# Introduction to Cosmology

ICTP School on Particle Physics, June 2025

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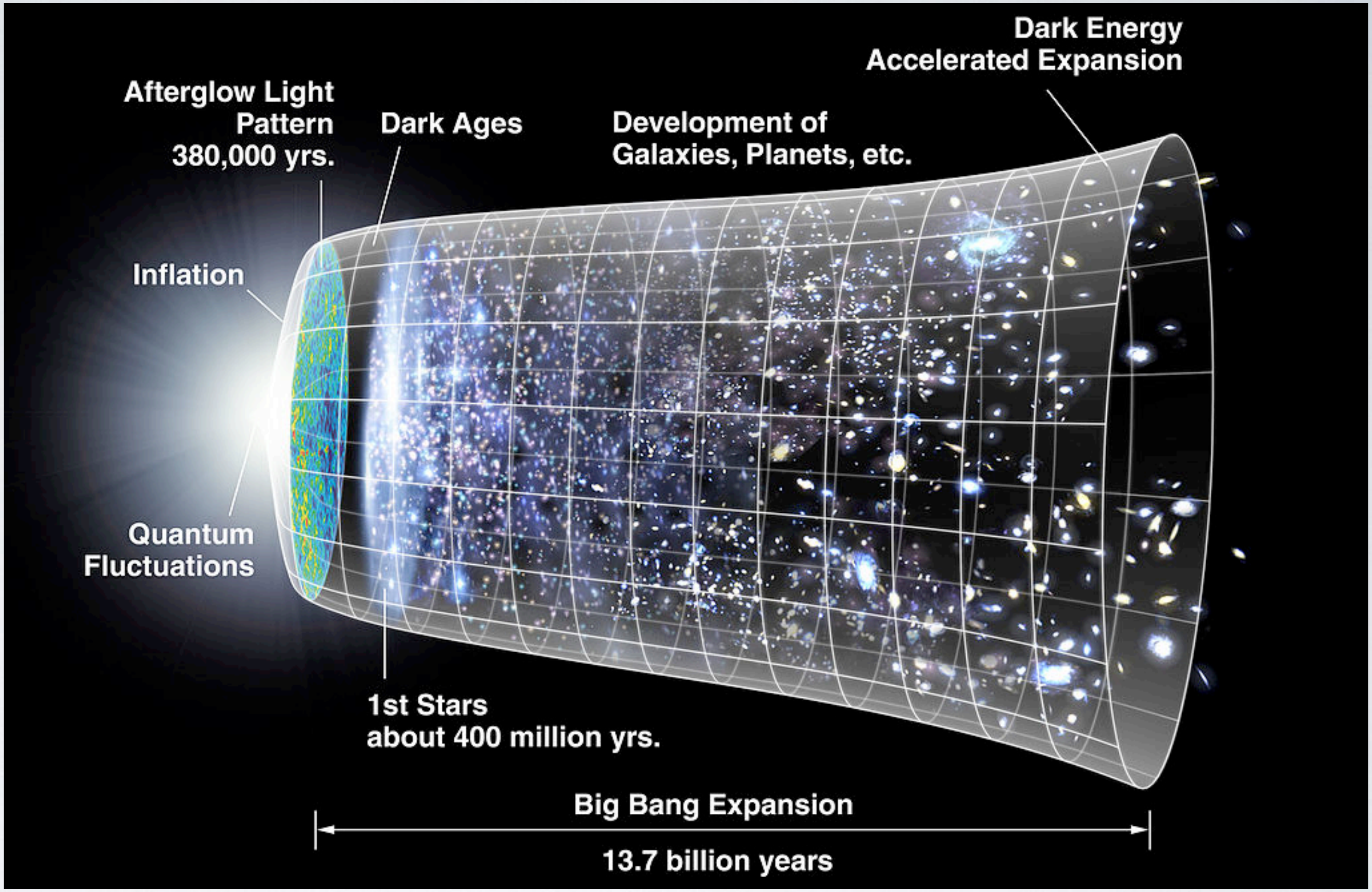
Lecture 1: overview, theoretical framework, observables, FRW metric

Lecture 2: homogenous universe, redshift, Hubble Law, energy densities, “our universe” = ΛCDM, DESI w0wa

Lecture 3/4: the thermal universe, equilibrium, big bang nucleosynthesis, recombination

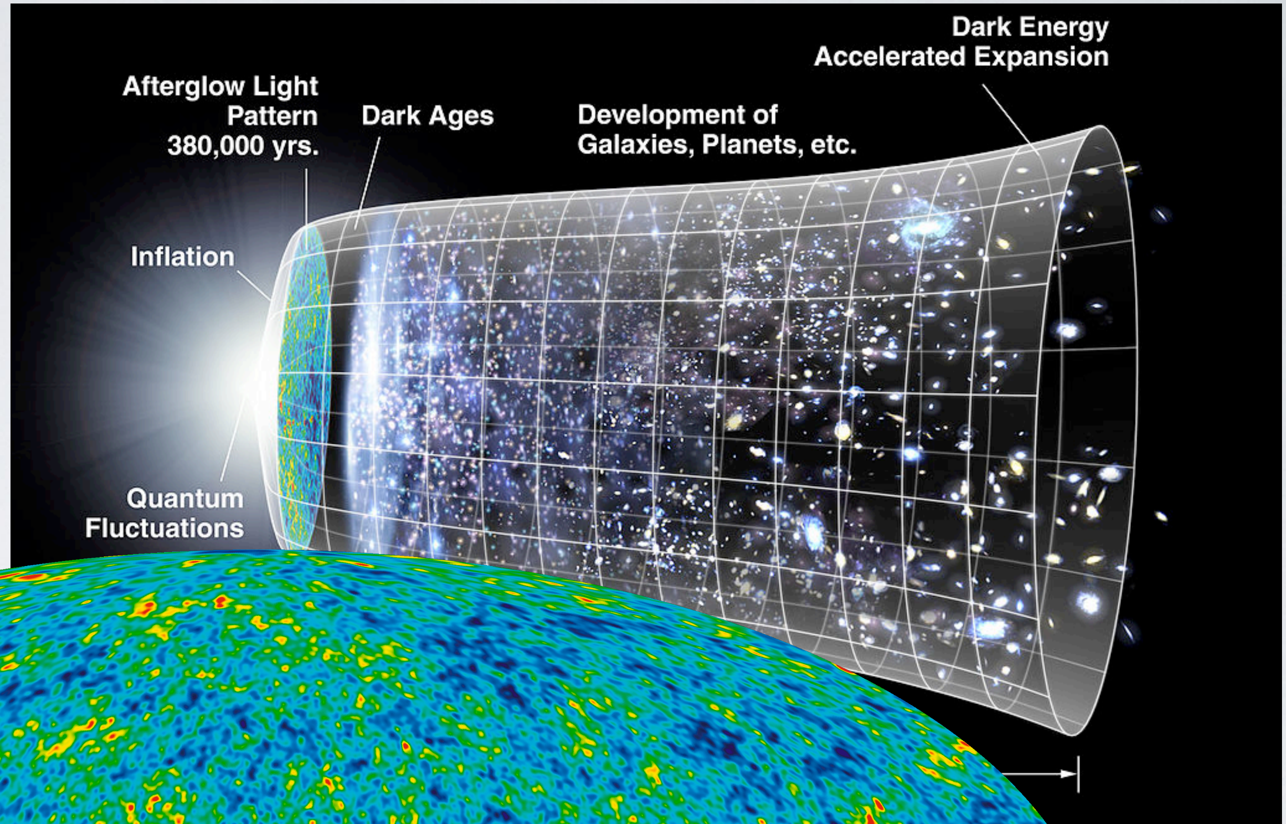
Lecture 4/5: inhomogenous universe, growth of perturbations, CMB, matter power spectrum, ΛCDM parameters

Resources: Dodelson book, Baumann lecture notes (Amsterdam) and book, Loverde TASI lectures ...


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

# OUR HISTORY

# CMB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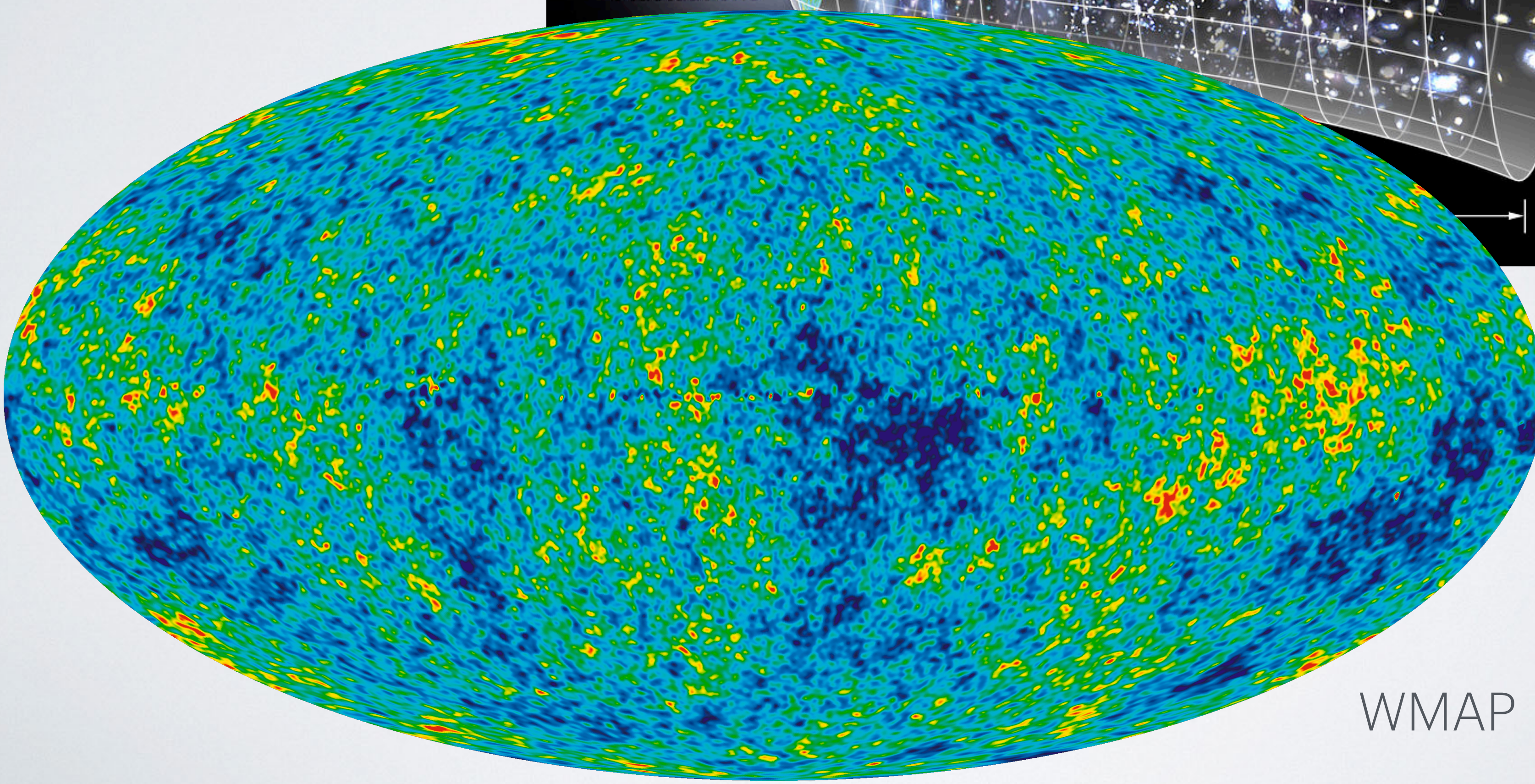
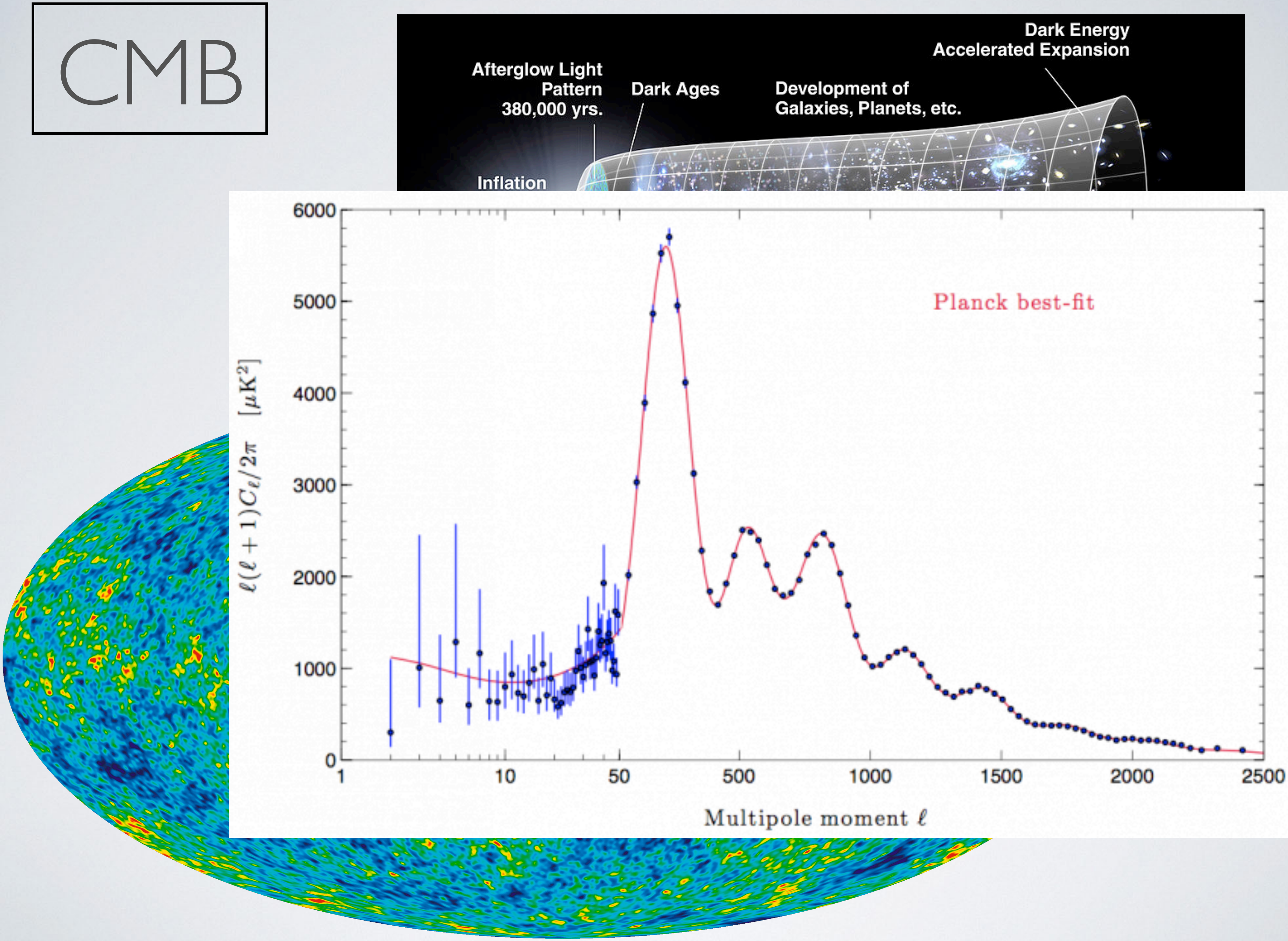


WMAP

# CMB


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Planck best-fit
$\ell(\ell+1)C_\ell/2\pi\ [\mu K^2]$
Multipole moment $\ell$
6000
5000
4000
3000
2000
1000
0
1
10
50
500
1000
1500
2000
2500

# Distribution of galaxies

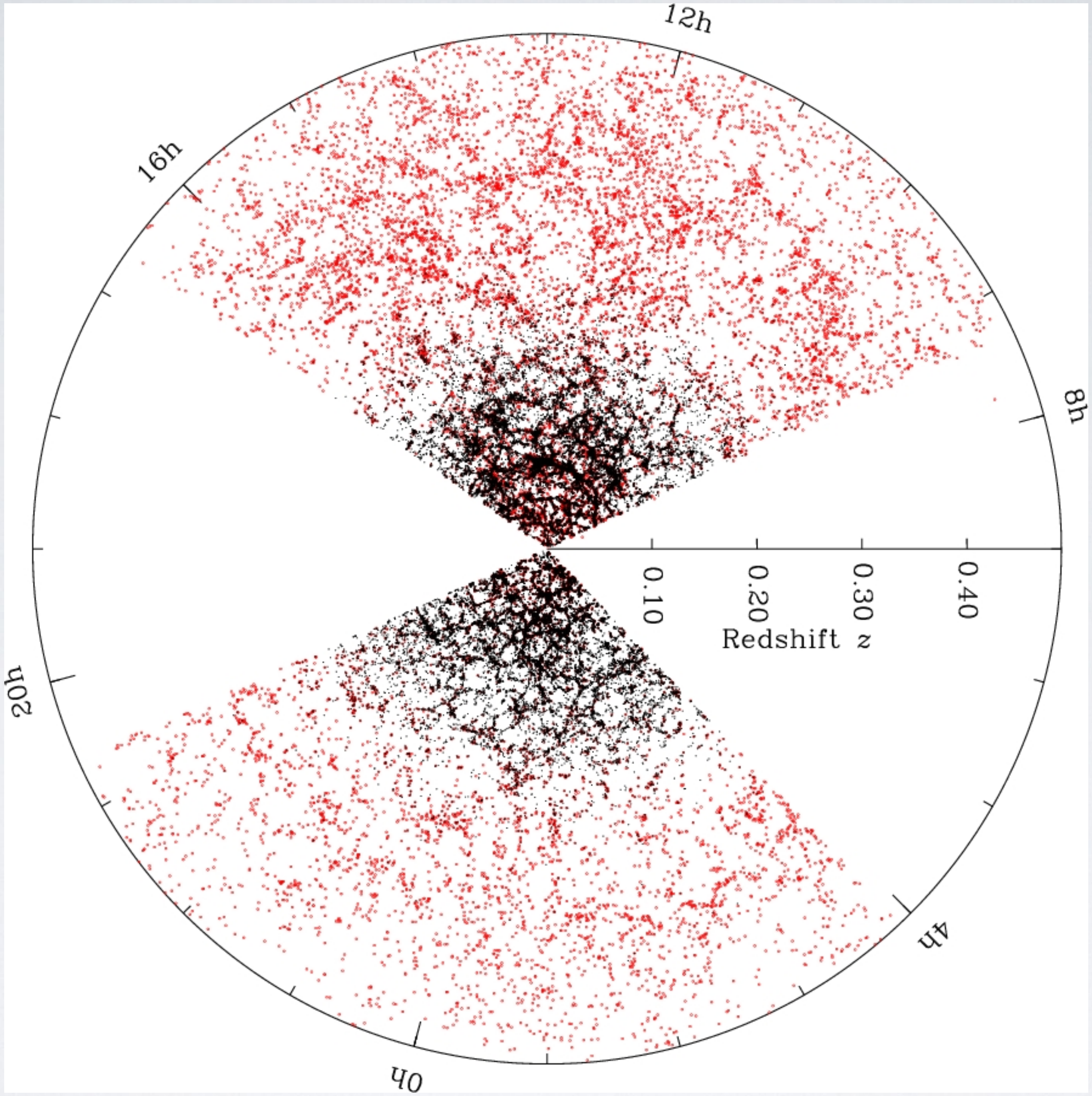



**Figure 1.1:** *The distribution of galaxies is clumpy on small scales, but becomes more uniform on large scales and at early times.*

Baumann notes

# Lecture 1.

Cosmology is the quantitative study of the evolution of the universe on large scales from the "beginning" to today.

Slide 1. Expanding universe

Slide 2. CMB $\frac{\delta T}{T} \sim 10^{-5}$ very homogeneous isotropic !

looks like noise but there are correlations which we can predict

Slide 3 planck

Slide 4 distribution of galaxies

note : 1. smooth on large scales, average density is homogeneous

2. speed of light is finite $\Rightarrow$ looking far out = looking backwards in time

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Assumptions:

1. Evolution governed by General Relativity (microphysics governed by QM, E&M, ... )

2. Cosmological principle : we are not in special place

3. laws of physics are space-time independent

4. separation of scales: physics at large scales independent of details on small scales $\Rightarrow$ averaging works, effective field theory

Assumptions are based on observations and we keep testing them.

General Relativity (underlies everything, we need very little)

space-time vectors $x^\mu = \begin{pmatrix} t \\ \vec{x} \end{pmatrix}$ ($\mu$ = 0,1,2,3) coordinate transformations mix $(\vec{x},t)$ rotations, boosts

metric: $g_{\mu\nu}(x)$

used to define space-time distances

$$ds^2 = \sum_{\mu,\nu} g_{\mu\nu}\, dx^\mu dx^\nu$$

e.g. 1 Minkowski metric, Minkowski space $g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$$\Rightarrow ds^2 = dt^2 - d\vec{x}^2$$

e.g. 2. $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$ Flat FRW metric, $a(t)$ scale factor

$g_{\mu\nu}(x)$ = 4x4 matrix $\Rightarrow$ 10 functions of space-time

Equations of motion = Einstein equations

Lecture 1:

1. $$\underbrace{G_{\mu\nu}}_{\text{Einstein}} = 8\pi G_N T_{\mu\nu}$$ 10 equations

$G_{\mu\nu}$: 2nd order derivatives of g, complicated

$T_{\mu\nu}$: Energy momentum Tensor, source of gravity

energy-momentum densities pressure

00: Newtonian limit: $\nabla^2 \phi = 4\pi G_N \rho$

$\phi$: grav potential

$\rho$: energy density (mass density)

Newton: know $\rho \Rightarrow$ solve for $\phi(x)$

$\Longrightarrow \vec{\nabla}\phi$ determines gravitational force

2. continuity equation: $\nabla_\mu T^{\mu\nu} = 0$

$\nabla_\mu$: metric inside $\left(\frac{\partial}{\partial x^\mu} + \text{"}\Gamma(g)\text{"}\right) T^{\mu\nu}$

Energy momentum conservation

$$\dot{\rho} = -\vec{\nabla}\cdot\vec{J}$$

$\rho$: charge density / energy density

$\vec{J}$: current density / momentum density

3. Geodesic equations $P^\mu \nabla_\mu P^\nu = 0$

End of GR.

Lecture 1:

Einstein $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

Cont. $\nabla_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = \sum_{a = \text{Dark Matter, photons, baryons}, \nu, \Lambda} T^{\mu\nu}$$

hard to solve in general $\longrightarrow$ perturbation theory

- $g_{\mu\nu} = \underbrace{g^{FRW}_{\mu\nu}}_{\substack{\text{homog. + isotropic} \\ \text{simple}}} + \delta g_{\mu\nu}$ ← small, complicated

- $T_{\mu\nu} = \langle T_{\mu\nu} \rangle + \delta T_{\mu\nu}$

  spatial average

  homog. + isotropic

  $\Longleftrightarrow \vec{x}$ independent

0th order:
$$\left.\begin{aligned} G^{FRW}_{\mu\nu} &= 8\pi G \langle T_{\mu\nu} \rangle \\ \nabla^{FRW}_\mu \langle T^{\mu\nu} \rangle &= 0 \end{aligned}\right\} \text{simple, today's lecture}$$

1st order:
$$\delta G_{\mu\nu} = 8\pi G\, \delta T_{\mu\nu}$$
$$\nabla^{FRW}_\mu \delta T^{\mu\nu} + \nabla^{\delta g}_\mu \langle T^{\mu\nu} \rangle = 0$$

higher order, non-perturbative, not needed for CMB, some of LSS observables

- redshifts $\dfrac{\lambda_{obs}}{\lambda_{emit}} \equiv 1+z = \dfrac{a(t_{obs})}{a(t_{emit})}$

$\underset{\text{redshift}}{\uparrow}$

- apparent luminosities
- angular sizes
- time intervals

$\Rightarrow$ distances

distance measurements are all indirect

- $T_{CMB}$ temperature of CMB + photon spectrum (departures from thermal blackbody)

- densities $\bar{\rho}_H$, $\dfrac{\bar{\rho}_{He}}{\bar{\rho}_H}$, $\bar{\rho}_{DM}$, $\bar{\rho}_{baryons}$, $\bar{n}_{galaxies}$

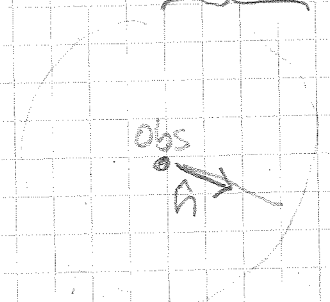
spectral line intensity — grav. lensing — $p^+, n, e^-$

Fluctuations: CMB: $\dfrac{\Delta T_{CMB}(\hat{n})}{\bar{T}_{CMB}} \equiv \dfrac{T_{CMB}(\hat{n}) - \bar{T}_{CMB}}{\bar{T}_{CMB}}$

+ polarization $E(\hat{n})$, $B(\hat{n})$

Matter distribution $\quad \dfrac{\delta\rho_m(\hat{n}, z)}{\bar{\rho}_m}$ ← matter

$\underbrace{r \leftrightarrow \Delta t}  \leftrightarrow z$




CMB photons are all from same distance $r_{CMB} \leftrightarrow z_{CMB} \simeq 1100$

$\dfrac{\delta n}{\bar{n}}$ | galaxies, Hydrogen, ...

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Homogeneous universe: (average over small scales, 10 Mpc – 3000 Mpc

↑ > galaxy clusters ↑ visible universe

Homog + isotropic $\qquad \text{Mpc} \sim 10^{22}\,\text{m}$

$$\Rightarrow \quad ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

FRW metric

scale r such that $a(t_0) \equiv a_0 = 1$

$k$ = curvature, constant

$k > 0$ closed

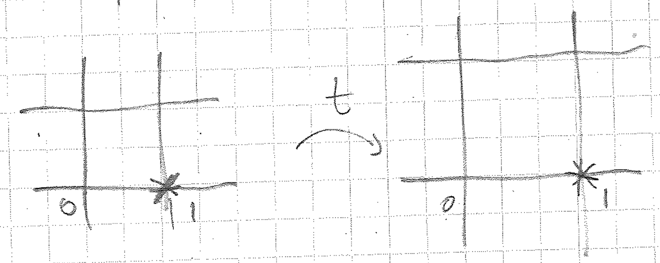
$k < 0$ open

our universe $\longrightarrow$ $k = 0$ flat

Inflation predicts $k \simeq 0$

observations find $k \simeq 0$

space expands



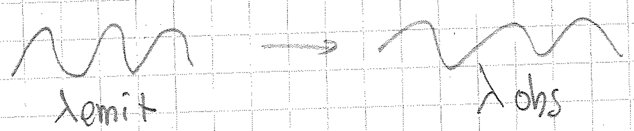

<u>metric distance</u> (co-moving) $r = \text{const.}$ for galaxy at rest

<u>physical distance</u> $dr_{phys} = \sqrt{g_{ij}\, dx^i dx^j} = a(t)\sqrt{d\vec{r}^{\,2}}$

grows with $a$

spatial only

<u>redshift of light</u> (relativistic particles)




$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a(t_{obs})}{a(t_{emit})} = \frac{f_{emit}}{f_{obs}} = \frac{E_{emit}}{E_{obs}} = \frac{P_{emit}}{P_{obs}}$$

$$\Rightarrow \boxed{P \propto \frac{1}{a}}$$

can also derive from geodesic equation $P^\mu \nabla_\mu P^\nu = 0$

valid for <u>massive</u> particles also

$$E = \sqrt{m^2 + p^2} \approx m + \frac{1}{2}\frac{p^2}{m} \approx m + \frac{1}{2} m v^2$$

$$E_{kin} \sim T \sim \frac{1}{a^2}, \quad v \sim \frac{1}{a} \quad \text{velocities redshift.}$$

$v \ll c$ today