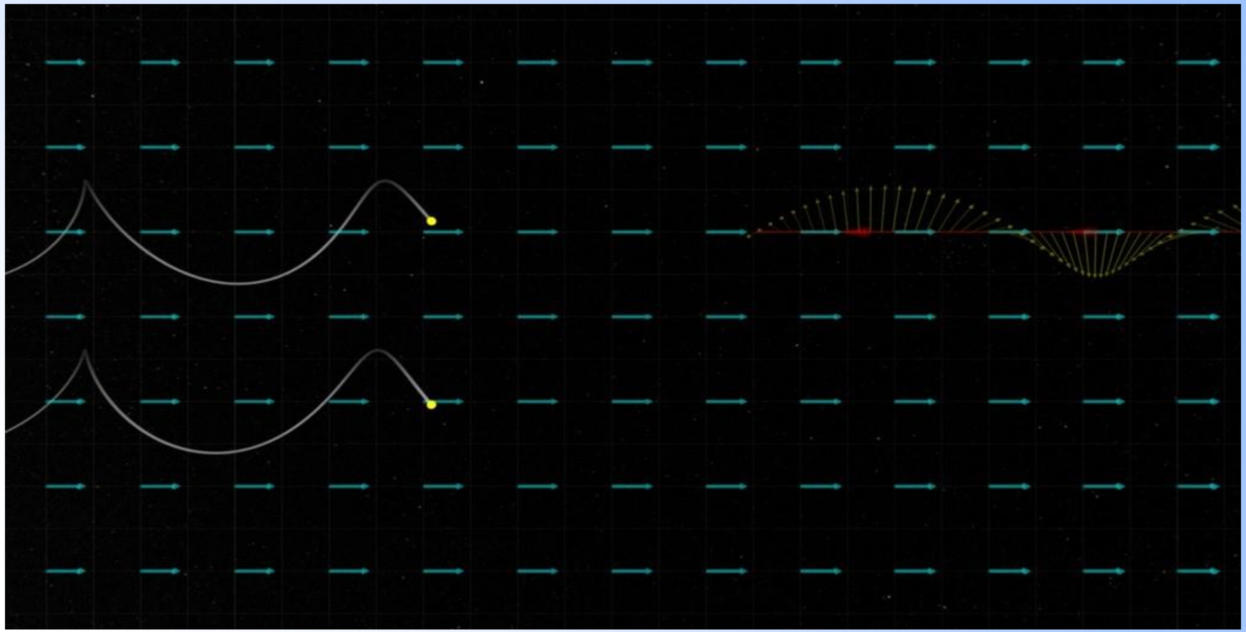
Alfven Waves

- Low-frequency ion oscillations in a magnetic field, and propagate along the magnetic field.

- Predicted by Hannes Alfvén in 1942.

- They are like waves on a magnetic string with plasma particles as beads.

- Laboratory and space plasmas.



$$\omega^2 = v_A^2 k^2$$

$$v_A = B_0/\sqrt{\mu_0 \rho}$$

Drift Alfven Waves

- Drift Alfven waves are Alfven wave in presence of non uniform medium.
- Laboratory and space plasma.

$$\omega^3 - \omega^2\omega^* - \omega\left(1 + k_y^2\rho_s^2\right)v_A^2k_z^2 + \omega^* v_A^2 k_z^2 = 0$$

$\omega^* = v_{ed}k_y$ is diamagnetic drift frequency.

$\rho_s = \frac{C_{si}}{\omega_{ci}}$ is Larmor Radius.

$C_{si} = \sqrt{\frac{k_B T_e}{m_i}}$ is ion acoustic speed.

$v_A = \frac{B_0}{\sqrt{\mu_0 n_0 m_i}}$ is Alfven wave.

$v_{ed} = \frac{k_B T_e}{e B_0}\mathrm{K}_n$ Diamagnetic drift

$\mathrm{K}_n = \frac{1}{n_0}\frac{\partial n_0}{\partial x}$ is inverse inhomogeneity length scale

Literature Review

- Pokhotelov in his paper derived two-fluid equation set captures the nonlinear dynamics of drift Alfvén waves in multicomponent dusty plasma, revealing the coupling between drift-Alfvén waves and drift convective cells in the linear limit[1].
- H. Saleem examines low-frequency electrostatic and electromagnetic linear modes in nonuniform cold quantum electron-ion plasma. Additionally, the impact of stationary dust on an electrostatic mode is explored. Quantum corrections in the linear dispersion relations for cold dense plasma are presented, accompanied by relevant equations [2].
- Onishchenko in a paper derived, The classical development of the theory of drift Alfvén waves, with a spatial scale comparable to the ion Larmor radius, has been undertaken. This includes investigating the dispersion relation and analyzing how plasma density perturbations vary with the wave frequency [3].

Literature Review

- A paper by Misra presents a concise analysis of Drift-Alfven modes in non-uniform, quantum dusty plasma, which consist of electron-ion and negatively charged dust grain. By solving linearized equations within the linear regime and using quantum hydrodynamics model and Fourier transformation they derived the dispersion relation[4].
- Haijun Ren, studied electromagnetic drift waves in a nonuniform quantum magnetized electron-positron-ion plasma. By using quantum hydrodynamics equations, and magnetic field of Wigner-Maxwell system, they derived a new dispersion relation which ion's motion are not considered[5].
- Qamar studied analytical description of drift Alfven modes in nonuniform bounded magnetized electron-positron-ion plasma. By considering Gaussian density profile linearized equations are solved[6].

Motivation

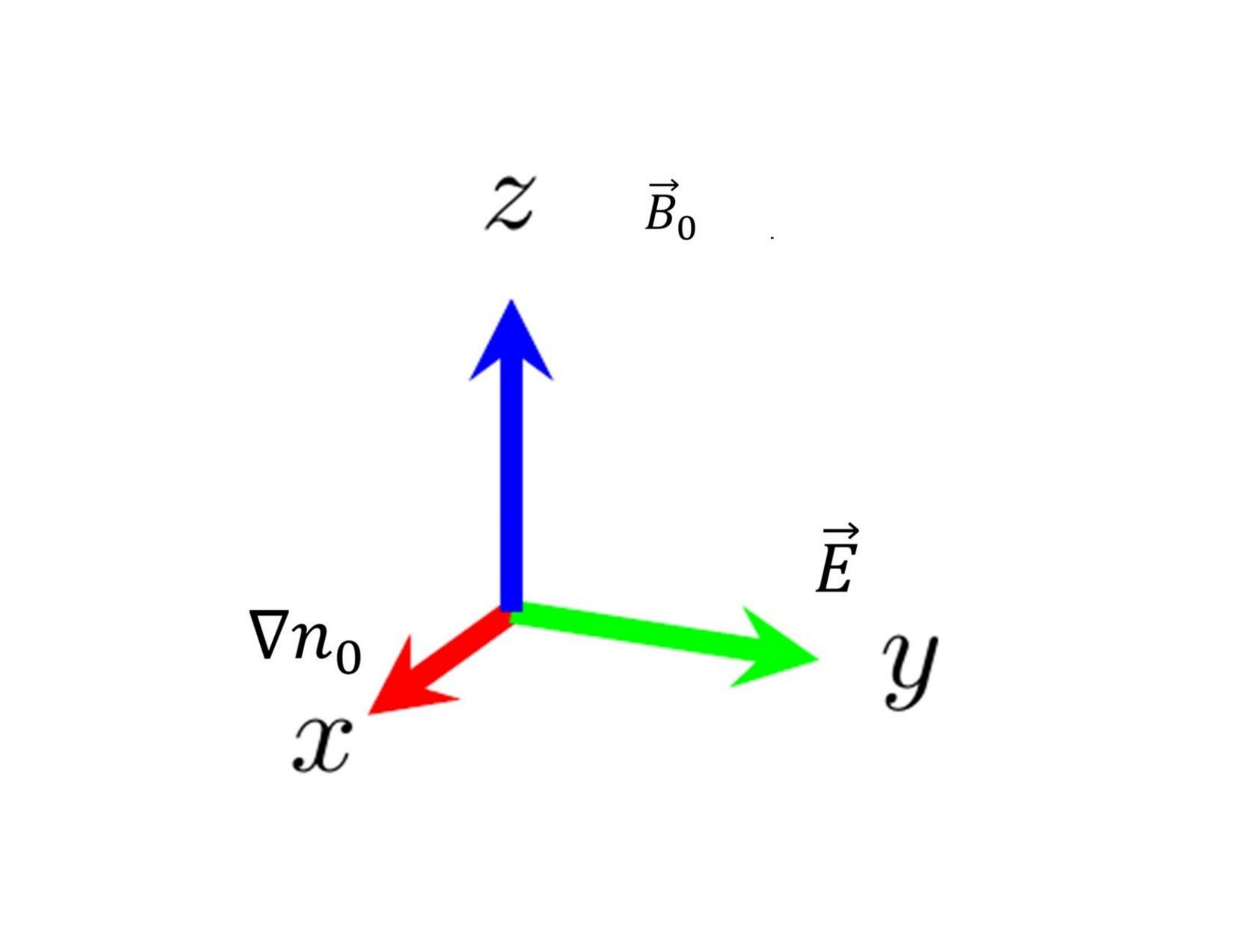
- Most research is conducted on multi-component plasmas (e.g., Electron-Positive Ion or Dusty plasma), whereas our study focuses on a two-species plasma (electron-ion).
- Most papers ignore the spin effect and only consider Fermi pressure and Bohm potential effects. However, we incorporated the spin term, providing a more comprehensive understanding of astrophysical objects by including all relevant corrections.
- The model manipulation and calculation methods are different. We can study it in three ways: Two-fluid theory, two-potential theory, and MHD theory. Most literature is based on two-potential theory, but we conducted our study using the two-fluid theory. Additionally, those who worked on the two-fluid theory employed different methods.

Abstract of the Work

- Our research introduces a more complete picture of spin quantum plasma, along with other previously unexplored corrections.
- In this academic study, we utilized some novel analytical techniques to identify new modes within spin quantum plasmas for the first time.
- These findings mark a significant advancement in the understanding of quantum plasma dynamics in dense astrophysical objects.

Discerption and Geometry of The Problem

- It is Electron Ion Plasma
- Quantum Mechanical regime
- Low Frequency perturbation is there ($\omega < \omega_{ci}$)
- (B_0) is in ($\hat{z}$) direction.
- We have two kinds of electric field ($E_{\perp}$) and ($E_{\parallel}$).
- ($E_{\parallel}$) is in ($\hat{z}$) direction and is in term of scaler and vector potential ($-\nabla_{\parallel}\varphi - \frac{\partial A_z}{\partial t}$).
- ($E_{\perp}$) is in ($\hat{y}$) direction and is just i term of scalar potential ($-\nabla_{\perp}\varphi$).



Our quantum mechanical study focuses on Fermi, Bohm, and Spin pressures with density variations, and considers electrons as inertia-less in low-frequency Drift Alfvén Waves (DAW).

For electron:

$$\vec{S} = -\frac{h^2\vec{M}}{2\mu_B n_e}$$

$$\vec{M} = \frac{n_e\mu_B^2}{E_F}\vec{B}$$

$$\in = \frac{B_0^2\mu_B^2}{E_F}$$

$$\vec{B}^2 = (\vec{B_0} + \vec{B_1})^2$$

$$-\vec{\nabla}\left[\frac{n_e\mu_B^2}{E_F}B_0^2\left(1+\frac{2\vec{B}_1}{\vec{B}_0}\right)^2\right]$$

$$mn_0\frac{\partial v}{\partial t} = en_0[E + v\times B] - \nabla P$$

$$0 = -en_0[E + v_e\times B] - \nabla P_{Fe} + \nabla P_B + \nabla P_s$$

$$0 = -e[E + v_e\times B] - \frac{\vec{\nabla}P_{Fe}}{n_e} + \frac{\hbar^2}{2m_e}\vec{\nabla}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) + \frac{2\mu_B n_e}{\hbar^2}\vec{\nabla}(\vec{S}\cdot\vec{B})$$

$$0 = -e[E + v_e\times B] - \frac{k_B T_{Fe}}{n_0}\vec{\nabla}n_0 + \frac{\hbar^2}{2m_e n_0}\vec{\nabla}(\Psi_0) - \vec{\nabla}\left(\frac{n_e\mu_B^2}{E_F}\vec{B}^2\right)$$

$$0 = e(\vec{\nabla}_\perp\Phi) - e\,(\vec{B}_0\times\vec{v}_{e\perp}) - \frac{k_B T_{Fe}}{n_0}\vec{\nabla}n_0 + \frac{\hbar^2}{2m_e n_0}\vec{\nabla}(\Psi_0) - \frac{\in}{n_0}\vec{\nabla}n_0$$

$$0 = e(\hat{z}\times\vec{\nabla}_\perp\Phi) - e\,\hat{z}\times(\vec{B}_0\times\vec{v}_{e\perp}) - \frac{k_B T_{Fe}}{n_0}(\hat{z}\times\vec{\nabla}n_0) + \frac{\hbar^2}{2m_e n_0}[\hat{z}\times\vec{\nabla}_\perp(\Psi_0)] - \frac{\in}{n_0}(\hat{z}\times\vec{\nabla}n_0)$$

$$\vec{v}_{e\perp} = \frac{1}{B_0}(\hat{z}\times\vec{\nabla}_\perp\Phi) - \frac{k_B T_{Fe}}{eB_0 n_0}(\hat{z}\times\vec{\nabla}n_0) + \frac{\hbar^2}{2m_e eB_0 n_0}(\hat{z}\times\vec{\nabla}_\perp\Psi_0) - \frac{1}{eB_0 n_0}\in_0(\hat{z}\times\vec{\nabla}n_0)$$

$$\vec{v}_{e\perp} = v_E\hat{x} + v_{eD}\hat{y} \qquad \dots\dots\dots \quad 1$$

Electric current density arise from electron movement, resulting in two types: free electron motion and bound electron motion. Thus, the total electric current density is the sum of these two types.

$$\vec{J}_t = \vec{J}_p + \vec{J}_B$$

$$\vec{J}_z = -en_0\vec{v}_{ez} + \left(\vec{\nabla} \times \vec{M}\right)_z$$

$$\vec{J}_z = -en_0\vec{v}_{ez} + \frac{3}{2}\frac{n_0\mu_B^2}{\epsilon_{Fe}}\left(\vec{\nabla} \times \vec{B}_1\right)_z$$

$$\vec{B}_1 = \vec{\nabla}A_z \times \hat{z}$$

$$\chi_e = \frac{3}{2}\frac{n_0\mu_B^2\mu_0}{\epsilon_F}$$

$$\vec{J}_z = -en_0\vec{v}_{ez} - \frac{3}{2}\frac{n_0\mu_B^2}{\epsilon_F}\nabla_\perp^2 A_z$$

$$\left(\vec{\nabla} \times \vec{\mathrm{B}}_1\right)_z = \mu_0\vec{J} = \mu_0\left(\vec{J}_z\right)$$

$$-\nabla_\perp^2 A_z = -\mu_0 en_0\vec{v}_{ez} - \frac{3}{2}\frac{n_0\mu_B^2\mu_0}{\epsilon_F}\nabla_\perp^2 A_z$$

$$\mu_0 en_0\vec{v}_{ez} = \nabla_\perp^2 A_z\left(1 - \frac{3}{2}\frac{n_0\mu_B^2\mu_0}{\epsilon_F}\right)$$

$$\vec{v}_{ez} = \frac{\nabla_\perp^2 A_z(1-\chi_e)}{\mu_0 en_0}$$

$$\vec{v}_e = \vec{v}_{e\perp} + \vec{v}_{ez} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

Electron Continuity Equation

We linearize the continuity equation for electrons, focusing on first-order perturbations and omitting second-order and background multiplications.

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} n_e \cdot \vec{v} + n_e \vec{\nabla} \cdot \vec{v} = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \vec{\nabla}_\perp n_{e0} \cdot \vec{v}_{e1\perp} + \vec{\nabla} n_{e1} \cdot \vec{v}_{e1\perp} + n_0\left(\vec{\nabla}_\perp \vec{v}_\perp + \vec{\nabla}_\parallel \vec{v}_\parallel\right) = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \vec{\nabla}_\perp n_{e0} \cdot \vec{v}_{e1\perp} + n_0 \frac{\partial}{\partial z} \vec{v}_{ez} = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \vec{\nabla}_\perp n_0 \cdot (-v_E \hat{x}) + n_0 \frac{\partial}{\partial z} \frac{\nabla_\perp^2 A_z (1 - \chi_e)}{\mu_0 e n_0} = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \frac{\partial n_0}{\partial x} \frac{1}{B_0}\left(\frac{\partial \varphi}{\partial y}\right) + \frac{(1-\chi_e)}{\mu_0 e} \frac{\partial}{\partial z} \nabla_\perp^2 A_z = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \frac{n_0}{n_0} \frac{k_B T_{Fe}}{e B_0} \frac{\partial n_0}{\partial x} \frac{\partial \varphi}{\partial y} \frac{e}{k_B T_{Fe}} + \frac{k_B T_{Fe}}{e^2 \mu_0} (1 - \chi_e) \frac{\partial}{\partial z} \nabla_\perp^2 \left(\frac{A_z e}{k_B T_{Fe}}\right) = 0$$

$$\frac{\partial n_{e1}}{\partial t} + n_0 \vec{v}_e^* \frac{\partial}{\partial y} \Phi_e + \frac{k_B T_e}{e^2 \mu_0} (1 - \chi_e) \frac{\partial}{\partial z} \nabla_\perp^2 (A_e) = 0$$

$$\frac{\partial n_{e1}}{\partial t} + n_0 \vec{v}_e^* \frac{\partial \Phi_e}{\partial y} + \frac{k_B T_{Fe}}{m_i} \frac{m_i^2}{e^2 B_0^2} \frac{B_0^2}{\mu_0 n_0 m_i} n_0 (1 - \chi_e) \frac{\partial}{\partial z} \nabla_\perp^2 (A_e) = 0$$

$$\frac{\partial n_{e1}}{\partial t} + n_0 \vec{v}_e^* \frac{\partial \Phi_e}{\partial y} + \frac{c_{si}^2}{\omega_{ci}^2} \vec{v}_{AM}^2 n_0 \frac{\partial}{\partial z} \nabla_\perp^2 A_e = 0$$

$$n_0^{-1} \frac{\partial n_{e1}}{\partial t} + \vec{v}_e^* \frac{\partial \Phi_e}{\partial y} + \rho_s^2 \vec{v}_{AM}^2 \frac{\partial}{\partial z} \nabla_\perp^2 A_e = 0 \quad \ldots\ldots\ldots\ldots \quad 3$$

$$\vec{v}_e^* = -\frac{k_s T_{Fe}}{e n_0 B_0}\left(\frac{\partial n_0}{\partial x}\right)$$

$$\vec{v}_{AM}^2 = \frac{B_0^2}{\mu_0 n_0 m_i}(1 - \chi_e)$$

$$v_E = \frac{1}{B_0}(-\nabla \varphi)$$

$$c_{si}^2 = \frac{k_B T_{Fe}}{m_i}$$

$$\omega_{ci}^2 = \frac{m_i^2}{e^2 B_0^2}$$

Electron Parallel Motion

In the parallel direction, we focus solely on Fermi pressure and exclude other pressures like Bohm potential and spin terms for simplicity. Linearized form of parallel motion is:

$$0 = e\left(\frac{\partial \varphi_1}{\partial z} + \frac{\partial \vec{A}_{z1}}{\partial t}\right) - e\left(\vec{v} \times \vec{B}\right)_{\parallel 1} - \frac{k_B T_{Fe}}{n_0}\vec{\nabla}_{\parallel} n_{e1}$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = \left(\vec{v}_{e0} \times \vec{B}_1 + \vec{v}_{e1} \times \vec{B}_1\right)_{\parallel}$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = \left(\vec{v}_{e\perp} \times B_1\right)_{\parallel}$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = \left(\vec{v}_E \times \vec{B}_1\right)_{\parallel} + \left(\vec{v}_{eD} \times \vec{B}_1\right)_{\parallel}$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = \left(\vec{v}_{eD} \times \vec{B}_1\right)_z$$

$$\vec{v}_{eD} = -\frac{k_B T_{Fe}}{e B_0 n_0}\left(\hat{z} \times \vec{\nabla}_{\perp} n_0\right) + \frac{h^2}{2 m_e e B_0}\left(\hat{z} \times \vec{\nabla}_{\perp} \Psi_0\right) - \frac{\epsilon_0}{e B_0 n_0}\left(\hat{z} \times \vec{\nabla} n_0\right)$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = -\frac{k_B T_{Fe}}{e B_0 n_0}\frac{\partial n_0}{\partial x}\left[(\hat{z} \times \hat{x}) \times \vec{B}_1\right]_z + \frac{h^2}{2 m_e e B_0}\frac{\partial \Psi_0}{\partial x}\left[(\hat{z} \times \hat{x}) \times \vec{B}_1\right]_z - \frac{\epsilon_z}{e B_0 n_0}\frac{\partial n_0}{\partial x}\left[(\hat{z} \times \hat{x}) \times \vec{B}_1\right]_z$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = \vec{v}_e^*\left[\hat{y} \times \left(\vec{B}_1\right)\right]_z + \vec{v}_{eB}^*\left[\hat{y} \times \left(\vec{B}_1\right)\right] - \vec{v}_{es}^*\left[\hat{y} \times \left(\vec{B}_1\right)\right]$$

$$\vec{B}_1 = \vec{\nabla} A_z \times \hat{z}$$

$$\vec{v}_e^* + \vec{v}_{eB}^* - \vec{v}_{es}^* = \vec{v}_{eDM}$$

$$\left(\vec{v} \times \vec{B}\right)_{\parallel 1} = -\left(\vec{v}_e^* + \vec{v}_{eB}^* - \vec{v}_{es}^*\right)\vec{\nabla}_{\perp} \mathrm{A_z}$$

$$0 = e\left(\frac{\partial \varphi_1}{\partial z} + \frac{\partial \vec{A}_{z1}}{\partial t}\right) - e\left(\vec{v}_{eDM}\vec{\nabla}_{\perp}\mathrm{A_z}\right) - k_B T_{Fe}\frac{\partial}{\partial z}\left(\frac{n_{e1}}{n_0}\right)$$

$$\frac{\partial \Phi_e}{\partial z} + \frac{\partial A_e}{\partial t} + \vec{v}_{eDM}\frac{\partial A_e}{\partial y} - \frac{\partial}{\partial z}\left(\frac{n_{e1}}{n_0}\right) = 0 \qquad \ldots\ldots\ldots\ldots \quad 4$$

Ions Continuity equation

In deriving the ion continuity equation, we include only the (E×B) drift in first term, and focus on the polarization drift in the third term.

$$\frac{\partial n_{i1}}{\partial t} + \vec{\nabla}_{\perp} n_0 \cdot \vec{v}_{i\perp} + n_0 \left(\vec{\nabla} \cdot \vec{v}_{\perp}\right) = 0$$

$$\vec{v}_{\mathrm{i}\perp} = \vec{v}_{iE} + \vec{v}_{iP}$$

$$\frac{\partial n_{i1}}{\partial t} + \vec{\nabla}_{\perp} n_0 \cdot \vec{v}_{iE} + n_0 \left(\vec{\nabla} \cdot \vec{v}_{\mathrm{iP}}\right) = 0$$

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial n_0}{\partial x} \cdot \frac{1}{B_0} \vec{E} + n_0 \left[\frac{\partial}{\partial \mathrm{y}} \left(- \frac{m_i}{e B_0^2} \frac{\partial^2 \varphi}{\partial t \partial y}\right)\right] = 0$$

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial n_0}{\partial x} \cdot \frac{1}{B_0} \left(-\vec{\nabla} \varphi\right) - n_0 \frac{m_i}{e B_0^2} \left[\frac{\partial}{\partial \mathrm{y}} \left(\frac{\partial^2 \varphi}{\partial t \partial y}\right)\right] = 0$$

$$\frac{\partial n_{i1}}{\partial t} - \frac{\partial n_0}{\partial x} \cdot \frac{1}{B_0} \frac{\partial \varphi}{\partial y} - n_0 \frac{m_i^2}{e^2 B_0^2} \frac{e}{m_i} \left[\frac{\partial}{\partial \mathrm{y}} \left(\frac{\partial^2 \varphi}{\partial t \partial y}\right)\right] = 0$$

$$\frac{\partial n_{i1}}{\partial t} - n_0 \frac{\partial n_0}{\partial x} \cdot \frac{1}{n_0 B_0} \frac{k_B T_e}{e} \frac{\partial}{\partial y} \left(\frac{e \varphi}{k_B T_e}\right) - n_0 \frac{m_i^2}{e^2 B_0^2} \frac{k_B T_e}{m_i} \left[\frac{\partial}{\partial \mathrm{t}} \left(\frac{\partial^2}{\partial y^2} \left(\frac{e \varphi}{k_B T_e}\right)\right)\right] = 0$$

$$\vec{v}_e^* = - \frac{\partial n_0}{\partial x} \cdot \frac{1}{n_0 B_0} \frac{k_B T_e}{e}$$

$$v_{th}^2 = \frac{k_B T_e}{m_i}$$

$$\frac{1}{\omega_{ci}^2} = \frac{m_i^2}{e^2 B_0^2}$$

$$\rho_{si}^2 = \frac{v_{th}^2}{\omega_{ci}^2}$$

$$\frac{\partial n_{i1}}{\partial t} + n_0 \vec{v}_e^* \frac{\partial \phi_e}{\partial y} - n_0 \frac{v_{th}^2}{\omega_{ci}^2} \left[\frac{\partial}{\partial \mathrm{t}} \left(\frac{\partial^2 \phi_e}{\partial y^2}\right)\right] = 0$$

$$n_0^{-1} \frac{\partial n_{i1}}{\partial t} + \vec{v}_e^* \frac{\partial \phi_e}{\partial y} - \rho_{si}^2 \frac{\partial}{\partial \mathrm{t}} \nabla_{\perp}^2 \phi_e = 0 \qquad \ldots\ldots\ldots \qquad 5$$

DAW Dispersion Relation

To find the Drift Alfvén Wave (DAW) dispersion relation, we subtract ions continuity equation from electron continuity equation.

$$n_0^{-1}\frac{\partial n_e}{\partial t}+\vec{v}_e^*\frac{\partial \Phi_e}{\partial y}+\rho_{si}^2\vec{v}_{AM}^2\frac{\partial}{\partial z}\nabla_\perp^2 A_e=0$$

$$n_0^{-1}\frac{\partial n_{i1}}{\partial t}+\vec{v}_e^*\frac{\partial \phi_e}{\partial y}-\rho_{si}^2\frac{\partial}{\partial \mathrm{t}}\nabla_\perp^2\phi_e=0$$

$$\frac{\partial}{\partial t}\left(\frac{n_{e1}-n_{i1}}{n_0}\right)+0+\rho_{si}^2\vec{v}_{AM}^2\frac{\partial}{\partial z}\vec{\nabla}_\perp^2 A_e+\rho_{si}^2\frac{\partial}{\partial t}\vec{\nabla}_\perp^2\Phi_e=0$$

$$\vec{v}_{AM}^2\frac{\partial}{\partial z}A_e+\frac{\partial}{\partial t}\Phi_e=0$$

$$A_e=\frac{\omega}{k_z\vec{v}_{AM}^2}\Phi_e \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad 6$$

$$n_e\cong n_i$$

$$\frac{\partial}{\partial z}\to ik_z \ \ and\ \ \frac{\partial}{\partial t}\to -i\omega$$

DAW Dispersion Relation…

By putting this value or A_e in electron parallel motion equation we can get:

$$A_e = \frac{\omega}{k_z \vec{v}_{AM}^2} \Phi_e$$

$$\frac{\partial \Phi_e}{\partial z} + \frac{\partial A_e}{\partial t} + \vec{v}_{eDM} \frac{\partial A_e}{\partial y} - \frac{\partial}{\partial z}\left(\frac{n_{e1}}{n_0}\right) = 0$$

$$\frac{\partial \Phi_e}{\partial z} + \frac{\omega}{k_z \vec{v}_{AM}^2} \frac{\partial}{\partial t} \Phi_e + \frac{\vec{v}_{eD}\omega}{k_z \vec{v}_{AM}^2} \frac{\partial}{\partial y} \Phi_e - \frac{\partial}{\partial z}\left(\frac{n_{e1}}{n_0}\right) = 0$$

$$(ik_z)\Phi_e + \frac{\omega}{k_z \vec{v}_{AM}^2}(-i\omega)\Phi_e + \frac{\vec{v}_{eD}\omega}{k_z \vec{v}_{AM}^2}(ik_y)\Phi_e - (ik_z)\left(\frac{n_{e1}}{n_0}\right) = 0$$

$$k_z^2 \vec{v}_{AM}^2 \Phi_e - \omega^2 \Phi_e + \vec{v}_{eD}\omega k_y \Phi_e - k_z^2 \vec{v}_{AM}^2 \left(\frac{n_{e1}}{n_0}\right) = 0$$

$$\frac{n_{e1}}{n_0} = \left(\frac{\vec{v}_{AM}^2 k_z^2 - \omega^2 + \omega_M^* \omega}{k_z^2 \vec{v}_{AM}^2}\right) \Phi_e \qquad \dots\dots\dots\dots\dots\dots \qquad 7$$

DAW Dispersion Relation…

We use the value of A_e and n_{e1}/n_0 in electron continuity equation.

$$n_0^{-1}\frac{\partial n_e}{\partial t}+\vec{v}_e^*\frac{\partial \Phi_e}{\partial y}+\rho_s^2\vec{v}_{AM}^2\frac{\partial}{\partial z}\nabla_\perp^2 A_e=0$$

$$\left(\frac{\vec{v}_{AM}^2k_z^2-\omega^2-\omega_M^*\omega}{k_z^2\vec{v}_{AM}^2}\right)\frac{\partial \Phi_e}{\partial t}+\vec{v}_e^*\frac{\partial \Phi_e}{\partial y}+\rho_s^2\vec{v}_{AM}^2\frac{\partial}{\partial z}\vec{\nabla}_\perp^2\left(\frac{\omega}{k_z\vec{v}_{AM}^2}\right)\Phi_e=0$$

$$\left(\frac{\vec{v}_{AM}^2k_z^2-\omega^2-\omega_M^*\omega}{k_z^2\vec{v}_{AM}^2}\right)(-i\omega)\Phi_e+(\vec{v}_e^*)(ik_y)\Phi_e+\rho_s^2\vec{v}_{AM}^2(ik_z)(-k_y^2)\left(\frac{\omega}{k_z\vec{v}_{AM}^2}\right)\Phi_e=0$$

$$-\omega\left(\frac{\vec{v}_{AM}^2k_z^2-\omega^2-\omega_M^*\omega}{k_z^2\vec{v}_{AM}^2}\right)+\vec{v}_e^*k_y-\rho_s^2\vec{v}_{AM}^2k_zk_y^2\left(\frac{\omega}{k_z\vec{v}_{AM}^2}\right)=0$$

$$-\omega(\vec{v}_{AM}^2k_z^2-\omega^2-\omega_M^*\omega)+\vec{v}_e^*k_yk_z^2\vec{v}_{AM}^2-\rho_s^2k_z^2\vec{v}_{AM}^2k_y^2\omega=0$$

$$-\omega\vec{v}_{AM}^2k_z^2+\omega^3+\omega^2\omega_M^*+\omega^*k_z^2\vec{v}_{AM}^2-\rho_s^2k_z^2\vec{v}_{AM}^2k_y^2\omega=0$$

$$\omega^3+\omega^2\omega_M^*-\omega\vec{v}_{AM}^2k_z^2-\rho_s^2k_z^2\vec{v}_{AM}^2k_y^2\omega+\omega^*k_z^2\vec{v}_{AM}^2=0$$

$$\omega^* = \vec{v}_e^*k_y$$

$$v_{AM}=B_0\sqrt{\frac{1-\chi_e}{\mu_0 m_i n_0}}$$

$$\rho_{si}=\frac{C_{si}}{\omega_{ci}}$$

$$\omega^3+\omega^2\omega_M^*-\omega\left(1+\rho_s^2k_y^2\right)k_z^2\vec{v}_{AM}^2+\omega^*k_z^2\vec{v}_{AM}^2=0 \qquad \ldots\ldots\ldots\ldots \qquad 8$$

Where:

$$\omega_M^* = k_y v_{eD}$$

$$v_{eD} = v_e^* + v_{eB}^* + v_{es}^*$$

$$\omega^* = k_y v_e^*$$

$$v_e^* = -\left(\frac{K_n C_{si}^2}{\omega_{ci}}\right)$$

$$v_{eB}^* = \frac{C_{sB}^2 K_n}{\omega_{ci}}$$

$$\rho_{si} = \frac{C_{si}}{\omega_{ci}}$$

$$v_{eS}^* = \frac{C_{sS}^2 K_n}{\omega_{ci}}$$

$$v_{AM} = B_0 \sqrt{\frac{1 - \chi_e}{\mu_0 m_i n_0}}$$

$$C_{Si} = \sqrt{\frac{\epsilon_F}{m_i}}$$

$$C_{Ss} = \sqrt{\frac{\epsilon_z}{m_i}}$$

$$\mu_0 = \frac{1}{c^2 \varepsilon_0}$$

$$\chi_e = \frac{3}{2}\left(\frac{n_0 \mu_B^2 \mu_0}{\epsilon_F}\right)$$

$$k_y = 2000\ cm^{-1}$$

$$K_n = 100\ cm^{-1}$$

$$\omega^3 + \omega^2\omega_M^* - \omega\left(1 + \rho_s^2 k_y^2\right)k_z^2 \vec{v}_{AM}^2 + \omega^* k_z^2 \vec{v}_{AM}^2 = 0$$

$$\omega^3 - \omega^2 a - \omega b + d = 0 \qquad \ldots\ldots\ldots\ldots \qquad 9$$

Where:

$$a = \omega_M^2$$

$$b = (1 + k_y^2 \rho_{si}^2) k_z^2 v_{AM}^2$$

$$d = \omega^* v_{AM}^2 k_z^2$$

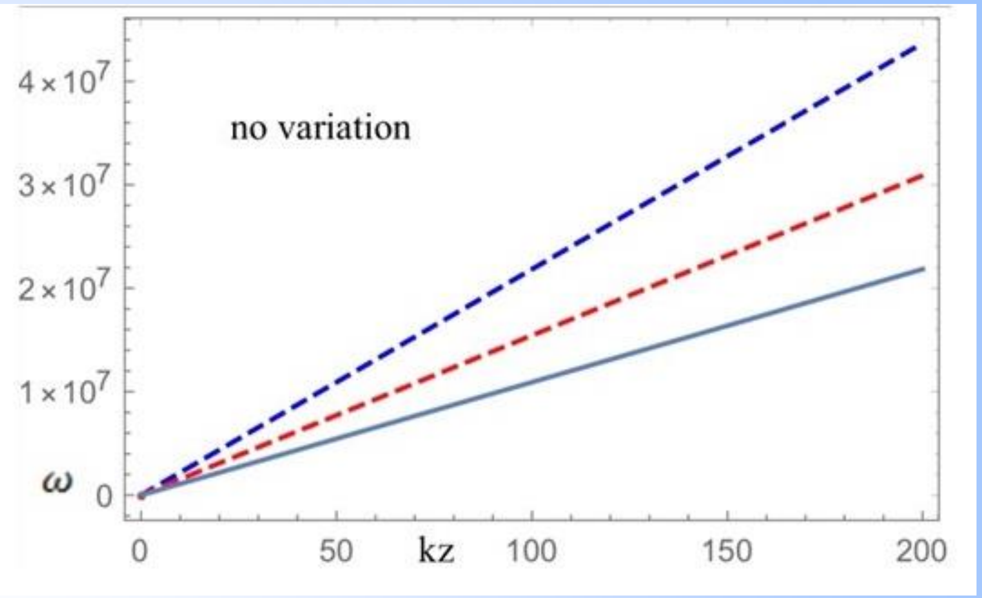
We're using Mathematica to solve this cubic equation and find its roots.

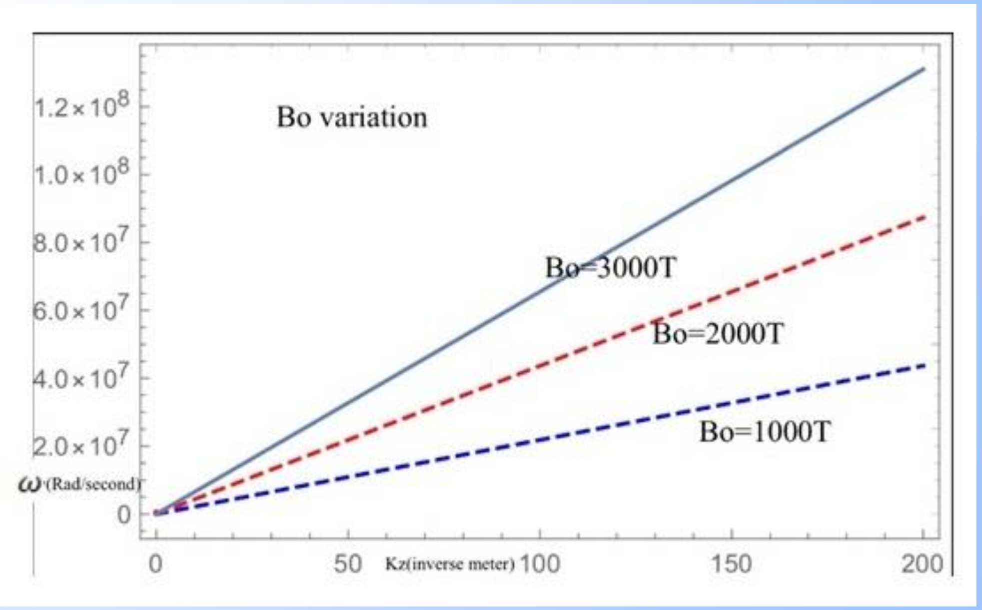
Graphical Results

$$\left\{\omega \to \frac{a}{3} - \frac{2^{1/3}(-a^2-3b)}{3\left(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2}\right)^{1/3}} + \frac{\left(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2}\right)^{1/3}}{3\,2^{1/3}}\right\},$$

$$\left\{\omega \to \frac{a}{3} + \frac{(1+i\sqrt{3})(-a^2-3b)}{3\,2^{2/3}\left(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2}\right)^{1/3}} - \frac{(1-i\sqrt{3})\left(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2}\right)^{1/3}}{6\,2^{1/3}}\right\},$$

$$\left\{\omega \to \frac{a}{3} + \frac{(1-i\sqrt{3})(-a^2-3b)}{3\,2^{2/3}(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2})^{1/3}} - \frac{(1+i\sqrt{3})(2a^3+9ab-27d+3\sqrt{3}\sqrt{-a^2b^2-4b^3-4a^3d-18abd+27d^2})^{1/3}}{6\,2^{1/3}}\right\}\}$$





References

1. O. A. Pokhotelov and O. G. Onishchenko, P. K. Shukla and L. Stenflo, Drift Alfven vortices in dusty plasmas, vol, 104, No. A9, Pages 19,797-19,800.
2. H. Saleem, Ali Ahmad, and S. A. Khan, Low frequency electrostatic and electromagnetic modes in nonuniform cold plasma, 15, 014503 (2008) doi: 10. 1063/1.2833596.
3. O. G. Onishchenko, O. A. Pokhotelov, V. V. Krasnoselskikh. And S. I. Shatolv, Drift Alfven waves in space plasma- theory and mode identification, Ann. Geophys, 27, 639-644, 2009.
4. A. P. Misra and C. Bhowmik, Coupled drift-Alfven-Shukla-Varma modes in quantum dusty plasmas, physics of plasma 16, 012103(2009), doi: 10.1063/1.3067827.
5. Haijun Ren, Zhengwei Wu, Jintao Cao and Paul K Chu, Electromagnetic drift waves in a nonuniform quantum magnetized (EPI) plasma, stacks.iop.org/JPhysA/41/115501.
6. Q Huque, S Yamin and Ali Ahmad, Drift-Alfven eigenmodes in inhomogeneous electron-positron-ion plasmas, phys. Scr. 83 (2011) 035501 (5pp).

Thank you
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