Minicourse 3: Nonlinear Fourier analysis and connections to quantum computing

Diogo Oliveira e Silva (IST, Lisbon)

Abstract: This course offers an introduction to the Nonlinear Fourier Transform (NLFT), a powerful analytic tool that extends classical Fourier analysis to nonlinear settings. In the first lecture, we present the definition of the NLFT and explore its functorial properties with an emphasis on symmetries. The second lecture focuses on the nonlinear Plancherel theorem, which serves as a nonlinear analogue for the classical Plancherel identity. In the third lecture, we discuss a deep and elegant equivalence: up to a change of variables, the NLFT coincides with a well-known algorithm in quantum signal processing. This reveals the versatility of the NLFT and its unifying power. The course assumes familiarity with basic (linear) Fourier analysis but aims to be accessible to a broad audience.