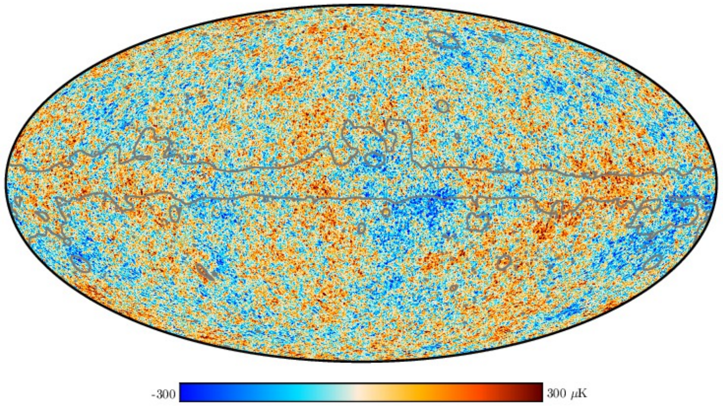
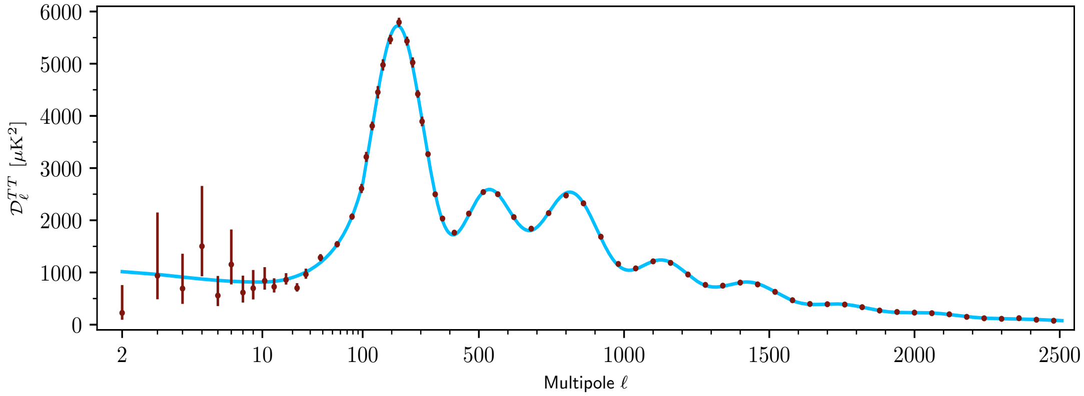
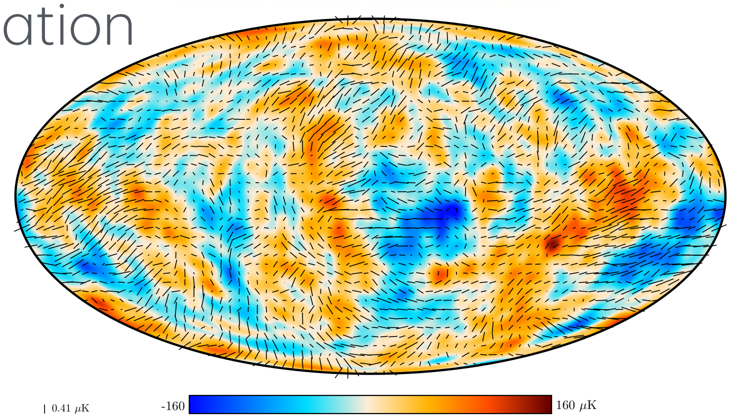
Planck 2018 results. I.
Overview and the cosmological legacy of *Planck*

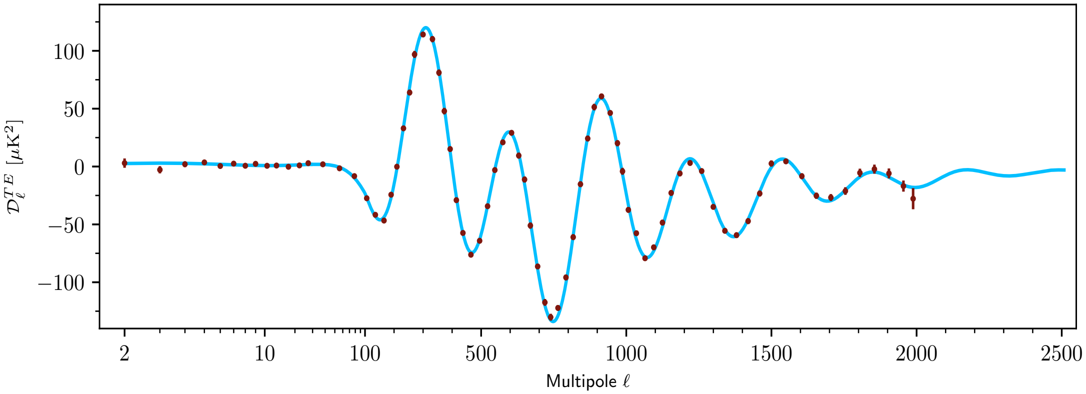
Temperature



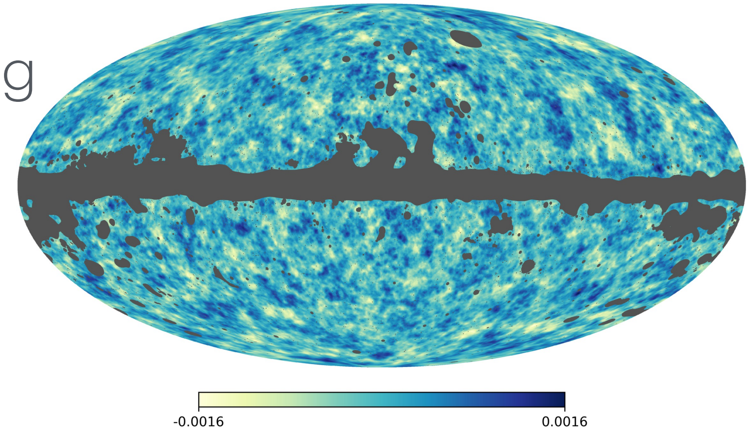


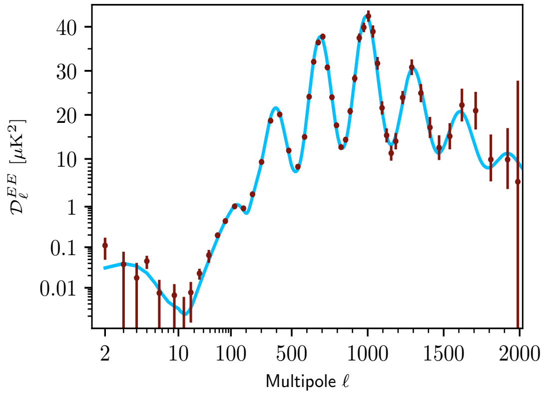
polarization

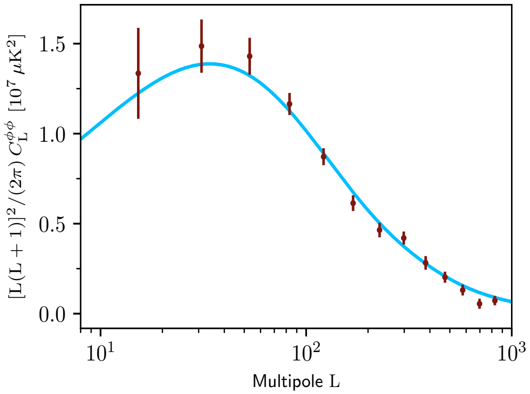


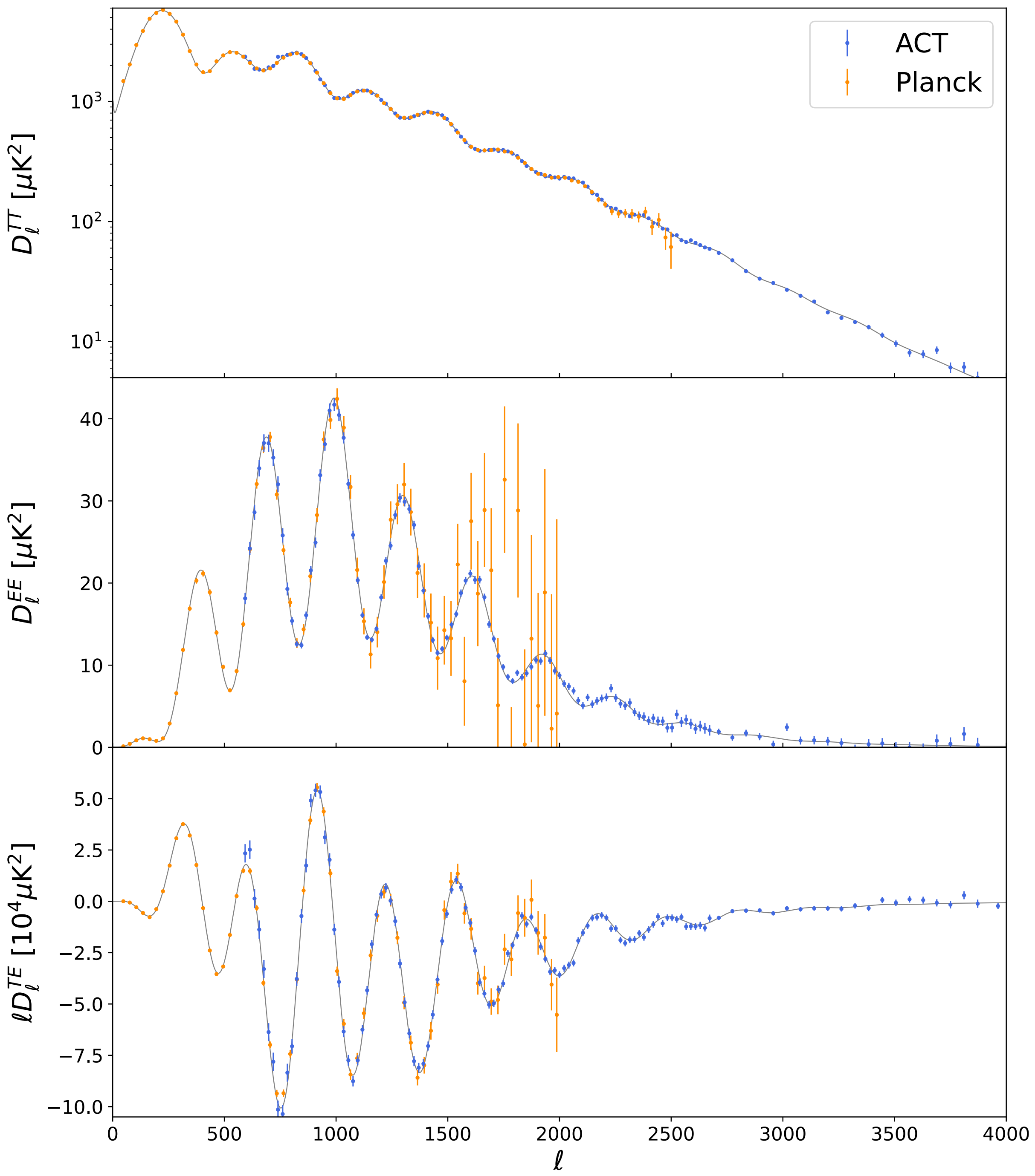


lensing



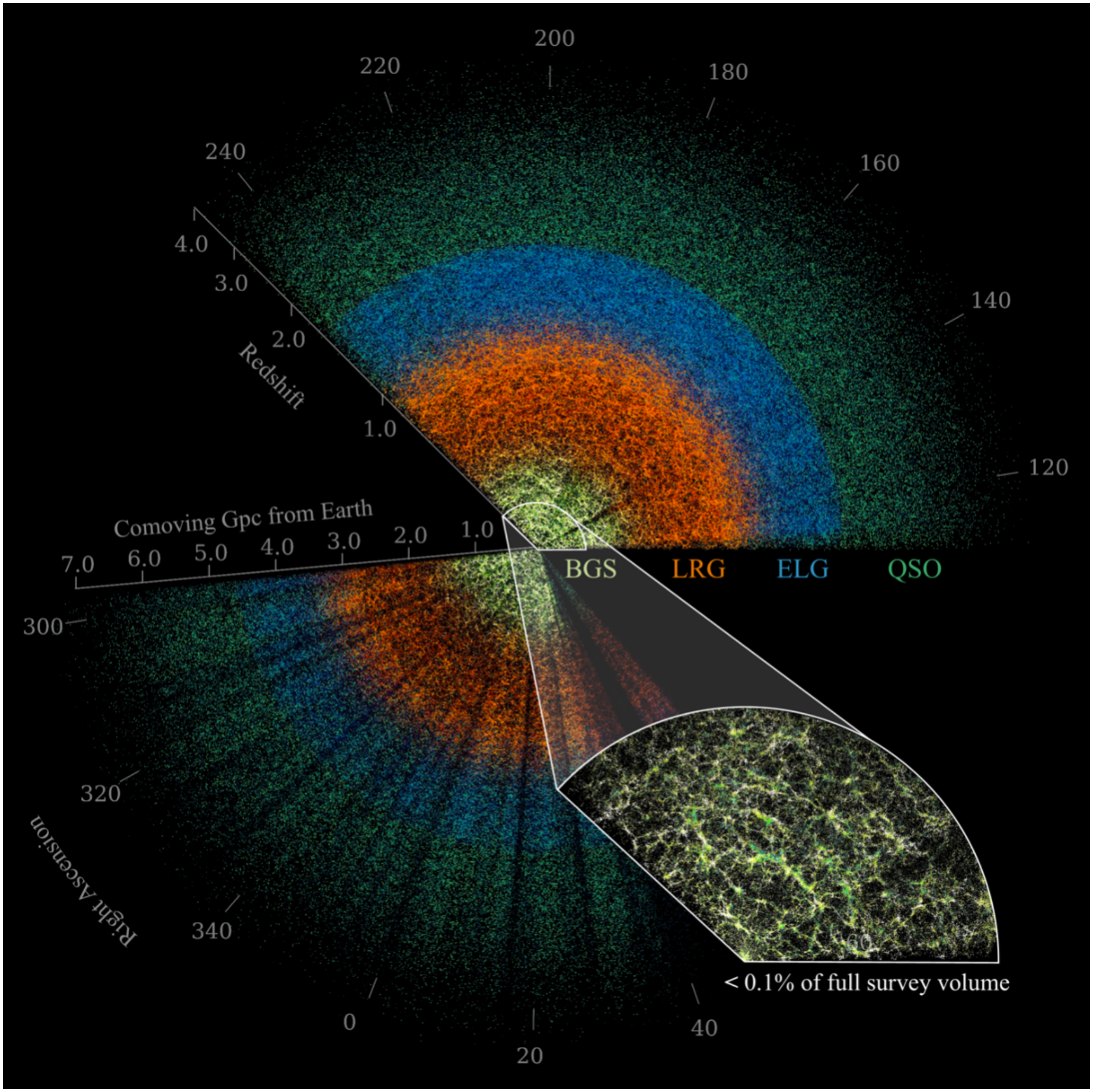





ACT
Planck
D_ℓ^{TT} [μK^2]
10^3
10^2
10^1
D_ℓ^{EE} [μK^2]
40
30
20
10
0
ℓD_ℓ^{TE} [$10^4\mu$K^2]
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
500
1000
1500
2000
2500
3000
3500
4000
ℓ

DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints

March 18, 2025



- CMB + LSS perturbations are sourced by quantum fluctuations during inflation, then classically evolve which can compute in perturbation theory.
- cannot predict individual perturbations, statistical averages over the sky

e.g. $\rho_m(\vec{x}) = \bar{\rho}_m + \delta\rho(\vec{x})$

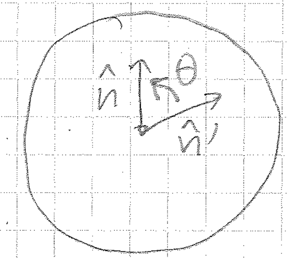
$\langle \delta\rho(x) \rangle = 0$ by definition of $\bar{\rho}$

↖ average over sky

$$\left\langle \frac{\rho(\vec{x})}{\bar{\rho}} \frac{\rho(\vec{y})}{\bar{\rho}} \right\rangle = f(|\vec{x}-\vec{y}|)$$

↖ homogneity + isotropy

CMB



$$\left\langle \frac{\delta T(\hat{n})}{\bar{T}} \frac{\delta(T(\hat{n}'))}{\bar{T}} \right\rangle = f(\cos\theta)$$

$$\equiv \sum_{\ell=2}^{\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

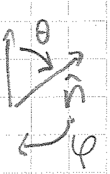
↖ Legendre polynomials

$\ell=0$ monopole $=0$ by def. of δT

$\ell=1$: dipole removed, measures our velocity relative to CMB ~ 370 km/s $\delta T \sim 3$ mK

$\ell \gtrsim 2$ $|\delta T| \lesssim 0.3$ mK

large $\ell \Leftrightarrow$ small angles $\ell \approx \frac{180^\circ}{\theta}$ $(\ell_{(1)}^{peak} \approx 200) \Leftrightarrow$ small scales how are C_ℓ's determined ?



spherical harmonic decomposition $Y_{\ell m}(\hat{n}) \sim P_\ell^m(\cos\theta)\, e^{im\varphi}$

$$\frac{\delta T}{T}(\hat{n}) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\hat{n})$$

$$a_{\ell m} = \int d\Omega\, Y^*_{\ell m}(\hat{n}) \frac{\delta T}{T}(\hat{n})$$

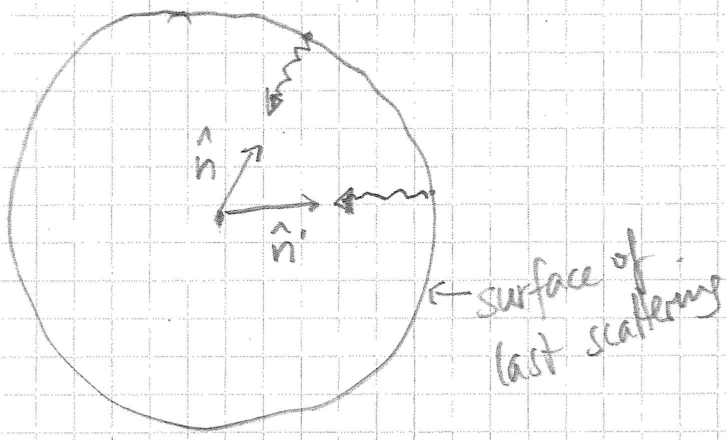
for statistically isotropic distribution $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$

$$C_\ell^{TT} = \sum_{m=-\ell}^{\ell} a_{\ell m} a^*_{\ell m} \frac{1}{2\ell+1}$$

Note: sum is an average over $2\ell+1$ statistically independent measurements of C_ℓ

$\Rightarrow$ statistical error bars $\propto \frac{1}{\sqrt{2\ell+1}}$

small $\ell \longleftrightarrow$ few m modes $\longleftrightarrow$ large "cosmic variance" predicting the C_ℓ's :



1. compute fluctuations in photon temperature at surface of last scattering

$$\rho_\gamma \sim T^4 \Rightarrow \left.\frac{\delta T}{T}\right|_{sols} \simeq \frac{1}{4} \left.\frac{\delta\rho_\gamma}{\rho_\gamma}\right|_{sols}$$

2. propagate (geodesic equation) photons from sols ($z=1100$) to $r=0$ ($z=0$) to get $\left.\frac{\delta T}{T}\right|_0$

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compute $\frac{\delta\rho}{\rho}$ with cosmological perturbation theory

CPT

expand metric $g = g^{FRW} + \delta g$

EM tensor: $T = \overline{T} + \delta T$

$\leftarrow$ conformal time

metric: $$ds^2 = a^2(\eta)\Big[(1+2\psi)\,d\eta^2 - 2(\hat{B}_i + \partial_i B)\,dx^i\,d\eta$$

$$- dx^i dx^j\Big\{\delta_{ij}(1-2\phi) + \hat{h}_{ij} + 2(\partial_i \hat{h}_j + \partial_j \hat{h}_i) + 2\left(\partial_i\partial_j - \tfrac{1}{3}\nabla^2\delta_{ij}\right)h\Big\}\Big]$$

4 scalars	$\psi(\vec{x},\eta)$, ϕ, B, h	4 degrees of freedom	10 d.o.f.
2 vectors	$\hat{B}_i$, $\hat{h}_i$	$2\cdot 2 = 4$	
tensor	$\hat{h}_{ij}$	2	

constraints: $$\left.\begin{array}{l}\nabla\cdot\hat{B} = \nabla\cdot\hat{h} = 0\\ \nabla_i \hat{h}_{ij} = 0\end{array}\right\}\ \text{divergenceless}$$

$\hat{h}_{ii} = 0$ traceless

at linear order scalars, vectors, tensor

- don't mix (rotational invariance)
- are sourced by different physics

scalars: $\frac{\delta T}{T}$, E modes, $\frac{\delta \rho_m}{\rho_m}$

vectors: usually small

tensor: gravity waves, B modes

focus on scalars

2 scalar coordinate transformations: $\eta \to \eta + \xi^0(x)$

$\vec{x} \to \vec{x} + \vec{\nabla}\xi(x)$

can use to eliminate 2 scalars

Conformal Newtonian gauge:

$$ds^2 = a^2\left[(1+2\psi)\,d\eta^2 - (1-2\phi)\,d\vec{r}^{\,2}\right]$$

grav potential

local curvature

$T^\mu{}_\nu$ (scalars) $\quad T^0{}_0 = \bar{\rho} + \delta\rho \equiv \bar{\rho}(1+\delta)$ density contrast

$T^i{}_0 = (\bar{\rho}+\bar{P})\,v^i \qquad \vec{v} = \vec{\nabla}V$ velocity perturbation

$T^i{}_j = -(\bar{P}+\delta P)\,\delta^i{}_j - \Pi^i{}_j \qquad \Pi^i{}_j = \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Pi$

anisotropic stress due to neutrinos, ignore

$T^\mu{}_\nu = \sum\limits_a (T_a)^\mu{}_\nu$ for multiple fluids $\Rightarrow \delta_a, V_a, \delta p_a, \Pi_a$

Einstein: $\delta G_{\mu\nu} = 8\pi G\,\delta T_{\mu\nu}$

$\Rightarrow$ EOM for ψ, ϕ in terms of $\delta\rho, \delta p, V, \Pi$

continuity: $\nabla_\mu T^\mu{}_\nu = 0$, 1st order, for each fluid "a"

$\nu = 0:$

<u>continuity</u>

$$\overset{\frac{d}{d\eta}}{\delta'} + 3\mathcal{H}\left(\frac{\delta p}{\delta\rho} - \frac{\bar{p}}{\bar{\rho}}\right)\delta = -\left(1+\frac{\bar{p}}{\bar{\rho}}\right)\vec{\nabla}\cdot\vec{v} - 3\phi'$$

$\nu = i:$

<u>Euler</u>

$$\vec{v}' + 3\mathcal{H}\left(\frac{1}{3} - \frac{\bar{p}'}{\bar{\rho}'}\right)\vec{v} = -\frac{\vec{\nabla}\delta p}{\bar{\rho}+\bar{p}} - \vec{\nabla}\psi - \frac{2}{3}\frac{\nabla^2\vec{\nabla}\pi}{\bar{\rho}+\bar{p}}$$

2 equations, 4 scalars $\Rightarrow$ need extra information

- $\pi = 0$
- $\delta p = c_s^2\,\delta\rho$ for <u>perfect fluids</u>

  c_s^2 ← adiabatic sound speed

$$c_s^2 = \begin{cases} 0 & \text{CDM} \\ 1/3 & \text{radiation} \\ \dfrac{1}{3\left(1+\dfrac{3\rho_b}{4\rho_\gamma}\right)} & \text{baryon-photon fluid} \end{cases}$$

not a perfect fluid? compute Boltzmann equation for distribution function $f(\vec{p}, \vec{x}, \eta)$

codes for computing all these and plotting CMB + MPS exist

CLASS, CAMB

include ν's, m_ν, decaying DM, iDM, ...

for qualitative understanding make simplifications matter $\delta p, \bar{p} = 0$

$$\left.\begin{array}{l}\delta' = 3\phi' - \vec{\nabla}\cdot\vec{v} \\ \vec{v}\,' = -\mathcal{H}\vec{v} - \vec{\nabla}\psi\end{array}\right\} \text{combine}$$

$$\delta_m'' + \underset{\substack{\uparrow \\ \text{Hubble friction}}}{\mathcal{H}\delta_m'} = \underset{\substack{\uparrow \\ \text{grav. potential}}}{\nabla^2\psi} + 3(\phi'' + \mathcal{H}\phi')$$

radiation $$\delta_r'' - \underset{\substack{\uparrow \\ \text{pressure}}}{\frac{1}{3}\nabla^2\delta_r} = \underset{\substack{\uparrow \\ \text{gravity}}}{\frac{4}{3}\nabla^2\psi} + 4\phi''$$

baryon-photon fluid:

not important

$$\delta_{b\gamma}'' + \underset{\substack{\uparrow \\ \text{damping}}}{\mathcal{H}\frac{R}{1+R}\delta_{b\gamma}'} - \underset{\substack{\uparrow \\ \text{pressure}}}{c_s^2\overset{-k^2}{\nabla^2}\delta_\gamma} = \frac{4}{3}\overset{-k^2}{\nabla^2}\psi + 4\left(\phi'' + \mathcal{H}\frac{R}{1+R}\phi'\right)$$

$$\boxed{R = \frac{3\rho_b}{4\rho_\gamma}}$$

driven harmonic oscillator: $$\underset{\substack{\uparrow \\ \text{mass}}}{m}\ddot{x} + \underset{\substack{\uparrow \\ \text{damping}}}{c}\,\dot{x} + \underset{\substack{\uparrow \\ \text{spring constant}}}{k}\,x = \underset{\substack{\uparrow \\ \text{driving force}}}{F(t)}$$

$$c_s^2 = \frac{1}{3(1+R)}$$

integrate from $\underset{z \simeq 10^9}{\eta_i \simeq 0} \longrightarrow \underset{z \simeq 1100}{\eta_{rec}}$, R small until η_{rec}

$\Rightarrow$ simplify $R \simeq 0$

$$\delta'' + \underset{\overset{\uparrow}{1/3}}{c_s^2} k^2 \delta = -\frac{4}{3} k^2 \Psi$$

<u>solution:</u> $\delta_{b\gamma} = \delta_{b\gamma}^{ini} \cos(c_s k \eta)$ ← baryonacoustic oscillation

$$+ \frac{1}{c_s^2} \int_0^{c_s k \eta} dy \, \frac{4}{3} \Psi(y) \sin(y - c_s k \eta)$$

↖ gravitational driving

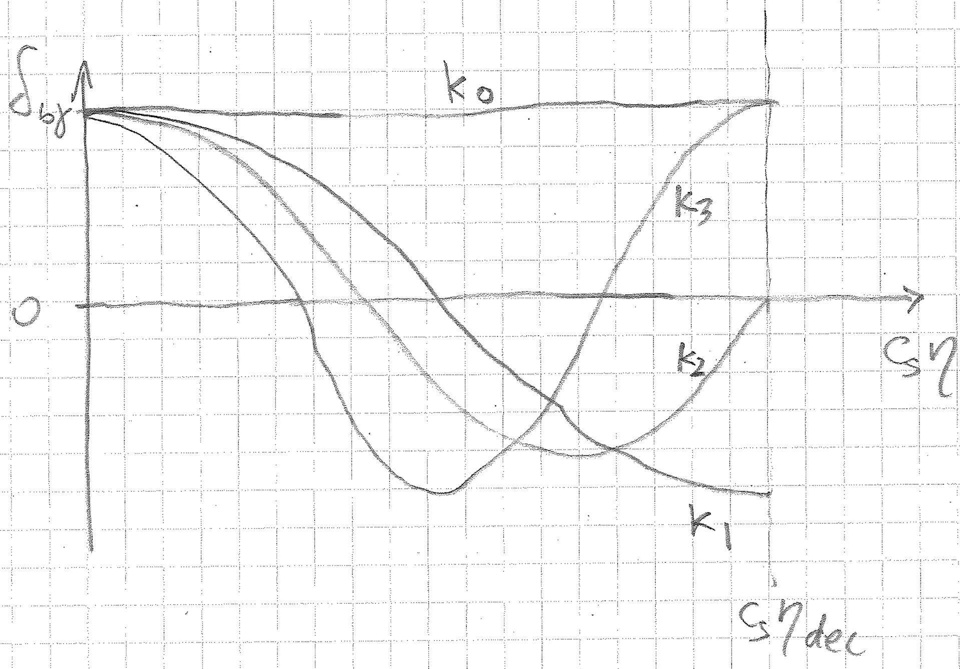
focus on cos, at recombination:

$$\delta_{b\gamma} = \delta_{b\gamma}^{ini} \cos(k \, c_s \eta_{rec})$$

$r_s = c_s \eta_{rec}$ = sound horizon

note: $r_s = \int_0^{\eta_{rec}} c_s \, d\eta$

$= \int_0^{z_{rec}} c_s(z) \frac{dz}{H(z)}$



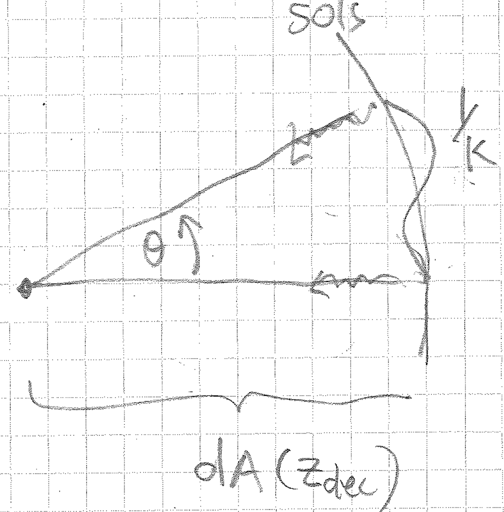
$k_0 \ll \frac{1}{c_s \eta_{dec}}$ $\quad k_2 \sim \frac{3}{2}\pi / c_s \eta_{dec}$

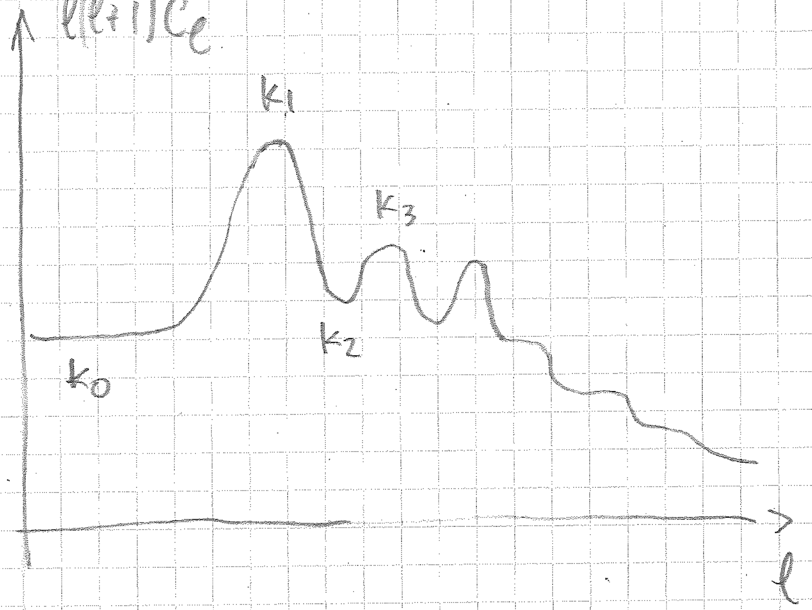
$k_1 \sim \pi / c_s \eta_{dec}$ $\quad k_3 \sim 2\pi / c_s \eta_{dec}$ ...

CMB

$$\left\langle \frac{\delta T}{T} \frac{\delta T}{T} \right\rangle \sim \langle \delta_{b\gamma} \delta_{b\gamma} \rangle$$

recall $\ell \sim \frac{1}{\theta} = k d_A(z_{dec})$





low ℓ: $\langle (\delta^i_{b\gamma})^2 \rangle \sim$ flat, scale invariant initial conditions from inflation

$$\sim A_s \left(\frac{k}{k_{pivot}} \right)^{n_s - 1}$$

$n_s - 1$ ← spectral index

k_{pivot} ← analysis choice $k = 0{,}05\ \mathrm{Mpc}^{-1}$ for CMB

A_s ← amplitude of initial perturbations $\sim 2 \cdot 10^9$

1st peak: $\ell_1 \sim d_A k_1 = d_A \frac{\pi}{c_s \eta_{dec}} \iff \theta_s \approx \frac{2\pi}{\ell_1}$ angular size of sound horizon

nth peak $\ell \sim d_A \frac{n\pi}{c_s \eta_{dec}}$

Dependence of ΛCDM parameters

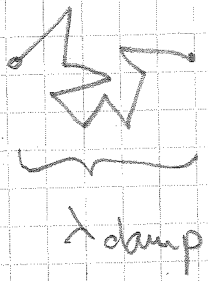
- Ω_b enters $R \rightarrow c_s$

$\Rightarrow$ even vs. odd peak heights

from shift of zero point of oscillator by $\frac{\Psi}{c_s^2}$

- damping of tail due to finite streaming length of photons near decoupling

$$\lambda_{\text{damp}} \sim \sqrt{N_{\text{scatters}}} \cdot \lambda_{\substack{\text{mean}\\ \text{free}\\ \text{path}}}$$



$$\lambda_{mfp} \sim \frac{1}{n_e \sigma_T a}$$

n_e ↗ electron number density

σ_T ↖ Thompson σ

$$N \sim \frac{\text{Hubble length}}{\lambda_{mfp}} \sim \frac{1}{H \lambda_{mfp}} \quad \text{(at decoupling)}$$

H ↑ depends on energy densities e.g. N_{eff} at dec.

- $\int \Psi$ driving term boosts amplitude of $\delta_{b\gamma}$ for modes which enter during RD but not during MD $\Longrightarrow \Omega_m$

ΛCDM parameters

$H_0, \Omega_m, \Omega_b, A_s, n_s, \tau_{reio}$

τ_{reio} optical depth parameterizes how many CMB photons are lost due to scattering at late times.

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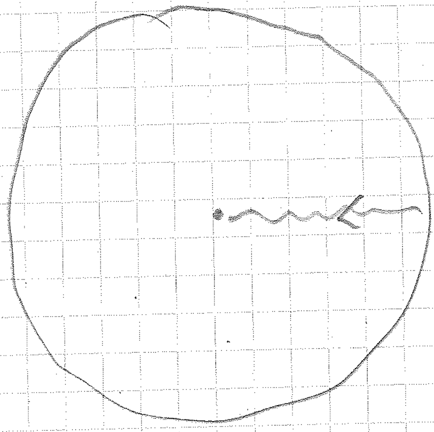
connection between $\delta_{b\gamma}(\eta_{dec})$ & $\frac{\delta T}{T}$?



integrate geodesic for photon of energy E

η_{dec} to η_0:

$$\ln(Ea)\Big|_0 = \ln(Ea)\Big|_{dec} + \Psi_{dec} - \Psi_0 + \int_{\eta_{dec}}^{\eta_0} d\eta\,(\Psi' + \Phi')$$

$$\langle Ea\rangle \propto Ta = \bar{T}a\left(1 + \frac{\delta T}{T}\right)$$

$$\Rightarrow \frac{\delta T}{T}\Big|_0 = \frac{\delta T}{T}\Big|_{rec} + \Psi_{rec} - \Psi_0 + \int_{\eta_{dec}}^{\eta_0} d\eta\,(\Psi' + \phi')$$

- Ψ_0 universal for all CMB photons $\Rightarrow$ ignore
- $\int d\eta\, \Psi' + \Phi'$ "integrated Sachs Wolfe" small

$$\rho_\gamma \propto T^4 \Rightarrow \left.\frac{\delta T}{T}\right|_{rec} = \frac{1}{4}\left.\frac{\delta\rho_{\gamma b}}{\rho_{\gamma b}}\right|_{rec} = \frac{\delta_{\gamma b}}{4}$$

$$\Rightarrow \left.\frac{\delta T}{T}\right|_0 = \left.\frac{\delta_{\gamma b}}{4} + \Psi\right|_{dec} \quad \text{Sachs-Wolfe}$$