Planckian Dynamics

$$\tau_r \geq \mathcal{C} \frac{\hbar}{k_B T}$$ (SS, *Quantum Phase Transitions*, 1999)

1. SYK as a solvable model of quantum matter without quasiparticles
2. Quantum critical dynamics of the Ising model in 2 spatial dimensions
3. Strange metal from metallic quantum phase transitions with spatial disorder

School on Quantum Dynamics of Matter, Light and Information
ICTP, Trieste
August 18, 19, 2025

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

SYK as a solvable model of quantum matter without quasiparticles

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

A simple model of a metal with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$

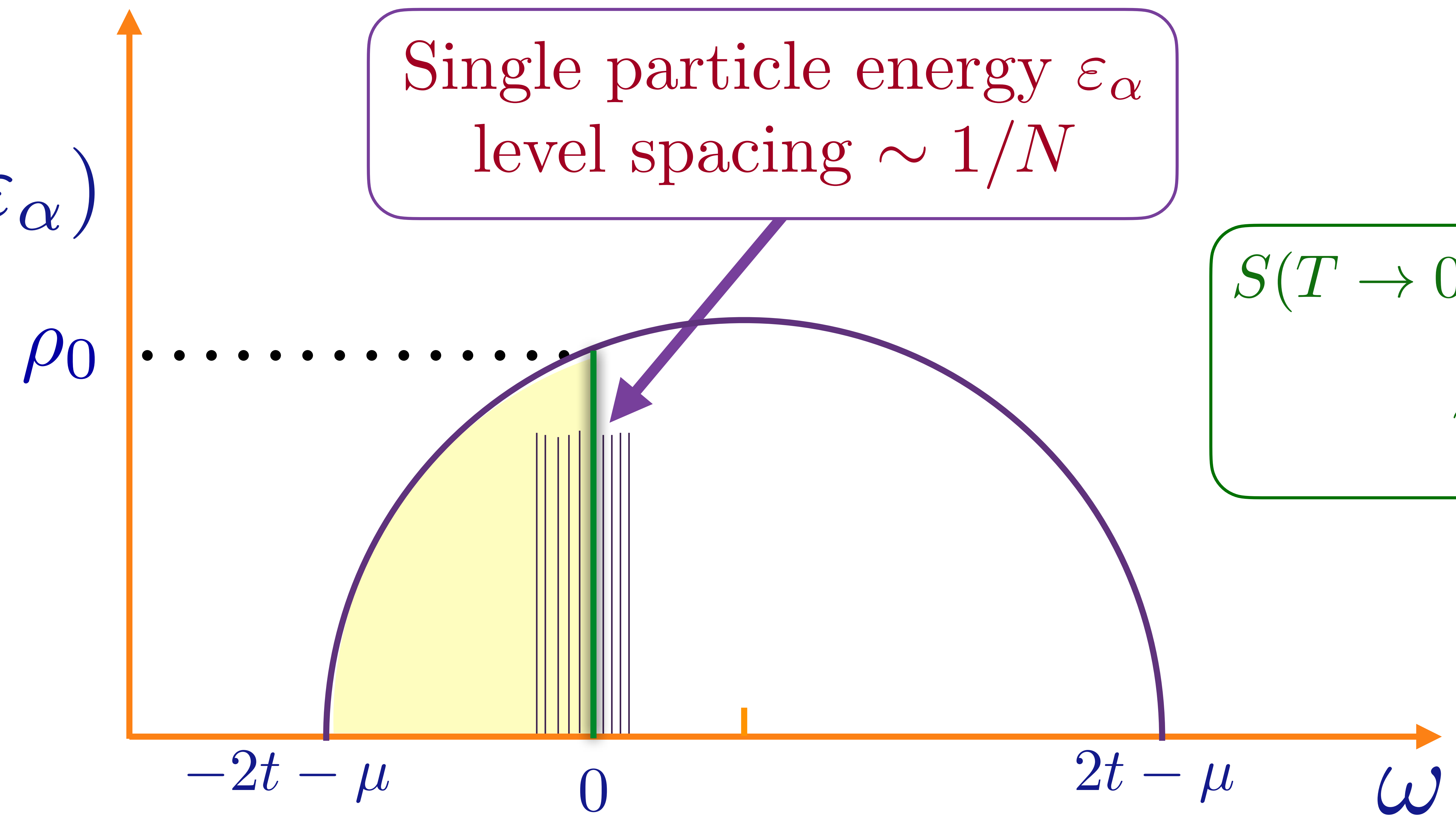
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.

A simple model of a metal with quasiparticles

$$\rho(\omega) = \frac{1}{N}\sum_\alpha \delta(\omega - \varepsilon_\alpha)$$

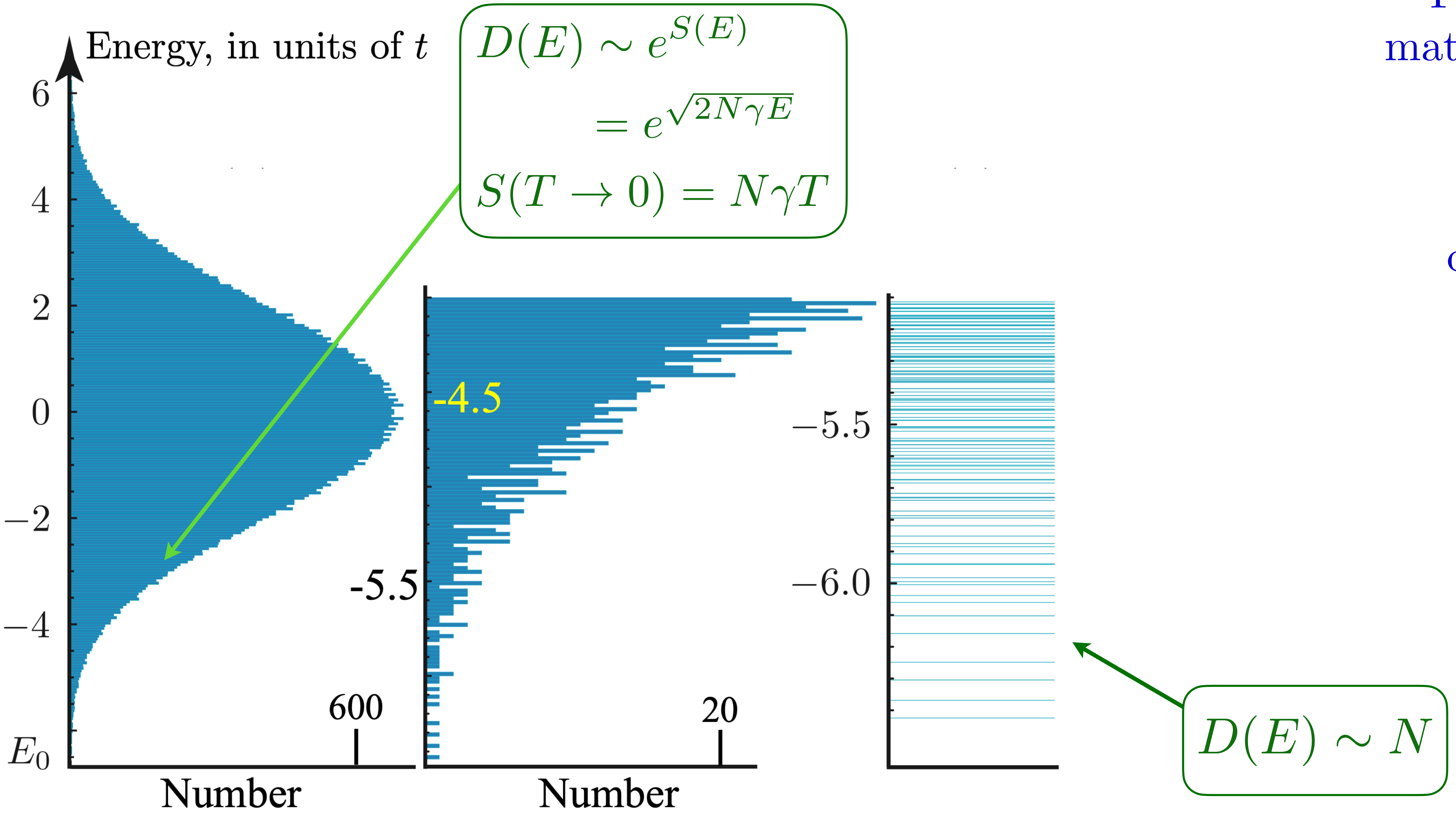


$$S(T \to 0) = N\gamma T$$

$$\gamma = \frac{\pi^2}{3}\rho_0$$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



For random matrix model:
$E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$
$n_\alpha = 0, 1,$ occupation number

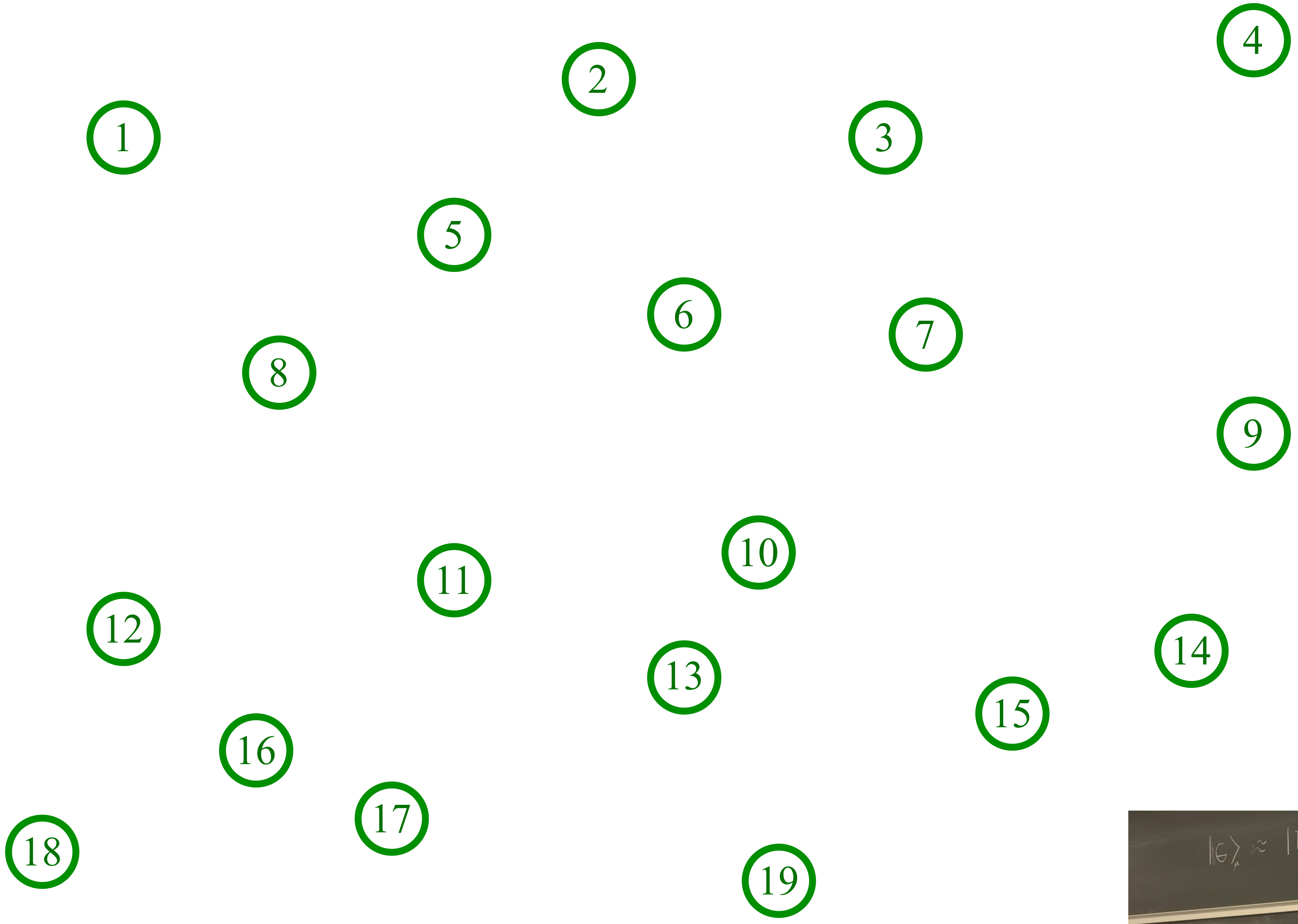
Random matrix model

1. Large-*N* theory of the SYK model
2. Finite-*N* theory of the SYK model
3. Quantum Einstein-Maxwell gravity theory of charged black holes

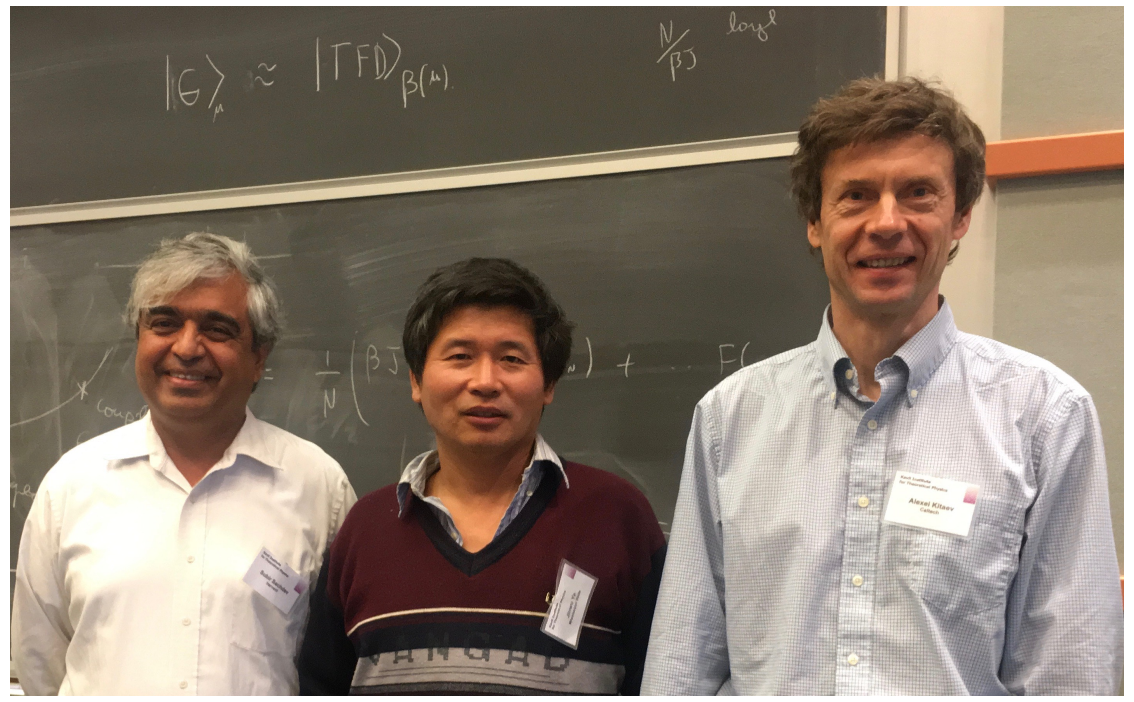
1. Large-N theory of the SYK model
2. Finite-N theory of the SYK model
3. Quantum Einstein-Maxwell gravity theory of charged black holes

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

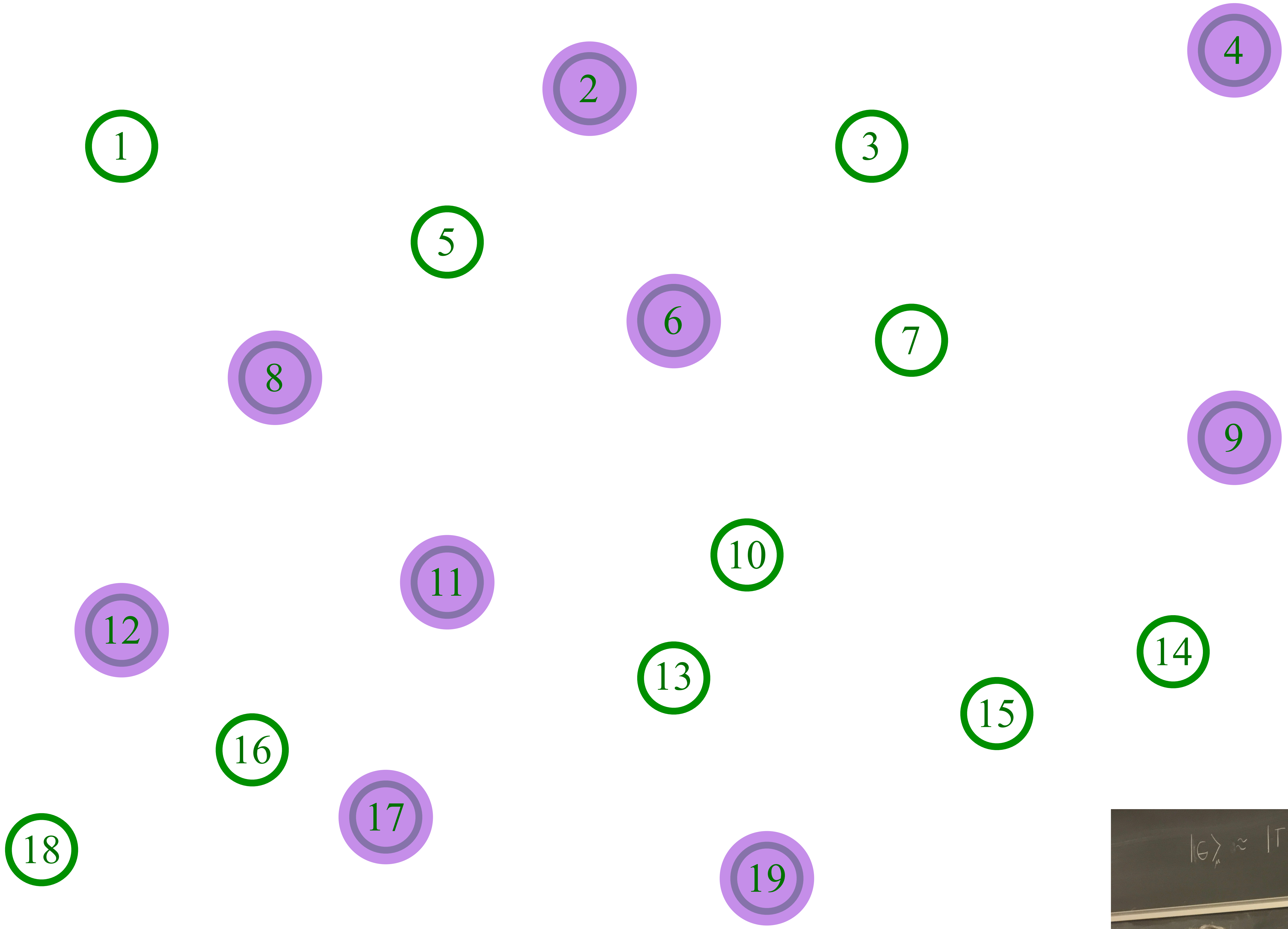


Pick a set of random positions

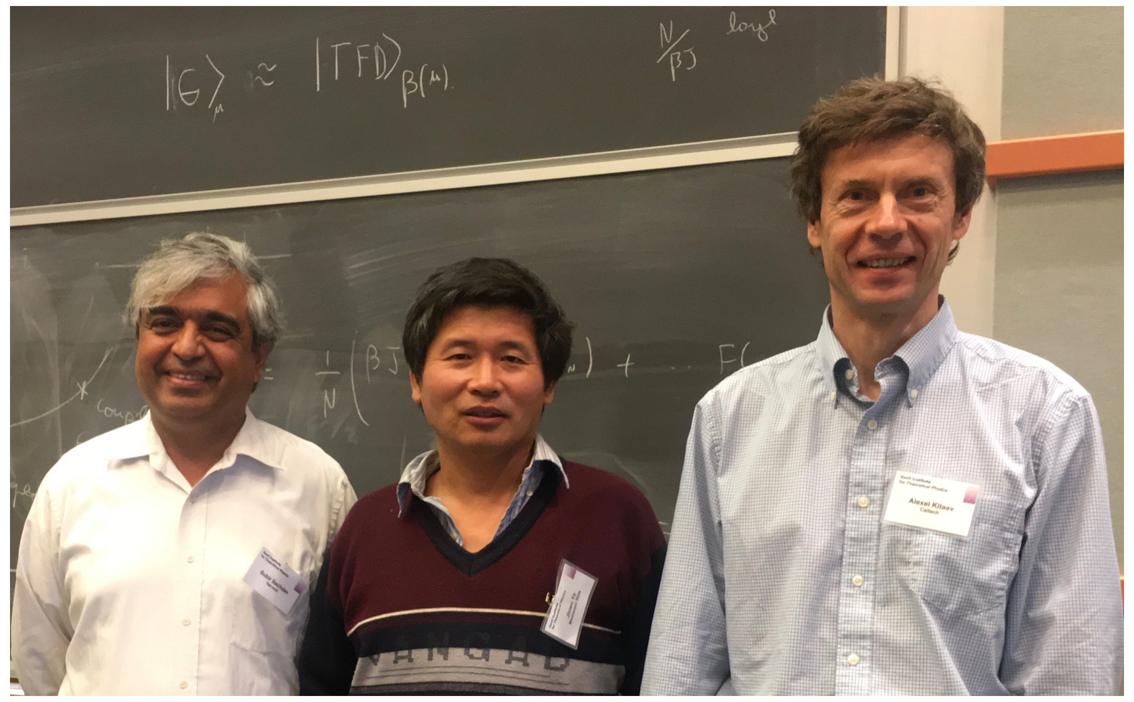


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



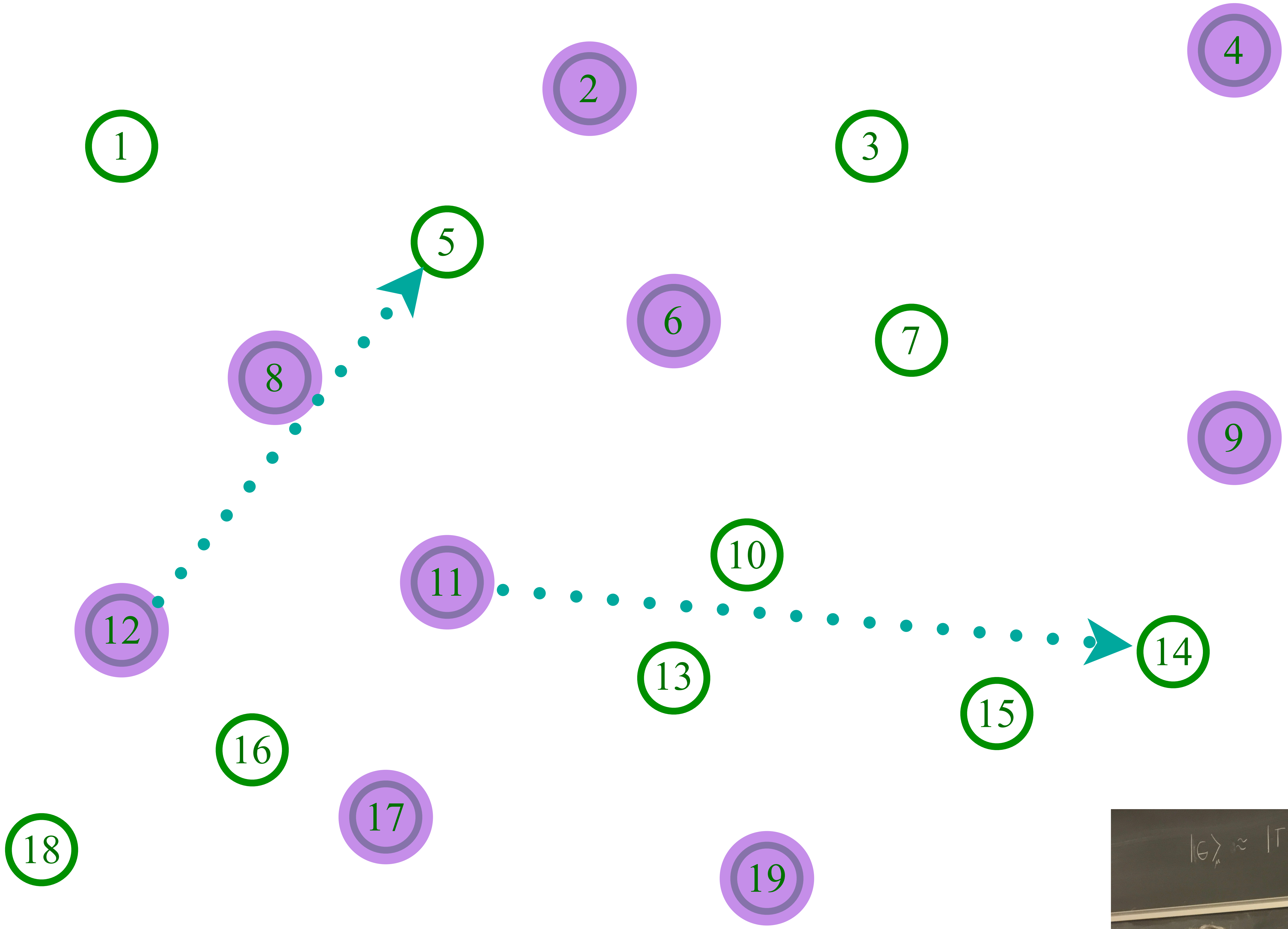
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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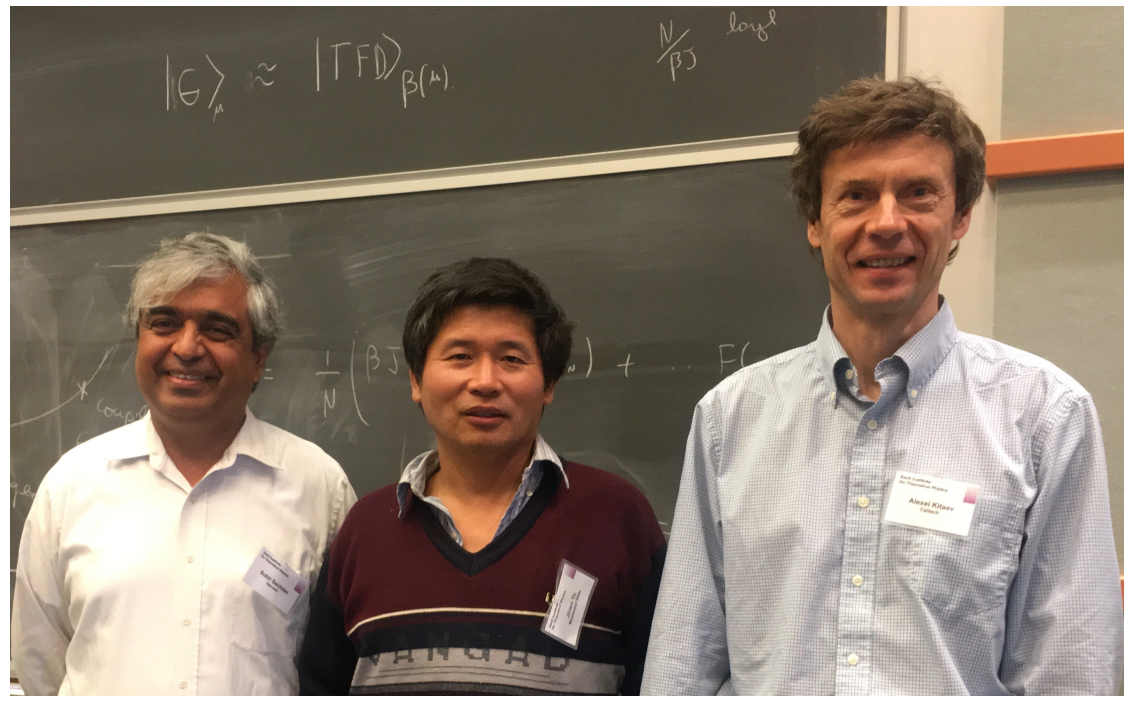

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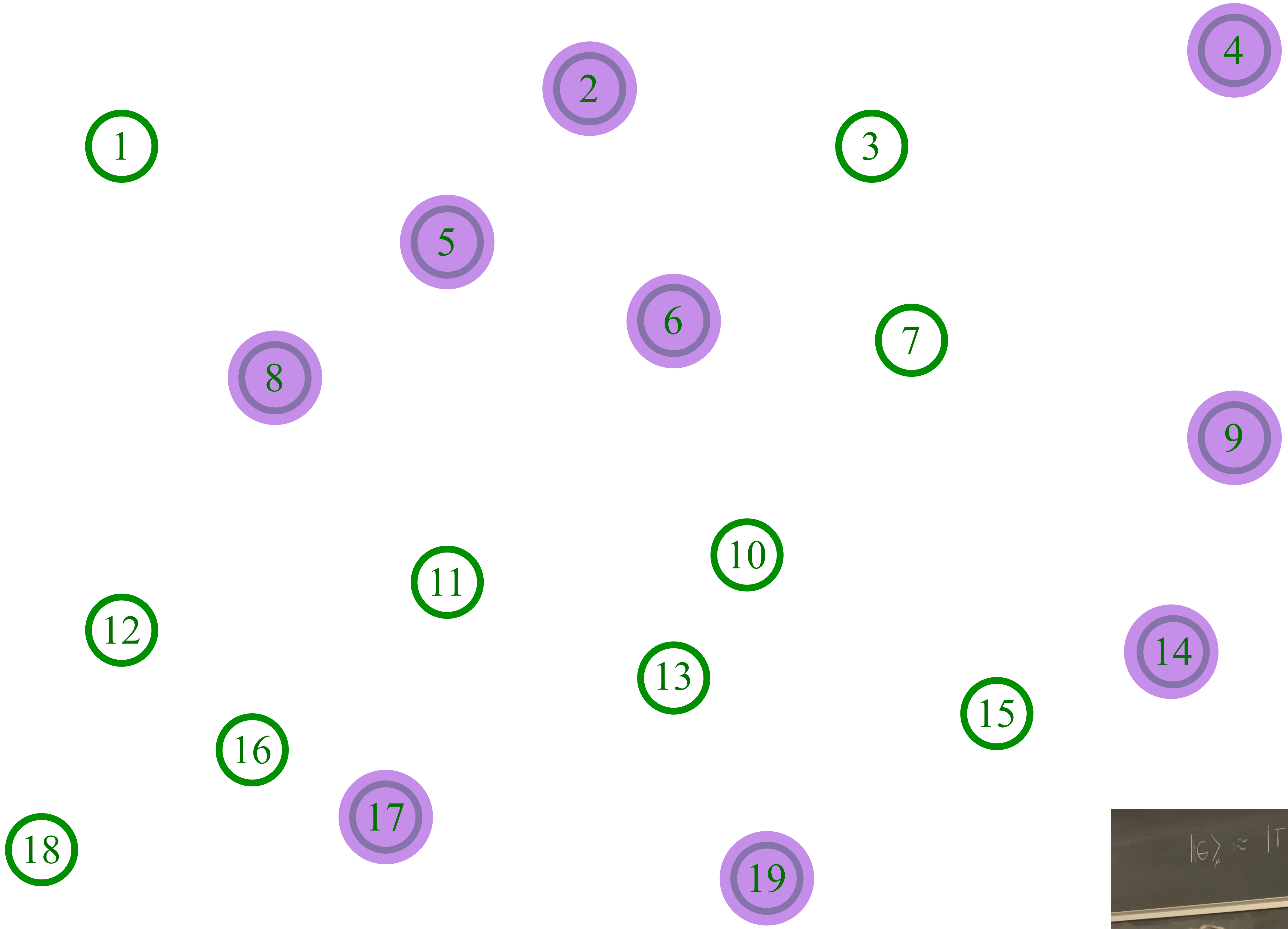
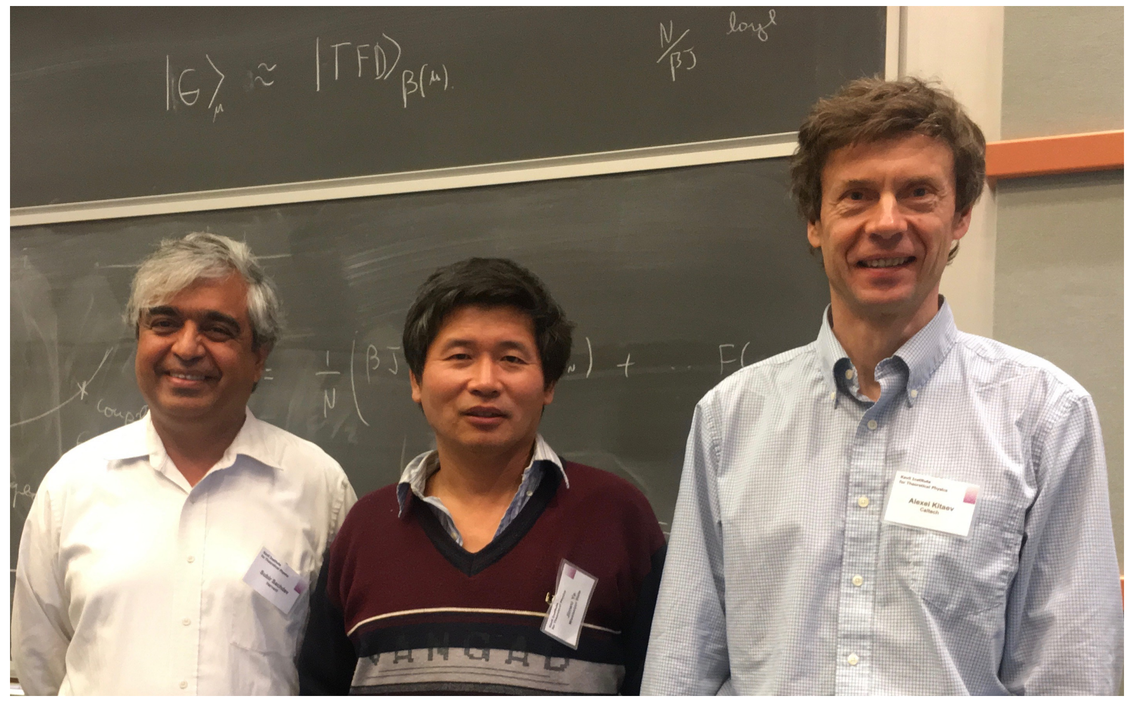


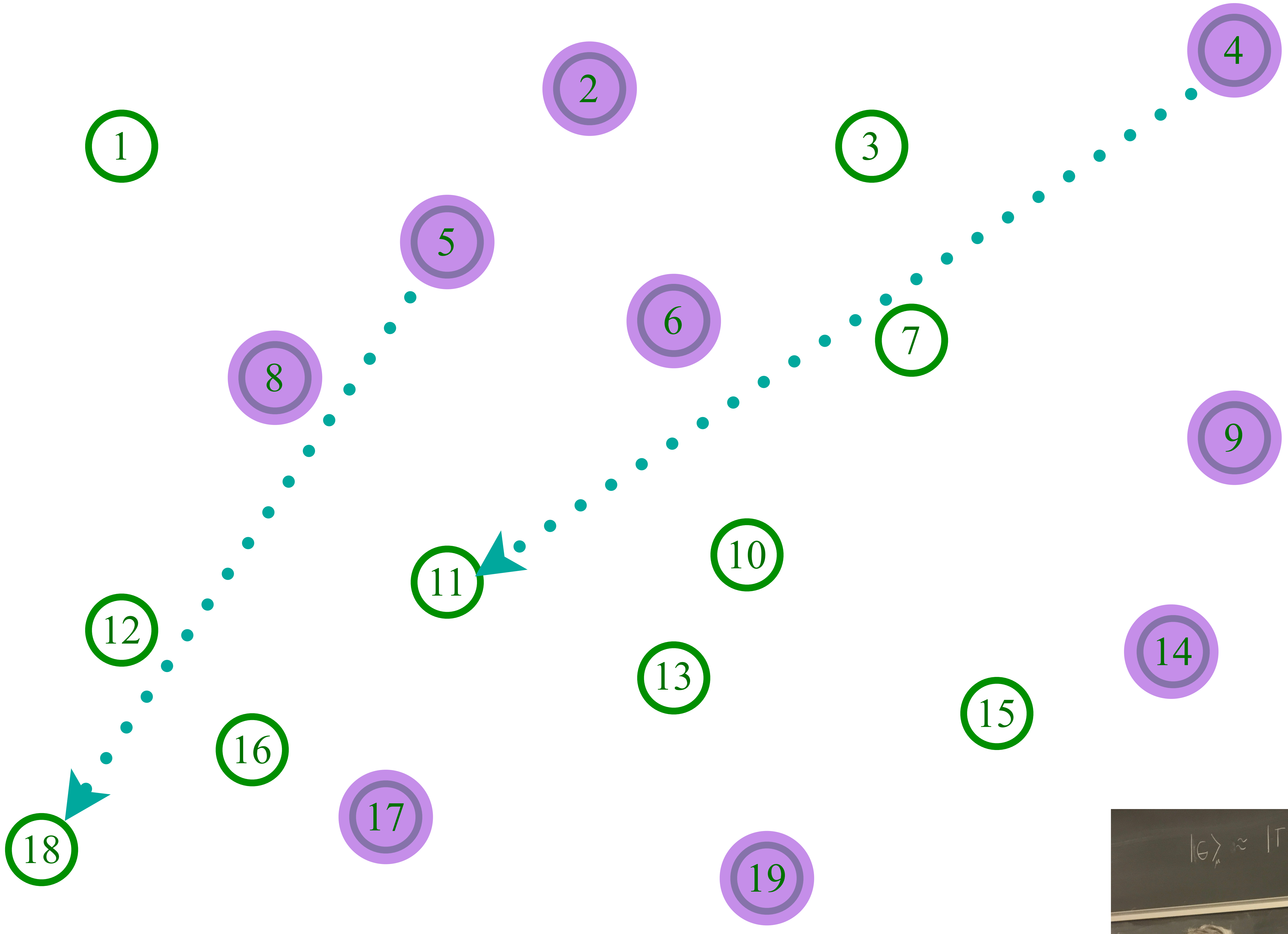
Entangle electrons pairwise randomly



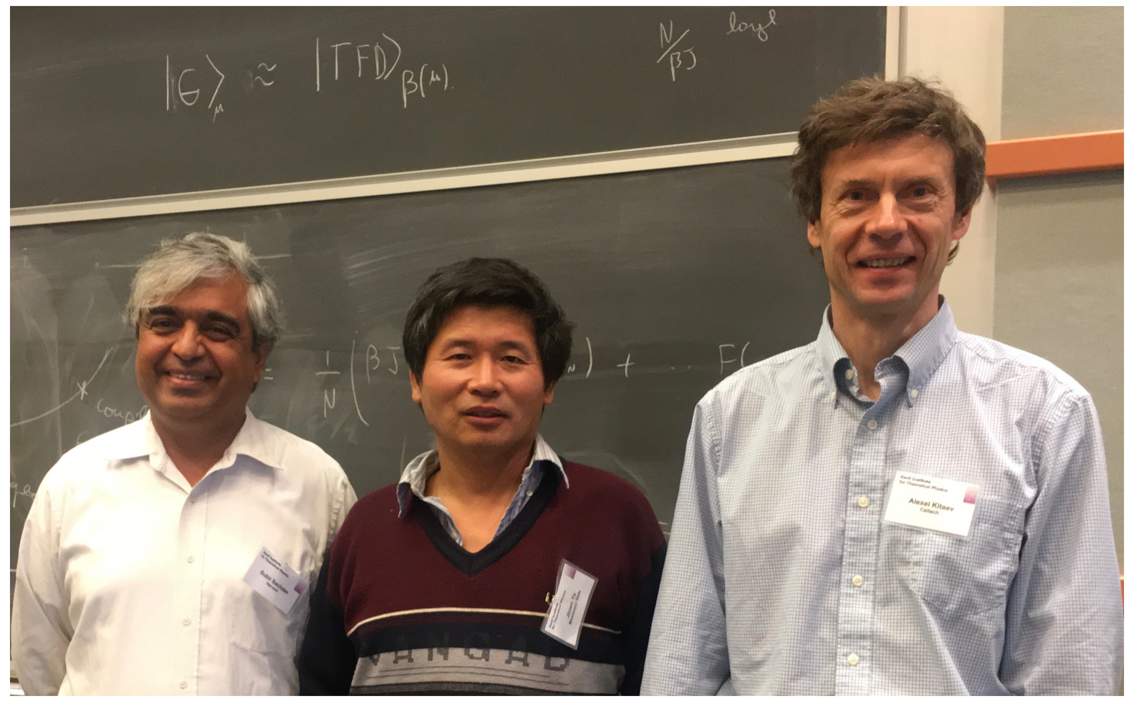
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



Entangle electrons pairwise randomly



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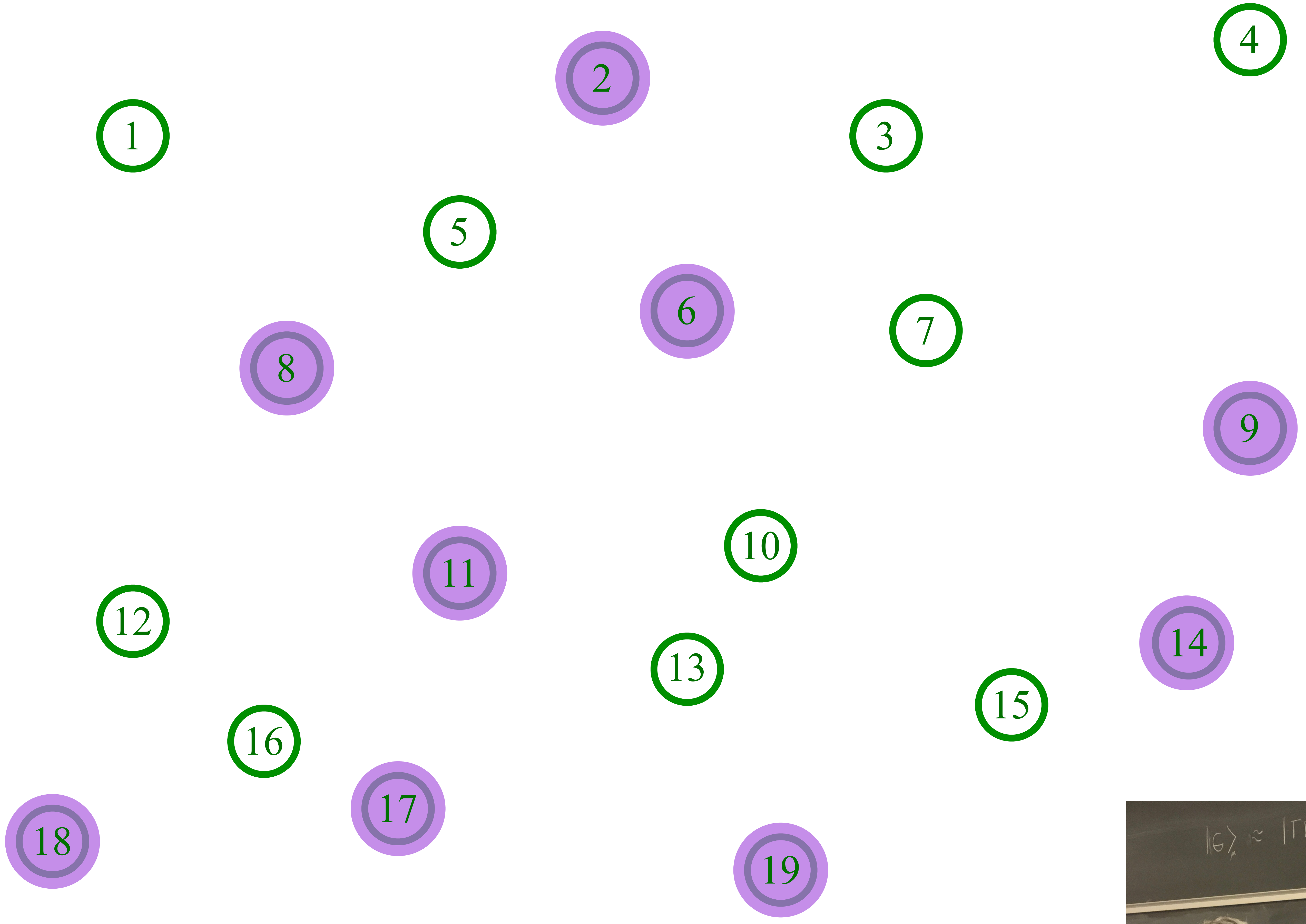
Sachdev, Ye (1993); Kitaev (2015)



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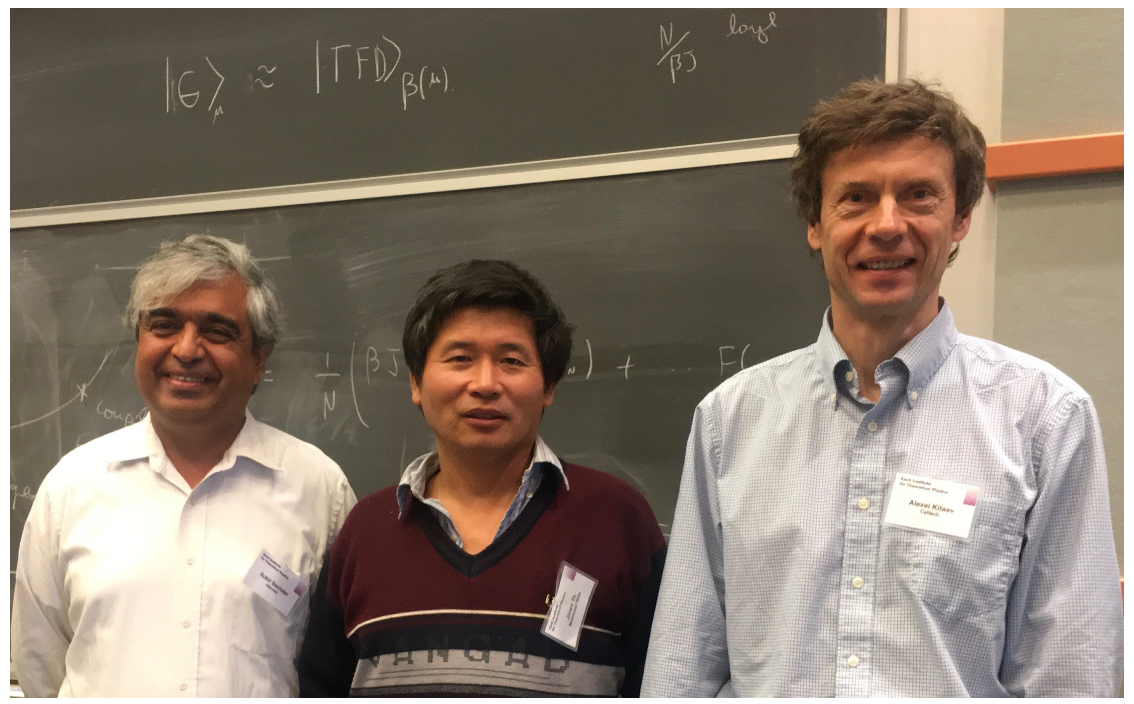
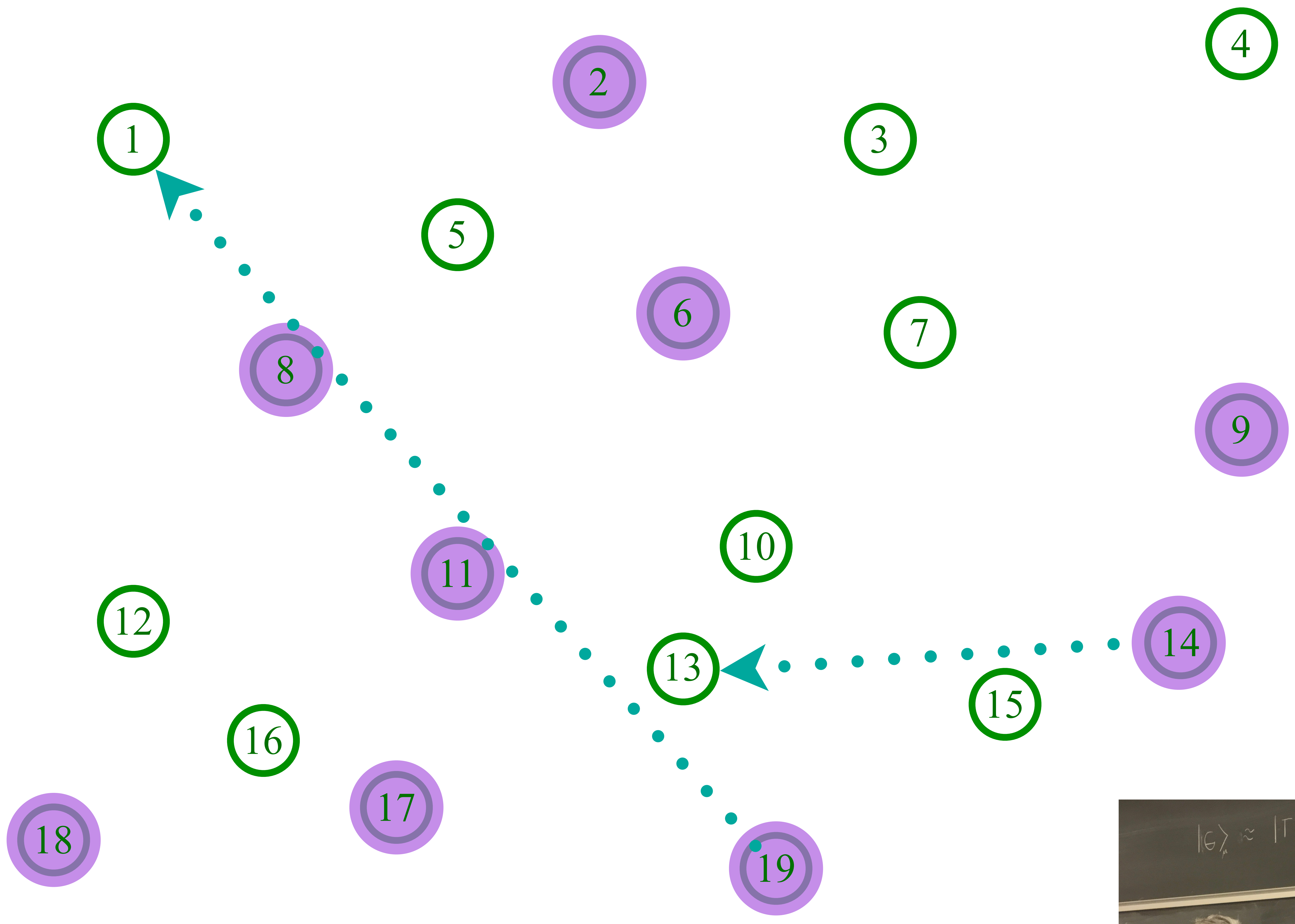


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



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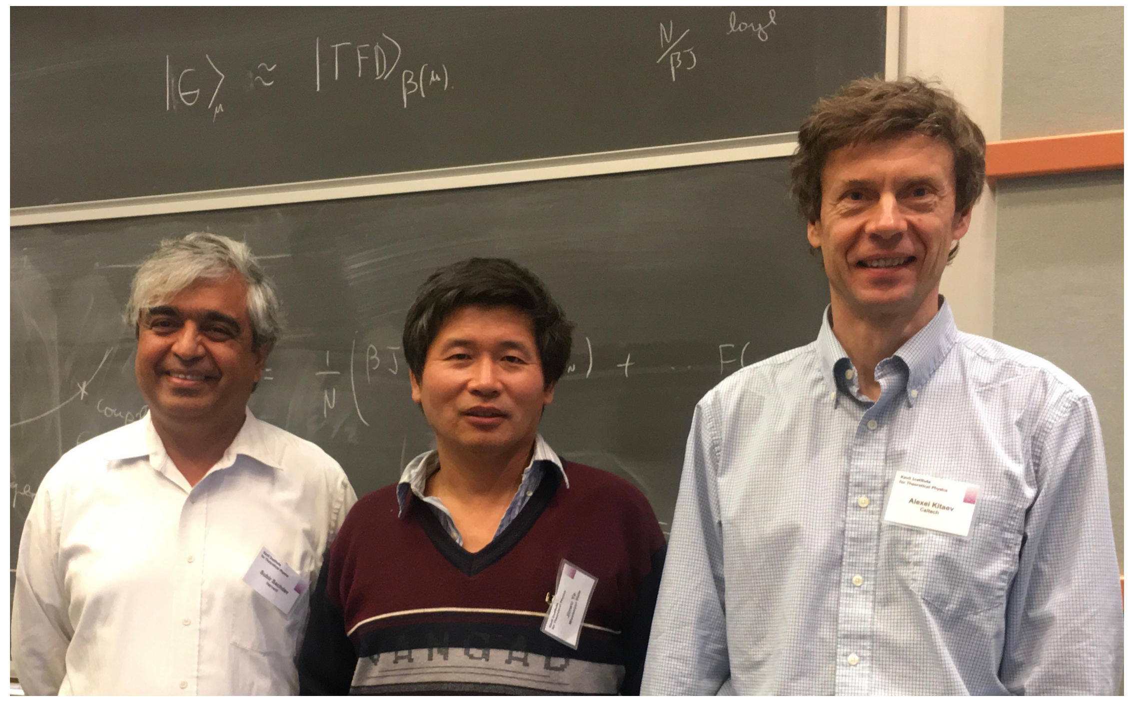

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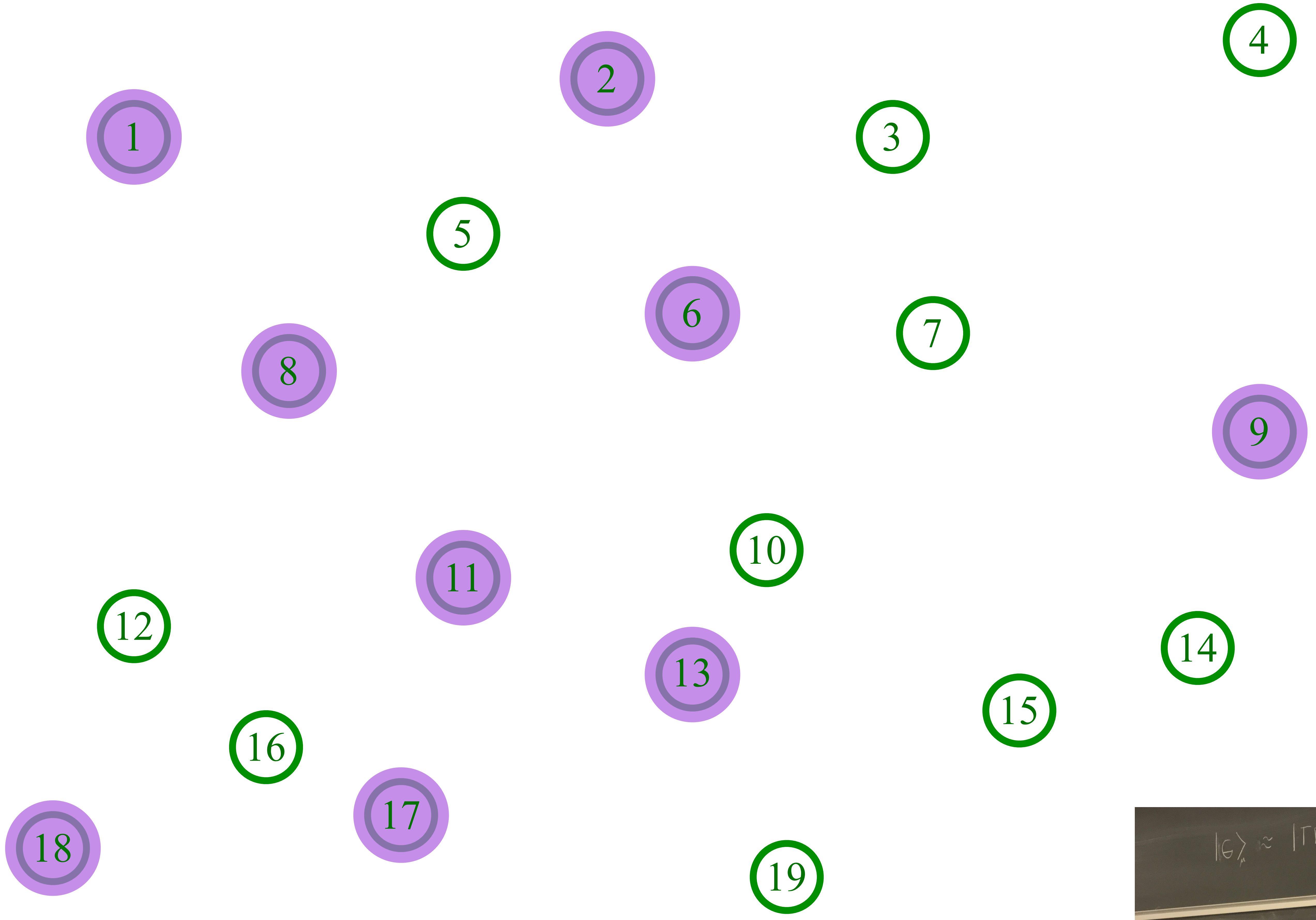

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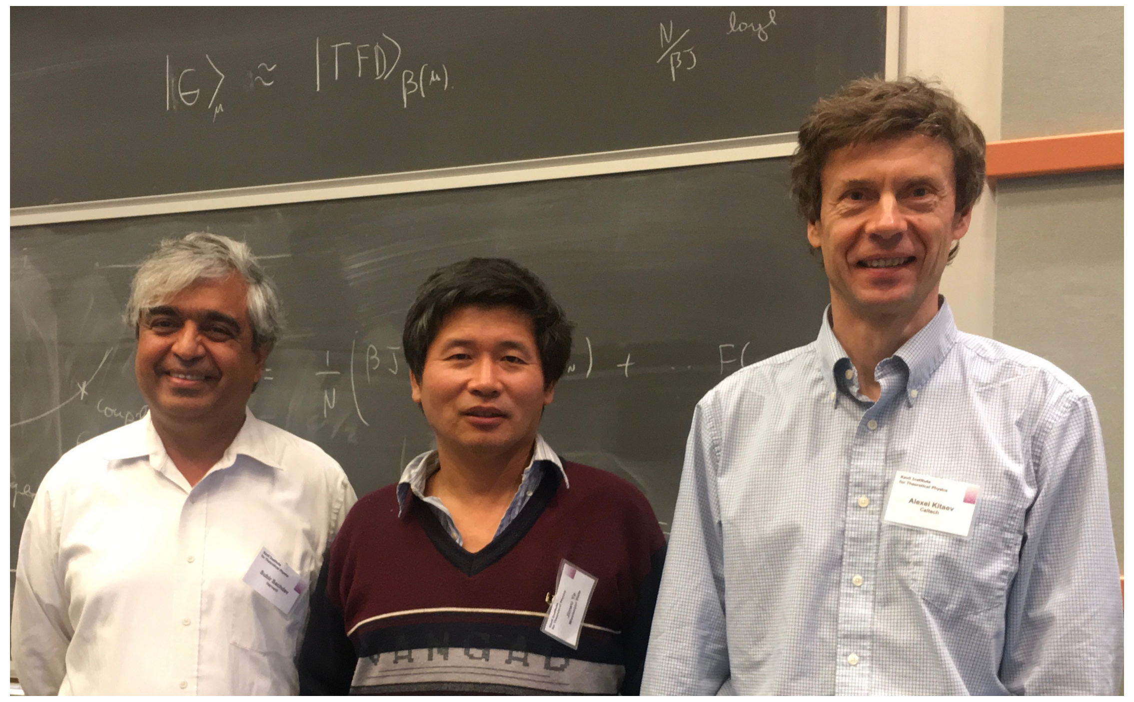
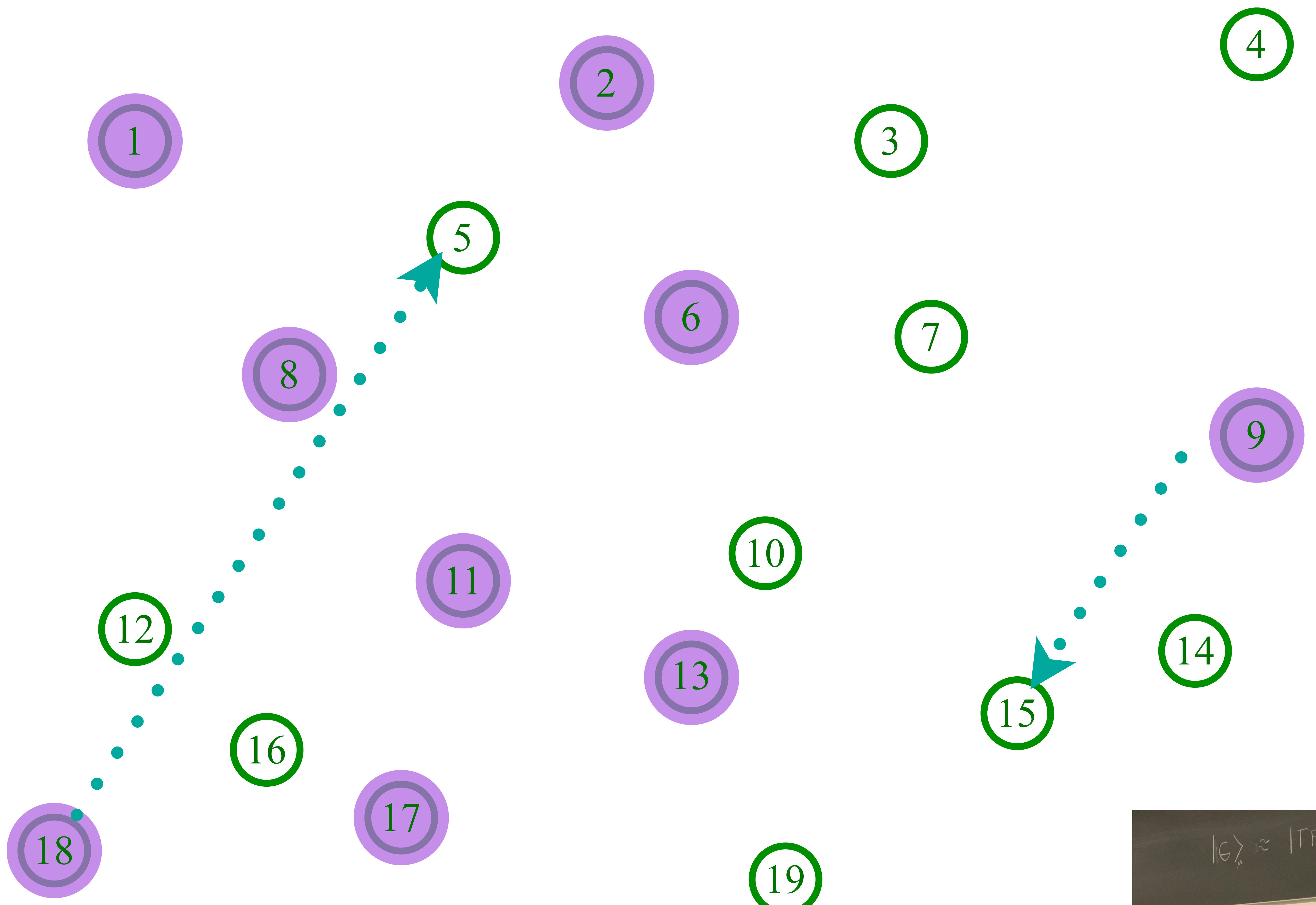


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



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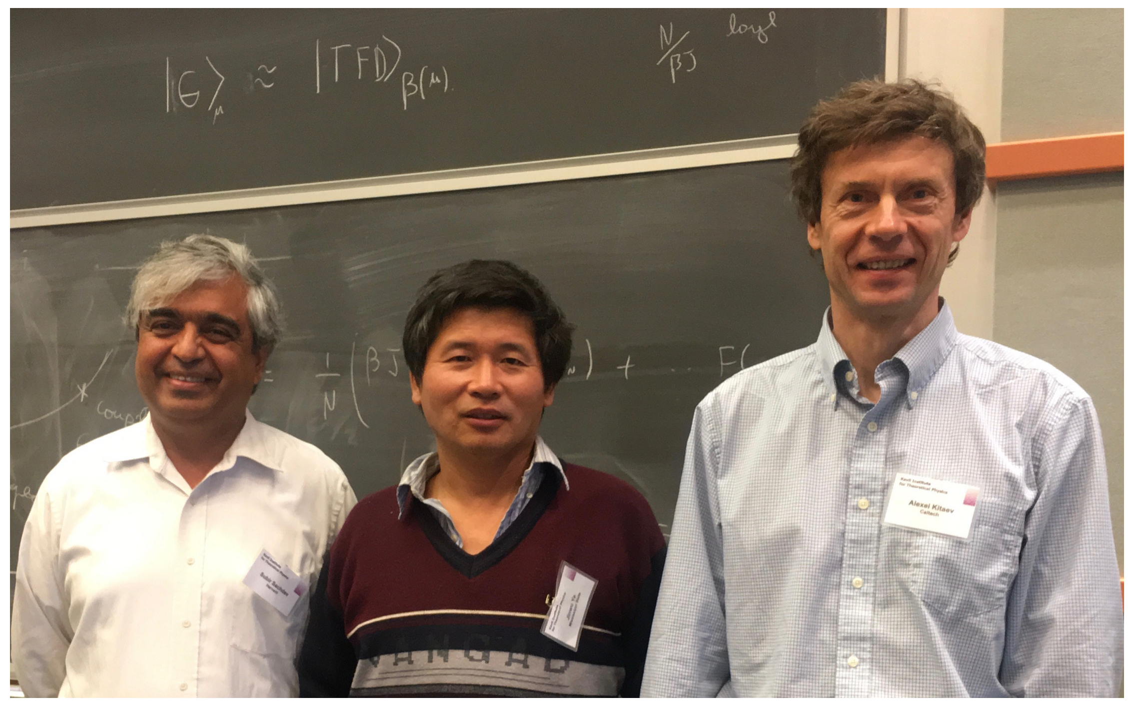

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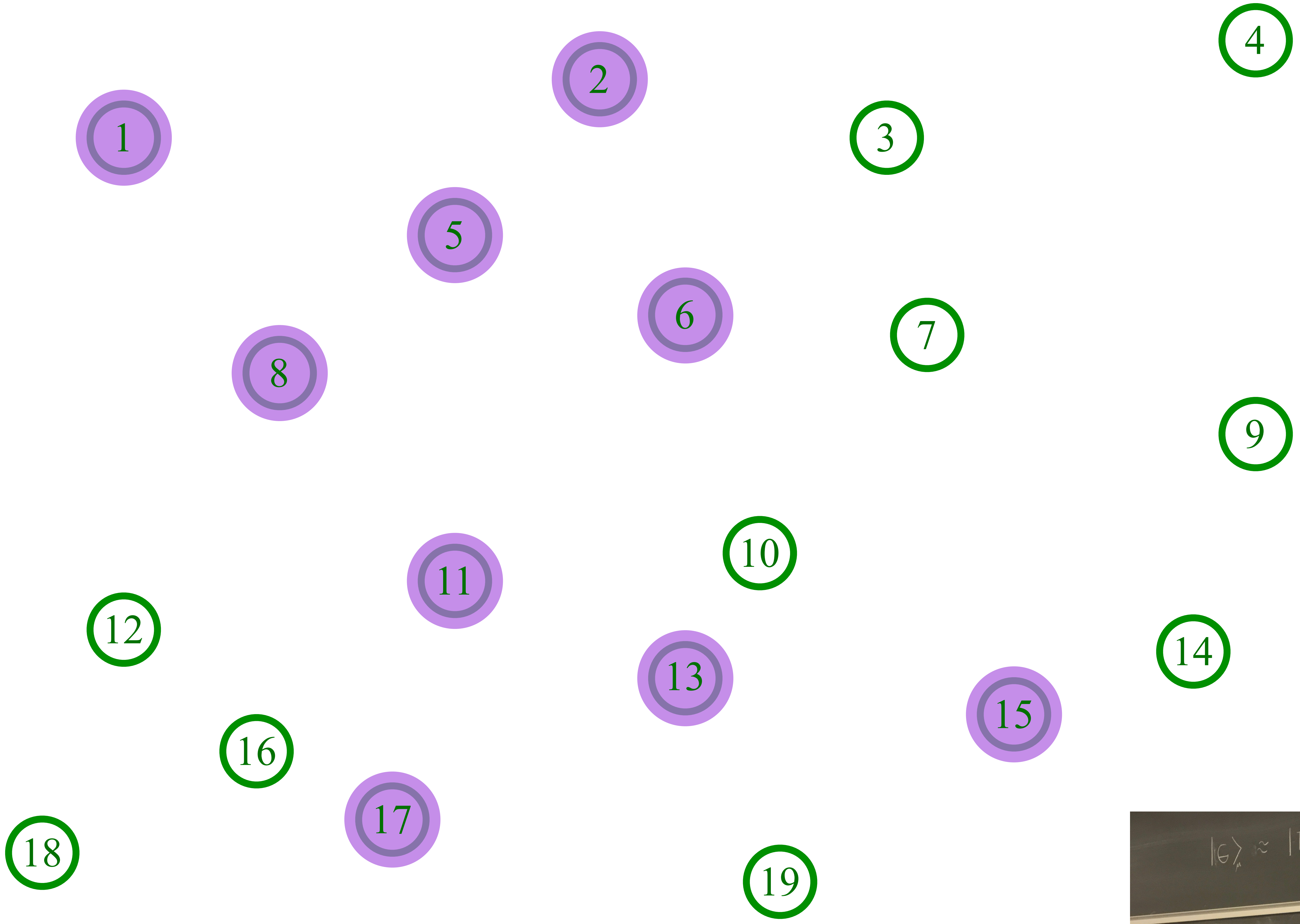

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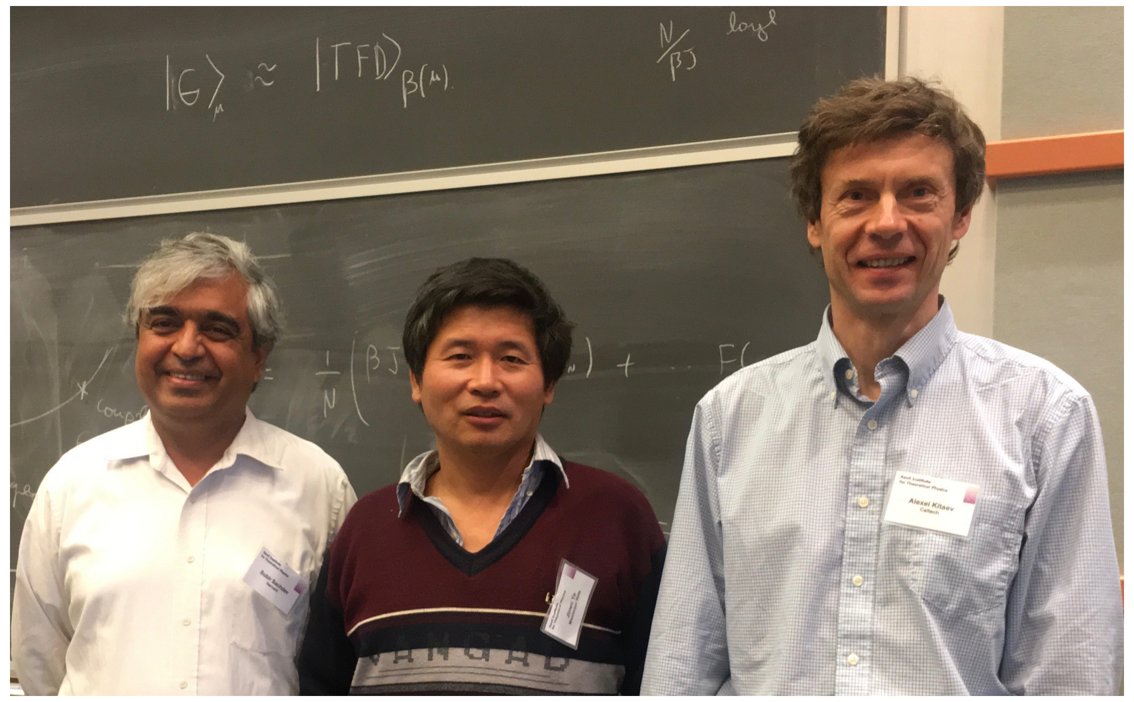
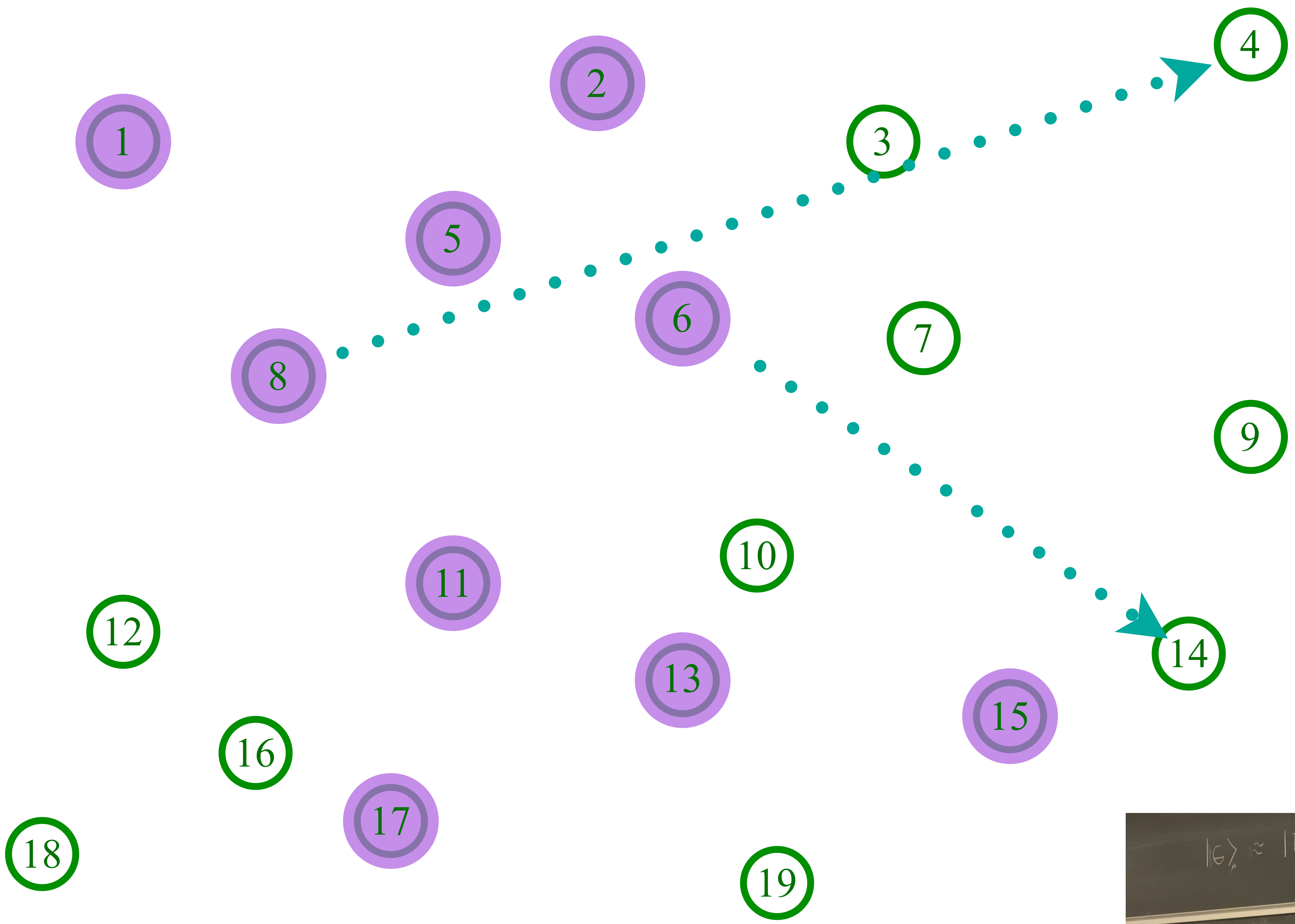


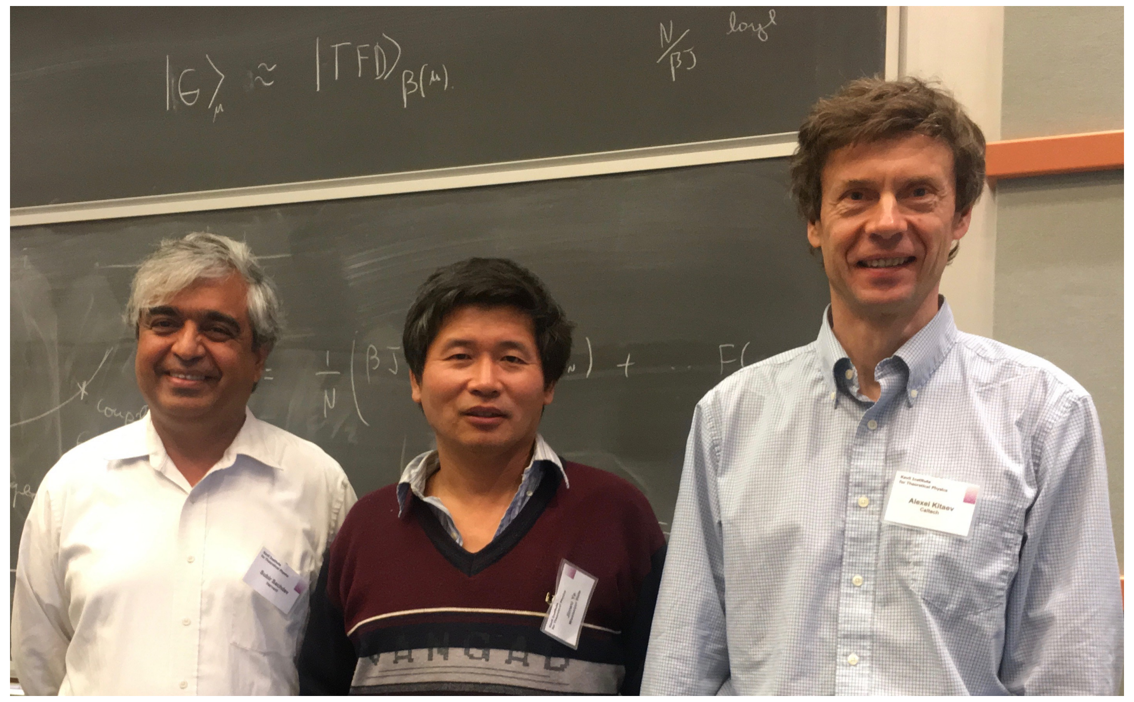
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



Entangle electrons pairwise randomly



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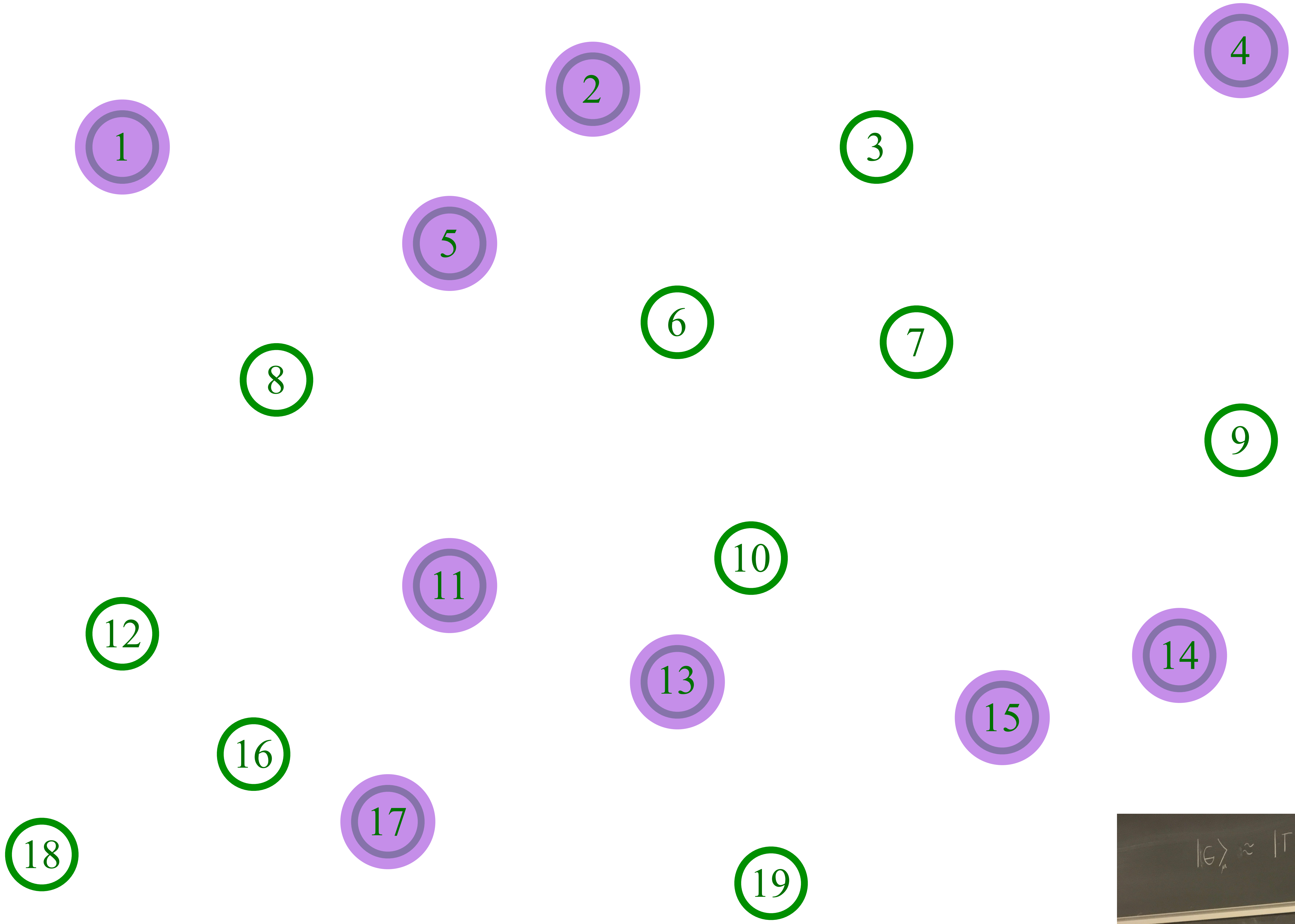

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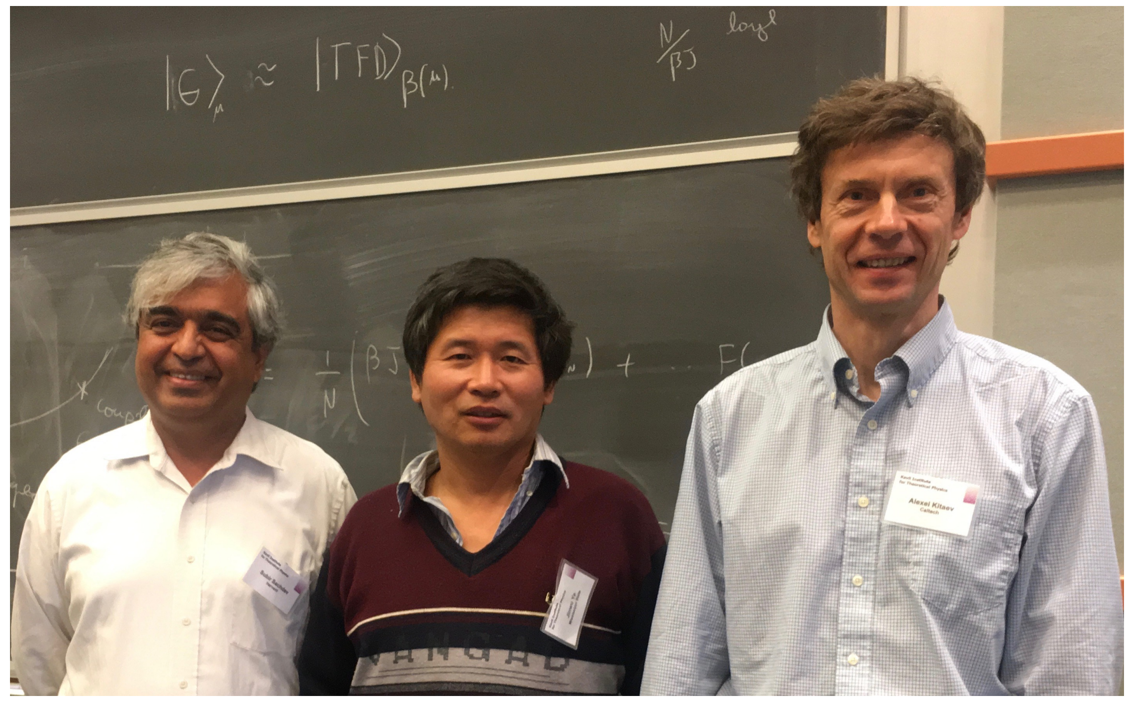


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

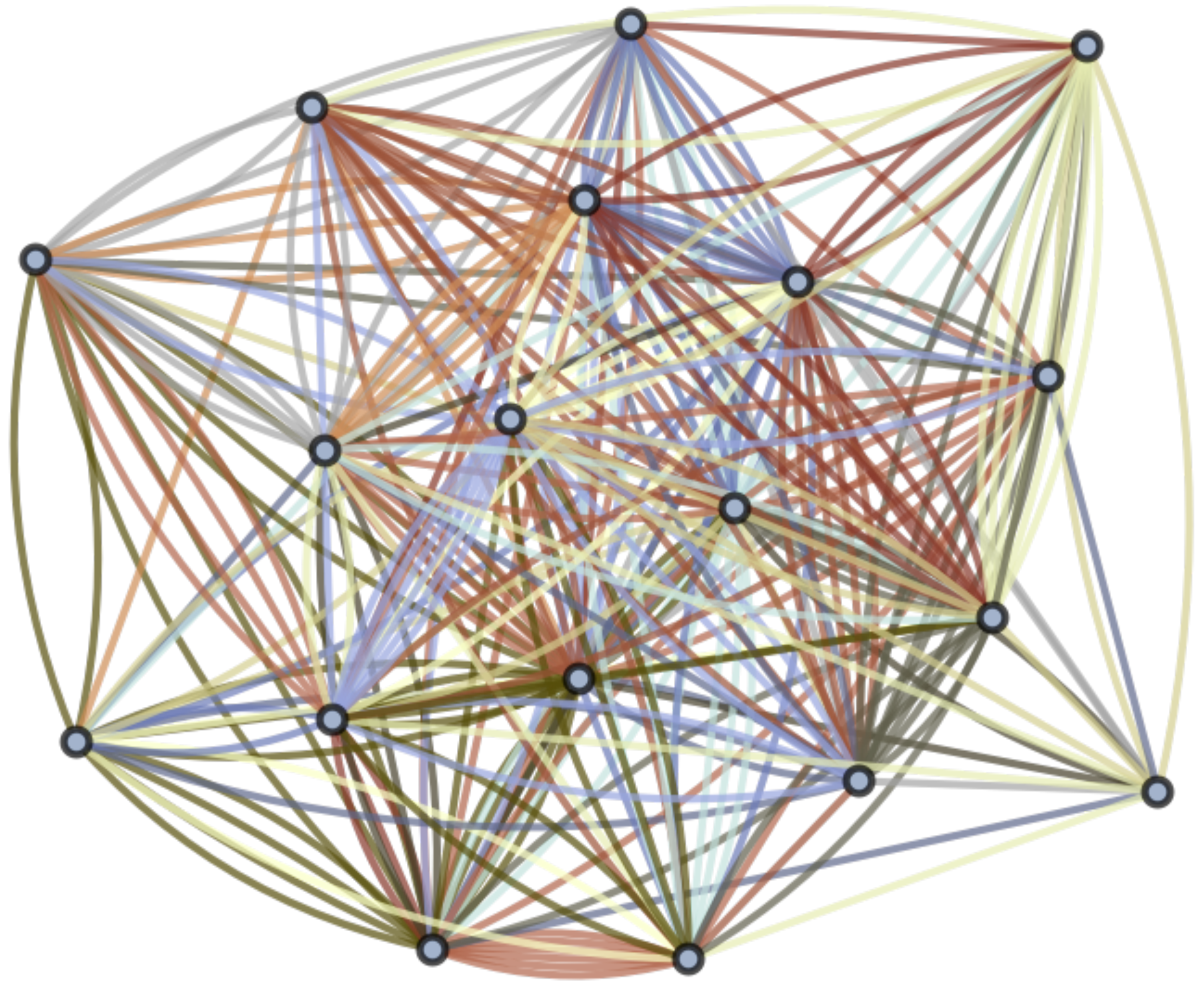
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

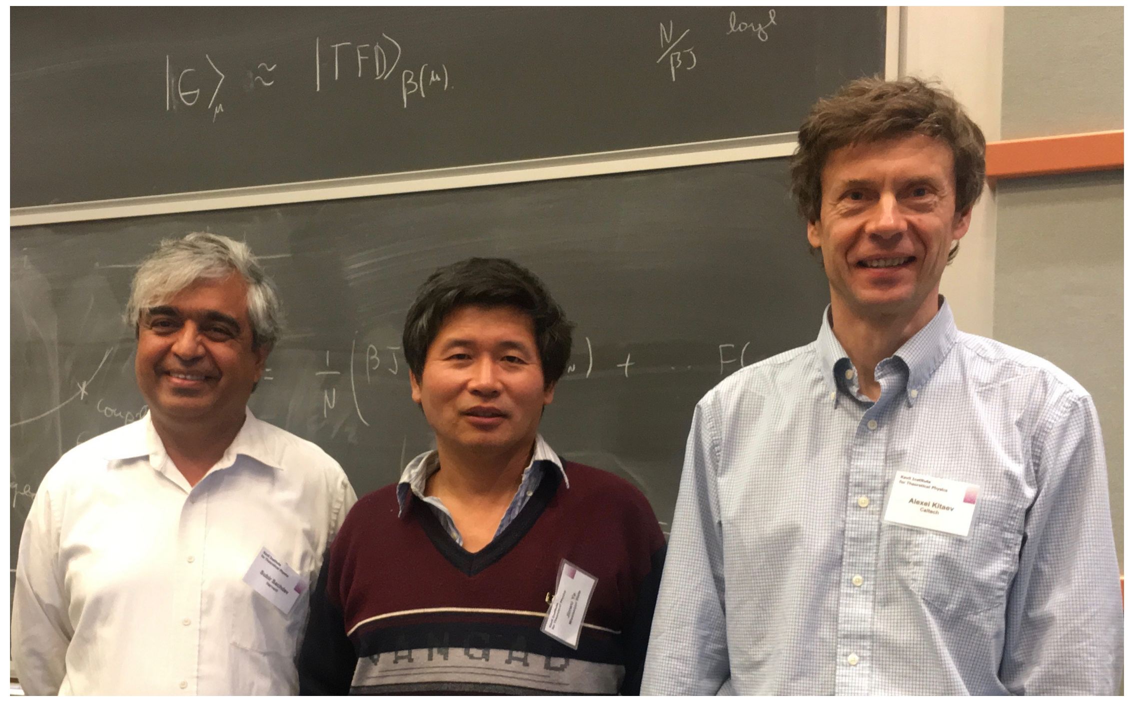
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The Sachdev-Ye-Kitaev (SYK) model

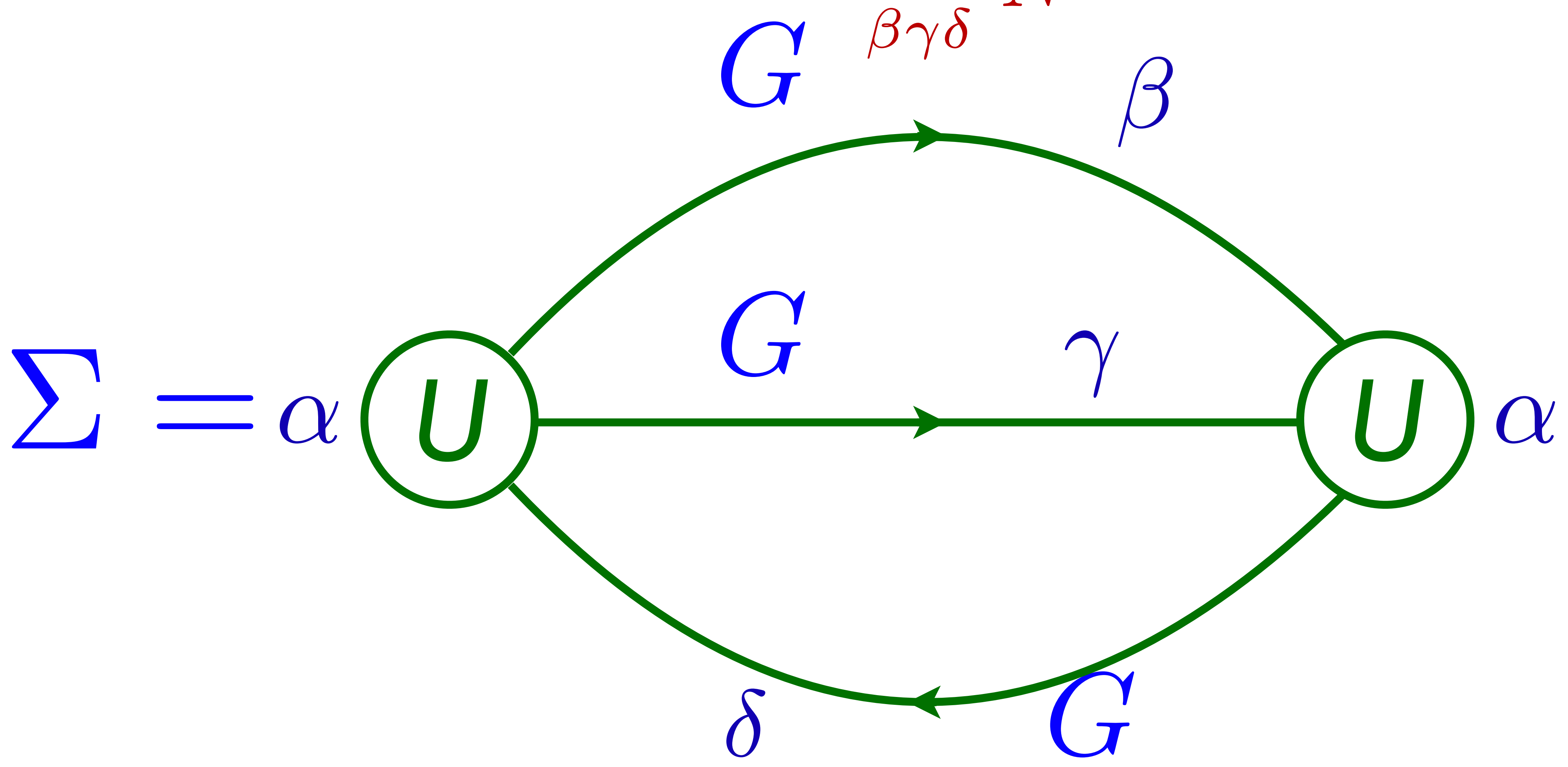
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

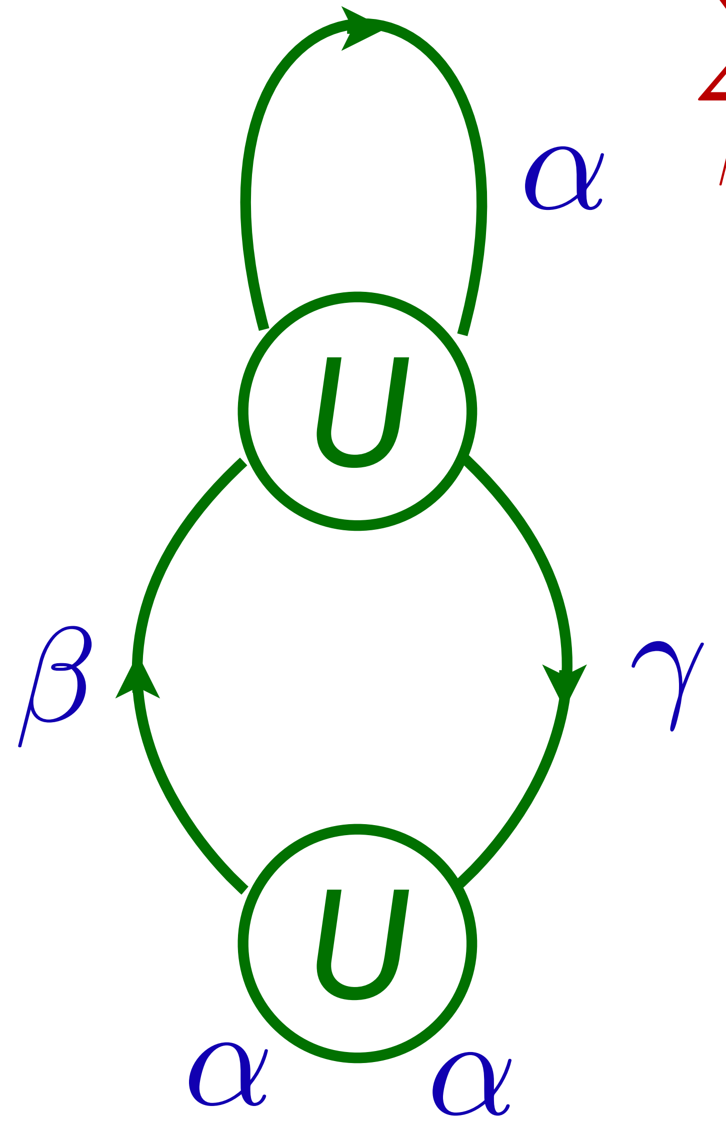
$$G(\tau = 0^-) = \mathcal{Q}.$$

Solution $G(\omega) \sim T^{-1/2} F(\hbar\omega / k_B T)$

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$



$$\sum_{\beta\gamma} \frac{U^2}{N^3} = \frac{U^2}{N}$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-1/2}$ ($\Rightarrow$ indication there are no quasiparticles)

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-1/2}$ ($\Rightarrow$ indication there are no quasiparticles)
- At general charge $\mathcal{Q}$, there is a spectral symmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \begin{cases} -\tau^{-1/2} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-1/2} & \tau < 0 \end{cases} \quad , \quad T = 0$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

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S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} \quad , \quad 0 < \tau < 1/T \,,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

$$G_*(\tau) = -C\frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2}.$$

$$G_*^R(\omega) =$$

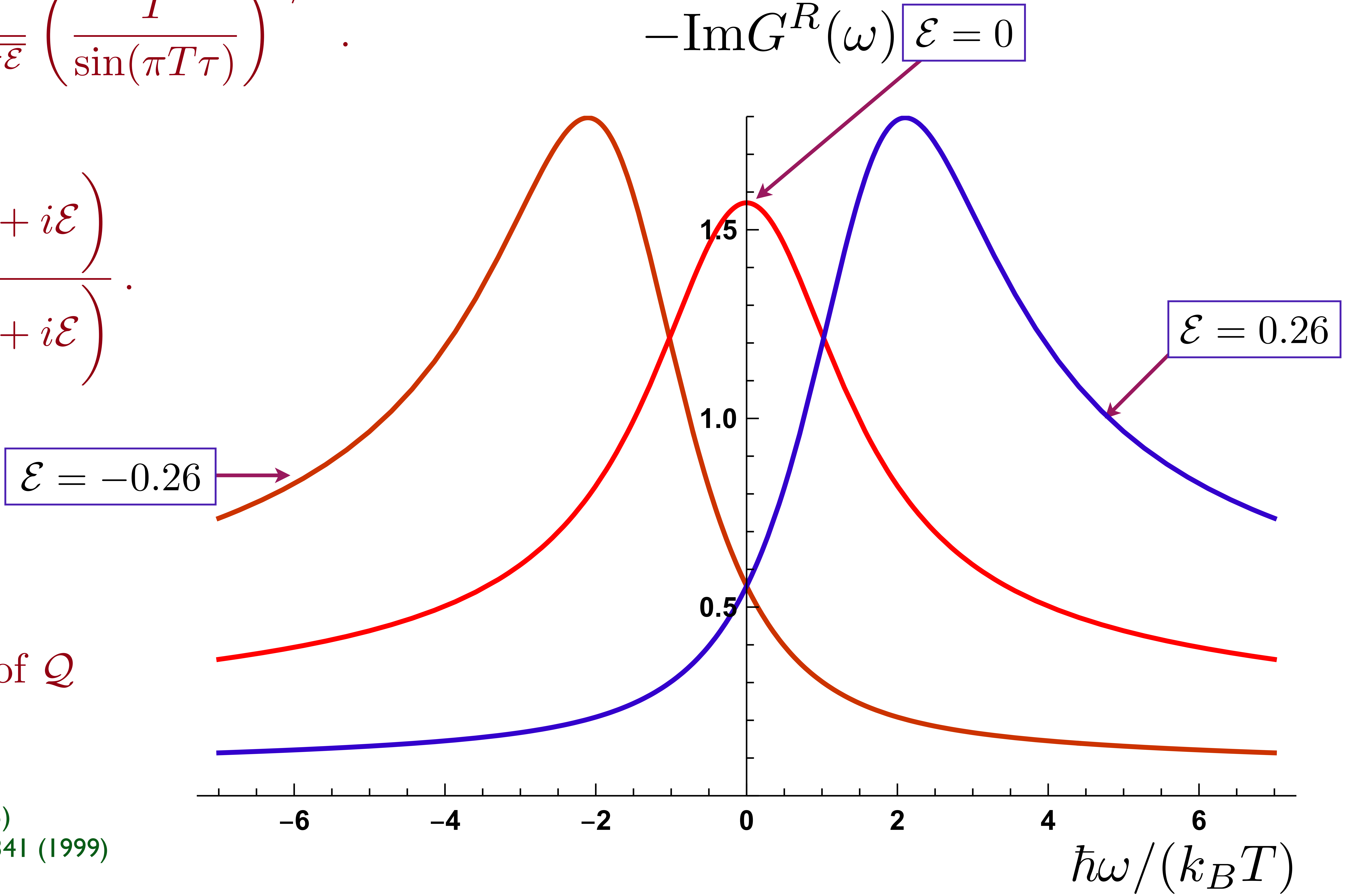
$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}}\frac{\Gamma\left(\frac{1}{4}-\frac{i\omega}{2\pi T}+i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4}-\frac{i\omega}{2\pi T}+i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$

(Luttinger relation)



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$$G_*(\tau) = -C \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} .$$

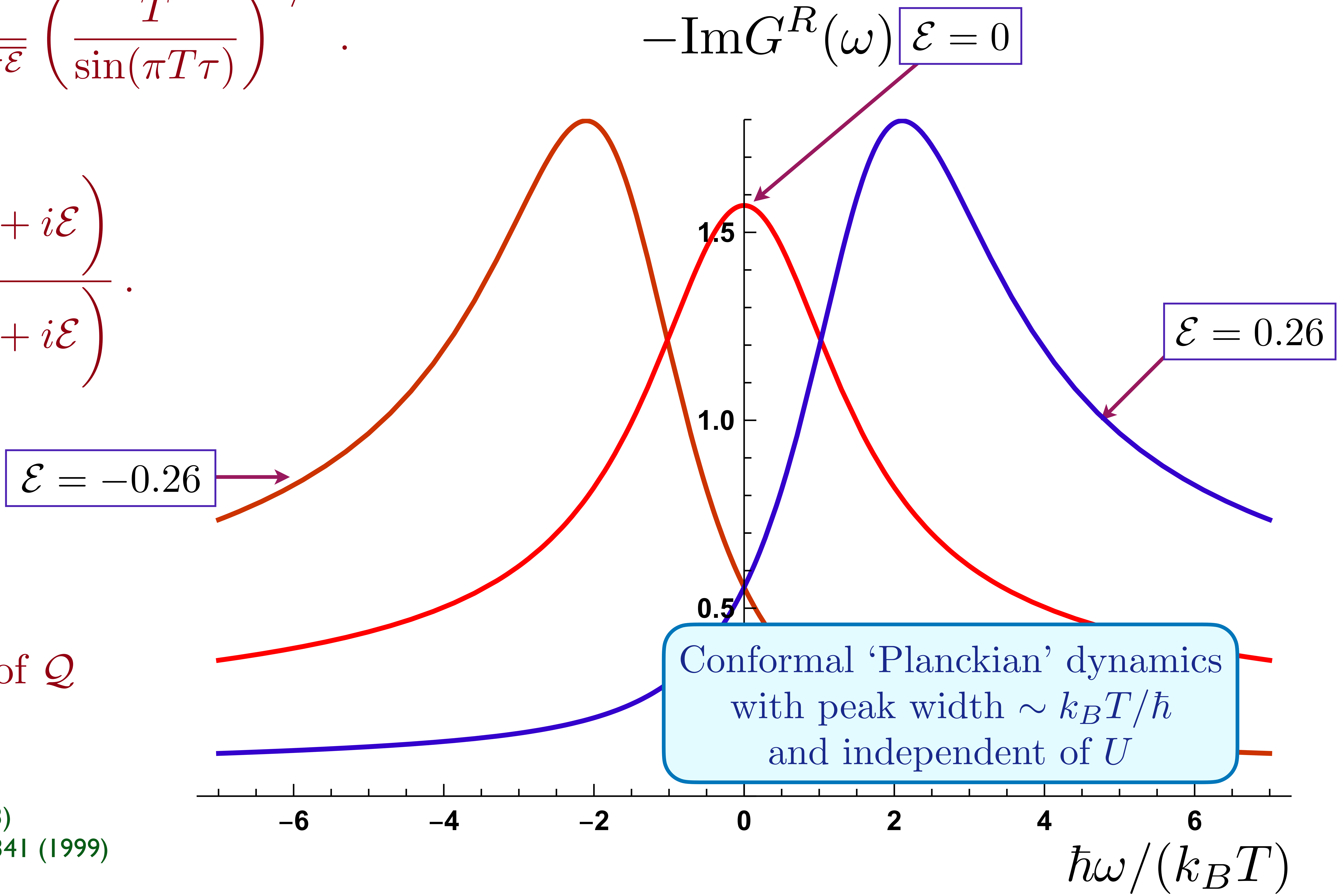
$$G_*^R(\omega) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)} .$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$ (Luttinger relation)



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

G-Σ path integral

After introducing replicas $a = 1 \ldots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{\alpha a}(\tau) \exp \left[-\sum_{ia} \int_0^\beta d\tau \, c_{\alpha a}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{\alpha a} \right.$$
$$\left. - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{\alpha a}^\dagger(\tau) c_{\alpha b}(\tau') \right|^4 \right] .$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left(G(\tau_2, \tau_1) \right. \right.$$
$$\left. \left. + \frac{1}{N} \sum_\alpha c_\alpha(\tau_2) c_\alpha^\dagger(\tau_1) \right) \right] .$$

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

G-Σ path integral

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

G-Σ path integral

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1-\tau_2)(\partial_{\tau_1}+\mu) - \Sigma(\tau_1,\tau_2)\right] + \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1,\tau_2)G(\tau_2,\tau_1) + (U^2/2)G^2(\tau_2,\tau_1)G^2(\tau_1,\tau_2)\right]$$

Saddle-point equations:

$$G(i\omega) = \frac{1}{i\omega+\mu-\Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2G^2(\tau)G(-\tau)$$

$$G(\tau=0^-) = \mathcal{Q}.$$

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK model

These results can also be obtained from the saddle-point of a G-Σ-D-Π action, obtained using replica methods as for the SYK model.

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-NS_{\text{all}})$$

$$\begin{aligned} S_{\text{all}} = & -\ln\det(\partial_\tau + -\mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \omega_0^2 - \Pi) \\ & + \int d\tau \int d\tau' \left[-\Sigma(\tau';\tau)G(\tau;\tau') + \frac{1}{2}\Pi(\tau';\tau)D(\tau;\tau') \right. \\ & \left. \qquad + \frac{g^2}{2}G(\tau;\tau')G(\tau';\tau)D(\tau;\tau') \right]. \end{aligned}$$

Yukawa-SYK model

These results can also be obtained from the saddle-point of a G-Σ-D-Π action, obtained using replica methods as for the SYK model.

$$\mathcal{Z} = \int \mathcal{D}G \, \mathcal{D}\Sigma \, \mathcal{D}D \, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

Saddle-point equations:

$$\Sigma(\tau) = g^2 D(\tau) G(\tau),$$
$$\Pi(\tau) = -g^2 G(-\tau) G(\tau),$$
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)},$$
$$D(i\Omega) = \frac{1}{\Omega^2 + \omega_0^2 - \Pi(i\Omega)}.$$

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

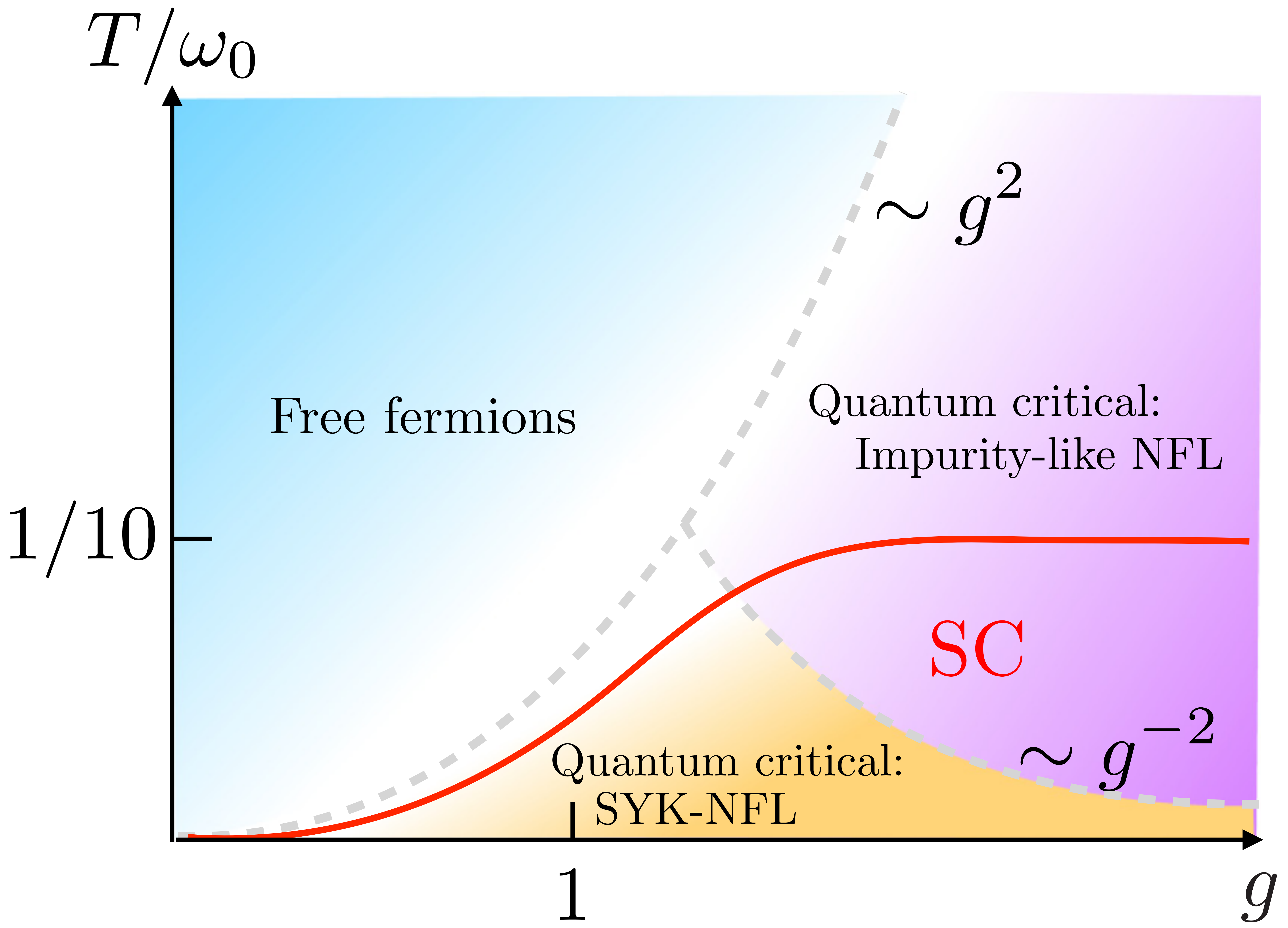
$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK model



I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

1. Large-N theory of the SYK model
2. Finite-N theory of the SYK model
3. Quantum Einstein-Maxwell gravity theory of charged black holes

G-Σ path integral

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NS)$$
$$S = \ln\det\left[\delta(\tau_1-\tau_2)(\partial_{\tau_1}+\mu)-\Sigma(\tau_1,\tau_2)\right]$$
$$+\int d\tau_1 d\tau_2\left[\Sigma(\tau_1,\tau_2)G(\tau_2,\tau_1)+(U^2/2)G^2(\tau_2,\tau_1)G^2(\tau_1,\tau_2)\right]$$

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$
$$G(\tau_1,\tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4}\frac{g(\sigma_1)}{g(\sigma_2)}\widetilde{G}(\sigma_1,\sigma_2)$$
$$\Sigma(\tau_1,\tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\widetilde{\Sigma}(\sigma_1,\sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

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where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

We can map the $T = 0$ solution to the $T > 0$ solution by

$$\tau = \frac{1}{\pi T} \tan(\pi T \sigma)$$

$$g(\sigma) = e^{-2\pi \mathcal{E} T \sigma}$$

G-Σ path integral

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) e^{-E_0/T + Ns_0 - NS_{\rm eff}[f,\phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Symmetries of the large N saddle point

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \quad \sim \quad (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \quad \sim \quad (\tau_1 - \tau_2)^{-3/2}.$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

Symmetries of the large N saddle point

- The saddle-point

$$G(\tau_1 - \tau_2) = -A \frac{e^{-2\pi\mathcal{E}T(\tau_1-\tau_2)}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T(\tau_1 - \tau_2))} \right)^{2\Delta}$$

is invariant only under PSL(2, R) transformations which map the thermal circle onto itself, and an associated gauge transformation

$$\frac{\tan(\pi T f(\tau))}{\pi T} = \frac{a \dfrac{\tan(\pi T \tau)}{\pi T} + b}{c \dfrac{\tan(\pi T \tau)}{\pi T} + d} \quad , \quad ad - bc = 1,$$

$$-i\phi(\tau) = -i\phi_0 + 2\pi\mathcal{E}T(\tau - f(\tau))$$

A. Kitaev, 2015

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

G-Σ path integral

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}}\left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

Low temperature thermodynamics: for $k_B T \ll U$

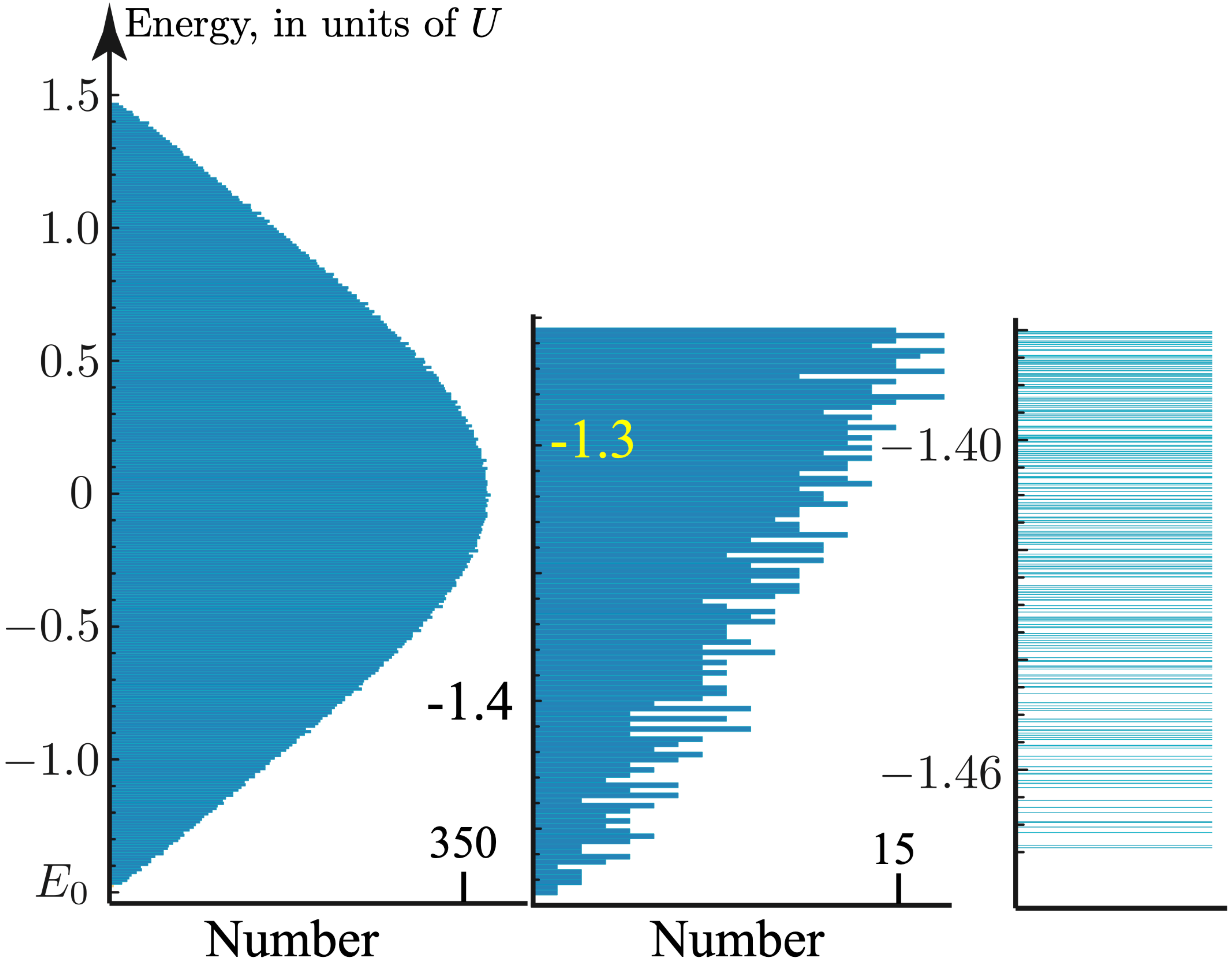
$$
\begin{aligned}
\mathcal{Z} &= \operatorname{Tr} \exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\
&= \exp\left(N\frac{S_0}{k_B}\right) \int \frac{\mathcal{D}f(\tau)\mathcal{D}\phi(\tau)}{||\text{SL(2,R)}||} \exp\left(-\frac{1}{\hbar} S_{\text{eff}}\left[f(\tau), \phi(\tau)\right]\right)
\end{aligned}
$$

- Feynman path integral over $f(\tau)$, the reparameterization of the time of the SYK model, and $\phi(\tau)$ a phase conjugate to the total charge $\mathcal{Q}$.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

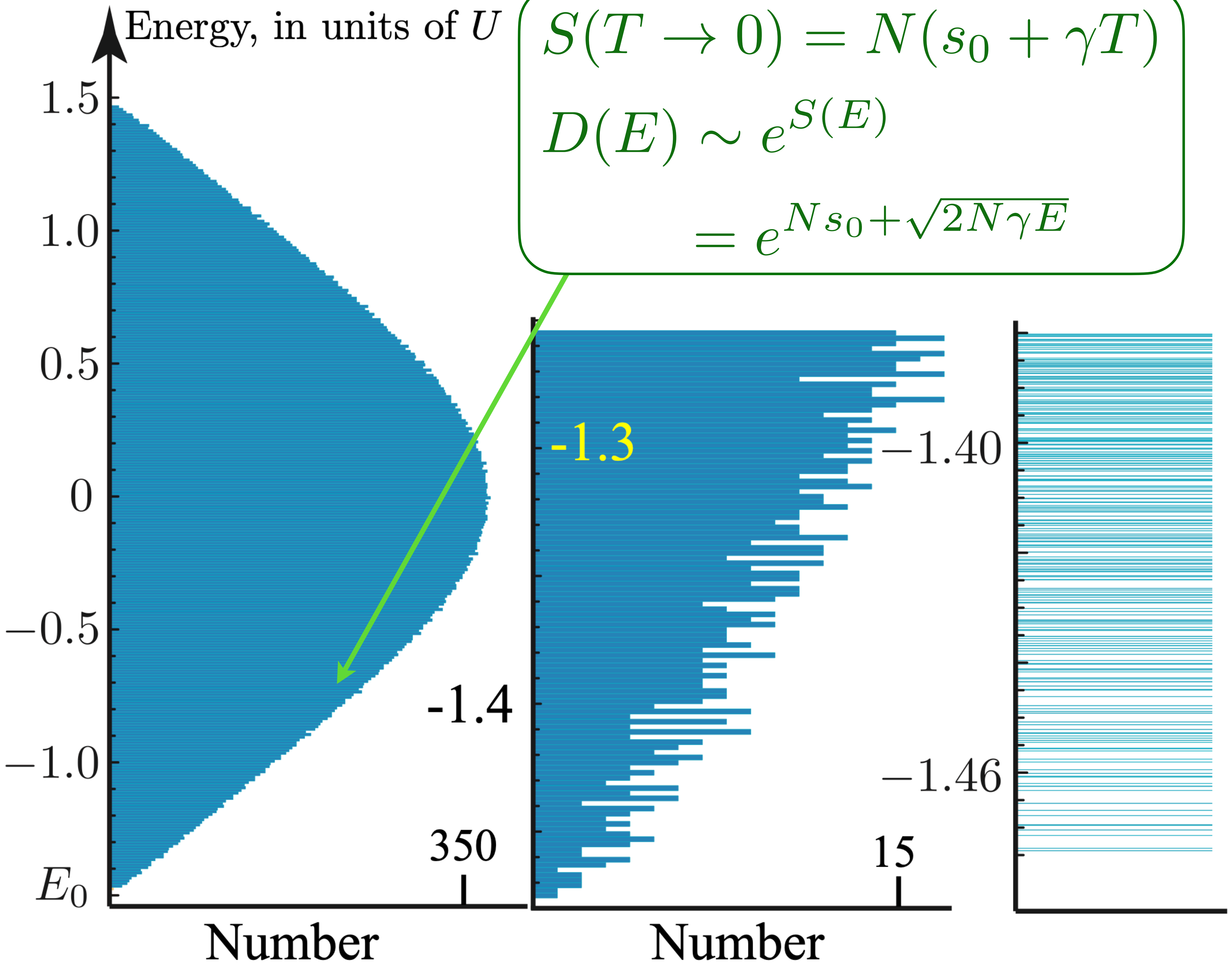
(Numerics: G. Tarnopolsky)

Many-body density of states

Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$
$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$



At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$
$$= 0.46484769917\ldots.$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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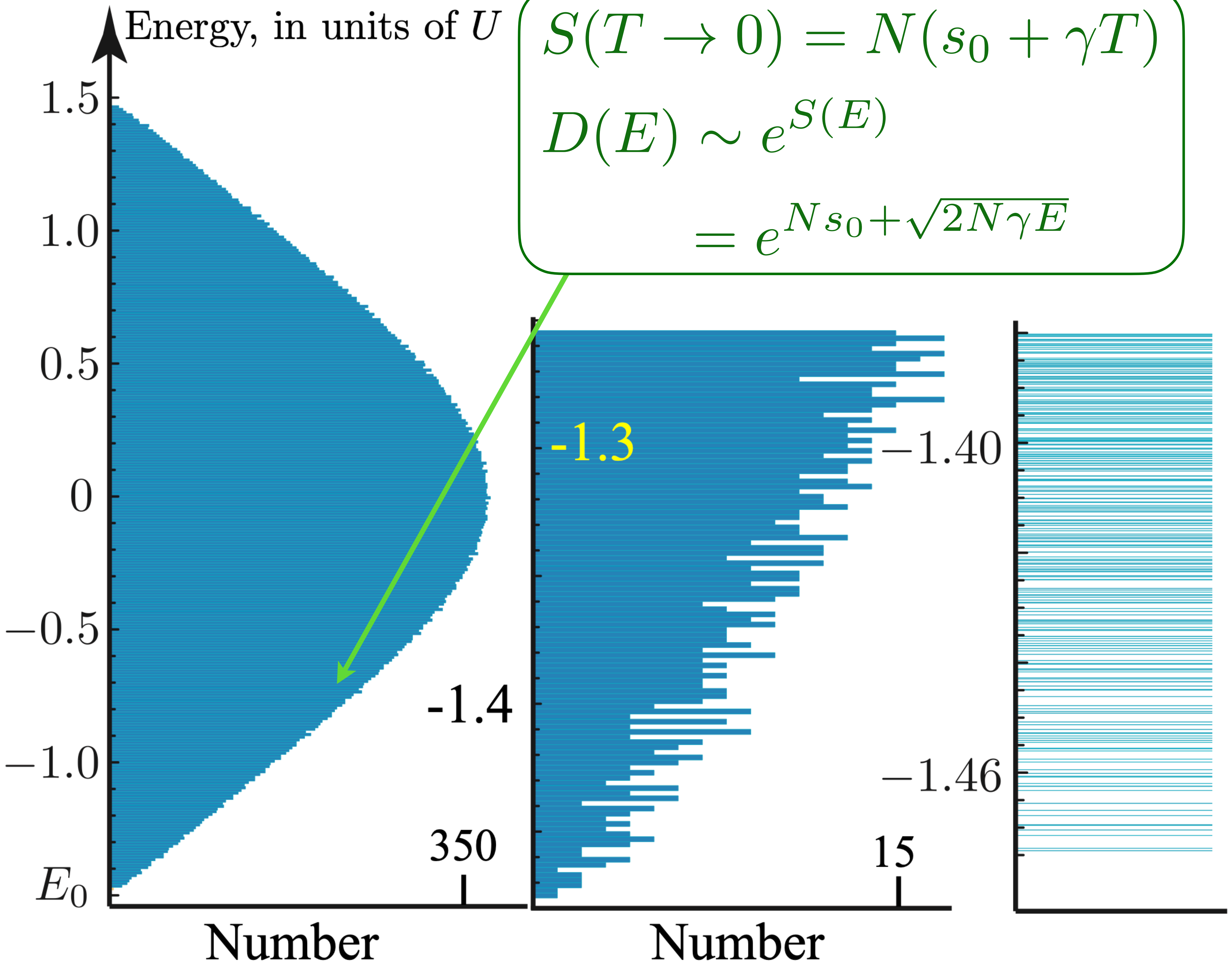

Energy level spacing $\sim e^{-N s_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

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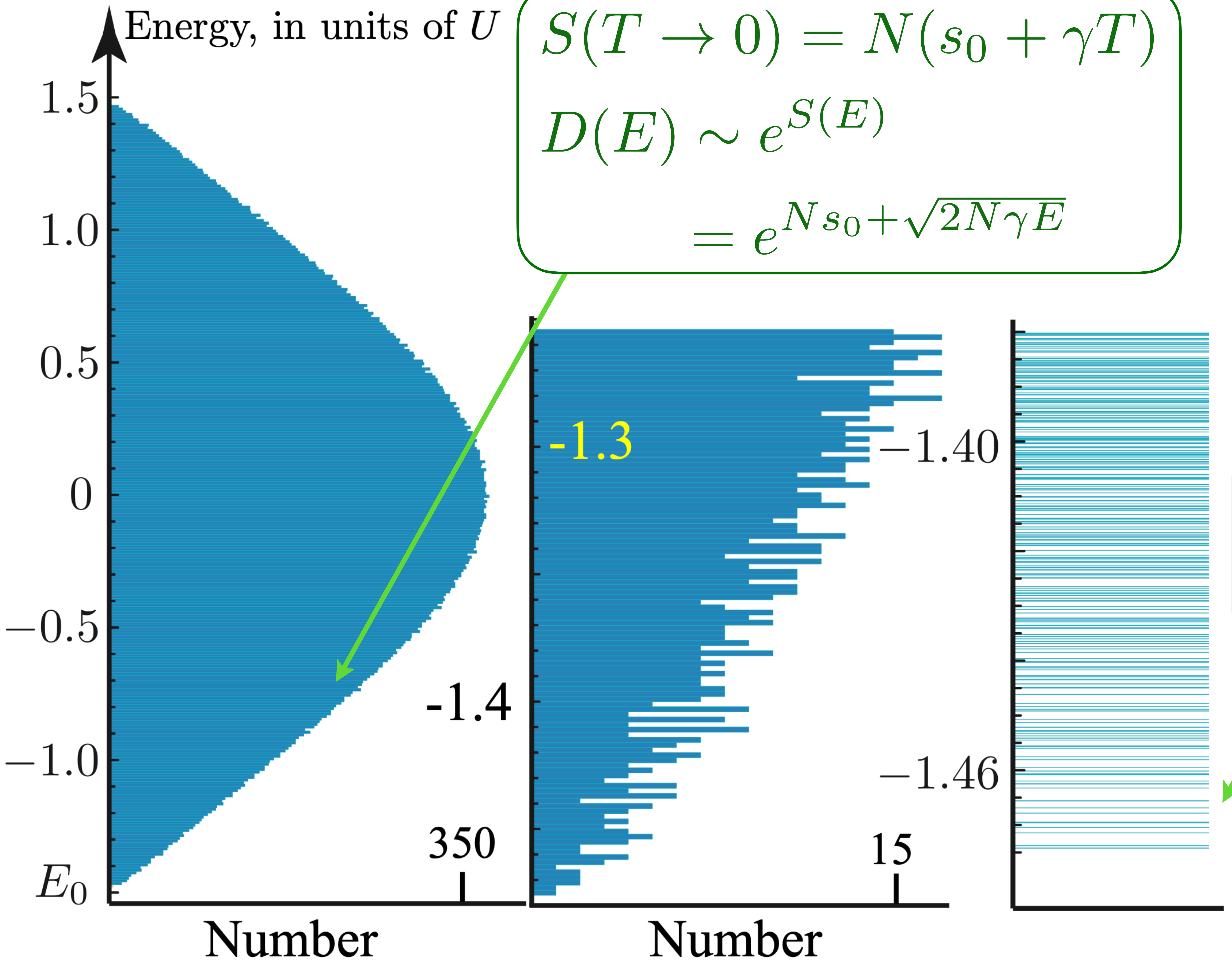

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E})$$

Complex SYK model

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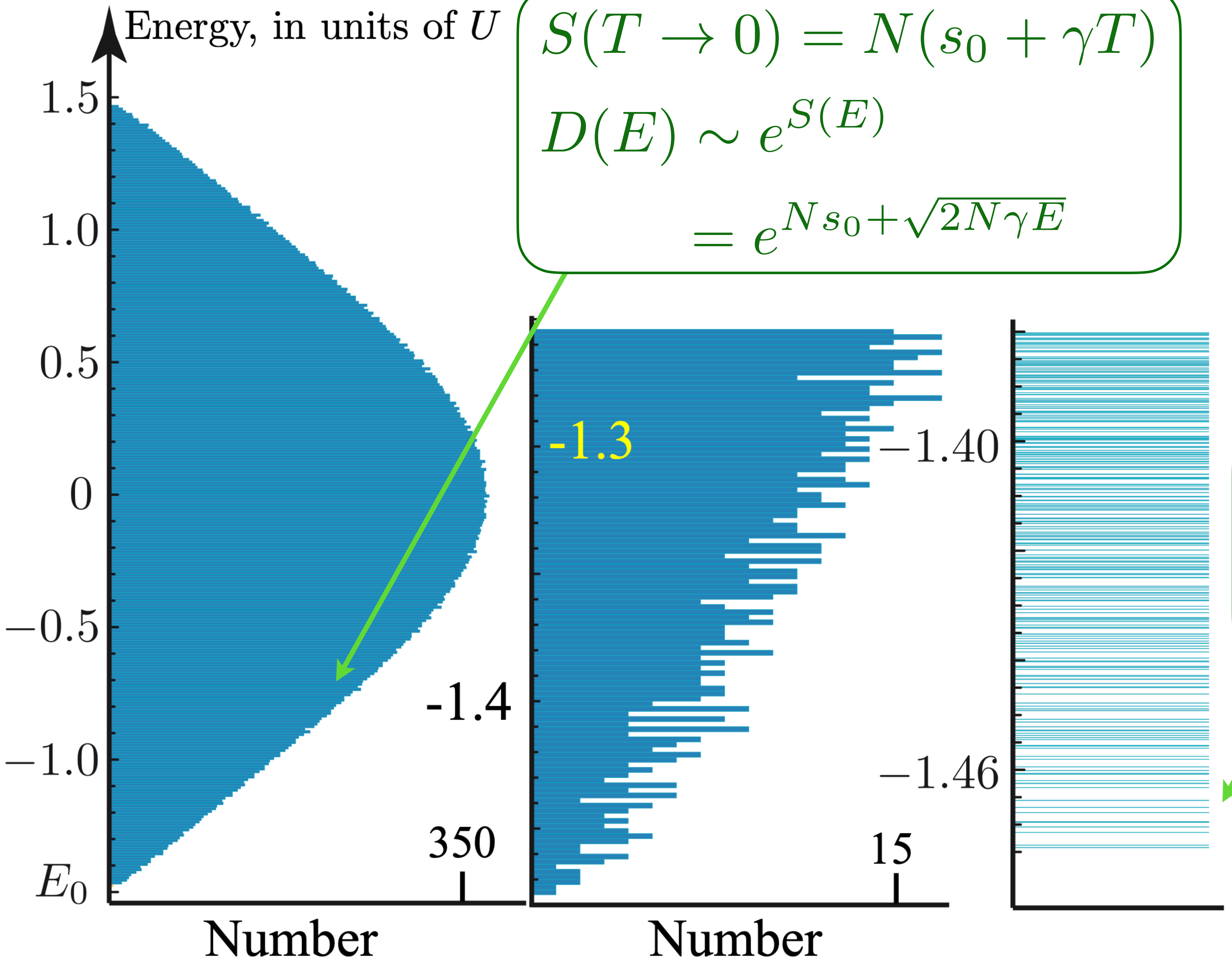

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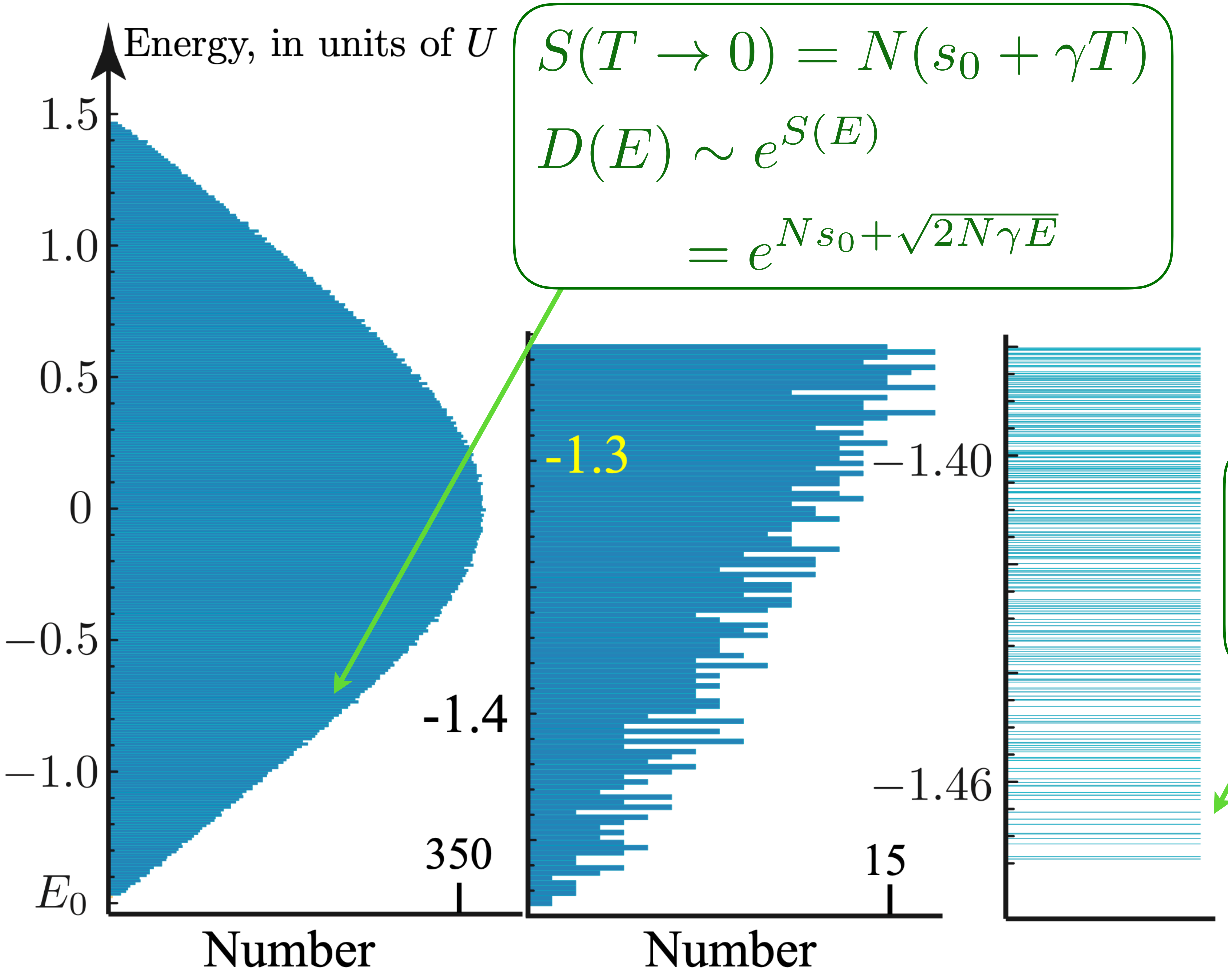
J. S. Cotler et al., JHEP 05 (2017) 118



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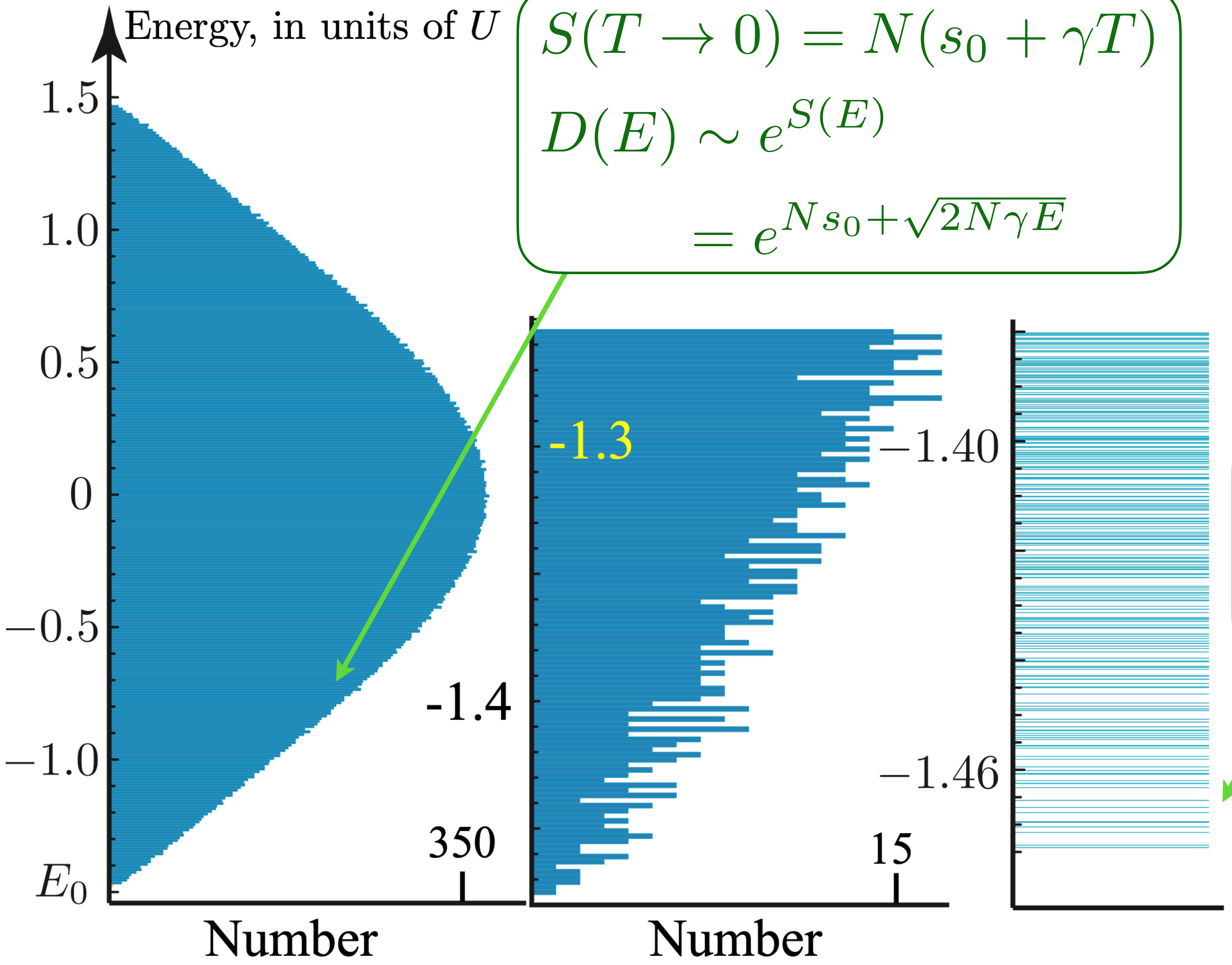
J. S. Cotler et al., JHEP 05 (2017) 118

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157



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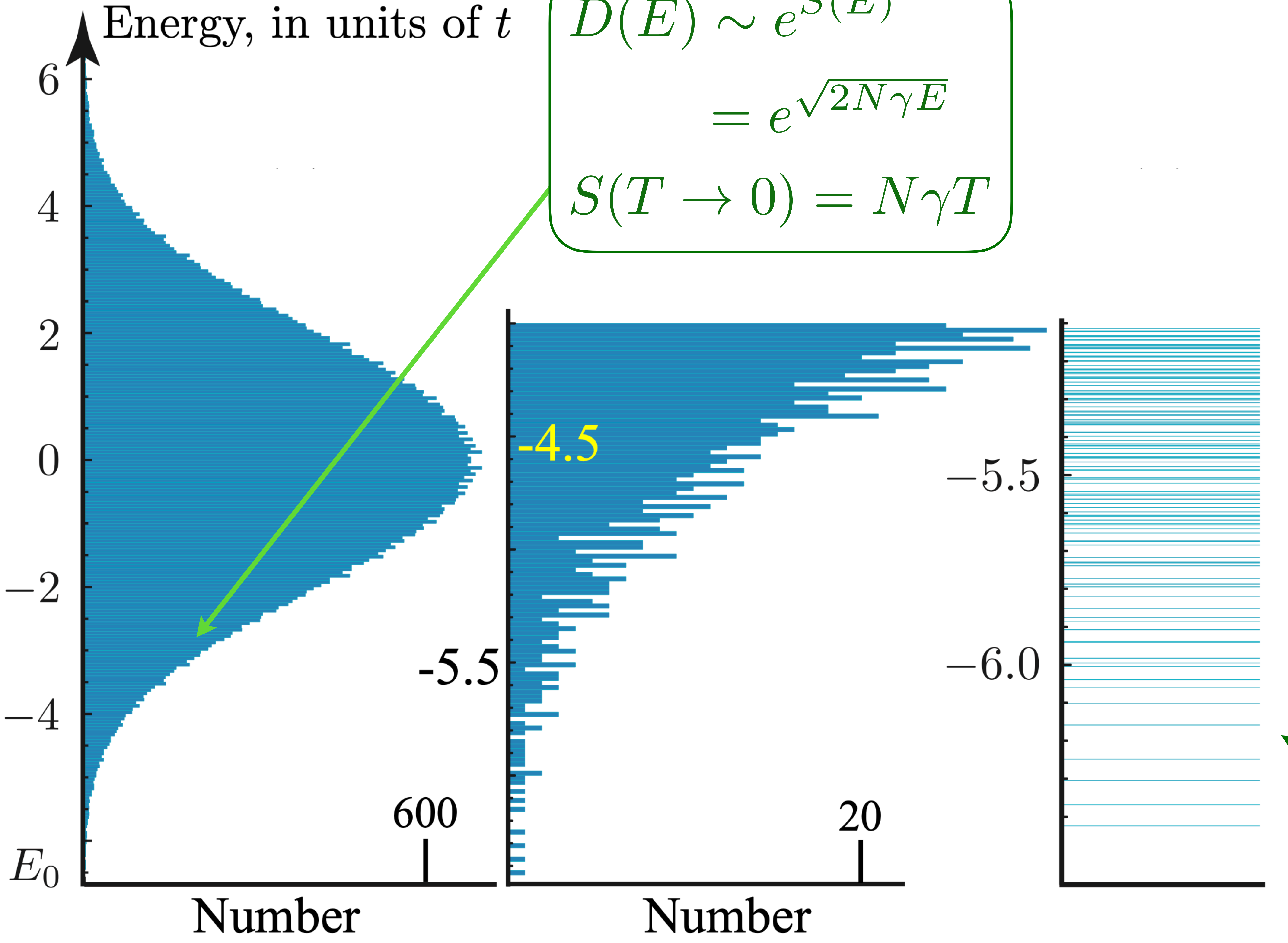
$$= e^{\sqrt{2N\gamma E}}$$
$$S(T \to 0) = N\gamma T$$



$$D(E) \sim N$$

For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$

$n_\alpha = 0, 1,$ occupation number

Random matrix model

1. Large-N theory of the SYK model
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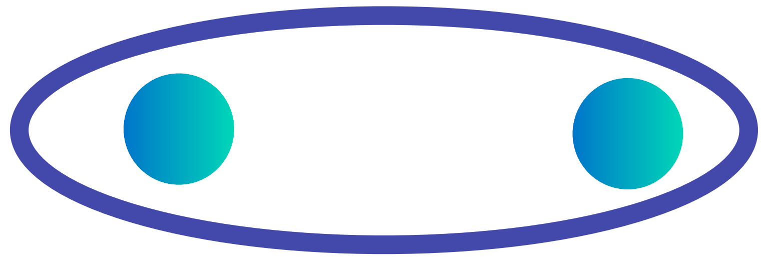
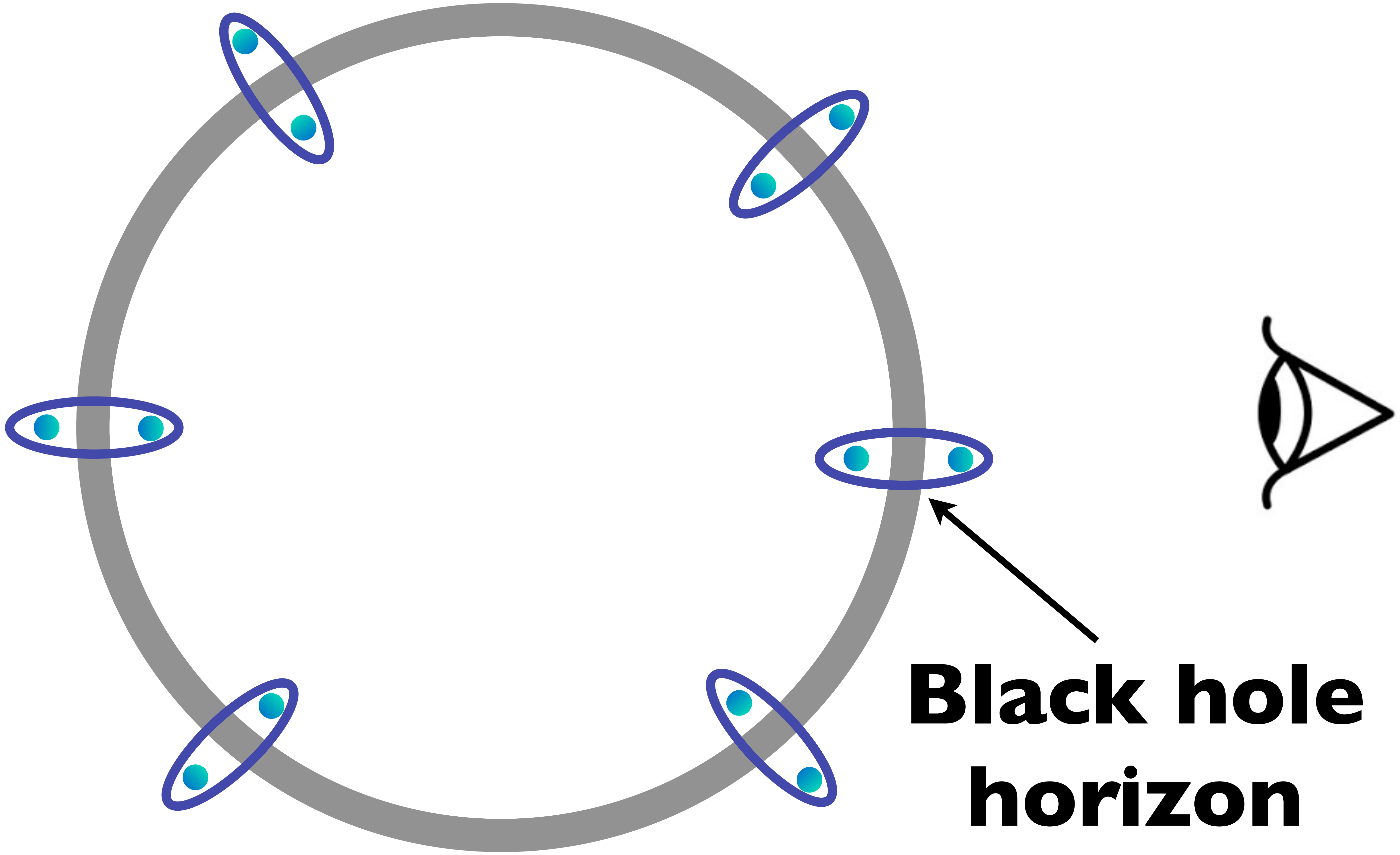
Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

The Einstein action for gravity in 3+1 dimensions is

$$I_E = \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\mathcal{R}_4\right] \quad , \quad \mathcal{Z} = \int \mathcal{D}g\,\exp(-I_E)\,,$$

where $\kappa^2 = 8\pi G_N$ is the gravitational constant, $\mathcal{R}_4$ is the Ricci scalar. The Schwarzschild solution of the saddle-point equations is

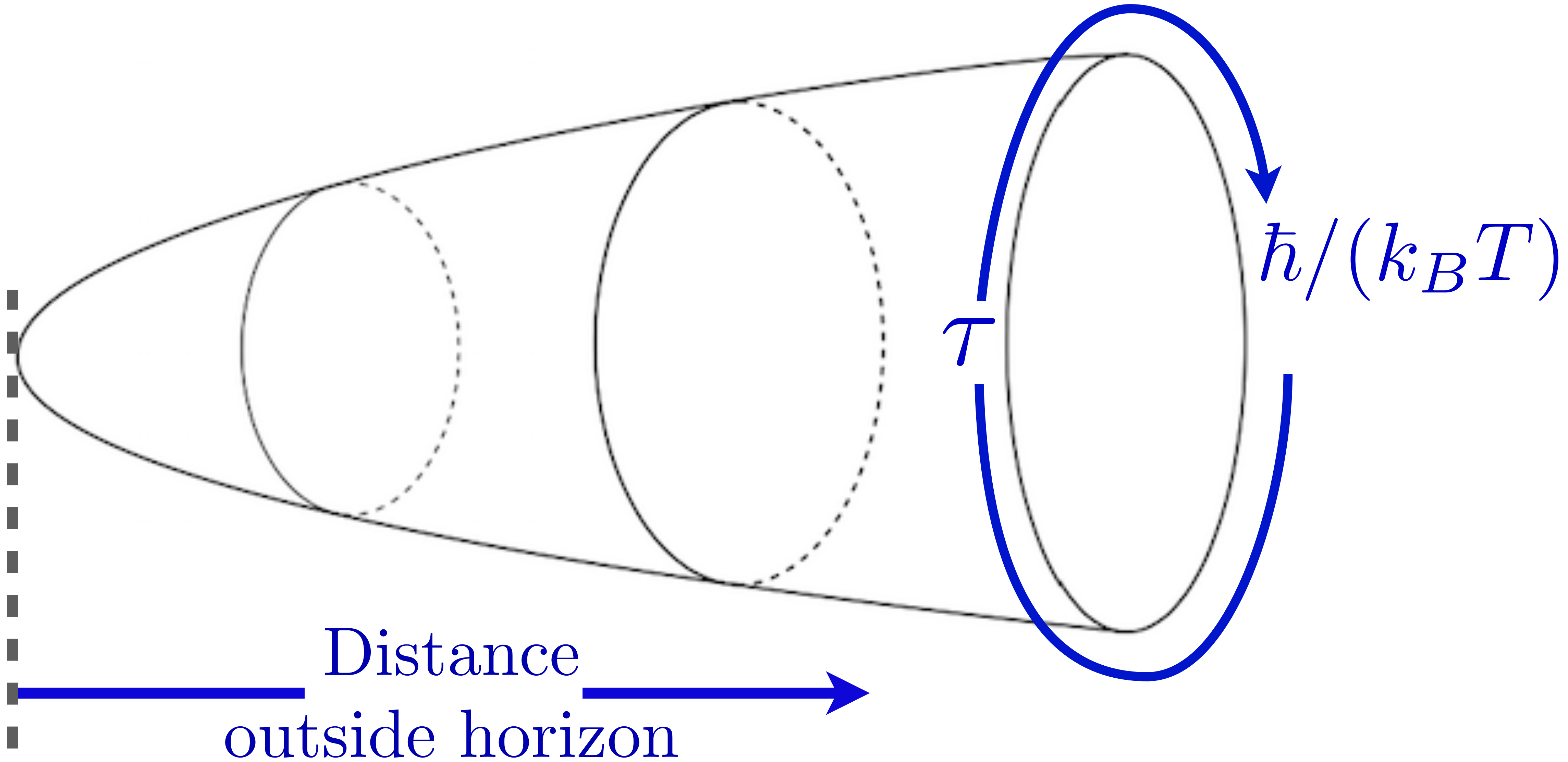
$$ds^2 = V(r)d\tau^2 + r^2 d\Omega_2^2 + \frac{dr^2}{V(r)}$$

where $d\Omega_2^2$ is the metric of the 2-sphere, and

$$V(r) = 1 - \frac{m}{r}.$$

The gravitational mass of the black hole is $M = 2G_N m$. The black hole horizon is at $r = r_0$ where $V(r_0) = 0$; so

$$r_0 = m$$



The $T > 0$ quantum partition function is obtained in a spacetime which is periodic as a function of τ with period $\hbar/(k_B T)$. We have to ensure that there is no singularity at the horizon r_0 where $V(r_0) = 0$. Let us change radial co-ordinates to y, where $r = r_0 + y^2$. Then for small y

$$ds^2 = \frac{4}{V'(r_0)} \left[\frac{(V'(r_0))^2}{4} y^2 d\tau^2 + dy^2 \right] + r_0^2 d\Omega_2^2 = \frac{4}{V'(r_0)} \left[y^2 d\theta^2 + dy^2 \right] + r_0^2 d\Omega_2^2$$

The expression in the square brackets is the metric of the flat plane in polar co-ordinates, with radial co-ordinate y and angular co-ordinate $\theta = V'(r_0)\tau/2$. Smoothness requires periodicity in θ with period 2π, and so

$$4\pi T = V'(r_0) = \frac{1}{m} \, .$$

The free energy $\beta F = I_E$, where $\beta = 1/T$. So the entropy is

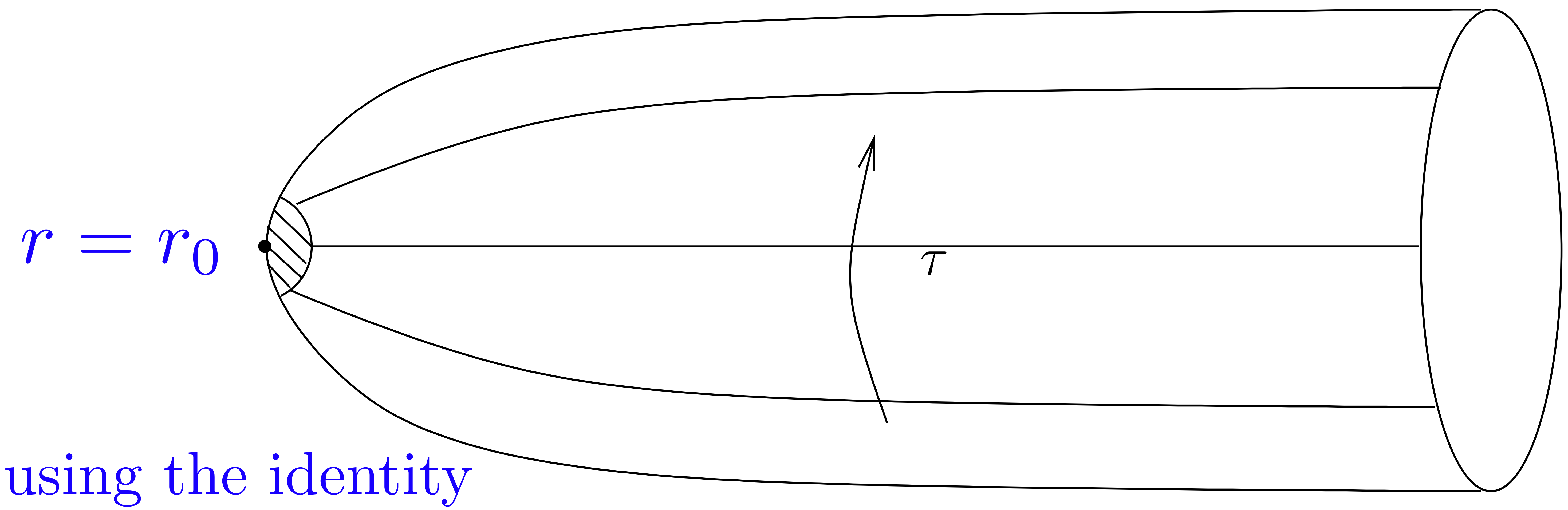
$$S = -\frac{\partial F}{\partial T} = \left(\beta \frac{\partial}{\partial \beta} - 1\right) I_E$$

However, the metric is τ-independent, and the only explicit dependence of the action is via $I_E = \beta H$. Such an action implies $S = 0$.

The entire contribution to the entropy comes from the vicinity of the co-ordinate singularity at $r = r_0$. We evaluate the action is the small region around this point

$$I_{\rm grav} = I_E + I_{GH} \quad , \quad I_{GH} = \int_\partial d^3x \sqrt{g_b} \left[-\frac{1}{\kappa^2} \mathcal{K}_3 \right] \quad , \quad \mathcal{Z} = \int \mathcal{D}g \exp(-I_{\rm grav}) \, ,$$

where $\mathcal{K}_3$ is the extrinsic scalar curvature of the 3-dimensional boundary of spacetime. I_{GH} is the Gibbons-Hawking boundary term, deduced by the requirement that the Euler-Lagrange equations of $I_{\rm grav}$ co-incide with the Einstein equations, with no additional boundary terms. The entire contribution to the entropy will come from I_{GH}.



We evaluate I_{GH} by using the identity

$$\int_\partial d^3x\sqrt{g_b}\,\mathcal{K}_3 = \frac{\partial}{\partial n}\int_\partial d^3x\sqrt{g_b}$$

where n is the Gaussian normal co-ordinate of the boundary. Evaluating at $y = \epsilon$, we have

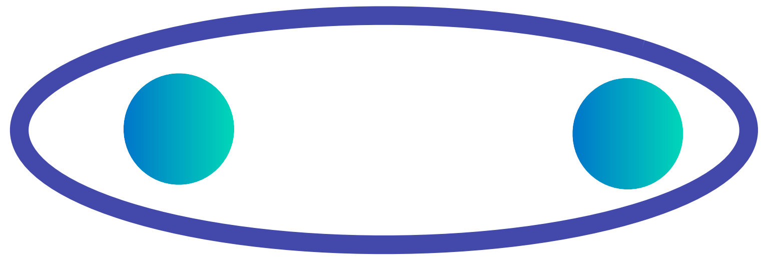
$$\int_\partial d^3x\sqrt{g_b} = 2\pi\epsilon\mathcal{A}$$

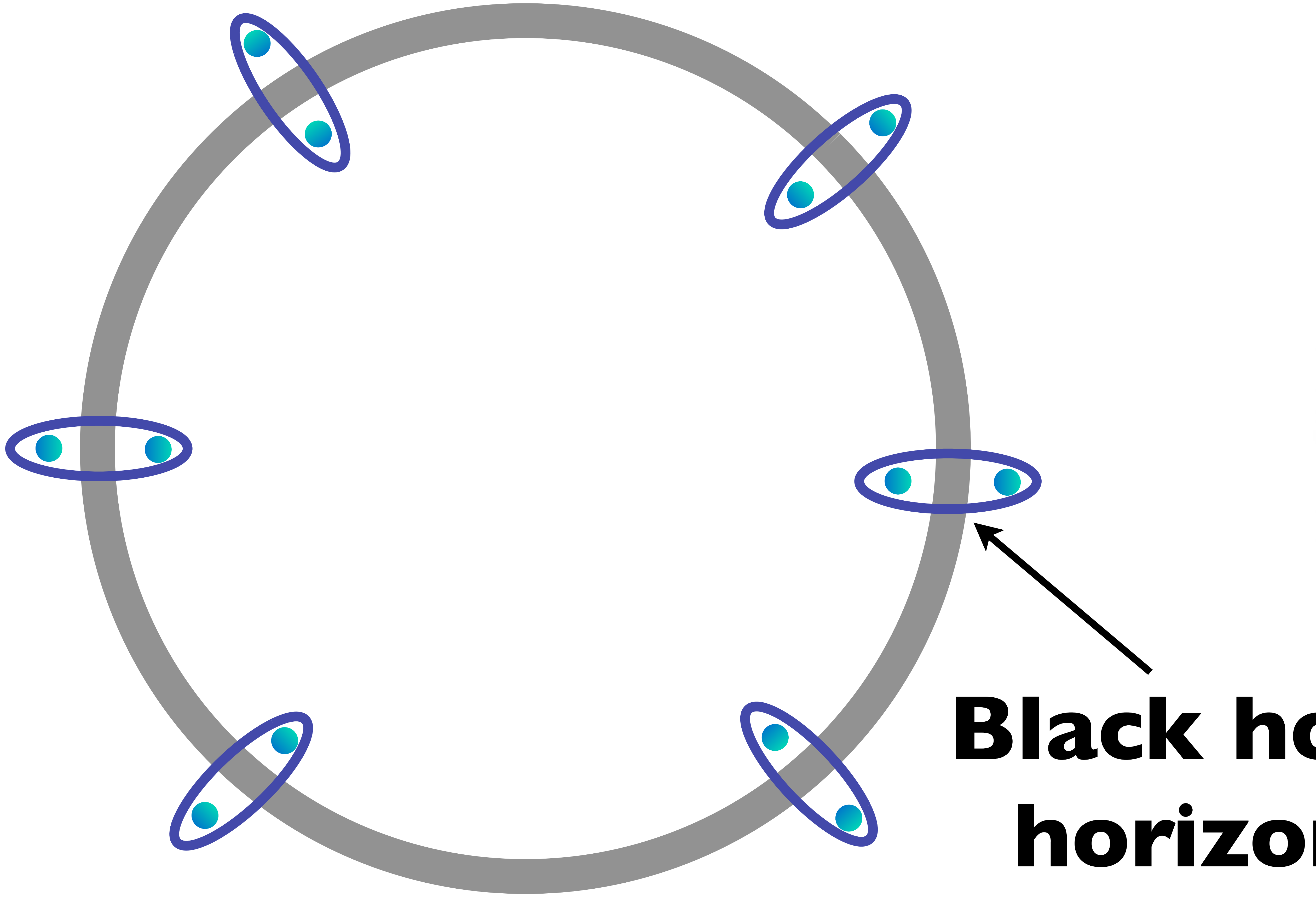
where $\mathcal{A} = 4\pi r_0^2$ is the area of the horizon. Combining everything, we have the famous result of Hawking

$$S = \frac{2\pi\mathcal{A}}{\kappa^2} = \frac{\mathcal{A}}{4G_N}.$$

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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Charged black holes

We consider a charged black hole in Einstein-Maxwell theory of g and a U(1) gauge flux $F = dA$

$$I_{EM} = \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\mathcal{R}_4 + \frac{1}{4g_F^2}F^2\right] \quad , \quad \mathcal{Z}_{\mathcal{Q}} = \int \mathcal{D}g\mathcal{D}A\,\exp(-I_{EM} - I_{GH})\,.$$

The saddle-point equations now yield a solution as before with

$$V(r) = 1 + \frac{\Theta^2}{r^2} - \frac{m}{r} \quad ; \quad A_\tau = i\mu\left(1 - \frac{r_0}{r}\right) \quad ; \quad \Theta = \frac{\kappa r_0}{\sqrt{2}g_F}\mu \quad ; \quad \mathcal{Q} = \frac{4\pi\mu r_0}{g_F^2} \quad ; \quad S = \frac{2\pi\mathcal{A}}{\kappa^2}$$

where $\mathcal{Q}$ is the total charge, the chemical potential is μ, and as before the horizon is where $V(r_0) = 0$, the temperature $T = V'(r_0)/(4\pi)$, and $\mathcal{A} = 4\pi r_0^2$.

This defines a two parameter family of charged black hole solutions of I_{EM} determined by T and $\mathcal{Q}$.

Charged black holes

Now we take the limit $T \to 0$ at fixed $\mathcal{Q}$. Then we find the remarkable feature that the horizon radius remains finite

$$R_h \equiv r_0(T \to 0, \mathcal{Q}) = \frac{\mathcal{Q}\kappa g_F}{4\pi}$$

In this limit, entropy becomes

$$S(T \to 0, \mathcal{Q}) = \frac{4\pi R_h^2}{G_N} + \gamma T \quad , \quad \gamma \equiv \frac{4\pi^2 R_h^3}{G_N}$$

For the near-horizon metric, it is useful to introduce the co-ordinate ζ

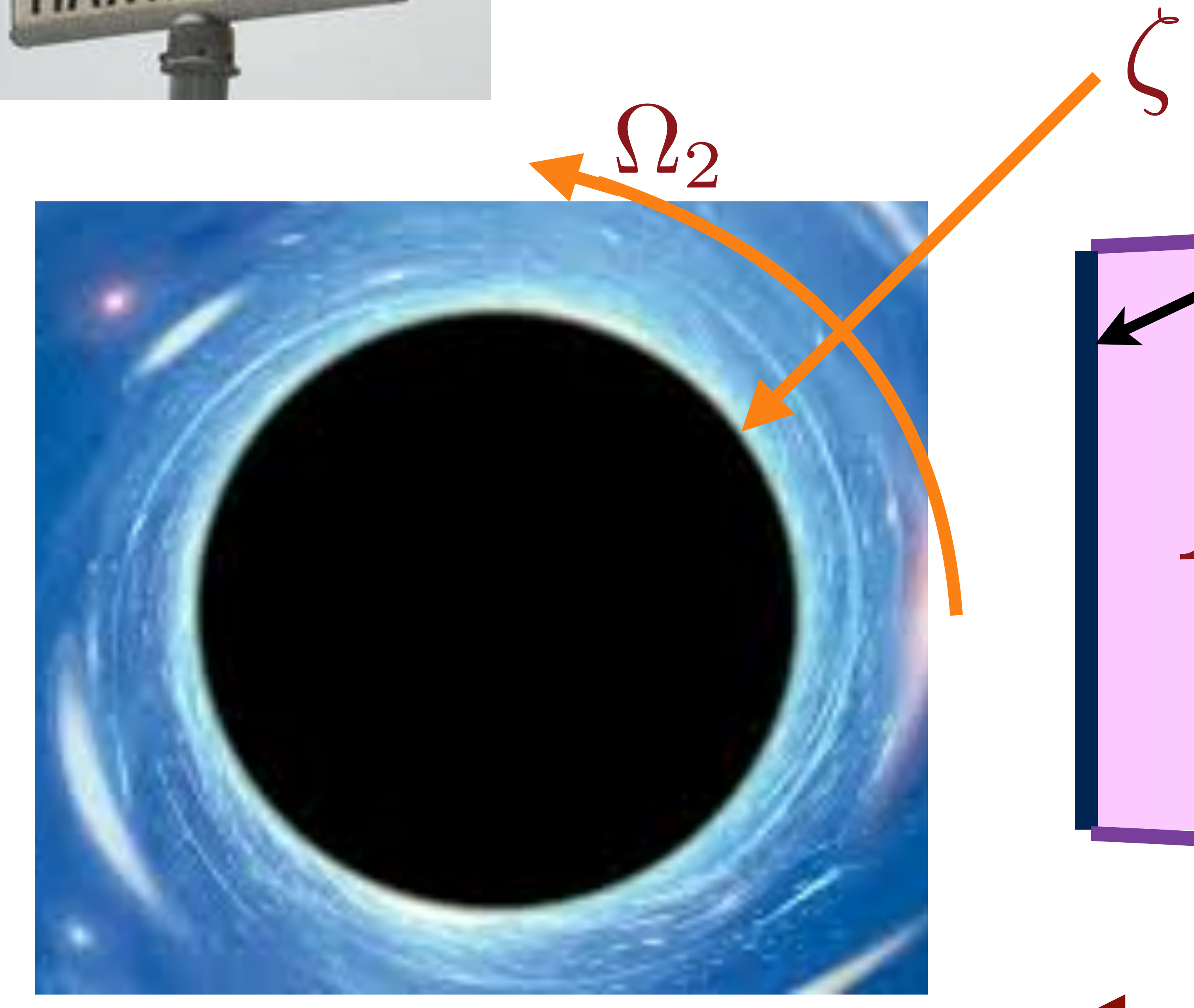
$$r = R_h + \frac{R_h^2}{\zeta}$$

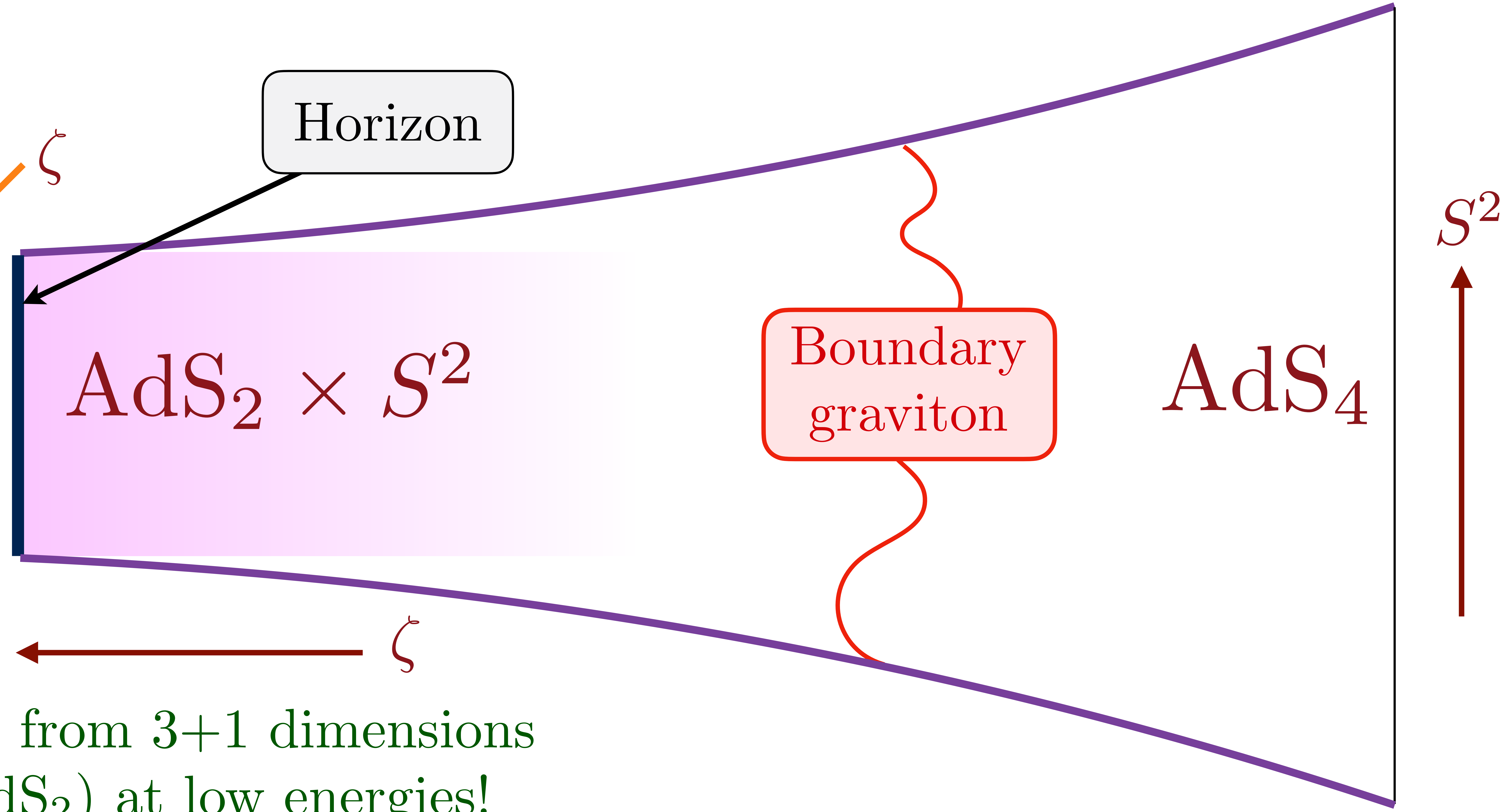
so that the horizon at $T = 0$ is at $\zeta = \infty$. Then in the near-horizon regime $R_h \ll \zeta < \infty$ the $T = 0$ metric is

$$ds^2 = R_h^2 \frac{d\tau^2 + d\zeta^2}{\zeta^2} + R_h^2 d\Omega_2^2$$

This spacetime is $\mathrm{AdS}_2 \times S^2$.

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

Saddle-point:

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} + \ldots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

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Maldacena, Stanford, Yang (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

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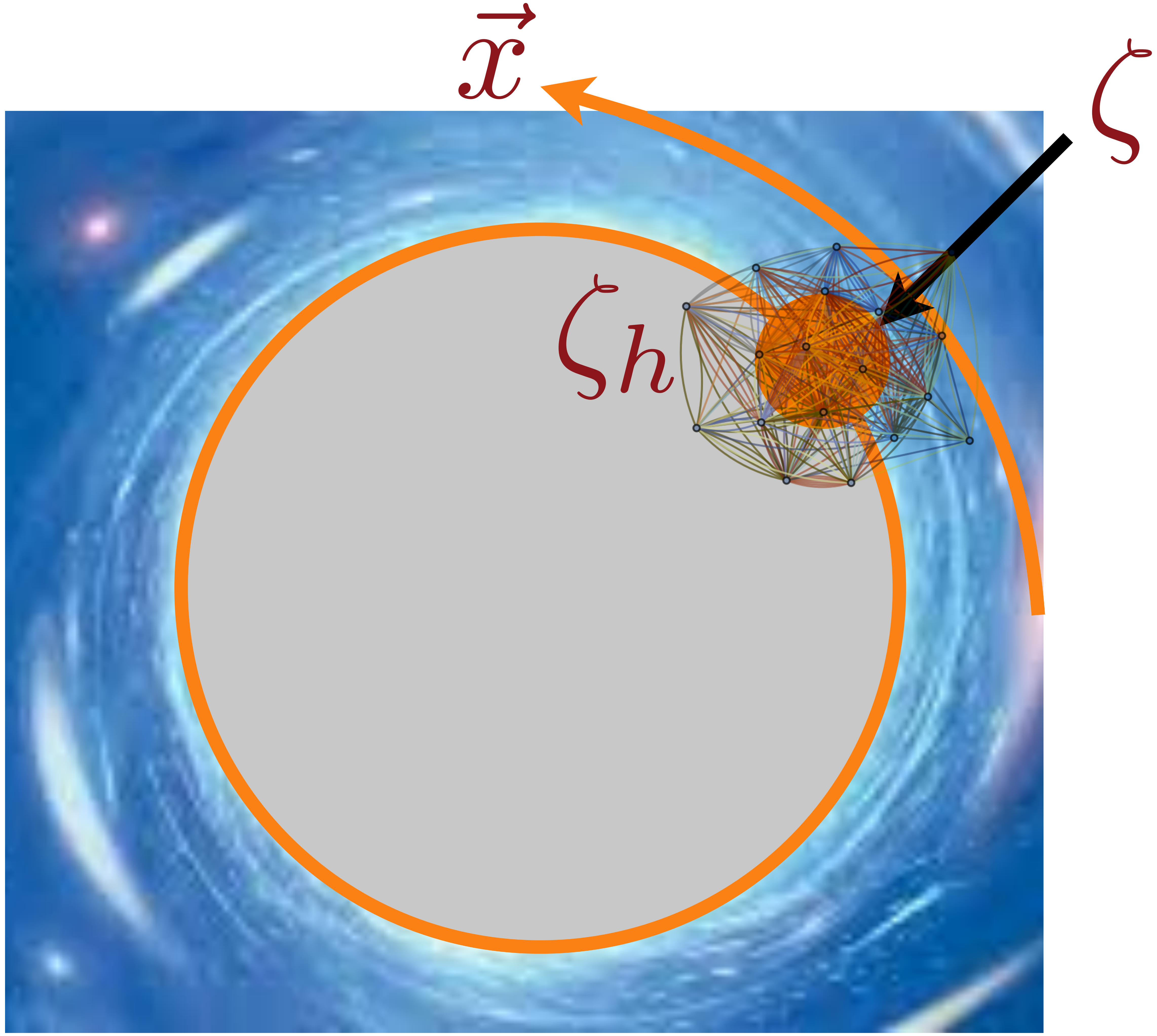
Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model!

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

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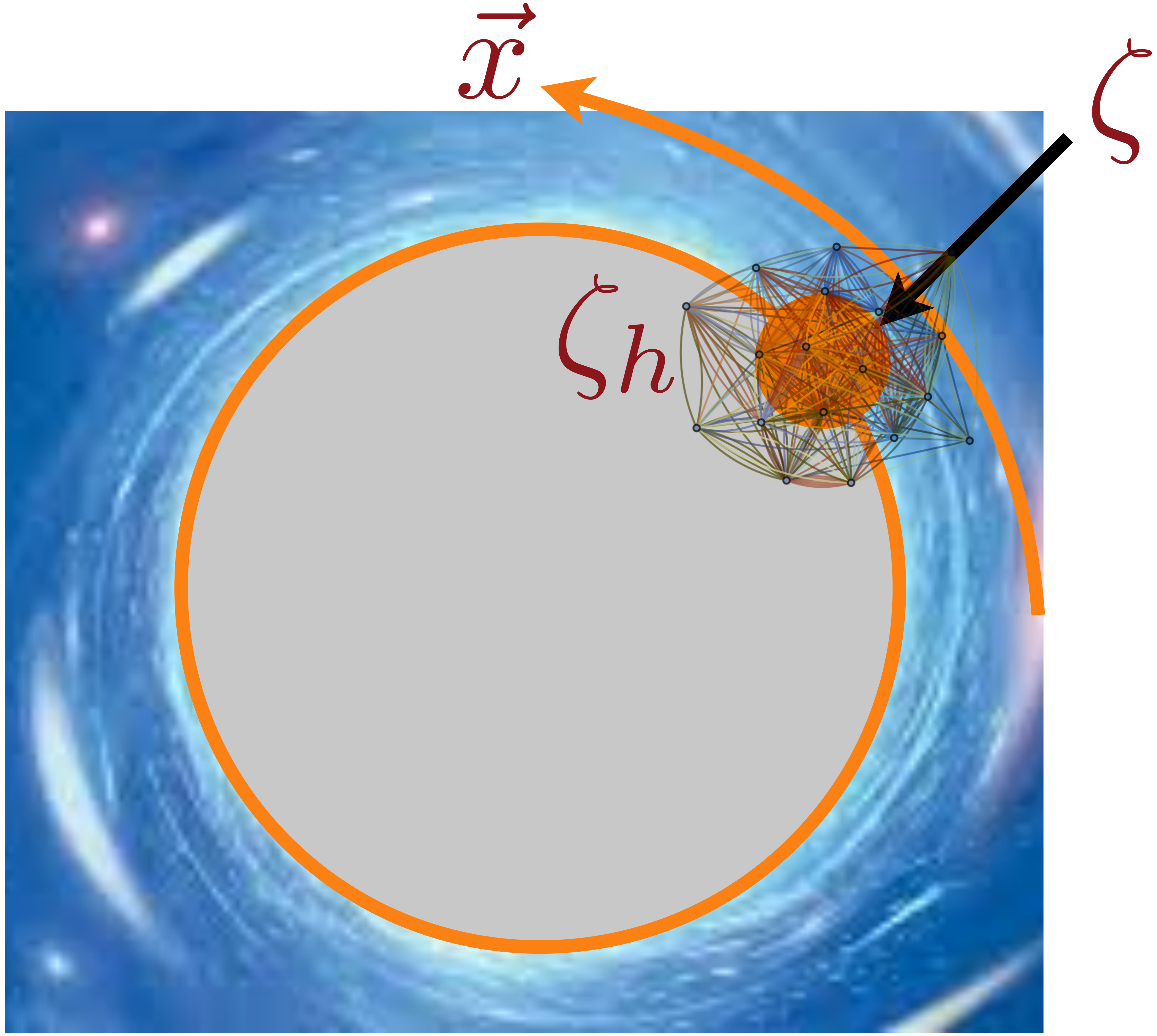

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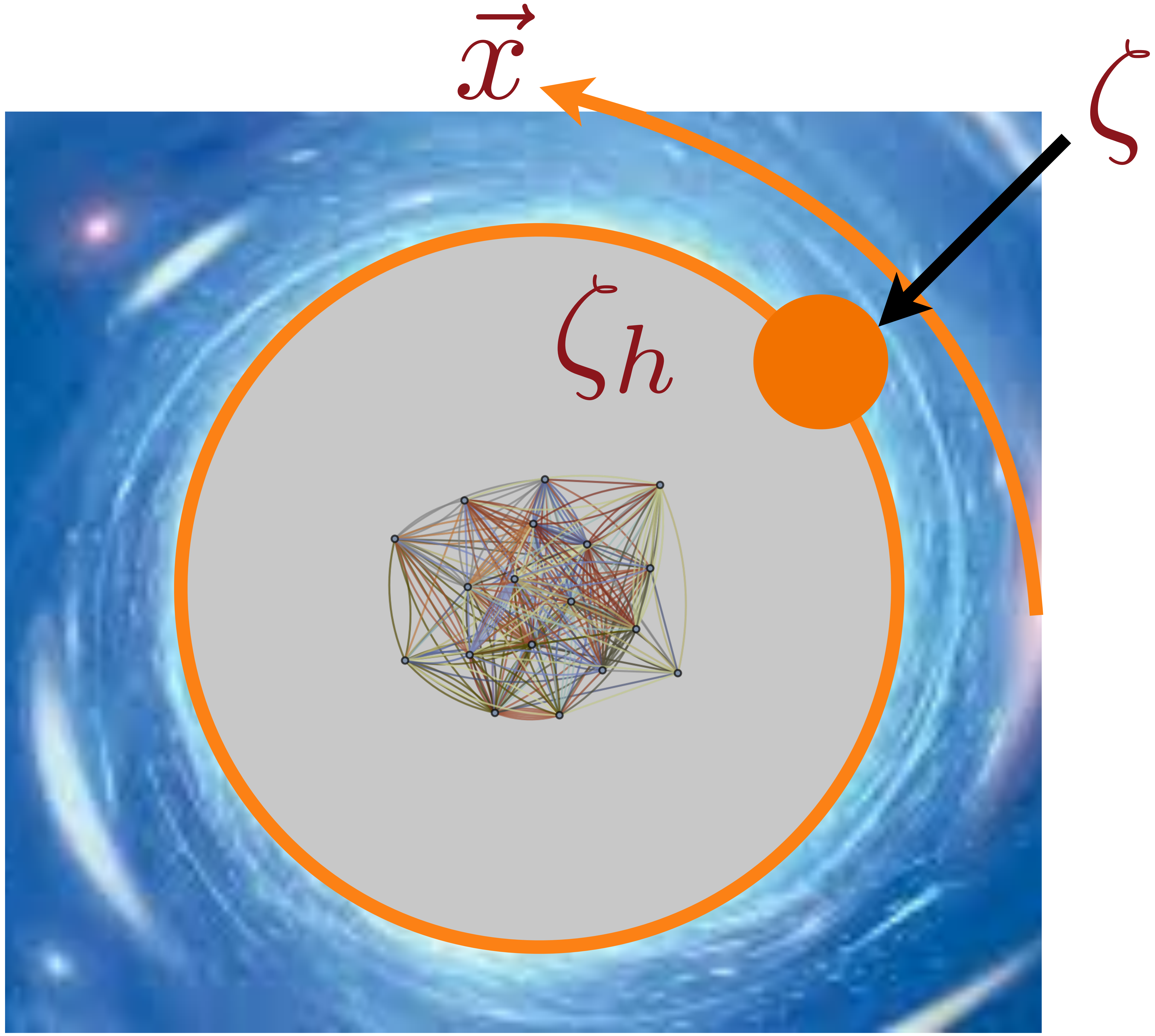

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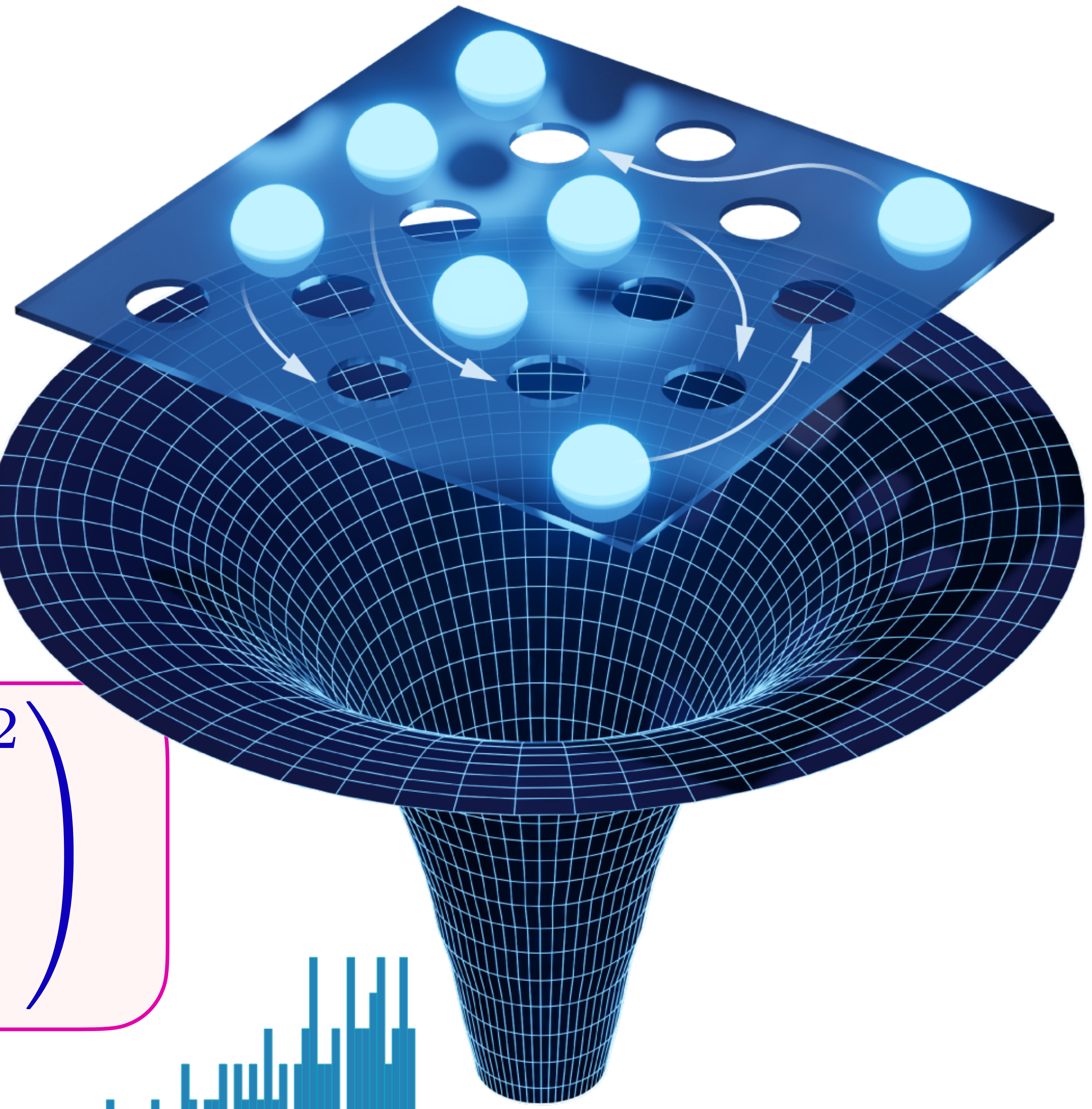
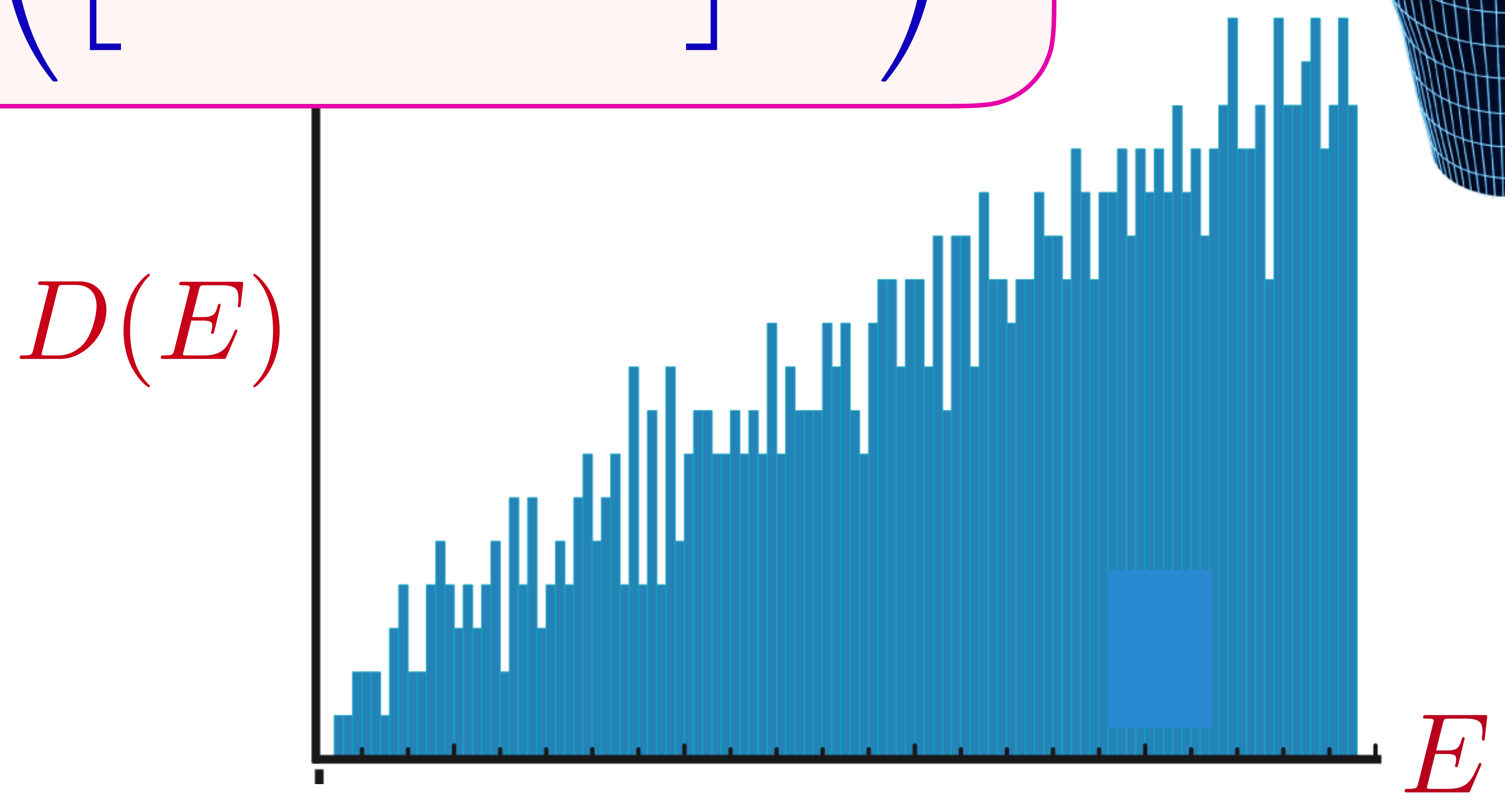
Quantum simulation of charged black holes by the SYK model



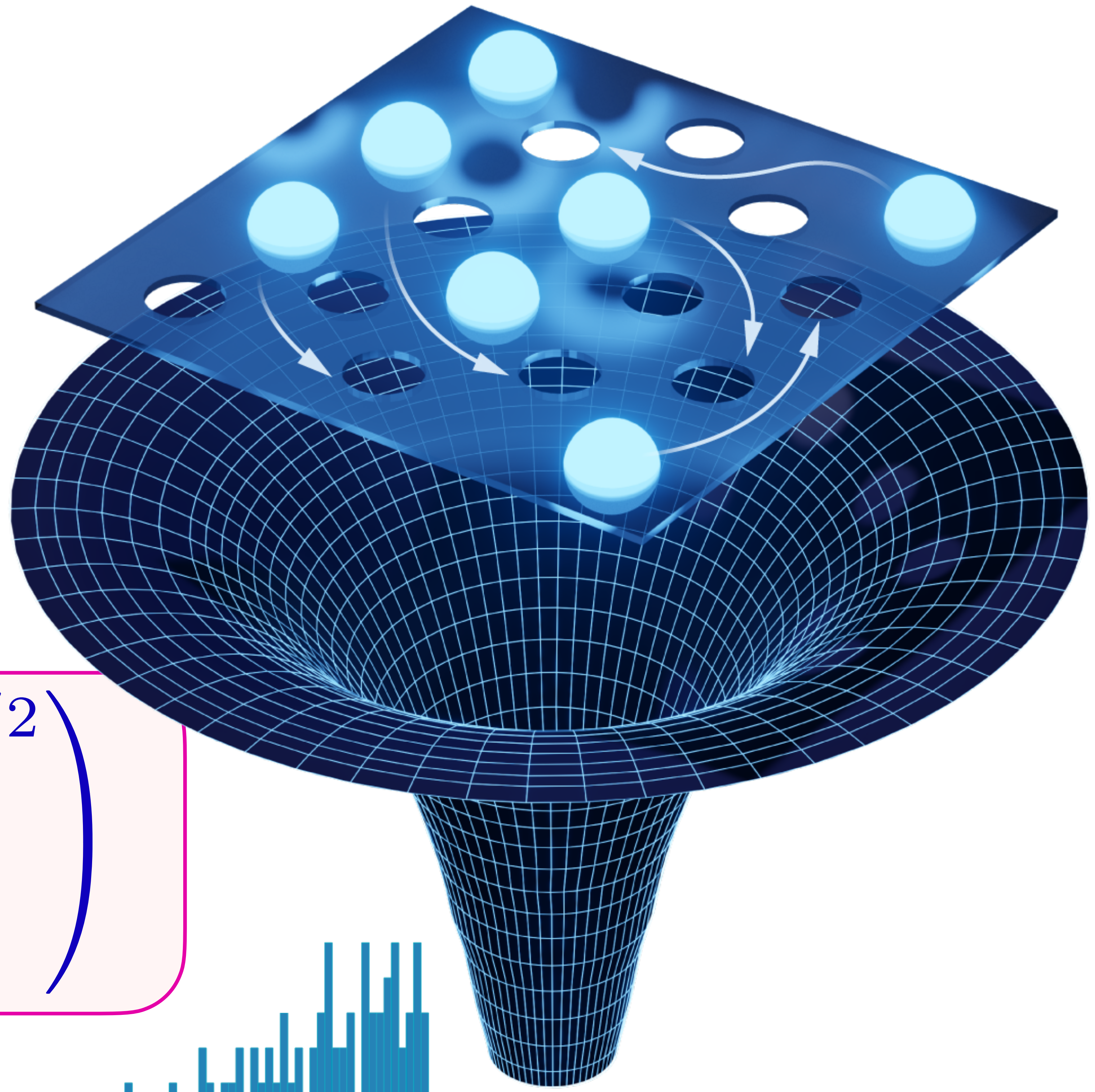
- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



Quantum simulation of charged black holes by the SYK model



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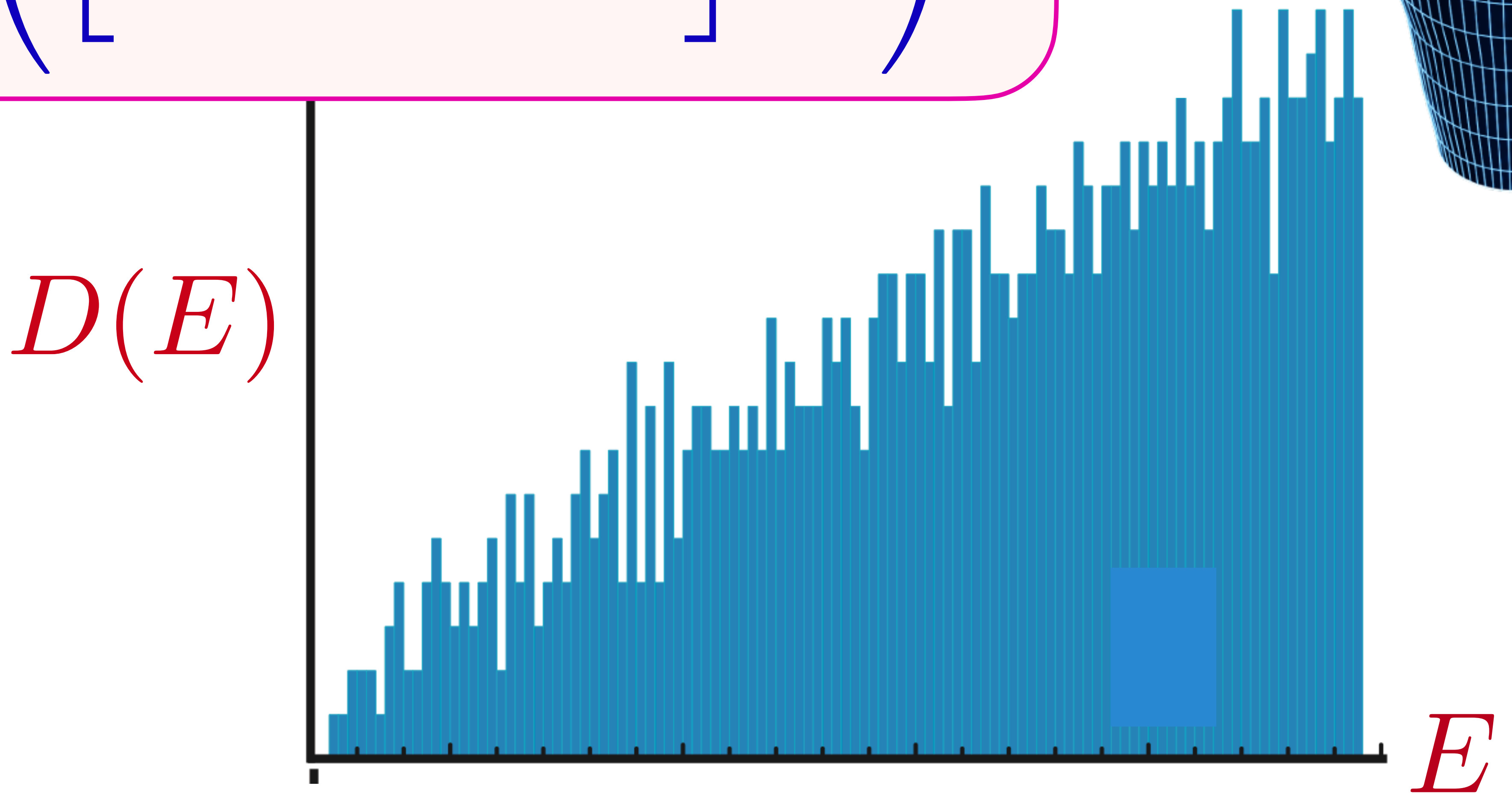

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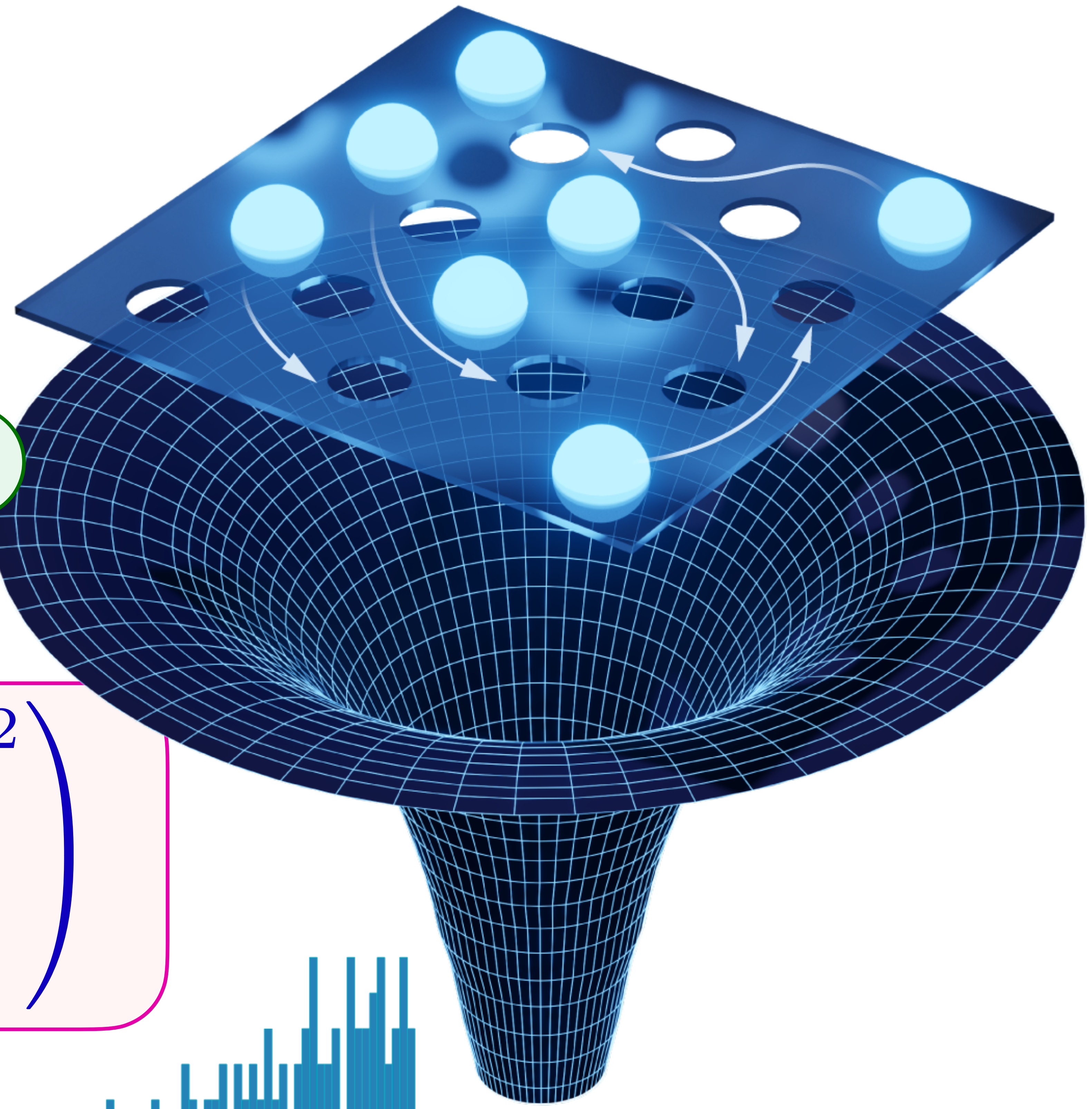

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Developments from the SYK model

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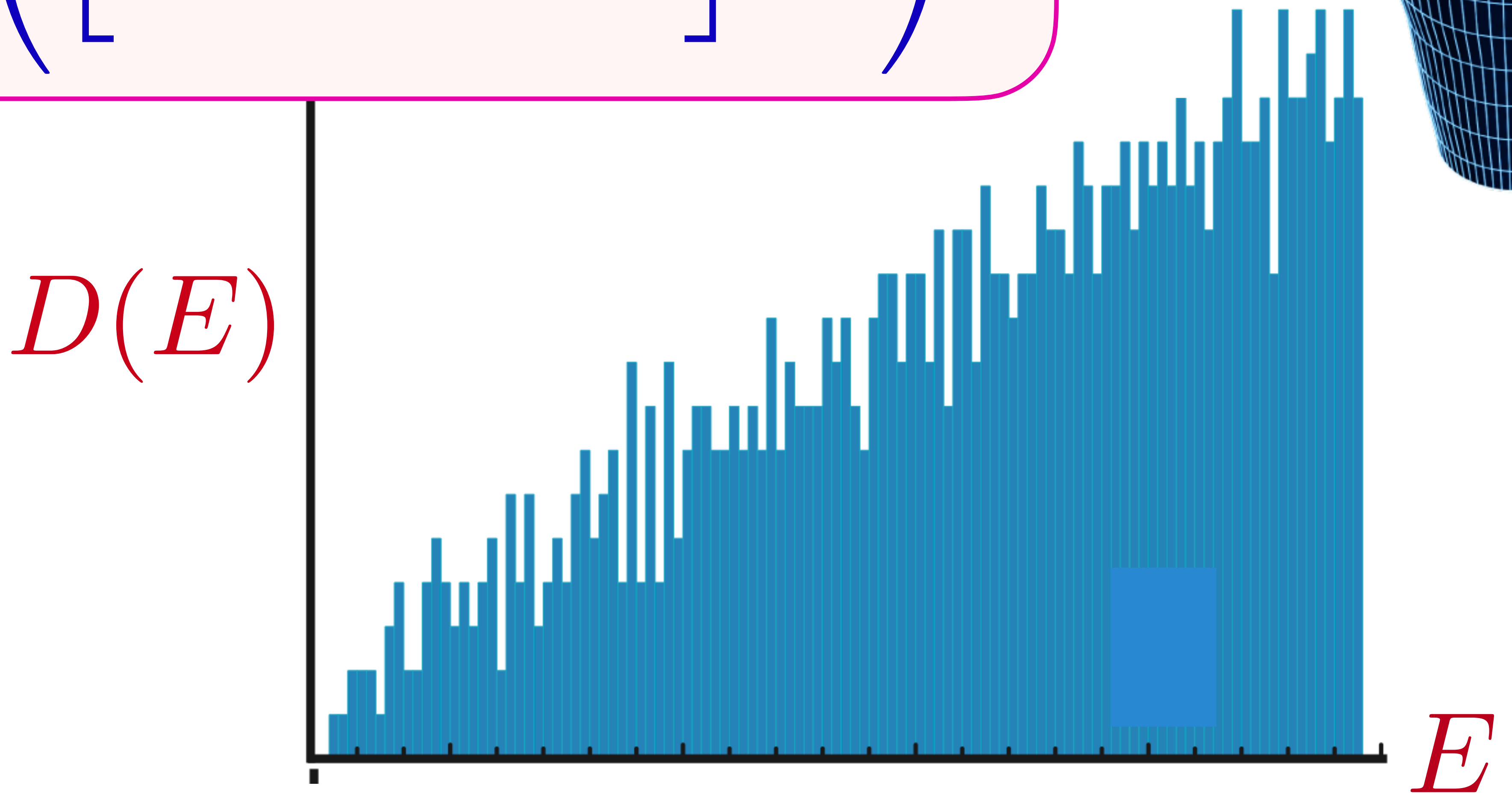

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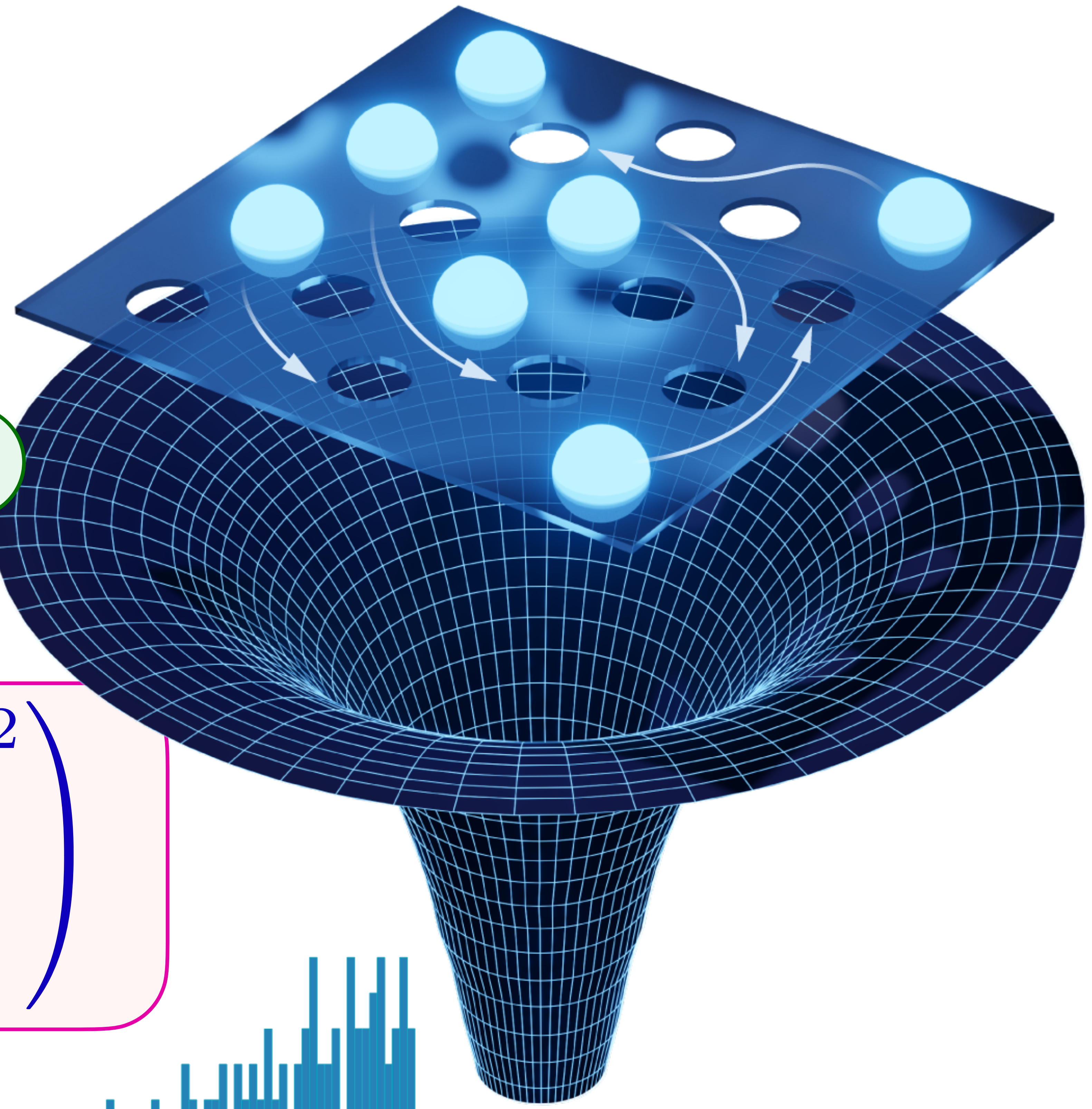

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Iliesiu, Murthy, Turiaci (2022) | Bekenstein-Hawking | Developments from the SYK model

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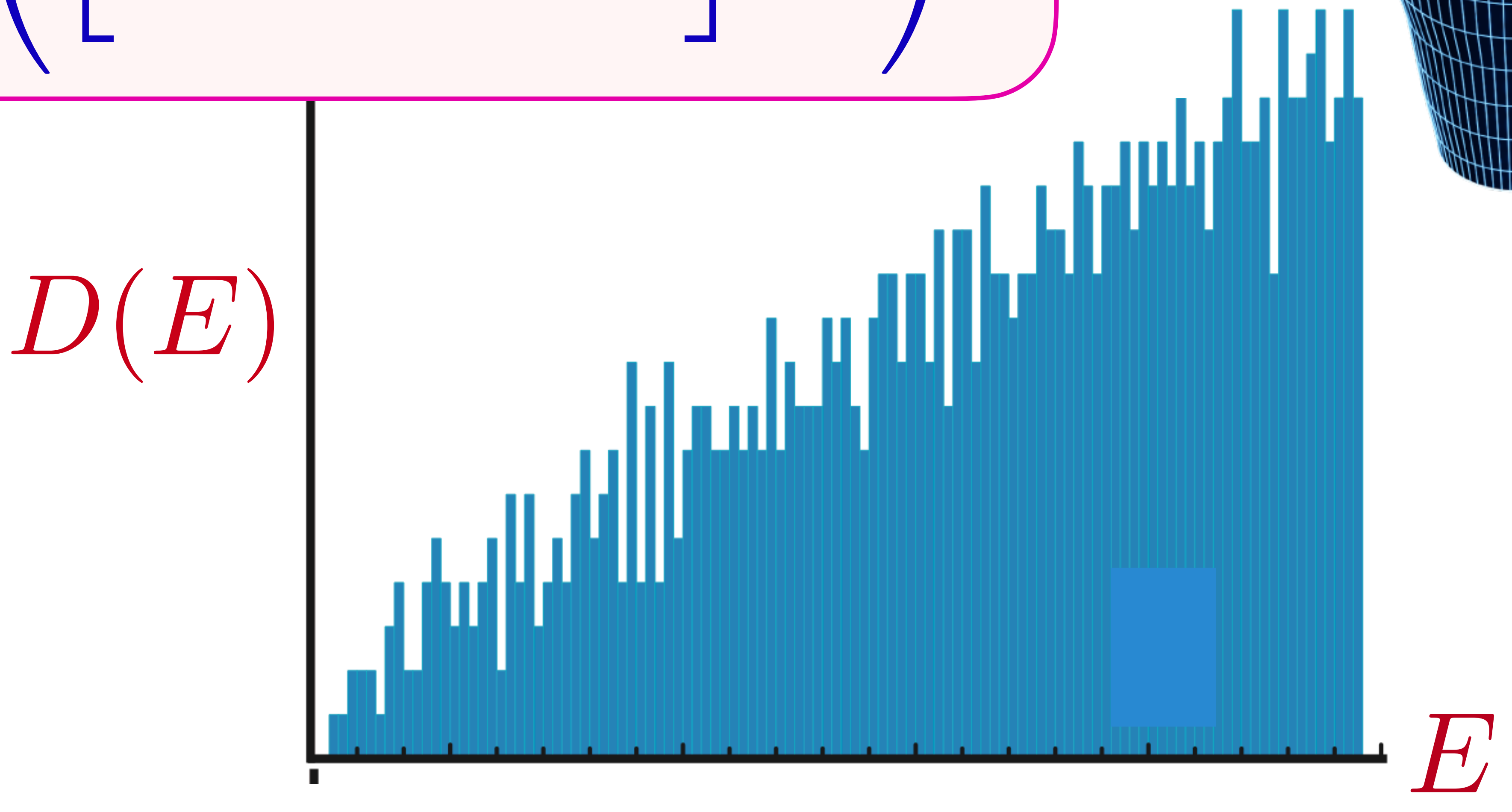


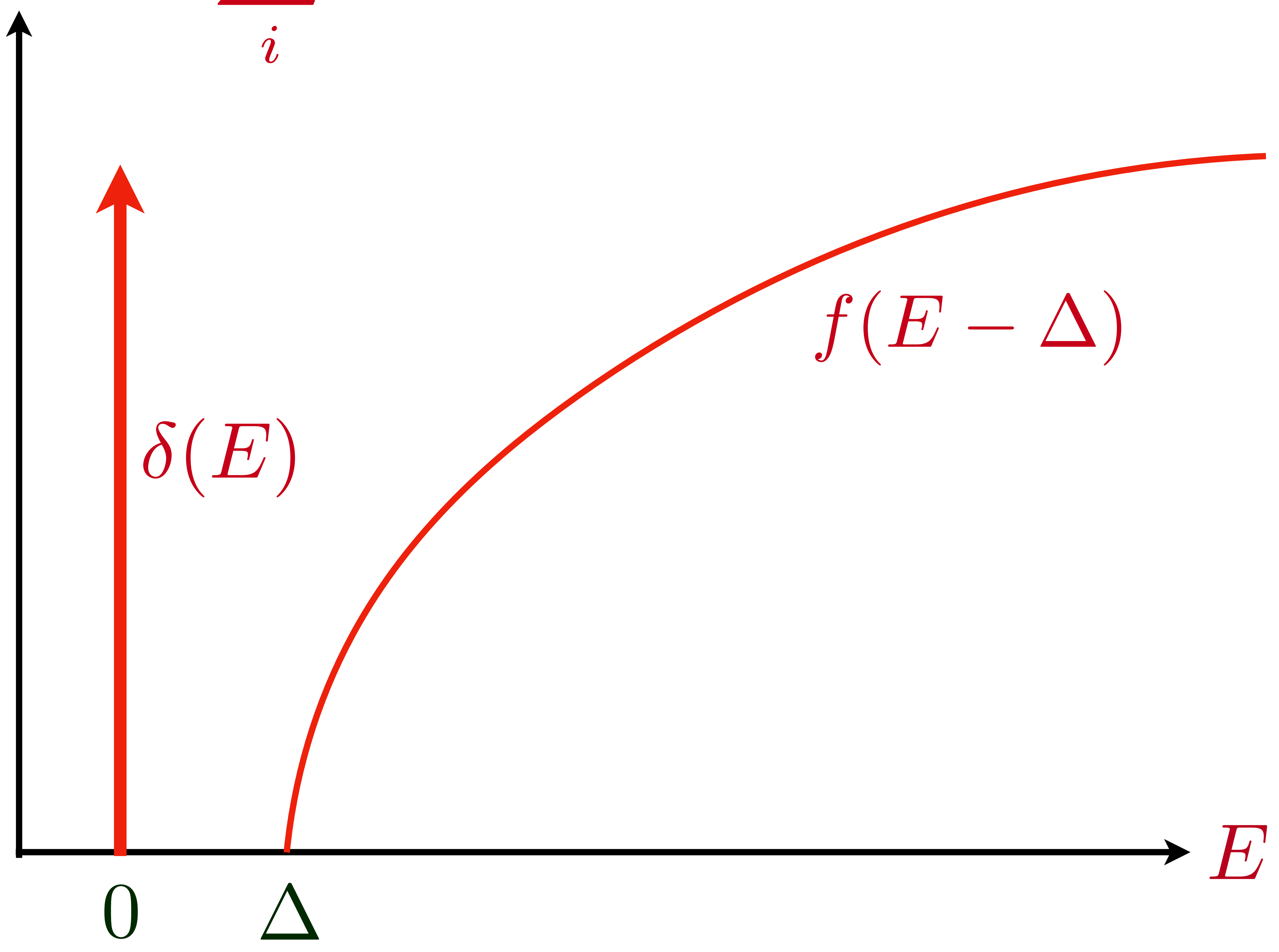
String theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots$$

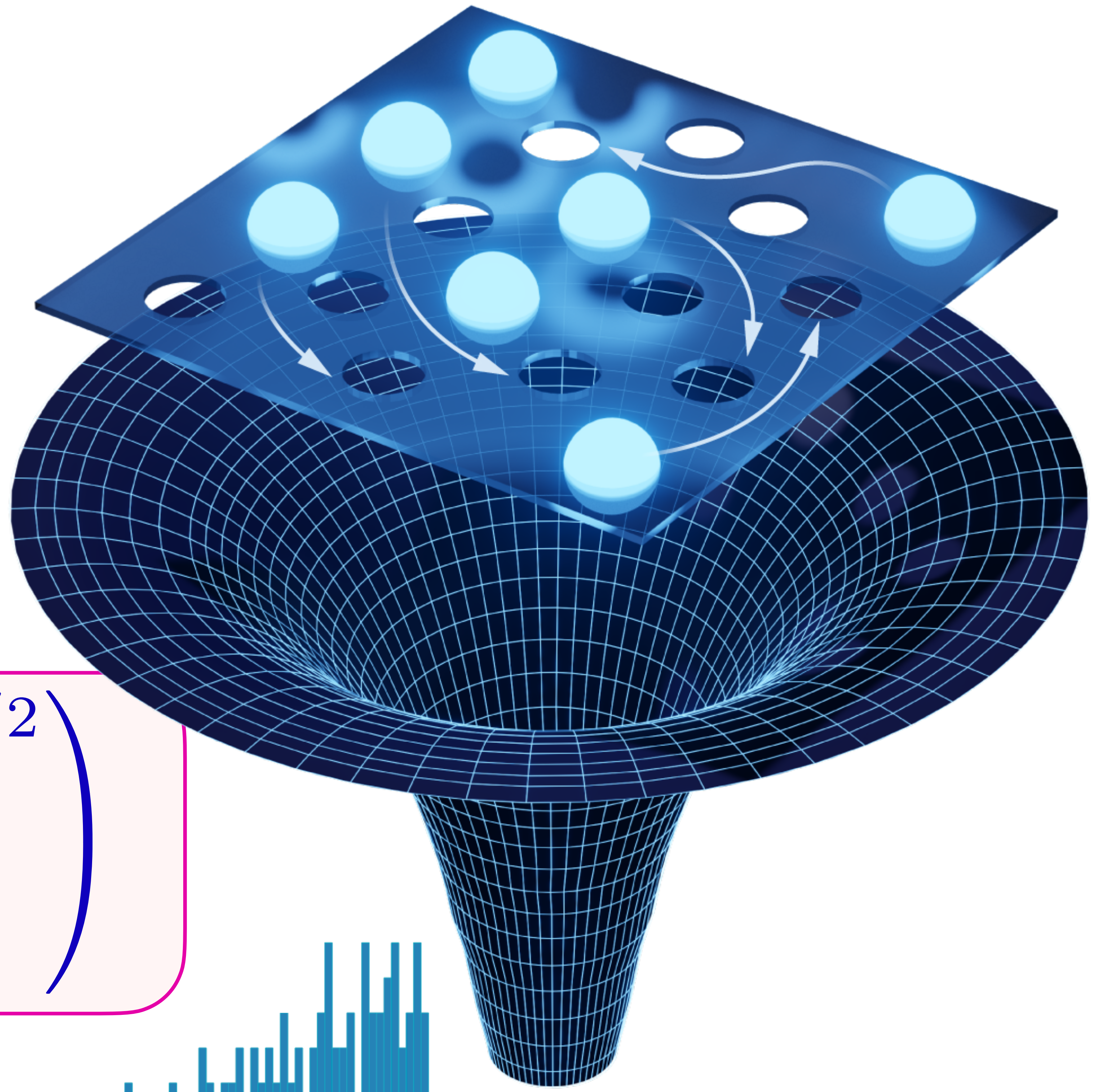
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Quantum simulation of charged black holes by the SYK model



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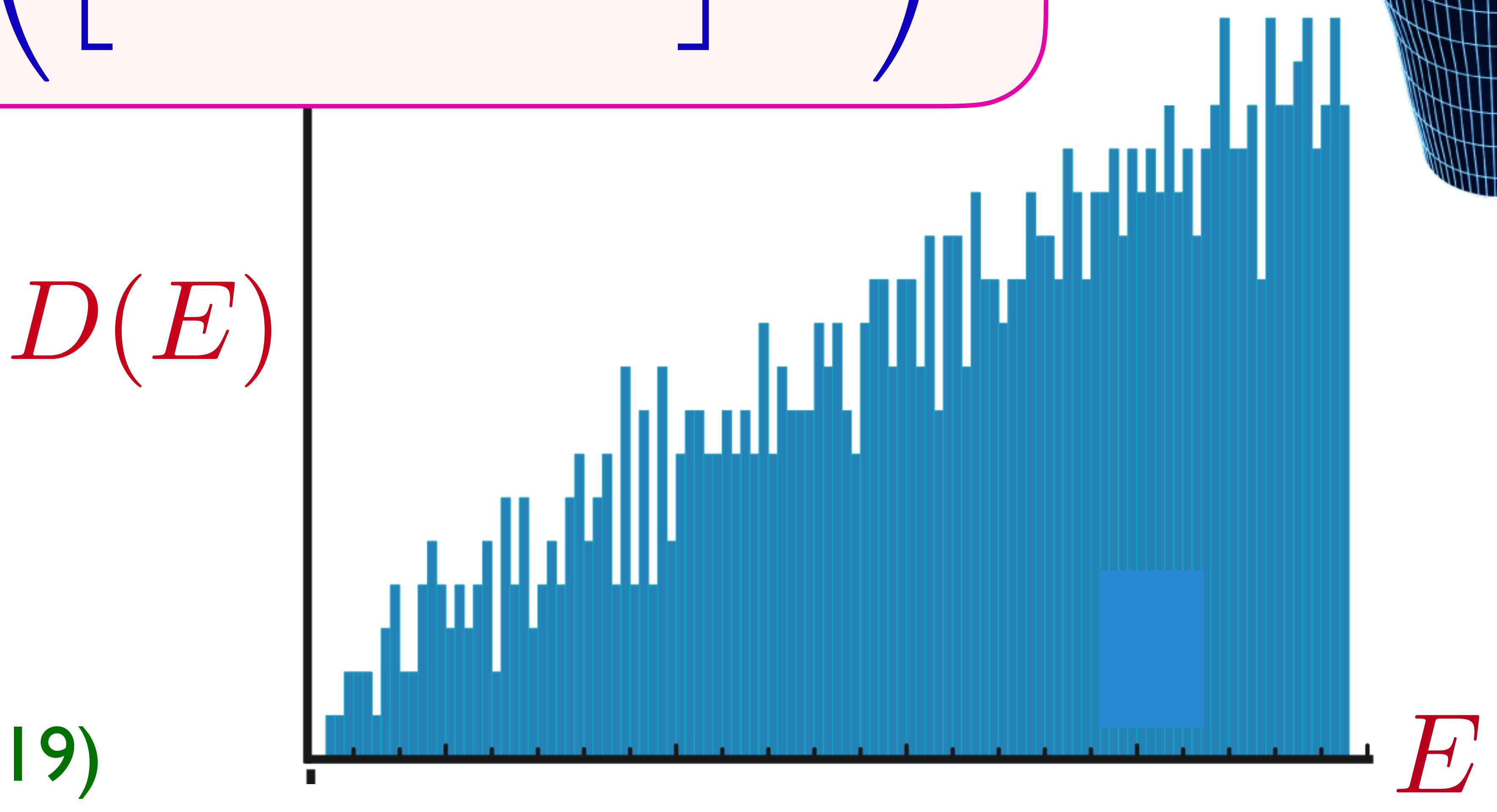
- 'Wormhole' contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.



Saad, Shenker, Stanford (2019)