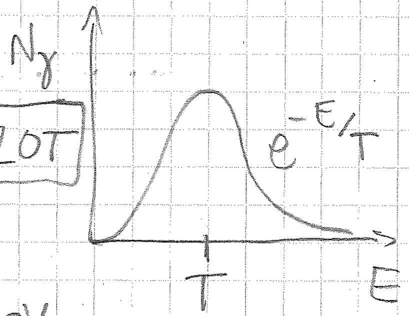
Thermal history

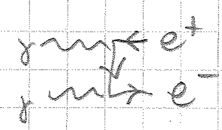
CMB photons have thermal spectrum $\boxed{\text{PLOT}}$

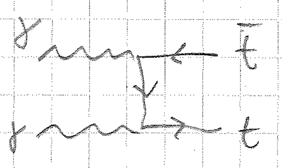


$T_0 \sim \langle E \rangle \sim 2.72548 \pm 0.00057\,K \approx 2.3\ 10^{-4}\,eV$

$\rho_\gamma \sim \langle E \rangle n_\gamma \sim T_\gamma^{\ 4}$

z	T_γ		
0	$10^{-4}\,eV$	we observe CMB	
~ 10	$10^{-3}\,eV$	stars, galaxies	
$\sim 10^4$	eV	Atomic binding energy $\Rightarrow$ no atoms $e^-, p^+, He^{++}, \ldots$	recombination CMB
10^{10}	MeV	nuclear binding $\rightarrow$ no nuclei $p^+, n, e^- e^+$	BBN $\frac{\rho_H}{\rho_{He}}$
10^{13}	GeV	QCD binding $\rightarrow$ quarks, gluons q, g, e^+, e^-	
10^{16}	TeV	weak scale, all SM particles are produced in thermal plasma	Baryogenesis, DM production





hot big bang

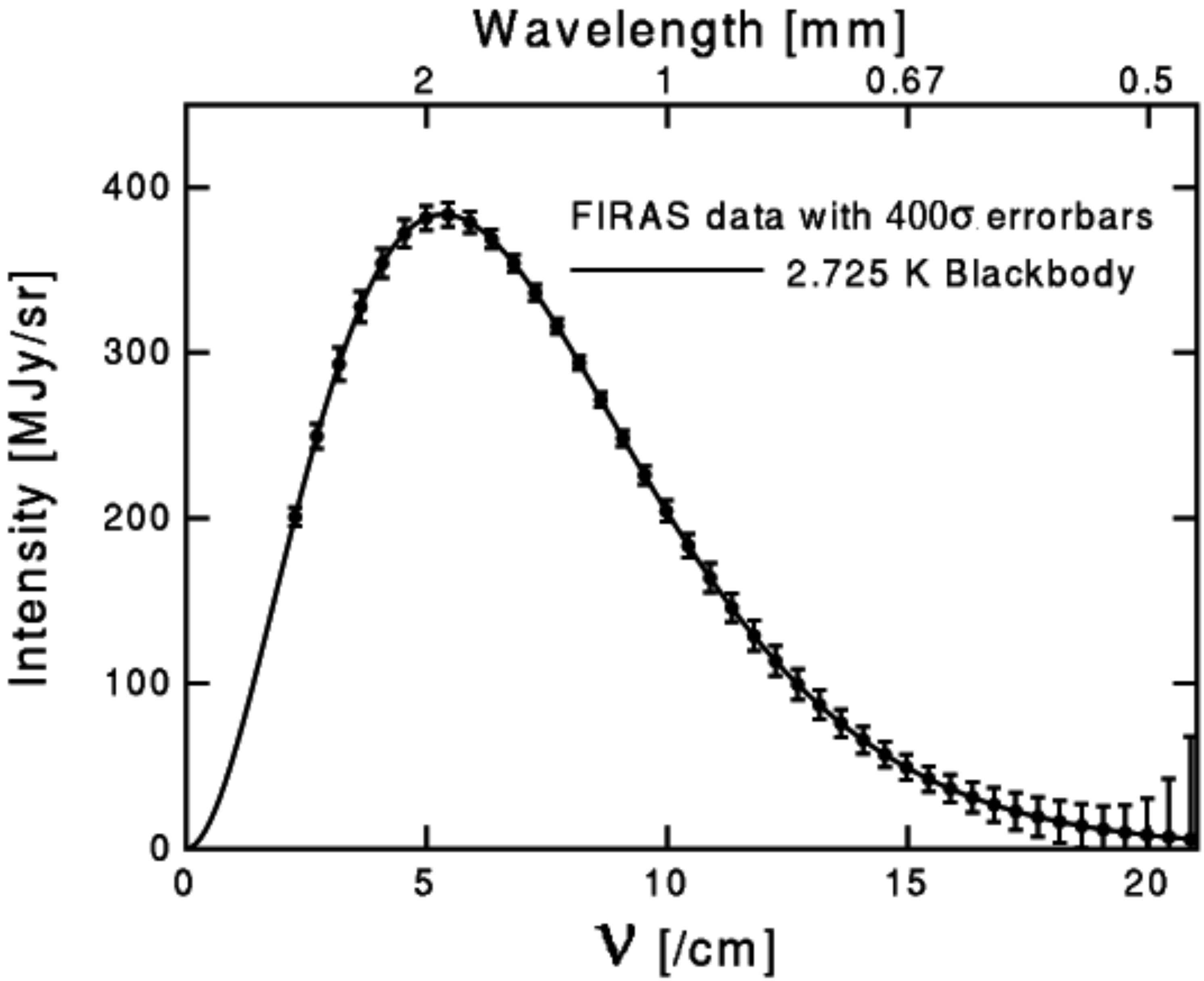
"Reheating" $\quad T \gg MeV$

Inflation

Reheating creates initial condition of hot big bang

radiation dominated universe $\quad H^2 = \frac{8\pi G}{3}\rho_r = \frac{1}{3 M_{pl}^2}\rho_r \sim \frac{T_\gamma^{\ 2}}{M_{pl}}$

CMB, perfect thermal photon spectrum



Thermal history

high density $\Rightarrow$ high scattering rate $\Gamma(T)$

key observation: $\Gamma(T)$ vs. $H(T)$

$T \uparrow$

$\Gamma \gg H$ many scatters before universe size changes $\Rightarrow$ equilibration!
$\Rightarrow$ particles are in thermal equilibrium

$T_{freeze\text{-}out}$ $\Gamma = H$ "complicated"

$\Gamma \ll H$ universe increases by multiples in size between scatters $\Rightarrow$ scatters are irrelevant $\rightarrow$ ignore Γ

thermal equilibrium:

scattering rates high enough that particles have equilibrium distributions $\rightarrow$ extremely predictive

$$n = g \int \frac{d^3p}{(2\pi)^3} f(p,T)$$

g: degrees of freedom e.g. 2 for γ

$f(p,T)$: distribution function

$$f_{B/F}(p) = \frac{1}{e^{E_p/T} \mp 1}$$ Bose Einstein / Fermi Dirac

$E_p = \sqrt{m^2+p^2}$

$$\rho = g \int \frac{d^3p}{(2\pi)^3} E f$$

$$p = \text{''} \quad \frac{p^2}{3E} f$$ (ignore chemical potential)

massless (or $m \ll T$)

$$\rho = g \frac{\pi^2}{30} T^4 \begin{cases} 1 & \text{boson} \\ 7/8 & \text{fermion} \end{cases}$$

e.g.

$$\Rightarrow \rho_\gamma = \frac{\pi^2}{15} T^4$$

today $\rho_{CMB} \approx 10^{-33} \frac{g}{cm^3}$ water 1, air 10^{-3}

$T_\gamma = 2.73K$

$$P_\gamma = \frac{\pi^2}{45} T^4 = \frac{\rho_\gamma}{3} \, !$$

$n_{CMB} \approx 400 \, \gamma/cm^3$

$$n_\gamma = 0.24 \, T^3$$

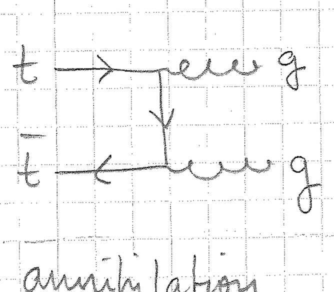
SM thermal plasma $T \gg m_{top}$

$$\rho_{SM} = \frac{\pi^2}{30} T^4 \underbrace{\left(\sum_{\text{bosons}} g_b + \sum_{\text{fermions}} \frac{7}{8} g_f \right)}_{\text{exercise: } 106.75}$$

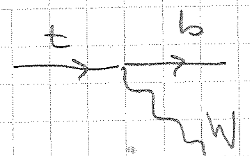
massive, $m \gg T$

$$\rho \sim e^{-m/T}$$ negligible compared with T^4

as temperature drops below mass particles disappear from the plasma. What happens?

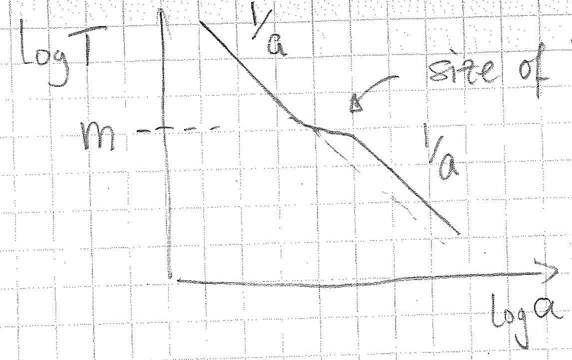


annihilation



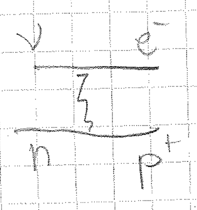
decay if unstable

Energy (Entropy) gets passed to rest of plasma



size of "step" can be computed from entropy conservation

$a^3 s = a^3 \frac{\rho + P}{T} = \text{const.}$



$n\langle\sigma v\rangle$

$T^3 \frac{T^2}{M_W^4} = \frac{T^2}{M_{pe}}$

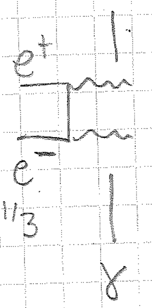
$\Rightarrow T \approx \left(\frac{M_W^4}{M_{pe}}\right)^{1/3} \approx \text{MeV}$

<u>example</u>: @ $T_\gamma = 2\,\text{MeV}$ neutrinos decouple

$\underbrace{e^+ e^- \gamma\, \nu}$ $\quad \Gamma_{weak} \ll H$

$e^+ e^- \gamma$ plasma $\qquad \nu \qquad T_\gamma = T_\nu$

0.5 MeV



$T_\nu \sim \frac{1}{a}$

ν

step: $\frac{T_{IR}}{T_{UV}} = \left(\frac{2 + 7/8 \cdot 4}{2}\right)^{1/3}$

$T_\gamma \sim \frac{1}{a} \cdot \text{"step"}$

entropy conservation: $\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$

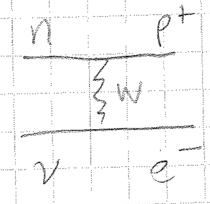
$$\Longrightarrow \frac{\rho_\nu}{\rho_\gamma} = 3 \cdot \frac{7}{8} \, \frac{2}{2} \left(\frac{4}{11}\right)^{4/3} = 0.68$$

$\frac{\rho_\nu}{\rho_r} \cong 40\%$ free streaming fraction

An example of beyond equilibrium physics: BBN

$$\frac{\rho_{He}}{\rho_b} = Y_p \quad \text{Helium abundance} \quad 0.245(3) \ \text{exp.}$$

- total # of nucleons "b" conserved, n_b dilutes $\sim \frac{1}{a^3}$
- at $T \gg \text{MeV}$, $n_p = n_n$



weak interactions equilibrium

- after BBN, $T \lesssim 0.1\,\text{MeV}$ all$^{0.999}$ neutrons are bound in ^4He, protons are in either ^4He or p^+

$$\Rightarrow \frac{\rho_{He}}{\rho_b} \cong \frac{2 n_n}{n_b} \equiv 2 X_n \quad \text{assuming } m_n \simeq m_p$$

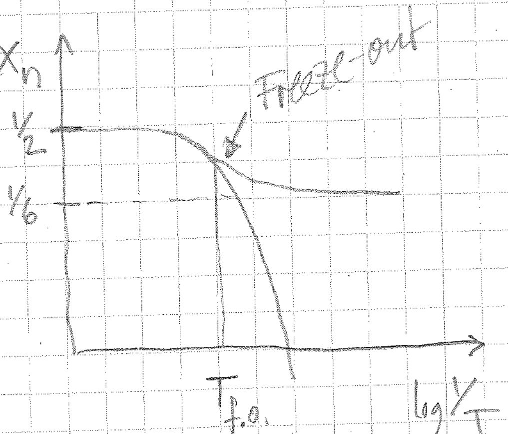
$\Rightarrow$ need to compute the fraction of neutrons at ^4He formation.

$\boxed{\text{PLOT}}$ Abundances

<u>3 steps:</u>

(1) weak interaction freeze-out

$$n + \nu_e \leftrightarrow p^+ + e^-$$



$$\frac{n_n}{n_p} \simeq e^{-\frac{\Delta E}{T}} \rightarrow e^{-\frac{0.7}{0.5}} \sim \frac{1}{5}$$

$$\Delta E = m_n - m_p - m_e \simeq 0.7\,\text{MeV}$$

Freezeout when $n_\nu \langle \sigma v \rangle \simeq T^3 \alpha_W^2 \frac{T^2}{M_W^4} = H \simeq \frac{T^2}{M_{pl}}$

$$\Rightarrow T \simeq \left(\frac{M_W^4}{M_{pl}} \frac{1}{\alpha_W^2} \right)^{1/3} \simeq 0.5\,\text{MeV} \qquad t \sim 1\,\text{sec}$$

Big Bang Nucleosynthesis - Helium

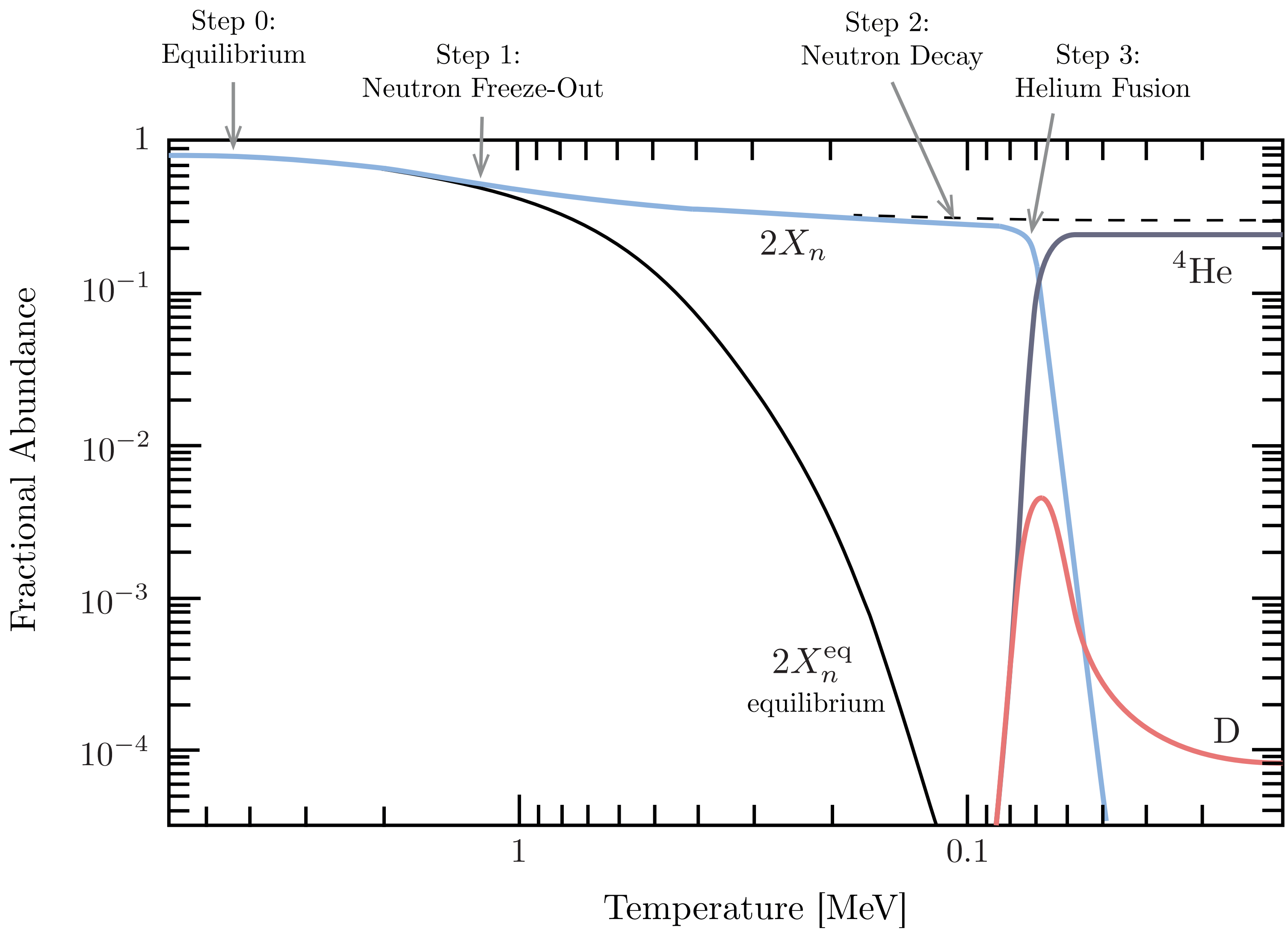


Figure 3.9: *Numerical results for helium production in the early universe.*

BBN

(2) free neutron decay until neutrons bind into Deuterium (at $t \sim 260$ sec)

$$X_n(t) = X_n(t_{\text{freeze-out}}) \cdot e^{-t/\tau_n}$$

neutron lifetime $\tau_n \hat{=} 880$ sec

$$\Rightarrow X_n(260) \approx \frac{1}{8} \quad \boxed{\Rightarrow Y_p \approx 1/4}$$

(3) neutron binding: $n + p^+ \longrightarrow D + \gamma$ in equilibrium

$$\frac{n_D}{n_n n_p} \approx \left(\frac{4\pi}{m_n T}\right)^{3/2} e^{\Delta E/T}$$

$$\Delta E = m_p + m_n - m_D = 2.2 \text{ MeV}$$

$$n_p \approx n_b \equiv \eta_b n_\gamma \approx \eta_b T^3$$

$$\Rightarrow \frac{n_D}{n_n} \approx \underbrace{\eta_b}_{10^{-9}} \underbrace{\left(\frac{4\pi T}{m_n}\right)^{3/2}}_{10^{-5}} e^{\Delta E/T} \overset{!}{\approx} 1 \quad \text{when} \quad \frac{\Delta E}{T} \approx 31$$

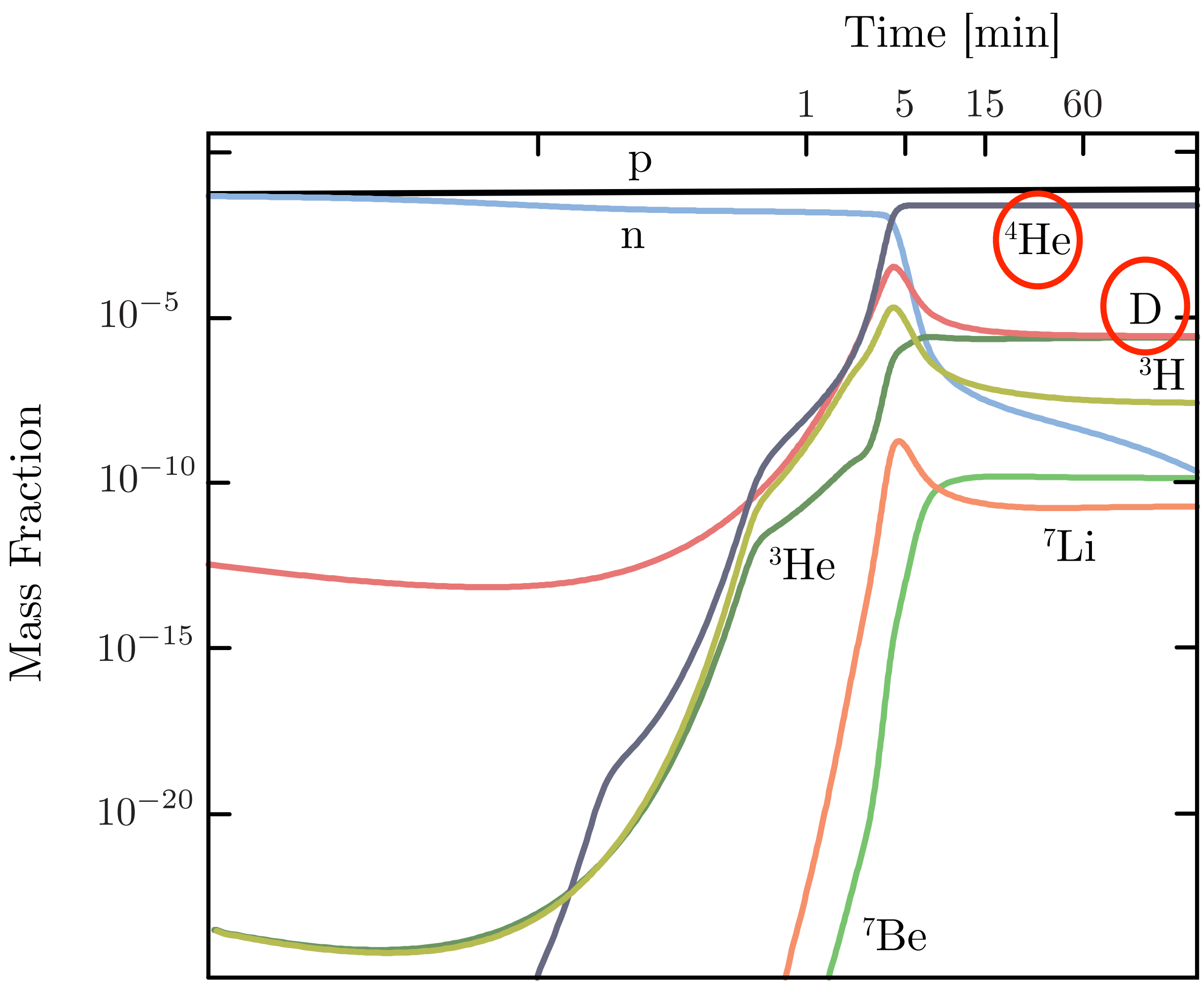
$$\Rightarrow T \approx 0.07 \text{ MeV}$$

Age of universe then? Friedmann I $\Rightarrow t \sim 260$ sec.

$\boxed{\text{PLOT}}$ Abundances

$\boxed{\text{PLOT}}$ observations vs $Y_p(\eta_b)$

Big Bang Nucleosynthesis - light elements



He/H ~ 1% precision

D/H ~ 1% precision

Figure 3.11: *Numerical results for the evolution of light element abundances.*

Big Bang Nucleosynthesis - constraints on new physics

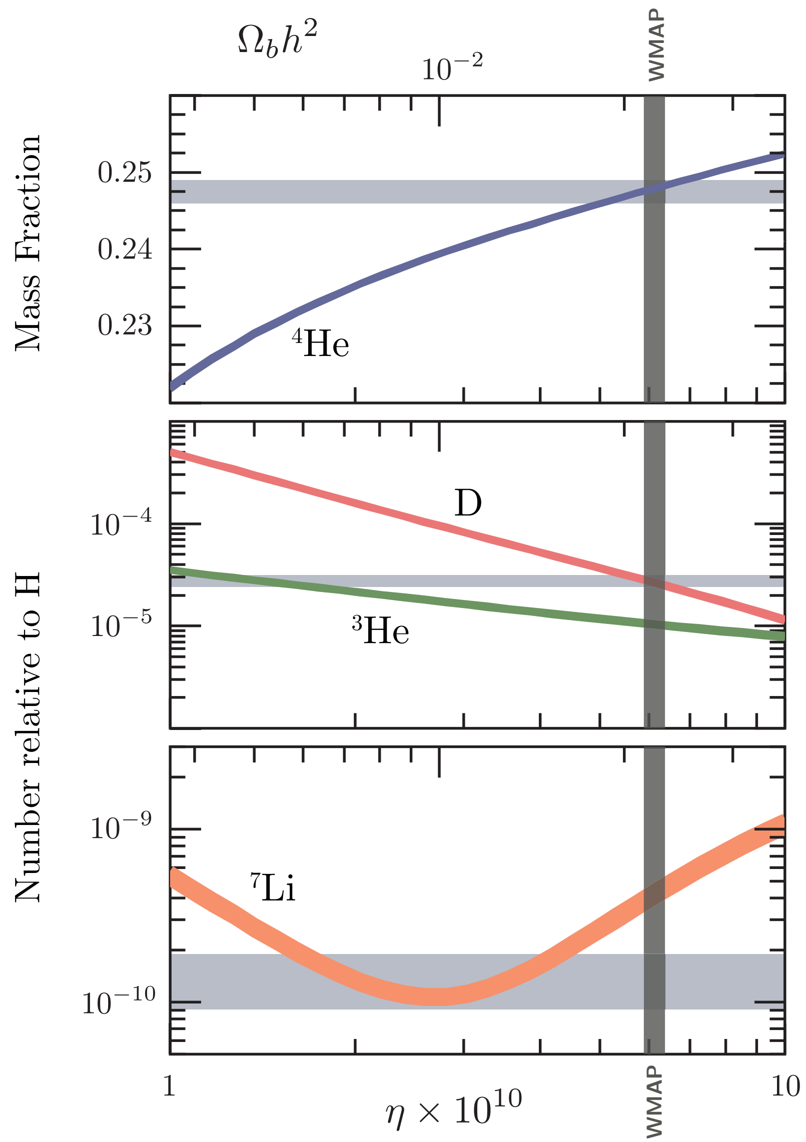


Figure 3.10: *Theoretical predictions (colored bands) and observational constraints (grey bands).*

Recombination $T \sim eV$

$$e^- + p^+ \to H + \gamma$$

(also He^{++})

$$X_e \equiv \frac{n_{e^-}}{n_b} = \text{"free electron fraction"}$$

find $$\frac{X_e^2}{1-X_e} \approx \underbrace{\left(\frac{m_e}{2\pi T}\right)^{3/2}}_{10^9} e^{-\frac{\Delta E}{T}} \underbrace{\frac{1}{\eta_b}}_{10^9 \sim e^{21}} \#$$

ΔE = 13.6 eV

$\Rightarrow X_e$ drops exponentially for $\frac{\Delta E}{T} \gtrsim 42$

$$\Rightarrow T_{rec} = \frac{13.6\,eV}{42} \sim 0.3\,eV \iff z = 1300$$

after recombination, "no more" free electrons

$\Longrightarrow$ photons are now free, propagate without scattering on geodesics $\to$ redshift $\to$ CMB.

$\boxed{\text{PLOT}}$

Recombination

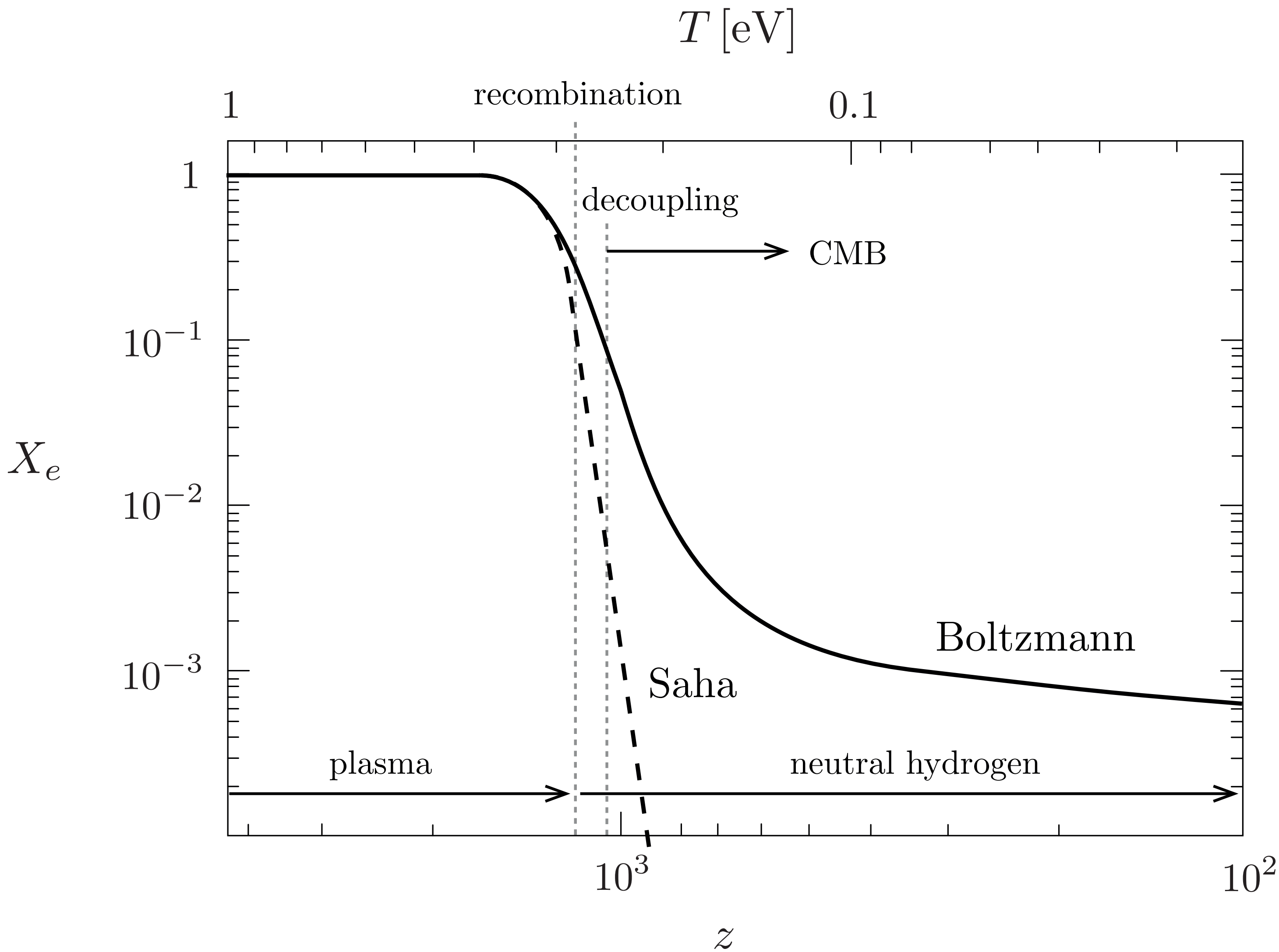


Figure 3.8: *Free electron fraction as a function of redshift.*