Investigating the Alternating Structures of TI-NI and TI-S to Realize Different Symmetries of the Superconducting Order Parameter in Weyl Semimetals

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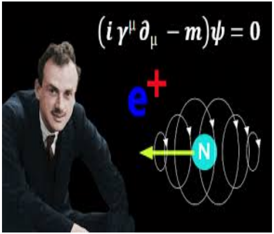
September 10, 2024

Outline

Introduction

- Dirac equations→ Dirac fermions
- Weyl equation→ Weyl fermions
- Weyl semimetals→ Weyl superconductors
- Method: analytical



Dirac and Weyl Equations

Dirac equation

1. $(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0.$
2. $\frac{\partial\psi}{\partial_t} + \alpha^r \frac{\partial\psi}{\partial x^r} + \beta m\psi = 0.$
   - γ^{μ} and α^r are 4 component matrices
   - saticfies the Clifford algebra.
   - $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$
   - predicts $\pm$E$\rightarrow$ ***s***

Weyl eqation

1. $i\sigma^{\mu}\partial_{\mu}\psi_R = 0.$
2. $i\bar{\sigma}^{\mu}\partial_{\mu}\psi_L = 0.$
3. $\frac{\partial\psi}{\partial_t} + \sigma^r \frac{\partial\psi}{\partial x^r} = 0.$
   - $\sigma^{\mu} = 1, \sigma_x, \sigma_y, \sigma_z$
   - $\bar{\sigma}^{\mu} = 1, -\sigma_x, -\sigma_y, -\sigma_z$
   - satisfies the Clifford algebra
   - $\{\sigma^{\mu}, \sigma^{\nu}\} = 2\eta^{\mu\nu}$

Chirality and Helicity

Handedness

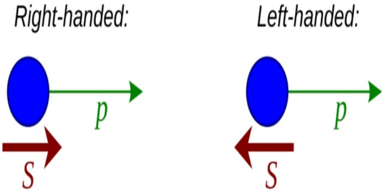
Chirality

- mathematical concept
- breaks M R symmetry.
- eigenvalue of $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
- $[\gamma_5, H_W] = 0$



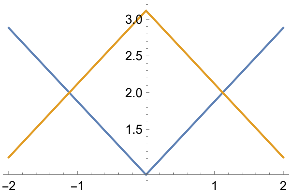
Helicity

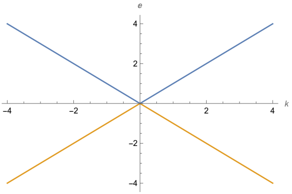
- projection of spin in momentum direction
- $h = \boldsymbol{s} \cdot \hat{p}$
- $[h, H_D] = 0$
- eigenvalue$=\pm 1$



Weyl Points and Weyl Semimetals

- Energy expansion: $E = \hbar v_F |\vec{k} - \vec{k}_0|$.
- Effective Hamiltonian: $\mathrm{H}{=}\varepsilon_0\sigma_0 \pm \hbar v_F(\vec{k} - \vec{k}_0)$.
- Weyl Hamiltonian ($\varepsilon_0{=}0$): $H = \pm\hbar v_F(\vec{k} - \vec{k}_0)$.





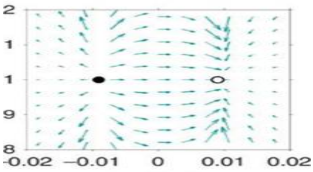
Important Properties of Weyl Semimetals

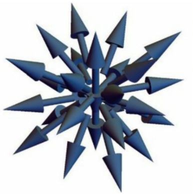
Berry curvature and Berry phase

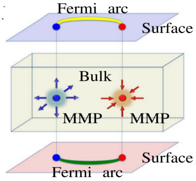
- Berry connection: $\vec{A}(k) = -i\sum_{n,occ}\langle u_{n,k}|\vec{\nabla}_k|u_{n,k}\rangle$.
- Berry curvature: $B\vec{(k)} = \vec{\nabla}_k \times A(\vec{k})$.
- Berry phase: $\Omega(\vec{k}) = \int d\vec{k}\langle u_{n,k}|\vec{\nabla}_k|u_{n,k}\rangle = \pm 2\pi$

$$H = \pm\hbar v_F(\vec{k} - \vec{k_0})$$

- Fermi arcs and chiral anomaly



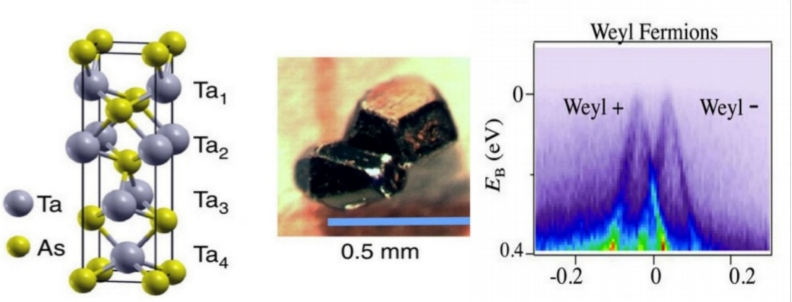




Realization of Weyl Semimetals

Experimental achievements

- *TaAs*: Tantalum arsenide
- $A_2Ir_2O_7$: Pyro-chloore iridate
- Na_3Bi: Sodium bismuthide



TI-NI Hetero Structure

$$H = \sum_{\boldsymbol{k}_\perp, ij} [v_F \tau^z (\hat{z} \times \boldsymbol{\sigma}) \cdot \boldsymbol{k}_\perp \delta_{i,j} + m\sigma^z \delta_{i,j} + \Delta_S \tau^x \delta_{i,j} + \frac{1}{2}\Delta_D \tau^+ \delta_{i,j+1} + \frac{1}{2}\Delta_D \tau^- \delta_{i,j-1}] c^\dagger_{\boldsymbol{k}_\perp i} c_{\boldsymbol{k}_\perp j}.$$

$m = 0$:

$$E_\pm^2 = v_F^2 (k_x^2 + k_y^2) + \Delta^2(k_z).$$

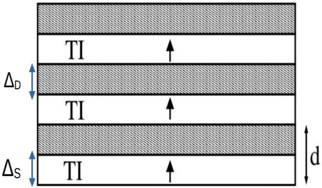
$$\Delta^2(k_z) = \Delta_S^2 + \Delta_D^2 + 2\Delta_S \Delta_D \cos(k_z d).$$

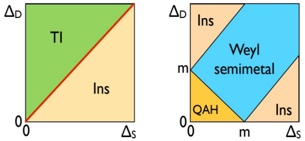
$m \neq 0$:

$$E_\pm^2 = v_F^2 (k_x^2 + k_y^2) + [m \pm \Delta(k_z)]^2.$$

$$(\Delta_S - \Delta_D)^2 < m^2 < (\Delta_S + \Delta_D)^2.$$

- $\Delta_D < \Delta_S \rightarrow$ NI
- $\Delta_D > \Delta_S \rightarrow$ TI
- $\Delta_S = \pm\Delta_D \rightarrow$ Dirac node





(a) m=0 (b) m≠0

TI-S Hetero-Structure

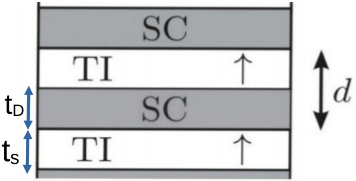
Scalar order parameter

$$H = \sum_{\vec{k}_\perp, ij} c^\dagger_{\vec{k}_\perp i} H_{ij} c_{k_\perp j} + H_{SC}.$$

$$H_{ij} = v_F \tau^z (\hat{z} \times \vec{\sigma}) \cdot \vec{k}_\perp \delta_{i,j} + m\sigma^z \delta_{i,j} + t_S \tau^x \delta_{i,j}.$$

$$+ \frac{1}{2} t_D \tau^+ \delta_{i,j+1} + \frac{1}{2} \tau^- \delta_{i,j-1}.$$

- $\sigma^\pm = \tau^z \sigma^\pm$
- $\tau^\pm = \sigma^z \tau^\pm$



$$H = \sum_{\vec{k} l = \pm} c^\dagger_{\vec{k}, l} H_l c_{\vec{k} l} + \sum_{l = \pm} H_{SC, l}.$$

$$H_\pm = v_F (\hat{z} \times \vec{\sigma}) \cdot \vec{k} + M_\pm(k_z) \sigma^z.$$

$$M_\pm(k_z) = m \pm \sqrt{t_S^2 + t_D^2 + 2 t_S t_D \cos(k_z d)}.$$

$$(t_S - t_D)^2 < m^2 < (t_S + t_D)^2.$$

$$\vec{k} = (0, 0, \frac{\pi}{2})$$

$$\vec{k} = (0, 0, \frac{\pi}{d} \pm k_0)$$

$$\vec{k} = (0, 0, \frac{\pi}{d} \pm k^\Delta_\pm)^T$$

Different Phases of Superconductivity

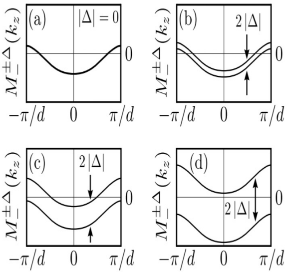
$$H_- = \frac{1}{2}\sum_{\vec{k}} \psi^\dagger_{\vec{k}}[(v_F(\hat{z}\times\vec{\sigma})\cdot\vec{k} + M_-(k_z)\sigma^z)\mathbf{I}_{\vec{k}} + \frac{1}{2}\sigma^z(\Delta\kappa^+ + \Delta^*\kappa^-)]\psi_{\vec{k}}.$$

$$H_- = \frac{1}{2}\sum_{\vec{k},n=\pm} \phi^\dagger_{\vec{k},n} H^{n\Delta}_- \phi_{\vec{k},n}.$$

$$H^{\pm\Delta}_- = v_F(\hat{z}\times\vec{\sigma})\cdot\vec{k} + M^{\pm\Delta}_-(k_z)\sigma^z.$$

$$M^{\pm\Delta}_-(k_z) = (m\pm|\Delta|) - \sqrt{t_S^2 + t_D^2 + 2t_S t_D\cos(k_z d)}.$$

- $m > 0$
- $t_S > t_D$



$$m > |\Delta| : (t_S - t_D) < m \pm |\Delta| < (t_S + t_D).$$

$$m < |\Delta| : (t_S - t_D) < |\Delta| \pm m < (t_S + t_D).$$

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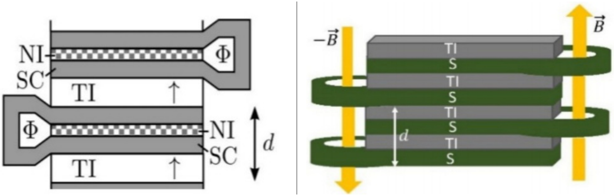
Different Symmetries of Superconductivity

- applying π phase difference $\rightarrow$ pseudo-scalar
- applying ϕ and $-\phi$ $\rightarrow$ two components order parameter

  (pseudo scalar and scalar)



Acknowledgements

Dr.Zahra Faraei: I would like to express my deepest gratitude to my supervisor, Dr. Zahra Faraei, for her invaluable guidance and support.

IASBS: I would also thanks from IASBS and all my teachers for their mentorship throughout my MSc studies.

ICTP: Additionally, I am grateful to ICTP for their unwavering support and the opportunity to present my work here.

Dr. Kate Shaw I have spacial thanks to Dr. Kate Shaw for her kind attention and dedication in this challenging situation.

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Thank you for your attention.