Gravitational Waves and Memory Effect

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Outline

- Gravitational Waves
- Asymptotically Flat Spacetime
- Memory Effect

Gravitational Waves

- First predicted by Einstein in 1916 as a result of General Relativity.
- GW as perturbations in flat spacetime.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad | \, h_{\mu\nu} \, | \ll 1$$

- The equations governing the dynamics of $h_{\mu\nu}$ are obtained by Einstein's Vacuum equations Outside of the source, where $T_{\mu\nu} = 0$.

$$R_{\mu\nu} = 0,$$

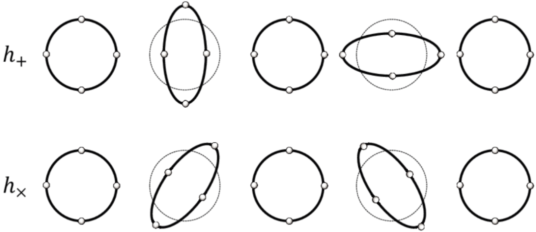
- By using appropriate gauge condition the Einstein's equation reduces to:

$$\Box \bar{h}_{\mu\nu} = 0, \quad \Rightarrow \quad \bar{h}_{\mu\nu} = Re\{A_{\mu\nu} e^{ik_\alpha x^\alpha}\}$$

- Where $\Box$ is D'Alembertian in flat spacetime ($\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2$)
- It shows that the perturbations of flat spacetime satisfy the wave equation.

GW polarization

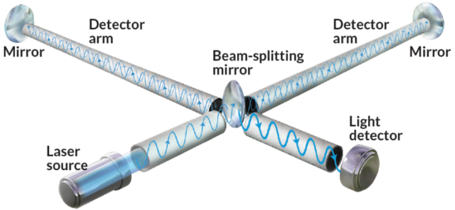
- GW has two degree of freedom proportional to two polarization states which are $\times$ and $+$.
- Particles displacement due to passage of GW with $\times$ and $+$ polarization.



[Krolak,Patil]

Detection of GW

- Detection of GW happened at 2015 by LIGO interferometers.
- LIGO work similar to Michelson interferometer, instead of light, beam laser beam is used and the arms are $\sim 4km$.



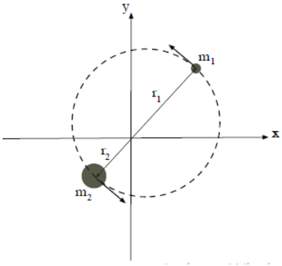
[LIGO.org]

- If the interferometer lie in the y and z direction and GW propagates in the x direction, with polarization $+$.
- Interference pattern is effected by the time delay in the light propagation produced by the GW.
- The time needed to cross the arms is effected by GW, when the rays join in the detector, there is a time delay:

$$\Delta t = t_{(y)} - t_{(z)} = \frac{2l_0}{c} h_+ \ .$$

Gravitational wave emitted by a binary system

- We consider a binary system composed of two stars moving at circular orbit.



[Ferrari,Pani]

- The GW emitted along direction n is equal:

$$h_{ij}^{TT} = -\frac{4\mu M G^2}{r l_0 c^4} \mathcal{P}_{ijkl} A_{kl} \; .$$

- For binary system PSR 1913+16, which consists of two neutron stars we have:

$$m_1 \sim m_2 = 1.4 M_\odot, \qquad l_0 = 1.9 \times 10^{11} cm,$$
$$P = 7h\, 45m\, 7s = 2.8 \times 10^4 s, \qquad \nu_K = \frac{\omega_K}{2\pi} = 3.58 \times 10^{-5} Hz \; .$$

- The GW amplitude is equal to:

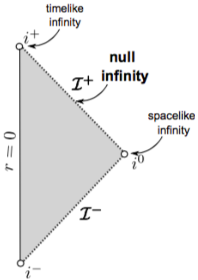
$$h_0 = \frac{4\pi M G^2}{r l_0 c^4} \sim 5 \times 10^{-23}$$

- The GW luminosity is equal to:

$$L_{GW} = \frac{dE_{GW}}{dt} = \sum_{k,n=1}^{3} \dddot{Q}_{kn} \dddot{Q}_{kn} = \frac{32}{5} \frac{G^4}{c^5} \frac{\mu^2 M^3}{l_0^5} = 5.2 \times 10^{30} erg/sec$$

Asymptotically flat spacetime

- 1962 Bondi, Metzner, Van Derberg, Sachs



[Physics.stackexchange]

- In Bondi gauge, the most general four dimensional metric can be written as:

$$ds^2 = -\frac{V}{r}e^{2\beta}du^2 - 2e^{2\beta}dudr + r^2\gamma_{AB}(dx^A - U^A du)(dx^B - U^B du).$$

- $V, \beta, \gamma_{AB}, U^A$ are functions of (u, θ, ϕ).

Boundary data

- Boundary data at $\mathcal{I}^+$: m_B, C_{AB}, N_A.
- m_B is masss aspect by which we can define Bondi mass:

$$M(u) = \frac{1}{4\pi} \oint_{s^2} m_B(u, \theta, \phi) sin\theta d\theta d\phi$$

- If we write Schwarzschild metric in Bondi coordinate Bondi mass is equal to black hole mass.
- C_{AB} is called shear tensor.
- News tensor ($N_{AB} = \partial_u C_{AB}$) contains information of gravitational waves.
- Bondi mass loss formula:

$$\frac{d}{du} M(u) = -\frac{1}{4\pi} \oint_{s^2} |N_{AB}|^2 sin\theta d\theta d\phi \ .$$

- The minus sign indicates that the radiating source loss mass in the form of gravitational wave energy.
- $N_A(u, x^A)$ is called angular momentum aspect, it's integral over S^2 gives angular momentum of source at $\mathcal{I}^+$.
- BMS group as asymptotic symmetry group of GR, which includes Poincare group, Supertranslations and Superrotations respectively.

Memory effect

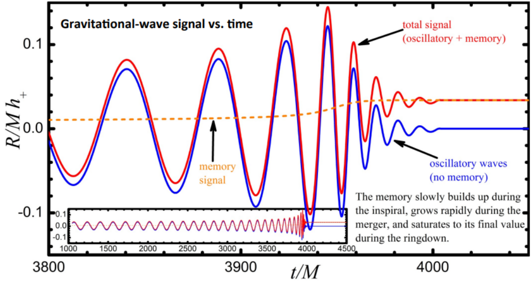
- **Memory effect** is defined as permanent change in physical characteristic (position, velocity, rotation angle...) of detector as (gravitational, electromagnetic...) waves passes.
- **Electromagnetic memory**: manifested as a kick, a residual velocity imparted on a charged particle by the electromagnetic field. For **weak field and slow motion** we have [Bieri, Garfinkle-2013]:

$$\Delta\vec{v} = \vec{v}(\infty) - \vec{v}(-\infty) = \frac{q}{m}\int_{-\infty}^{\infty}\vec{E}dt = \frac{q}{mr}P\left[\frac{d}{dt}\vec{d}_{EM}(t=\infty) - \frac{d}{dt}\vec{d}_{EM}(t=-\infty)\right]$$

- This equation give us information about the changes of dipole moment of source.

Gravitational memory effect

- Known since 1974 by the work of Polnerav and Zeldovic.
- We think of GW signals as having oscillatory amplitude that starts small at early times, build to some maximum and decay back to zero.
- BUT in reality all GW source posses some form of GW memory

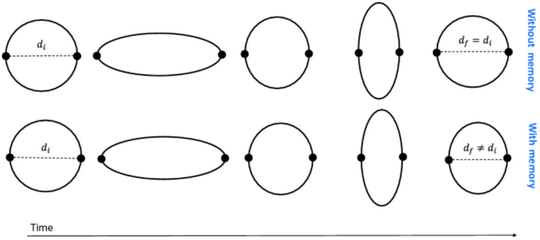


[M.Favata]

- **Displacement memory effect**: The passage of gravitational radiation past a pair of inertial detectors stationed near $\mathcal{I}^+$ cause permanent displacement which remembers certain moments of the energy flux.
- In weak-field and slow-motion case:

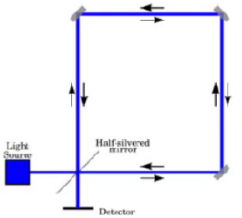
$$\Delta S^j = \frac{S}{r} P \left[\sum_k m_k v_k^i v_k^j (t = \infty) - \sum_k m_k v_k^i v_k^j (t = -\infty) \right],$$

- The change in quadrupole moment of the source cause the displacement.



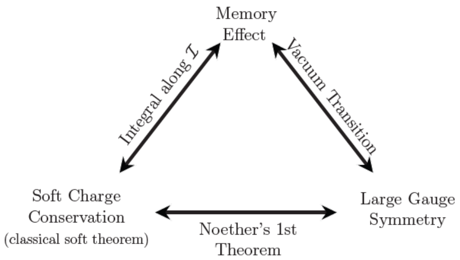
[Lambiase,Papini]

- **Spin memory effect**: this effect is sourced by Bondi angular momentum aspect [Pasterski-2016]:

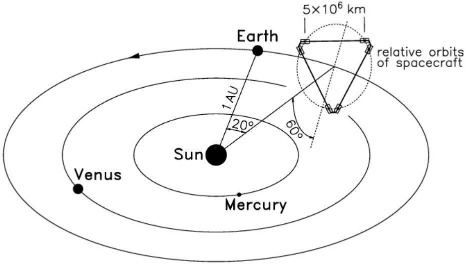


- **Gyroscope memory effect**: finite burst of gravitational radiation leaves permanent imprint (a memory) of its passage on the gyroscope's orientation [Seraj, Oblak-2020]:

- Memory, Symmetry and Charge conservation are related by Infrared triangle [Strominger-2018]



- Possible detection: Analysis of $\sim$ 100 event at advanced LIGO, or 2037 LISA [Lasky, Thrane-2016].



Conclusions and Outlooks

- Studying gravitational wave and memory effect help us in better understanding of GR and give us information about the source.
- GW memory is a new approach to study modified gravity, in modified theories of gravity, the memory can differ from the GR predication. in Brans-Dicke theory we have a contribution to memory form scalar term [Hou-2021]:
- HSP proposal of black hole soft hair and its potential as a solution to the black hole information paradox [Hawking, Strominger, Perry-2016].
- BMS supertranslations symmetries imply an infinite number of conservation charges (soft hair), each charge label different states of black hole.
- Memory effect has been studied in classical Yang-Mills theory, it shows itself as color change of quarks (color memory effect) [Pate, Strominger-2018]
- Thank You!