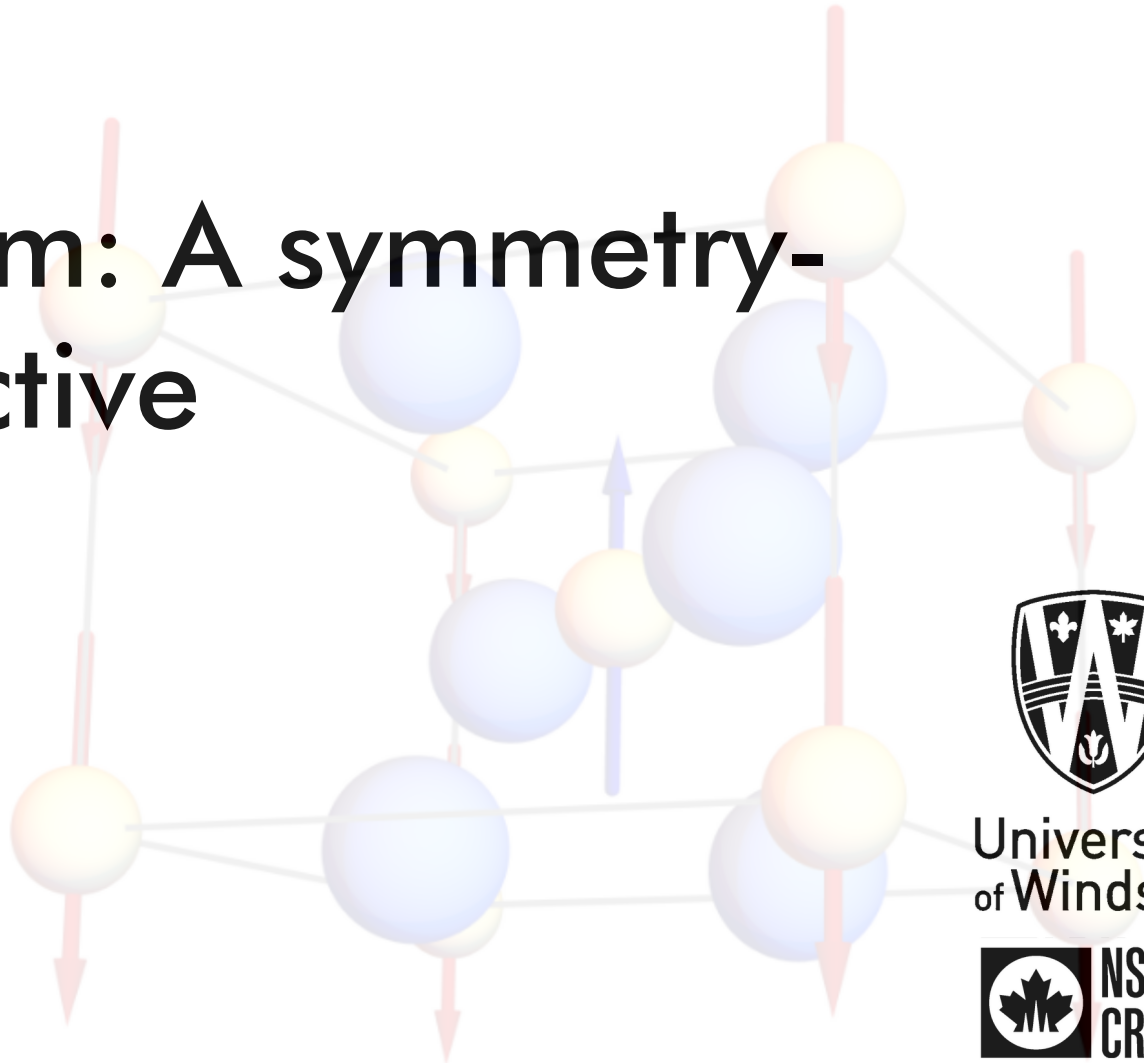


Tutorial:

Altermagnetism: A symmetry-based perspective

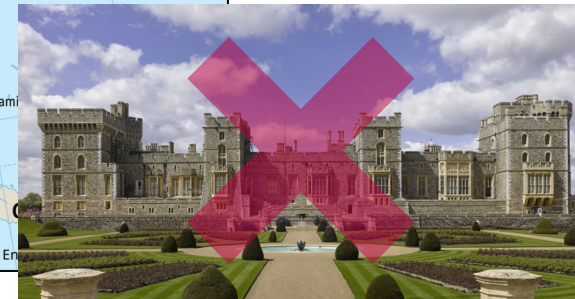
Jeffrey G. Rau

University of Windsor

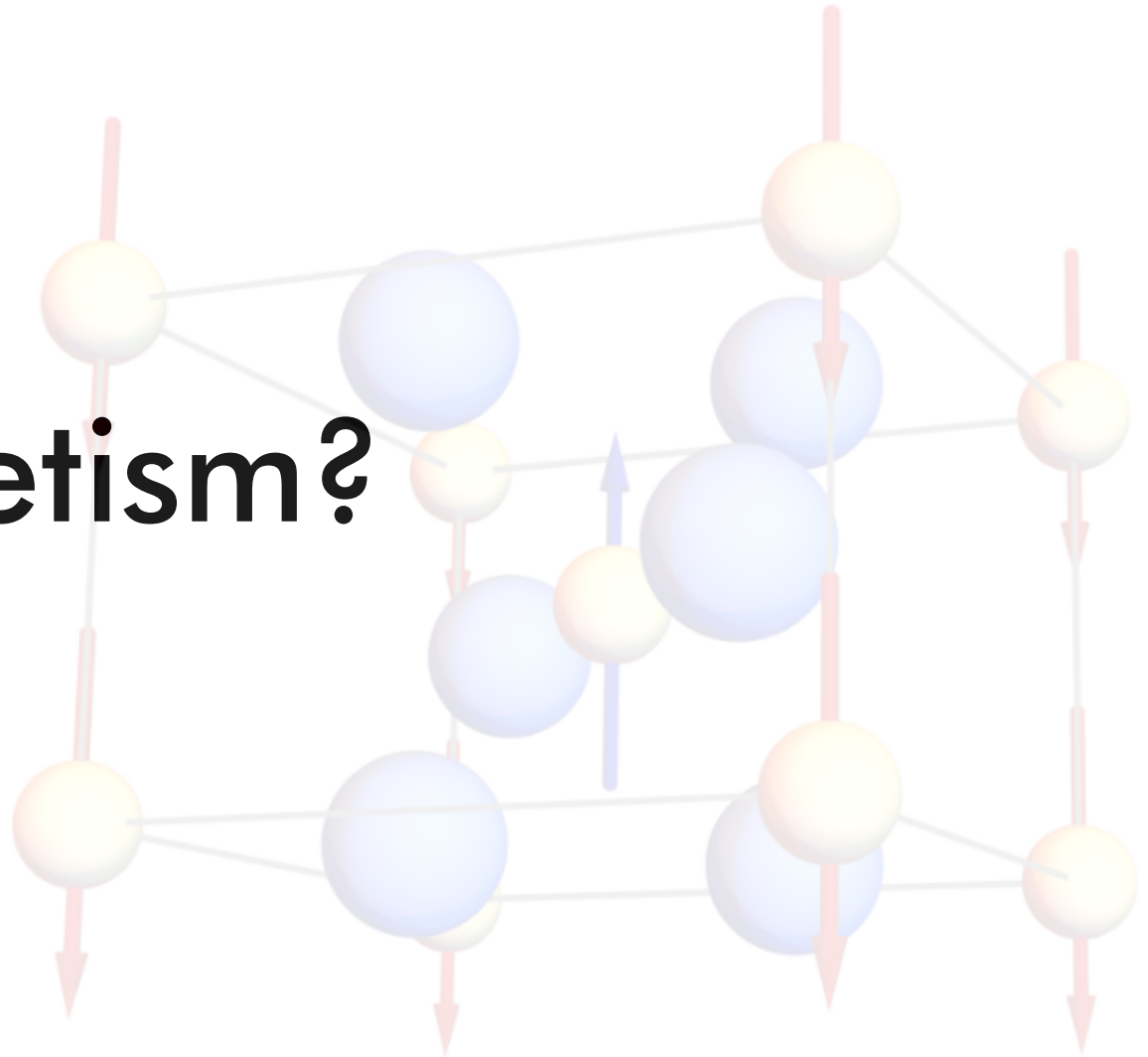


University
of Windsor





Altermagnetism?





Altermagnetism Then and Now

Igor Mazin
Physics and Astronomy Department, George Mason University, Fairfax, Virginia
January 8, 2024 • *Physics* 17, 4

Recent theoretical work has identified the possibility of a new and fundamental form of magnetism.

Experimental Evidence for a New Type of Magnetism

January 18, 2024 • *Physics* 17, s10
Spectroscopic data suggest that thin films of a certain semiconducting material can exhibit altermagnetism, a new and fundamental form of magnetism.

Condensed-matter physics

New type of magnetism splits from convention

Carmine Autieri

Magnetic materials with zero net magnetization fall into two classes: conventional antiferromagnets and altermagnets. Physicists have identified a property in altermagnets that widens the divide between the two groups. **See p.517 & p.523**

nature

NEWS | PHYSICS

Researchers discover new kind of magnetism

More than 200 materials could be “altermagnets,” predicted just a few years ago

6 FEB 2024 • 3:30 PM ET • BY ZACK SAVITSKY



Scientists Have Confirmed the Existence of a Third Form of Magnetism

This could change the game.

BY [CAROLINE DELBERT](#) PUBLISHED: DEC 18, 2024 1:00 PM EST

SAVE ARTICLE



POPULAR



Physics

We've discovered a new kind of magnetism. What can we do with it?

Researchers have found the first new type of magnet in nearly a century. Now, these strange "alt magnets" could help us build an entirely new type of computer

By [Jacklin Kwan](#)

📅 15 July 2025

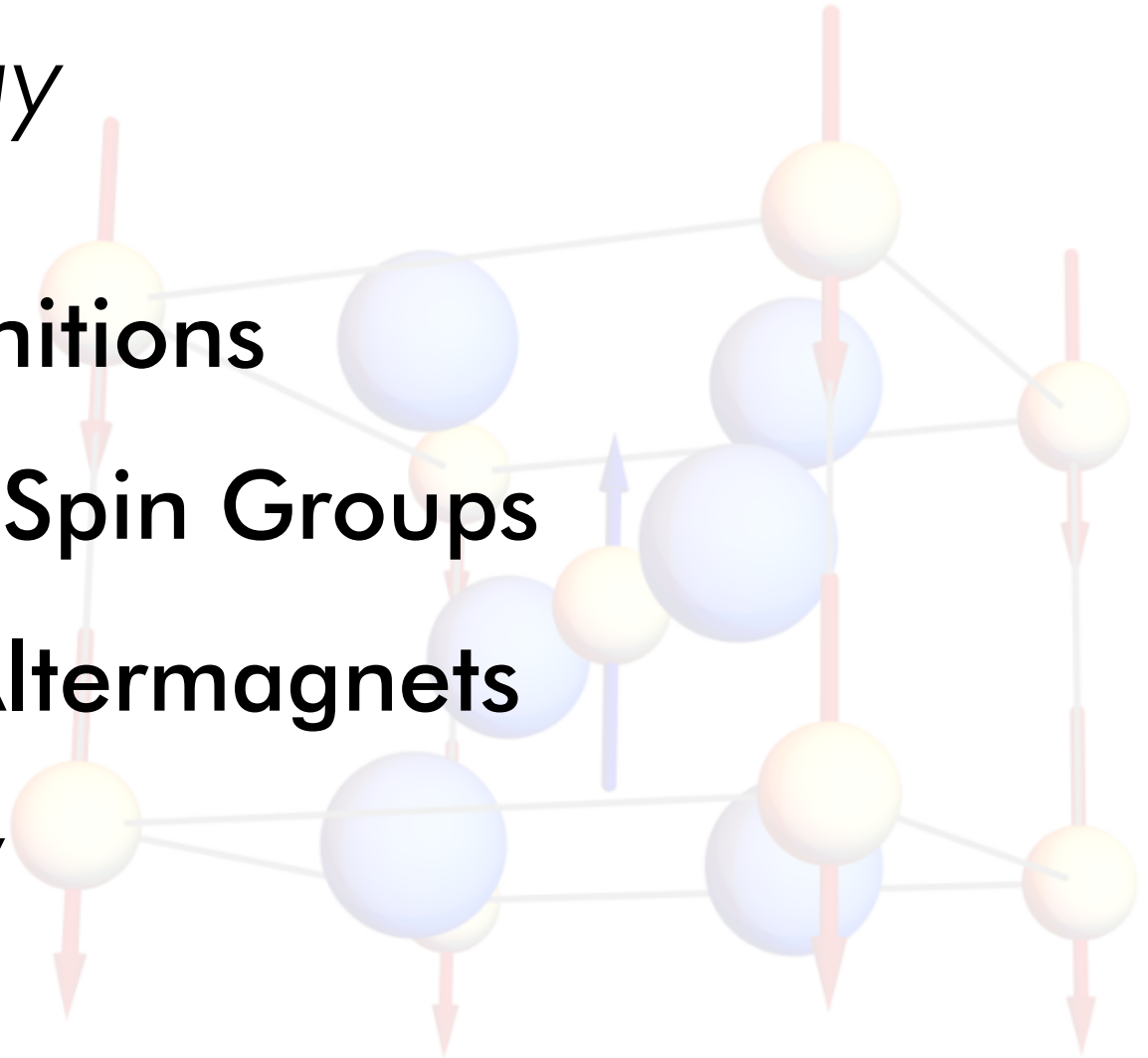
NewScientist

Questions

- ◆ Is this really a **third kind of magnetism**?
 - *Why only two? What about spirals and non-collinear or coplanar orders, etc*
- ◆ Is this more like a ferromagnet or more like an antiferromagnet?
 - *When is it really distinct from either of those?*
- ◆ How does this relate to large body of knowledge on magnetic symmetries?
 - *Like space groups, there are magnetic space groups that have been well-understood for decades*

Outline - *Today*

1. History & Definitions
2. Magnetic and Spin Groups
3. Symmetry in Altermagnets
4. Landau theory

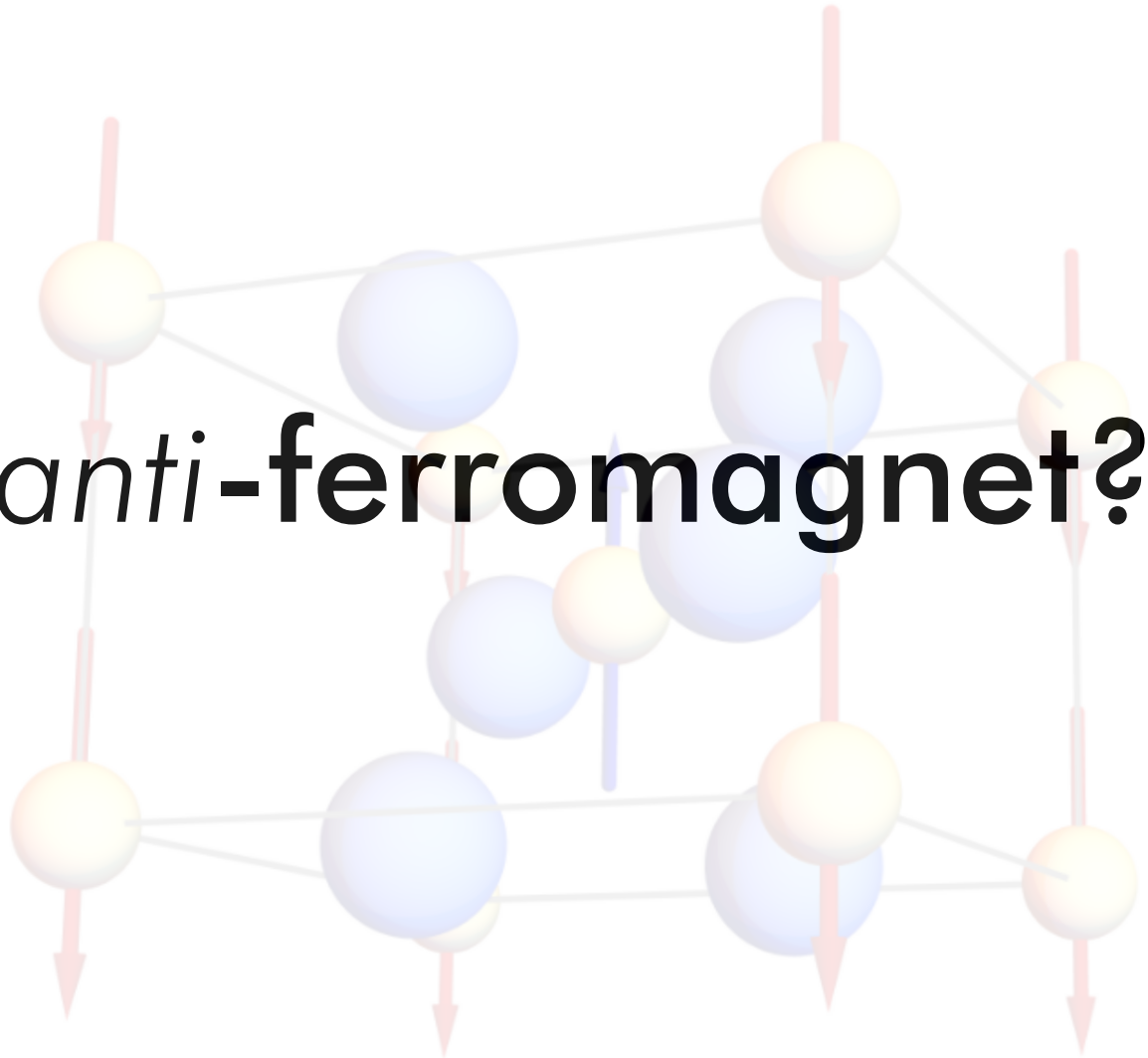


Outline - *Tomorrow (tentative)*

1. Insulating altermagnets
2. Altermagnetic magnons
3. Chiral magnon bands
4. Detection, realization & characterization



What is an *anti*-ferromagnet?

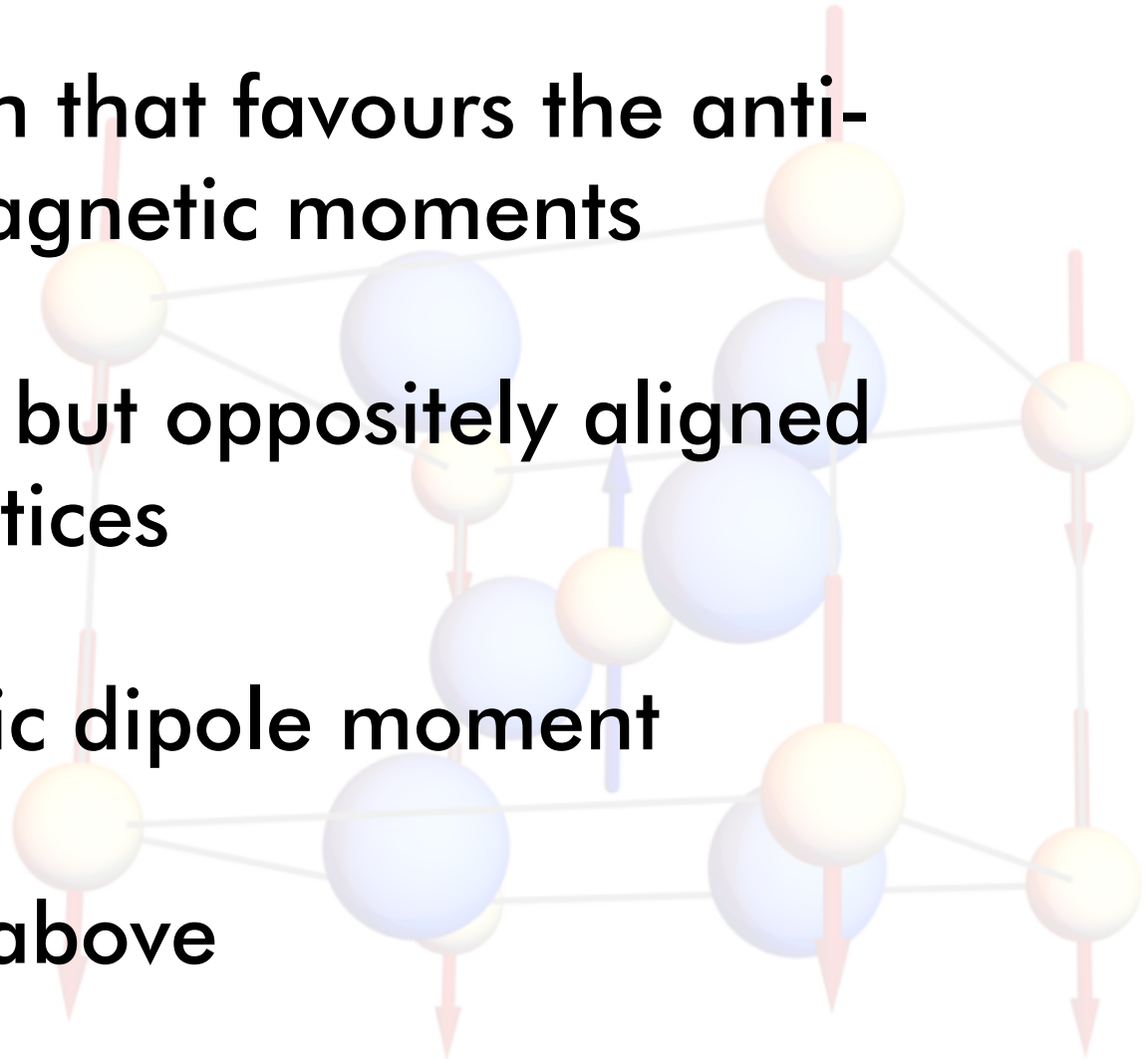


1. An interaction that favours the anti-alignment of magnetic moments

2. Two identical but oppositely aligned magnetic sublattices

3. Zero magnetic dipole moment

4. None of the above





ChatGPT 4o ▾

What is an antiferromagnet?

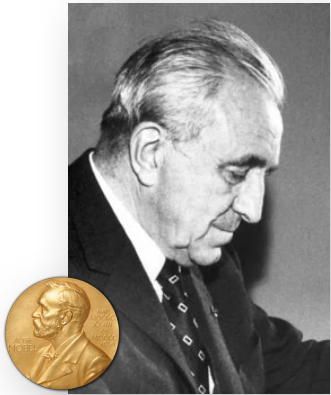
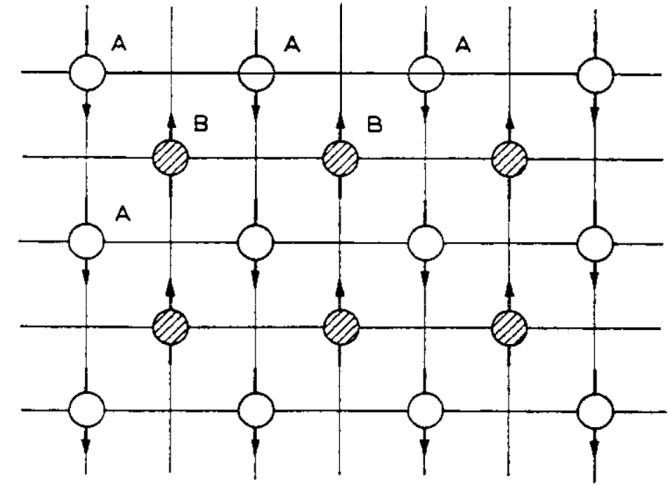
An **antiferromagnet** is a material in which adjacent atomic magnetic moments (spins) align in opposite directions, resulting in **no net macroscopic magnetization** in the absence of an external field. This arrangement minimizes the system's magnetic energy due to **strong exchange interactions** that favor antiparallel spin alignment.

Key Characteristics:

- **Magnetic Ordering:** Below a critical temperature called the **Néel temperature** T_N , the material exhibits long-range magnetic order with alternating spin directions.
- **Zero Net Magnetization:** In contrast to ferromagnets, the opposing magnetic moments cancel out, leading to zero or negligible total magnetization in the bulk.
- **Response to External Fields:** Susceptibility is generally small and decreases as temperature drops below T_N , unlike ferromagnets which show spontaneous magnetization.
- **Examples:** MnO , FeO , NiO , Cr_2O_3 , and hematite (Fe_2O_3) under certain conditions.

Still in 1936, I had the idea⁵ of applying the theory of the local molecular field to the two sub-lattices A and B with magnetizations J_A and J_B used in the previous papers, and of representing the interactions by imaginary fields H_A and H_B with, at low temperature and for $H=0$, the fundamental relationship $J_B = -J_A$.

The result was that the two sub-lattices had to acquire spontaneous magnetizations in opposite directions, disappearing at a certain transition temperature θ_N , known nowadays as the Néel temperature, following a proposal made by C. J. Gorter. We were hence faced with a new kind of magnetic material, composed of the sum of two interlaced identical ferromagnetics spontaneously magnetized in opposite directions. Effects depending on the square of






LOUIS NÉEL

Magnetism and the local molecular field

Nobel Lecture, December 11, 1970




Definitions

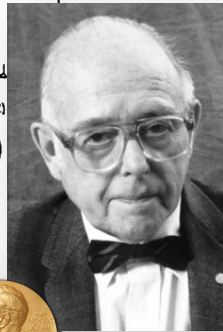
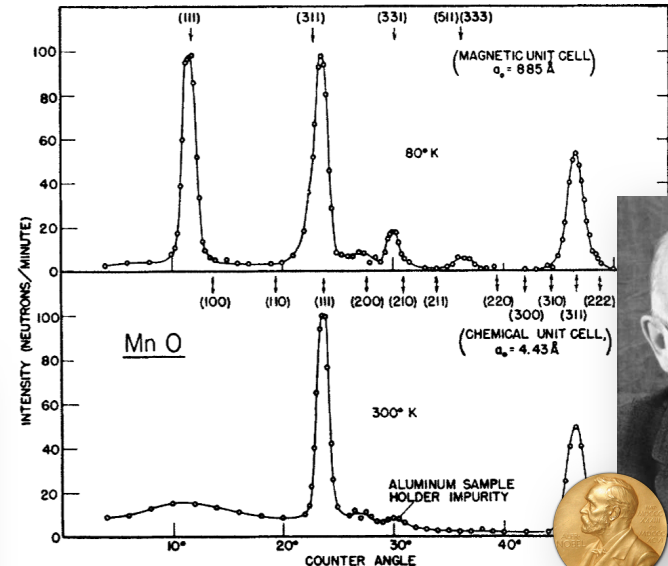
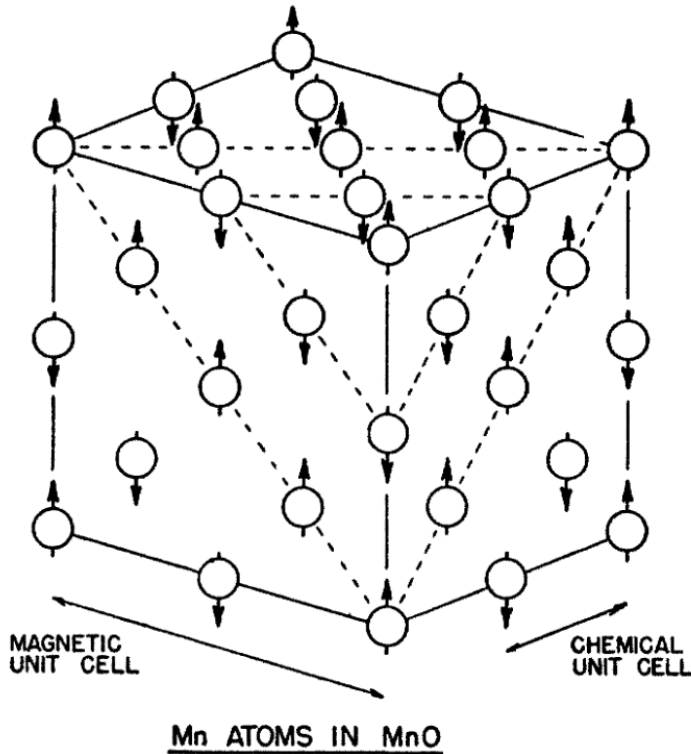
*Assume magnetic order
(spontaneous breaking of
time-reversal symmetry,
smoothly connected to trivial
state)*

- ◆ **Ferromagnetism:** Spontaneous development of a non-zero magnetic dipole moment
- ◆ **Antiferromagnetism?** Some options
 - An interaction that favours the anti-alignment of magnetic moments  *Tendency to anti-align*
 - Two *identical* but oppositely aligned magnetic sublattices*  *Néel's definition*
 - Zero magnetic dipole moment  *“Modern” definition*

**Néel's definition implies the modern one, but not vice-versa*

Manganese Oxide (MnO)

- ◆ Antiferromagnetic interactions? 
- ◆ Identical, anti-aligned sublattices? 
- ◆ Zero net moment? 



PHYSICAL REVIEW

VOLUME 83, NUMBER 2

JULY 15, 1951

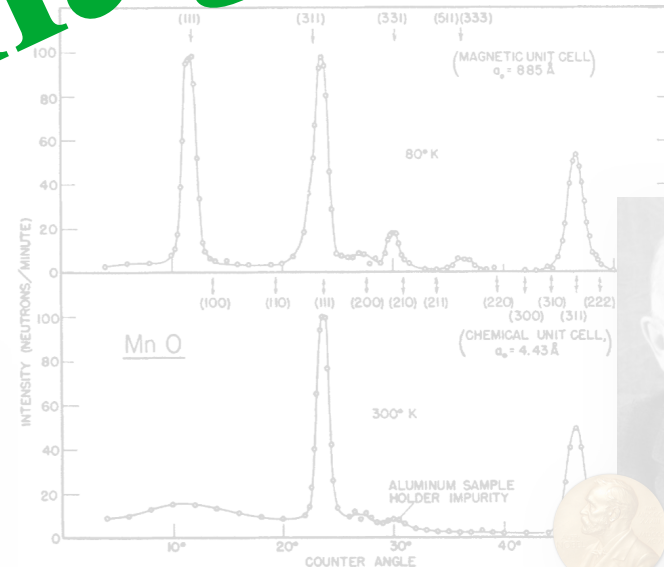
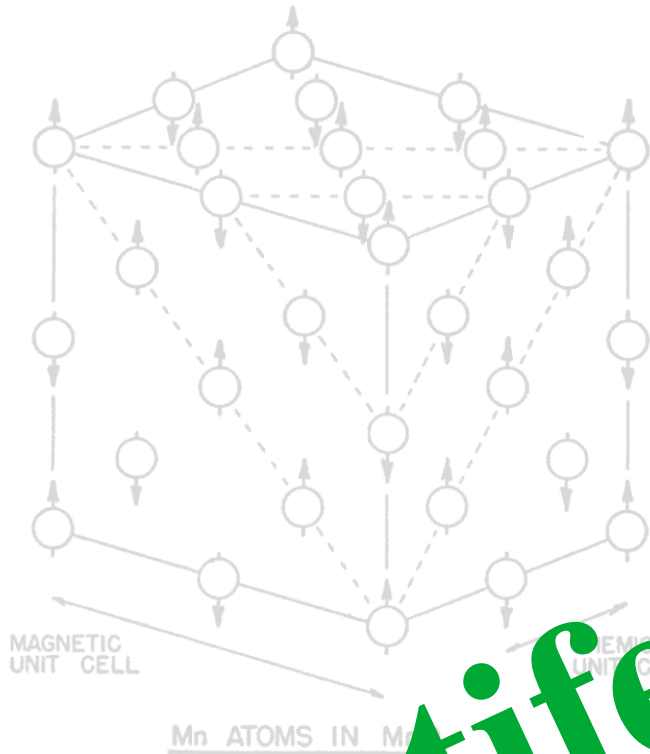
Neutron Diffraction by Paramagnetic and Antiferromagnetic Substances

C. G. SHULL, W. A. STRAUSSER, AND E. O. WOLLAN
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received March 2, 1951)

Neutron scattering and diffraction studies on a series of paramagnetic and antiferromagnetic substances are reported in the present paper. The paramagnetic diffuse scattering predicted by Halpern and Johnson has been studied, resulting in the determination of the magnetic form factor for Mn^{++} ions. From the form factor, the radial distribution of the electrons in the 3d-shell of Mn^{++} has been determined, and this is com-

Manganese Oxide (MnO)

- ◆ Antiferromagnetic interactions? ☒
- ◆ Identical, anti-aligned sublattices? ☒
- ◆ Zero net moment. ☒



PHYSICAL REVIEW VOLUME 83, NUMBER 2 JULY 15, 1951

Neutron Diffraction by Paramagnetic and Antiferromagnetic Substances

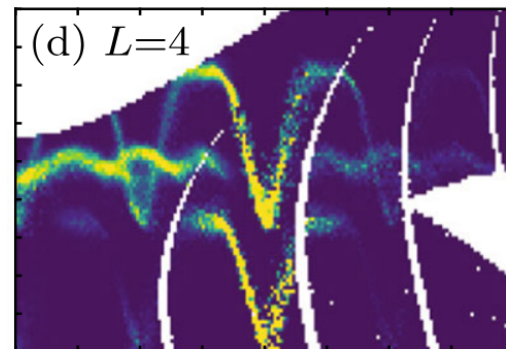
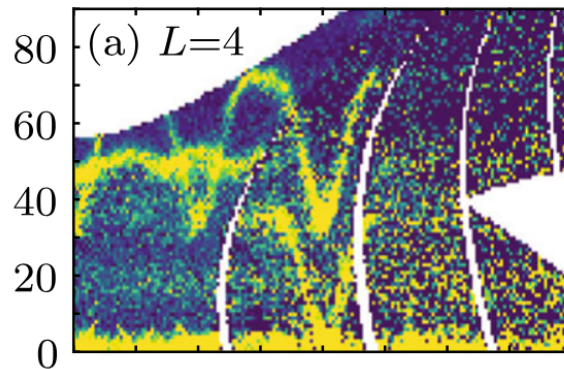
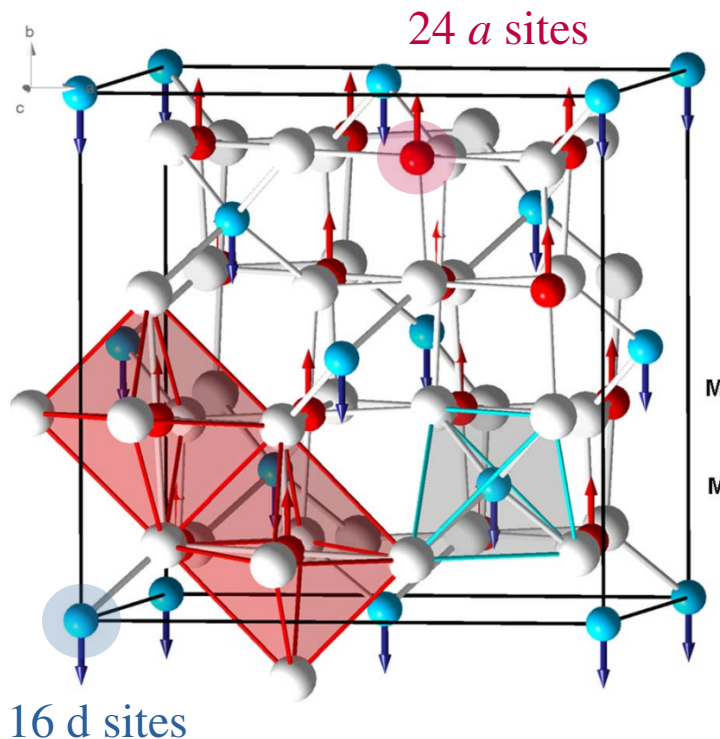
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Yttrium Iron Garnet

“YIG” ($\text{Y}_3\text{Fe}_5\text{O}_{12}$)

“the fruitfly of magnetism” (C. Kittel)



Net magnetic moment
 $\sim 40 \mu_B$ / unit cell

This work (meV)

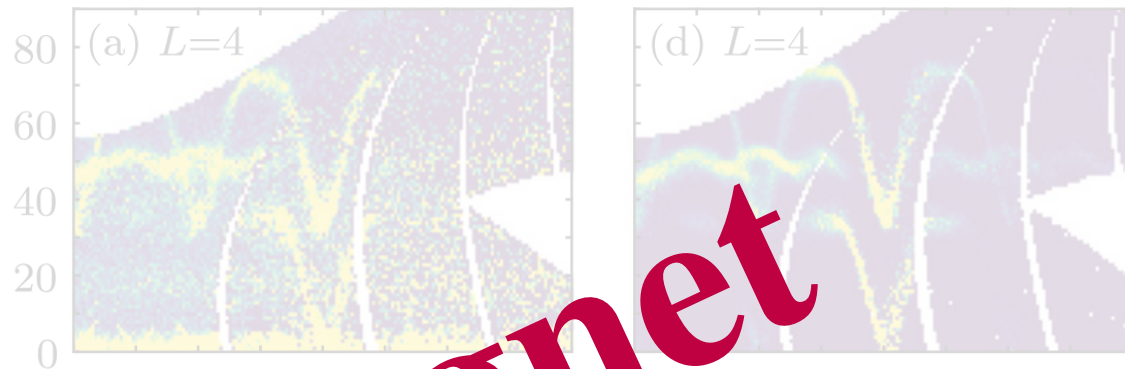
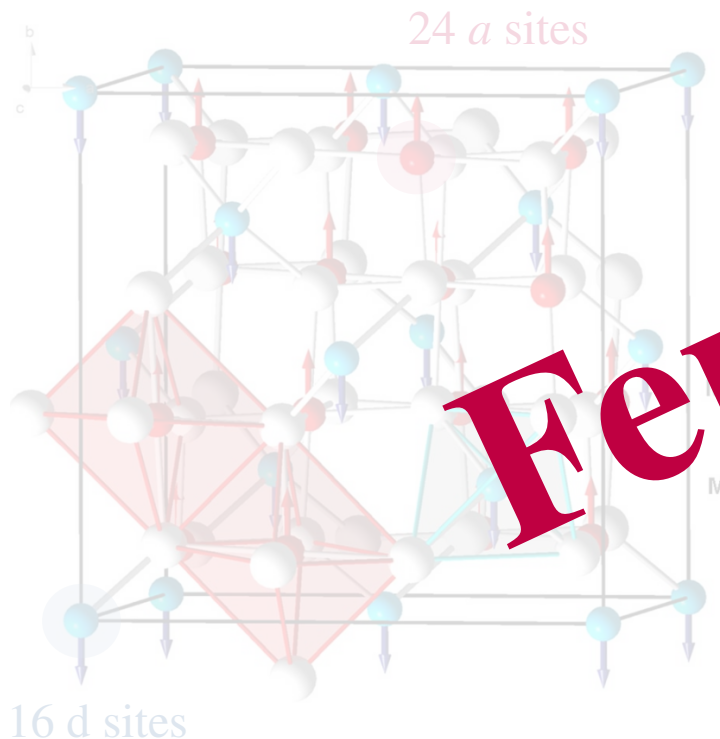
J_1	6.8(2)
J_2	0.52(4)
J_{3a}	0.0(1)
J_{3b}	1.1(3)

- ◆ Antiferromagnetic interactions? ☒
- ◆ Identical, anti-aligned sublattices? ☐
- ◆ Zero net moment? ☐

Yttrium Iron Garnet

“YIG” ($\text{Y}_3\text{Fe}_5\text{O}_{12}$)

“the fruitfly of magnetism” (C. Kittel)



Net magnetic moment
 $\sim 0 \mu_B$ / unit cell

Ferrimagnet
(Ferrimagnet)

This work (meV)

J_1	6.8(2)
J_2	0.52(4)
J_{3a}	0.0(1)
J_{3b}	1.1(3)

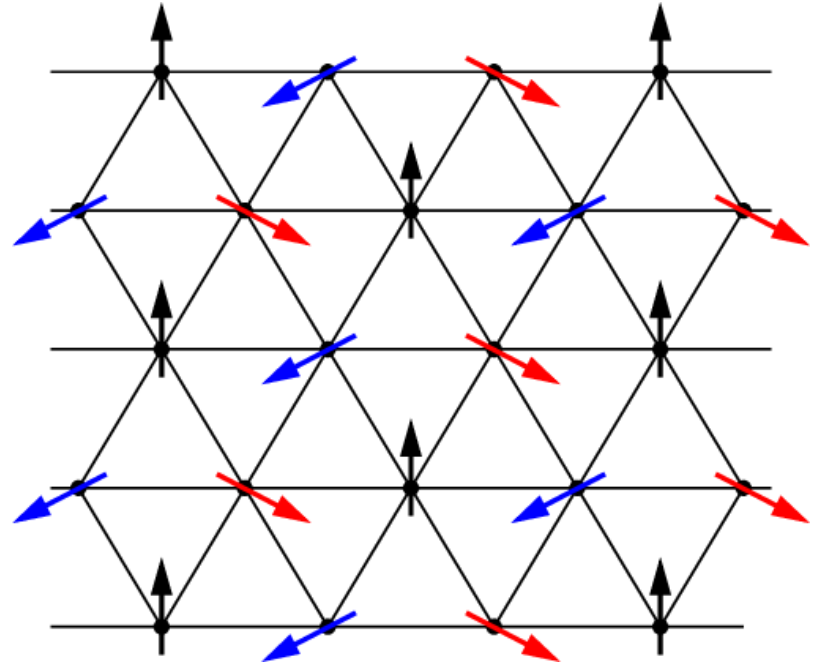
- ♦ Antiferromagnetic interactions? ✓
- ♦ Identical, anti-aligned sublattices? ✗
- ♦ Zero net moment? ✗

Triangular HAFM

- ◆ Ground state of antiferromagnetic Heisenberg model on the triangular lattice

$$J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- ◆ *Frustrated* – can't all anti-align
 - Compromise: Three sublattice 120° order
- ◆ **Compensated** (no net moment)
 - Sublattices cancel



- ◆ Antiferromagnetic interactions? 
- ◆ Identical, anti-aligned sublattices? 
- ◆ Zero net moment? 

Triangular HAFM

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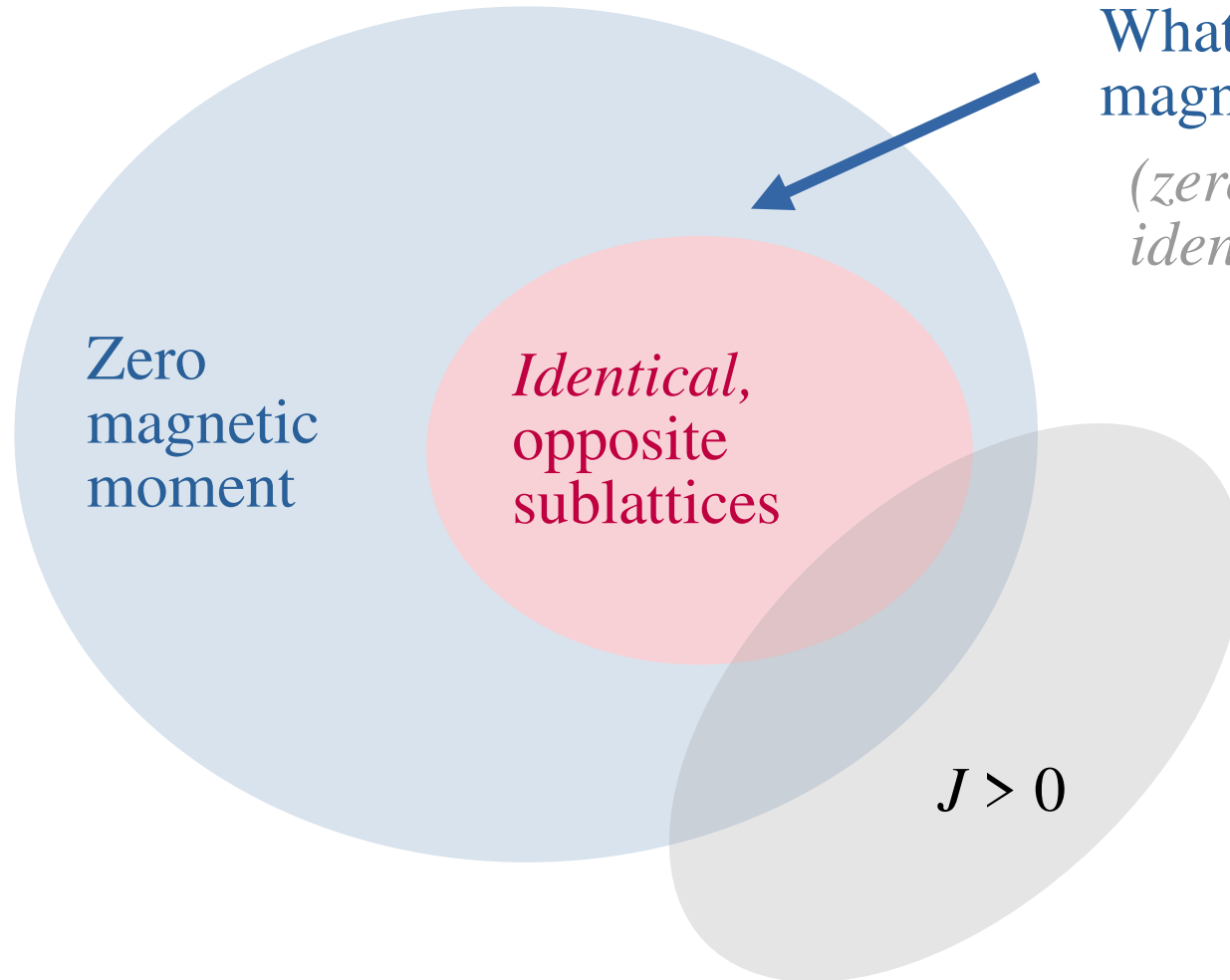
◆ Antiferromagnetic interactions? 

◆ Identical, anti-aligned sublattices? 

◆ Zero net moment? 



Definitions not equivalent

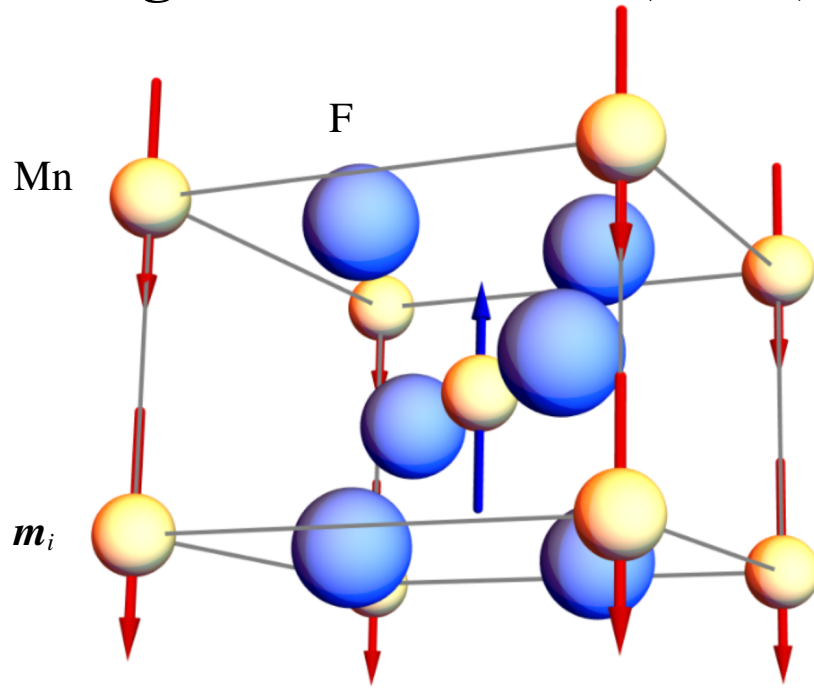


What kind of interesting magnetic structures live *here*?

(zero net moment, but not identical sublattices)

What do we actually mean by *identical*?

Manganese Difluoride (MnF_2)



- ◆ Antiferromagnetic interactions?
- ◆ *Identical*, anti-aligned sublattices?
- ◆ Zero net moment?

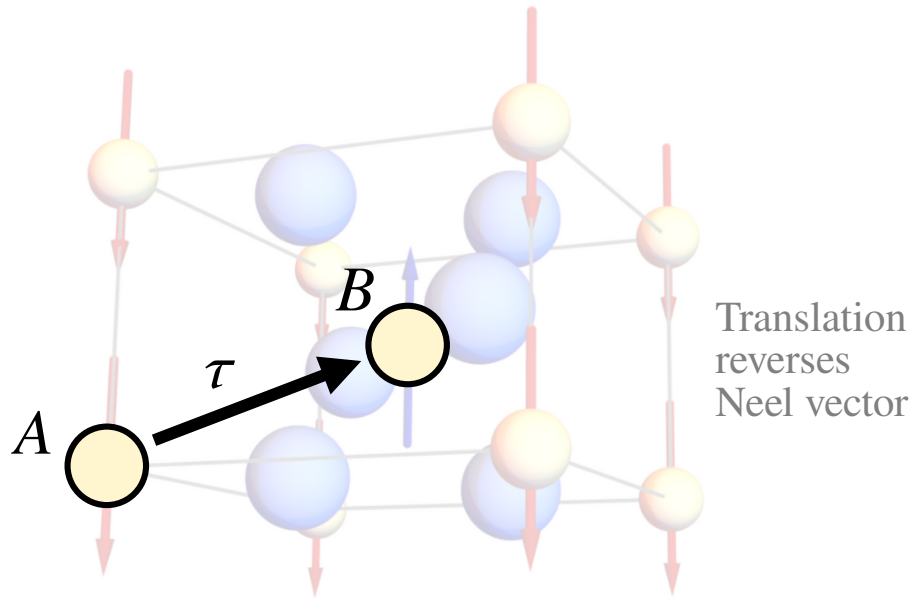
- ◆ Insulating $S = 5/2$ magnet on a tetragonal lattice
- ◆ AF exchange, nearly classical
- ◆ Two-sublattice order, oppositely aligned
- ◆ Well-studied for 60+ years

TUTORIAL / ARTICLE DIDACTIQUE

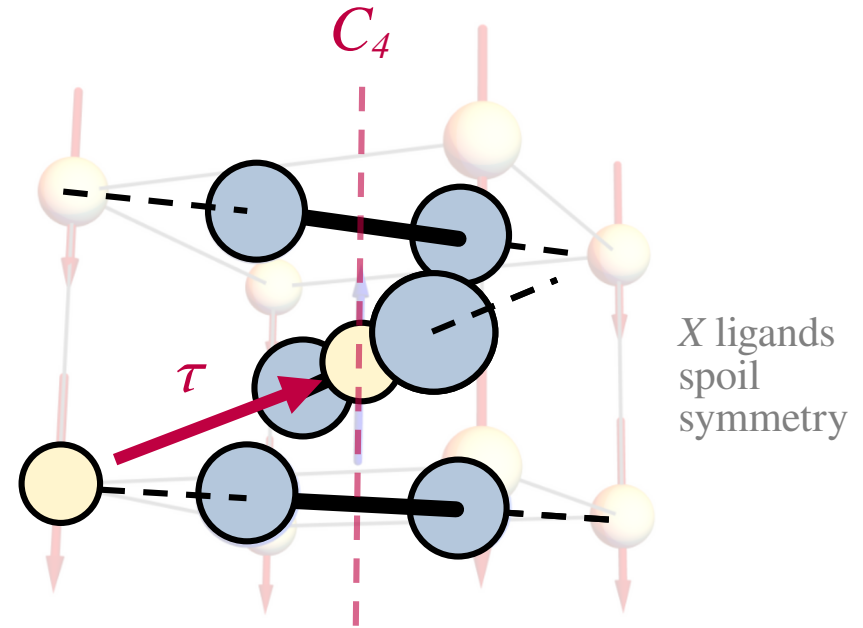
Neutron scattering study of the classical antiferromagnet MnF_2 : a perfect hands-on neutron scattering teaching course¹

Z. Yamani, Z. Tun, and D.H. Ryan

Abstract: We present the classical antiferromagnet MnF_2 as a perfect demonstration system for teaching a remarkable wide variety of neutron scattering concepts. The nature of antiferromagnetism and the magnetic Hamiltonian in this classical antiferromagnet are discussed. The transition temperature to the Neel state, the value of magnetic moment in the ordered state, the critical scattering close to the phase transition, spin waves associated with the ordering of the moment



Translation connecting A
and B sublattice *not a*
symmetry ...

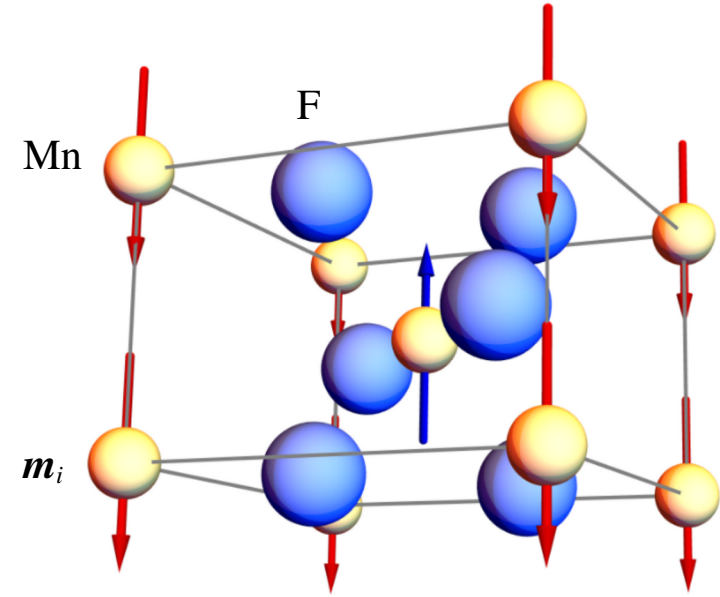


... need to follow with a
four-fold rotation about
z-axis

... magnetic dipole moments are identical, but crystal *environment* is not

Definition of an *alter*-magnet

- ◆ Colinear antiferromagnetic arrangement of magnetic moments
 - Not exactly “identical”
- ◆ Opposite sublattices *symmetry-related*
- ◆ ... but by a *non-trivial spatial symmetry* (rotation or reflection)
 - *Not* translation
 - *Not* inversion
- ◆ Can be defined with or without spin-rotation symmetry, in metal or insulator



Moment structure in MnF₂

More signatures of altermagnetism

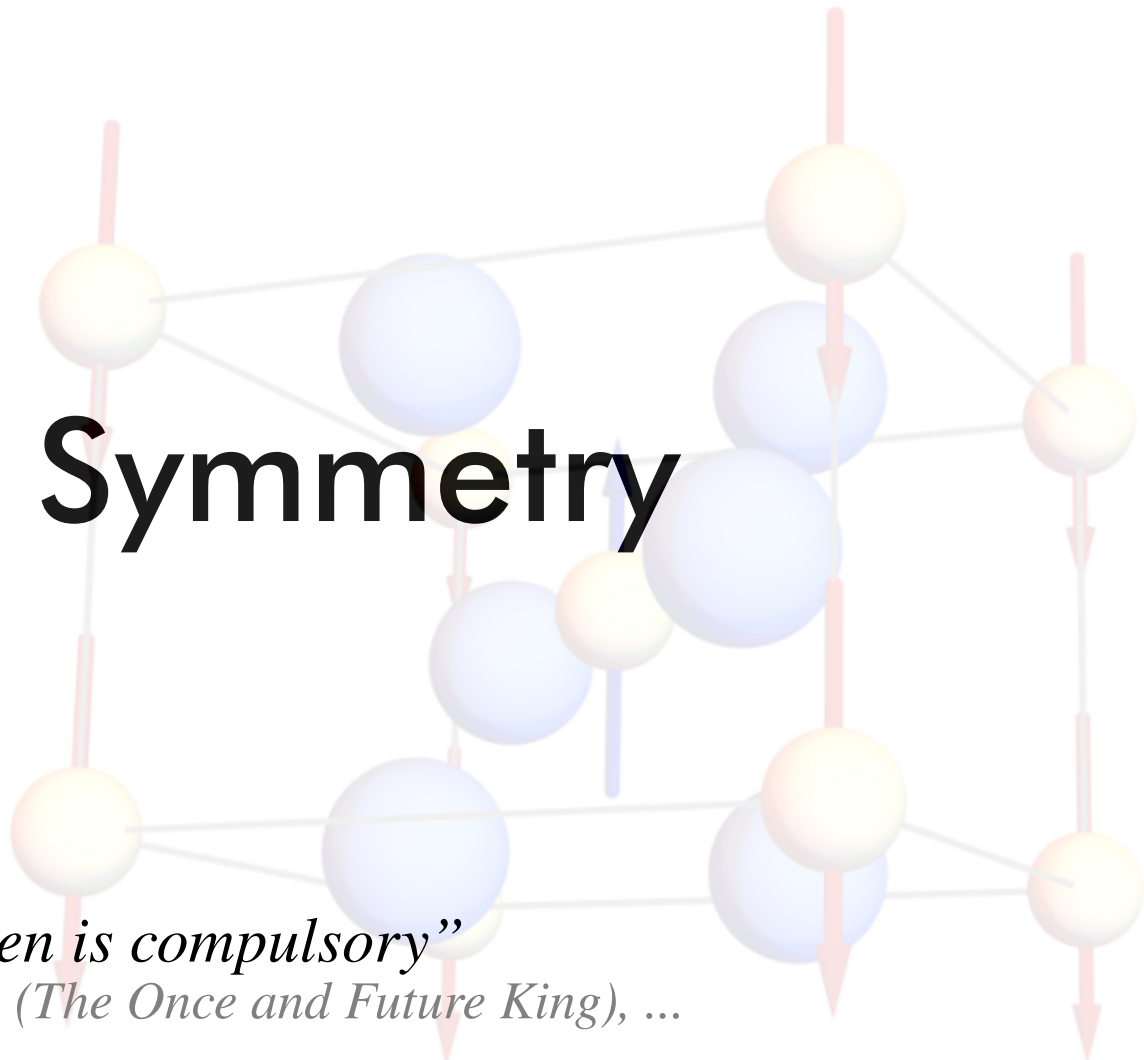
- ◆ Think of as antiferromagnets – **no net moment** - but with many properties *usually* associated with *ferromagnets* *No stray fields*
- ◆ Features that are ferromagnetic-like
 - Spin-split bands
 - Anomalous Hall Effect
 - Magneto-optical response
 - Piezomagnetism
 - Spin-polarized currents, spin-transfer torque, ...
- ◆ **New features?** Induced multipolar order?

*Applications in
Spintronics?*

Magnetic Symmetry

“Everything not forbidden is compulsory”

- M. Gellman, T. H. White (The Once and Future King), ...



Crystal Symmetries

- ◆ Symmetries are combinations of *rotations, inversions & translations*

$$\{\pm \mathbf{R} | \boldsymbol{\tau}\}$$

Rotation part,
without or
without inversion

Translation
part

- ◆ **Space group:** All such operations that leave the crystal invariant
 - *Point group:* Space group with translations modded out

Acts as expected on position \mathbf{r}

$$\{\pm \mathbf{R} | \boldsymbol{\tau}\} \mathbf{r} = \pm \mathbf{R} \mathbf{r} + \boldsymbol{\tau}$$

Pseudo-vector invariant
under inversion

$$\{\pm \mathbf{R} | \boldsymbol{\tau}\} [\mathbf{m}(\mathbf{r})] = \mathbf{R} \mathbf{m}(\pm \mathbf{R}^{-1}(\mathbf{r} - \boldsymbol{\tau}))$$

Pseudo-vector field $\mathbf{m}(\mathbf{r})$

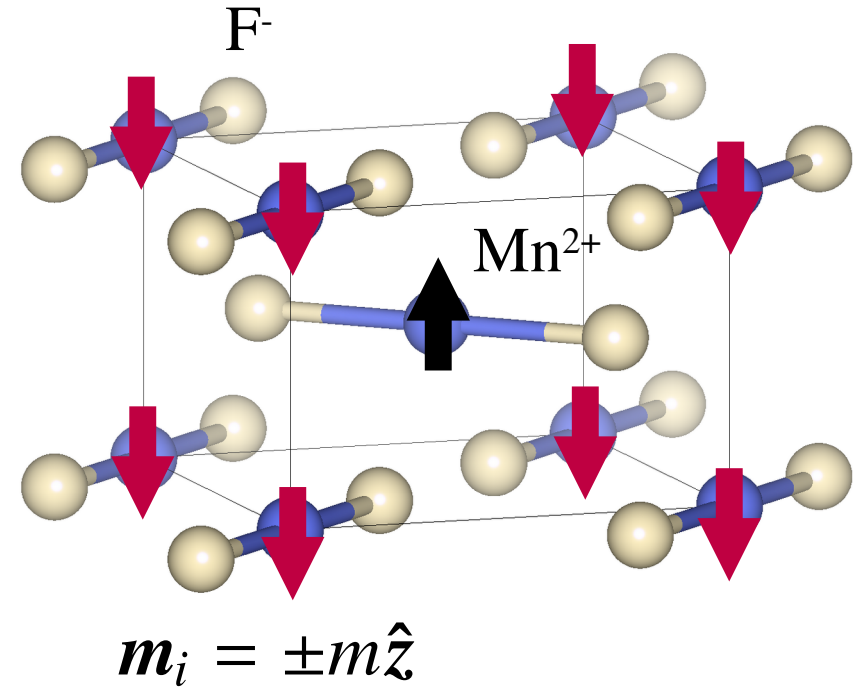
$I =$ inversion

$C_{n,\hat{\mathbf{v}}} = 2\pi/n$ rotation along \mathbf{v}

$\sigma_{\hat{\mathbf{v}}} =$ reflect in plane $\perp \mathbf{v}$

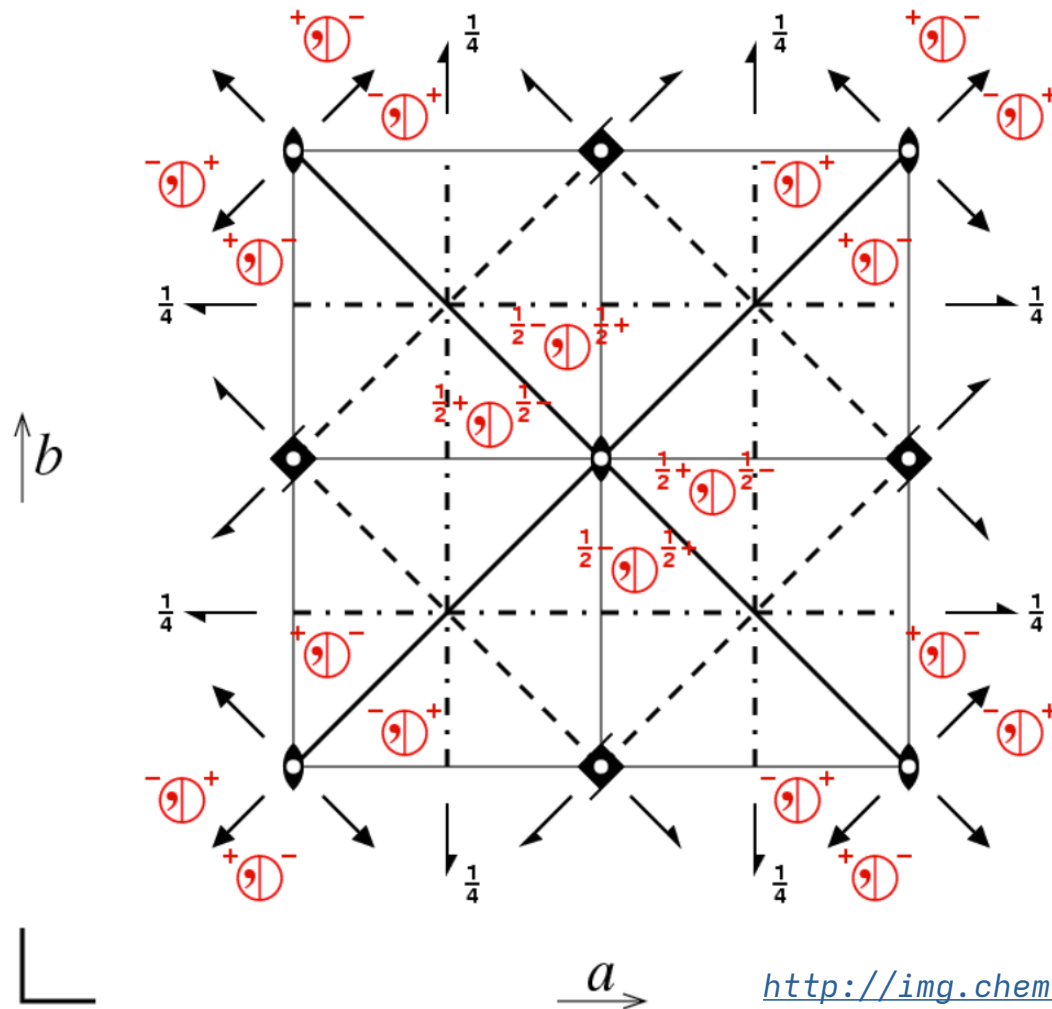
Example: MnF_2

- ◆ Space group of *crystal* is $P4_2/mnm$ (#136)
- ◆ Unit cell contains:
 - Two Mn^{2+} ions (magnetic)
 - Four F^- ions (non-magnetic)
- ◆ Magnetic ordering is *within* the unit cell
 - Propagation vector is $\mathbf{k} = \mathbf{0}$
- ◆ Experimentally $\mathbf{m}_i = \mathbf{m}(\mathbf{r}_i)$ at each site along z



$P4_2/mnm$ $P4_2/m\ 2_1/n\ 2/m$ $4/mmm$

No. 136



Id.

1	x, y, z
2	\bar{x}, \bar{y}, z
3	$\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} + z$
4	$\frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} + z$
5	$\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} + z$
6	$\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} + z$
7	y, x, z
8	\bar{y}, \bar{x}, z
9	$\bar{x}, \bar{y}, \bar{z}$
10	x, y, \bar{z}
11	$\frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} - z$
12	$\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} - z$
13	$\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} - z$
14	$\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} - z$
15	$\bar{y}, \bar{x}, \bar{z}$
16	y, x, \bar{z}

 C_{2z} $C_{4z} + \tau, C_{4z}^{-1} + \tau$ $C_{2x} + \tau, C_{2y} + \tau$ $C_{2,x+y}, C_{2,x-y}$

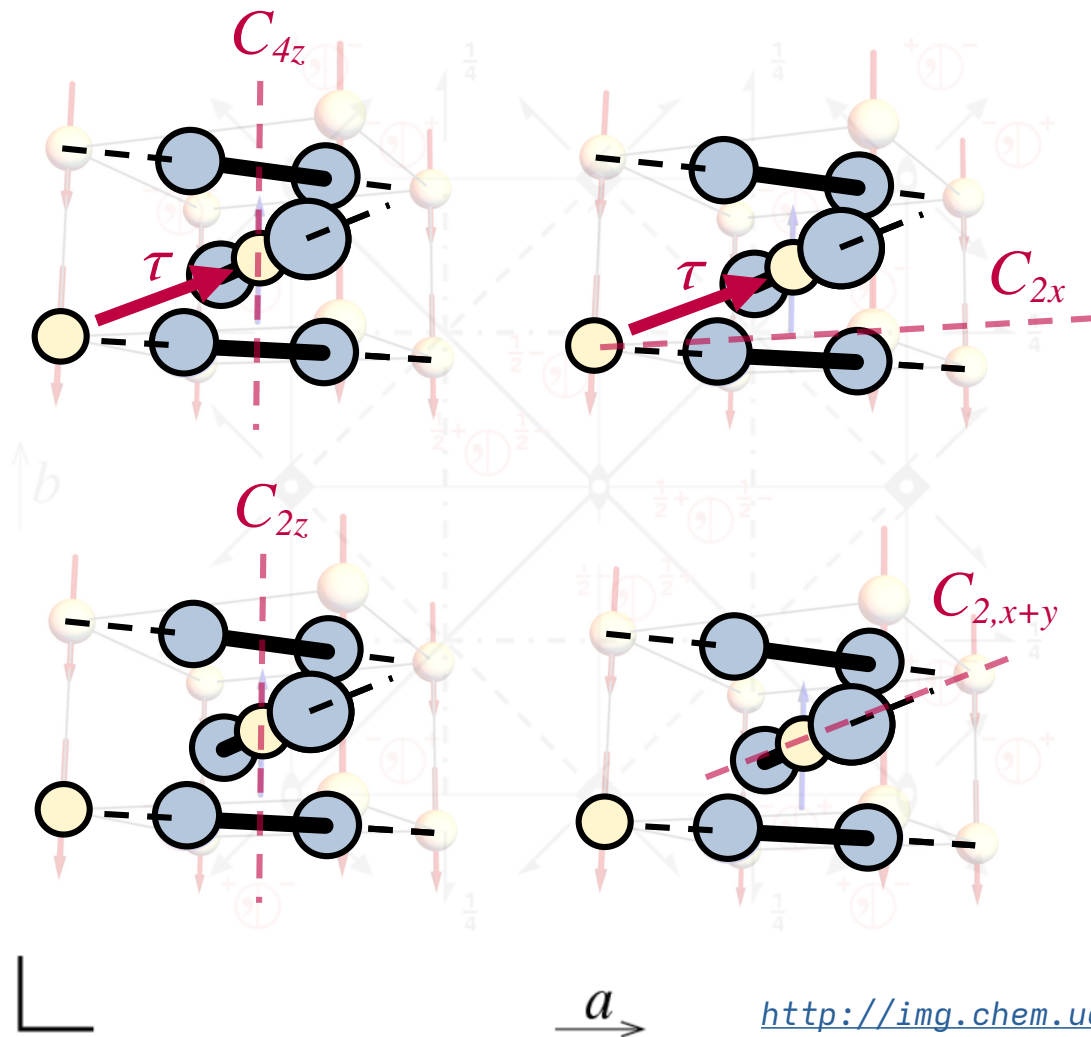
... same
elements +
inversion

... + all
translations



$P4_2/mnm$ $P 4_2/m 2_1/n 2/m$ $4/mmm$

No. 136



Id.

1	x, y, z
2	\bar{x}, \bar{y}, z
3	$\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} + z$
4	$\frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} + z$
5	$\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} + z$
6	$\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} + z$
7	y, x, z
8	\bar{y}, \bar{x}, z
9	$\bar{x}, \bar{y}, \bar{z}$
10	x, y, \bar{z}
11	$\frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} - z$
12	$\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} - z$
13	$\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} - z$
14	$\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} - z$
15	$\bar{y}, \bar{x}, \bar{z}$
16	y, x, \bar{z}

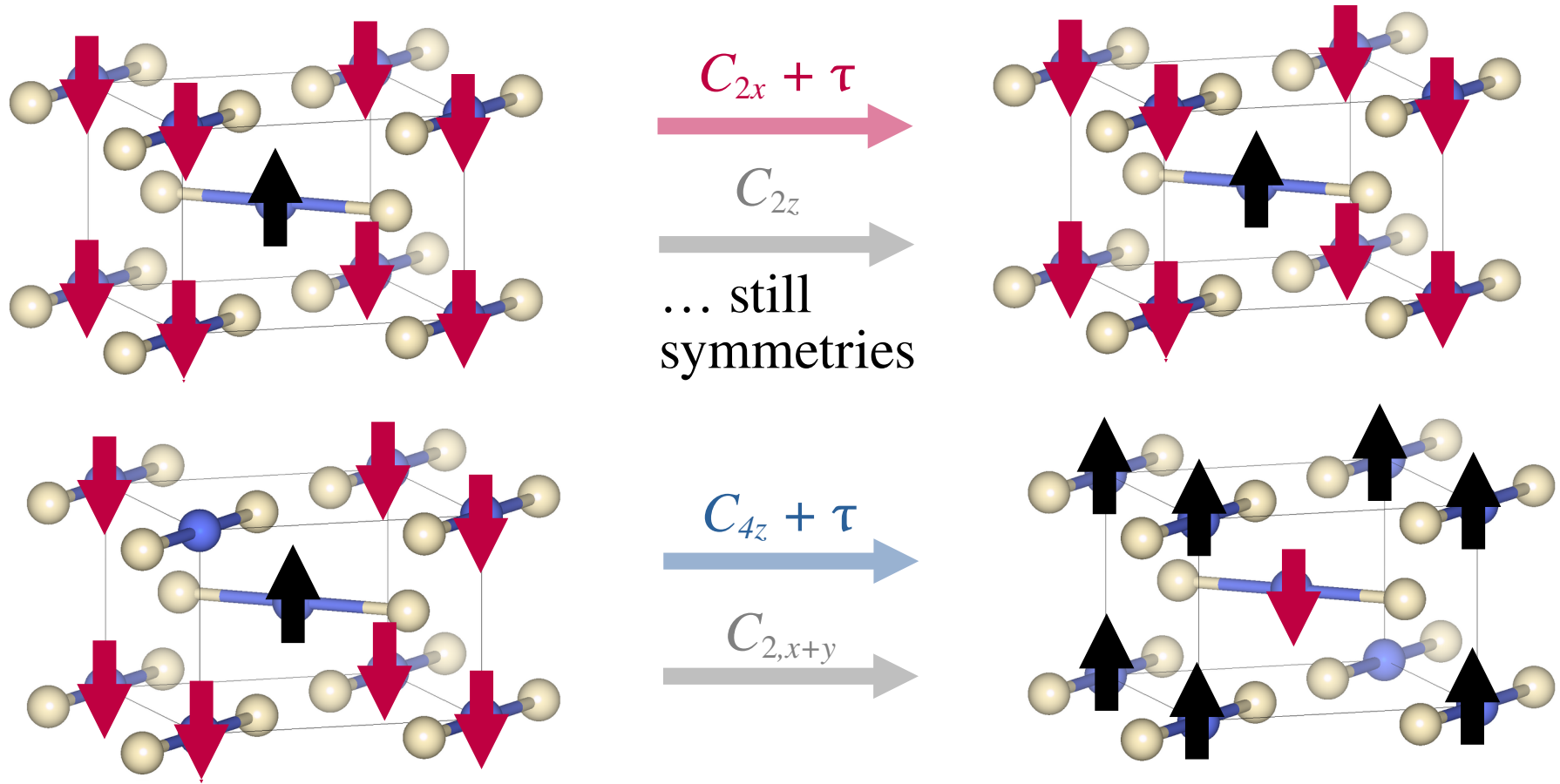
 C_{2z} $C_{4z} + \tau, C_{4z}^{-1} + \tau$ $C_{2x} + \tau, C_{2y} + \tau$ $C_{2,x+y}, C_{2,x-y}$

... same
elements +
inversion

... + all
translations



What about symmetries of *magnetic structure*?



Symmetry when combined with **time reversal**

- ◆ Space group promoted to **magnetic space group**
 - Subset of elements paired with *time-reversal*
- ◆ **MSG: $P4_2'/mnm'$ (#136.499)**

$$C_{2z}$$

$$C_{4z} + \tau, C_{4z}^{-1} + \tau$$

$$C_{2x} + \tau, C_{2y} + \tau$$

$$C_{2,x+y}, C_{2,x-y}$$

Anything involving these comes with T.R.

Easy to find on Bilbao Crystallographic Server (MAGNDATA)

<https://www.cryst.ehu.es/magndata/>

MAGNDATA: A Collection of magnetic structures with portable cif-type files

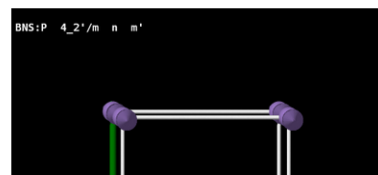
[Log in](#)

Element search (separate with space or comma): ☒ AND ☐

OR [View Full Database](#) [Advanced Search](#)

To upload any published structure click [HERE](#)

Enter the label of the structure:



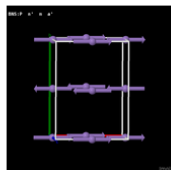
MnF₂ (#0.15)

[view in Jmol](#)

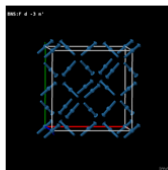
[Download mcif file](#)

[Download vosta file](#)

Zero propagation vector



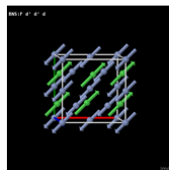
0.1 LaMnO_3



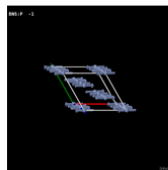
0.2 $\text{Cd}_2\text{Os}_2\text{O}_7$



0.3 $\text{Ca}_3\text{LiOsO}_6$



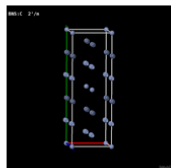
0.4 NiCr_2O_4



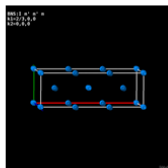
0.5 Cr_2S_3

[Click to expand/shrink back the rest](#)

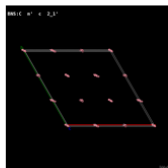
Non-zero propagation vector (magnetic space groups of Type I or Type III)



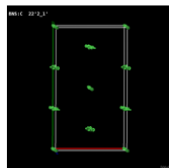
1.0.1 Ag_2CrO_2



1.0.2 $\text{URu}_{0.96}\text{Rh}_{0.04}\text{Si}_2$



1.0.3 CsCoBr_3



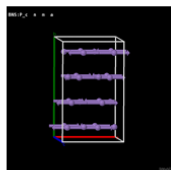
1.0.4 CsNiCl_3



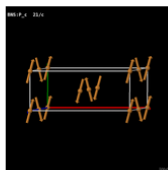
1.0.5 $\text{Sr}_3\text{CoIrO}_6$

[Click to expand/shrink back the rest](#)

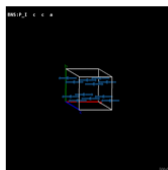
Non-zero propagation vector (magnetic space groups of Type IV)



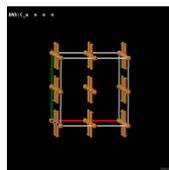
1.1 Mn_3O_4



1.2 CuSe_2O_5



1.3 Sr_2IrO_4



1.4 $\text{YBa}_2\text{Cu}_3\text{O}_{6+d}$



1.5 $\text{YBa}_2\text{Cu}_3\text{O}_{6+d}$

[Click to expand/shrink back the rest](#)

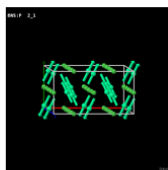
Two propagation vectors



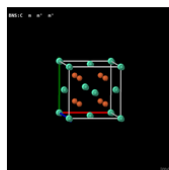
2.1 EuFe_2As_2



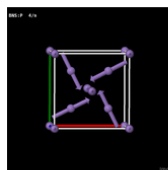
2.2 $\text{Sr}_2\text{Fe}_2\text{OS}_2$



2.3 HoNiO_3



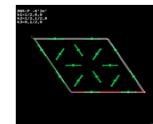
2.4 $\text{Eu}(\text{Fe}_{0.82}\text{Co}_{0.18})\text{As}_2$



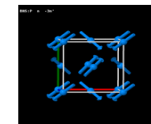
2.5 Mn_3CuN

Aside

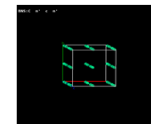
Three or more propagation vectors



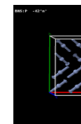
3.1 TmAgGe



3.2 UO_2



3.3 Ho_2RhIn



3.4 M

[Click to expand/shrink back the rest](#)

INCOMMENSURATE STRUCTURES

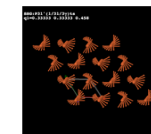
[Click to expand/shrink back the rest](#)

INCOMMENSURATE STRUCTURES

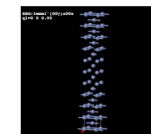
One propagation vector



1.1.1 Cs_2CuCl_4



1.1.2 $\text{RbFe}(\text{MoO}_4)_2$



1.1.3 Cr



1.1.4

[Click to expand/shrink back the rest](#)

N	Standard/Default Setting			
	(x,y,z) form	Matrix form	Geom. interp.	Seitz notation
1	x, y, z, +1 m_x, m_y, m_z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	1 <u>+1</u>	{ 1 0 }
2	x+1/2, -y+1/2, -z+1/2, +1 $m_x, -m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (1/2,0,0) x,1/4,1/4 <u>+1</u>	{ 2 ₁₀₀ 1/2 1/2 1/2 }
3	-x+1/2, y+1/2, -z+1/2, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (0,1/2,0) 1/4,y,1/4 <u>+1</u>	{ 2 ₀₁₀ 1/2 1/2 1/2 }
4	-x, -y, z, +1 $-m_x, -m_y, m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	2 0,0,z <u>+1</u>	{ 2 ₀₀₁ 0 }
5	-x, -y, -z, +1 m_x, m_y, m_z	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	-1 0,0,0 <u>+1</u>	{ -1 0 }
6	-x+1/2, y+1/2, z+1/2, +1 $m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (0,1/2,1/2) 1/4,y,z <u>+1</u>	{ m ₁₀₀ 1/2 1/2 1/2 }
7	x+1/2, -y+1/2, z+1/2, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (1/2,0,1/2) x,1/4,z <u>+1</u>	{ m ₀₁₀ 1/2 1/2 1/2 }
8	x, y, -z, +1 $-m_x, -m_y, m_z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	m x,y,0 <u>+1</u>	{ m ₀₀₁ 0 }

... still
symmetries

9	-y+1/2, x+1/2, z+1/2, -1 $m_y, -m_x, -m_z$	$\begin{pmatrix} 0 & -1 & 0 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	4 ⁺ (0,0,1/2) 0,1/2,z <u>-1</u>	{ 4 ⁺ ₀₀₁ 1/2 1/2 1/2 }
10	y+1/2, -x+1/2, z+1/2, -1 $-m_y, m_x, -m_z$	$\begin{pmatrix} 0 & 1 & 0 & 1/2 \\ -1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	4 ⁻ (0,0,1/2) 1/2,0,z <u>-1</u>	{ 4 ⁻ ₀₀₁ 1/2 1/2 1/2 }
11	y, x, -z, -1 $-m_y, -m_x, m_z$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	2 x,x,0 <u>-1</u>	{ 2' ₁₁₀ 0 }
12	-y, -x, -z, -1 m_y, m_x, m_z	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	2 x,-x,0 <u>-1</u>	{ 2' ₁₋₁₀ 0 }
13	y+1/2, -x+1/2, -z+1/2, -1 $m_y, -m_x, -m_z$	$\begin{pmatrix} 0 & 1 & 0 & 1/2 \\ -1 & 0 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	-4 ⁺ 1/2,0,z 1/2,0,1/4 <u>-1</u>	{ -4 ⁺ ₀₀₁ 1/2 1/2 1/2 }
14	-y+1/2, x+1/2, -z+1/2, -1 $-m_y, m_x, -m_z$	$\begin{pmatrix} 0 & -1 & 0 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	-4 ⁻ 0,1/2,z 0,1/2,1/4 <u>-1</u>	{ -4 ⁻ ₀₀₁ 1/2 1/2 1/2 }
15	-y, -x, z, -1 $-m_y, -m_x, m_z$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	m x,-x,z <u>-1</u>	{ m' ₁₁₀ 0 }
16	y, x, z, -1 m_y, m_x, m_z	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	m x,x,z <u>-1</u>	{ m' ₁₋₁₀ 0 }

Symmetry when combined
with **time reversal**

Magnetic Symmetry

- ◆ All physical quantities must transform appropriately under these symmetries
- ◆ **Example:** Net magnetization should be *invariant*

$$\mathbf{M} \xrightarrow{C_{4z}+T} -M_y\hat{x} + M_x\hat{y} + M_z\hat{z} \xrightarrow{T} M_y\hat{x} - M_x\hat{y} - M_z\hat{z}$$

- ◆ Implies immediately that $\mathbf{M} = \mathbf{0}$.
- ◆ Symmetry *requires* that MnF₂ is compensated; similarly **AHE vanishes**
- ◆ Also implies that $N_x = N_y = 0$ – Neel vector must be along z

Piezomagnetism

- ◆ Other observables can be analyzed in same way; *piezomagnetism* is magnetization induced by applied strain

$$\mathbf{M} = \sum_{\mu\nu} \mathbf{C}_{\mu\nu} \epsilon_{\mu\nu}$$

- ◆ Strain transforms like a tensor $\epsilon \rightarrow \mathbf{R}^{-1} \epsilon \mathbf{R}$

$$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{yz}, \epsilon_{xz}, \epsilon_{xy} \xrightarrow{C_{4z}+T} \epsilon_{yy}, \epsilon_{xx}, \epsilon_{zz}, -\epsilon_{xz}, \epsilon_{yz}, -\epsilon_{xy}$$

- ◆ Using all these symmetries: $\mathbf{M} = C' \left(\epsilon_{yz} \hat{\mathbf{x}} + \epsilon_{xz} \hat{\mathbf{y}} \right) + C \epsilon_{xy} \hat{\mathbf{z}}$

Other examples are the antiferromagnetics MnF_2 , CoF_2 , and FeF_2 . In accordance with Ref. 4, their magnetic symmetry class consists of

$$C_2, 2C_4R, 2U_2, 2U_2'R, I, \sigma_h, 2S_4R, 2\sigma_v, 2\sigma_v'R.$$

This symmetry group leaves invariant the following term in the expression for Φ :

$$\Phi = -\lambda(\sigma_{xz}H_y + \sigma_{yz}H_x),$$

whence we get for the magnetic moment

$$m_x = \lambda\sigma_{yz}, \quad m_y = \lambda\sigma_{xz}.$$

THE PROBLEM OF PIEZOMAGNETISM

I. E. DZIALOSHINSKII

Physical Problems Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 20, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 807-808 (September, 1957)

IN the recent literature one encounters assertions that piezomagnetic bodies in general cannot exist in nature. This conclusion has been based on the invariance of the equations of mechanics with respect to

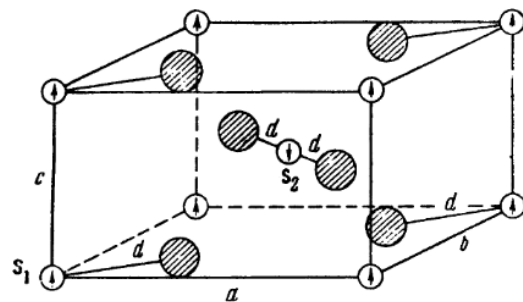


FIG. 1. The magnetic structure of MnF_2 and CoF_2 . Open circles – Mn, Co; shaded circles – F.

*In Dzyaloshinskiĭ paper⁵ a term was omitted in the expression for the thermodynamic potential responsible for the appearance of a piezo-magnetic moment along the z axis. A complete analysis of this problem was given in his dissertation.¹⁸

but only on applying shear stresses

$$m_x^p = \Lambda_1 \sigma_{yz}, \quad m_y^p = \Lambda_1 \sigma_{xz}, \quad m_z^p = \Lambda_2 \sigma_{xy}. \quad (1)$$

SOVIET PHYSICS JETP

VOLUME 11, NUMBER 4

OCTOBER, 1960

PIEZOMAGNETISM IN THE ANTIFERROMAGNETIC FLUORIDES OF COBALT AND MANGANESE

A. S. BOROVIK-ROMANOV

Institute of Physical Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 6, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1088-1098 (April, 1960)

A special magnetic balance and press were constructed to observe piezomagnetism experimentally. In agreement with theoretical predictions, piezomagnetic moments m_j^p were found

Multipoles

- ◆ Only symmetry distinct if *spherically symmetric*
- ◆ A definition of a magnetic octupole

Always induce
higher multipoles
in crystal

$$\mathbf{O}_{\mu\nu} \equiv \int d^3r \, r_\mu r_\nu \mathbf{m}(\mathbf{r})$$

$$O_{xy}^z \xrightarrow{C_{4z}+T} +O_{xy}^z$$

$$O_{zz}^z \xrightarrow{C_{4z}+T} -O_{zz}^z$$

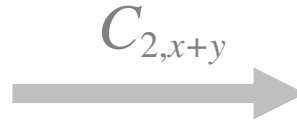
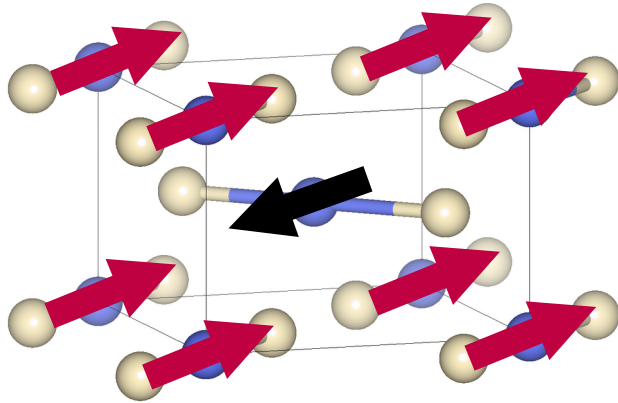
$$O_{yz}^x \xrightarrow{C_{4z}+T} -O_{xz}^y$$

- ◆ Two fully symmetric components can be constructed after apply all operations

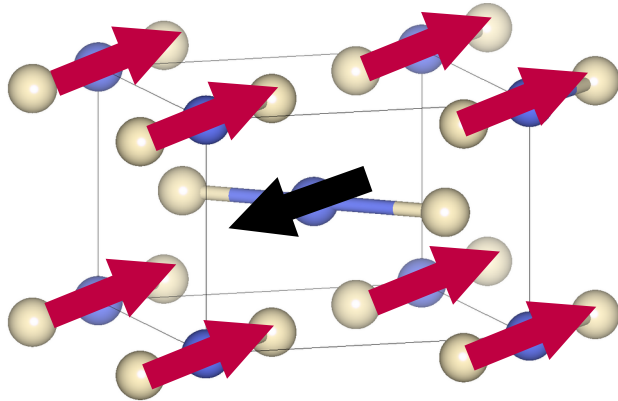
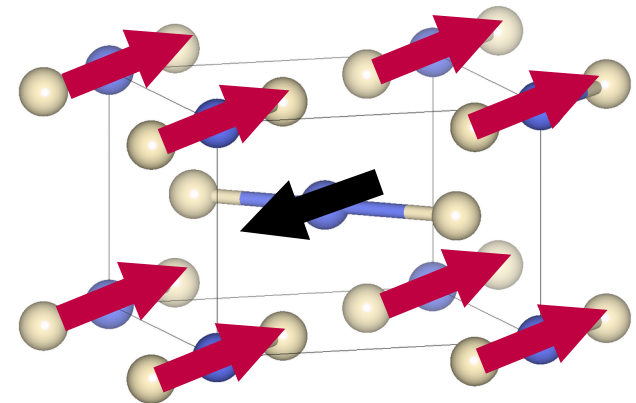
$$O_{xy}^z, \quad O_{yz}^x + O_{xz}^y$$

Expect to be
non-zero in
the ordered
phase

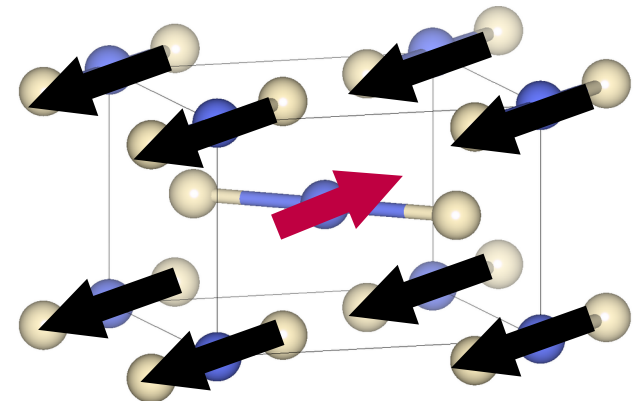
Symmetry depend on **moment direction**



... still a
symmetry



... with T.R.



... rest of four-fold and two-fold operations are simply *not symmetries*

Magnetic Symmetry (cont.)

- ♦ Moment *direction* affects the magnetic space group
- ♦ Net magnetization is less constrained by symmetry for in-plane N

$$\mathbf{M} \xrightarrow{C_{2z}+T} M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}} - M_z \hat{\mathbf{z}} \quad M_z = 0$$

$$\mathbf{M} \xrightarrow{C_{2,x-y}+T} M_y \hat{\mathbf{x}} + M_x \hat{\mathbf{y}} \quad M_x = M_y \equiv M$$

$$\mathbf{M} \xrightarrow{C_{2,x+y}} \frac{M}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

(weak) Ferromagnet
not antiferromagnetic

Simpler antiferromagnet

- ◆ Consider a case where these *are* identical: we have a **translation** connecting the two sublattices

$$\mathbf{M} \xrightarrow{\tau+T} -\mathbf{M}$$



$$\mathbf{M} = 0$$

$$\mathbf{M} = \sum_{\mu\nu} \mathbf{C}_{\mu\nu} \boldsymbol{\epsilon}_{\mu\nu} \xrightarrow{\tau+T} -\mathbf{M} = \sum_{\mu\nu} \mathbf{C}_{\mu\nu} \boldsymbol{\epsilon}_{\mu\nu}$$



$$\mathbf{C}_{\mu\nu} = 0$$

$$\mathbf{O}_{\mu\nu} \xrightarrow{\tau+T} -\mathbf{O}_{\mu\nu}$$



$$\mathbf{O}_{\mu\nu} = 0$$

Interlude

- ◆ So for a given crystal and magnetic structure, can work out a **magnetic space group**
- ◆ There are **1651 distinct magnetic space groups** (well understood)
 - Many more than the usual 230 space groups
- ◆ So are altermagnets just some subset of these groups?
 - Some anti-ferromagnetic (zero moment) some weak ferromagnets?
 - Was this just hidden in MSG tables from the 1960s and no one noticed?
 - What is *new* in this definition of an altermagnet?

Answer: Yes *and* no

Are altermagnets *really* new?

- ◆ **Altermagnets are well-defined in the non-relativistic limit** (*zero* spin-orbit coupling, *zero* dipolar interactions)
 - In this limit they are distinct from FMs and AFMs
 - Not a ferromagnet since zero moment
 - Symmetry acts differently than simplest AFMs
- ◆ Symmetries are higher in non-relativistic limit: **Spin space groups**
 - Spin rotations can act independently than spatial symmetries
 - Classification goes beyond usual 230 space groups or the 1651 magnetic space groups

Brinkman & Elliot, Proc. Roy. Soc. A **294**, 343 (1966), Litvin & Opechowski, Physica **76**, 538 (1974), Corticelli *et al*, Phys. Rev. B 105, 064430 (2022), Xiao *et al* Phys. Rev. X 14, 031037 (2024), Chen *et al*, Phys. Rev. X 14, 031038 (2024), ...

Questions

- ◆ Is this really a **third kind of magnetism**? **No**
 - *Why only two? What about spirals and non-collinear or coplanar orders, etc*
- ◆ Is this more like a ferromagnet or more like an antiferromagnet? **Yes** (no SOC)
 - *When is it really distinct from either of those?*
- ◆ How does this relate to large body of knowledge on magnetic symmetries? **No** (w/ SOC)
 - *Like space groups, there are magnetic space groups that have been well-understood for decades*

Spin Space Groups (no SOC)

With SOC? Magnetic groups

Non-relativistic symmetries

- ◆ Altermagnets *are* distinct in the non-relativistic limit
 - Absent spin-orbit coupling, dipole-dipole interactions, crystal field effects
- ◆ Lots of magnetic materials *naturally* near this limit
 - Elements high up the periodic table (Cu, Ni, ...)
 - Elements with spin only moments (Mn^{2+} , Fe^{3+} , Eu^{2+} , Gd^{3+} , ...)
 - ... sometimes it happens accidentally (e.g. some Yb^{3+} compounds)
- ◆ **Never actually zero**
 - *Essential* to delineate relativistic and non-relativistic contributions

Non-relativistic symmetries

- Without spin orbit coupling, **spin and space can transform independently**

$$\{\pm R, \tau\}[m(r)] = Rm(\pm R^{-1}(r - \tau))$$

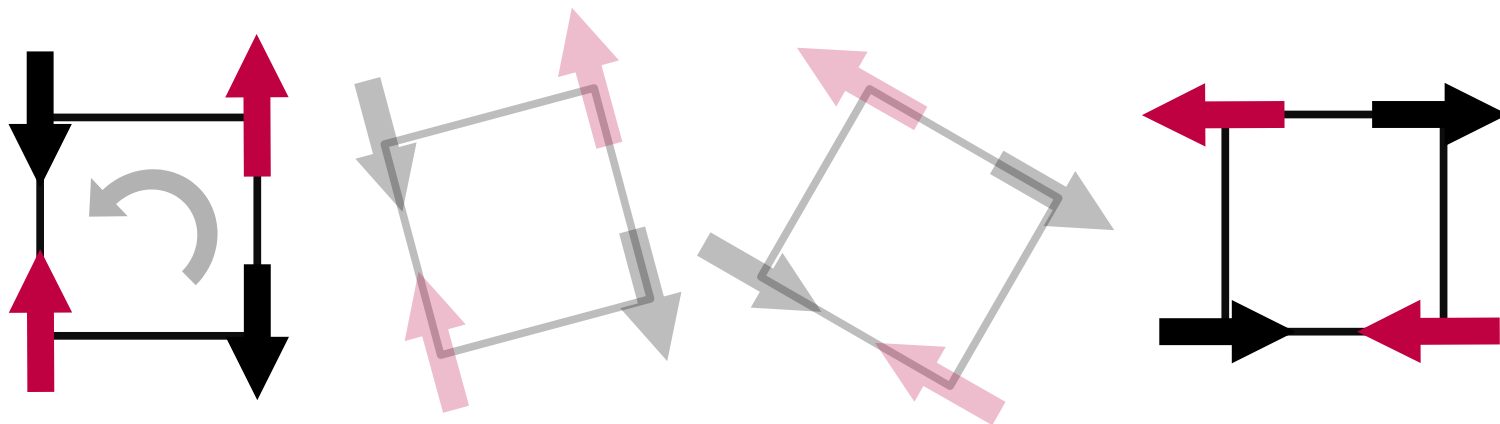
Rotation & translation in space

$$\{M || \pm R, \tau\}[m(r)] = Mm(\pm R^{-1}(r - \tau))$$

Separate rotation on spin

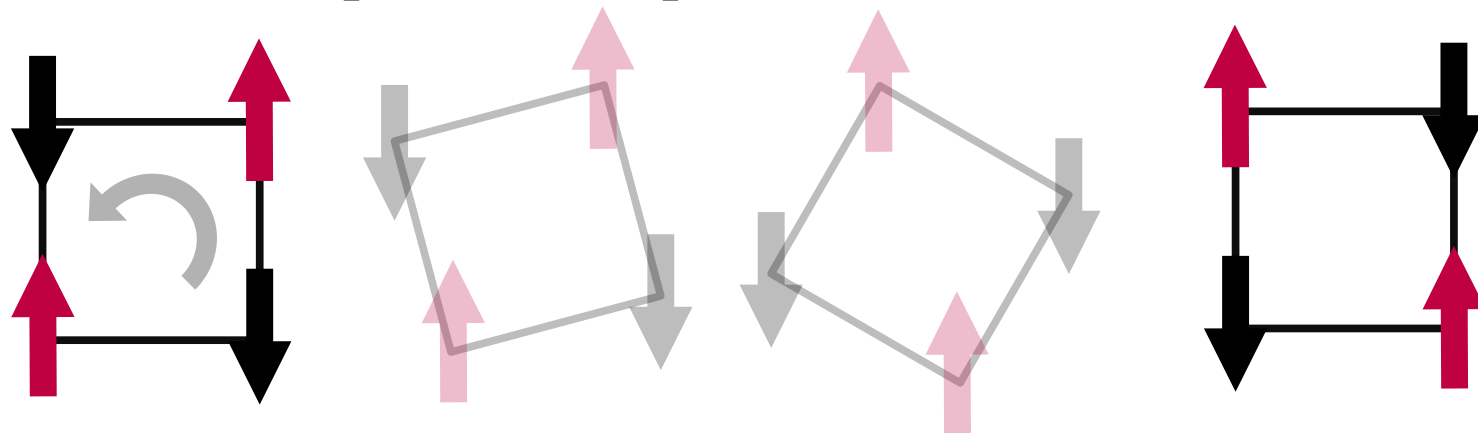
- Symmetry groups of this form are called **Spin Space Groups** (for infinite lattices) and **Spin Point Groups** (for finite objects)

Rotate spin and space together



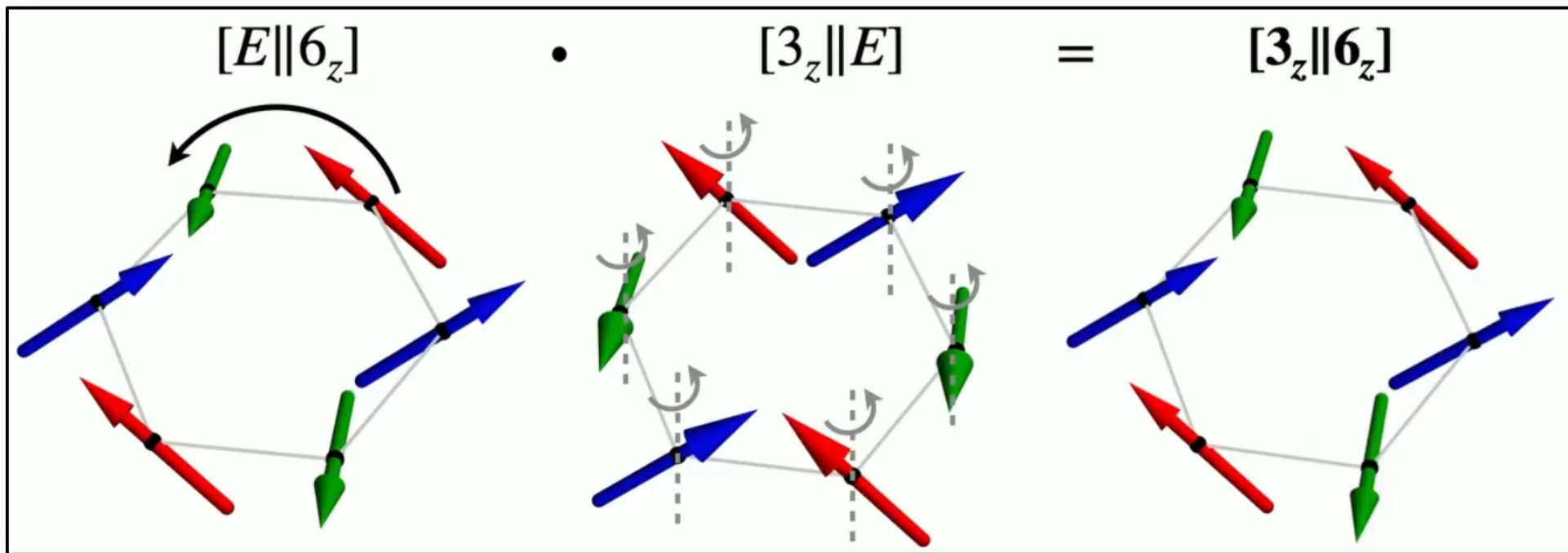
$\{C_4 \parallel C_4\}$

Rotate *just* space, leave spin alone



$\{1 \parallel C_4\}$

Example* $3_z 6/1 m^{m_x} m^{m_{xy}} m$



Atoms related by 60 degree rotation,
spins by **120 degree rotation**

* stolen from H. Schiff

Classification of Spin Space Groups

- ◆ *Infinite* number of spin space groups; Partial classifications from several groups

- **Colinear orders:** $1421^{a,b,c}$

- **Coplanar orders:** 16383^a or 9542^b or 24788^c

- **Non-coplanar orders:** 87308^a or 56512^b or 157289^c

Q. Liu's group^a
(Supercells size 8)
Z. Song's group^b
(Families mod k)
C. Fang's group^c
(Supercells size 12)

- ◆ Colinear case is simplest

Xiao *et al* Phys. Rev. X 14, 031037 (2024), Chen *et al*, Phys. Rev. X 14, 031038 (2024), Jiang *et al*, Phys. Rev. X (2024), ...

<https://findspingroup.com/>

FINDSPINGROUP: Identify Spin Group and Related Properties

Online Program: FINDSPINGROUP [1]

[Start here](#)

Manual: International notation of Spin Space Group and How to use FINDSPINGROUP

<https://cmpdc.iphy.ac.cn/ssg>

Spin Space Group Database

Home

Welcome to the Spin Space Group Database

Spacegroup Number [Select](#)

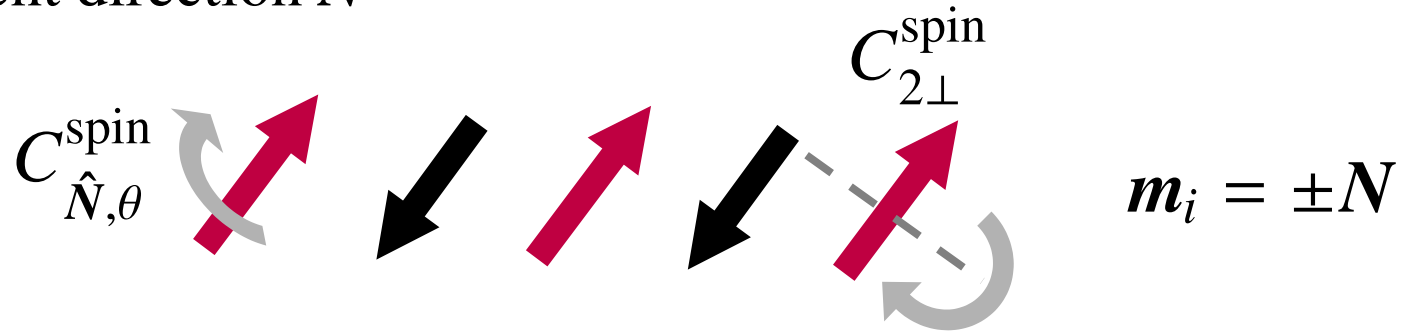
H Spacegroup Number [Select](#)

All SSG eg. 230.1.1 or 230.1.1.1.L [Search](#)

SSG #	Dimension	Group number of pure lattice symmetry	Point group of Q	Point group of spin part
1.1.1.1	3D	1	1	1

Colinear Spin Space Groups

- ◆ Altermagnets are *colinear*; subset of these 1421 SSGs
- ◆ All colinear spin space groups have $U(1)$ continuous symmetry along the moment direction N



- ◆ Projection of spin along this axis is good quantum number
- ◆ Additional symmetry in all these groups $C_{2\perp}^{\text{spin}} + \text{T.R.}$

Types of colinear SSGs

Šmejkal *et al* Phys. Rev. X **12**, 031042 (2022), Šmejkal *et al*, Phys. Rev. X **12**, 040501 (2022), Xiao *et al* Phys. Rev. X **14**, 031037 (2024), Chen *et al*, Phys. Rev. X **14**, 031038 (2024), Jiang *et al*, Phys. Rev. X (2024), ...

- ◆ These can be grouped into three types:

$\{1 \parallel g\}, g \in \text{SG}$

230 groups

Ferromagnets

$\{1 \parallel g\}$ or $\{C_{2,\perp} \parallel gh\}, g \in \text{SG mod } h$

Zero dipole moment



Rotation or reflection

674 groups

Altermagnets



Translation or inversion

517 groups

Conventional Antiferromagnets

Non-relativistic

Colinear Spin
Space Groups

Ferromagnets

Zero dipole moment

Altermagnets

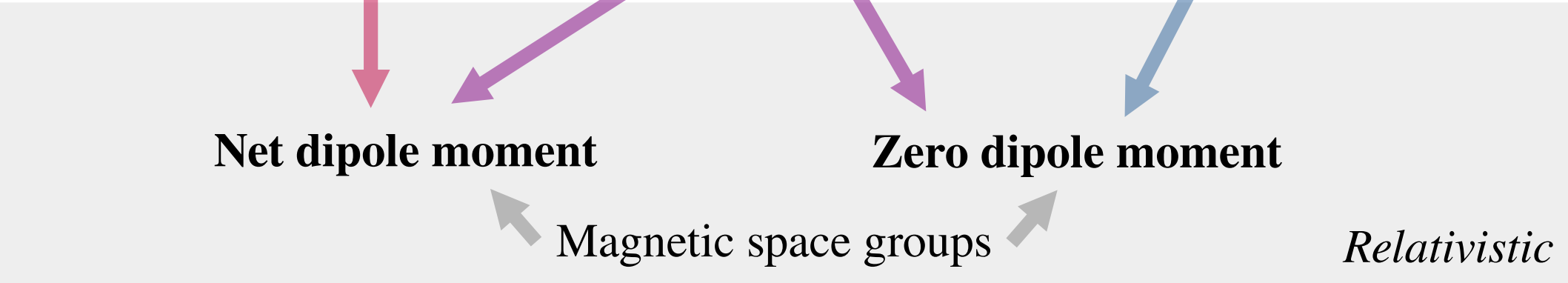
**Conventional
antiferromagnets**

Net dipole moment

Zero dipole moment

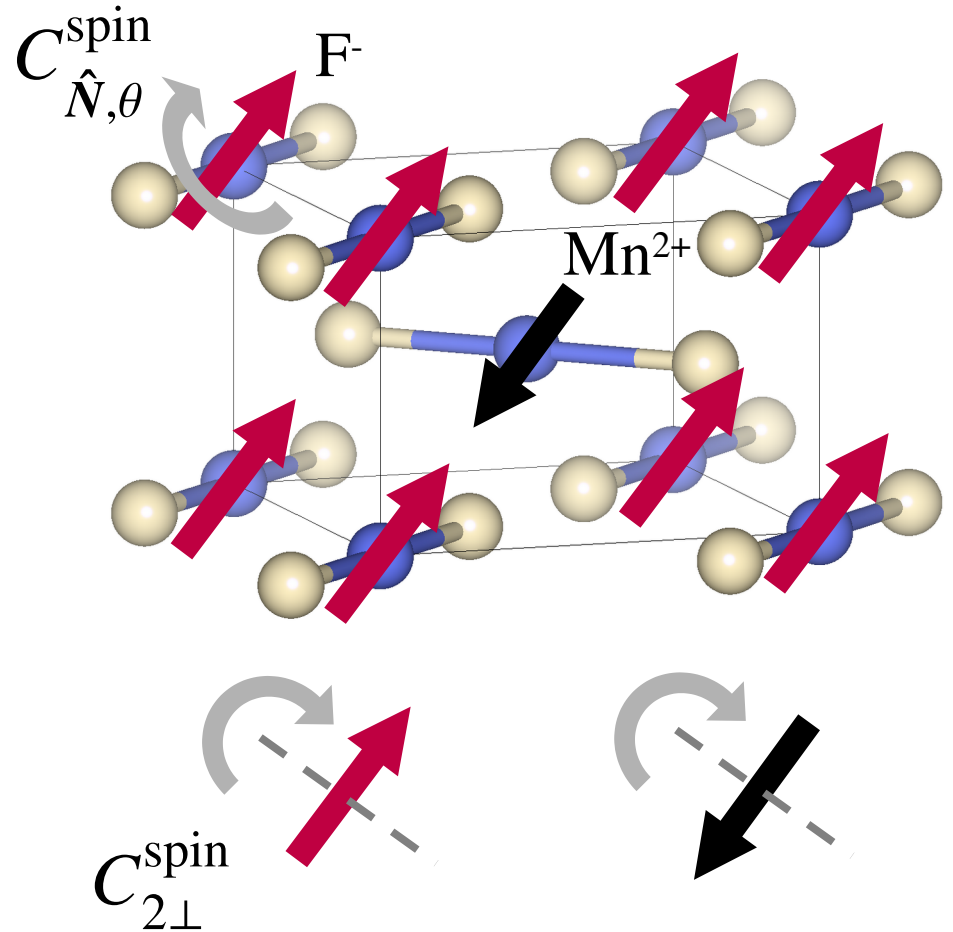
Magnetic space groups

Relativistic



Example: MnF_2

- ◆ Spatial operations don't change the spins anymore!
- ◆ Two possibilities:
 - Swaps sublattices: $C_{4z} + \tau$, $C_{2,x+y}$
 - Doesn't swap: $C_{2x} + \tau$, C_{2z}
- ◆ Follow those that swap with **spin rotation** $\perp N$ *or* time reversal
- ◆ Still have continuous spin rotations about N and two-fold spin rotation + T. R.



Example: MnF_2

$$\{C_{2,\perp} \parallel C_{4z} + \tau\} \quad \{C_{2,\perp} \parallel C_{2,x+y}\} \quad \{1 \parallel C_{2x} + \tau\} \quad \{1 \parallel C_{2z}\}$$

- ◆ Symmetries are *much more constraining*

$$\mathbf{M} \xrightarrow{\{C_{\hat{N},\theta} \parallel 1\}} \mathbf{R}_{\hat{N}}(\theta)\mathbf{M}$$

$$\mathbf{M} = \mathbf{M}\hat{N}$$

$$\mathbf{M} \xrightarrow{\{C_{2,\perp} \parallel C_{4z} + \tau\}} \mathbf{M} \xrightarrow{\{1 \parallel C_{2\perp}\}} -\mathbf{M}\hat{N}$$

$$\mathbf{M} = \mathbf{0}$$

- ◆ *Always* compensated in non-relativistic limit, independent of direction

Multipoles

$$\mathbf{O}_{\mu\nu} \equiv \int d^3r \, r_\mu r_\nu \mathbf{m}(\mathbf{r})$$

- ◆ Do *not* rule out induced multipoles

$$\mathbf{O}_{\mu\nu} \xrightarrow{\{C_{\hat{N},\theta} \parallel 1\}} \mathbf{R}_{\hat{N}}(\theta) \mathbf{O}_{\mu\nu} \quad \Rightarrow \quad \mathbf{O}_{\mu\nu} = \mathbf{O}_{\mu\nu} \hat{N}$$

$$\begin{aligned} & O_{xx}, O_{yy}, O_{yz}, O_{xz}, O_{xy} \\ & \xrightarrow{\{C_{2,\perp} \parallel C_{4z} + \tau\}} \\ & -O_{yy}, -O_{xx}, -O_{xz}, +O_{yz}, +O_{xy} \end{aligned} \quad \begin{aligned} & \Rightarrow O_{xx} = -O_{yy} \\ & \Rightarrow O_{yz} = O_{xz} = 0 \end{aligned}$$

- ◆ Remaining symmetries eliminate O_{xx}

$$O_{xy} \propto \hat{N}$$

Piezomagnetism

$$\mathbf{M} = \sum_{\mu\nu} \mathbf{C}_{\mu\nu} \epsilon_{\mu\nu}$$

- ◆ Does *not* rule out piezomagnetism

$$\begin{aligned} \mathbf{M} &\xrightarrow{\{C_{\hat{N},\theta} \parallel 1\}} \mathbf{R}_{\hat{N}}(\theta) \mathbf{M} \quad \Rightarrow \quad \mathbf{M} = \mathbf{M} \hat{N} \\ \epsilon_{\mu\nu} &\xrightarrow{\{C_{\hat{N},\theta} \parallel 1\}} \epsilon_{\mu\nu} \quad \Rightarrow \quad \mathbf{C}_{\mu\nu} = \mathbf{C}_{\mu\nu} \hat{N} \end{aligned}$$

- ◆ Strain transforms almost identically to \mathbf{O}

$$\mathbf{M} = \mathbf{C} \epsilon_{xy} \hat{N}$$

$$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{yz}, \epsilon_{xz}, \epsilon_{xy} \xrightarrow{\{C_{2,\perp} \parallel C_{4z+\tau}\}} \epsilon_{yy}, \epsilon_{xx}, \epsilon_{zz}, -\epsilon_{xz}, \epsilon_{yz}, -\epsilon_{xy}$$

Implications

- ◆ Spin symmetries remove *some* of these responses, but all of them
 - Piezomagnetism remains
 - Magnetic octupoles remain
 - Spin-splitting remains
- ◆ Can be understood using spin space groups & spin point groups
- ◆ **Prescription is clear:**
 - Classify observables, states, bands, ... in terms representations of spin groups
 - Make predictions in the spin-orbit free limit

Take-aways

- ◆ Altermagnets *are* a new class of magnetic ordering, **in the non-relativistic limit**
 - *Not* once relativistic effects are included, *but many properties of non-relativistic limit are dictated by non-relativistic case*
- ◆ Properties of altermagnets are (mostly) determined by symmetries of the non-relativistic limit: **spin groups**
 - Symmetries where operations on space and on spin act *independently*
- ◆ *Some* of the properties of altermagnets can be understood from the paramagnetic phase, avoiding spin groups