T

$q, \bar{q}, g, \mu^\pm, \tau^\pm, e^\pm, \gamma, \nu$ + small asymmetry in B

GeV QCD phase transition

$\pi, \mu^\pm, e^\pm, \gamma, \nu$ + 10^{-9} p, n

10 MeV e^+, e^-, γ, ν + 10^{-9} p, n

2 MeV ν decoupling

BBN

0.5 MeV

0.1 MeV γ, ν 10^{-9} p^+, He^{++}, e^-
+ tiny amounts of $D, T, {}^3He$, Li, Be

Recombination 0.3 eV γ, ν 10^{-9} H, He

Big Bang Nucleosynthesis - Helium

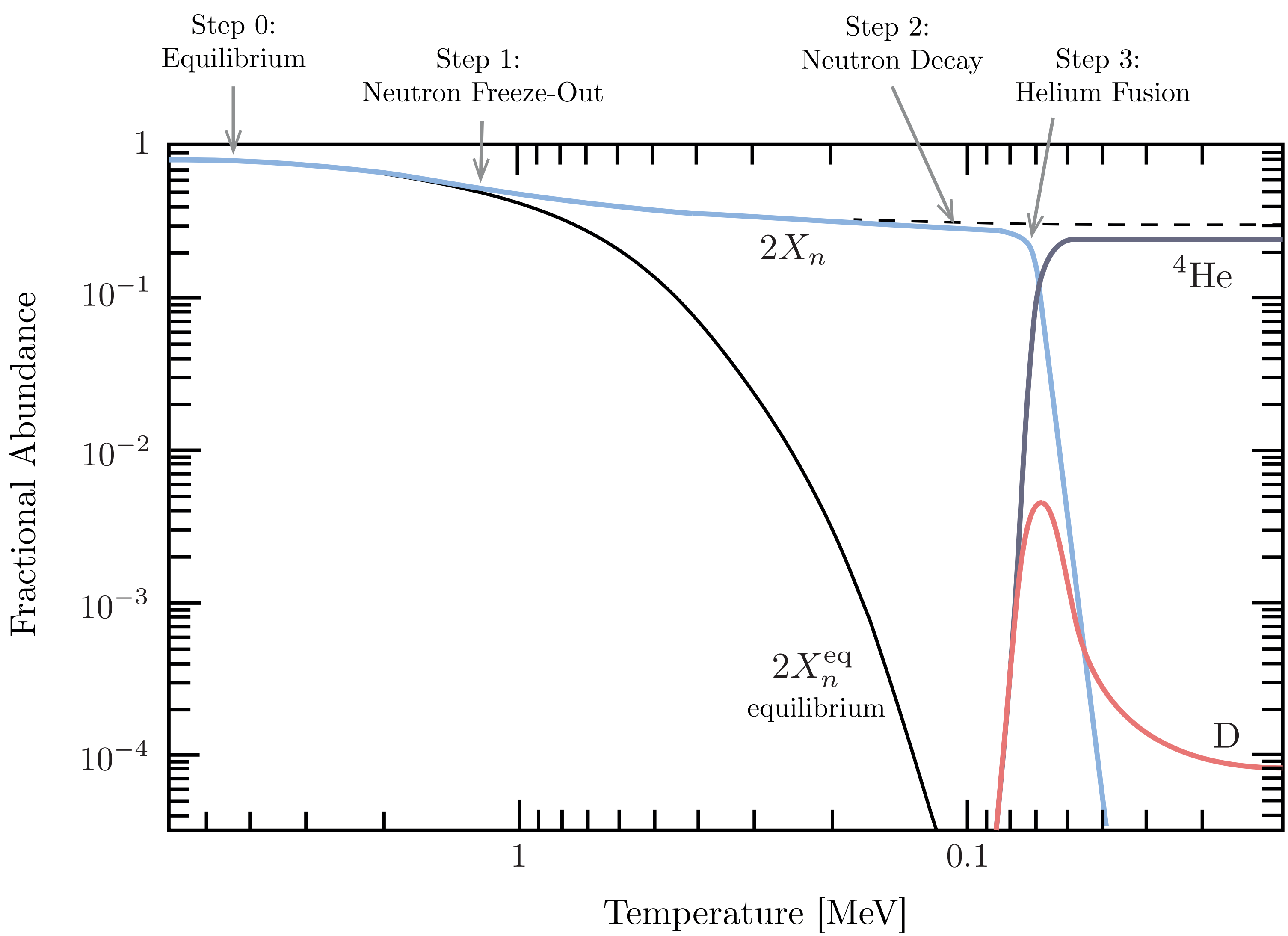
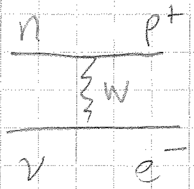


Figure 3.9: *Numerical results for helium production in the early universe.*

An example of beyond equilibrium physics: BBN

$$\frac{\rho_{He}}{\rho_b} = Y_p \quad \text{Helium abundance} \quad 0.245(3) \text{ exp.}$$

- total # of nucleons "b" conserved, n_b dilutes $\sim \frac{1}{a^3}$
- at $T \gg \text{MeV}$, $n_p = n_n$



weak interactions equilibrium

- after BBN, $T \lesssim 0.1\,\text{MeV}$ all (0.999) neutrons are bound in ^4He, protons are in either ^4He or p^+

$$\Rightarrow \quad \frac{\rho_{He}}{\rho_b} \simeq \frac{2 n_n}{n_b} \equiv 2 X_n \qquad \text{assuming } m_n \simeq m_p$$

$\Rightarrow$ need to compute the fraction of neutrons at ^4He formation.

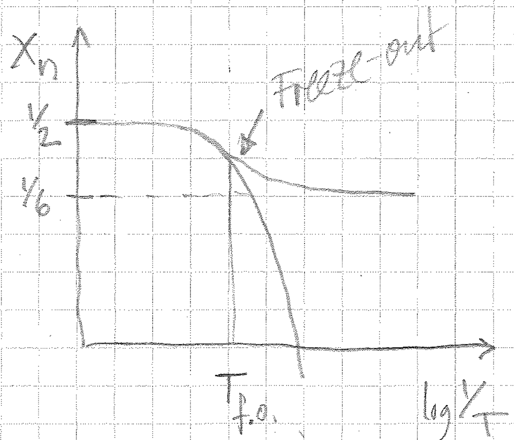
$\boxed{\text{PLOT}}$ Abundances

3 steps:

(1) weak interaction freeze-out

$$n + \nu_e \leftrightarrow p^+ + e^-$$

$$\frac{n_n}{n_p} \simeq e^{-\frac{\Delta E}{T}} \rightarrow e^{-\frac{0.7}{0.5}} \sim \frac{1}{5}$$



$$\Delta E = m_n - m_p - m_e \simeq 0.7\,\text{MeV}$$

Freezeout when $\quad n_\nu \langle \sigma v \rangle \simeq T^3 \alpha_W^2 \frac{T^2}{M_W^4} = H \simeq \frac{T^2}{M_{pe}}$ $\leftarrow$ sensitivity to Neff!

$$\Rightarrow T \simeq \left(\frac{M_W^4}{M_{pe}} \frac{1}{\alpha_W^2} \right)^{1/3} \simeq 0.5\,\text{MeV} \qquad t \sim 1\,\text{sec}$$

BBN

(2) free neutron decay until neutrons bind into Deuterium (at $t \sim 260$ sec)

$$X_n(t) = X_n(t_{\text{freeze-out}}) \cdot e^{-t/\tau_n}$$

neutron lifetime $\tau_n \approx 880$ sec

$$\Rightarrow X_n(260) \simeq \frac{1}{8} \qquad \boxed{\Rightarrow Y_p \approx 1/4}$$

(3) neutron binding: $n + p^+ \longrightarrow D + \gamma$ in equilibrium

$$\frac{n_D}{n_n n_p} \cong \left(\frac{4\pi}{m_n T}\right)^{3/2} e^{\Delta E/T} \qquad \Delta E = m_p + m_n - m_D = 2.2 \text{ MeV}$$

$$n_p \approx n_b \equiv \eta_b \, n_\gamma \approx \eta_b \, T^3$$

$$\Rightarrow \frac{n_D}{n_n} \approx \underbrace{\eta_b}_{10^{-9}} \underbrace{\left(\frac{4\pi T}{m_n}\right)^{3/2}}_{10^{-5}} e^{\Delta E/T} \overset{!}{\approx} 1 \quad \text{when} \quad \frac{\Delta E}{T} \approx 31$$

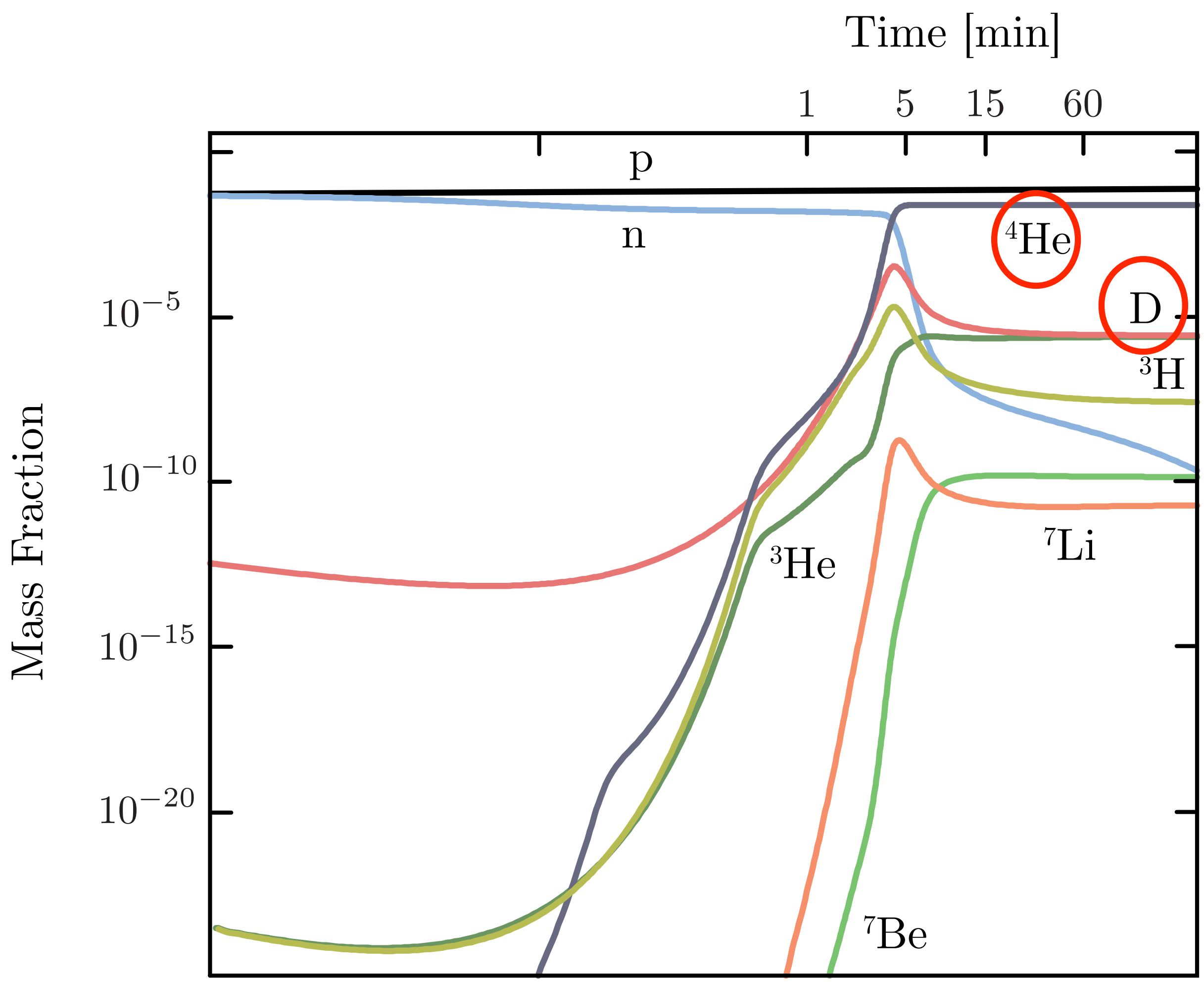
$$\Rightarrow T \approx 0.07 \text{ MeV}$$

Age of universe then? Friedmann I $\Rightarrow t \sim 260$ sec.

$\boxed{\text{PLOT}}$ Abundances

$\boxed{\text{PLOT}}$ observations vs $Y_p(\eta_b)$

Big Bang Nucleosynthesis - light elements



He/H ~ 1% precision

D/H ~ 1% precision

Figure 3.11: *Numerical results for the evolution of light element abundances.*

Big Bang Nucleosynthesis - constraints on new physics

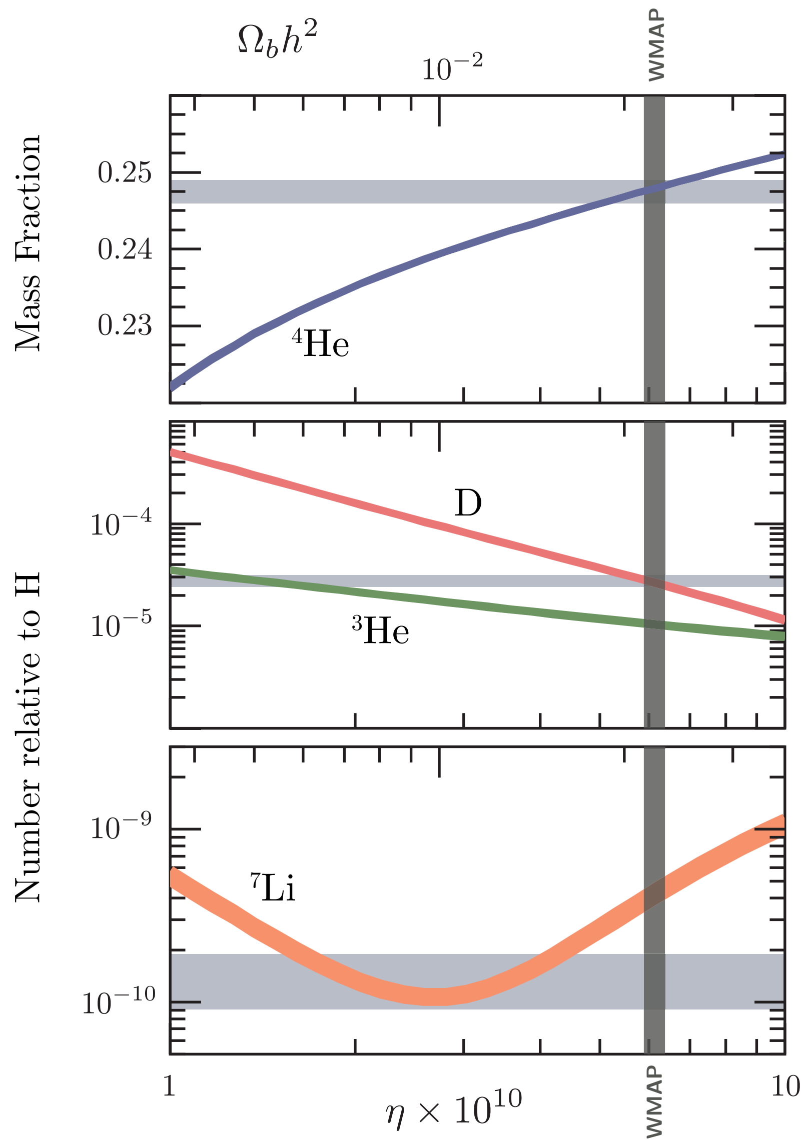


Figure 3.10: *Theoretical predictions (colored bands) and observational constraints (grey bands).*

Recombination $T \sim eV$

$$e^- + p^+ \to H + \gamma$$

(also He^{++})

$$X_e \equiv \frac{n_{e^-}}{n_b} = \text{"free electron fraction"}$$

from equil. dist. derive Saha Eq.:

$$\frac{X_e^2}{1-X_e} \approx \left(\frac{m_e}{2\pi T}\right)^{3/2} e^{-\frac{\Delta E}{T}} \frac{1}{\eta_b} \quad \#$$

ΔE: 13.6 eV

$10^9 \qquad 10^9 \sim e^{21}$

$\Rightarrow X_e$ drops exponentially for $\frac{\Delta E}{T} \gtrsim 42$

$$\Rightarrow T_{rec} = \frac{13.6\,eV}{42} \sim 0.3\,eV \iff z = 1300$$

after recombination, "no more" free electrons

$\Longrightarrow$ photons are now free, propagate without scattering on geodesics $\to$ redshift $\longrightarrow$ CMB.

$\boxed{\text{PLOT}}$

Recombination

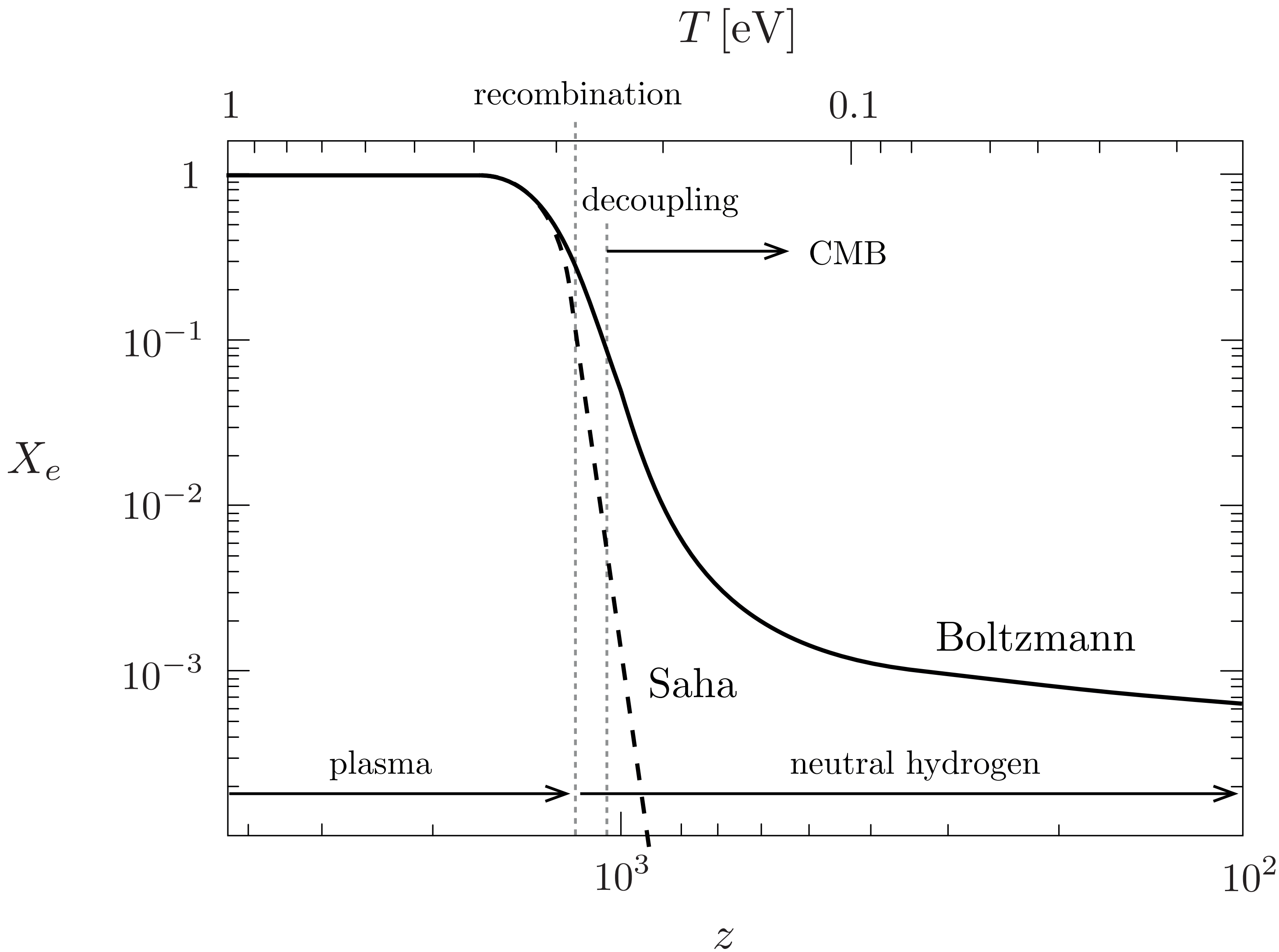
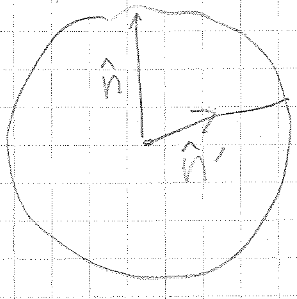


Figure 3.8: *Free electron fraction as a function of redshift.*

CMB is very isotropic (and presumably homogeneous)



$$\frac{\delta T}{T} = \frac{T(\hat{n}) - T}{T} \sim 10^{-5}$$

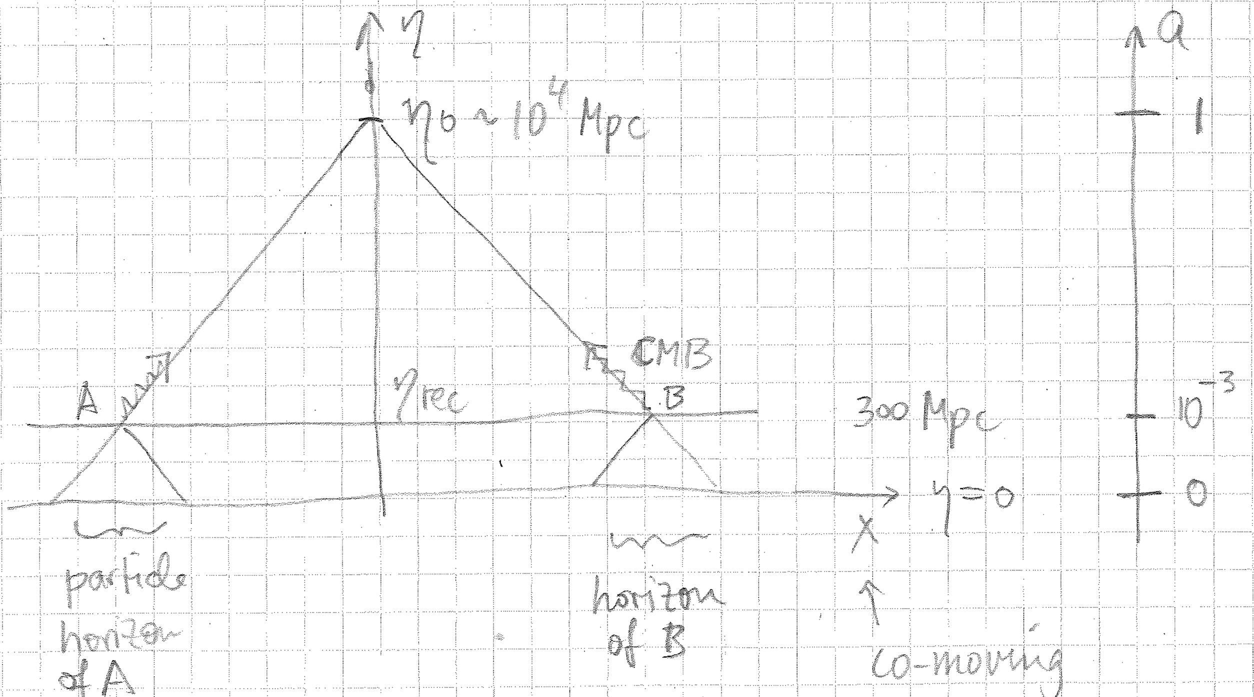
it's a thermal spectrum $\Rightarrow$ can thermal equilibration explain isotropy? Answer: No!

<u>Horizon problem (causality)</u>

change to "conformal time" $dt = a\,d\eta$

$$\Rightarrow ds^2 = a^2 (d\eta^2 - d\vec{r}^{\,2})$$

light travels on $ds^2 = 0 \Rightarrow d\eta = |d\vec{r}|$



particle horizons of A & B do not overlap
$\Rightarrow$ they cannot have equilibrated

there was not enough time because $a \to 0$ before horizons of A&B overlap...

proof: (assume RD)

$$H = \frac{\dot a}{a} \propto \sqrt{\rho_r} \propto \frac{1}{a^2} \Rightarrow a^2 \sim t \Rightarrow a \to 0 \text{ as } t \to 0$$

negative t is inaccessible, before singularity

conformal time: $\frac{1}{a}\frac{da}{\underbrace{a\,d\eta}_{dt}} = \frac{\dot a}{a} \propto \frac{1}{a^2} \Rightarrow \frac{da}{d\eta} = \text{const}$

$a \sim t \Rightarrow a \to 0$ as $\eta \to 0$ ∎

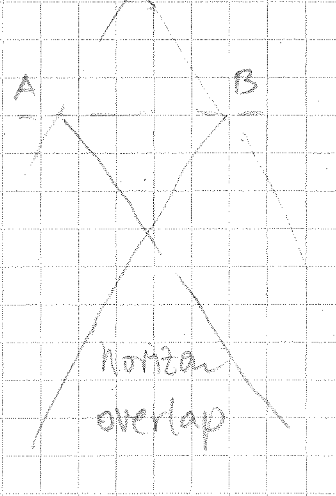
define: $a' = \frac{da}{d\eta}$ $\quad \mathcal{H} = \frac{a'}{a} = aH = \dot a$

RD: $\mathcal{H} = \frac{1}{\eta}$

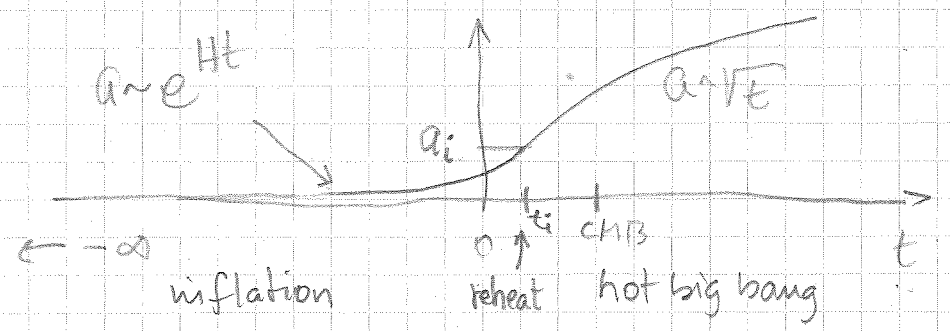
MD: $\mathcal{H} = \frac{2}{\eta}$

causality problem solution requires more time before CMB

idea: at some small a_i, before singularity, a period of vacuum (potential) energy domination



vacuum energy dom: $\frac{\dot a}{a} = H = \text{const}$



during inflation: $H^2 \propto \left(\rho_{vacuum} + \cancel{\frac{\rho_m}{a^3}} + \cancel{\frac{\rho_r}{a^4}} + \cancel{\frac{\rho_k}{a^2}} \right)$

- all energy except vacuum energy scales to zero
  $\Rightarrow$ empty universe, homogeneous + isotropic, cold
  no monopoles, domain walls, ...
- need a "reheating" process which converts $\rho_{vacuum} \hat{=} \rho_\phi$
  into ρ_r^{SM} to start hot big bang
- added bonus: quantum fluctuations during inflation
  get stretched to enormous sizes "outside the horizon"
  $\Rightarrow$ spectrum of small fluctuations on all scales are
  initial conditions for perturbations.