Data Analysis and Filter Optimization for Online Pulse-Amplitude Measurement

A Case Study on High-Resolution X-ray Spectroscopy

Andres Cicuttin

ICTP - Multidisciplinary Laboratory

Trieste, Italy

ICTP The Abdus Salam International Centre for Theoretical Physics

UNESCO
United Nations Educational, Scientific and Cultural Organization

IAEA
International Atomic Energy Agency

Trieste

Main Areas of Expertise

- Read-out electronics and high performance **Digital Signal Processing**.
- Advanced **FPGA Design** and **Programmable Systems-on-Chip**.
- Reconfigurable virtual instrumentation for **Particle Detectors**.
- Novel architectures for **Supercomputing Based on FPGA**.
- Instruments and methods for **X-Ray Imaging and Analytical Techniques**.

Outline

- Introduction
- Pulsed signals: Description levels
- Processing chain: Detector/Sensor, Preamplification, Pulse shaping, Data acquisition, transmission, . . .
- Digital Pulse Processor (DPP): Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .
- DPP Optimization
  - Data analysis
  - Pulse modeling
  - Digital Penalized Least Mean Squares (DPLMS) method for filtering optimization
- Discussion and Conclusions

More technical details in

sensors MDPI

Article

Data Analysis and Filter Optimization for Pulse-Amplitude Measurement: A Case Study on High-Resolution X-ray Spectroscopy

Kasun Sameera Mannatunga [1,2,3], Bruno Valinoti [1,4,5,*], Werner Florian Samayoa [1,5], Maria Liz Crespo [1,*], Andres Cicuttin [1], Jerome Folla Kamdem [1,6], Luis Guillermo Garcia [1,5] and Sergio Carrato [5]

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Digital Penalized LMS method for filter synthesis with arbitrary constraints and noise

E. Gatti*, A. Geraci, S. Riboldi, G. Ripamonti

Politecnico di Milano, Dipartimento di Elettronica e Informazione, Piazza Leonardo da Vinci 32, Milano 20133, Italy

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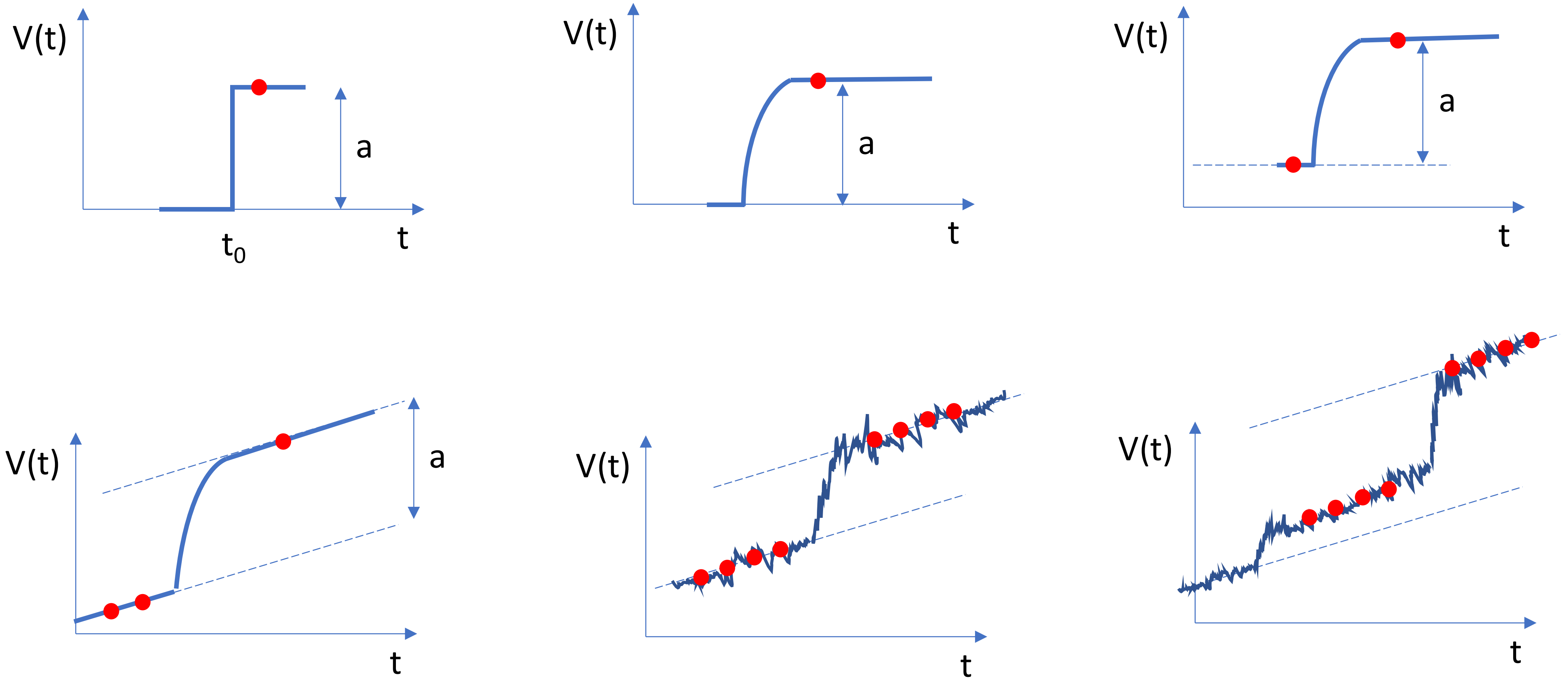
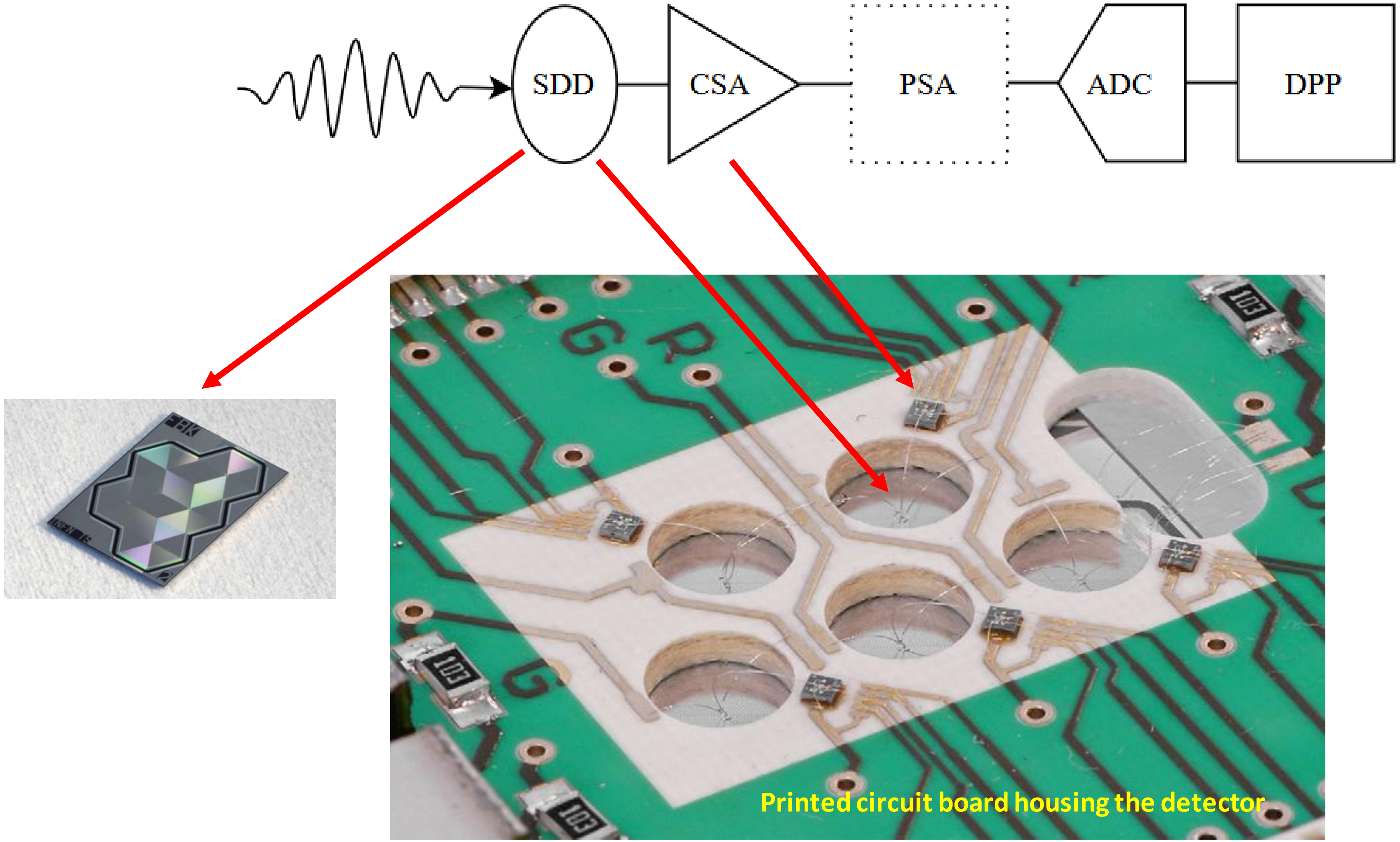
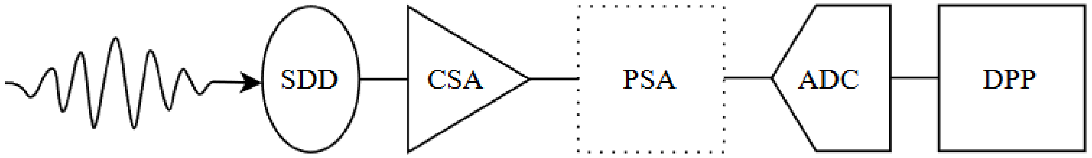
Pulsed signals: Description levels

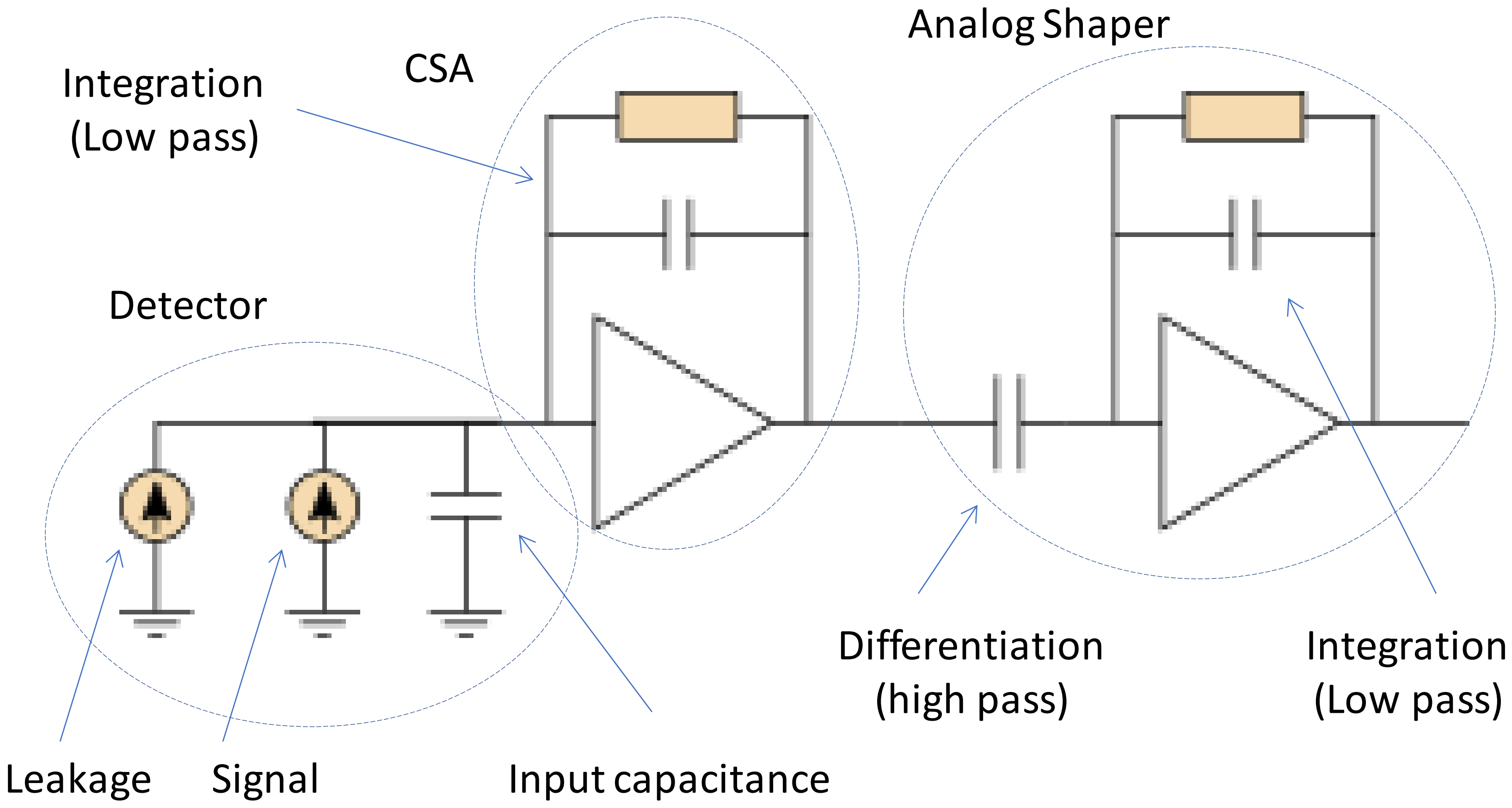


Processing chain

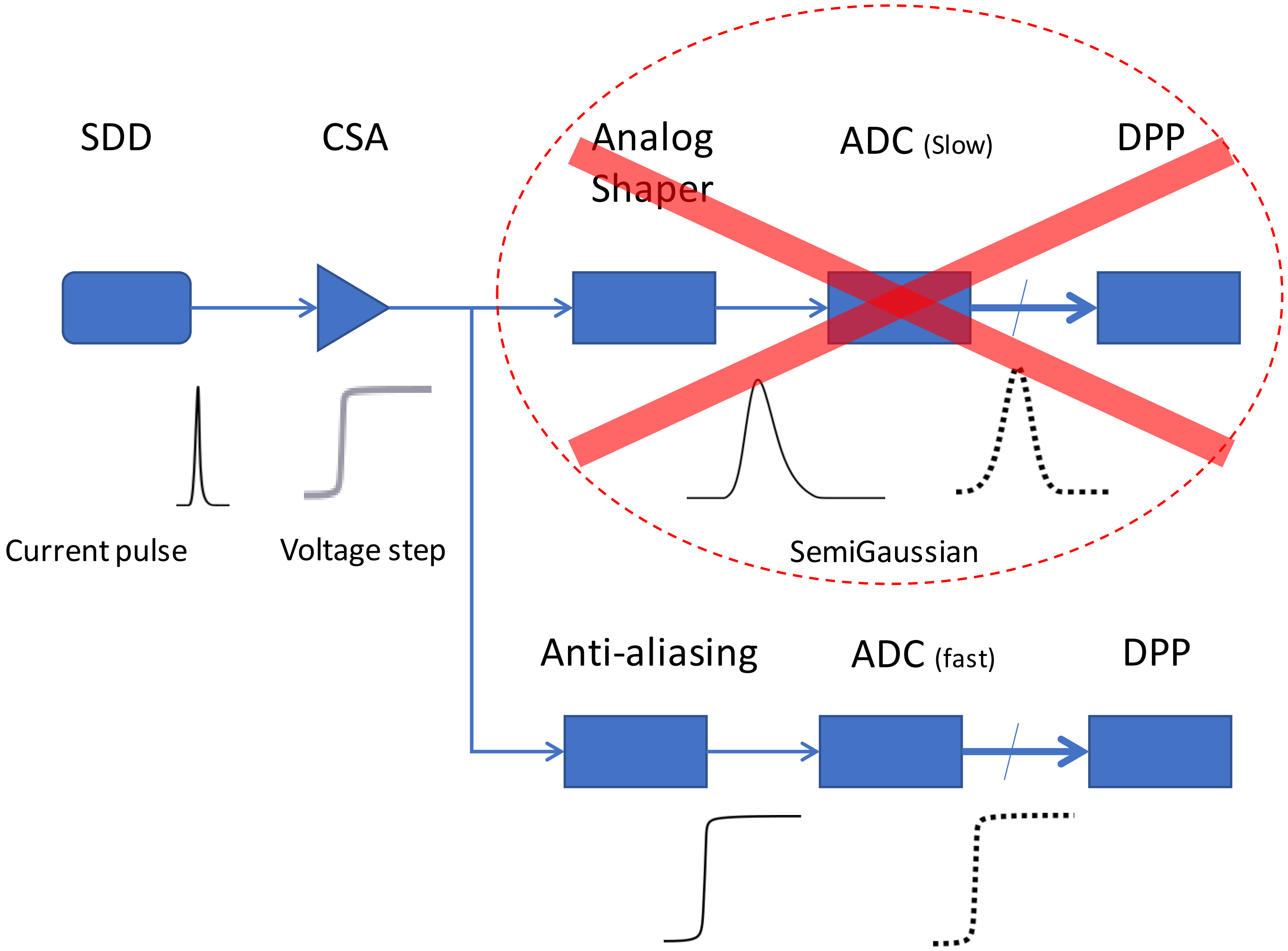


Detector, CSA, Pulse Shaper





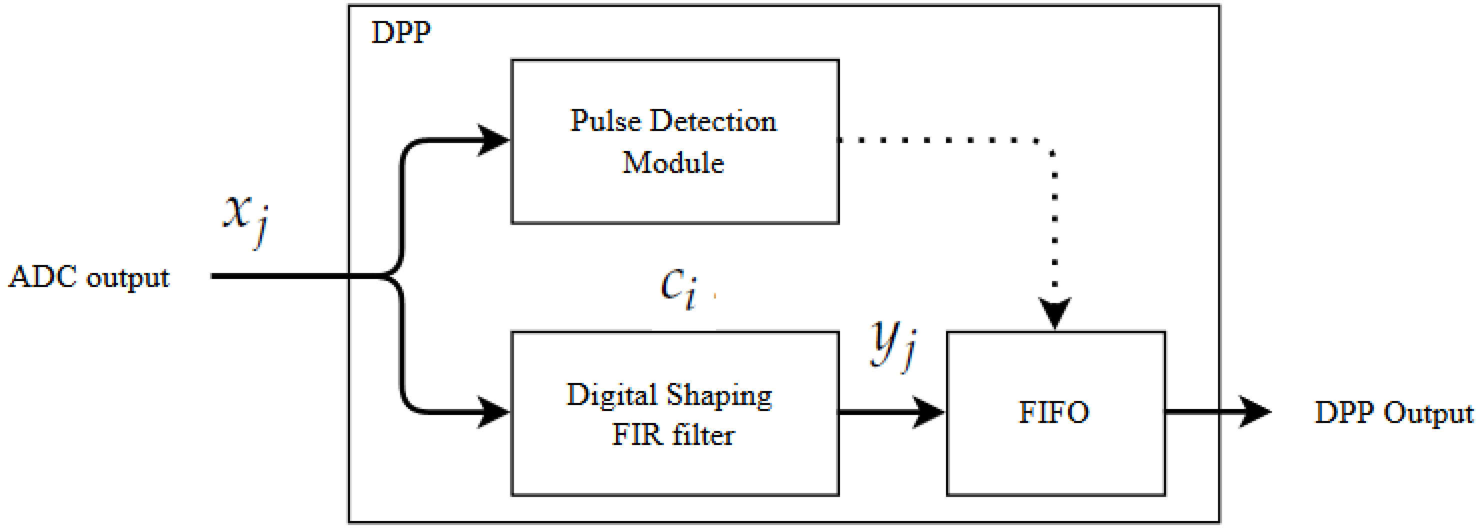
Pulse Processing Chain



Digital Pulse Processor (DPP)

Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .

Minimal version

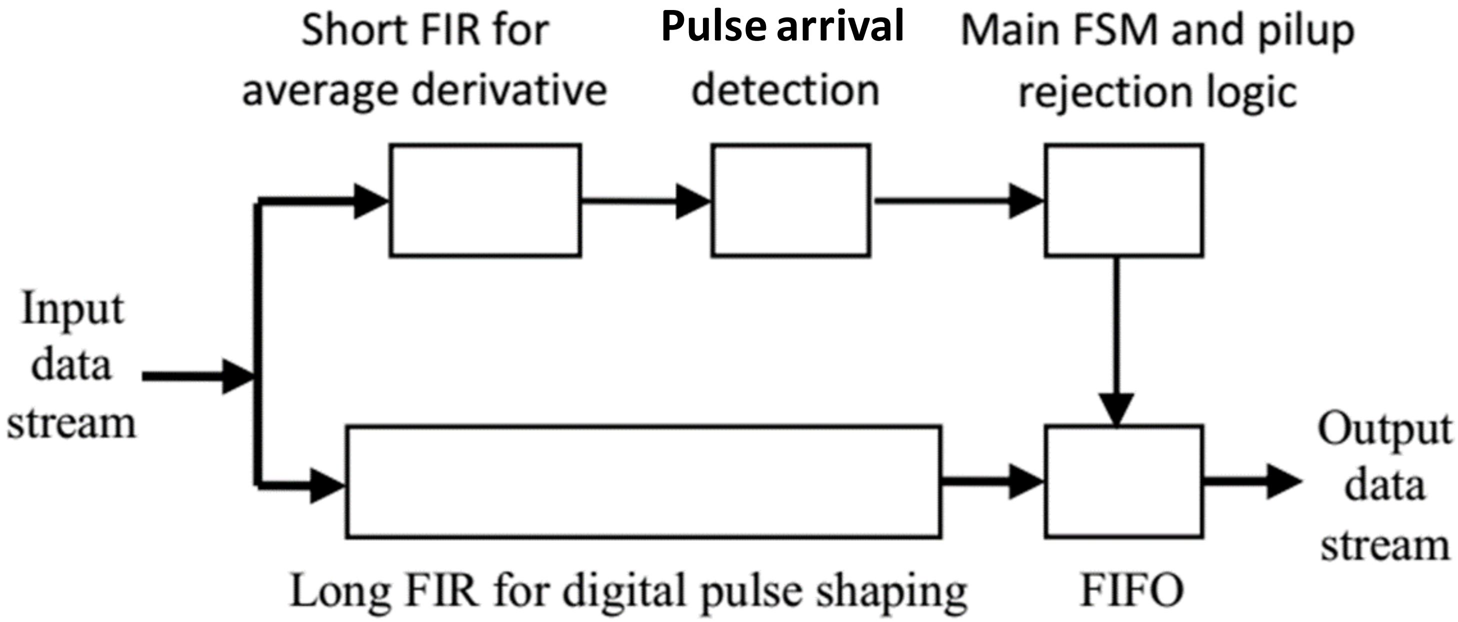


convolution

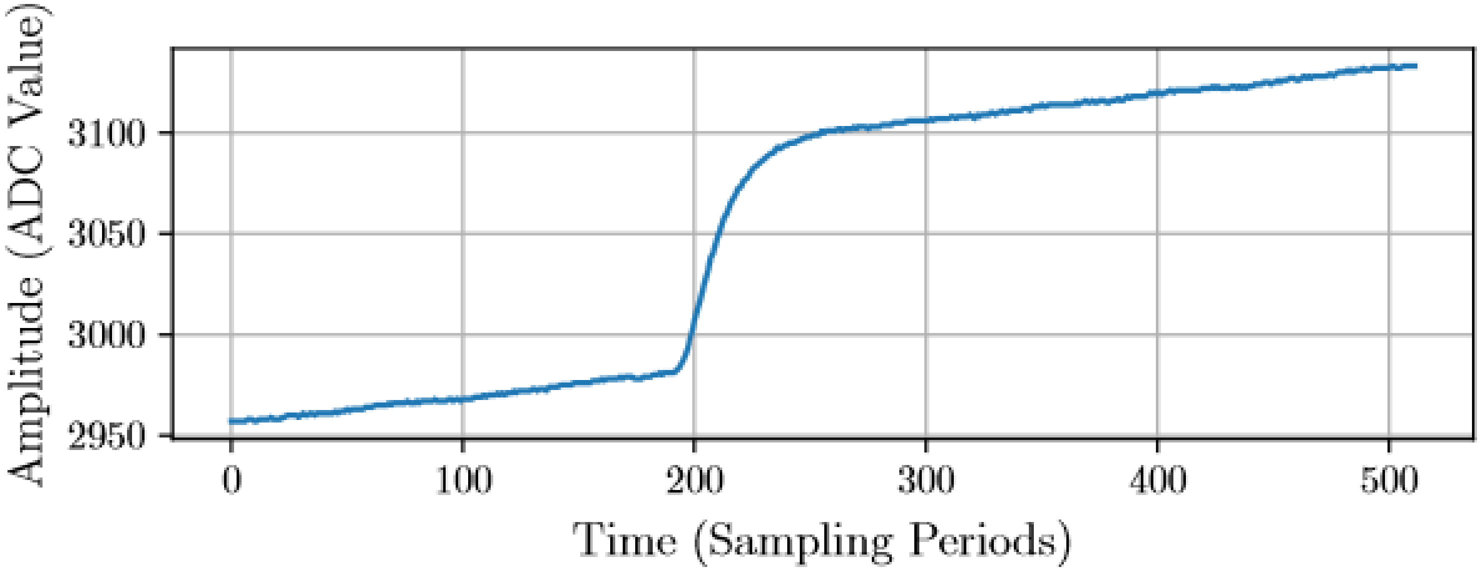
$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

Digital Pulse Processing Strategy for High-Resolution and High-Performance Amplitude Measurement

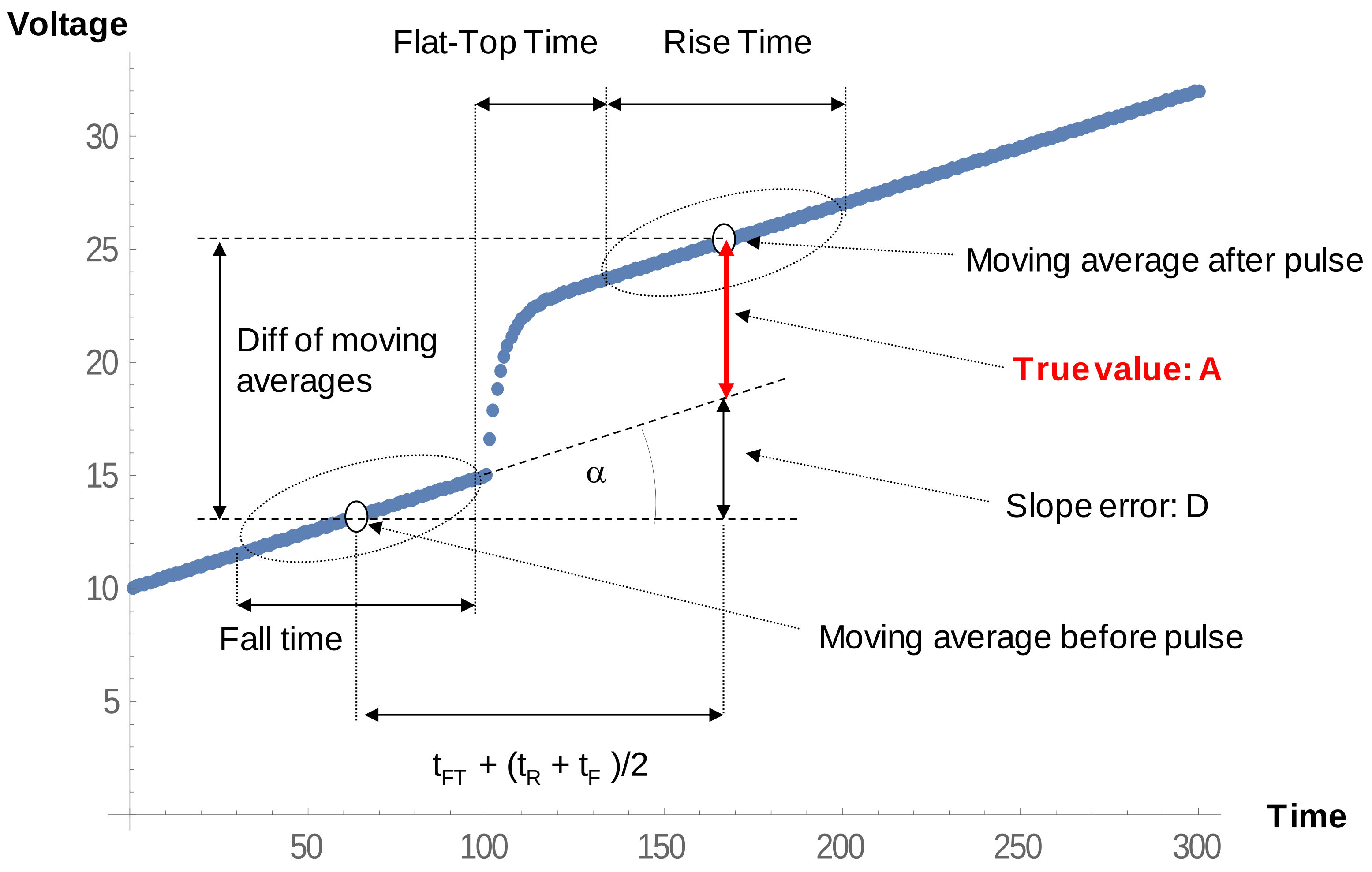
$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$



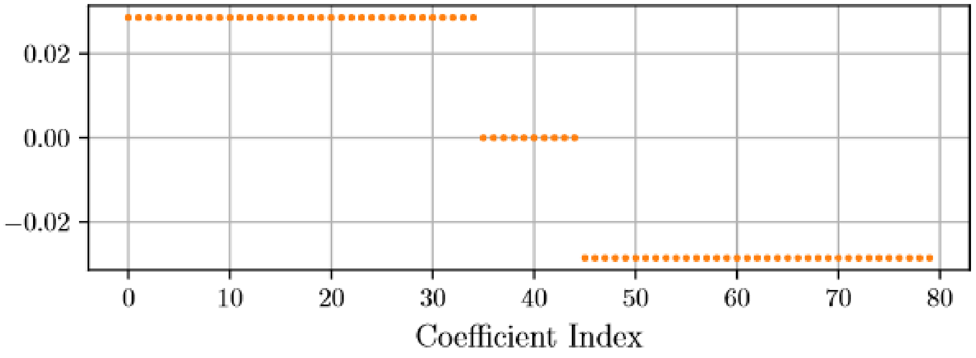
A typical experimental pulse

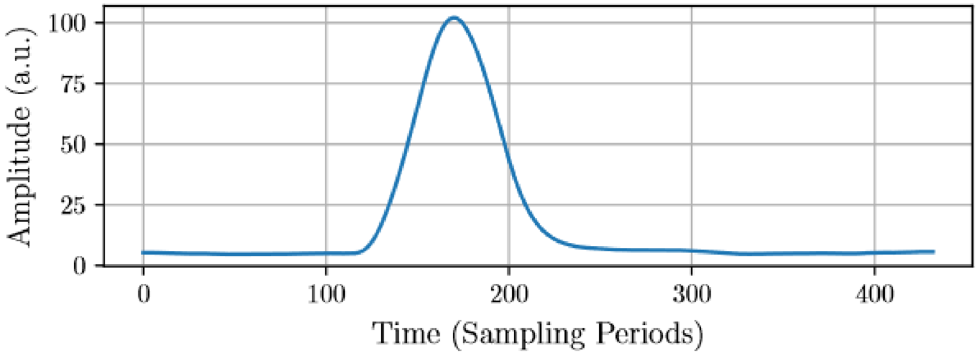


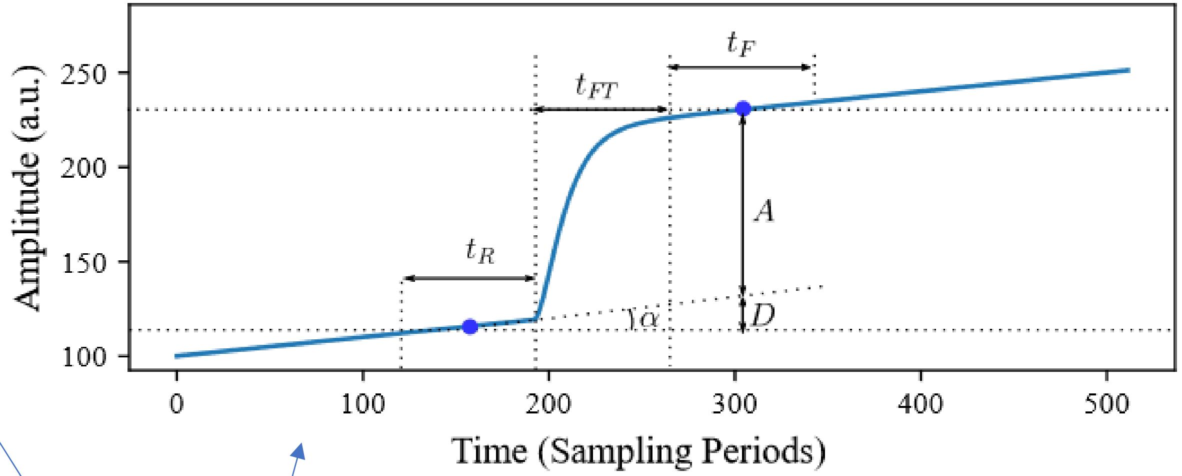
Pulse amplitude measurement



A simple trapezoidal shaper







$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

convolution

$$tan\ \alpha = \frac{D}{\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F}$$

$$tan\ \alpha \approx \sum_{i=0}^{t_R-1} -6\left(\frac{1 + t_R - 2i}{t_R^3 - t_R}\right) x_i$$

Errata corrige: t_R ⇄ t_F

After some algebra . . .

$$D = \sum_{i=0}^{t_R-1} -6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)x_i$$

$$A = \frac{1}{t_F}\sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} x_i - \frac{1}{t_R}\sum_{i=0}^{t_R-1} x_i - \sum_{i=0}^{t_R-1} -6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)x_i$$

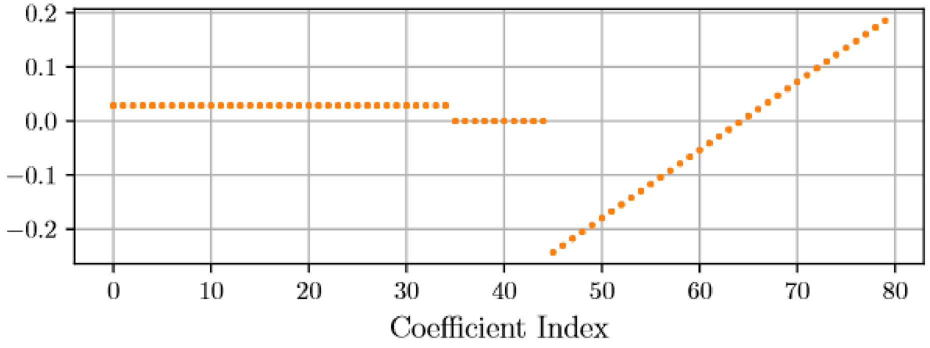
$$A = \sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} \frac{1}{t_F}x_i + \sum_{i=0}^{t_R-1}\left[-\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)\right]x_i$$

← Linear combination

$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leqslant i < t_F; \\ 0, & t_F \leqslant i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right), & t_F + t_{FT} \leqslant i < t_F + t_{FT} + t_R; \end{cases}$$

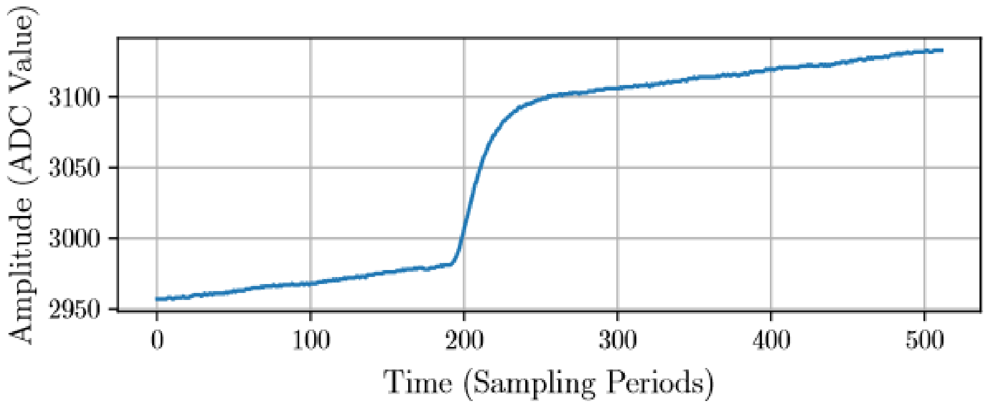
Errata corrige: t_R ⇄ t_F

Geometrically Derived FIR Filter

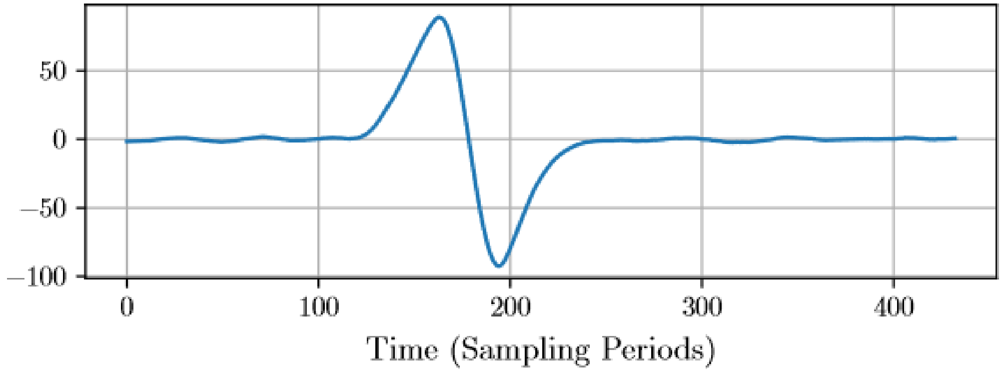


$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leqslant i < t_F; \\ 0, & t_F \leqslant i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F\right), & t_F + t_{FT} \leqslant i < t_F + t_{FT} + t_R; \end{cases}$$

Input pulse



Output pulse



DPP Optimization

Pulse modeling

$$V(t) = \begin{cases} 0, & t \leqslant t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}), & t > t_0; \end{cases}$$

$$V(t) = \begin{cases} B_0 + B_1 t + n(t), & t \leqslant t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}) + B_0 + B_1 t + n(t), & t > t_0; \end{cases}$$

$$x_i = \begin{cases} B_0 + B_1 i + n_i, & i \leqslant t_0; \\ A(1 - e^{\frac{-(i-t_0)}{\tau}}) + B_0 + B_1 i + n_i, & i > t_0; \end{cases}$$

DPP Optimization

Pulse modeling

Deterministic component
(ideal pulse)

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

$$S_i = \begin{cases} B_0 + iB_1, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1, & i > t_0 \end{cases}$$

Increasing information

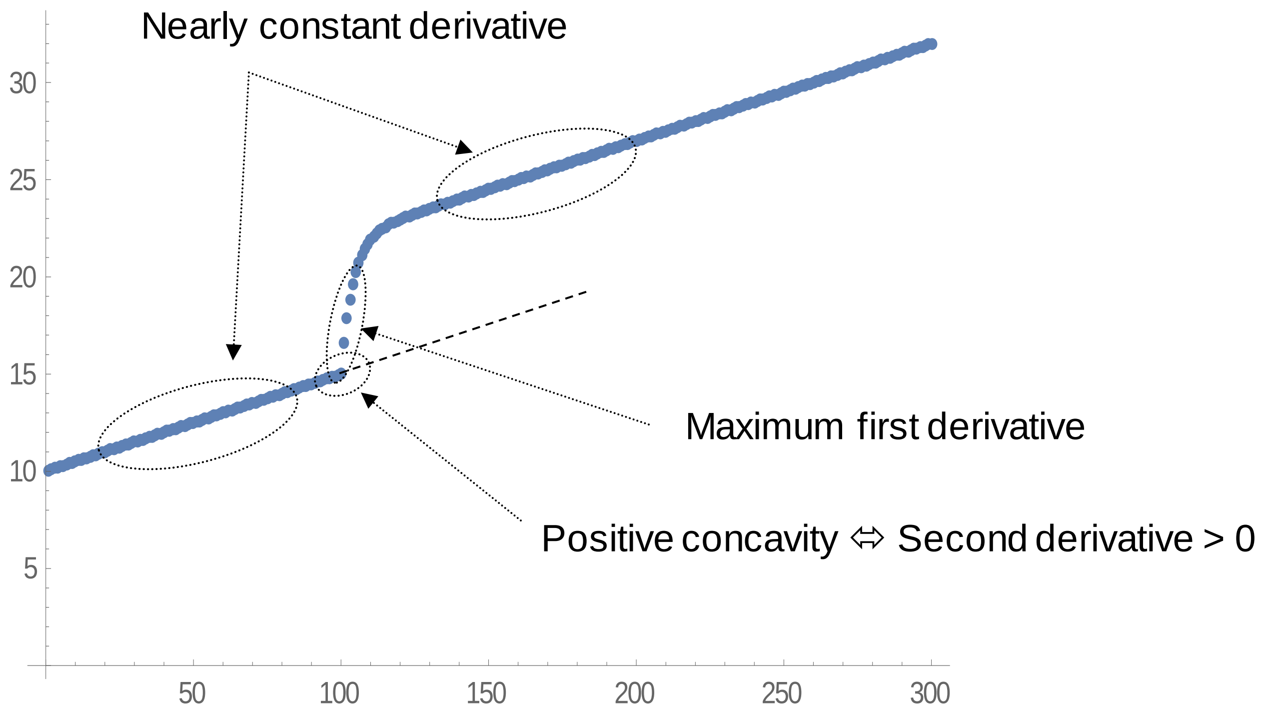
Stochastic component

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right), & i > t_0 \end{cases}$$

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A, & i > t_0 \end{cases}$$

Deterministic component

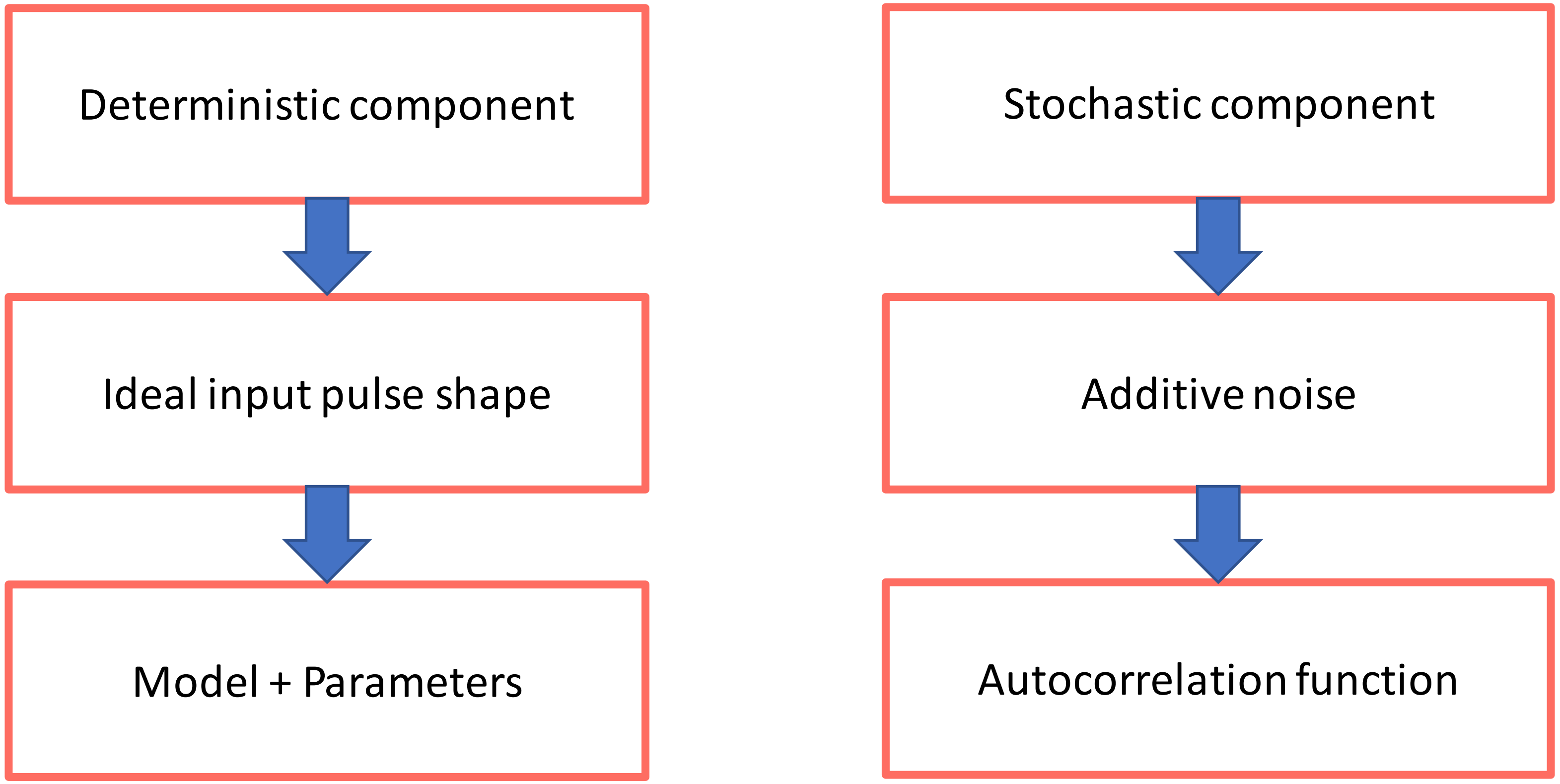
Digital Pulse Processing: Detecting Arrival Time



A short FIR can compute different discrete derivatives

FIR Design and Optimization

Input signal analysis



FIR Design and Optimization

Input pulse modeling I

The ideal case corresponding to a single photon detection is represented by the step function S_i

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A, & i > t_0 \end{cases}$$

The finite frequency response of the CSA determines a limited rise time that could be modeled (1st aprox) as an exponential growth

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}), & i > t_0 \end{cases}$$

A constant detector leakage current determines a baseline with a steady slope and a variable offset on top of which the signal segment must be processed

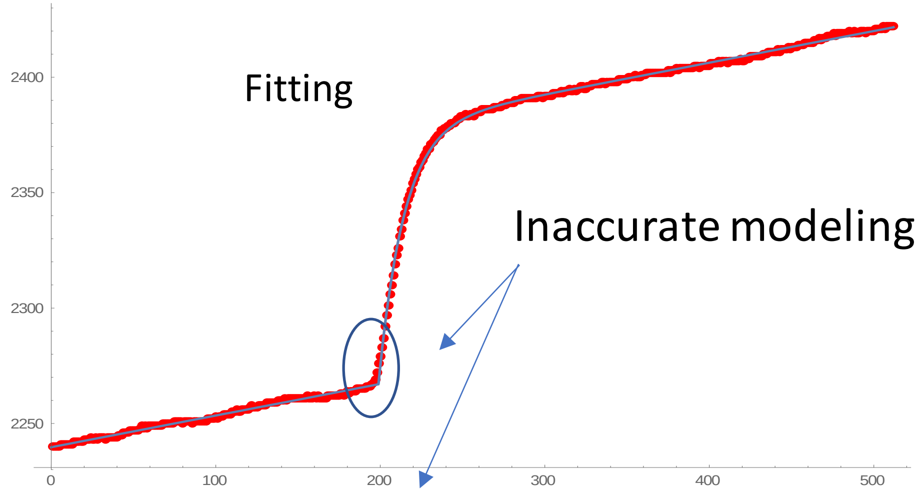
$$S_i = \begin{cases} B_0 + iB_1, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1, & i > t_0 \end{cases}$$

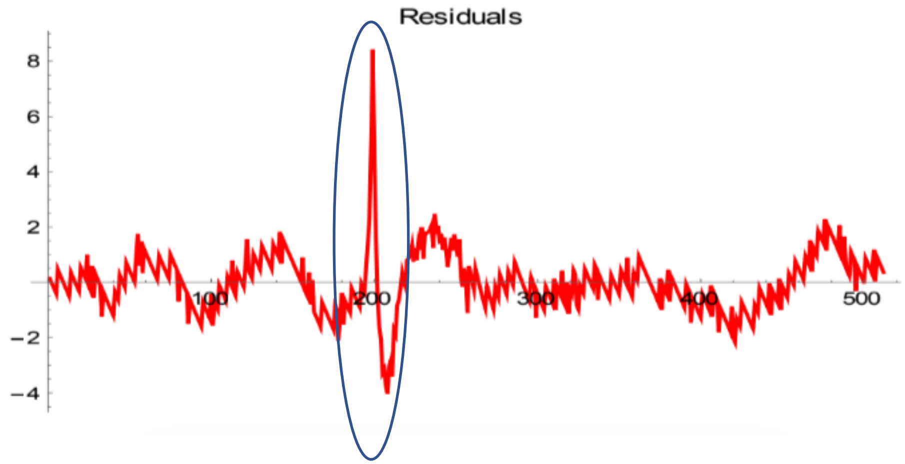
Several sources of noise will contribute with an additive spurious signal n_i that degrades the voltage step measurement

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

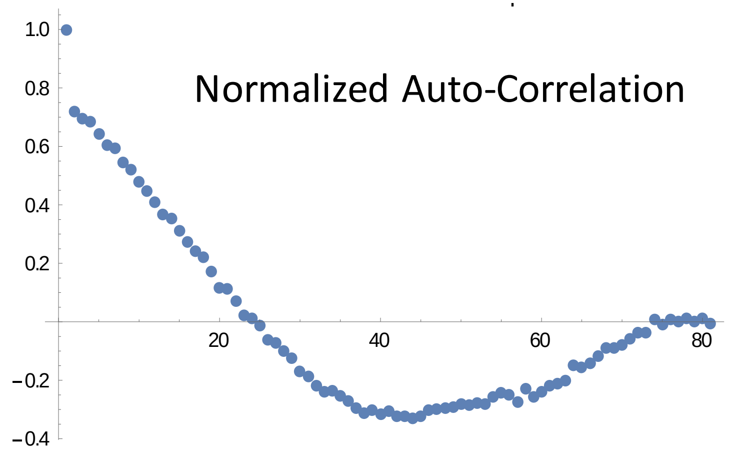
FIR Design and Optimization

Input noise characterization





$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i \, x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$



Some statistic results from the extracted parameters after fitting 1447 segments with the special bi-exponential function.

The proposed signal model has five (quite independent) parameters

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0\left(2\left(1 - e^{-(i-t_0)/\tau}\right) - \left(1 - e^{-2(i-t_0)/\tau}\right)\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

(1) Amplitude,

(2) Arrival Time,

(3) Exponential Time,

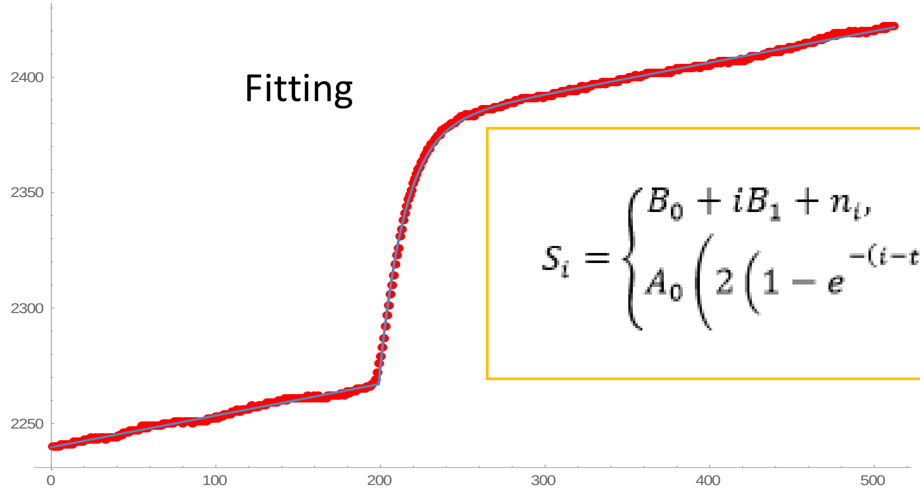
(4) Offset,

(5) Slope Coefficient

FIR Design and Optimization

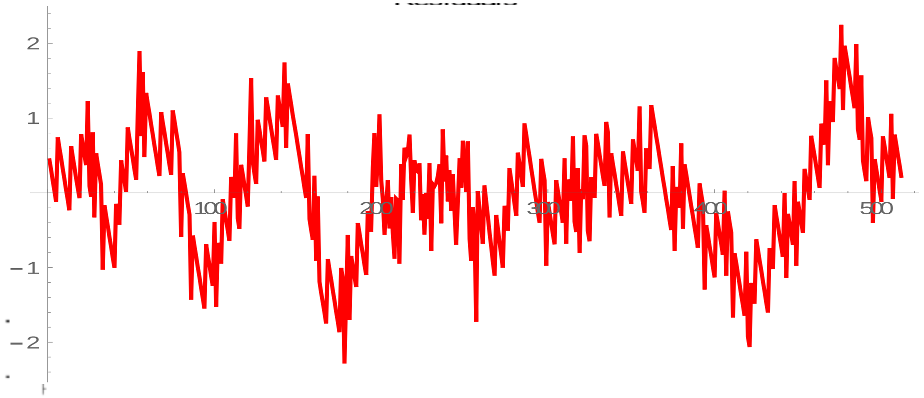
Input noise characterization

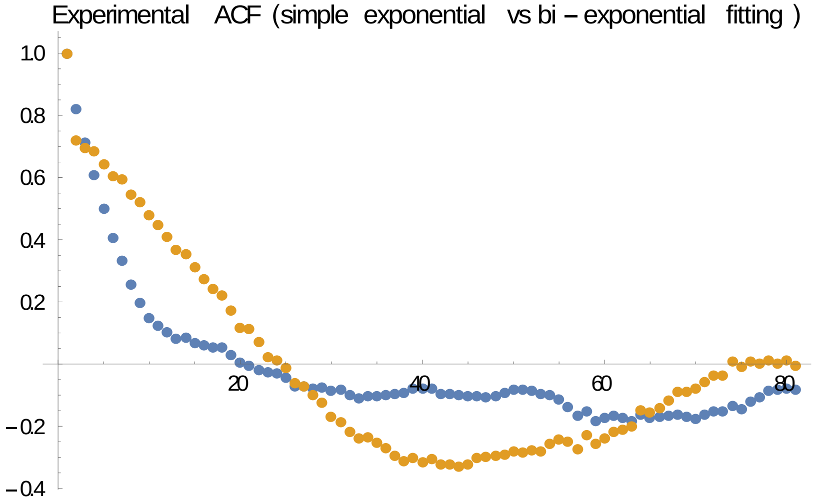
Fitting



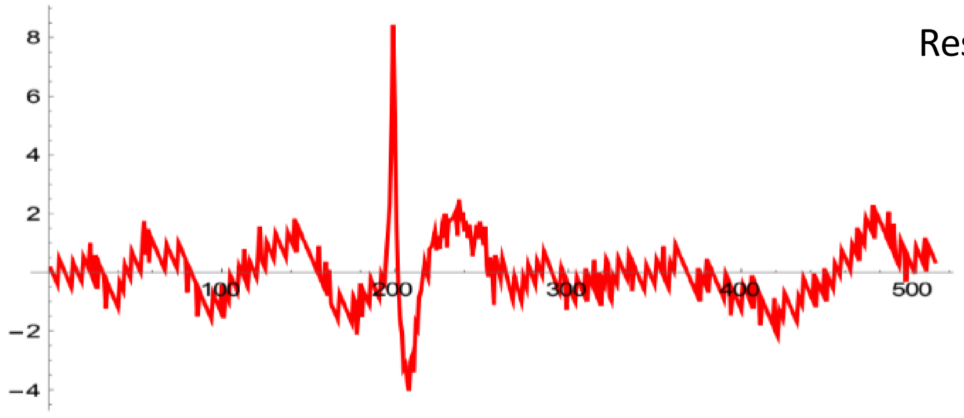
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \le t_0 \\ A_0\left(2\left(1 - e^{-(i-t_0)/\tau}\right) - \left(1 - e^{-2(i-t_0)/\tau}\right)\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

Residuals (noise)

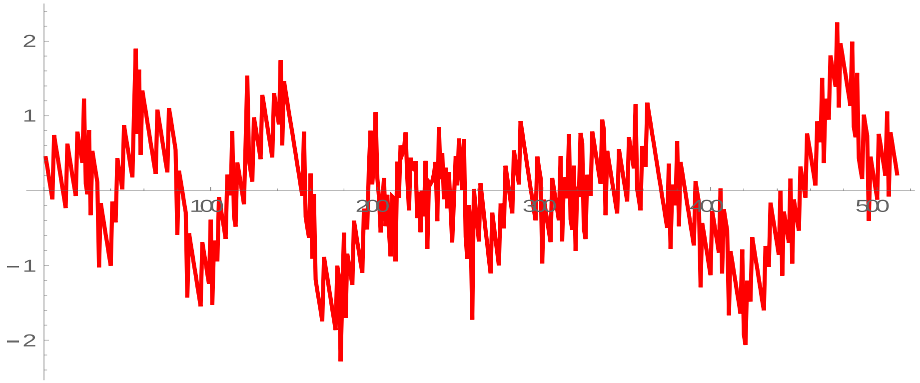




Pulse models comparison

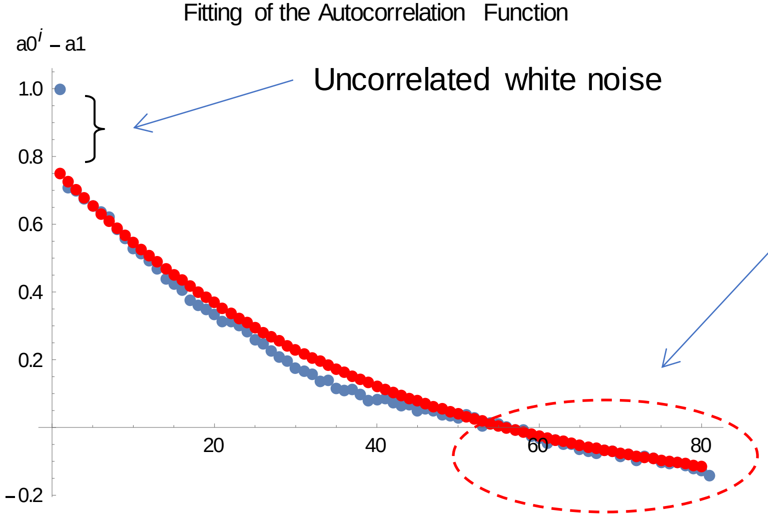


Residuals (noise)

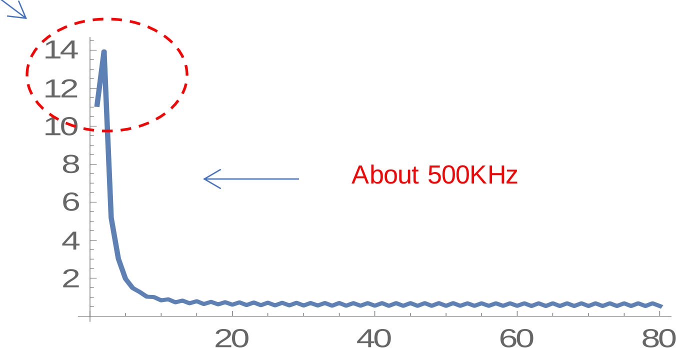


	Exponential Model	Bi-Exponential Model
Mean quadratic residuals	6201	5914
Mean peak-to-peak residuals	13.7	6.6
Mean Akaike information criterion	1720	1397

Autocorrelation model



Negative values determine a peak in the PSD
(via the Winner-Khintchine theorem)



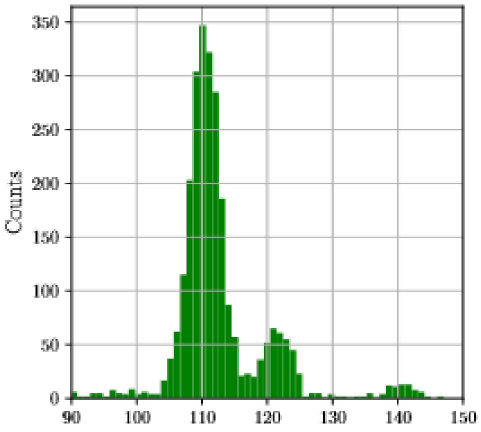
$$ACF(i, a_0, a_1) = \begin{cases} 1 & , \quad i = 0 \\ {a_0}^i + a_1 & , \quad i \geq 1 \end{cases}$$

In this case the average normalized ACF can be approximated with $\boldsymbol{a_0=0.965}$ and $\boldsymbol{a_1=-0.2}$

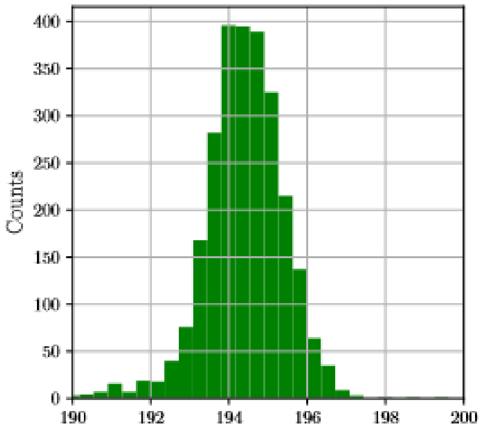
Histograms of fitting parameters corresponding to the bi-exponential model.

They looks nice but . . .

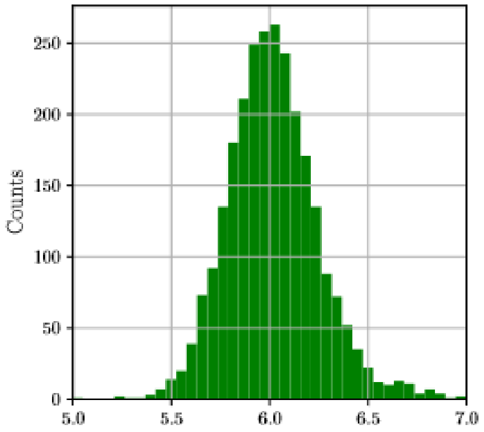
. . . scattered plots and correlation distances between pair of parameters may reveal non idealities of the DAQ system.



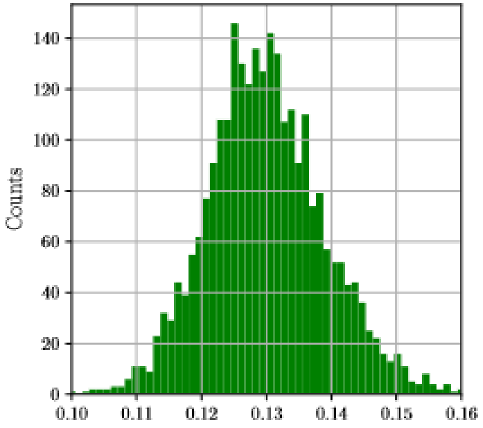
(a) Amplitude (A)



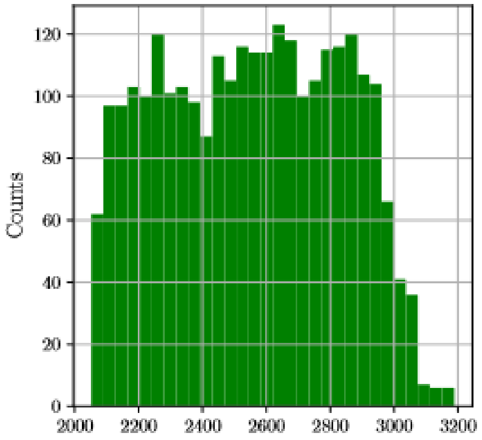
(b) Arrival time (t_0)



(c) Exponential time (τ)

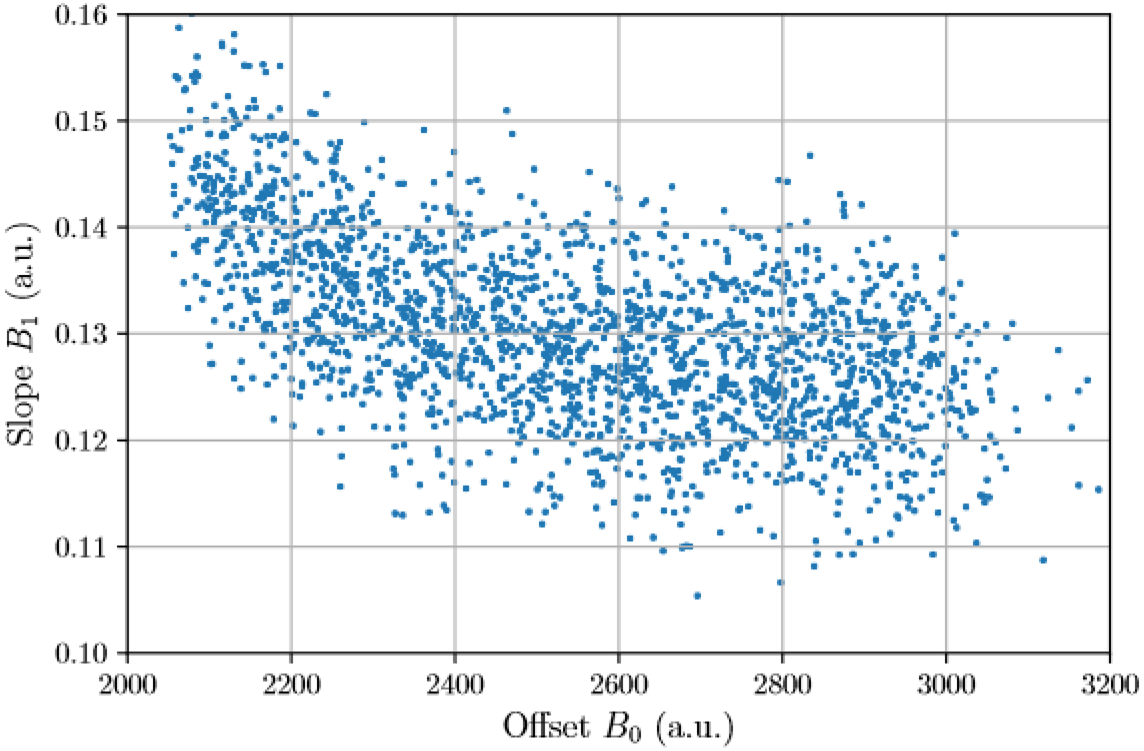


(d) Slope (B_1)

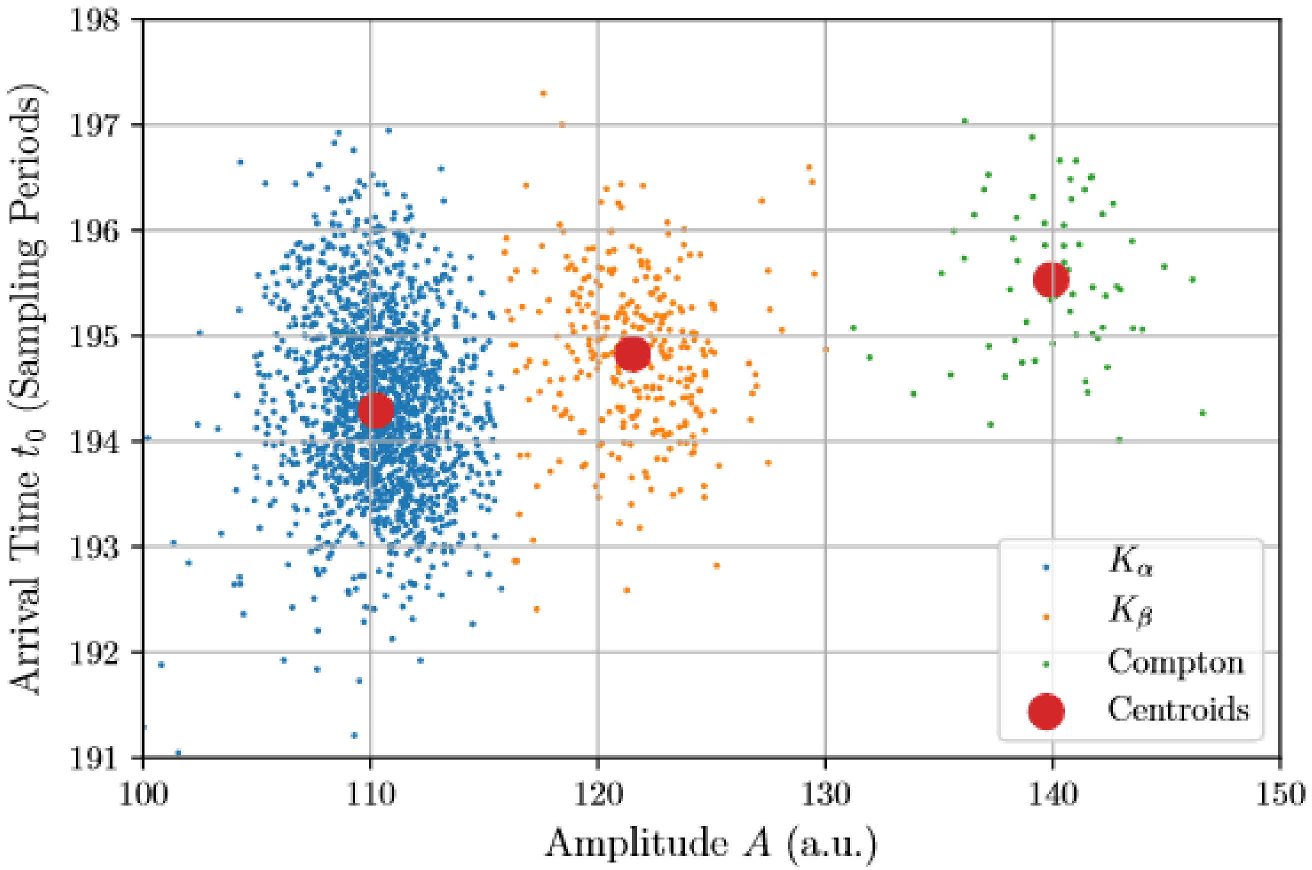


(e) Offset (B_0)

Non linearity: Amplification gain depends on offset (!)



Detection arrival time depends on pulse amplitude (!)



DPP: Digital Penalized LMS Method for filtering optimization

$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

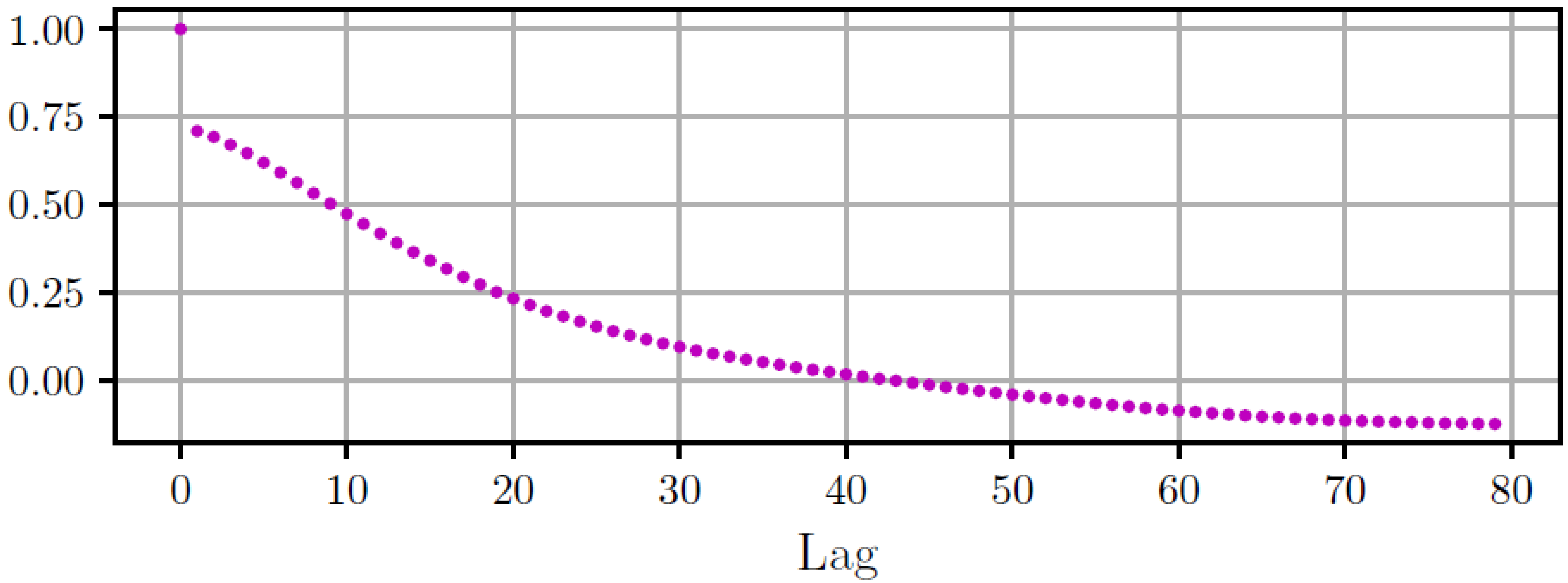
$$\sigma_y^2 = \left\langle (y - \langle y \rangle)^2 \right\rangle = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} c_i \, c_j \underbrace{\langle x_i - \langle x_i \rangle \rangle \langle x_j - \langle x_j \rangle \rangle}_{\text{Covariance Matrix } V_{i,j}}$$

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i \, c_j \, ACF(|i-j|)$$

$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i \, x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$

DPP: Digital Penalized LMS Method for filtering optimization

Normalized average ACF



DPP: Digital Penalized LMS Method for filtering optimization

$$\sum_{i=0}^{k-1} c_i = 0$$

Offset rejection

$$\sum_{i=0}^{k-1} c_i \, i = 0$$

Slope rejection

Ideal requirements

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i \, c_j \, ACF(|i-j|)$$

Output noise

$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i} = A, \qquad j \in [t_R, t_R + t_{FT} - 1]$$

Output flat top

DPP: Digital Penalized LMS Method for filtering optimization

$$\Psi(c_0, c_1, \ldots, c_{k-1}) = \alpha_1 \left(\sum_{i=0}^{k-1} c_i \right)^2 + \alpha_2 \left(\sum_{i=0}^{k-1} c_i \, i \right)^2 + \alpha_3 \sum_{j=t_R}^{t_R+t_{FT}-1} \left(\sum_{i=0}^{k-1} c_i \, x_{k+j-i} - A \right)^2 +$$

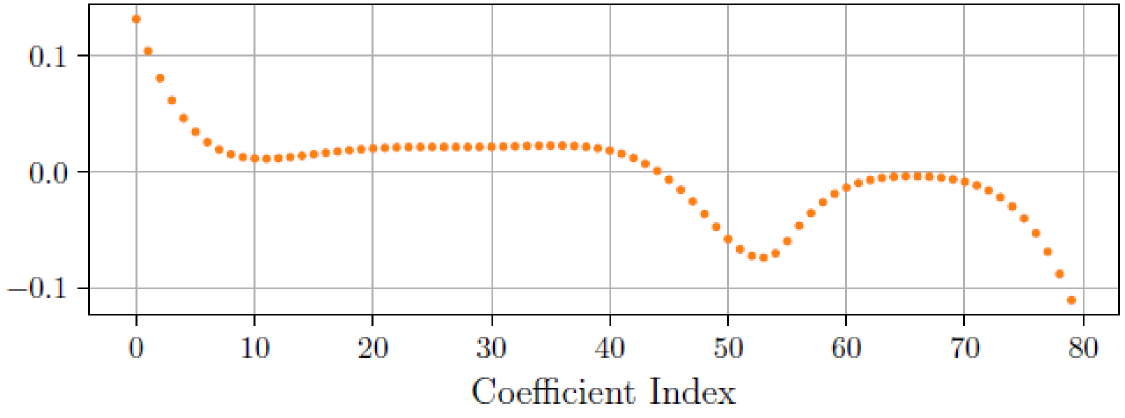
$$\alpha_4 \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i \, c_j \, ACF_{|i-j|}$$

$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i} = A, \qquad j \in [t_R, t_R + t_{FT} - 1]$$

$$\{c_0, c_1, \ldots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \ldots, c_{k-1}\}}{argmin} \; \Psi(c_0, c_1, \ldots, c_{k-1})$$

DPP: Digital Penalized LMS Method for filtering optimization

$$\{c_0, c_1, \ldots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \ldots, c_{k-1}\}}{argmin} \; \Psi(c_0, c_1, \ldots, c_{k-1})$$



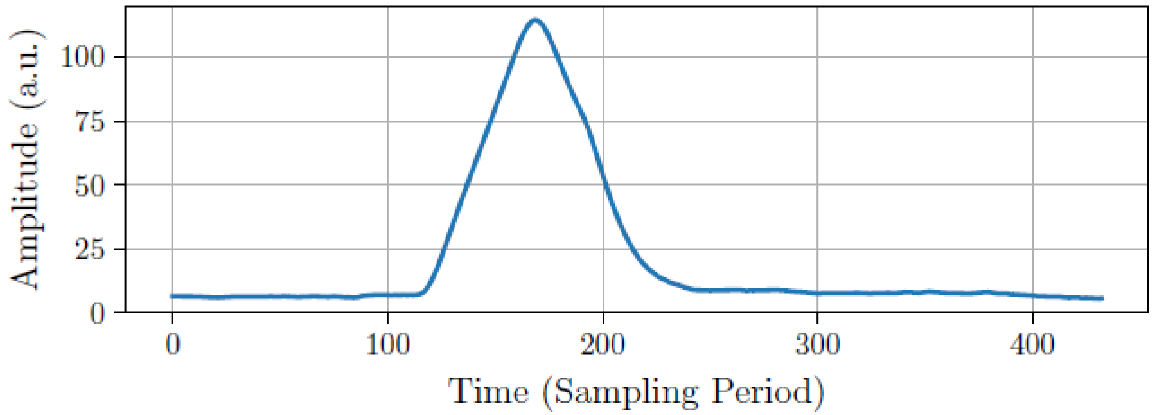


Table 3. Comparison of energy resolutions with different methods to estimate the energy spectrum.

Method	FWHM K_α [eV]	FWHM K_β [eV]	Slope-Error Correction
GD FIR	286 ± 4	316 ± 16	yes
Fitting †	267 ± 4	288 ± 17	yes
Trapezoidal FIR	207 ± 3	247 ± 17	no
DPLMS FIR	202 ± 2	233 ± 12	no

† These results correspond to the histogram of the amplitudes obtained by fitting all available photon traces.

Conclusions

- High-resolution pulse amplitude measurement can be achieved by considering concrete experimental noise and accurate pulse modeling.
- DPP can be optimized through DPLMS method allowing satisfactory trade-off among competing requirements that cannot be all simultaneously satisfied.
- An appropriate data analysis provides the necessary information to apply the DPLMS method, and it may also provide information about the quality the frontend electronics and data acquisition system.

Tank you !

Data Analysis and Filter Optimization for Online Pulse-Amplitude Measurement

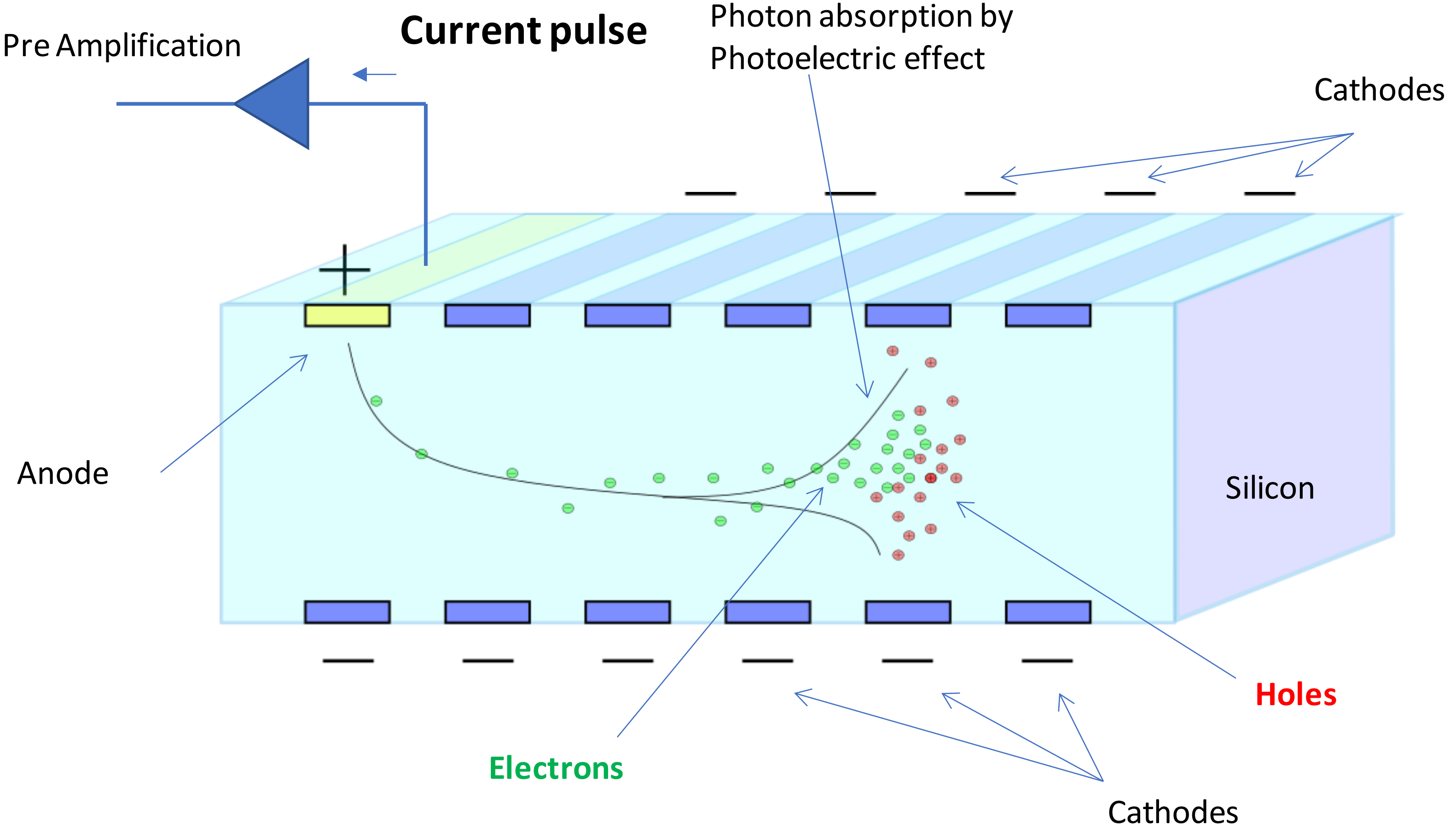
A Case Study on High-Resolution X-ray Spectroscopy

Andres Cicuttin

ICTP - Multidisciplinary Laboratory

Trieste, Italy

Backup slides: X-Ray Photon detection with Silicon Drift Detectors (SDD)



Backup slides: Pile up (1)

Pile up: Being a Poissonian process, two or more photons could be absorbed in the SDD within any arbitrary small time window. The superposition of two photons absorbed at times t_0 and t_1 and respectively with amplitudes A_0 and A_1 is then given by

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \le t_0 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & t_0 < i \le t_1 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + A_1\left(1 - e^{-(i-t_1)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_1 \end{cases}$$

Backup slides: Pile up (2)

. . . and in general for *m+1* photons

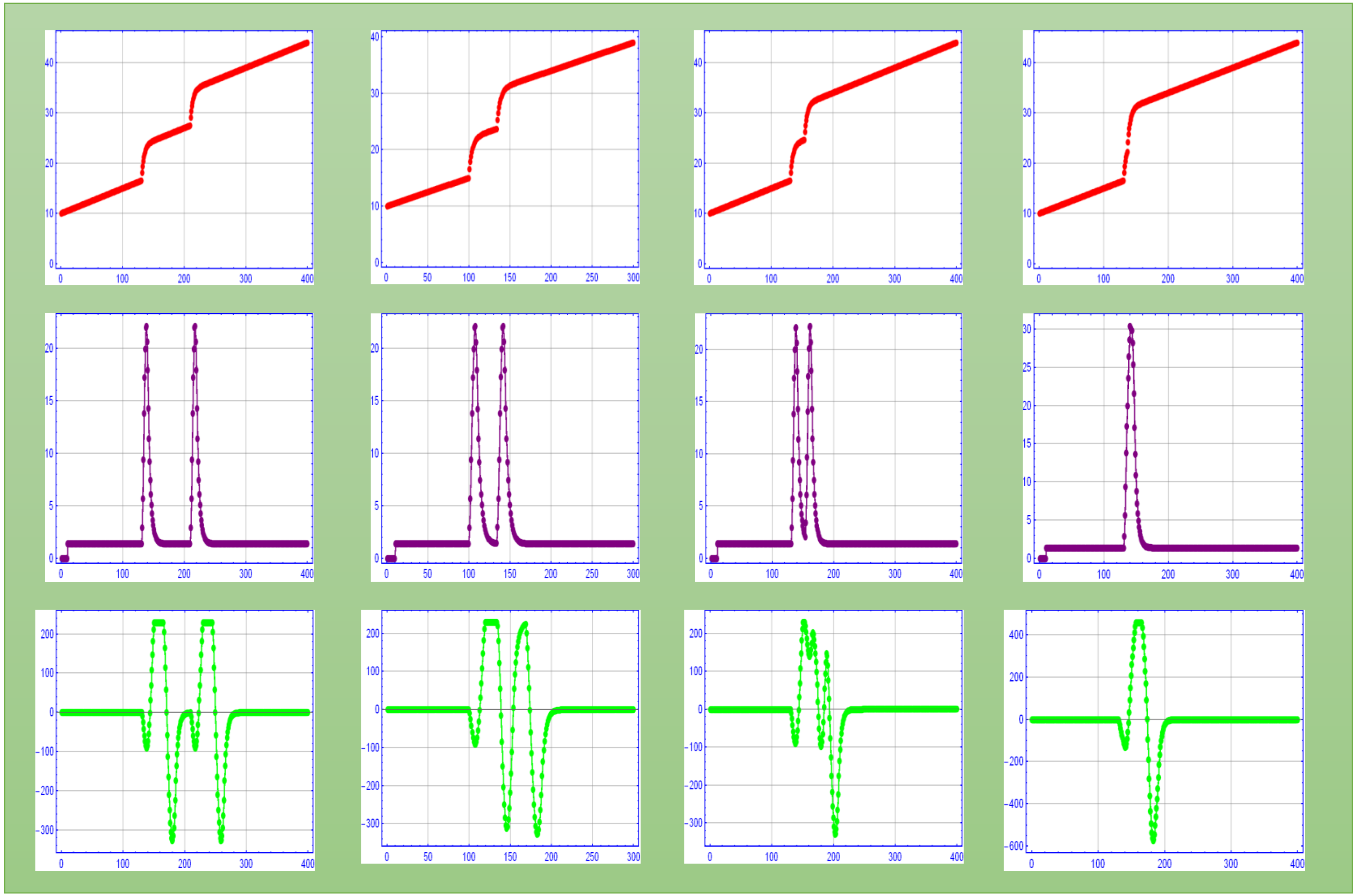
$$
S_i = \begin{cases}
B_0 + iB_1 + n_i, & i \le t_0 \\
A_0\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & t_0 < i \le t_1 \\
A_0\left(1 - e^{-(i-t_0)/\tau}\right) + A_1\left(1 - e^{-(i-t_1)/\tau}\right) + B_0 + iB_1 + n_i, & t_1 < i \le t_2 \\
\vdots & \\
\sum_{j=0}^{m} A_j\left(1 - e^{-(i-t_j)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_m
\end{cases}
$$

Backup slides: Pileup rejection

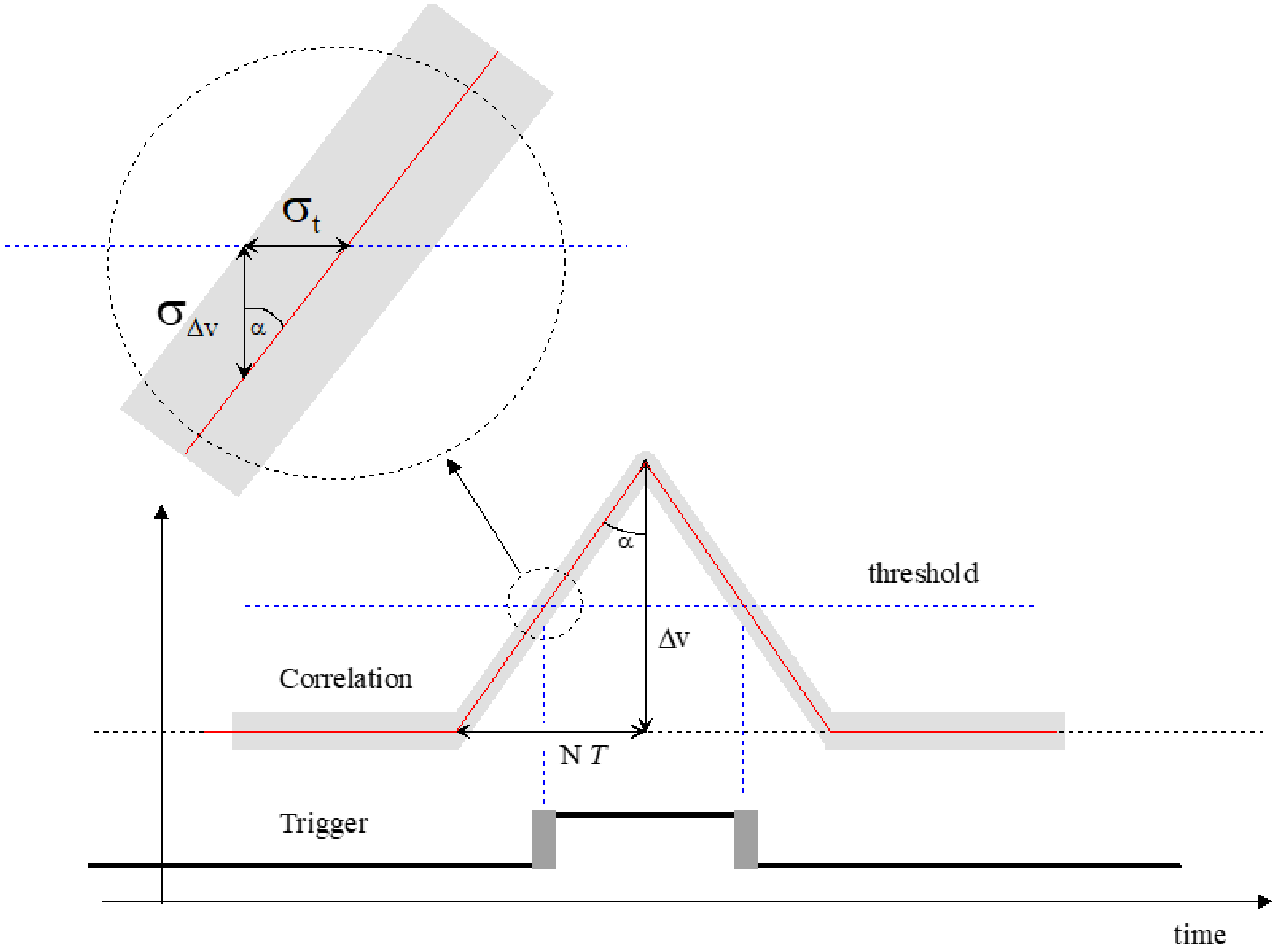
Input Pulses

First Derivs

Shaped Pulses



Backup slides: Uncertainty relation between **Energy** and **Time**



$$\sigma_t \sigma_{\Delta V} = \frac{2T}{\Delta V} \sigma_x^{\ 2}$$