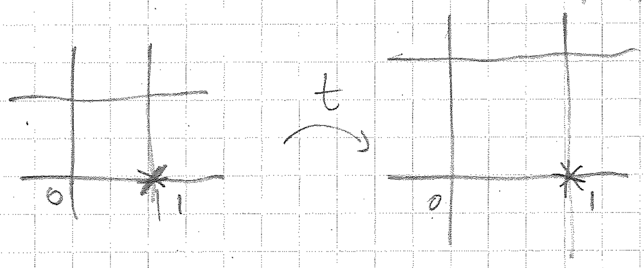
space expands

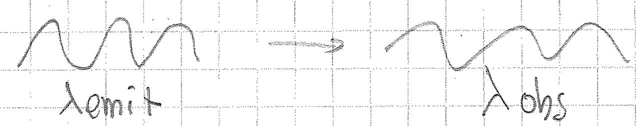


metric distance (co-moving) r = const. for galaxy at rest

physical distance $dr_{phys} = \sqrt{g_{ij}\, dx^i dx^j} = a(t)\sqrt{d\vec{r}^{\,2}}$
grows with a

spatial only

redshift of light (relativistic particles)



physical distances

$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a(t_{obs})}{a(t_{emit})} = \frac{f_{emit}}{f_{obs}} = \frac{E_{emit}}{E_{obs}} = \frac{P_{emit}}{P_{obs}}$$

$$\Longrightarrow \boxed{P \propto \frac{1}{a}}$$

Can also derive from geodesic equation $P^\mu \overset{FRW}{\nabla_\mu} P^\nu = 0$

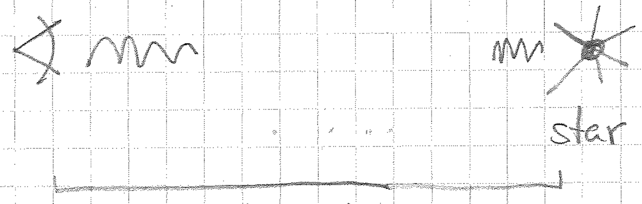
valid for massive particles also

$$E = \sqrt{m^2 + p^2} \approx m + \frac{1}{2}\frac{p^2}{m} \approx m + \frac{1}{2} m v^2$$

$$E_{kin} \sim T \sim \frac{1}{a^2}, \quad v \sim \frac{1}{a} \quad \text{velocities redshift.}$$

$v \ll c$ today.

Hubble's law



$$H(t) \equiv \frac{\dot a(t)}{a(t)}$$

$$d = c(t_0 - t)$$

$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{\overset{=1}{a_0}}{a(t_{emit})} \overset{\text{Taylor}}{=} \frac{\cancel{a_0}}{\cancel{a_0}\left(1 + \frac{\dot a}{a}\Big|_0 (t-t_0) + \mathcal{O}(t-t_0)^2\right)}$$

$$\cong 1 + \frac{\dot a}{a}\Big|_{t_0} (t_0 - t) = 1 + H_0 \frac{d}{c}$$

$$\Rightarrow \boxed{d(z) = \frac{c}{H_0} z}$$ distance-redshift relation

hard (d) easy (z)

$\boxed{\text{PLOT:}}$ Hubble plot

SHOES 2025 $\quad H_0 = 73 \pm 1 \, \frac{km/s}{Mpc}$ $\leftarrow c$, $\leftarrow d$

$$\equiv h \; 100 \, \frac{km/s}{Mpc}$$

"5σ Hubble tension": $H_0 = 67.5 \pm 0.5$ CMB + ΛCDM model

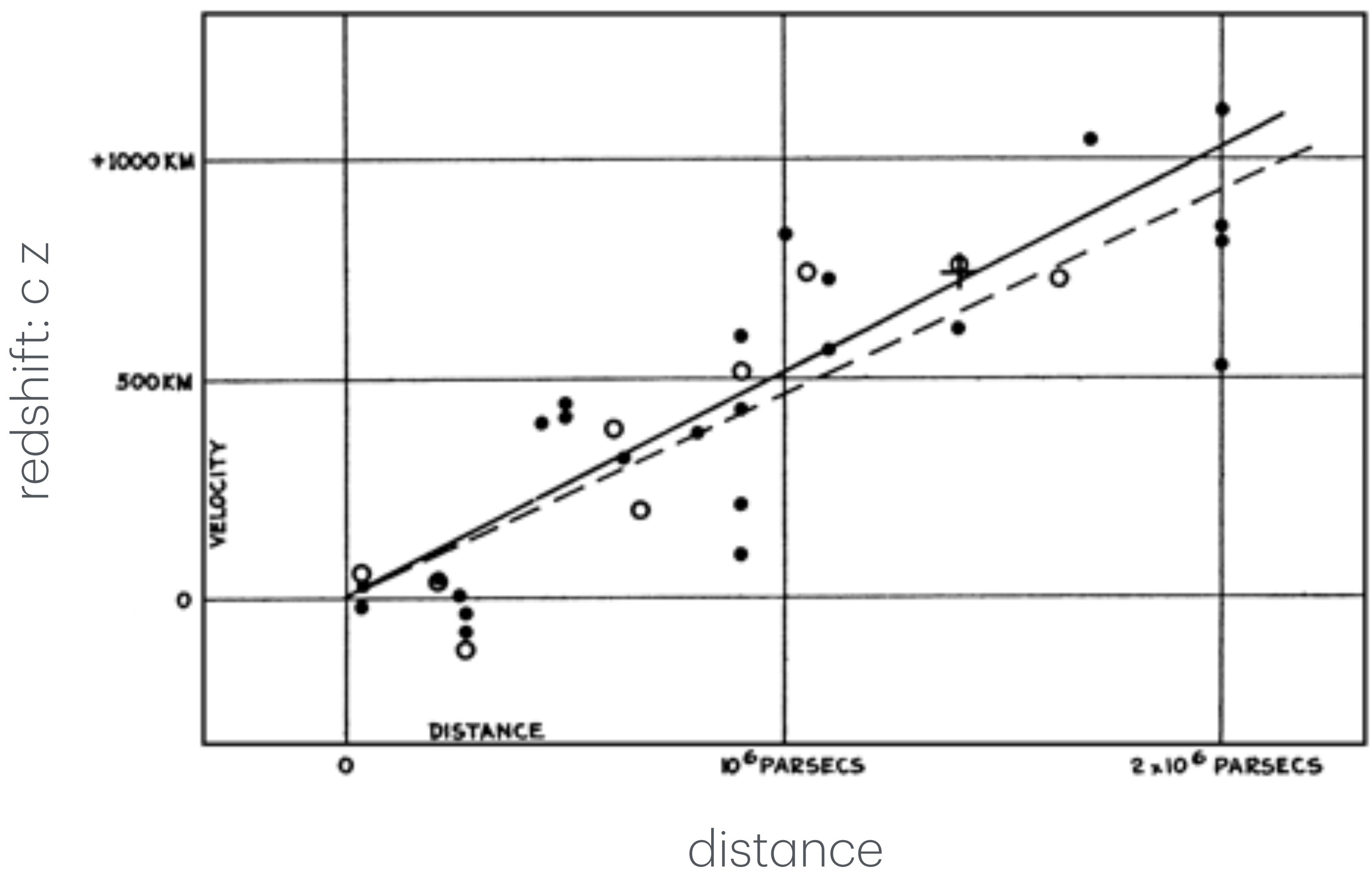
we expanded for small z ($d H_0$)

large redshift: light travels on geodesic $0 = ds^2 = dt^2 - a^2 dr^2$

comoving distance (not measurable) $\Rightarrow d = \int_0^d dr = \int_{a_{emit}}^1 \frac{dt}{a} = \int \frac{da}{\underbrace{a \dot a}_{a^2 H}} = \int_0^z \frac{dz'}{H(z')}$

Luminosity distance: $d_L = (1+z) \int_0^z \frac{dz'}{H(z')}$

Hubble's original data: H_0~500 km/s/Mpc



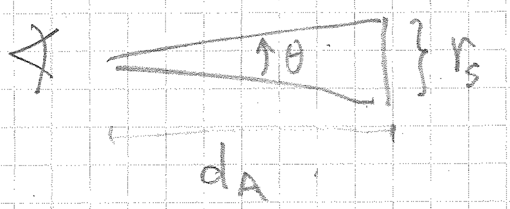
Hubble, 1929

Angular diameter distance: $d_A = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}$

if you know the size of an enormous scale

"standard ruler": BAO scale $= r_s =$ sound horizon

$$= \int_0^{t_{dec}} c_s \frac{dt}{a(t)}$$



$\Rightarrow$ know r_s, measure θ, infer d_A

$\Rightarrow$ can measure $H(z) = \frac{\dot{a}}{a}$

<u>Q</u>: Can we also <u>calculate it</u>? Yes! Einstein

$$\underset{\substack{\uparrow \\ \text{Einstein} \\ \text{Tensor} \\ \sim \partial^2 g}}{G_{\mu\nu}} = \underset{\substack{\uparrow \\ \text{Energy} \\ \text{Momentum} \\ \text{tensor}}}{T_{\mu\nu}} \qquad T = \begin{pmatrix} \rho(t) & & & \\ & -p(t) & & \\ & & -p(t) & \\ & & & -p(t) \end{pmatrix} \leftarrow \text{homogeneity, isotropy}$$

energy density ($\rho(t)$)

pressure ($p(t)$)

$$00: \boxed{\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}} \quad \text{Friedmann } \underline{1}$$

curvature (k)

$$i0: \quad 0 = 0$$

$$ij: \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad F2$$

(redundant)

can solve for $H(t)$ if we know $\rho(t)$

continuity equation: $\nabla^{FRW}_\mu T^\mu{}_\nu = 0$ $\quad \nabla_\mu T^\mu{}_\nu = \partial_\mu T^\mu{}_\nu + \Gamma^\mu_{\mu\lambda} T^\lambda{}_\nu - \Gamma^\lambda_{\mu\nu} T^\mu{}_\lambda$

$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\alpha\mu} - \partial_\lambda g_{\mu\nu})$

$\dot a = \frac{da}{dt}$ $\qquad g = \begin{pmatrix} +1 & \\ & a(t)^2 \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \end{pmatrix}$

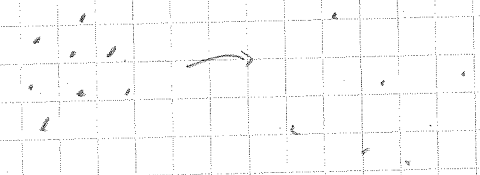
$$\nu = 0: \quad \dot\rho = -3\frac{\dot a}{a}(\rho + p) \iff a\frac{d\rho}{da} = -3(\rho + p)$$

$\Rightarrow$ need also p, assume $\quad p = w\rho \quad \Rightarrow \quad \frac{d\rho}{d\ln a} = -3(1+w)\rho$

"equation of state" parameter

<u>cold matter</u> ("pressureless dust") $\quad w = 0 = p$

$$\frac{d\rho}{\rho} = -3\frac{da}{a} \Rightarrow \rho \sim a^{-3}$$

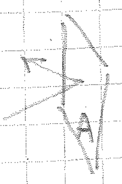


volume dilution

$\rho = m\, n$

energy density $\qquad$ number density

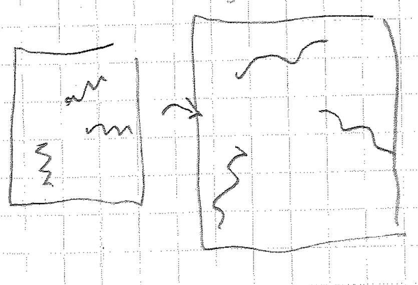
<u>radiation</u> (relativistic particles) $\quad w = \frac{1}{3} \quad \Rightarrow p = \frac{1}{3}\rho$



$P = \frac{F}{A}$

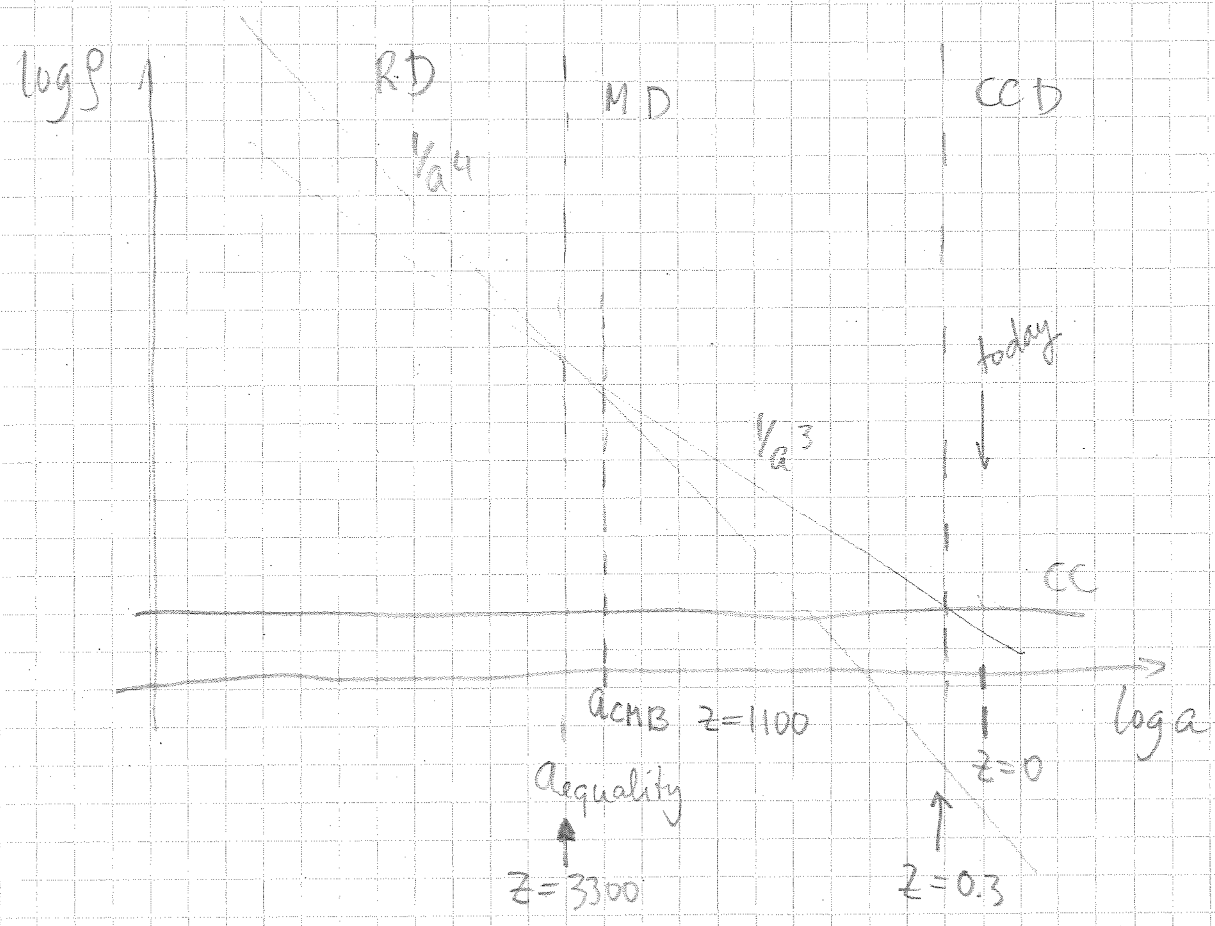
$\rho \sim a^{-4} = E\, n$

E: energy redshift $\sim \frac{1}{a}$ $\qquad n$: $\frac{1}{a^3}$



Cosmological constant : $w = -1 \Rightarrow \rho = \text{const}$

	w	ρ
matter	0	a^{-4}
radiation	$1/3$	a^{-3}
CC	-1	const
general w = const	w	$a^{-3(1+w)}$
$w_0 w_a$	$w_0 + w_a(1-a)$	$a^{-3(1+w_0+w_a)}\, e^{-3w_a(1-a)}$



Friedmann I for single component dominance :

$$\left(\frac{\dot a}{a}\right)^2 \propto \rho(a) \sim a^{-3(1+w)} \Rightarrow \begin{cases} a \sim t^{\frac{2}{3(1+w)}} & w \neq -1 \\ a \sim e^{Ht} & w = -1 \end{cases}$$

Our Universe

$$\text{FI:}\quad H(a) = \sqrt{\frac{8\pi G}{3}}\sqrt{\frac{\rho_r^0}{a^4} + \frac{\rho_m^0}{a^3} + \rho_\Lambda \underbrace{\cancel{- \frac{3k}{8\pi G}\frac{1}{a^2}}}_{\text{curvature just like another energy density with } w=-1/3}}$$

ignore

$$\equiv H_0 \sqrt{\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda}$$

with $\Omega_r + \Omega_m + \Omega_\Lambda = 1$

$\Omega_r = 9.4 \quad 10^{-5}$ 3:2 γ, ν's

$\Omega_m = 0.32 \pm 0.02$ 5:1 DM : baryons (n, p^+, e^-)

$\Omega_\Lambda = 0.68 \pm 0.02$

$|\Omega_K| \leq 0.01$

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(a=\frac{1}{1+z'})}$$

$\boxed{\text{PLOT}}$ supernovae $z \in [0, 1.2]$

$$H \cong H_0\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$$

DESI (2025) : assume "dark energy" is not CC

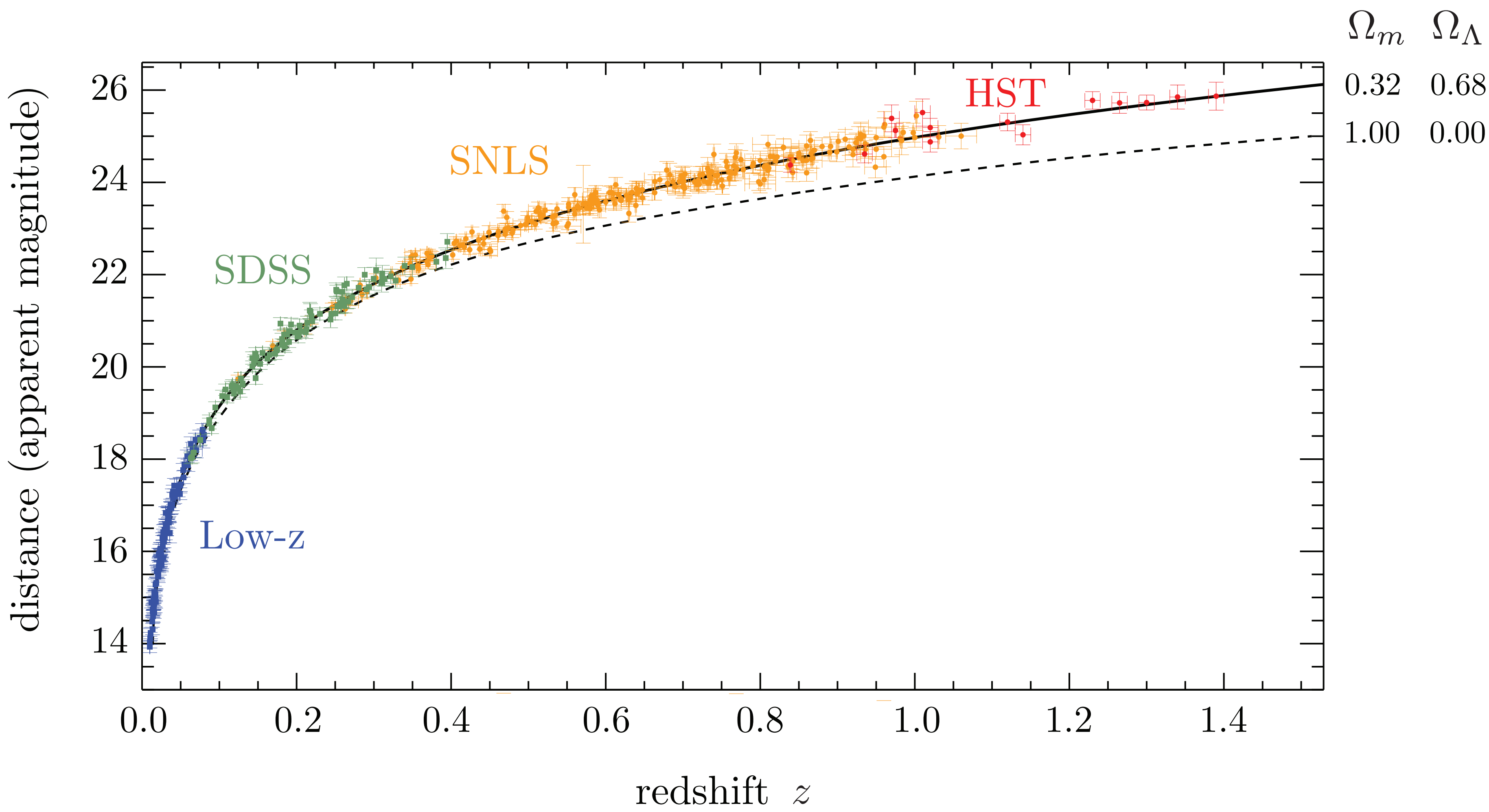
but $w = w_0 + w_a(1-a)$

combined fit SN + DESI BAO prefers

$w_0 = -0.7$
$w_a = -0.7$ $\sim 3\sigma$ from $w=-1$ CC

$\boxed{\text{PLOTs}}$ from DESI

Hubble diagram from Supernova 1a

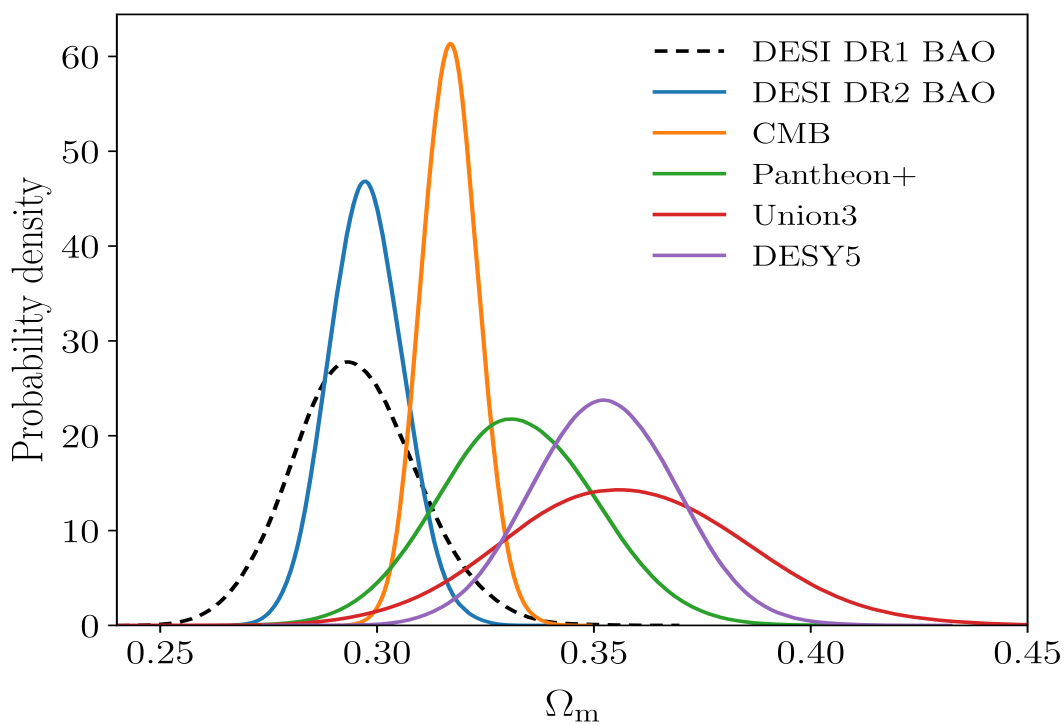


Baumann notes, Fig 1.7

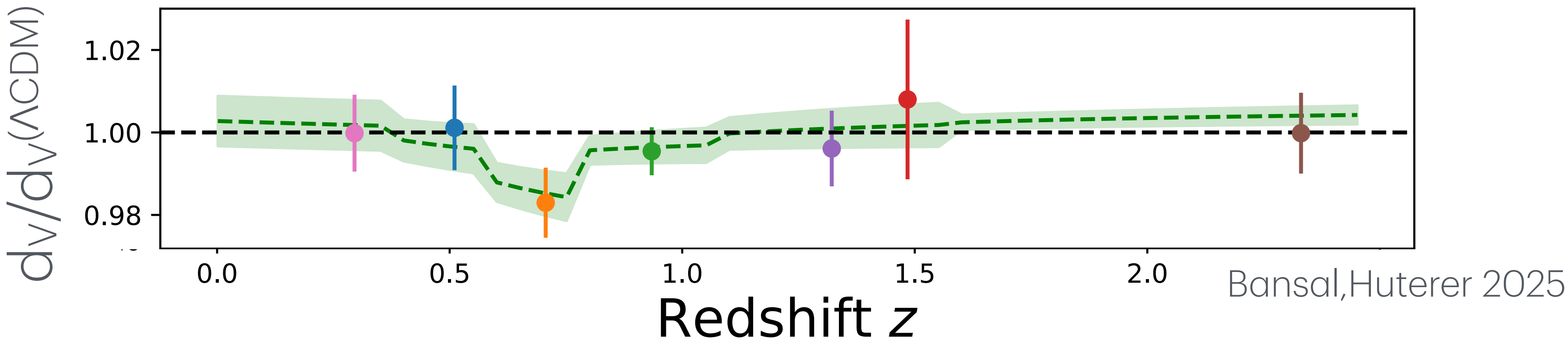
DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints

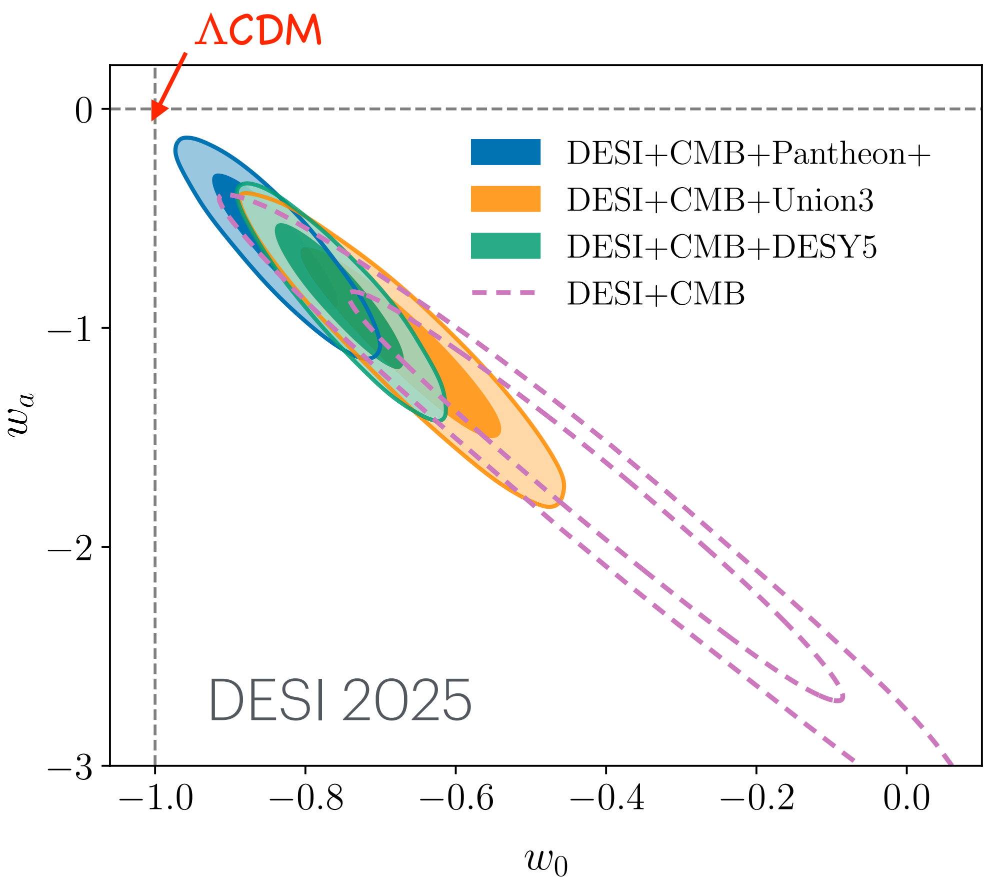
arXiv:2503.14738v2

Model/Dataset	Ω_m	H_0 [km s^{-1} Mpc^{-1}]	$10^3\Omega_\mathrm{K}$
ΛCDM			
DESI	0.2975 ± 0.0086	—	—
ΛCDM+Ω_K			
CMB	$0.354^{+0.020}_{-0.023}$	63.3 ± 2.1	$-10.7^{+6.4}_{-5.3}$
DESI	0.293 ± 0.012	—	25 ± 41
DESI+CMB	0.3034 ± 0.0037	68.50 ± 0.33	2.3 ± 1.1



DESI (2025) w0wa fit





dark energy equation of state:

$$w = w_0 + w_a(1 - a)$$

$$\frac{H(a)}{H_0} = \sqrt{\frac{\Omega_m}{a^3} + \frac{\Omega_{de}}{a^{3(1+w_0+w_a)}} e^{-3w_a(1-a)}}$$