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Workshop on Differential Geometry | (SMR 4060)

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P01 - CHAVEDAR DE SOUZA ARAUJO Murilo

A Serrin's type Problem in Weighted Manifolds: Soap Bubble Results and Rigidity in Generalized Cones

P02 - BASTOS Henrique Nogueira

Uniqueness of capillary discs

P03 - BUSTAMANTE BIANCHI Ignacio

Deriving Perelman's Entropy from Colding's Monotonic Volume

P04 - DA SILVA Danilo Ferreira

New Maximum Principles for Laplacian Drift with Applications to Uniqueness of Hypersurface in Weighted Lorentzian Product Spaces

P05 - SOUSA FRANCA Alan

Submanifolds with the Joachimsthal property

P06 - PAES GALINDO NETO Giovane

Bundle type sub-Riemannian structures on holonomy bundle

P07 - FERREIRA GOMES Wallace

NONEXISTENCE OF MEAN CURVATURE FLOW SOLITONS WITH POLYNOMIAL VOLUME GROWTH IMMERSED IN CERTAIN SEMI-RIEMANNIAN WARPED PRODUCTS

P08 - NAVARRO GUAJARDO Diego Alonso

Isometric deformations of hypersurfaces in high codimensions

P09 - LEITE Ary Vinícius

Umbilicity and the First Stability Eigenvalue of a Subclass of CMC Hypersurfaces Immersed in Certain Einstein Manifolds

P10 - LIMA Marcos César De Vasconcelos

On free boundary minimal annuli in geodesic balls of S^3_+ and H^3.

P11 - MARTINEZ LEIRANES Marcos

The Asymptotic Plateau Problem for The Hyperbolic Space

P12 - RISTOW MONTES Rodrigo

The contact angle for Immersed Surfaces in the Special Linear Group

P13 - DO NASCIMENTO Guilherme Francisco

Rigidity conditions through the infinite-Laplacian operator: a study of the Lorentzian distance function

P14 - PINILLOS PRADO Sergio Andrés

Deformable hypersurfaces in \mathbb{R}^{k}\times\mathbb{S}^{n-k+1}

P15 - JOAQUIM SANTOS Elisa

Rigidity results for complete conformally flat A2-manifolds

P16 - SOARES Matheus Nunes

A Liouville-type theorem for the p-Laplacian on complete non-compact Riemannian manifolds

P17 - SOUZA Vinicius Guardiano

Workshop on differential geometry

P18 - TORO CARDONA Carlos Andrés

Non-existence of free boundary minimal MÃ¶bius bands in the unit three-ball

P19 - CHAVES BARBOZA Weiller Felipe

Stochastically complete, Parabolic and $L1$-Liouville spacelike submanifolds with parallel mean curvature vector

P01

A Serrin's type Problem in Weighted Manifolds: Soap Bubble Results and Rigidity in Generalized Cones

Murilo Araújo *
Departamento de Matemática
Universidade Federal da Paraíba

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Abstract

An important and famous type of overdetermined problem is presented in [2] when Serrin proved the following celebrated result:

If there exists a positive solution $u \in C^2(\overline{\Omega})$ to the overdetermined problem:

$$\begin{cases} \Delta u = -1 & in \quad \Omega \\ u = 0 & on \quad \partial\Omega \\ u_\nu = -c & on \quad \partial\Omega \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with boundary $\partial\Omega$ of class C^2, ν denotes the unit normal to $\partial\Omega$ and u_ν denote the normal derivative of u, then Ω must be a ball and u is radially symmetric.

An "overdetermined problem" refers to a partial differential equation (PDE) in which "too many" boundary conditions are assigned, such as both the Dirichlet and Neumann conditions, as indicated in (1). This means that the system of conditions is "overdetermined" relative to the degrees of freedom in the PDE.

In this short talk we investigate a weighted overdetermined problem within the framework of Riemannian manifolds with density. Initially, by examining a Poisson problem associated with the drift Laplacian, we derive a Heintze-Karcher inequality and a soap bubble theorem that characterize geodesic balls in these spaces. Subsequently, by imposing both Dirichlet and Neumann boundary conditions, we establish a Serrin-type result in generalized cones, identifying metric balls as the unique solutions to this underlying overdetermined problem.

This short talk is based on joint works with A. Freitas and M. Santos ([1]).

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*e-mail: murilo.chavedar@academico.ufpb.br

P02

Uniqueness of capillary discs

Henrique Nogueira Bastos[1]

[1] *Instituto de Matemática Pura e Aplicada - IMPA*

One of the most influential classification problems in differential geometry is Hopf's theorem, which states that every CMC sphere in R^3 is necessarily a round sphere. In 2016, Gálvez and Mira [1] presented a generalized version of this theorem. Compared to the case of spheres in Euclidean space, their result applies to any 3-manifold that admits a transitive family of surfaces described by elliptic ODEs.

Similarly, in this poster, I discuss the classification problem of capillary disks in the Euclidean ball, Nitsche's theorem [2], and draw parallels with the theorem presented by Gálvez and Mira. I introduce the concept of transitive families with capillary boundary conditions and propose a generalized version of this theorem.

This is a work in preparation.

[1] José A. Gálvez and Pablo Mira. "Uniqueness of immersed spheres in three-manifolds". In: Journal of Differential Geometry 116.3 (2020), pp. 459–480.

[2] Johannes C. C. Nitsche. "Stationary partitioning of convex bodies". In: Archive for Rational Mechanics and Analysis 89 (1985), pp. 1–19.

P03

Deriving Perelman's Entropy from Colding's Monotonic Volume

Ignacio Bustamante[1] **and Martin Reiris**[1]

[1]*(Presenting author underlined) Universidad de la República*

In his groundbreaking work from 2002, Perelman introduced two fundamental monotonic quantities: the reduced volume and the entropy. While the reduced volume was motivated by the Bishop-Gromov volume comparison applied to a suitably constructed N-space, which becomes Ricci-flat as $N \to \infty$, Perelman did not provide a corresponding explanation for the origin of the entropy. In this article, we demonstrate that Perelman's entropy emerges as the limit of Colding's monotonic volume for harmonic functions on Ricci-flat manifolds, when appropriately applied to Perelman's N-space.

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P04

New Maximum Principles for Laplacian Drift with Applications to Uniqueness of Hypersurface in Weighted Lorentzian Product Spaces

Danilo Ferreira da Silva

[1] ***Universidade Regional do Cariri-URCA***

In this work, we establish some maximum principles related to the drift Laplacian in weighted Riemannian manifolds and, under suitable constraints on the Bakry-Émery-Ricci tensor, we apply them to study the uniqueness of complete spacelike hypersurfaces immersed with constant f-mean curvature in a weighted Lorentzian product space. Calabi-Bernstein type results concerning spacelike entire graphs defined on the Riemannian base of the ambient space are also given.

The results presented are contained in the paper [3] of author Henrique F. L, Eraldo A. L. JR and presenting author, in which some results are extention to the more general scenario of weighted manifolds and to hypersurfaces with constant f-curvature, in particular, for f-maximal in scenario pondered Lorentzian. The references [2] and [1] provided the main theoretical tools to obtain the results

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P05

Submanifolds with the Joachimsthal property

França, A.[1], **Tojeiro, R.**[2].

[1,2] *Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Av. Trabalhador São Carlense, 400, São Carlos, 13560-970, Brazil*

An umbilic-free surface $f\colon M^2 \to \mathbb{R}^3$ is said to be a *Joachimsthal surface* if the curvature lines corresponding to one of the principal curvatures are contained in planes that intersect along a common line. Accordingly, we say that an isometric immersion $f\colon M^n \to \mathbb{R}^N$ has the *Joachimsthal property* if there exists a gradient vector field $Z \in \mathfrak{X}(M)$ that is a principal direction for every normal direction $\xi \in \Gamma(N_fM)$, such that the image by f of the Z-curvature lines (that is, the integral curves of Z) are contained in two-dimensional planes that intersect along a straight line. We present a description of all immersions $f\colon M^n \to \mathbb{R}^{2n-1}$ with the Joachimsthal property and show that the images by f of the leaves of $\{Z\}^\perp$ are contained in hyperspheres centered on that axis. We also obtain a result that generalizes the classical fact that every Enneper surface in $\mathbb{R}^3$ with nonzero constant Gaussian curvature is a Joachimsthal surface.

P06

Bundle Type Sub-Riemannian Structures on Holonomy Bundles

Eder Correa[1], **Giovane Galindo**[1] and Lino Grama[1]

[1]*(Presenting author underlined) Universidade estadual de Campinas (Unicamp)*

In this presentation we summarize the work done on [1], combining the Rashevsky-Chow-Sussmann (orbit) theorem with the Ambrose-Singer theorem, we introduce the notion of controllable principal connections on principal G-bundles. Using this concept, under a mild assumption of compactness, we estimate the Gromov-Hausdorff distance between principal G-bundles and certain reductive homogeneous G-spaces. In addition, we prove that every reduction of the structure group G to a closed connected subgroup gives rise to a sequence of Riemannian metrics on the total space for which the underlying sequence of metric spaces converges, in the Gromov-Housdorff topology, to a normal reductive homogeneous G-space

More precisely we obtain the following results:

Theorem 0.1. *If $\theta \in \Omega^1(P, \mathfrak{g})$ is a principal connection, such that $\mathrm{Hol}_u(\theta) \subset G$ is a closed subgroup, then*

$$d_{GH}\big((P, d_{g_t}), (G/\mathrm{Hol}_u(\theta), d_{g_T})\big) \leq \frac{\mathrm{diam}\big(P_u(\theta), \mathrm{dist}_{\mathcal{H}}\big) f(t)}{2}, \tag{1}$$

$\forall t \in [0, T)$, where $P_u(\theta)$ is the holonomy bundle through $u \in P$ and $\mathrm{dist}_{\mathcal{H}}$ is the Carnot-Carathéodory metric induced by the bundle type sub-Riemannnian structure $(\ker(\theta), g_P)$ restricted to $P_u(\theta)$.

From above theorem, we immediately have the following corollaries.

Corollary 0.1. *Under the hypotheses of the last theorem, if $\lim_{t\to T} f(t) = 0$, then*

$$\lim_{t\to T} d_{GH}\big((P, d_{g_t}), (G/\mathrm{Hol}_u(\theta), d_{g_T})\big) = 0. \tag{2}$$

Theorem 0.2. *Let $\pi\colon P \to M$ be a principal G-bundle, such that M and G are both compact and connected, and $\dim(M) \geq 2$. Consider the subset $\mathcal{M}(P) \subset (\mathcal{M}, d_{GH})$ defined by*

$$\mathcal{M}(P) := \big\{(P, d_g) \mid d_g \text{ is the distance induced by } g \in \mathrm{Sym}^2_+(T^*P)\big\}. \tag{3}$$

If $\pi\colon P \to M$ is reducible to a closed connected subgroup $H \subset G$, then $(G/H, d_{g_T}) \in \overline{\mathcal{M}(P)}^{GH}$.

[1] Correa, Eder M., Giovane Galindo, and Lino Grama. ”Bundle type sub-Riemannian structures on holonomy bundles.” arXiv preprint arXiv:2407.01427 (2024).

P07

NONEXISTENCE OF MEAN CURVATURE FLOW SOLITONS WITH POLYNOMIAL VOLUME GROWTH IMMERSED IN CERTAIN SEMI-RIEMANNIAN WARPED PRODUCTS

P08

Abstract

Diego Guajardo[1]

[1] *USP-ICMC*

Sbrana and Cartan locally classified the Euclidean hypersurfaces $M^n \subseteq \mathbb{R}^{n+1}$ which admit another isometric immersions in $\mathbb{R}^{n+1}$ for $n \geq 3$. In this work we extend their classification to higher codimensions. Our main result is a complete description of the moduli space of genuine deformations of hypersurfaces of rank $(p+1)$ in $\mathbb{R}^{n+p}$ and $\mathbb{R}^{n+p+1}$ for $p \leq n-2$ and generic nullity.

P09

Umbilicity and the First Stability Eigenvalue of a Subclass of CMC Hypersurfaces Immersed in Certain Einstein Manifolds

Ary V.F. Leite[1], Henrique F. de Lima[1], and Marco A.L. Velásquez [1]

[1] *Universidade Federal de Campina Grande*

We study the umbilicity of constant mean curvature (CMC) complete hypersurfaces immersed in an Einstein manifold satisfying appropriate curvature constraints. In this setting, we obtain new characterization results for totally umbilical hypersurfaces via suitable maximum principles which deal with the notions of convergence to zero at infinity and polynomial volume growth. Afterwards, we establish optimal estimates for the first eigenvalue of the stability operator of CMC compact hypersurfaces in such an Einstein manifold. In particular, we derive a nonexistence result concerning strongly stable CMC hypersurfaces.

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P10

On free boundary minimal annuli in geodesic balls of $\mathbb{S}^3_+$ and $\mathbb{H}^3$

César Lima[1,2]

[1]*Universidade Federal do Rio Grande do Sul*
[2]*Instituto Federal do Ceará*

We consider Σ an embedded free boundary minimal annulus in a geodesic ball in the round hemisphere $\mathbb{S}^3_+$ or in the hyperbolic space $\mathbb{H}^3$. Under the hypothesis of invariance due to an antipodal map on the geodesic ball and using the fact that this surface satisfies the Steklov problem with frequency, we prove that Σ is congruent to a critical rotational annulus.

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P11

The Asymptotic Plateau Problem for The Hyperbolic Space

Marcos Martínez, Sébastien Alvarez

Universidad de la República, Uruguay

For a κ between 0 and 1, and a Jordan curve C on the asymptotic boundary of the three-dimensional hyperbolic space, there exist only two surfaces with constant Gauss curvature equal to κ that have C as their boundary at infinity. This theorem was previously proven by Rosenberg and Spruck in 1994. For my master's thesis, my advisor and I are attempting to provide a different approach to the same theorem.

The purpose of this poster is to highlight the main tools we are using to prove the theorem. The central idea is to consider one of the connected components of the sphere joined with C, and to construct a convex set that is the asymptotic volume minimizer of a family of convex sets with Gauss curvature greater than or equal to κ in the weak sense.

The Contact Angle of Surfaces in the Special Linear Group

P12

Rodrigo Ristow Montes

Universidade Federal do Paraná

ristow@ufpr.br

Abstract

In this paper we establish the equation for the Gaussian curvature of a constant mean curvature surface in the special linear group $Sl(2,\mathbb{R})$. Using the Gauss equation we prove that constant mean curvature surfaces in $Sl(2,\mathbb{R})$ with constant contact angle have constant Gaussian curvature. Also, we provide a congruence theorem for constant mean curvature surfaces of $Sl(2,\mathbb{R})$. Finally , we give an example of a minimal surface in $Sl(2,\mathbb{R})$ with non constant contact angle.

Introduction

Surfaces making constant angles with certain directions are interesting and they are intensively studied by several authors in different ambient spaces. An interesting characterization of constant angle surfaces in the special linear group $Sl(2,\mathbb{R})$ was showed by Montaldo, Onnis and Passamani, see [5]. The study in Bianchi-Cartan-Vranceanu spaces was completed in [5] and in [6]. Also, in [2] and [3], Dillen and others have studied constant angle surfaces in product spaces $S^2 \times \mathbb{R}$ and $H^2 \times \mathbb{R}$, namely those surfaces for which the unit normal makes a constant angle with the tangent direction to $\mathbb{R}$. Recently, Munteanu, Fastenakels and van der Veken, in [10], extended the notion of constant angle surfaces in $S^2 \times \mathbb{R}$ and $H^2 \times \mathbb{R}$ to general Bianchi–Cartan–Vranceanu spaces and they showed that these surfaces have constant Gaussian curvature, also they gave a complete local classification in the Heisenberg group.

Results

In [7] we introduced the notion of contact angle, which can be considered as a new geometric invariant useful for investigating the geometry of immersed surfaces in S^3. Also in [7], we derived formulae for the Gaussian curvature and the Laplacian of an immersed minimal surface in S^3, and we gave a characterization of the Clifford torus as the only minimal surface in S^3 with constant contact angle. In [8], we construct a family of minimal tori in S^5 with constant contact angle and constant holomorphic angle. These tori are parametrized by the following circle equation

$$a^2 + \left(b - \frac{\cos\beta}{1+\sin^2\beta}\right)^2 = 2\frac{\sin^4\beta}{(1+\sin^2\beta)^2}. \tag{1}$$

I assume that β is the contact angle.
In particular, when $a = 0$, we recover the examples found by Kenmotsu. These examples are defined for $0 < \beta < \frac{\pi}{2}$. Also, when $b = 0$, we find a new family of minimal tori in S^5, and these tori are defined for $\frac{\pi}{4} < \beta < \frac{\pi}{2}$. For $\beta = \frac{\pi}{2}$, we give an alternative proof of this classification of a Theorem from Blair, and Yamaguchi, Kon and Miyahara. For Legendrian minimal surfaces in S^5 with constant Gaussian curvature. Also in [9] we provide a congruence theorem for minimal surfaces in S^5 with constant contact angle using Gauss-Codazzi-Ricci equations. More precisely, we prove that Gauss-Codazzi-Ricci equations for minimal surfaces in S^5 with constant contact angle satisfy an equation for the Laplacian of the holomorphic angle.

Definition 1. *We denote $d\beta(e_1) = \beta_1$, $d\beta(e_2) = \beta_2$, $dH(e_1) = H_1$ and $dH(e_2) = H_2$.*

In this notes, we show that the Gaussian curvature K of a constant mean curvature surface of $Sl(2,\mathbb{R})$. with contact angle β is given by:

$$K = 1 - |\nabla\beta + (\cosh^2(\beta) + \sinh^2(\beta))e_1|^2 - 2H\beta_2. \tag{2}$$

Using the equation (2), we have proved the following theorem.

Theorem 1. *The Gaussian curvature for constant mean curvature surfaces in $Sl(2,\mathbb{R})$ with constant contact angle is constant.*

Main Result

More in general, we have the following congruence result.

Theorem 2. *Consider S a Riemannian surface, e a vector field on S, and $\beta : S \to]0, \frac{\pi}{2}[$ a function over S that satisfies the following equation*

$$\Delta(\beta) = -\tanh(\beta)|\nabla\beta + 2(\cosh(\beta)^2 + \sinh(\beta)^2)e|^2 + 2H_2$$

then there exist one, up to isometries of S into $Sl(2,\mathbb{R})$, immersion of mean curvature H , such that, e is the characteristic vector fied, and β is the contact angle of this immersion.

A particular case for $0 < \beta < \frac{\pi}{2}$ and supposing that we have a compact surface with constant mean curvature H produce the following result.

Corollary 1. *In particular, when H is constant and for $0 < \beta < \frac{\pi}{2}$, we have that compact surfaces in the special linear group $Sl(2,\mathbb{R})$ have constant contact angle β.*

Remark 1. *For non-compact surfaces, there is an example that is the hyperbolic space H^2, for this case the contact angle is $\beta = \operatorname{arccosh}(x_2)$.*

Remark 2. *For minimal surfaces in product spaces $S^2 \times \mathbb{R}$ and $H^2 \times \mathbb{R}$, the author proves the following theorem bellow (see [1]).*

Theorem 3. *(**Benoit Daniel**)*
Let Σ be a minimal surface in $M^2(c) \times \mathbb{R}$. Then its angle function $\nu :\to [-1,1]$ satisfies
(M1) $\|\nabla\nu\|^2 = -(1-\nu^2)(K - c\nu^2)$,
(M2) $\Delta\nu - 2K\nu + c(1+\nu^2)\nu = 0$,
where K denotes the intrinsic curvature of Σ.
Conversely, let Σ be a real analytic simply connected Riemannian surface and $\nu :\to [-1,1]$ a smooth function satisfying (M1) and (M2) where K is the curvature of Σ. Then there exists an isometric minimal immersion $f : \Sigma \to M^2(c) \times \mathbb{R}$ whose angle function is ν. Moreover, if $g : \Sigma \to M^2(c) \times \mathbb{R}$ is another isometric minimal immersion whose angle function is ν, then f and g are associate.

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P13

Rigidity conditions through the infinite-Laplacian operator: a study of the Lorentzian distance function

G.F. do Nascimento[1], **and E.A. Lima jr**[1]

[1]*Universidade Federal da Paraíba*

In this research we study the behavior of the Lorentzian distance function through the infinity-Laplacian operator focusing on totally trapped submanifolds. Here, we impose certain conditions on the submanifolds and use one of the most classical tools of geometric analysis, the Hessian comparison theorem. The rigidity results obtained here involve the determination of submanifolds within *level sets*. Hence, we are dedicated to study the complete totally trapped submanifolds in spacetimes using the Hessian comparison in [4] and a maximum principle for the infinity Laplacian in [2]. We also studied the behavior of such submanifolds, accordingly the asymptotic conditions found on maximum principles found in [1]. This procedure has physical importance since it is a type of compactification of the trapped submanifold and it is similar to the asymptotic condition assumed on the type of positive mass theorems; see [3].

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P14

Deformable hypersurfaces in \mathbb{R}^{k}\times\mathbb{S}^{n-k+1}

P15

Rigidity results for complete conformally flat $\mathcal{A}_2$ -manifolds

Elisa Joaquim Santos[1], **Fábio Reis dos Santos**[2]

[1]*(PhD student) Departamento de Matemática, Universidade Federal de Pernambuco*
[2] *Departamento de Matemática, Universidade Federal de Pernambuco*

In this poster presentation, we consider complete Riemannian manifolds with nonnegative scalar curvature and constant, nonnegative second Schouten curvature, which we refer to as $\mathcal{A}_2$-manifolds. For these manifolds, we obtain an estimate for the Cheng-Yau operator acting on the norm of the traceless Ricci tensor. As an application and under certain conditions, we prove that the manifold has a universal cover isometric to either $\mathbb{R}^n$ or $\mathbb{S}^n$. Moreover, if an estimate involving the norm of the traceless Ricci tensor and the second Schouten curvature occurs, the manifold either has a universal cover isometric to $\mathbb{R} \times \mathbb{S}^{n-1}$, or is covered isometrically by $\mathbb{S}^1 \times \mathbb{S}^{n-1}$ equipped with the product metric. Separately, we characterize the cases of dimensions three and four.

Keywords: $\mathcal{A}_2$-manifolds; Cheng-Yau operator, Conformally flat manifolds; Einstein manifolds.

[1] L.J. Alías, S.C. García-Martínez, and M. Rigoli, *A maximum principle for hypersurfaces with constant scalar curvature ad applications*, Ann. Glob. Anal. Geom. 41 (2012), 307–320.

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P16

A Liouville-type theorem for the p-Laplacian on complete non-compact Riemannian manifolds

Matheus N. Soares[1]

[1] *Universidade Federal de Pernambuco*

In this work, we present a Liouville-type result via the first eigenvalue of the $p-$Laplacian operator. The main tool is using a divergent-type linearlized version of this operator introduced by Soares and dos Santos [1]. As an application, we proof some results concerning complete non-compact hypersufaces immersed in a suitable warped product manifold. The results summarized here are available as a preprint in [2].

[1] F.R. dos Santos and M.N. Soares, Lower bounds for the length of the second fundamental form via the first eigenvalue of the p-Laplacian, *Nonlinear Analysis.* **232** (2023), 113251.

[2] F.R. dos Santos and M.N. Soares, A Liouville-type theorem for the p-Laplacian on complete non-compact Riemannian manifolds. arXiv preprint arXiv:2501.06952 (2025).

P17

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[1] A. Author, B. Coauthor, J. Sci. Res. **13**, 1357 (2012).
[2] A. Author, B. Coauthor, J. Sci. Res. **17**, 7531 (2013).

P18

Non-existence of free boundary minimal Möbius bands in the unit three-ball

Carlos Andrés Toro Cardona[1]

[1]*Instituto de Matemática Pura e Aplicada*

We prove the impossibility of constructing free boundary minimal Möbius bands in the Euclidean ball $\mathbb{B}^3$. This answers in the negative a question proposed by I. Fernández, L. Hauswirth and P. Mira.

[1] C. Toro, *Non-existence of free boundary minimal Möbius bands in the unit three-ball*. Preprint https://arxiv.org/abs/2404.11101 (2024).

P19

Stochastically complete, parabolic and L^1-Liouville spacelike submanifolds with parallel mean curvature vector (Poster)

Barboza, W.F.C[1], **H.F. de Lima**[2] **and M.A.L. Velásquez**[2]

[1]*(Barboza, W.F.C.) Universidade Federal de Campina Grande, Paraíba, Brazil.*
[2] *Universidade Federal de Campina Grande,Paraíba, Brazil.*

We deal with n-dimensional spacelike submanifolds immersed with parallel mean curvature vector h in a pseudo-Riemannian space form $L_q^{n+p}(c)$ of index $1 \leq q \leq p$ and constant sectional curvature $c \in \{-1, 0, 1\}$. Considering the cases when h is either spacelike or timelike, we are able to prove that such a spacelike submanifold is either totally umbilical or it holds a lower estimate for the supremum of the norm of its traceless second fundamental form, occurring equality if the spacelike submanifold is pseudo-umbilical and its principal curvatures are constant. In our approach, we use three main core concepts: Stochastic completeness, parabolicity and L^1-Liouville property.

[1] Barboza, W.F.C., de Lima, H.F. and Velásquez, M.A.L., *Stochastically Complete, Parabolic and L1-Liouville Spacelike Submanifolds with Parallel Mean Curvature Vector*, Potencial Analysis, **60**, 27-43 (2024).https://doi.org/10.1007/s11118-022-10043-8.