

Planckian Dynamics

$$\tau_r \geq C \frac{\hbar}{k_B T} \quad (\text{SS, Quantum Phase Transitions, 1999})$$

1. SYK as a solvable model of quantum matter without quasiparticles
2. Quantum critical dynamics of the Ising model in 2 spatial dimensions
3. Strange metal from metallic quantum phase transitions with spatial disorder

School on Quantum Dynamics of Matter, Light and Information

ICTP, Trieste

August 18, 19, 2025

Subir Sachdev

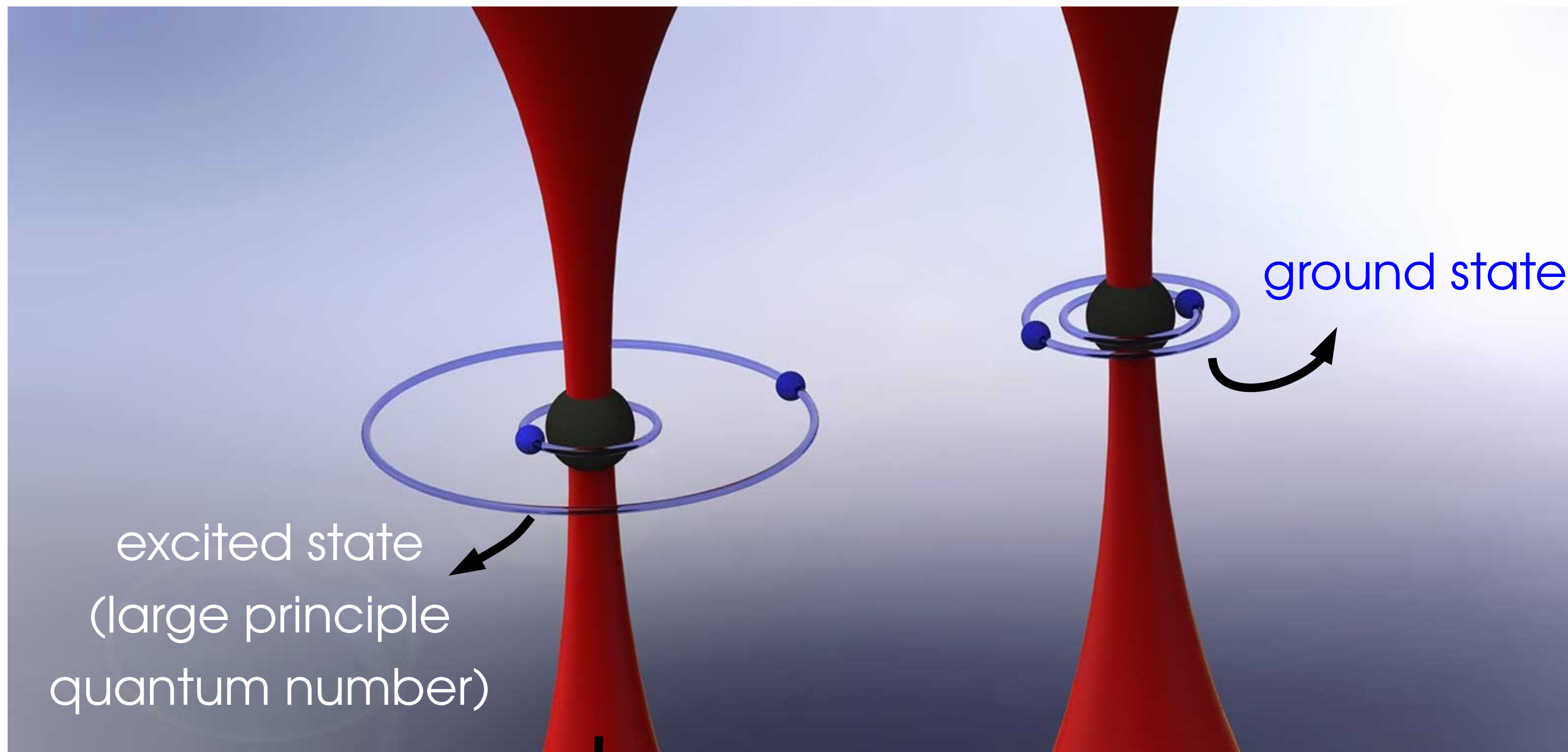


Quantum critical dynamics of the Ising model in 2 spatial dimensions

School on Quantum Dynamics of Matter, Light and Information
ICTP, Trieste
August 19, 2025

Subir Sachdev





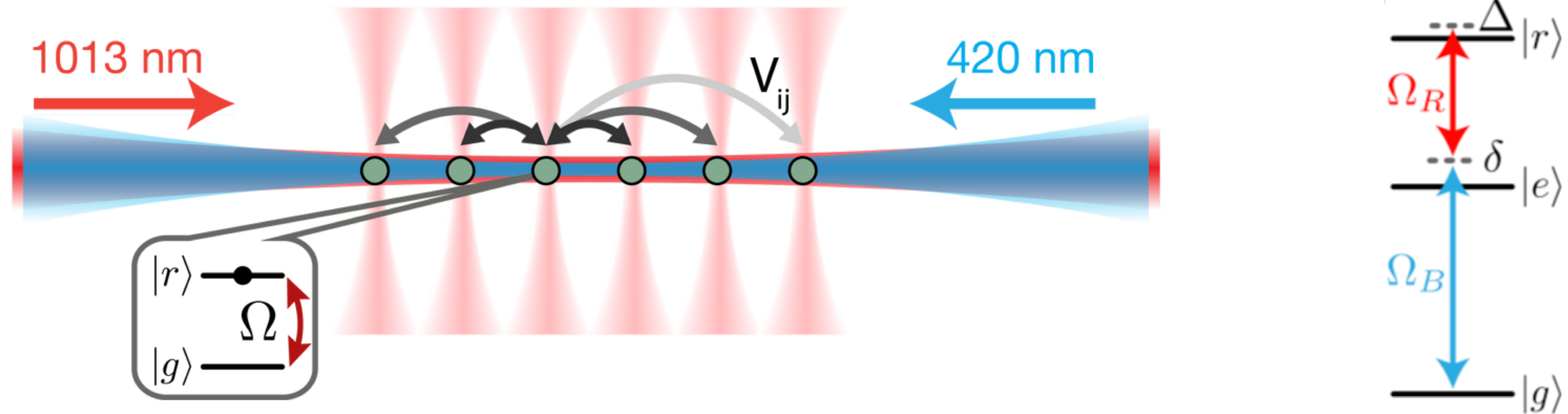
$$V_{|i-j|} \sim |i-j|^{-6}$$

optical tweezer (traps atom)

Fig: <https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms>

$$H_{\text{Ryd}} = \sum_i \left[\frac{\Omega}{2} (|g\rangle\langle r| + |r\rangle\langle g|)_i - \Delta |r\rangle\langle r| \right] + \sum_{(i,j)} V_{|i-j|} \left(|r\rangle\langle r|_i \otimes |r\rangle\langle r|_j \right)$$

QPTs in a Rydberg quantum simulator



$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} (b_j + b_j^\dagger) - \Delta n_j \right] + \sum_{i < j} V_{|i-j|} n_i n_j \quad V_{|i-j|} \sim \frac{1}{|r_i - r_j|^6}$$

$$n_j \equiv b_j^\dagger b_j$$

$$n_j = 0, 1 \quad \text{'hard core' bosons}$$

The PXP model: ($V_1 = \infty, V_{j \geq 2} = 0$, $\mathcal{P}$ projects out nearest-neighbor boson states)

$$\mathcal{H}_{\text{PXP}} = \sum_j \left[\frac{\Omega}{2} \mathcal{P} \left(b_j + b_j^\dagger \right) \mathcal{P} - \Delta b_j^\dagger b_j \right]$$

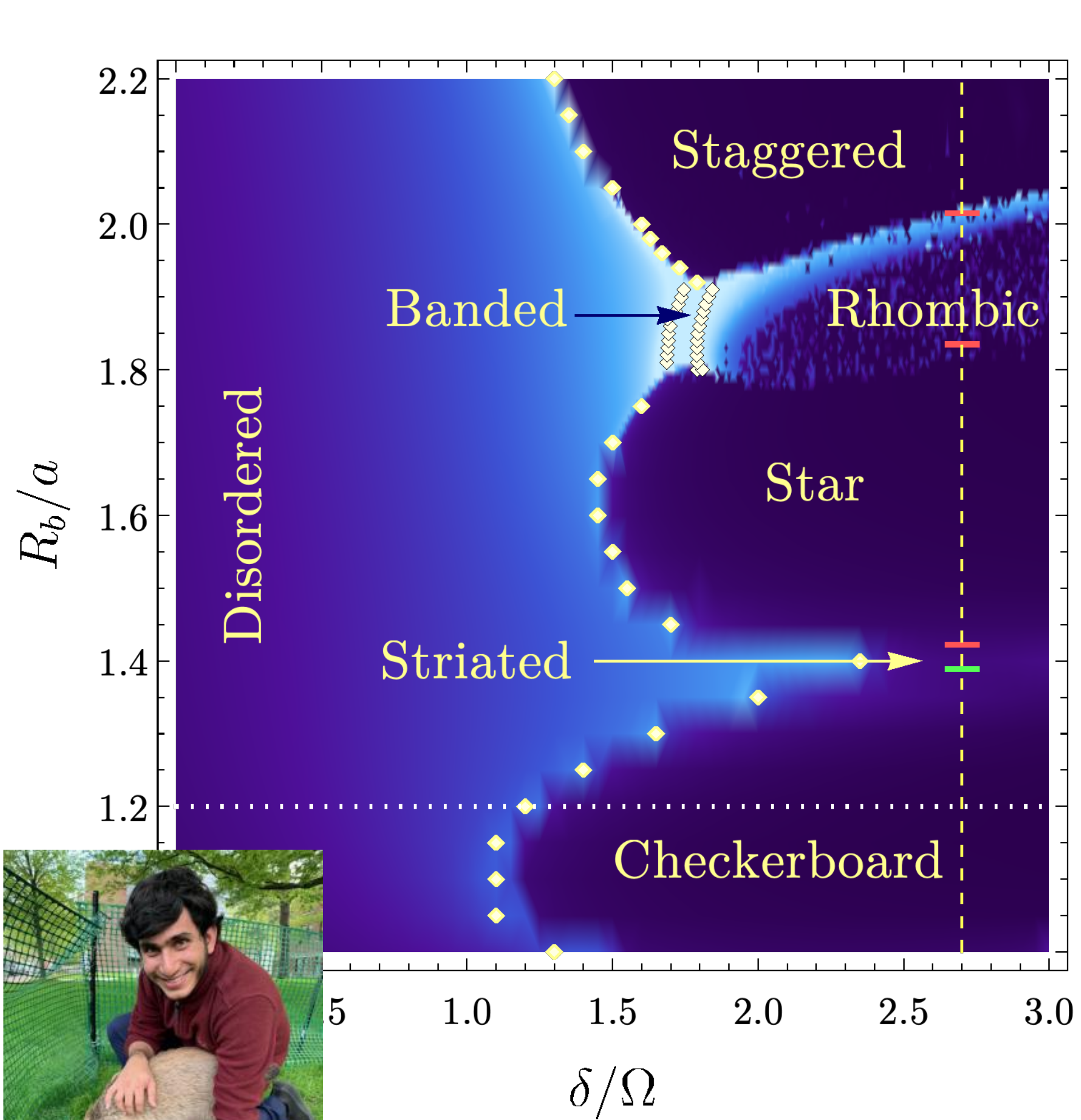
These models were motivated by ‘tilted lattices’ of bosonic atoms, and predicted a $\mathbb{Z}_2$ quantum transition which was observed in J. Simon, W. S. Bakr, Ruichao Ma, M. Eric Tai, P. M. Preiss, M. Greiner, Nature **472**, 307 (2011).

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i < j} V_{|i-j|} n_i n_j \qquad V_{|i-j|} \sim \frac{1}{|r_i - r_j|^6}$$

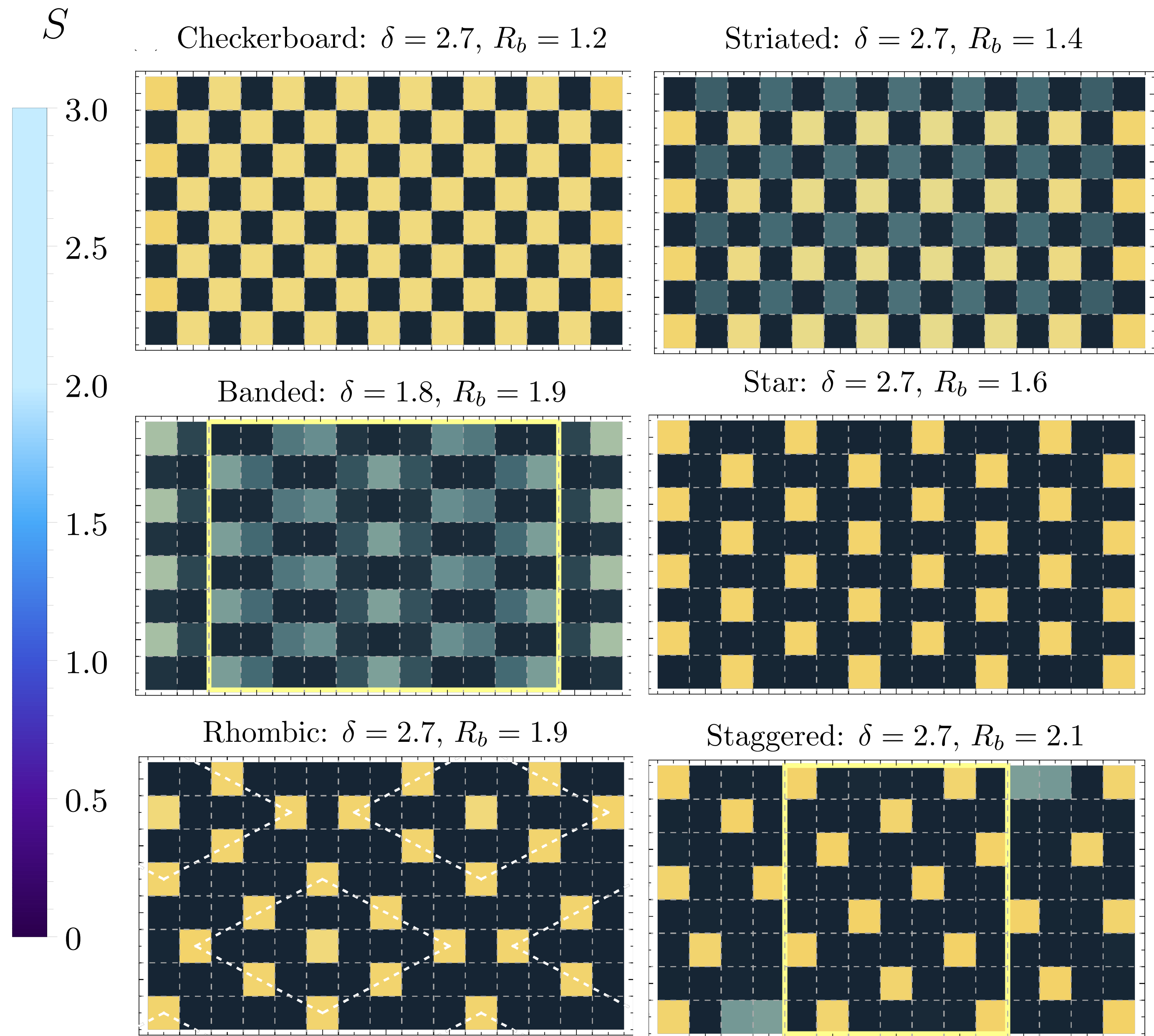
$$n_j \equiv b_j^\dagger b_j$$

$$n_j = 0, 1 \quad \text{‘hard core’ bosons}$$

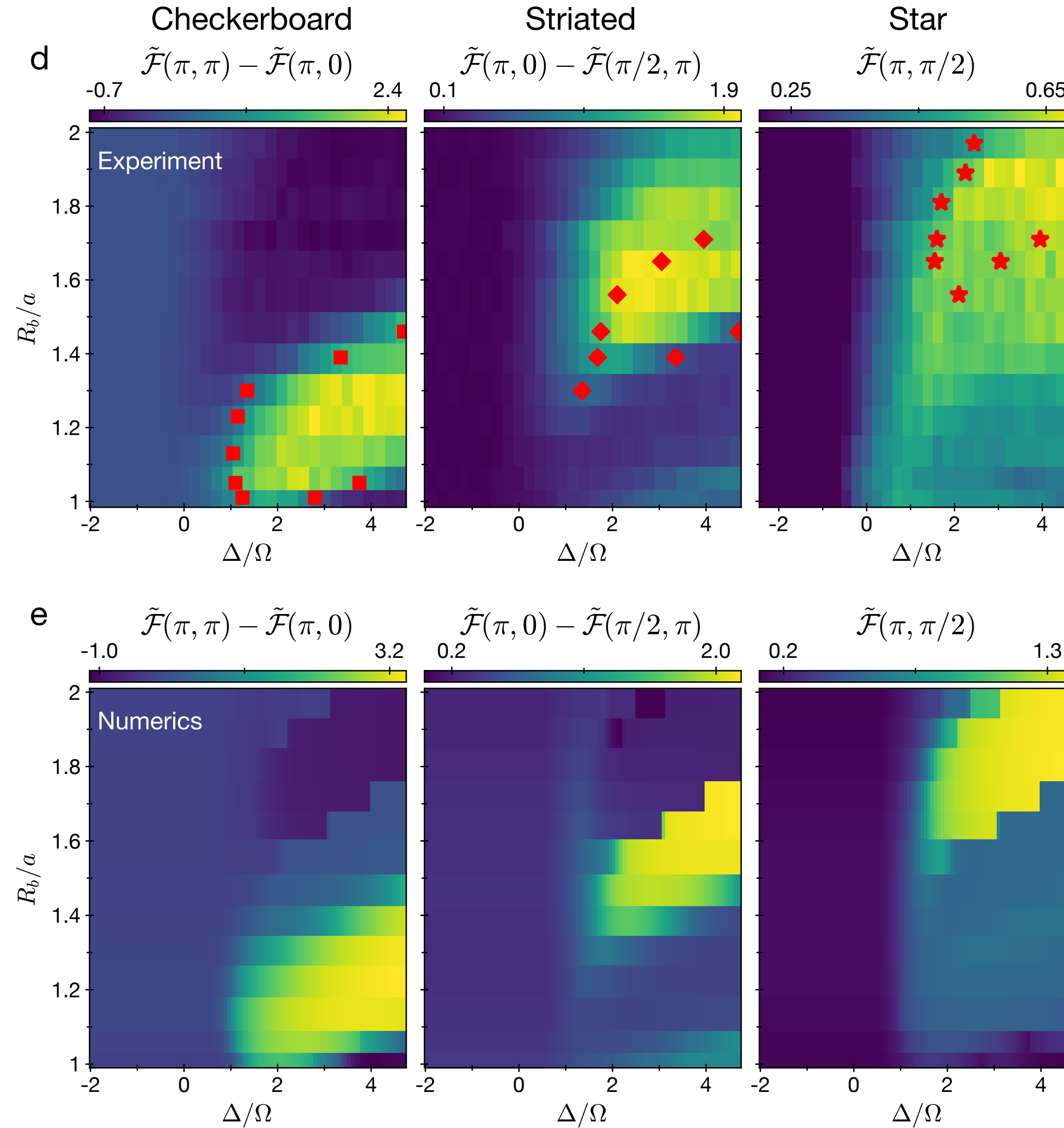
Rydberg atoms on the square lattice: theory



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PRL **124**, 103601 (2020)

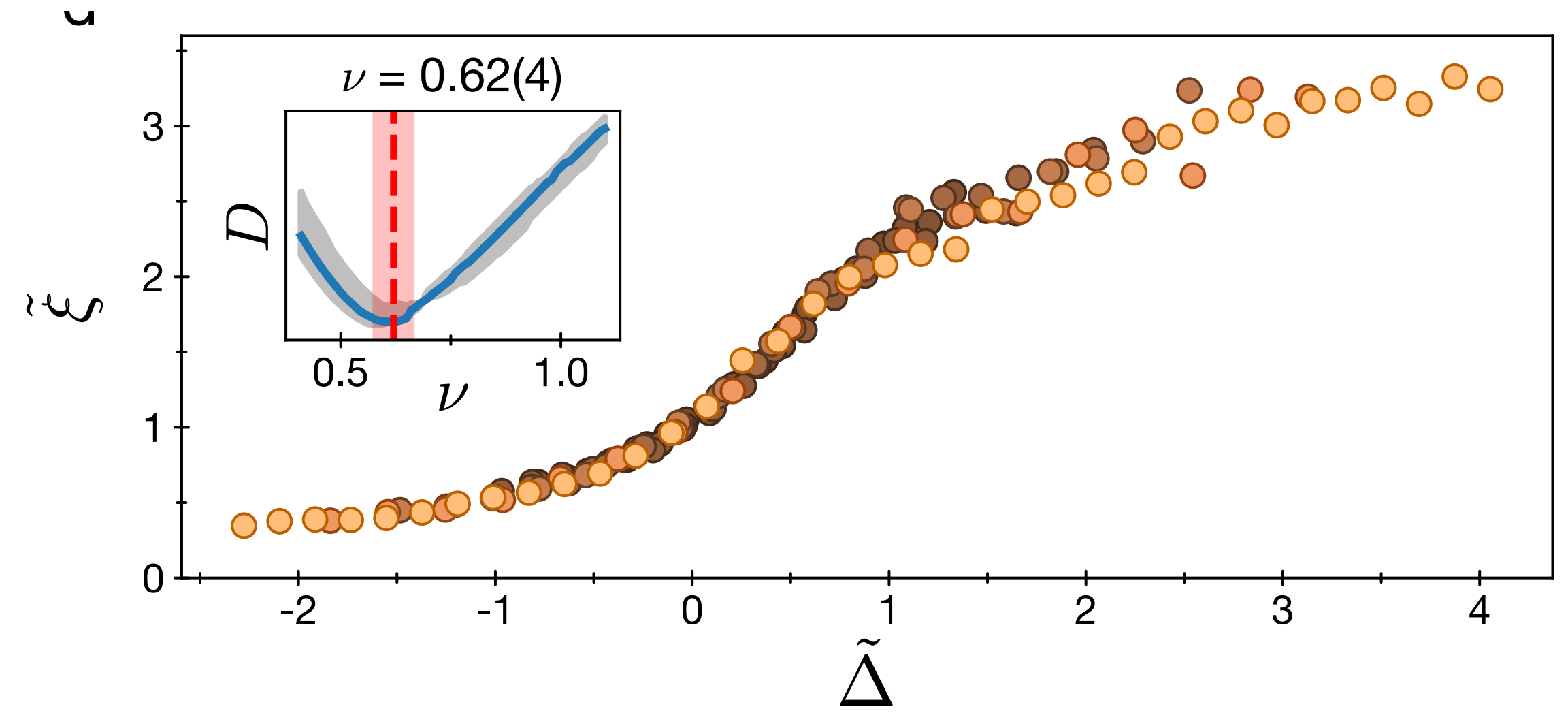
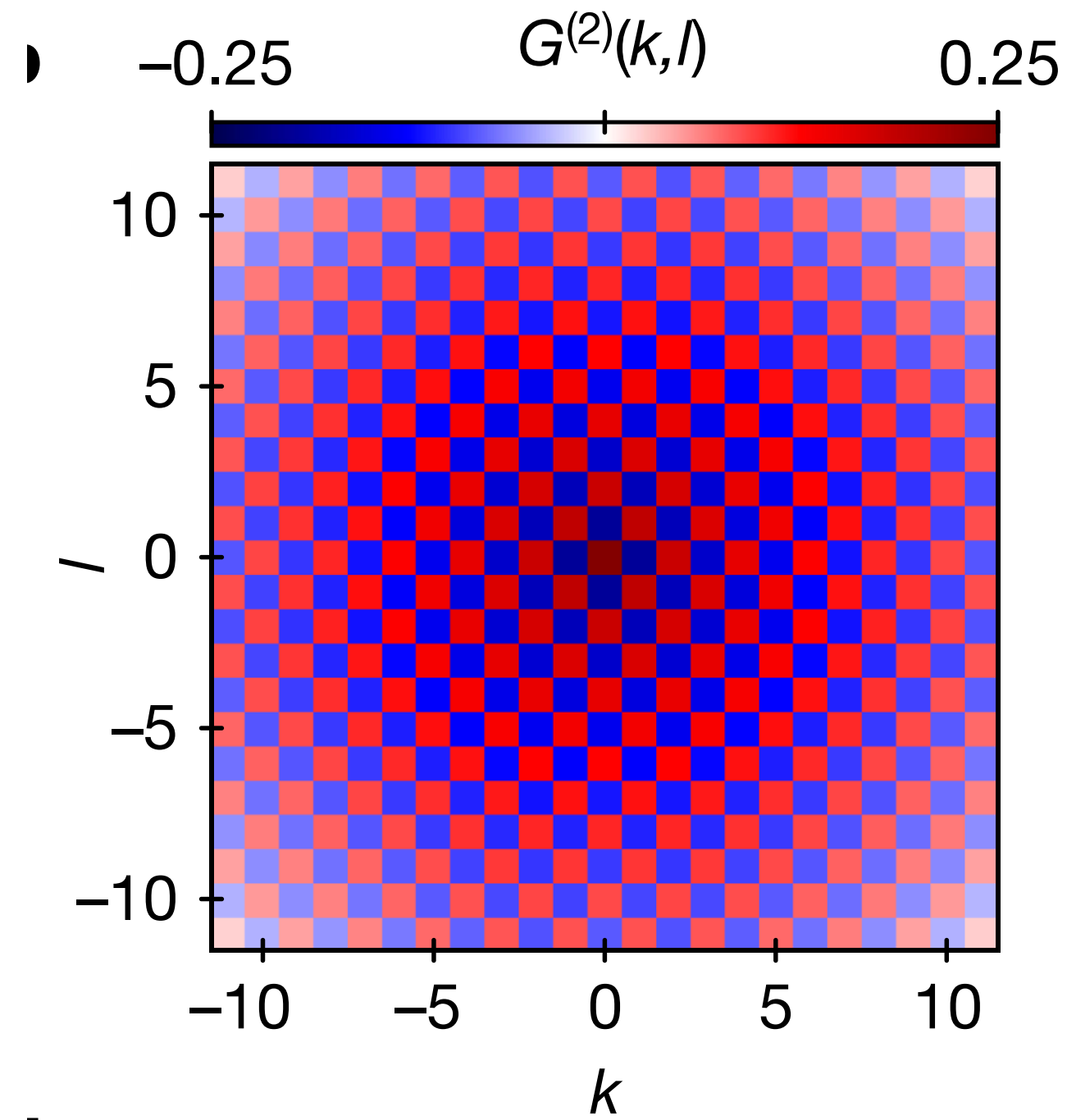
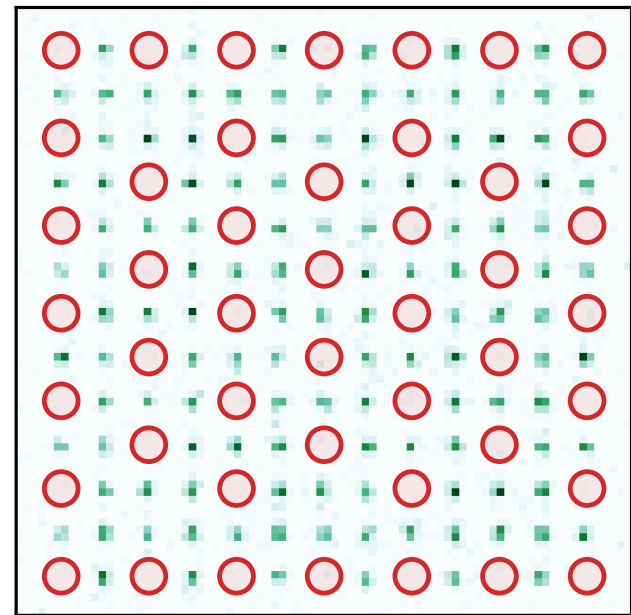
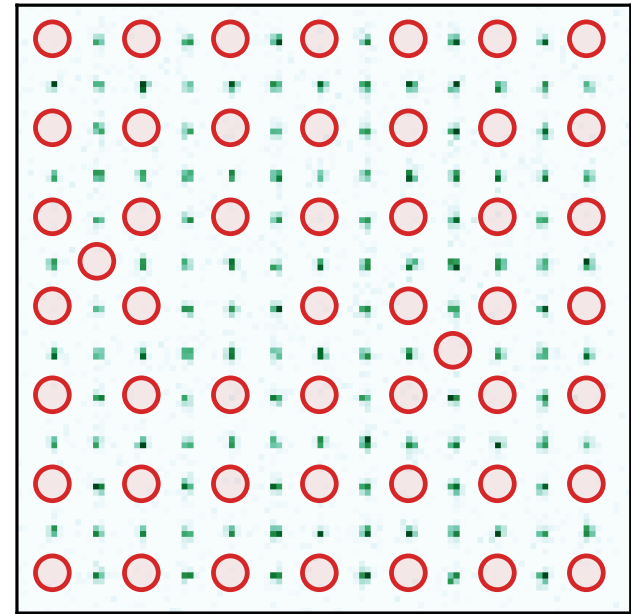
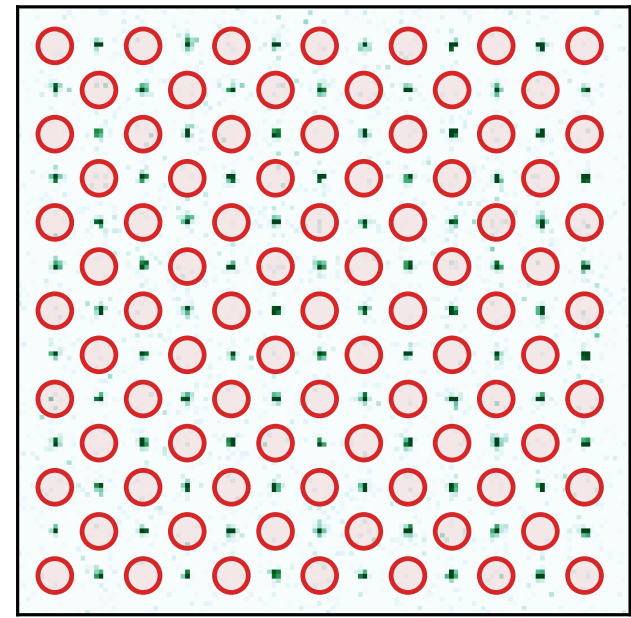


Rydberg atoms on the square lattice: experiment



Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator, Sepehr Ebadi, Tout T. Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Samajdar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, Nature **595**, 227 (2021); Pascal Scholl et al. Nature **595**, 233 (2021)

Rydberg atoms on the square lattice: experiment

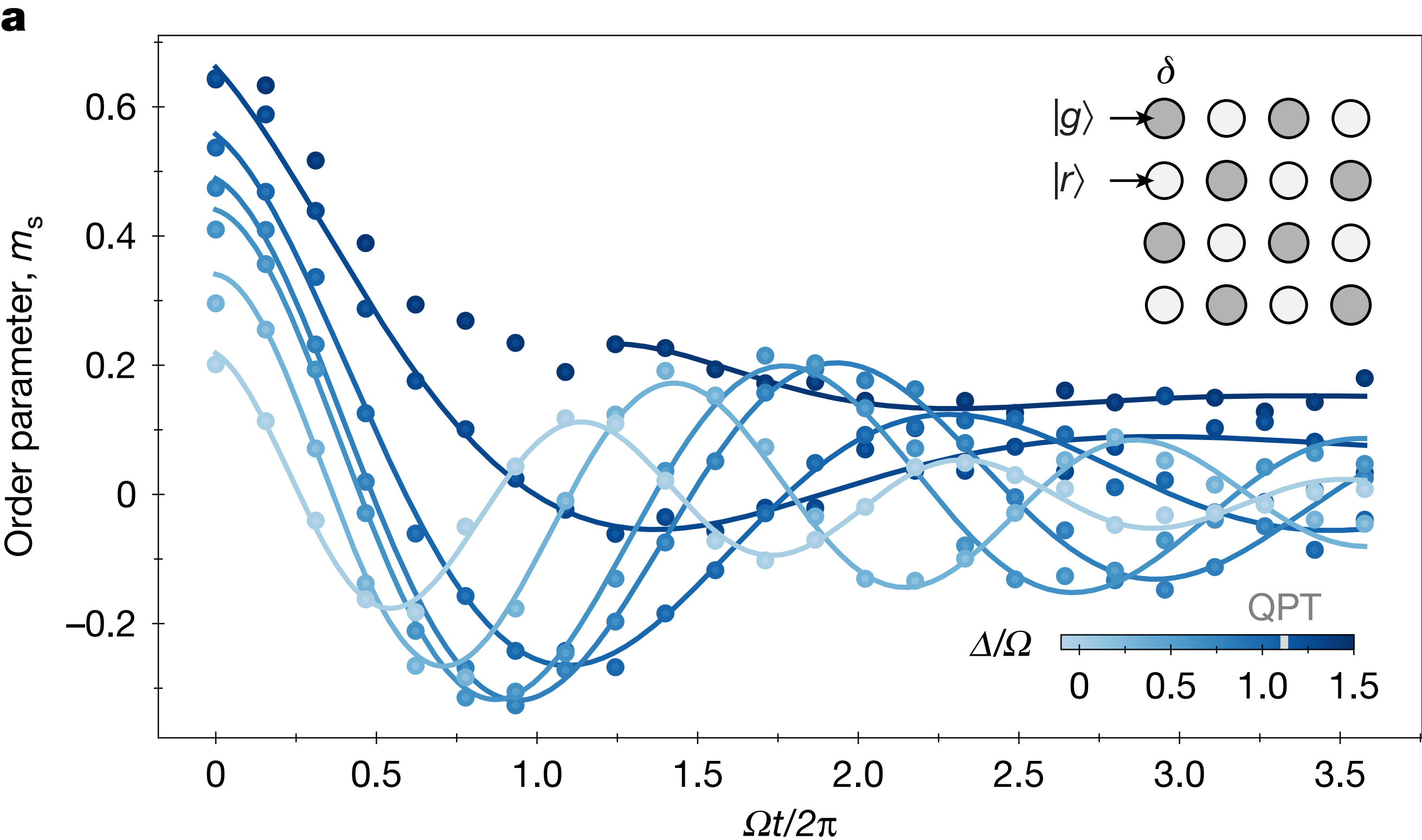


First observation of Ising
quantum phase transition
in 2+1 dimensions

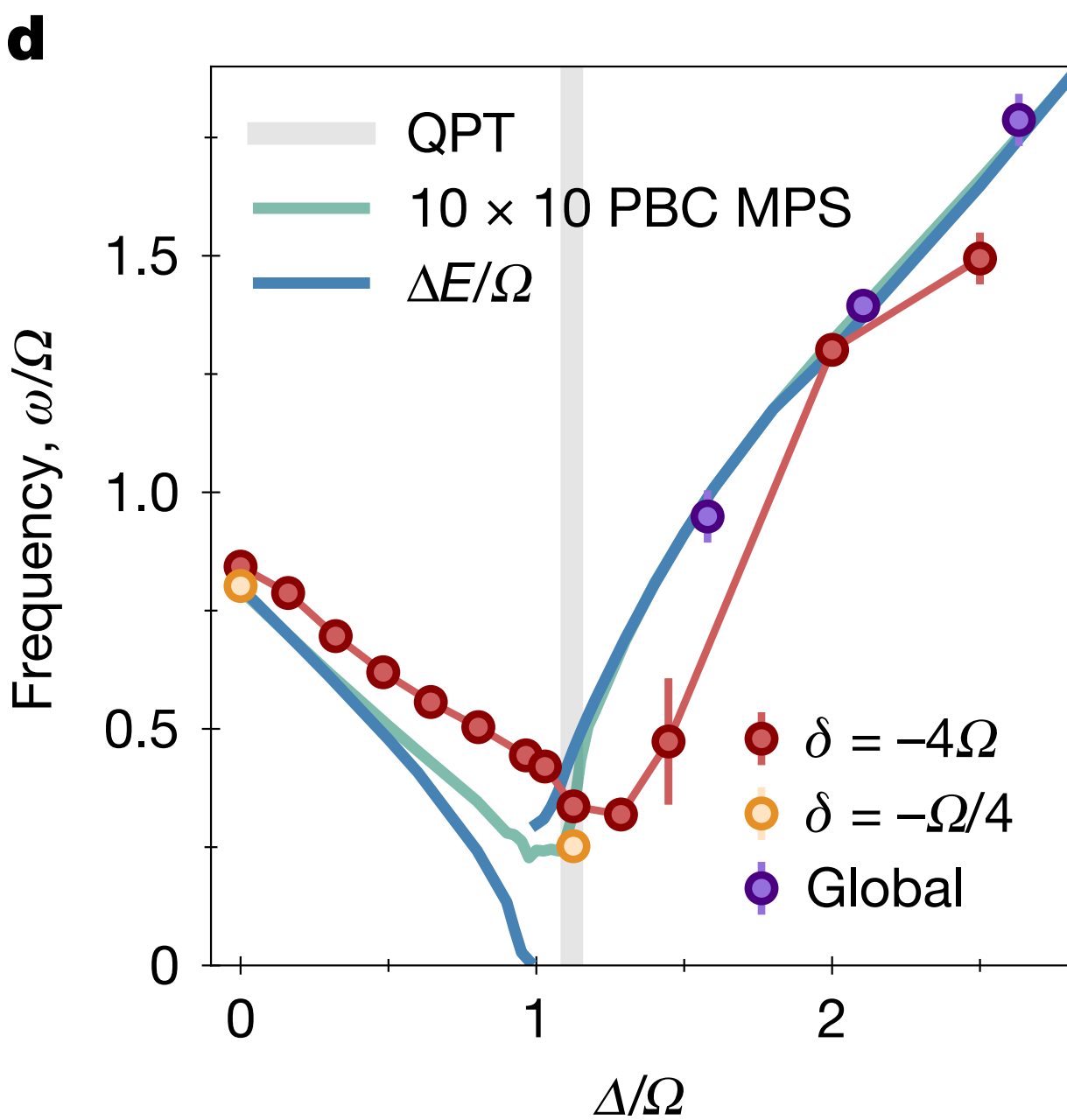
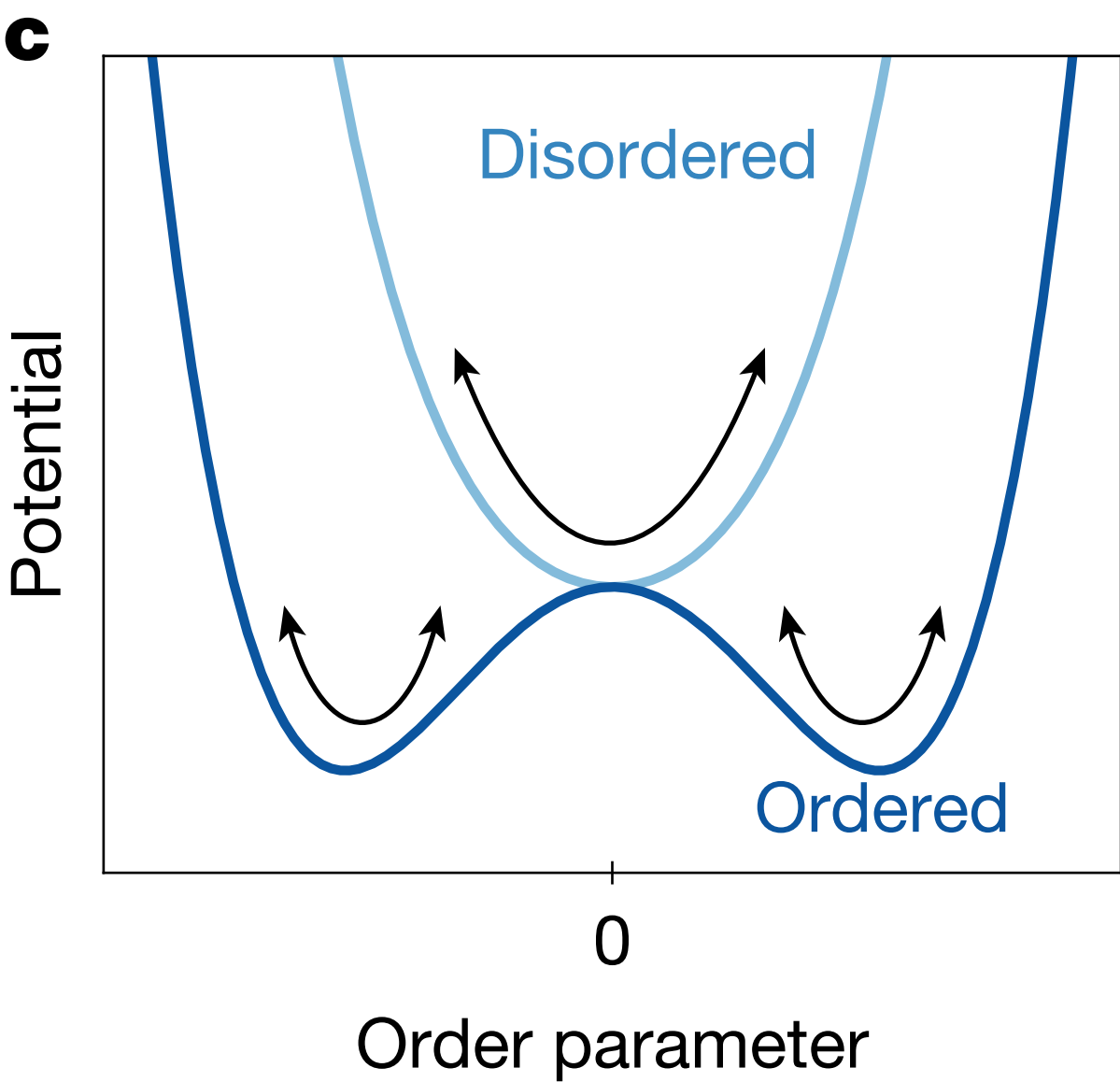
Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator, Sepehr Ebadi, Tout T. Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Samajdar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, Nature **595**, 227 (2021); Pascal Scholl et al. Nature **595**, 233 (2021)

Quantum coarsening and collective dynamics on a programmable simulator

Tom Manovitz^{1,8}, Sophie H. Li^{1,8}, Sepehr Ebadi^{1,7,8}, Rhine Samajdar^{2,3}, Alexandra A. Geim¹, Simon J. Evered¹, Dolev Bluvstein¹, Hengyun Zhou^{1,4}, Nazli Ugur Koyluoglu^{1,5}, Johannes Feldmeier¹, Pavel E. Dolgirev¹, Nishad Maskara¹, Marcin Kalinowski¹, Subir Sachdev¹, David A. Huse², Markus Greiner¹, Vladan Vuletić⁶ & Mikhail D. Lukin¹✉



$$f(x) \equiv \frac{\omega(\Delta - \Delta_c = x)}{\omega(\Delta_c - \Delta = x)} \approx 1.9$$



$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - Jg \sum_i X_i$$

$i \rightarrow d$ -dimensional hypercubic lattice

$X_i, Z_i \rightarrow$ Pauli operators :

$$J > 0, g > 0.$$

Determine phase diagram as a function of g .

Will find a quantum phase transition at

$$g = g_c \text{ for all } d.$$

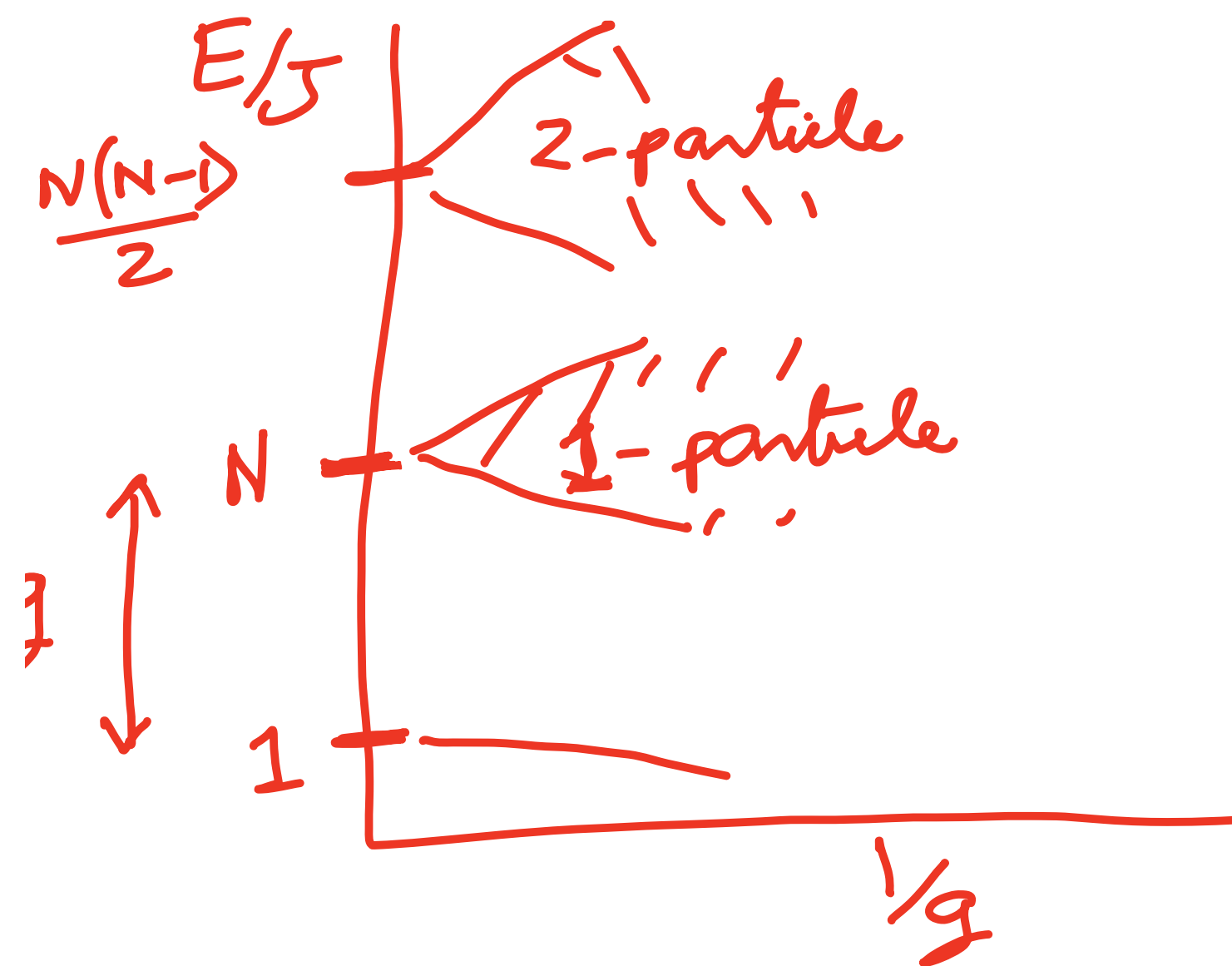
(i) $g \gg 1$.

Unique ground state $|\rightarrow \rightarrow \rightarrow \rightarrow \dots\rangle$

"Quantum paramagnet" ; $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
 $|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$

N -fold degenerate first excited states (N sites)
 at $g = \infty$

$|i\rangle = |\rightarrow \rightarrow \rightarrow \leftarrow_i \rightarrow \rightarrow \rightarrow \dots\rangle$



Use degenerate perturbation theory in $1/g$.

1- particle states

$$|\vec{k}\rangle = \frac{1}{\sqrt{N}} \sum_i e^{i\vec{k} \cdot \vec{r}_i} |i\rangle$$

$$\frac{\varepsilon_k}{J} = -Ng + 2g - 2 \sum_{\alpha=1}^d \cos(k_\alpha a)$$

2- particle states

$$|\vec{k}_1, \vec{k}_2\rangle$$

Scattering and bound states...

(ii) $g \ll 1$

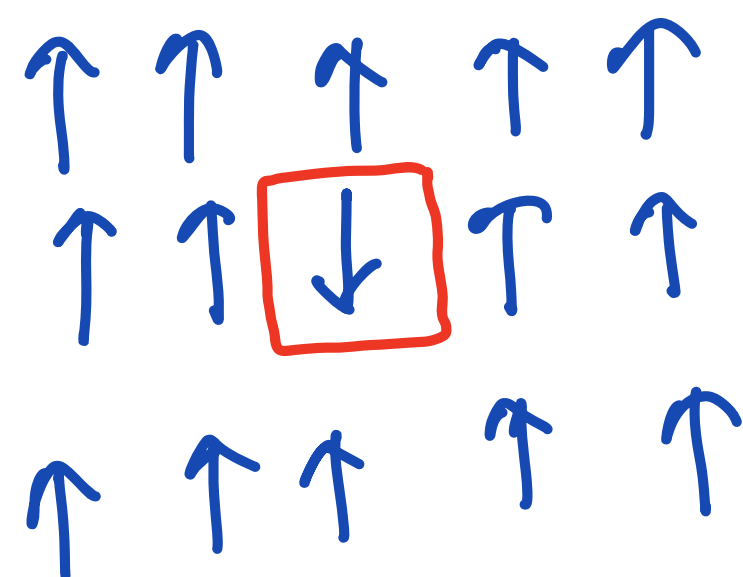
2-fold degenerate ferromagnetic ground states.

$$|\uparrow\uparrow\rangle = |\uparrow\uparrow\uparrow\uparrow\dots\rangle$$

$$|\downarrow\downarrow\rangle = |\downarrow\downarrow\downarrow\downarrow\dots\rangle$$

Cannot be smoothly connected to $g \gg 1$ at $N=\infty$
 $\Rightarrow$ must be a quantum phase transition(s) at intermediate g

First excited states



"Flipped spin"
particle

$$E = 2dJ$$

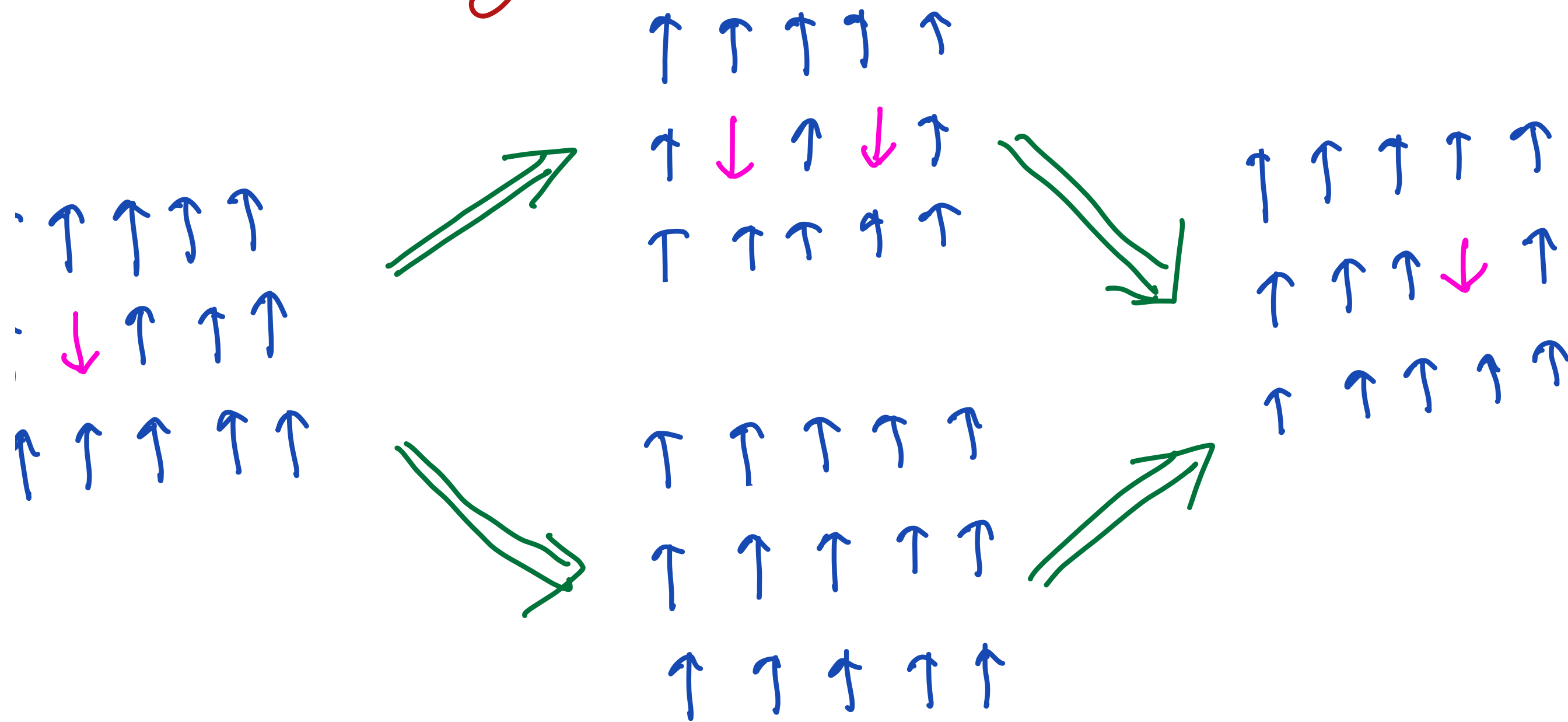
$d=1$ is special

$\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow \Rightarrow \uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow$

Flipped spin "fractionalizes" into
2 domain-wall particles.

Dispersion of flipped spin particle.

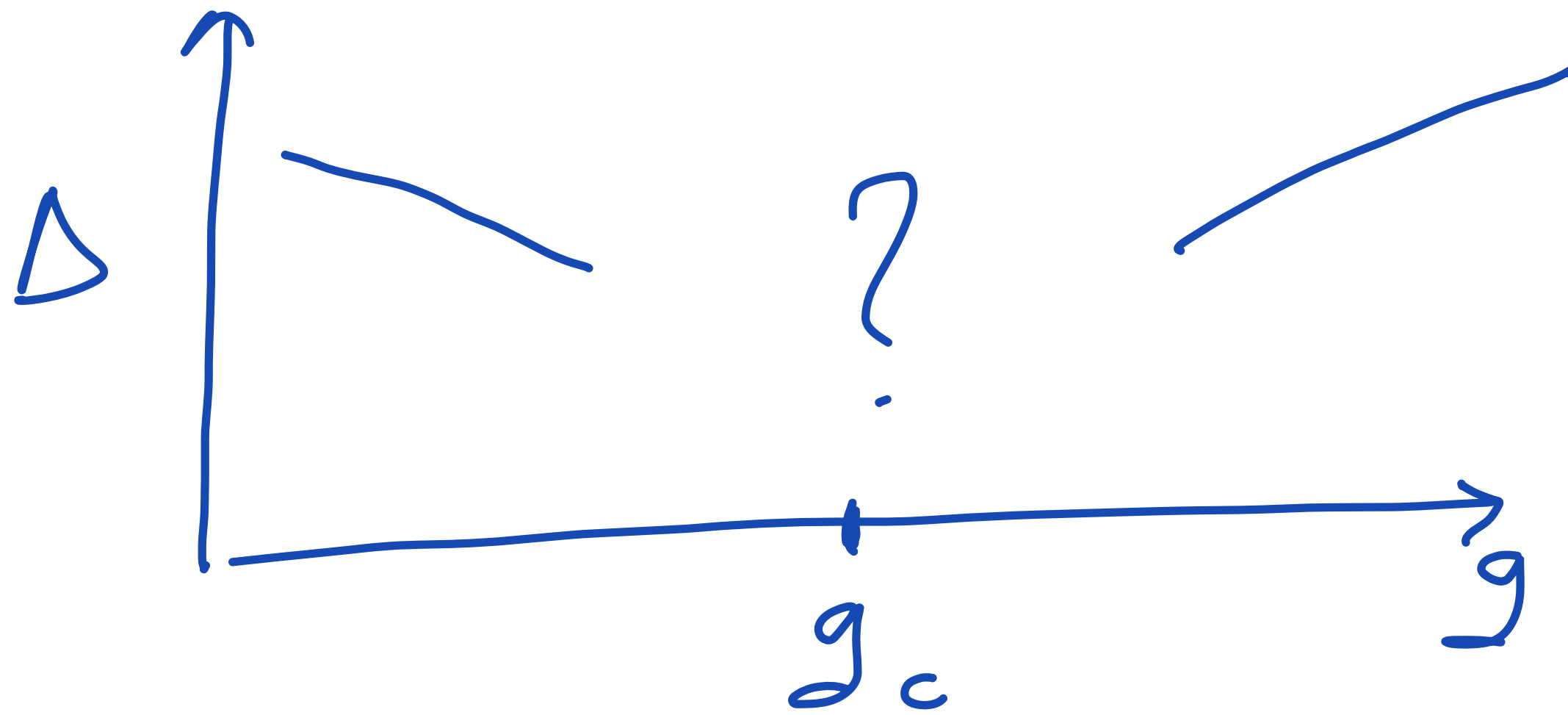
Have to use effective Hamiltonian method to perform perturbation theory to order g^2 .



Total matrix element is zero unless initial and final flipped spins are nearest neighbors.

$$\epsilon_k = 2dJ + cJg^2 \sum_{\alpha=1}^d \cos(k_\alpha a)$$

Δ = gap to first excited state



A single quantum phase transition at $g = g_c$ where Δ vanishes as

$$\Delta \sim |g - g_c|^z$$

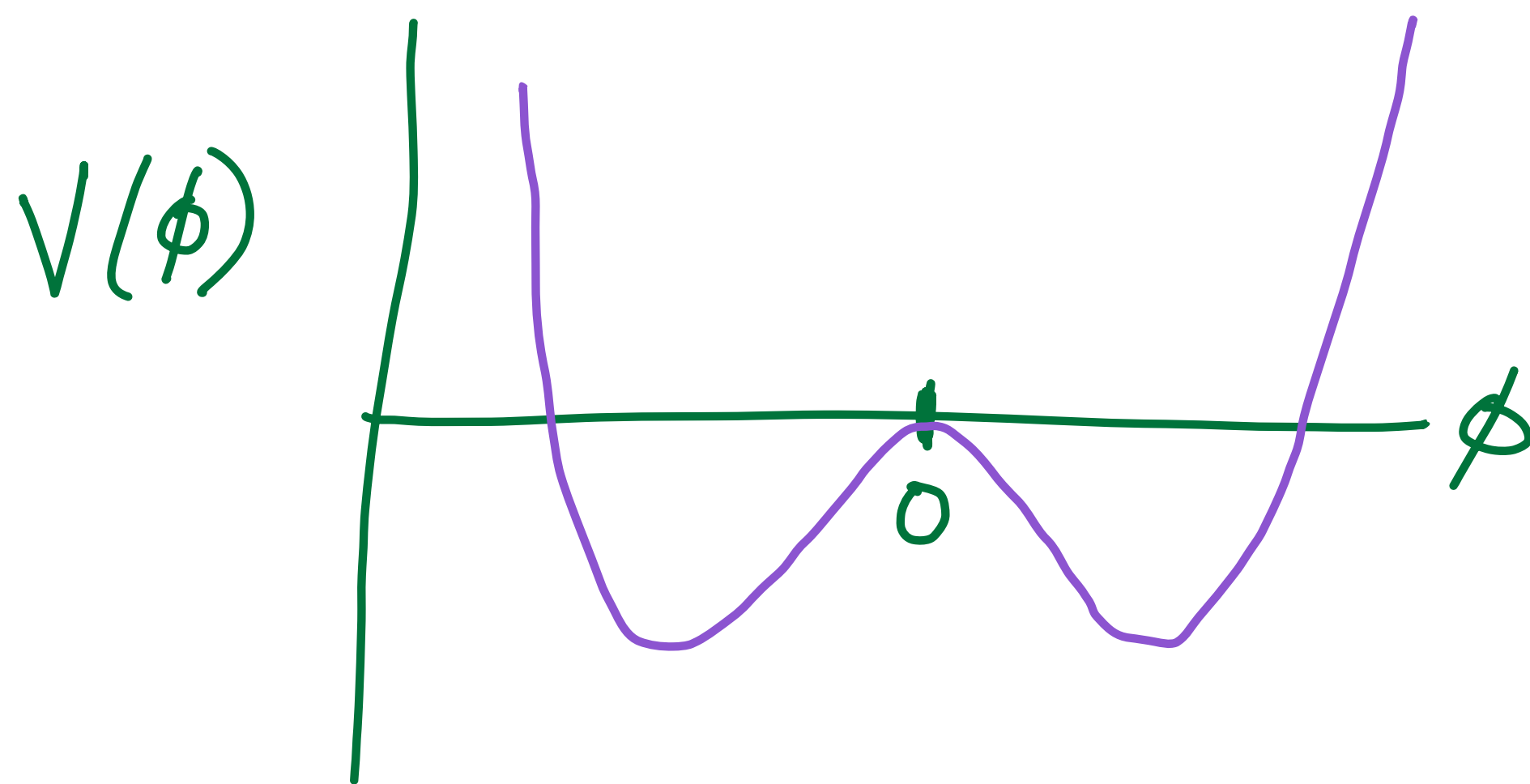
with $z = 1$.

Mapping to ϕ^4 quantum field theory

Consider the following non-linear oscillator

$$H = \frac{1}{2m} \pi_\phi^2 + V(\phi)$$

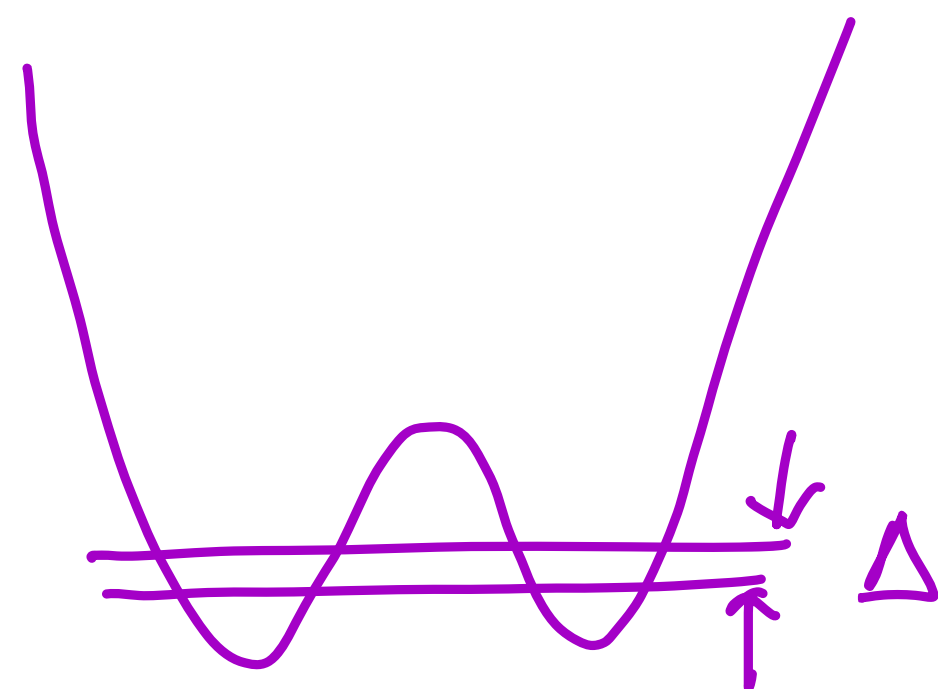
$$[\phi, \pi_\phi] = i$$

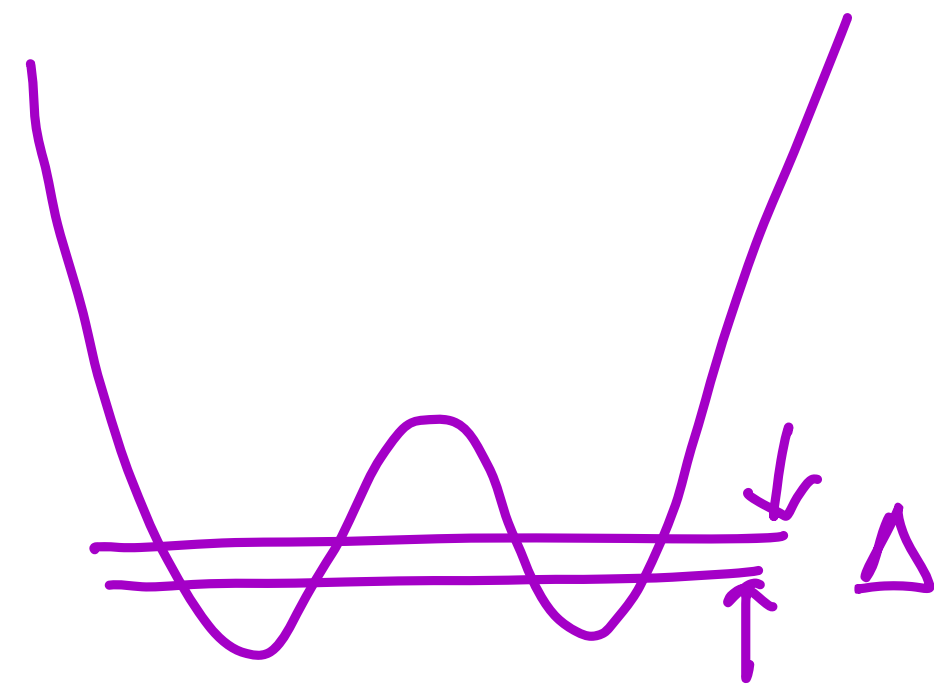


Focus on 2 lowest energy states



Tunneling under the barrier will lift the degeneracy and produce a gap Δ





ground state

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} (\psi_u + \psi_D) \equiv \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}} (\psi_u - \psi_D) \equiv \frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle)$$

low energy Hamiltonian

$$H = -\frac{\Delta}{2} X$$

What is the operator ϕ in this
low energy subspace?

$$\phi \mapsto \Rightarrow \frac{\phi}{\sqrt{2}} (\psi_u(\phi) + \psi_d(\phi))$$

π odd under $\phi \rightarrow -\phi$

$$\langle \rightarrow | \phi \mapsto \rangle = 0$$

$$\langle \leftarrow | \phi \leftarrow \rangle = 0$$

$$\begin{aligned} \langle \leftarrow | \phi \mapsto \rangle &= \int_{-\infty}^{\infty} d\phi \frac{\phi}{2} (\psi_u^*(\phi) - \psi_d^*(\phi)) (\psi_u(\phi) + \psi_d(\phi)) \\ &= K \neq 0 \end{aligned}$$

$\Rightarrow \phi$ maps to Z !

Now consider a lattice of such
non-linear oscillators and couple them
together

$$H = \sum_i \frac{1}{2m} \pi_i^2 + \sum_i V(\phi_i) - \sum_{\langle ij \rangle} \tilde{J} \phi_i \phi_j$$

Provided the $\tilde{J}$ is not too-large,
we can use the single-site mapping
to obtain

$$H_{\text{eff}} = -\frac{\Delta}{2} \sum_i X_i - \tilde{J} K^2 \sum_{\langle ij \rangle} Z_i Z_j$$

From now on we will consider

$$H_\phi = \sum_i \frac{1}{2m} \pi_{\phi_i}^2 + \sum_i \left(\frac{\lambda}{2} \phi_i^2 + \frac{u}{24} \phi_i^4 \right) - J \sum_{\langle ij \rangle} \phi_i \phi_j$$

In momentum space we can write

$$\phi_k = \frac{1}{\sqrt{N}} \sum_i \phi_i e^{i k r_i}$$

$$-J \sum_{\langle ij \rangle} \phi_i \phi_j = -J \sum_k |\phi_k|^2 \left(\sum_{\alpha} 2 \omega(k_{\perp} a) \right)$$

$$\approx -2J \sum_k |\phi_k|^2 (2d - k^2 \dots) \quad \text{as } k \rightarrow 0.$$

Now take the continuum limit while
defining

$$\phi(x_i) = \frac{1}{\sqrt{a^d}} \phi_i$$

$$\pi_\phi(x_i) = \frac{1}{\sqrt{a^d}} \pi_{\phi_i}$$

so that

$$[\phi(x), \pi_\phi(y)] = i \int (x-y)$$

$\text{as } a \rightarrow 0.$

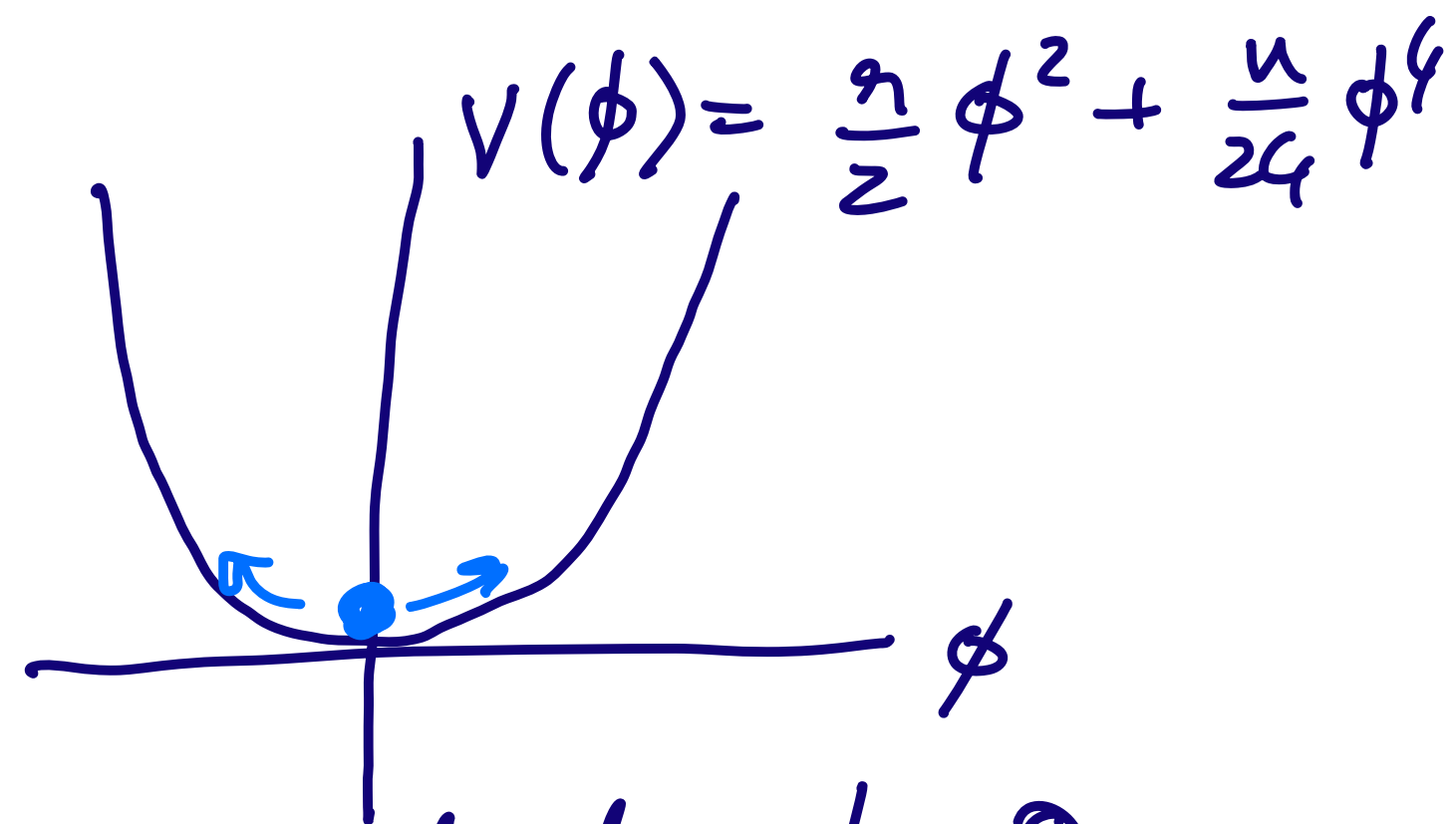
Then after various re-scalings

$$H_\phi = \int d^d x \left[\frac{1}{2} \pi_\phi^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{s}{2} \phi^2 + \frac{u}{24} \phi^4 \right]$$

ϕ^4 quantum field theory!

Examine perturbation theory in u for
 $s < 0$ and $s > 0$.

(i) $r > 0$



ϕ will fluctuate about $\phi = 0$.

$$H = \sum_k \left(\frac{\pi_k^2}{2} + \frac{\phi_k^2}{2} (r + k^2) \right)$$

One oscillator for each k

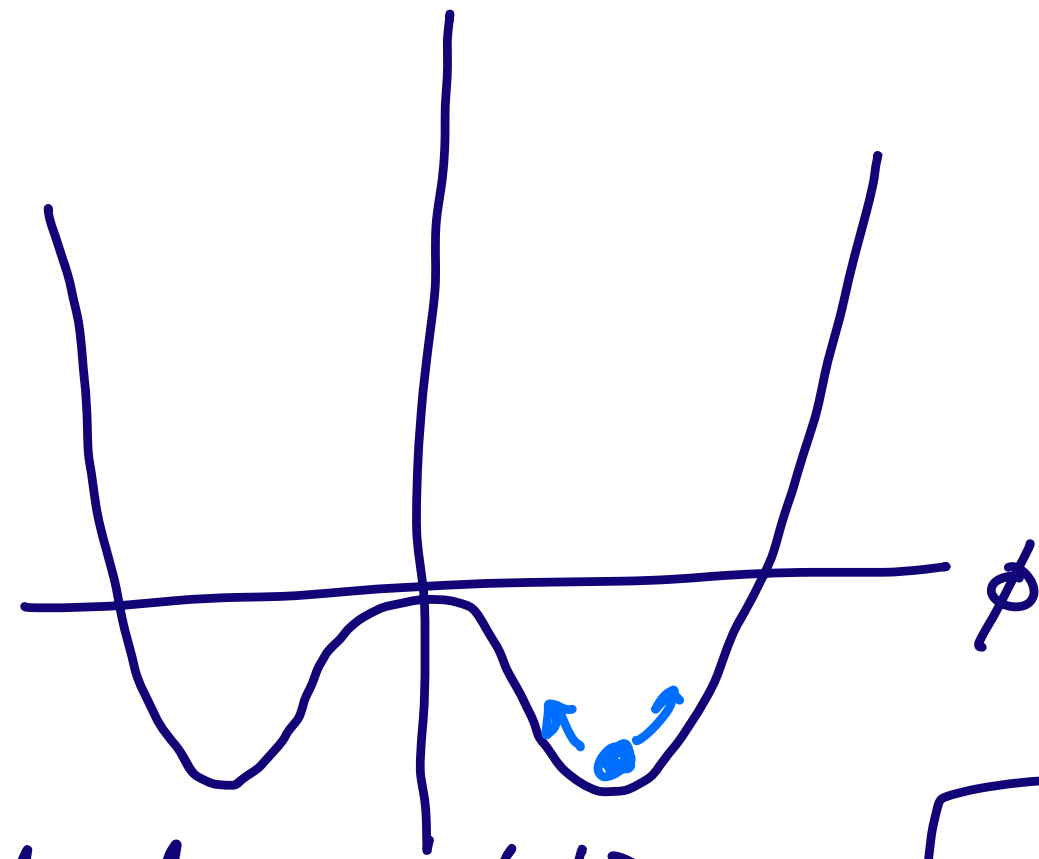
$$\epsilon_k = \sqrt{r + k^2}$$

Very similar to $g \gg 1$ Ising model

gap vanishes as $r \rightarrow 0$

$$\Delta = \sqrt{r} \quad ; \quad Z\nu = \frac{1}{2}.$$

(ii) $\eta < 0$



ϕ fluctuates about $\langle \phi \rangle = \sqrt{-\frac{6\eta}{u}}$

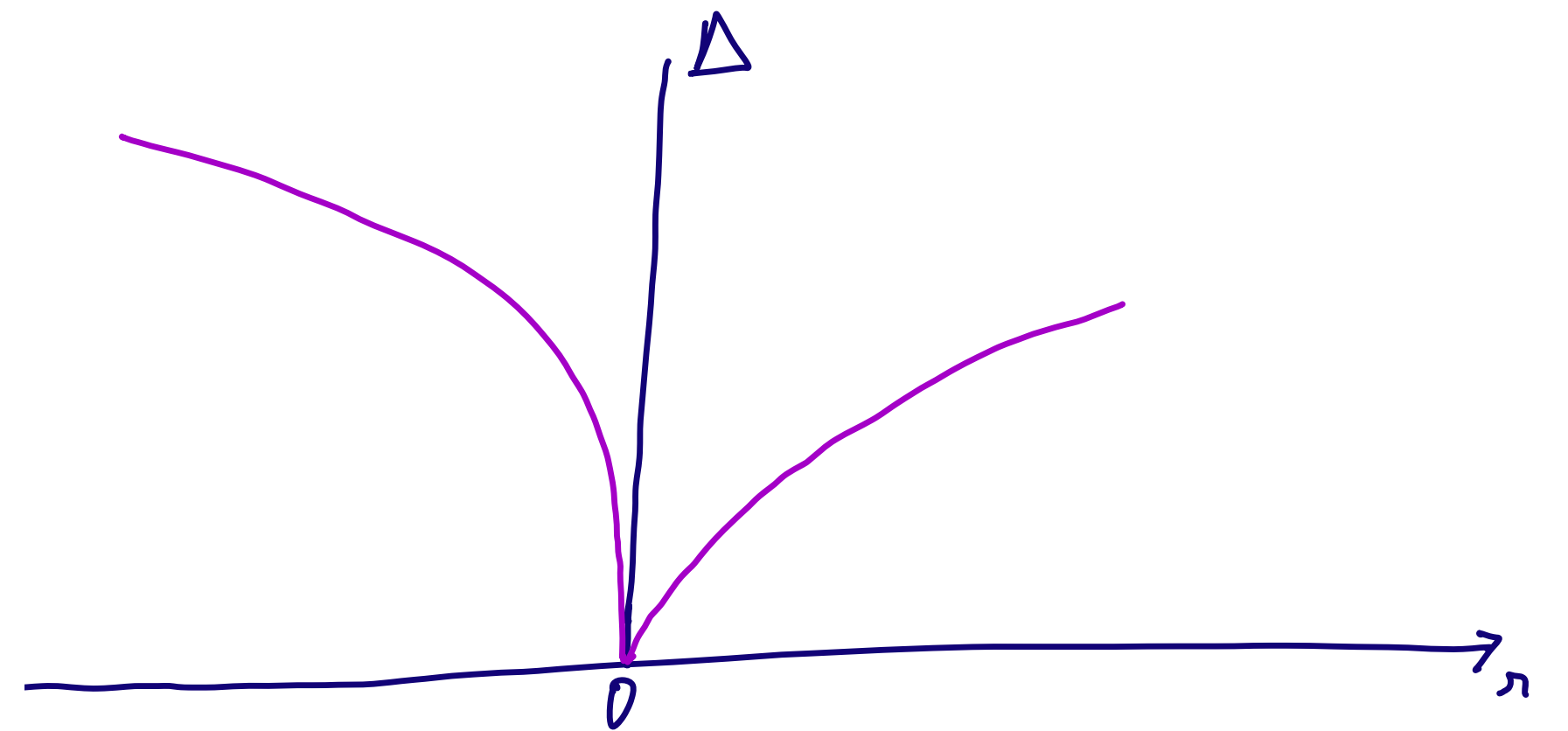
$$H_\phi = \sum_k \frac{|\phi_k|^2}{2} (2|\eta| + k^2) + \sum_k \frac{\pi_k^2}{2}$$

Similar to flipped spin quasiparticles at $g \ll 1$.

$$\epsilon_k = \sqrt{k^2 + 2|\eta|}$$

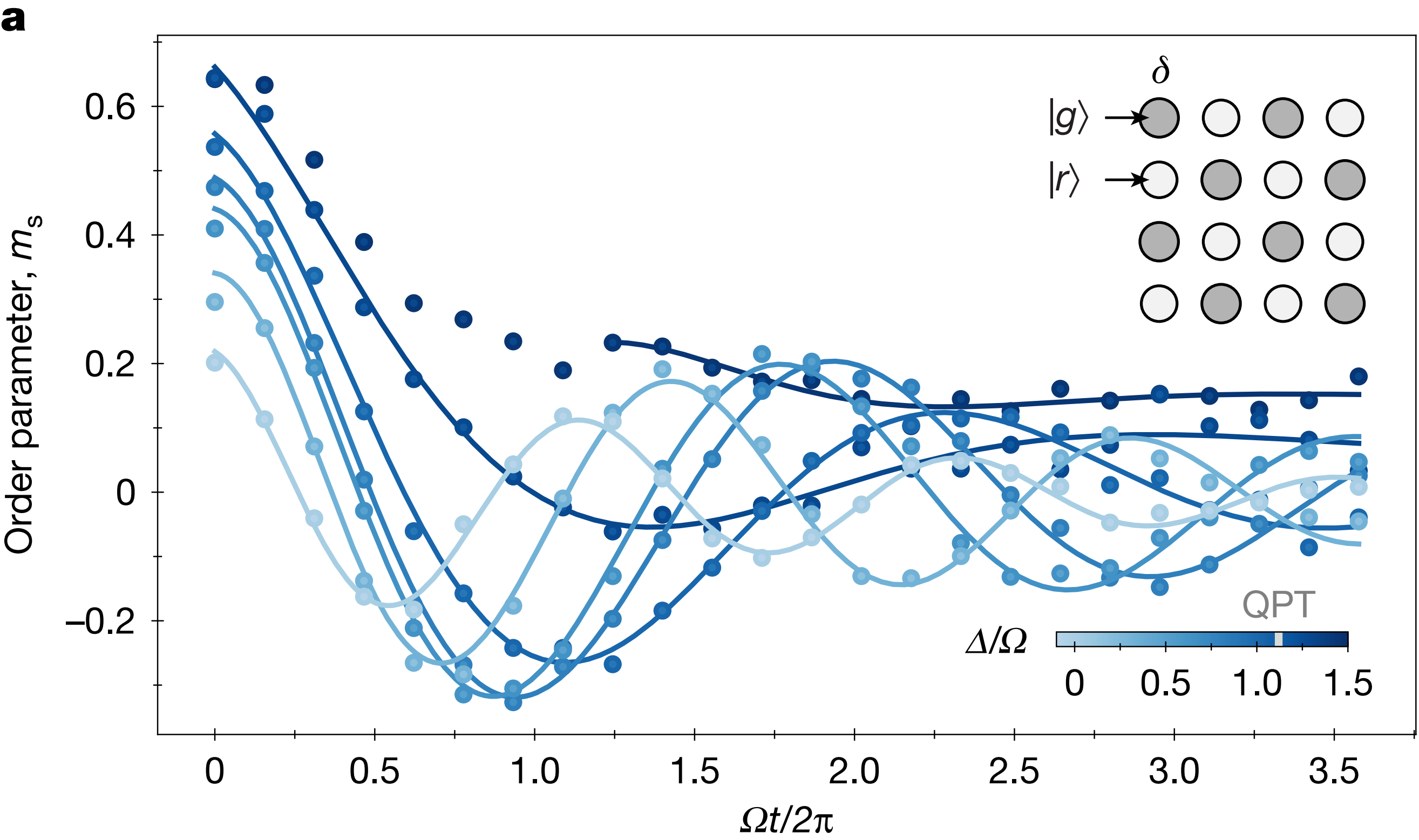
Energy gap vanishes as

$$\Delta = \sqrt{2|\eta|}$$

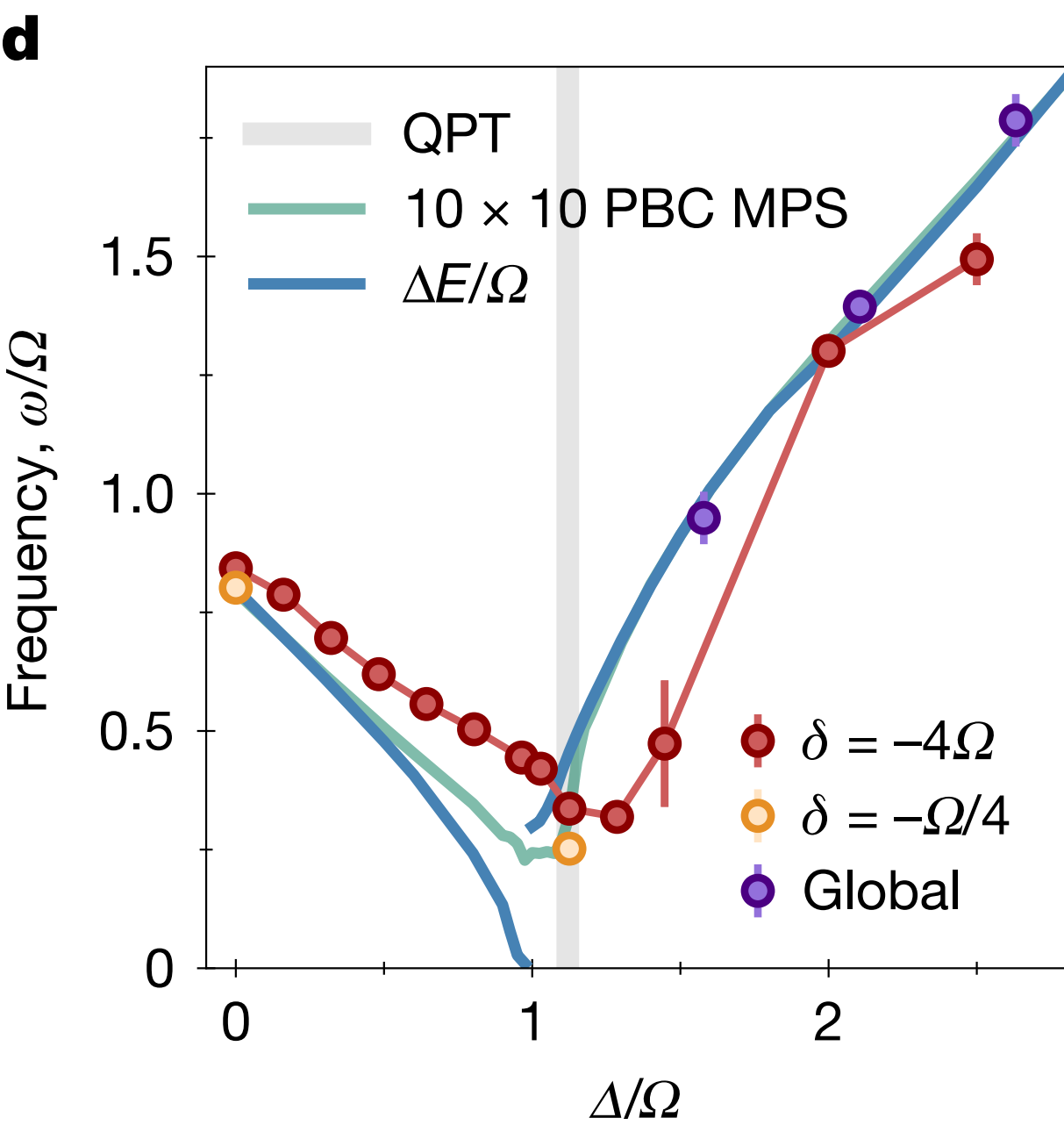
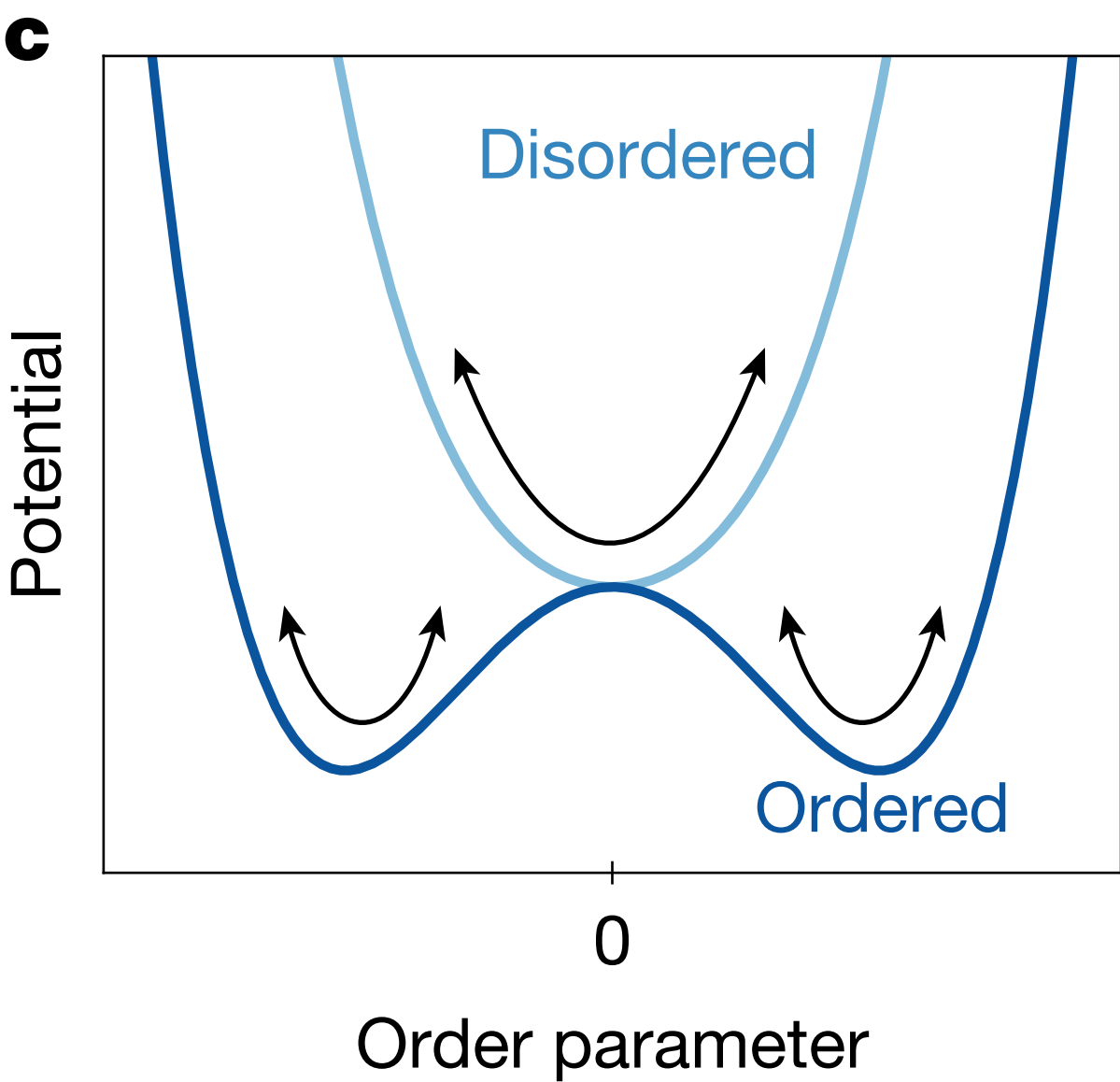


Quantum coarsening and collective dynamics on a programmable simulator

Tom Manovitz^{1,8}, Sophie H. Li^{1,8}, Sepehr Ebadi^{1,7,8}, Rhine Samajdar^{2,3}, Alexandra A. Geim¹, Simon J. Evered¹, Dolev Bluvstein¹, Hengyun Zhou^{1,4}, Nazli Ugur Koyluoglu^{1,5}, Johannes Feldmeier¹, Pavel E. Dolgirev¹, Nishad Maskara¹, Marcin Kalinowski¹, Subir Sachdev¹, David A. Huse², Markus Greiner¹, Vladan Vuletić⁶ & Mikhail D. Lukin¹✉



$$f(x) \equiv \frac{\omega(\Delta - \Delta_c = x)}{\omega(\Delta_c - \Delta = x)} \approx 1.9 \quad f(x) = \sqrt{2} \text{ in mean-field theory.}$$



Quantum critical point at $g=0$

$$\varepsilon_k = k$$

Massless relativistic boson

$$Z=1$$

Imaginary time path integral

Single particle path integral

$$H = \frac{m}{2} \Pi_\phi^2 + V(\phi)$$

$$Z = \text{Tr} e^{-\beta H} = \int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi(\tau) e^{-S[\phi]}$$

$$S[\phi] = \int_0^\beta d\tau \left[\frac{m}{2} \left(\frac{d\phi}{d\tau} \right)^2 + V(\phi) \right]$$

$$H_\phi = \int d^d x \left[\frac{1}{2} \Pi_\phi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{g}{2} \phi^2 + \frac{u}{24} \phi^4 \right]$$

$$Z_\phi = \text{Tr} e^{-\beta H_\phi} = \int_{\phi(x,\beta)=\phi(x,0)} \mathcal{D}\phi(x,\tau) \exp \left(- \int d^d x d\tau \mathcal{L}[\phi] \right)$$

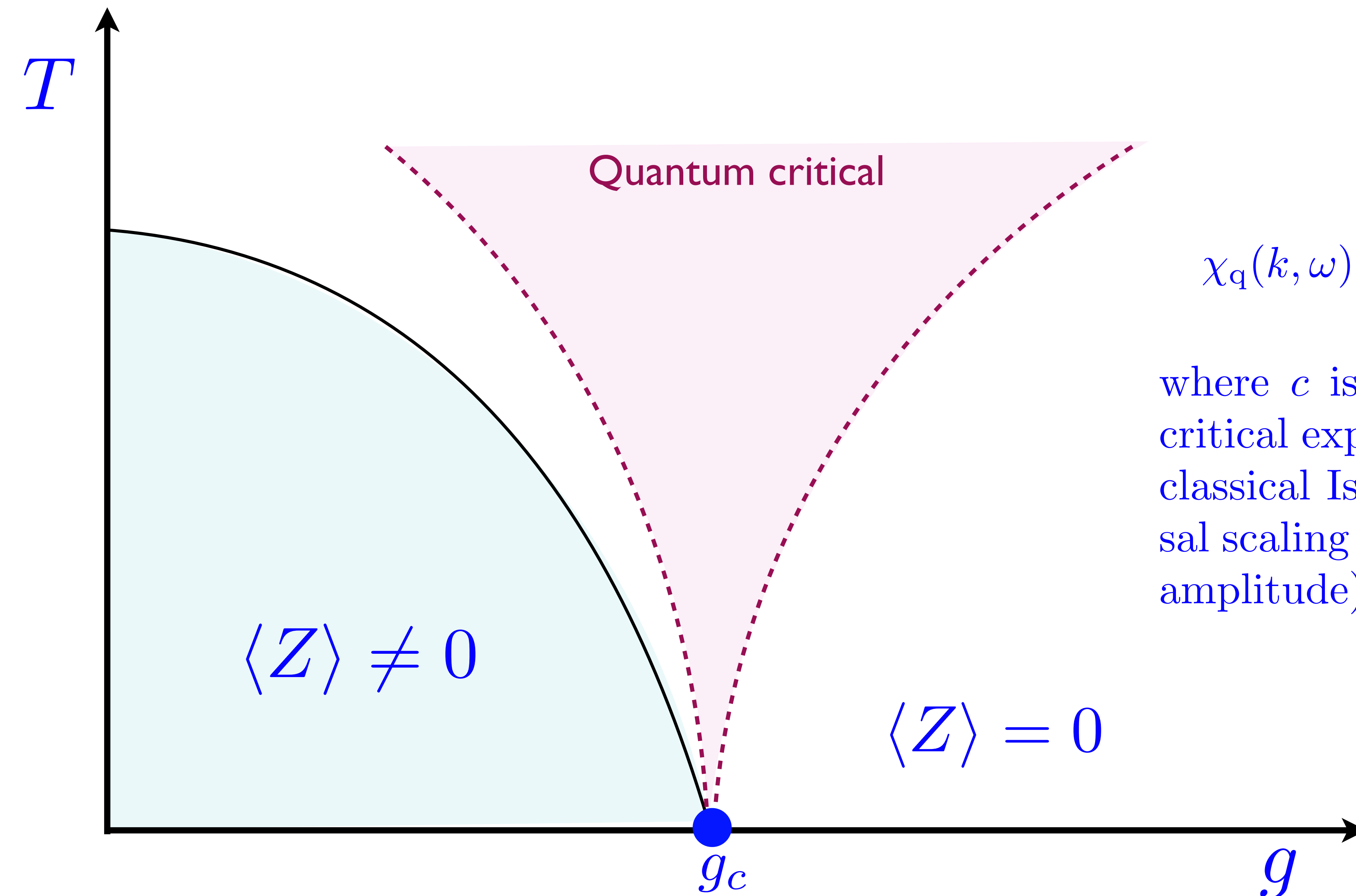
$$Z_\phi = \text{Tr} e^{-\beta H_\phi} = \int_{\phi(x, \beta) = \phi(x, 0)} \mathcal{D}\phi(x, \tau) \exp\left(-\int_0^\beta \int_x^d \mathcal{L}[\phi]\right)$$

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla \phi)^2 + \frac{g}{2}\phi^2 + \frac{u}{24}\phi^4$$

$$= \frac{1}{2}(\partial_\mu \phi)^2 + \frac{g}{2}\phi^2 + \frac{u}{24}\phi^4$$

Relativistic quantum field theory in
 $d+1$ spacetime dimensions.

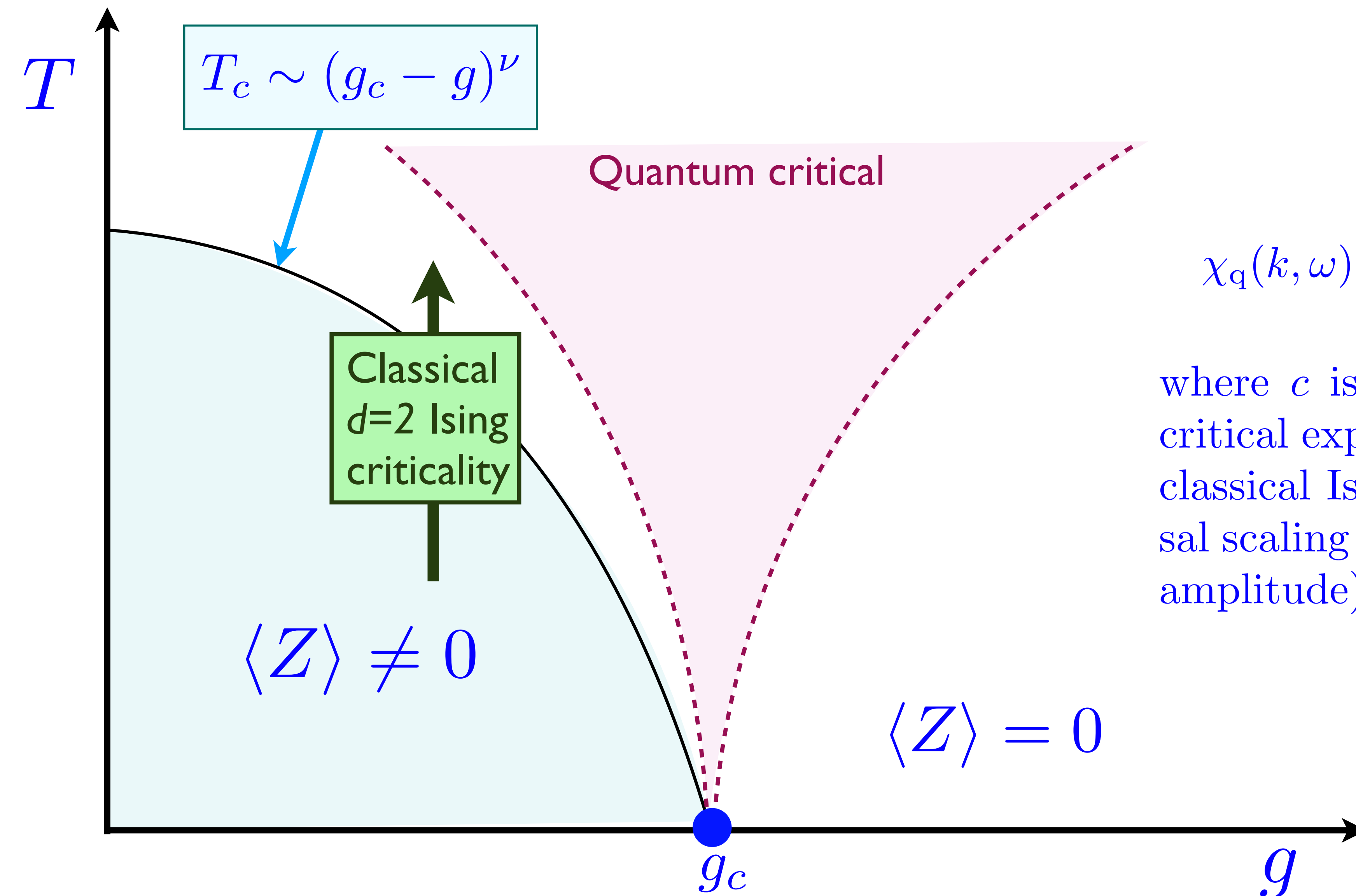
Quantum criticality of Ising model in 2+1 dimensions



$$\chi_q(k, \omega) = \frac{1}{T^{2-\eta}} \Phi_I \left(\frac{ck}{T}, \frac{\omega}{T}, \frac{g - g_c}{T^{1/\nu}} \right)$$

where c is the velocity of 'light', ν is a critical exponent of the three-dimensional classical Ising model, and Φ_I is a universal scaling function (apart from an overall amplitude).

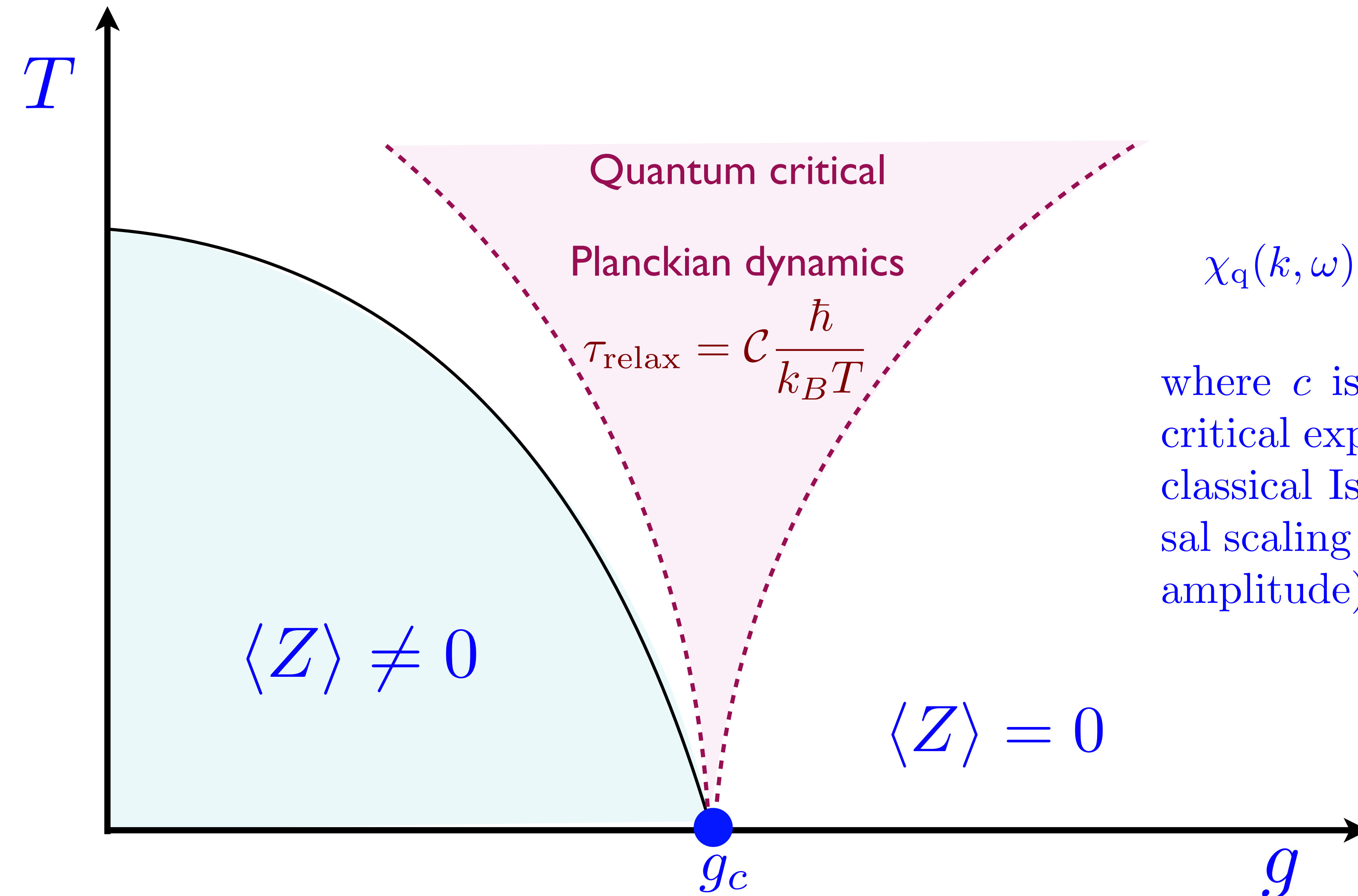
Quantum criticality of Ising model in 2+1 dimensions



$$\chi_q(k, \omega) = \frac{1}{T^{2-\eta}} \Phi_I \left(\frac{ck}{T}, \frac{\omega}{T}, \frac{g - g_c}{T^{1/\nu}} \right)$$

where c is the velocity of 'light', ν is a critical exponent of the three-dimensional classical Ising model, and Φ_I is a universal scaling function (apart from an overall amplitude).

Quantum criticality of Ising model in 2+1 dimensions



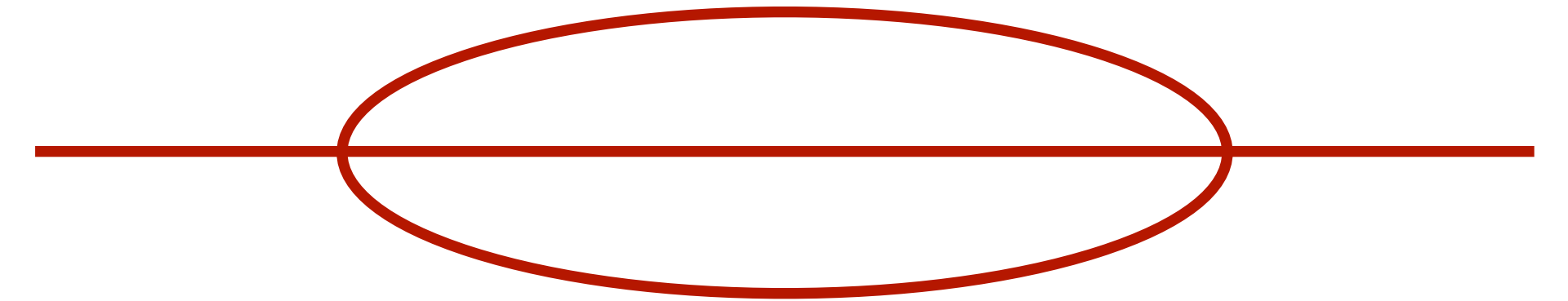
$$\chi_q(k, \omega) = \frac{1}{T^{2-\eta}} \Phi_I \left(\frac{ck}{T}, \frac{\omega}{T}, \frac{g - g_c}{T^{1/\nu}} \right)$$

where c is the velocity of ‘light’, ν is a critical exponent of the three-dimensional classical Ising model, and Φ_I is a universal scaling function (apart from an overall amplitude).

Quantum criticality of Ising model in 2+1 dimensions

Quantum critical dynamic susceptibility in an expansion in $\epsilon = 3 - d$

$$\chi(k, \omega) = \frac{1}{c^2 k^2 - \omega^2 + \epsilon \frac{2\pi^2 T^2}{9} - i \operatorname{sgn}(\omega) \epsilon^2 \frac{4\pi^3 T^2}{27}}$$



Quantum criticality of Ising model in 2+1 dimensions

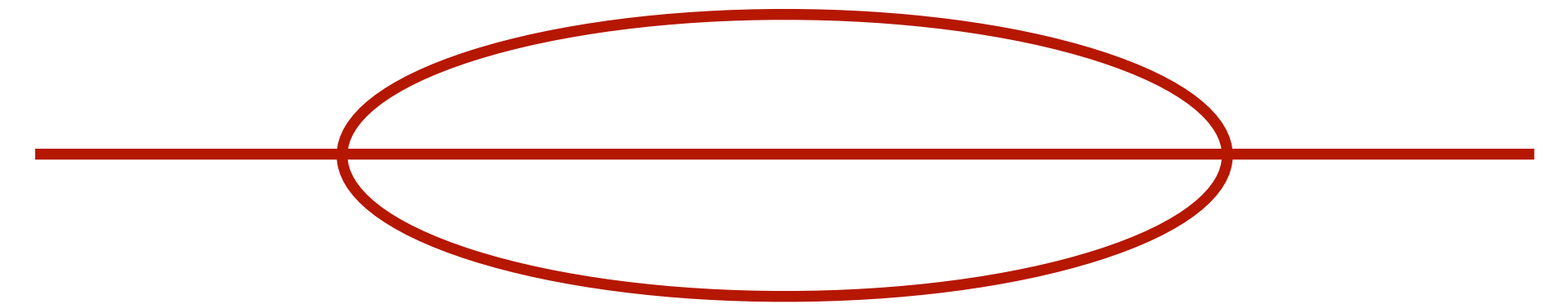
Quantum critical dynamic susceptibility in an expansion in $\epsilon = 3 - d$

$$\chi(k, \omega) = \frac{1}{c^2 k^2 - \omega^2 + \epsilon \frac{2\pi^2 T^2}{9} - i \operatorname{sgn}(\omega) \epsilon^2 \frac{4\pi^3 T^2}{27}}$$

This indicates a breakdown of the ϵ expansion for $\hbar\omega \sim \sqrt{\epsilon} k_B T$. Expected low frequency form is

$$\chi(k, \omega) = \frac{1}{c^2 k^2 - \omega^2 + \epsilon \frac{2\pi^2 T^2}{9} - i\Gamma\omega T}$$

with Γ a universal constant.



Quantum criticality of Ising model in 2+1 dimensions

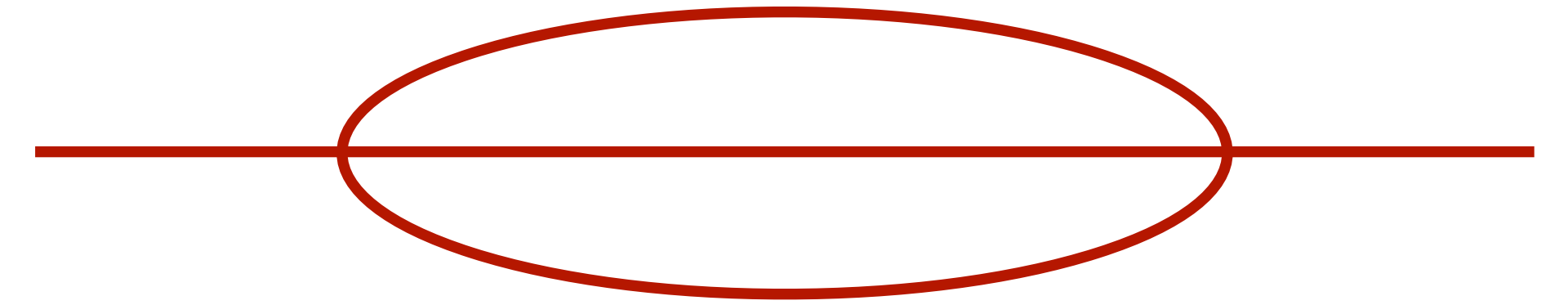
Quantum critical dynamic susceptibility in an expansion in $\epsilon = 3 - d$

$$\chi(k, \omega) = \frac{1}{c^2 k^2 - \omega^2 + \epsilon \frac{2\pi^2 T^2}{9} - i \operatorname{sgn}(\omega) \epsilon^2 \frac{4\pi^3 T^2}{27}}$$

This indicates a breakdown of the ϵ expansion for $\hbar\omega \sim \sqrt{\epsilon} k_B T$. Expected low frequency form is

$$\chi(k, \omega) = \frac{1}{c^2 k^2 - \omega^2 + \epsilon \frac{2\pi^2 T^2}{9} - i\Gamma\omega T}$$

with Γ a universal constant.



However, the ϵ expansion works well at $T = 0$, and yields systematic corrections to $f(x) \equiv \frac{\omega(\Delta - \Delta_c = x)}{\omega(\Delta_c - \Delta = x)}$ which agree with observations.

Quantum criticality of Ising model in 2+1 dimensions

For $\hbar\omega \sim \sqrt{\epsilon}k_B T$ and ϵ small, we can use a **semi-classical approach**.

Equal time correlators

In a classical theory, equal time correlators are independent of the dynamics, and depend only upon the potential energy. The classical Φ^4 field theory has potential energy

$$\mathcal{V}[\Phi] = \int d^2x \left[\frac{1}{2} (\nabla\Phi)^2 + \frac{\tilde{R}}{2} \Phi^2 + \frac{U}{24} \Phi^4 \right]$$

59, 14054 (1999)

The partition function at a temperature T is

$$\mathcal{Z} = \int \mathcal{D}\Phi \exp \left(-\frac{\mathcal{V}[\Phi]}{T} \right)$$

and a typical correlator of interest is the equal time structure factor

$$S_{\text{cl}}(k) = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \exp \left(-\frac{\mathcal{V}[\Phi]}{T} \right) \int d^2x e^{-ikx} \Phi(x) \Phi(0)$$

Quantum criticality of Ising model in 2+1 dimensions

Matching to determine couplings

Let us assume a square lattice of spacing a . A key assertion is that results are independent of the lattice as $a \rightarrow 0$ a single mass renormalization. This renormalization is equivalent to replacing $\tilde{R}$ by R , where

$$\tilde{R} = R - \frac{TU}{2} \int_{-\pi/a}^{\pi/a} \frac{dk_x dk_y}{4\pi^2} \frac{1}{(2/a^2)[2 - \cos(k_x a) - \cos(k_y a)] + R}$$

After all lattice results are expressed in terms of R , then the claim is that no further renormalizations are needed, and the limit $a \rightarrow 0$ exists, and is independent of the lattice choice. Notice that we assume that $R > 0$, and that as R ranges from 0 to $+\infty$, $\tilde{R}$ ranges from $-\infty$ to $+\infty$. The values of R and U are determined by matching to static properties of the quantum critical point.

Quantum criticality of Ising model in 2+1 dimensions

Matching to determine couplings

Let us assume a square lattice of spacing a . A key assertion is that results are independent of the lattice as $a \rightarrow 0$ a single mass renormalization. This renormalization is equivalent to replacing $\tilde{R}$ by R , where

$$\tilde{R} = R - \frac{TU}{2} \int_{-\pi/a}^{\pi/a} \frac{dk_x dk_y}{4\pi^2} \frac{1}{(2/a^2)[2 - \cos(k_x a) - \cos(k_y a)] + R}$$

After all lattice results are expressed in terms of R , then the claim is that no further renormalizations are needed, and the limit $a \rightarrow 0$ exists, and is independent of the lattice choice. Notice that we assume that $R > 0$, and that as R ranges from 0 to $+\infty$, $\tilde{R}$ ranges from $-\infty$ to $+\infty$. The values of R and U are determined by matching to static properties of the quantum critical point.

Classical dynamics

We introduce a conjugate momentum field $\Pi(x)$ with the Poisson bracket

$$\{\Phi(x), \Pi(y)\}_{P.B.} = \delta^2(x - y)$$

and the Hamiltonian

$$\mathcal{H}[\Phi, \Pi] = \int d^2x \frac{c^2 \Pi^2}{2} + \mathcal{V}[\Phi]$$

Quantum criticality of Ising model in 2+1 dimensions

where c is a velocity. The partition function is now extended to

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{D}\Pi \exp \left(-\frac{\mathcal{H}[\Phi, \Pi]}{T} \right) .$$

The equations of motion are

$$\begin{aligned} \frac{\partial \Phi}{\partial t} &= c^2 \Pi \\ \frac{\partial \Pi}{\partial t} &= \nabla^2 \Phi - \tilde{R} \Phi - \frac{U}{6} \Phi^3 \end{aligned}$$

By integrating these equations of motion, we can obtain the dynamic structure factor

$$S_{\text{cl}}(k, \omega) = \int d^2x \int dt \langle \Phi(x, t) \Phi(0, 0) \rangle e^{-i(kx - \omega t)} .$$

Here the average $\langle \cdot \rangle$ is over thermal initial conditions defined by the partition function $\mathcal{Z}$.

Quantum criticality of Ising model in 2+1 dimensions

where c is a velocity. The partition function is now extended to

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{D}\Pi \exp \left(-\frac{\mathcal{H}[\Phi, \Pi]}{T} \right) .$$

The equations of motion are

$$\begin{aligned} \frac{\partial \Phi}{\partial t} &= c^2 \Pi \\ \frac{\partial \Pi}{\partial t} &= \nabla^2 \Phi - \tilde{R} \Phi - \frac{U}{6} \Phi^3 \end{aligned}$$

By integrating these equations of motion, we can obtain the dynamic structure factor

$$S_{\text{cl}}(k, \omega) = \int d^2x \int dt \langle \Phi(x, t) \Phi(0, 0) \rangle e^{-i(kx - \omega t)} .$$

Here the average $\langle \cdot \rangle$ is over thermal initial conditions defined by the partition function $\mathcal{Z}$.

Precise numerical results to appear by
Julia Wei, Matthew Dodelson, Anders Sandvik...

S. Sachdev, PRB **59**, 14054 (1999)