Planckian Dynamics

$$\tau_r \geq \mathcal{C} \frac{\hbar}{k_B T} \qquad \text{(SS, \textit{Quantum Phase Transitions}, 1999)}$$

1. SYK as a solvable model of quantum matter without quasiparticles
2. Quantum critical dynamics of the Ising model in 2 spatial dimensions
3. Strange metal from metallic quantum phase transitions with spatial disorder

School on Quantum Dynamics of Matter, Light and Information
ICTP, Trieste
August 18, 19, 2025

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Strange metal from metallic quantum phase transitions with spatial disorder

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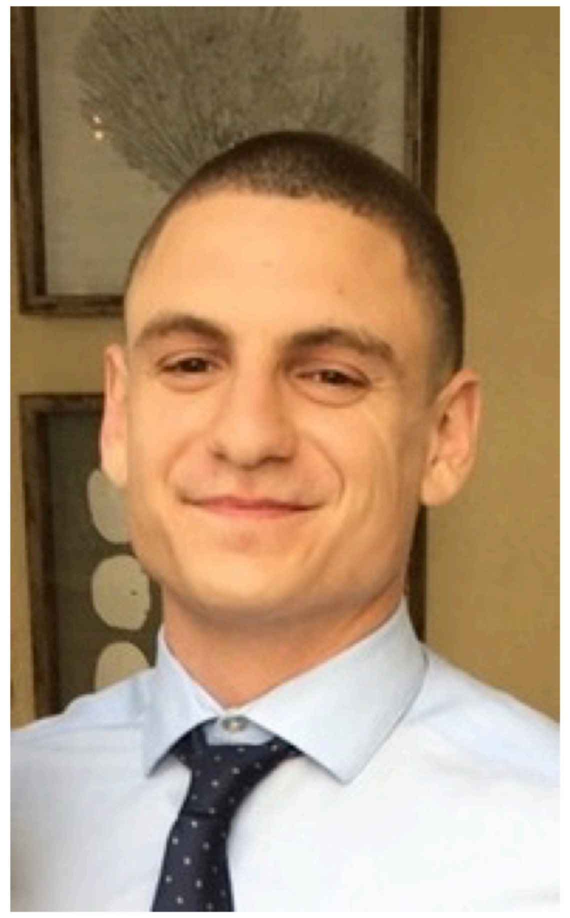


Ilya Esterlis
Wisconsin

Haoyu Guo
Cornell

Aavishkar Patel
Flatiron

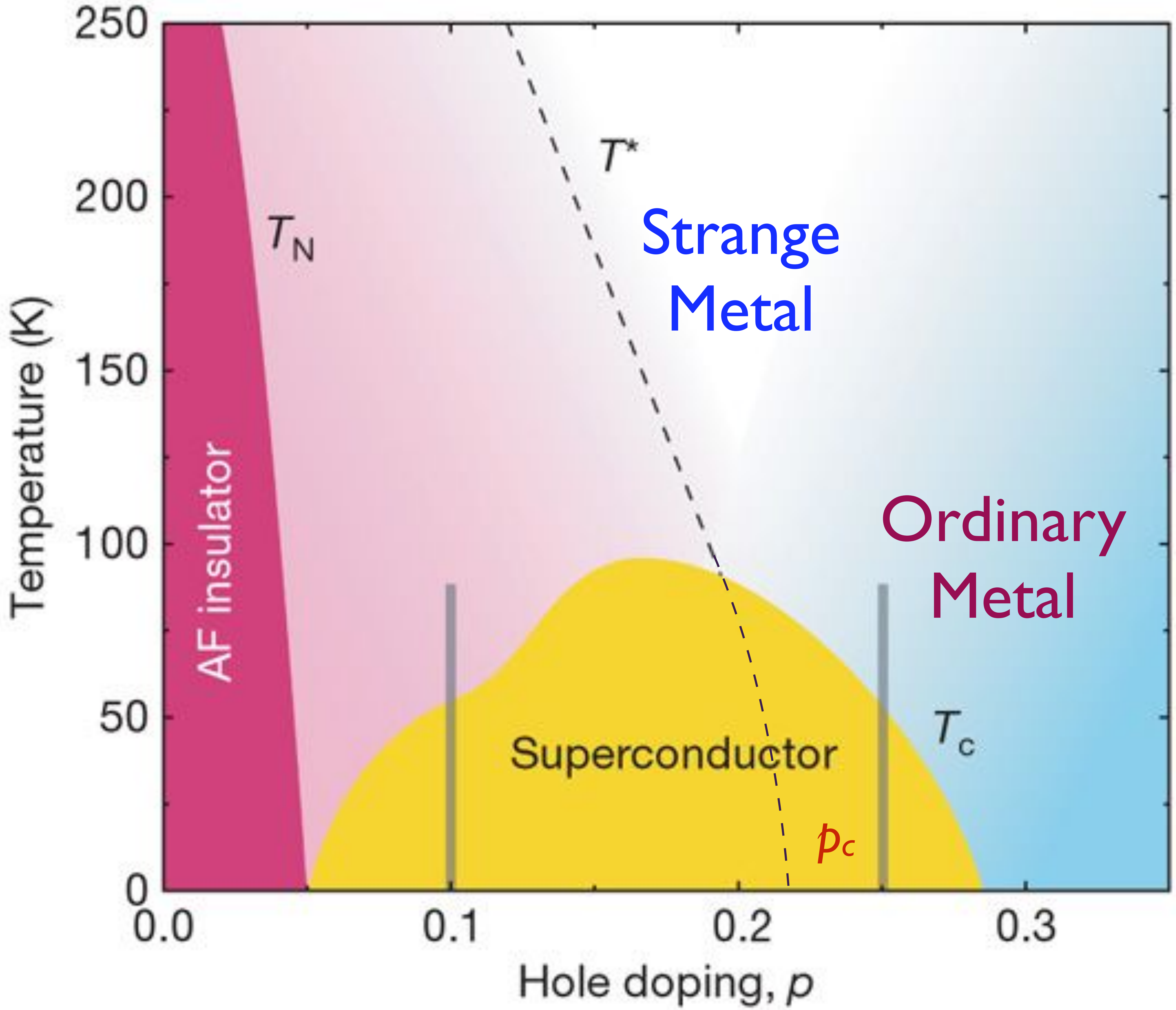
Chenyuan Li
Harvard → Rice

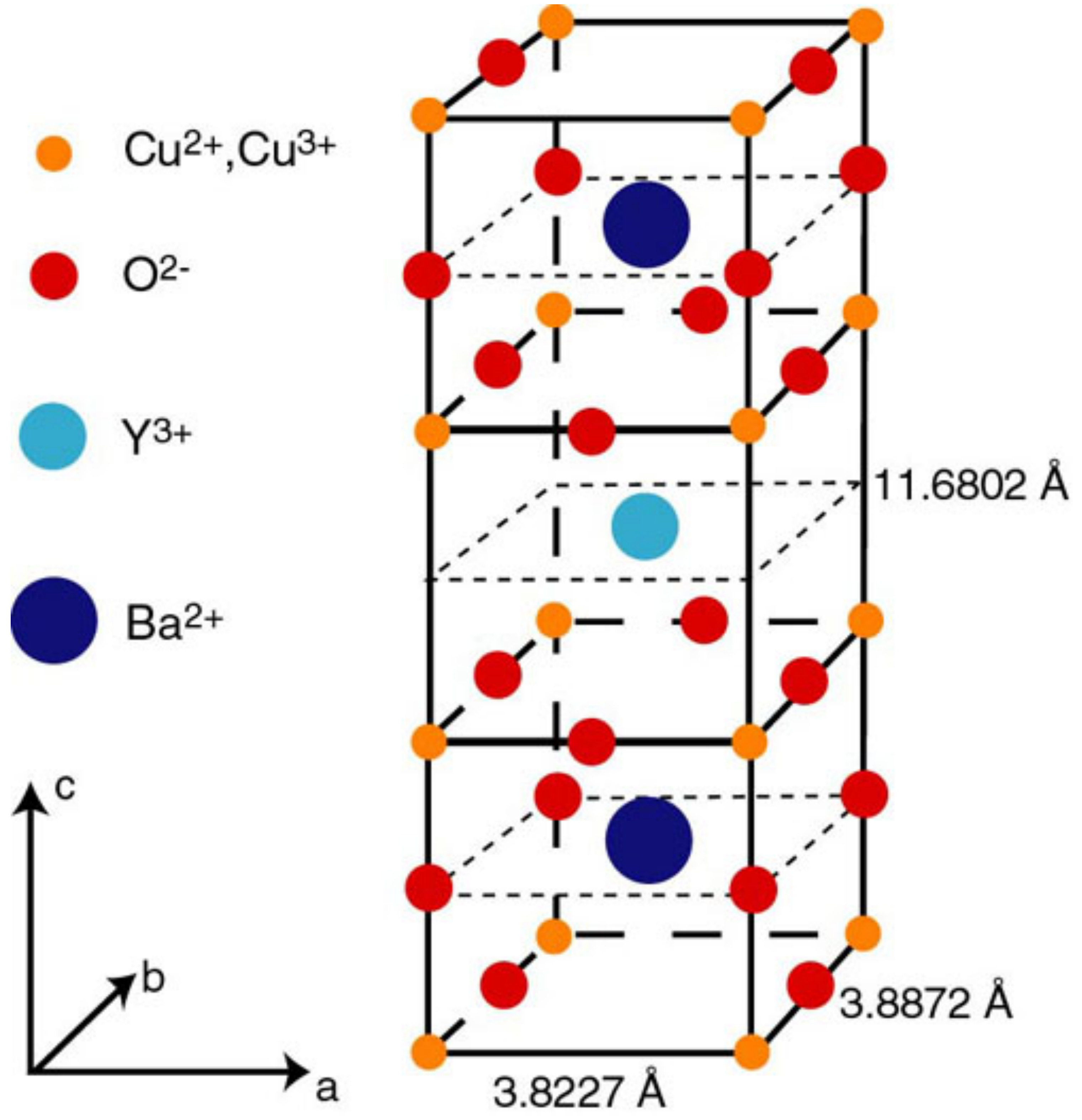
Davide Valentinis
KIT

Joerg Schmalian
KIT

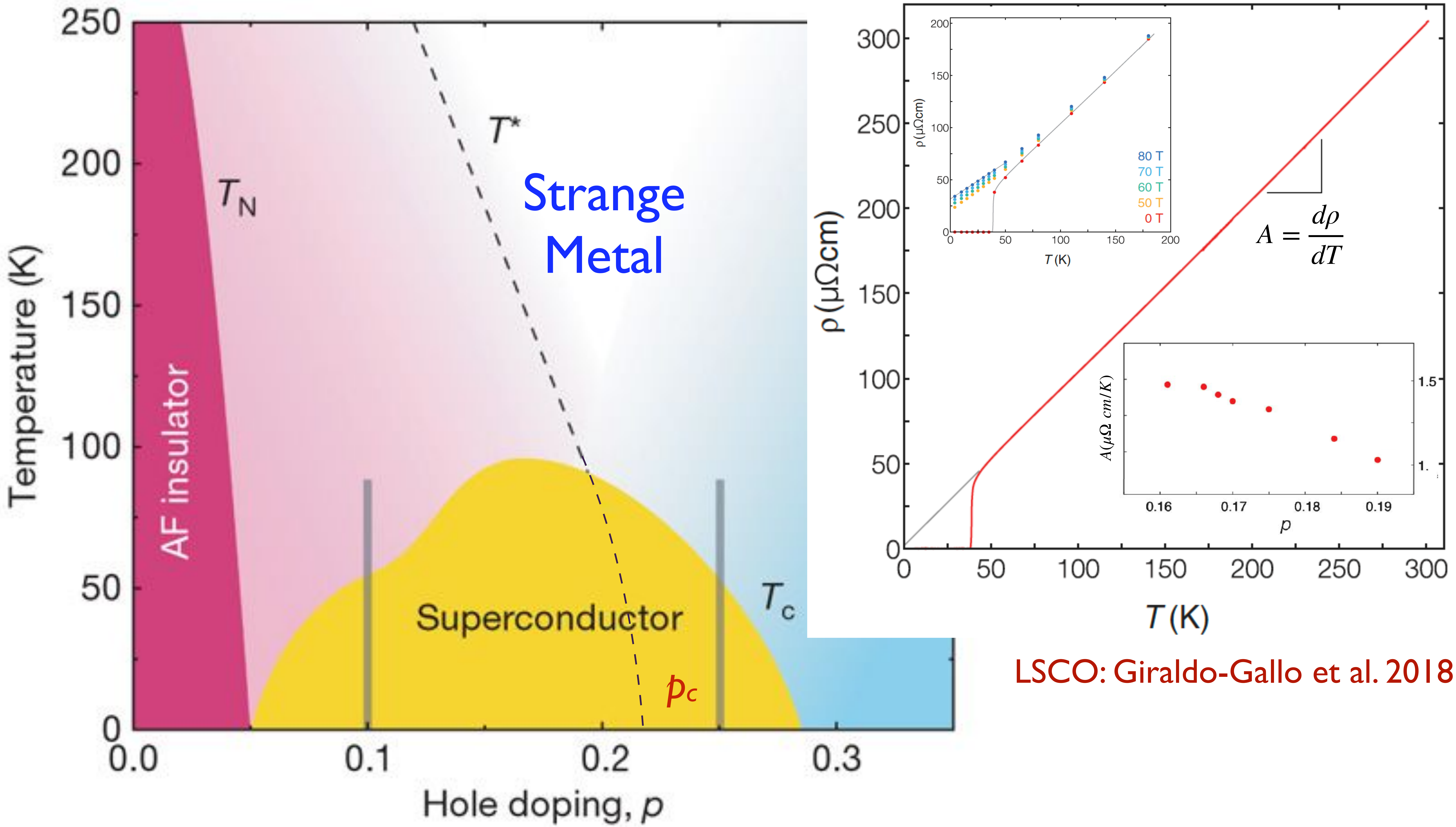
Peter Lunts
Harvard

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



LSCO: Giraldo-Gallo et al. 2018

Transport properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

Electronic properties of a marginal Fermi liquid:

1. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP (2022)

1. Ising-nematic ordering in a disordered metal

2. FL*-FL quantum-criticality in the cuprates

3. Theory of the "foot":
   quantum Griffiths SDW phase

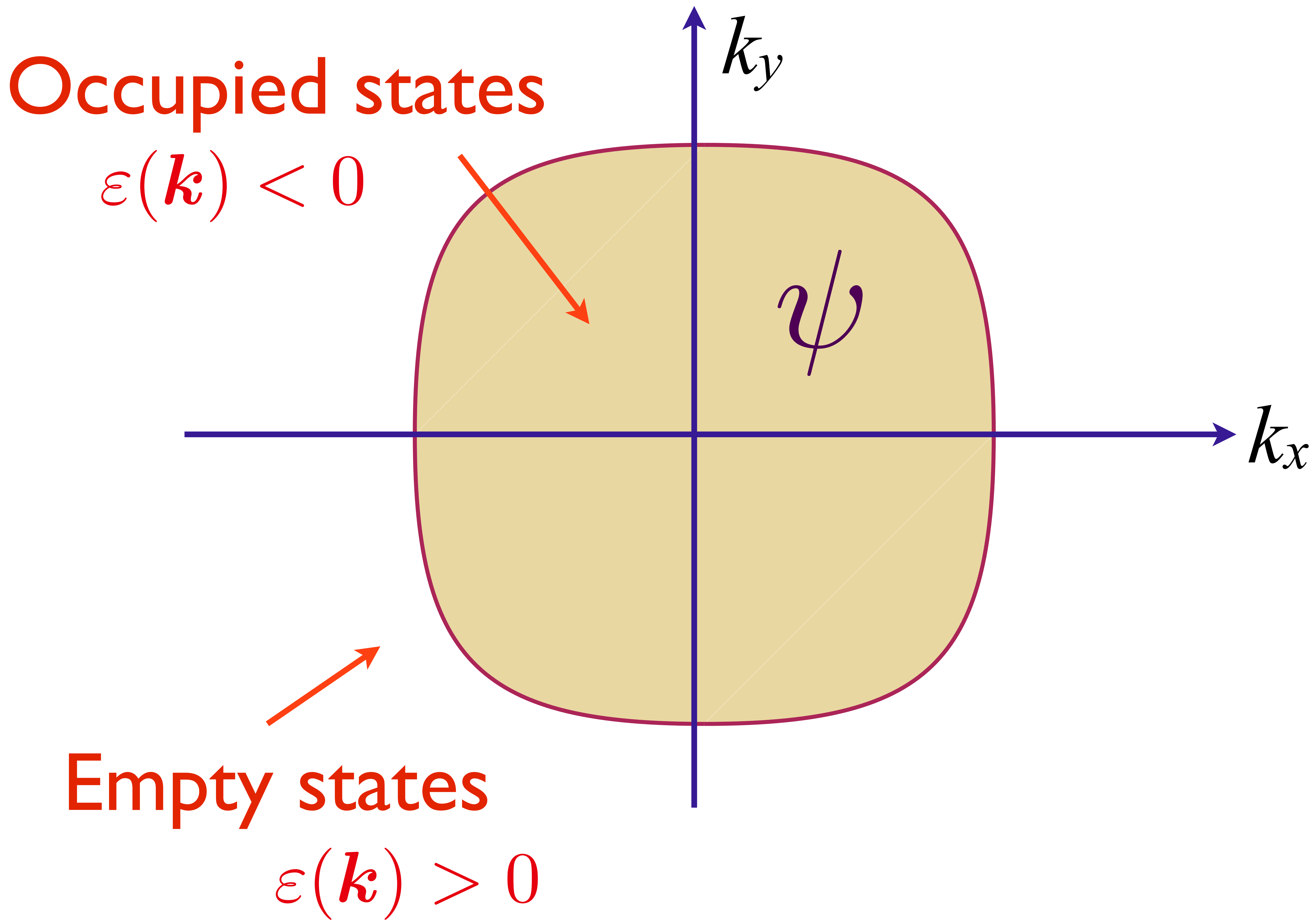
1. Ising-nematic ordering in a disordered metal

2. FL*-FL quantum-criticality in the cuprates

3. Theory of the "foot":

quantum Griffiths SDW phase

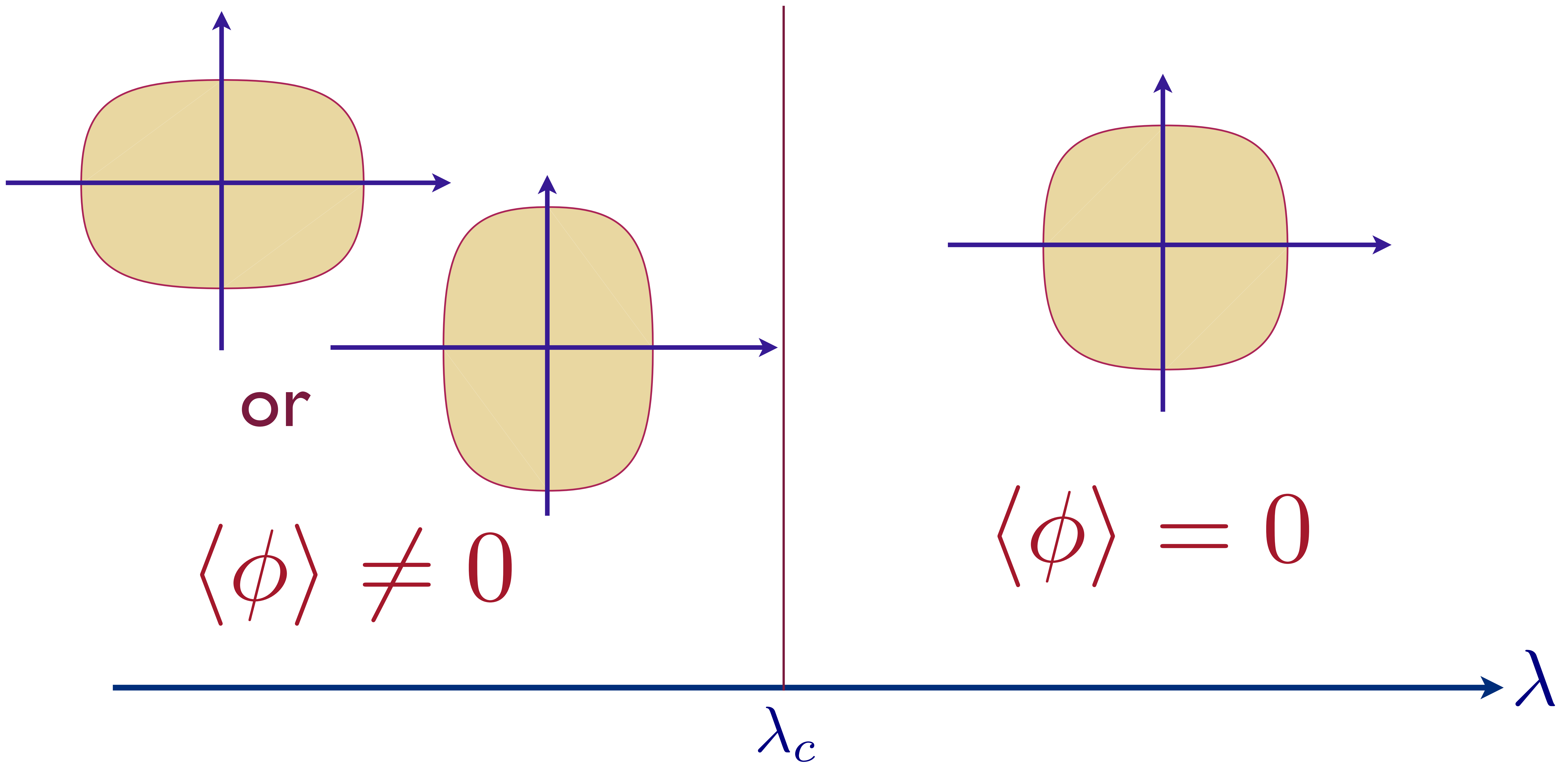
Fermi surface coupled to a critical boson



+

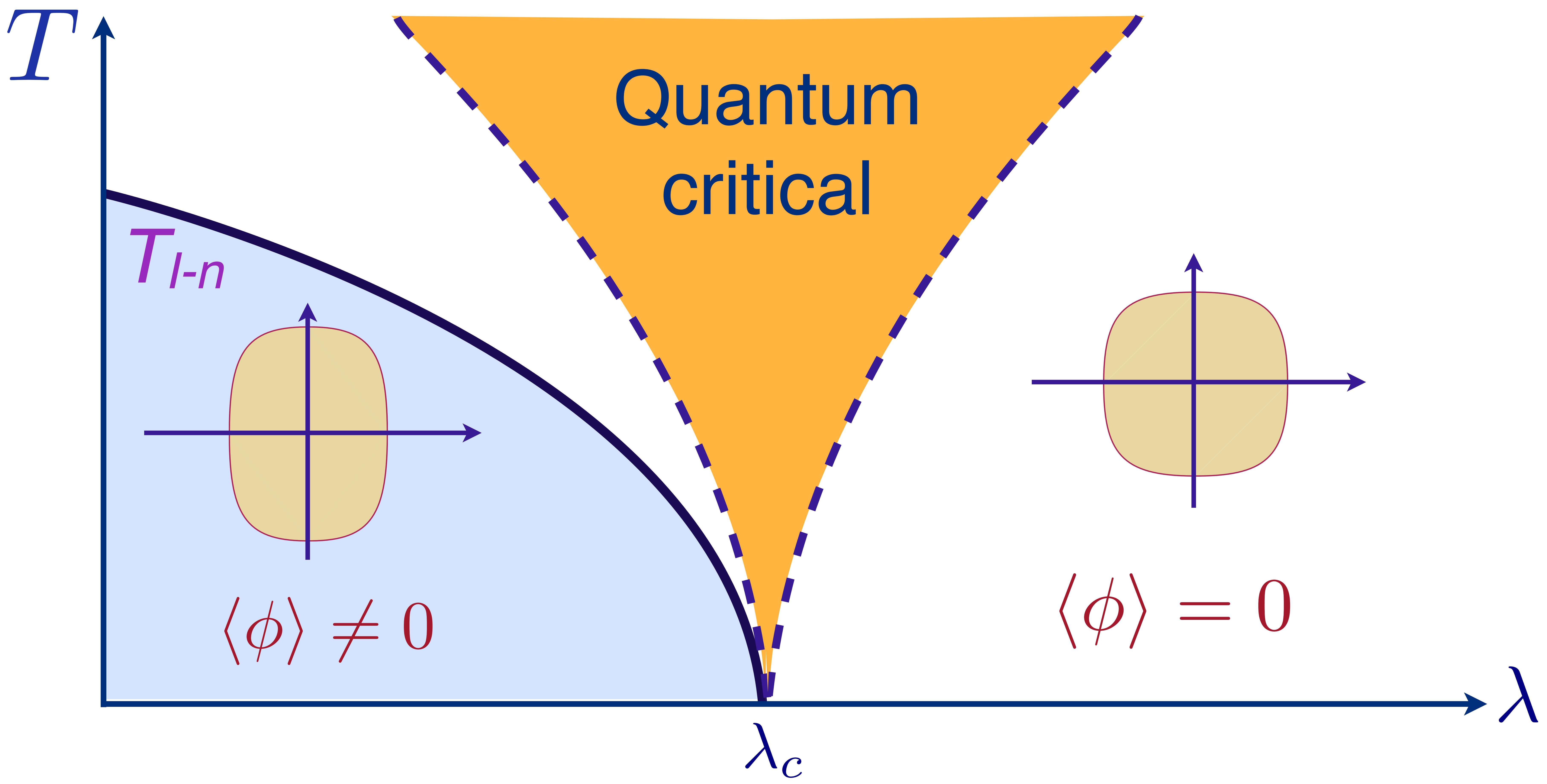
a
critical
boson
ϕ

Quantum criticality of Ising-nematic ordering in a metal



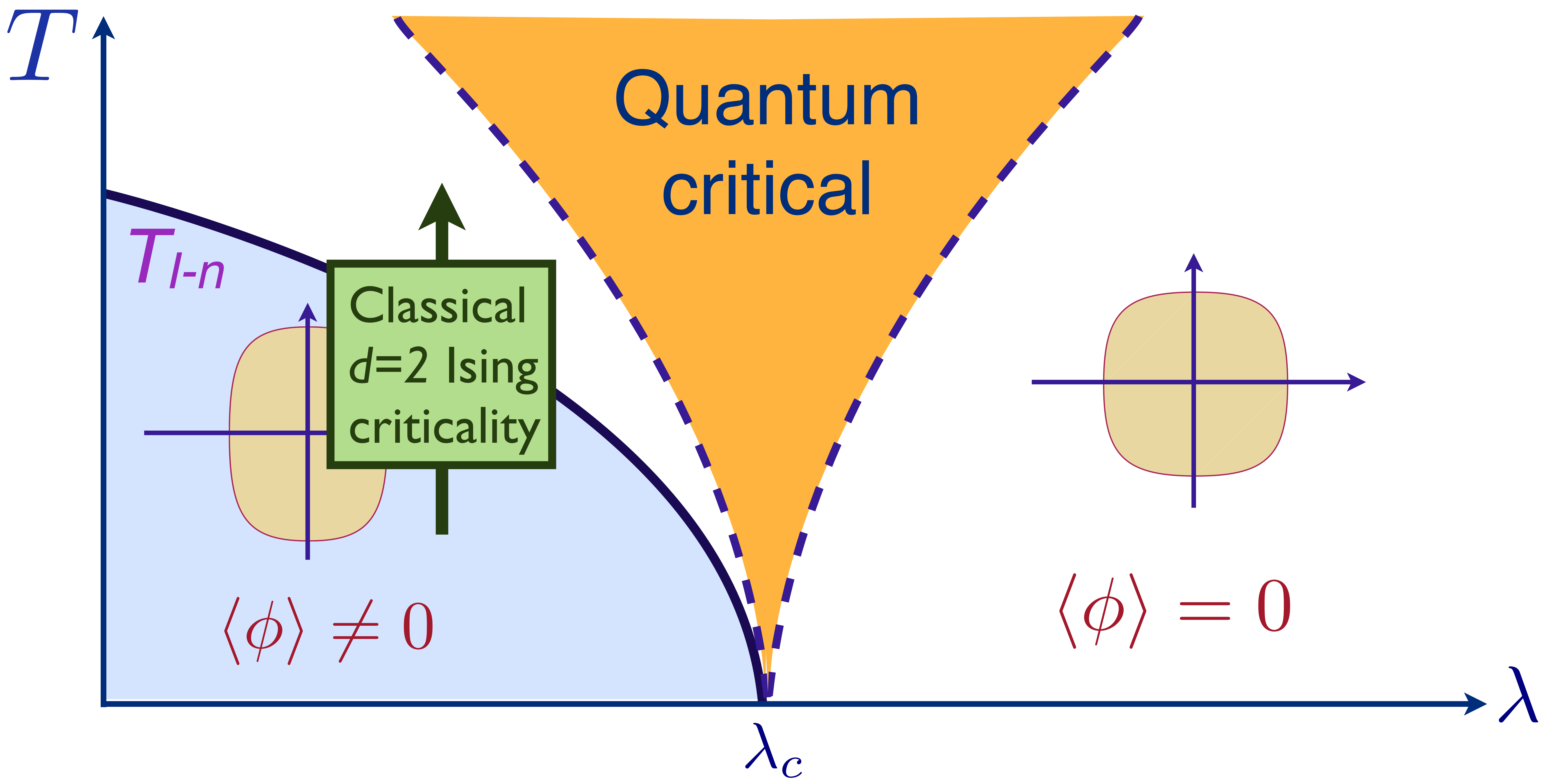
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



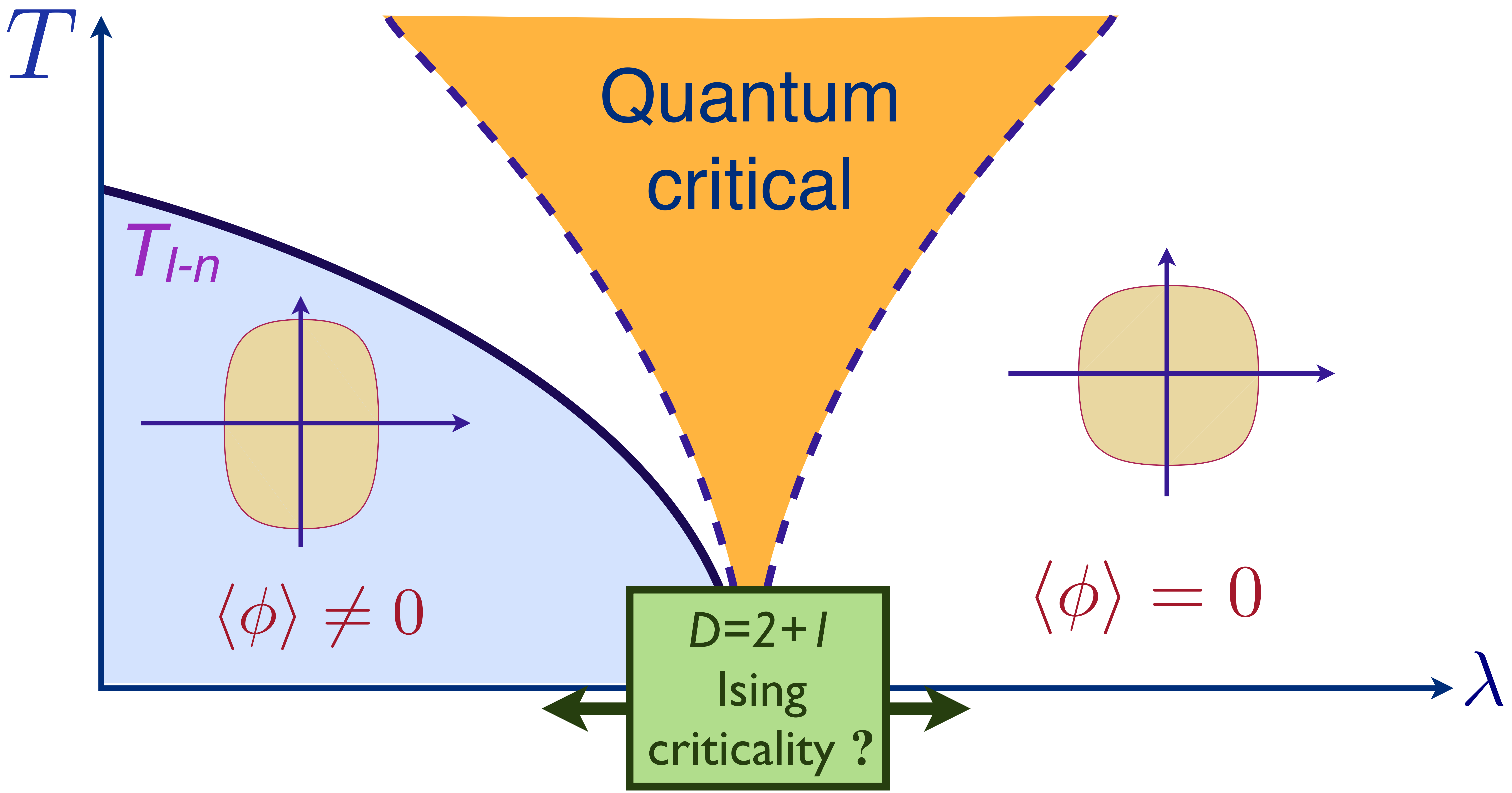
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



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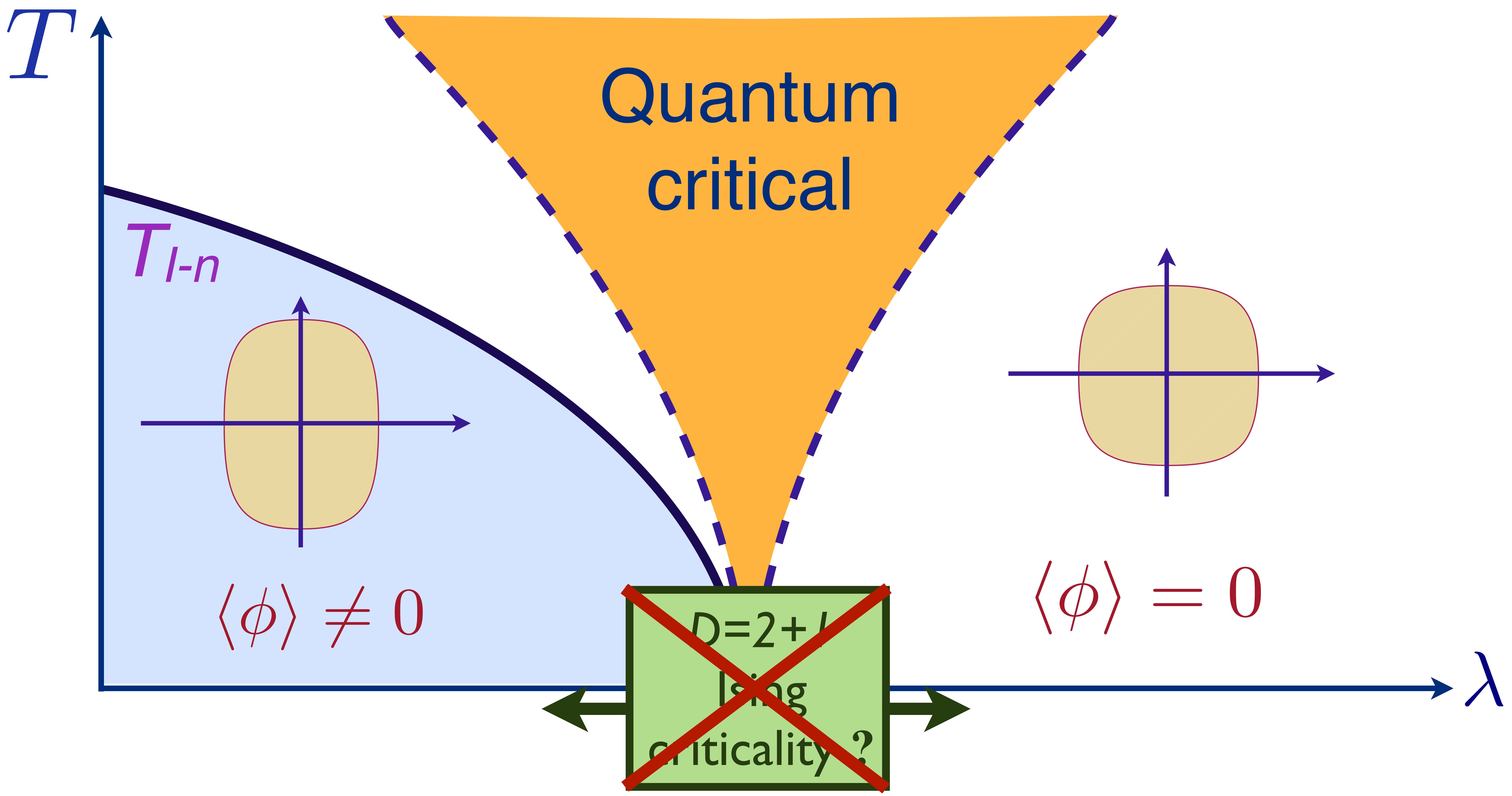
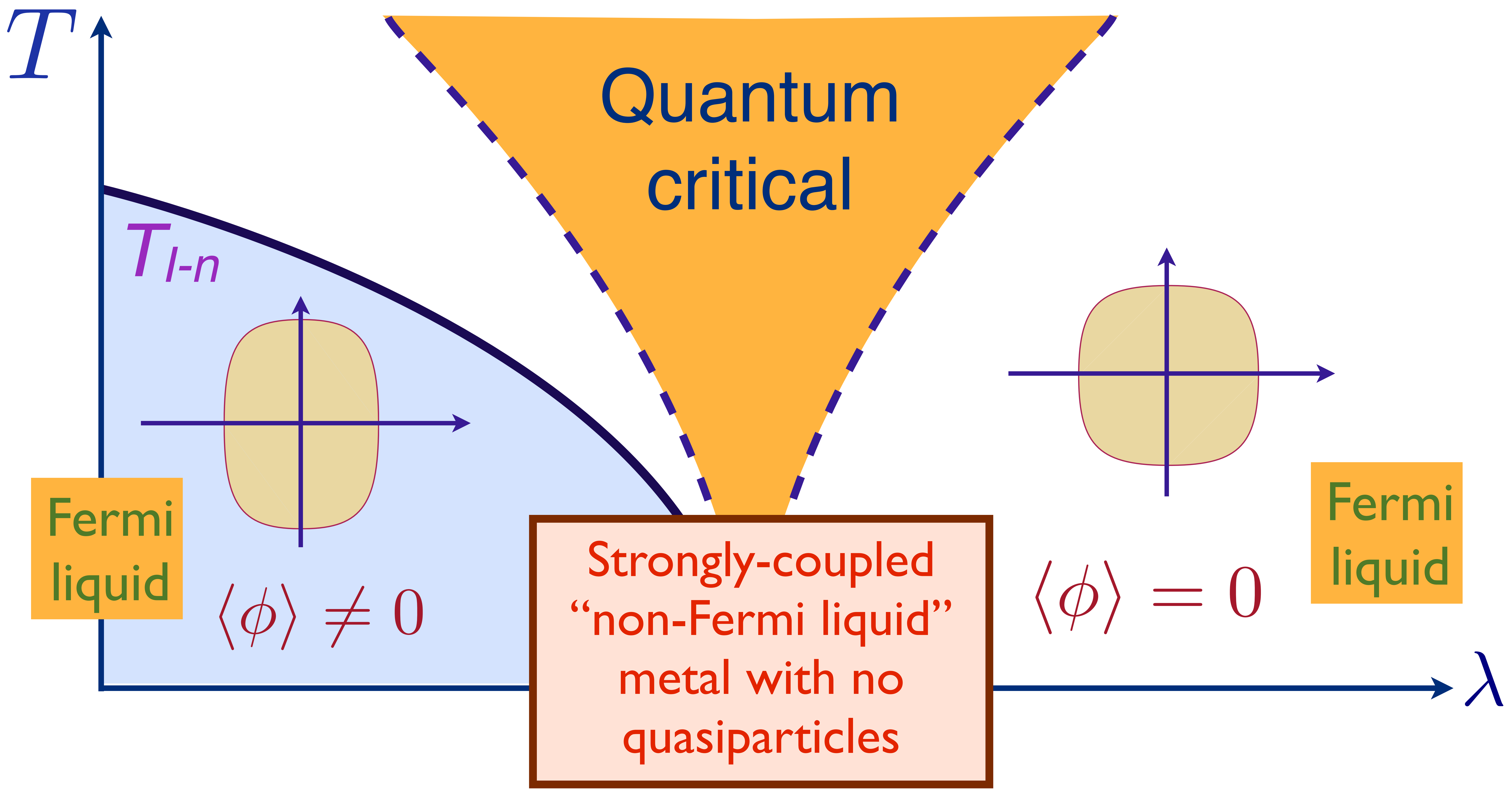


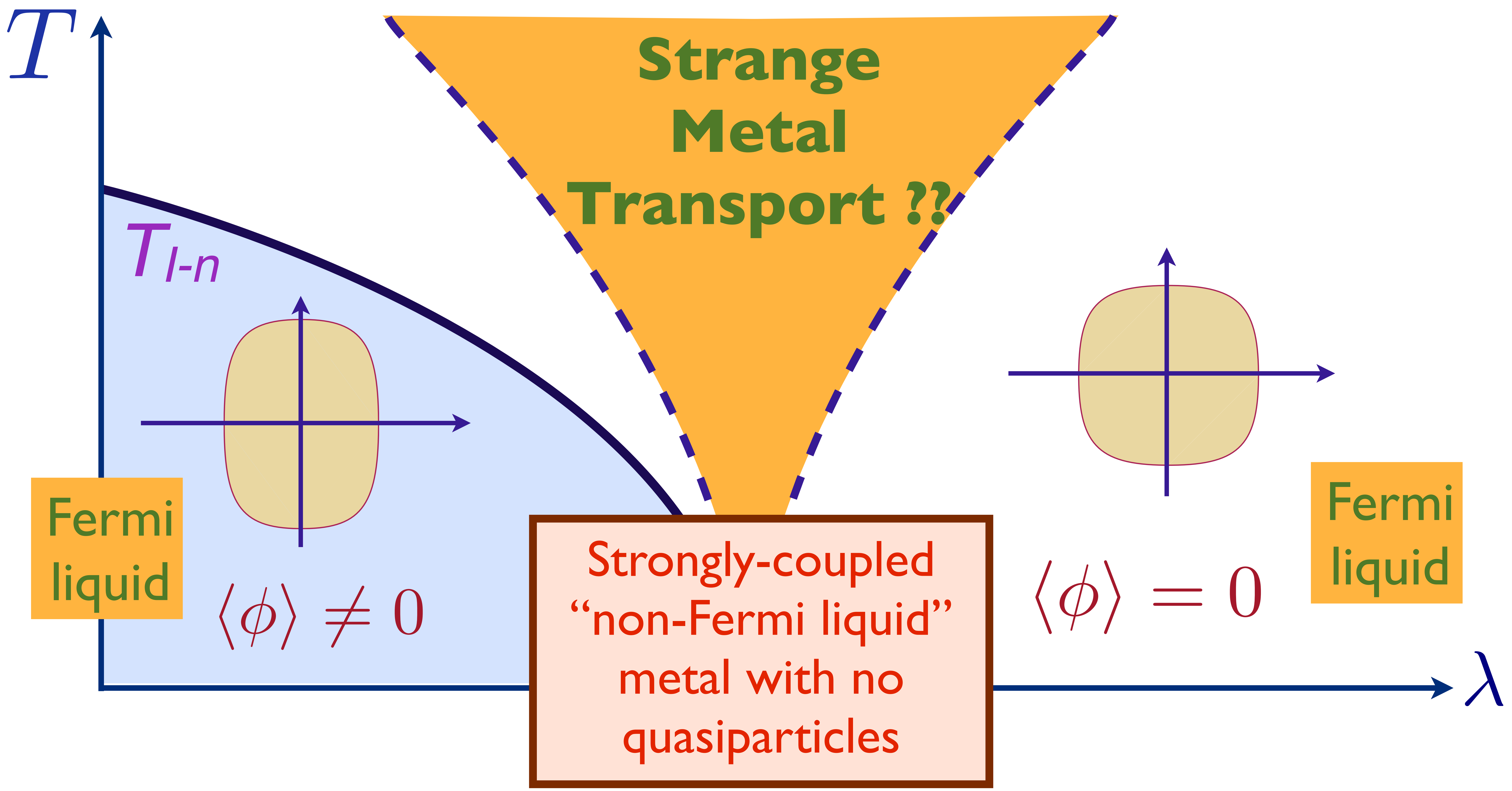
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



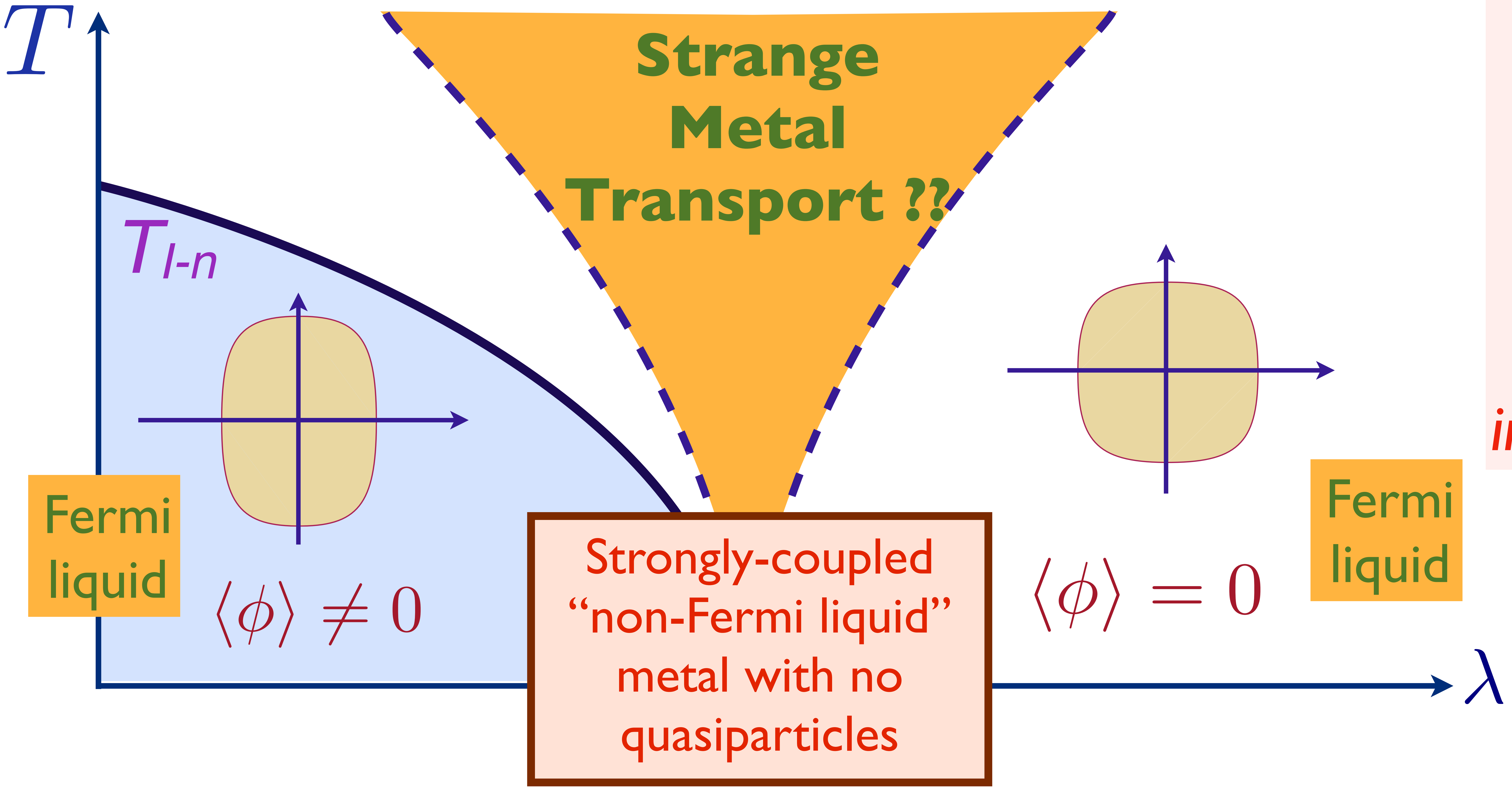
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



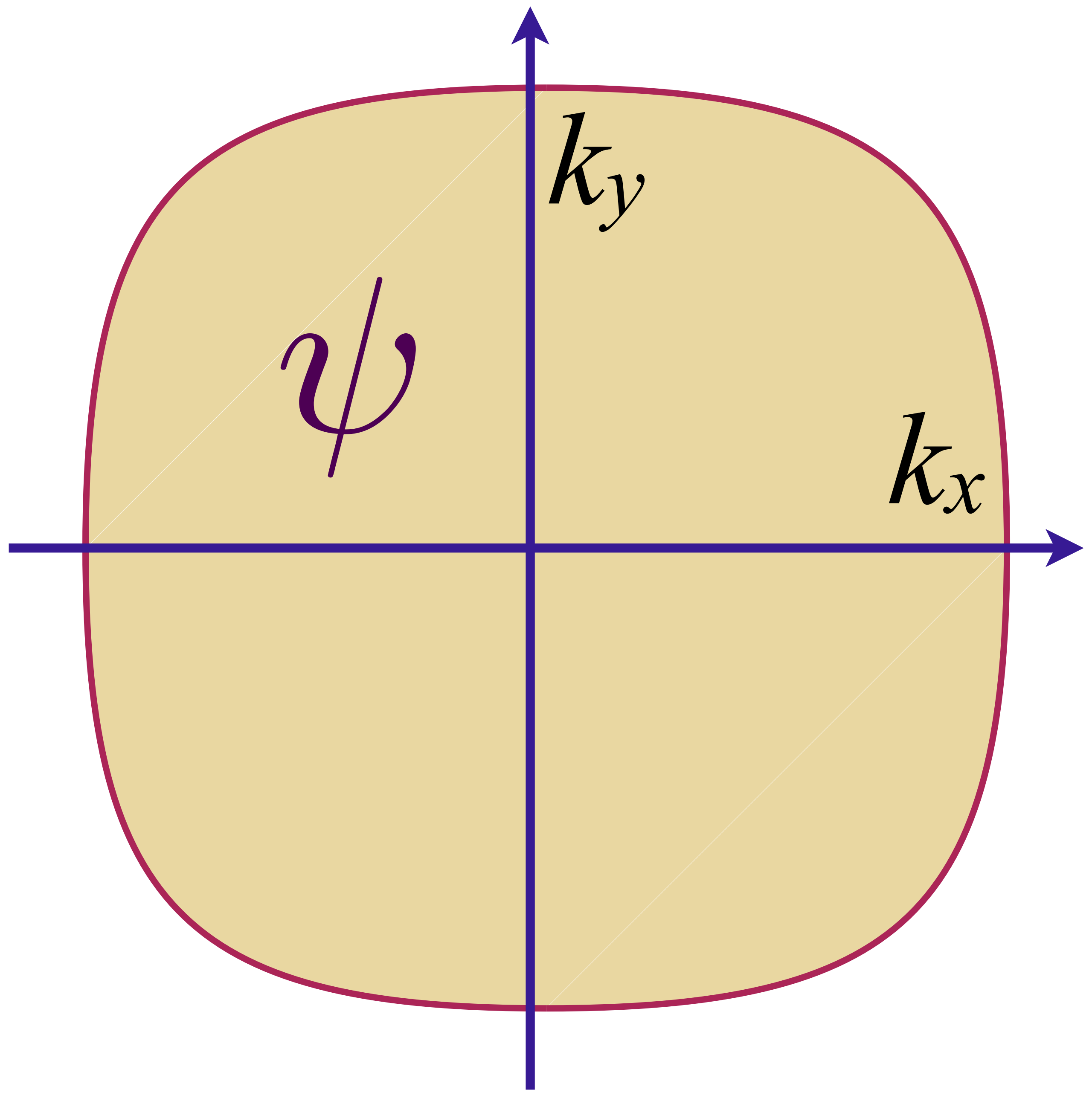
No.

Need spatial *disorder* in the *interactions*

Phase diagram as a function of T and λ

Fermi surface

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

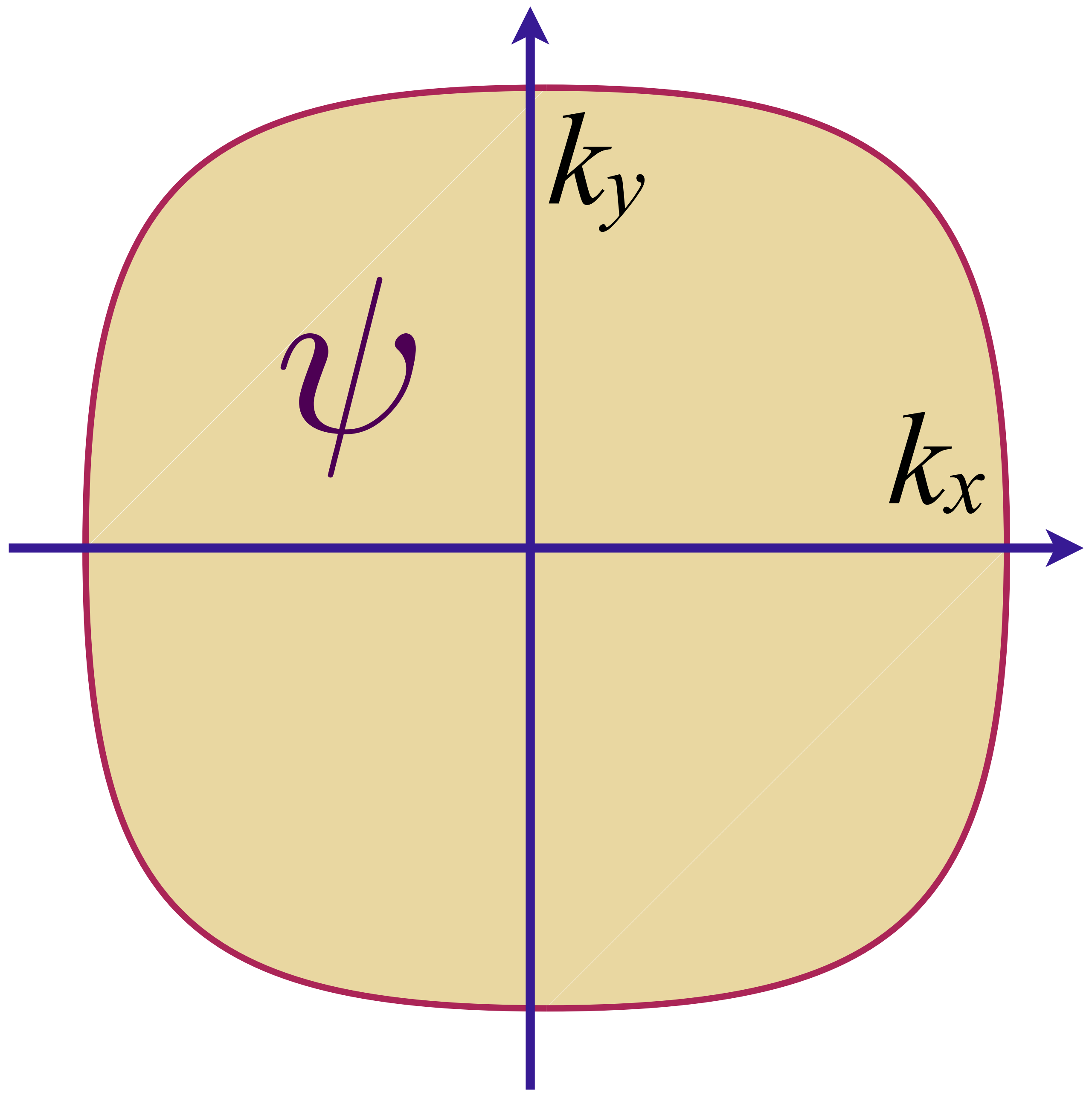


$$-J\, \psi^\dagger(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



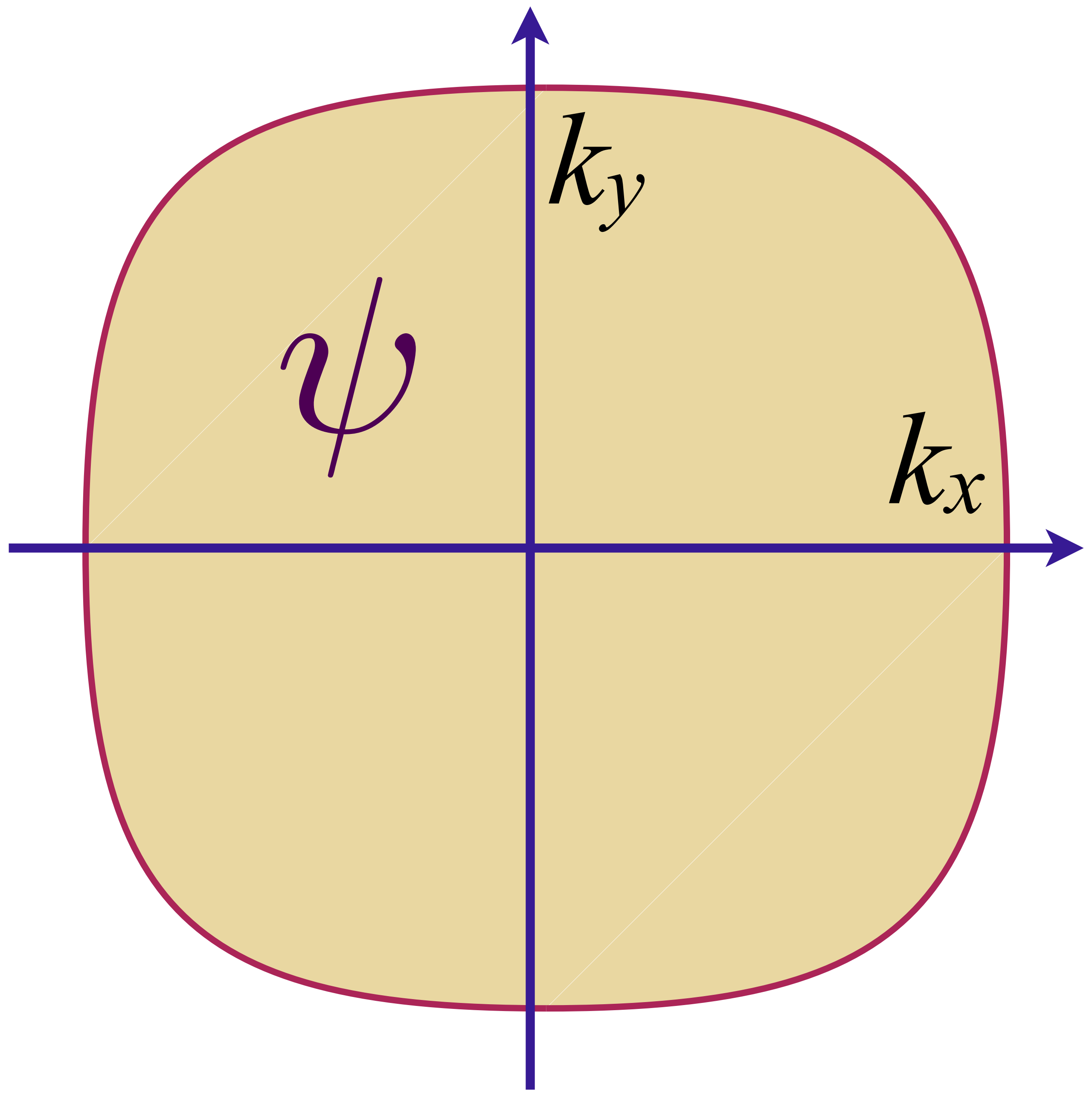
$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

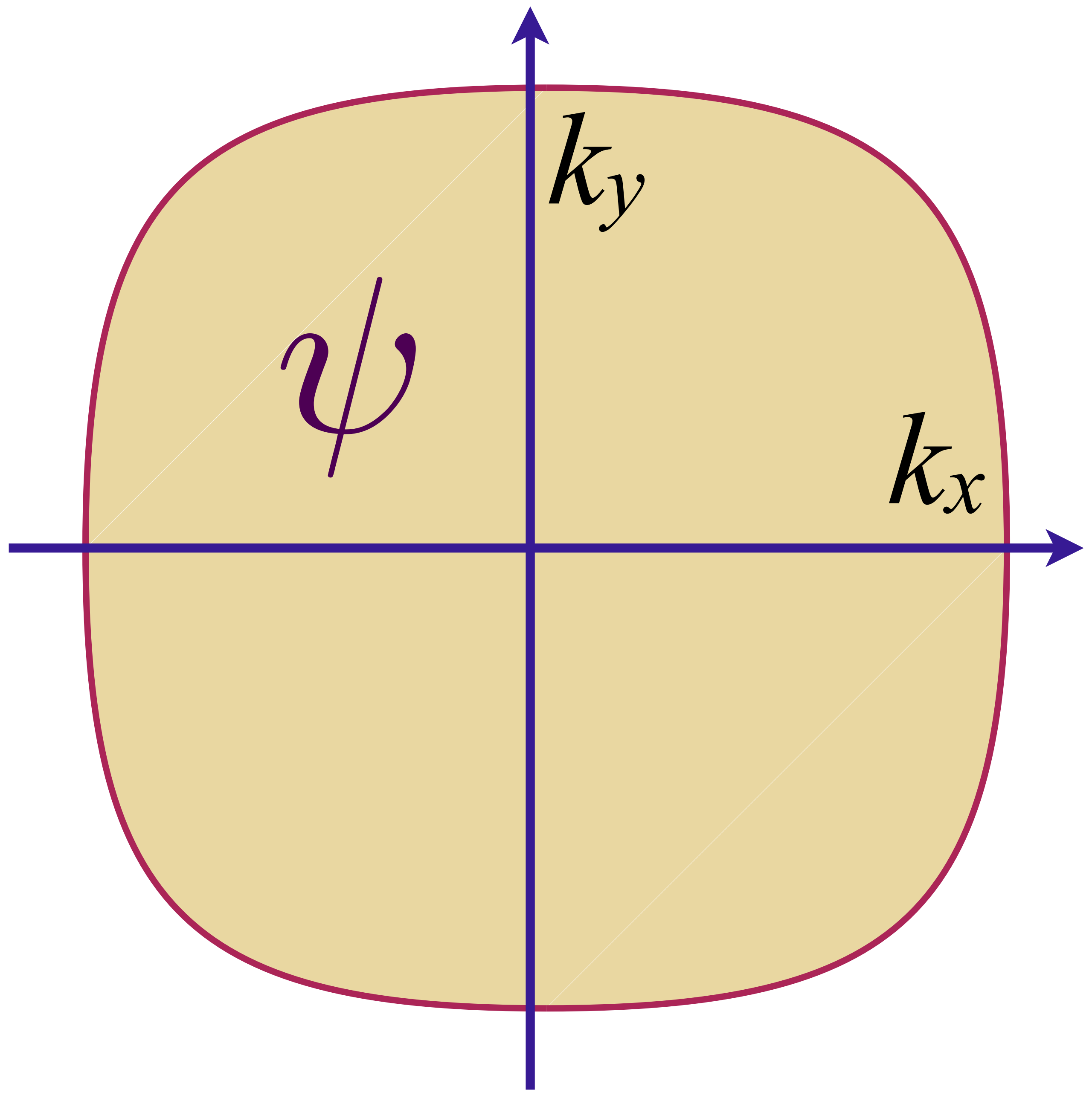
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

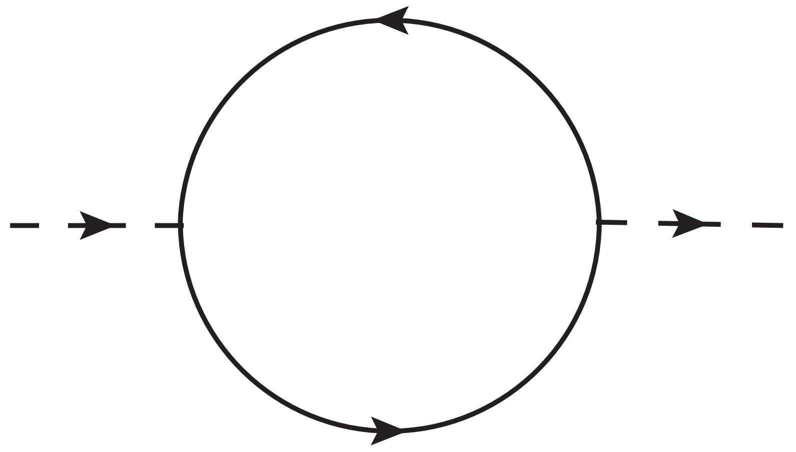
$$[\phi(\boldsymbol{r})]^2 + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Large N limit of:
$$\frac{g_{\alpha\beta\gamma}}{N}\,\psi^\dagger_\alpha(\boldsymbol{r})\psi_\beta(\boldsymbol{r})\,\phi_\gamma(\boldsymbol{r})$$
with $\alpha, \beta, \gamma = 1 \ldots N$
and $g_{\alpha\beta\gamma}$ random in flavor space,
as in *Yukawa-SYK* models
of fermions and bosons

Fermi surface coupled to a critical boson

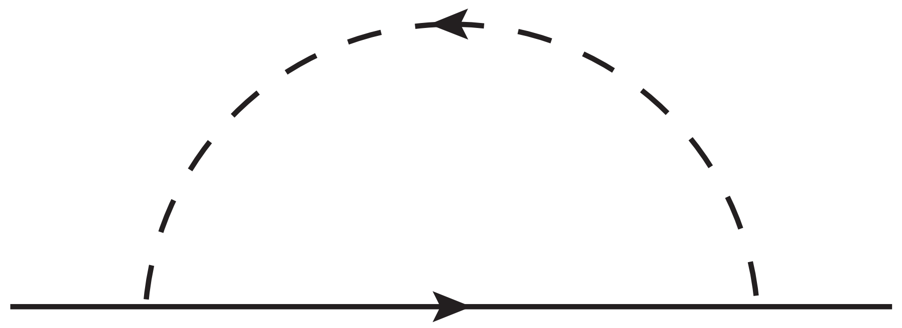
"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Boson self energy $\Pi(q, i\Omega) \sim -g^2 \frac{|\Omega|}{q}$ (Landau damping)



Boson Green's function $D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|/q}$

Fermion self energy $\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}$



Fermion Green's function $G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$

P.A. Lee (1989)

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface + critical boson

These results can also be obtained from the saddle-point and response functions of a G-Σ-D-Π action. Such an action can formally be obtained in a Yukawa-SYK-like large-N limit of a theory with couplings which are random in an additional (fictitious) flavor space.

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

$$S_{\text{all}} = -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi)$$
$$+ \int d\tau d^2r \int d\tau' d^2r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}')$$
$$+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}')\Big] \,.$$

Fermi surface + critical boson

These results can also be obtained from the saddle-point and response functions of a G-Σ-D-Π action. Such an action can formally be obtained in a Yukawa-SYK-like large-N limit of a theory with couplings which are random in an additional (fictitious) flavor space.

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

Saddle-point equations:

Migdal-Eliashberg

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}),$$

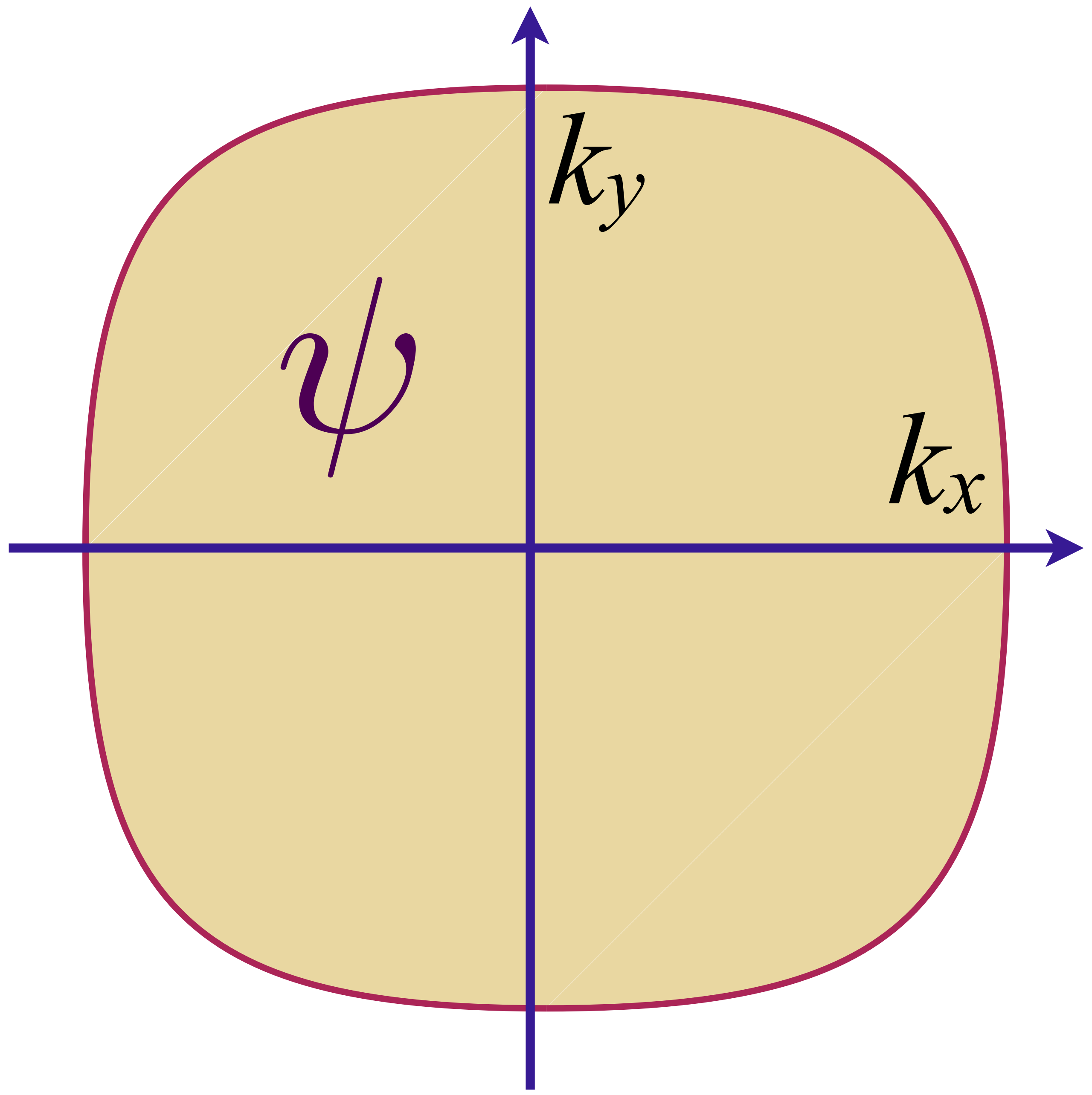
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

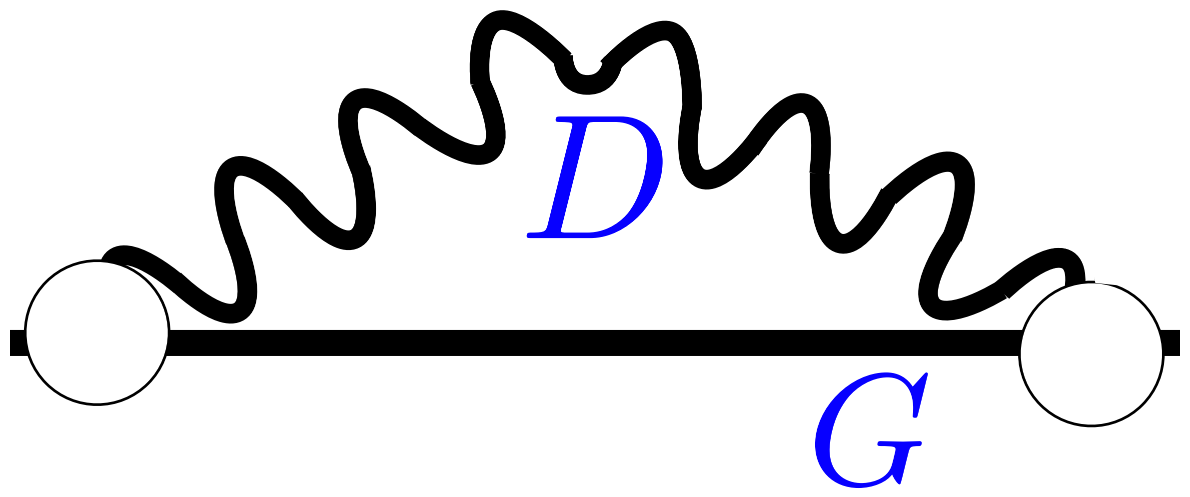
Fermi surface + critical boson

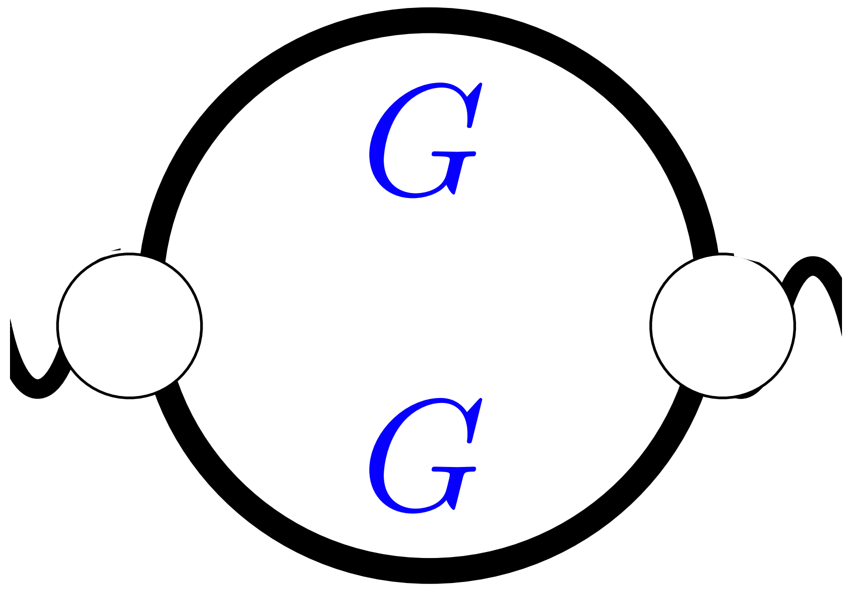
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$\Sigma =$ 

$\Pi =$ 

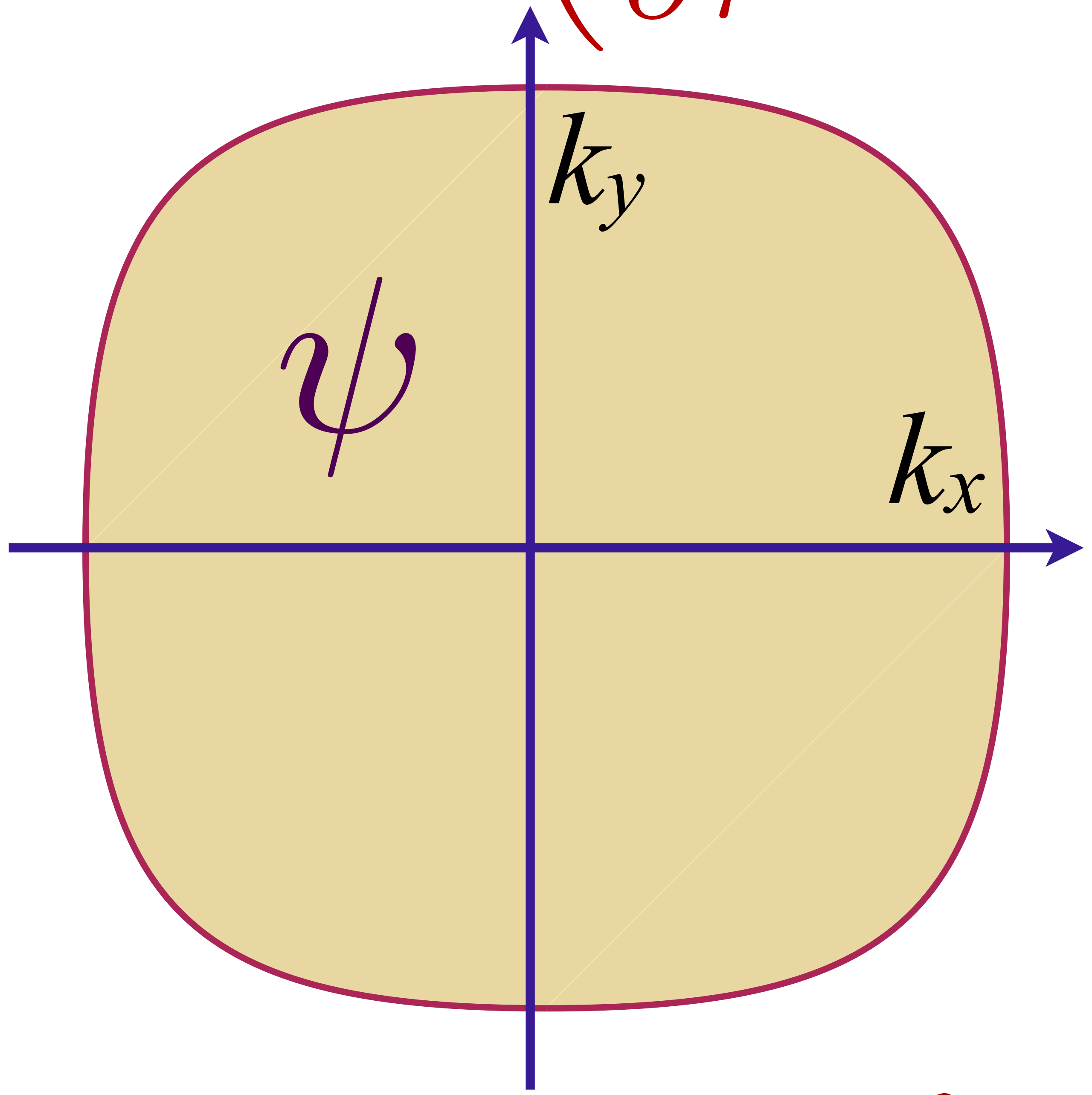
Solution of Migdal-Eliashberg equations for electron (G) and boson (D) Green's functions at small ω:

P.A. Lee (1989)

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i \text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$[\phi(\boldsymbol{r})]^2 + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

Transport—a perfect metal!

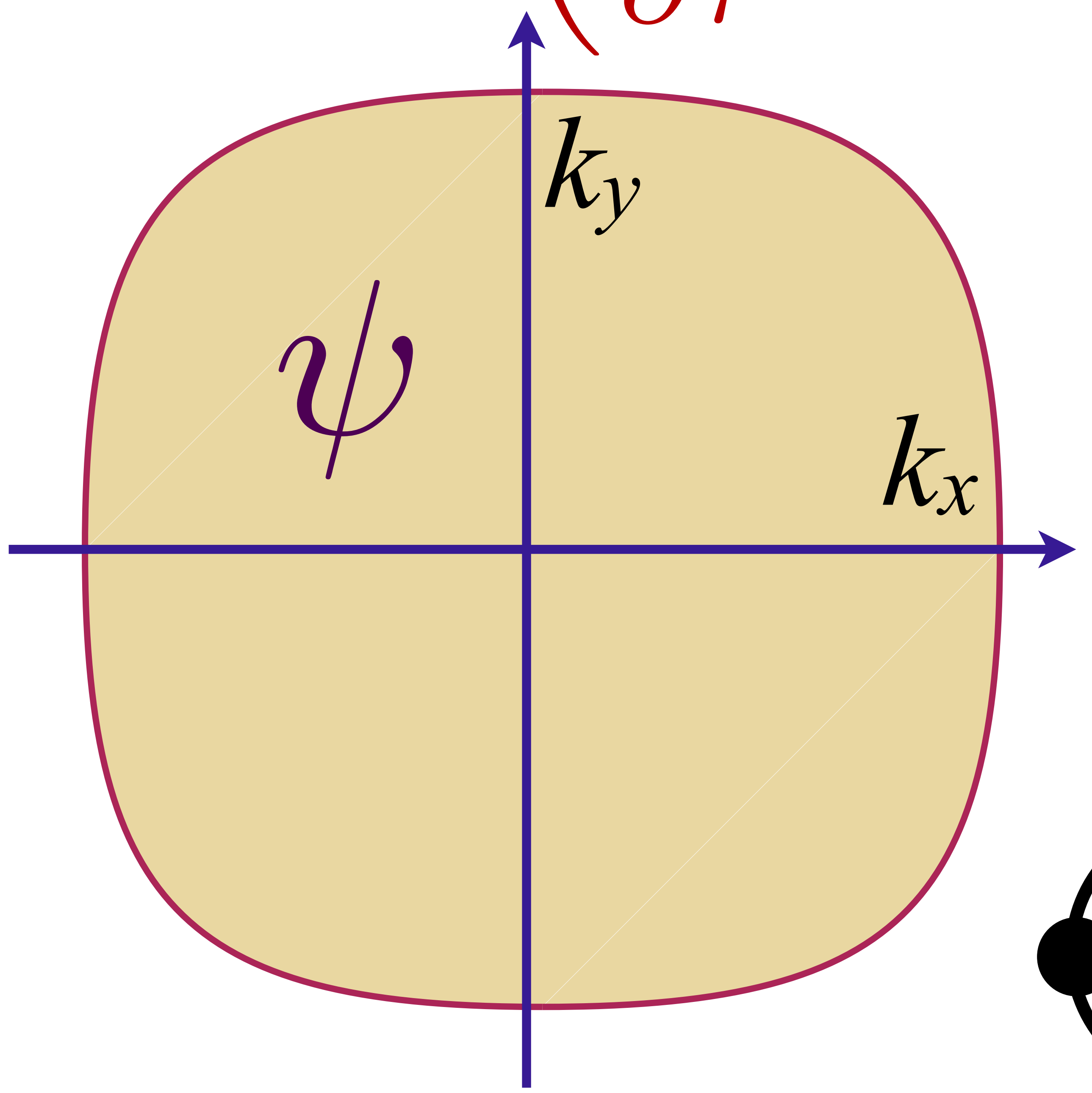
Conservation of momentum and fermion-boson drag imply:

$$\text{Re}\,[\sigma(\omega)] = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Fermi surface + critical boson

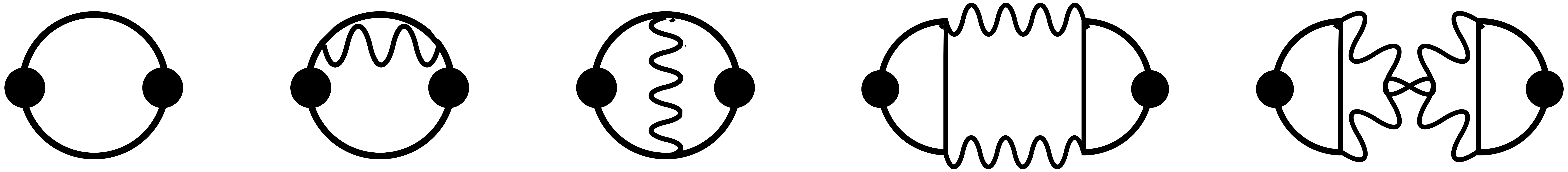
$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$[\phi(\boldsymbol{r})]^2 + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

Optical conductivity—Diagrams



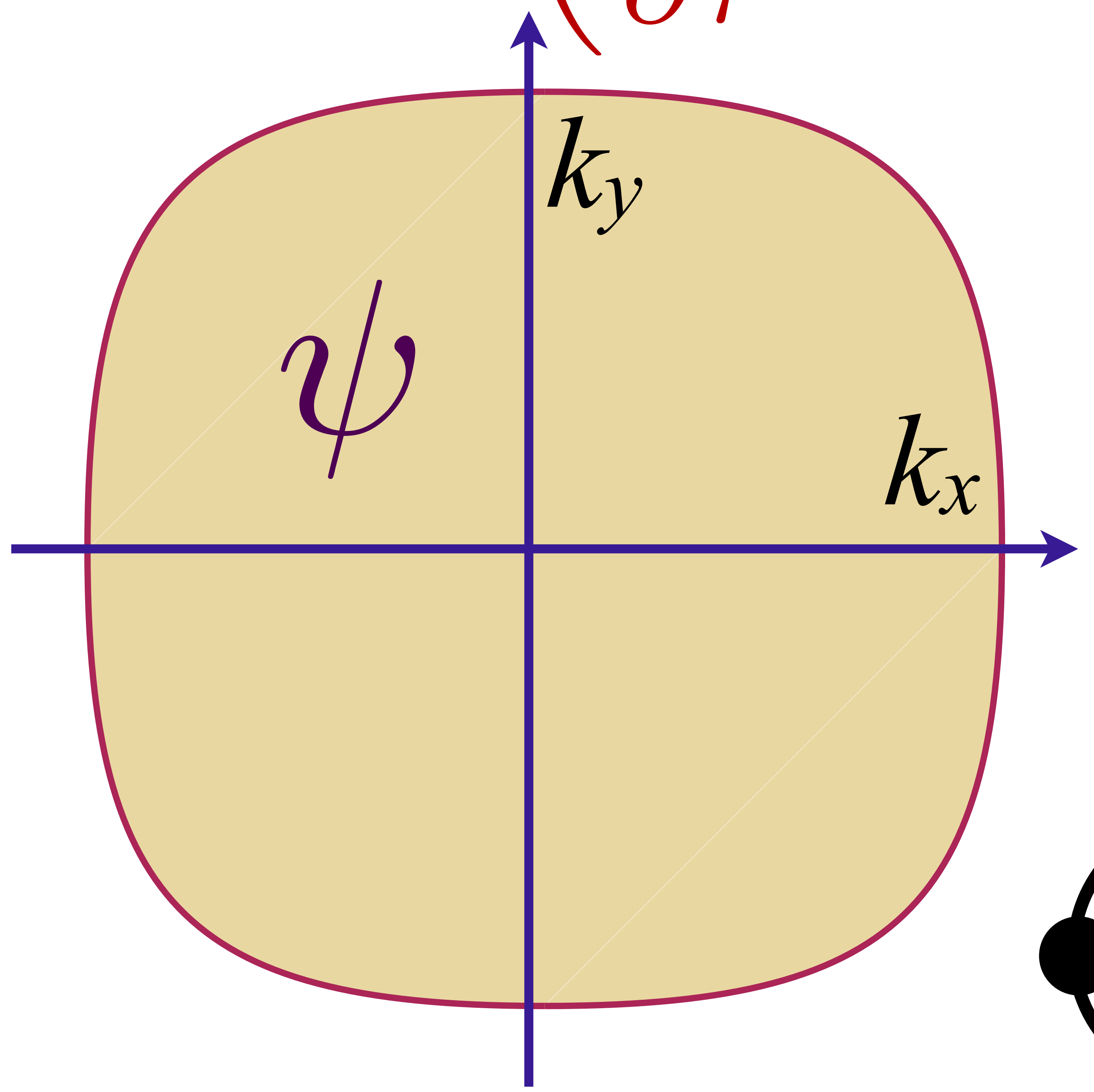
$$\mathrm{Re}\left[\sigma(\omega)\right] = C\, |\omega|^{-2/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

Fermi surface + critical boson

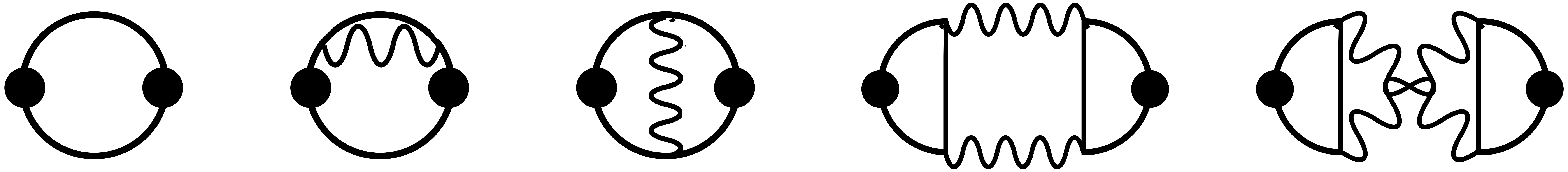
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Optical conductivity—Diagrams



$$\mathrm{Re}\,[\sigma(\omega)] = C\,|\omega|^{-2/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

$\boxed{C = 0}$; $\quad \sigma(\omega) \sim i/(\omega) + \omega^0 + \ldots$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)
Z. Darius Shi, D.V. Else, H. Goldman and T. Senthil, SciPost Phys. **14**, 113 (2023)

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

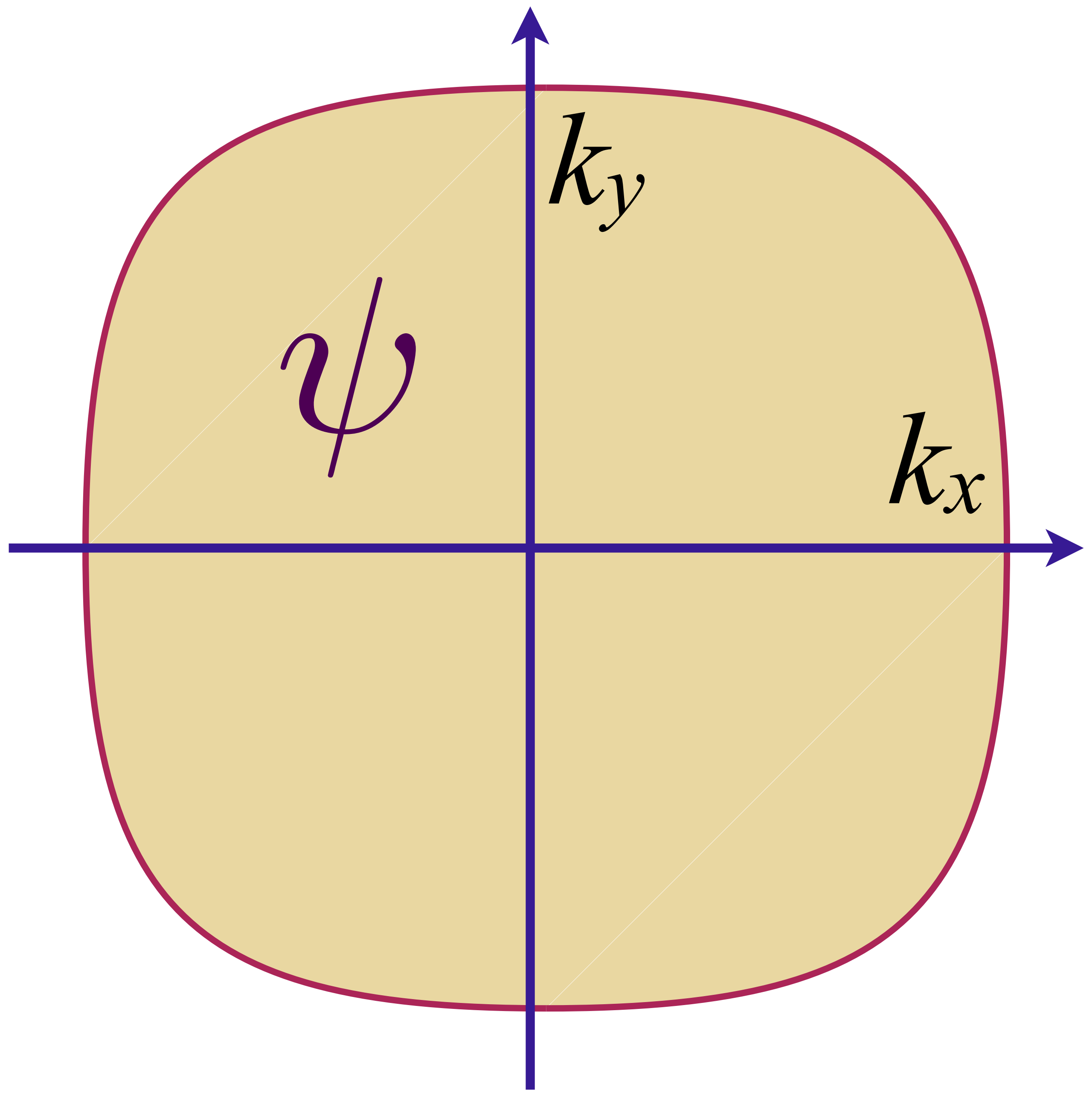
A marginal Fermi liquid but NO strange metal transport

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

$$S_{\text{all}} = -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi)$$
$$+ \int d\tau d^2 r \int d\tau' d^2 r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}')$$
$$+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \Big]$$

Fermi surface + critical boson with potential disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi\,\exp(-NS_{\rm all})$$

Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

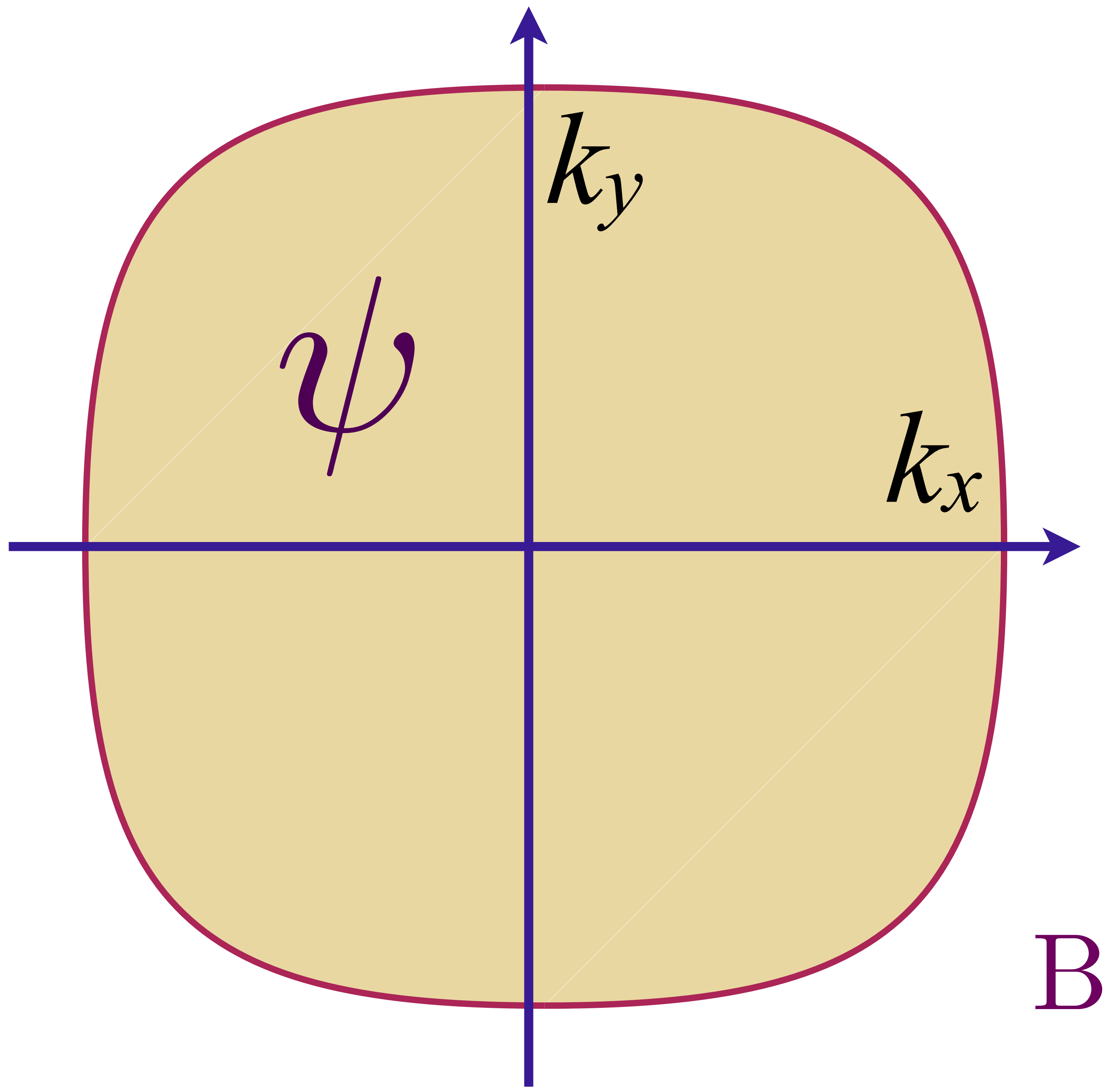
$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$
$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson self energy: $\Pi \sim -\frac{g^2}{v^2}|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$

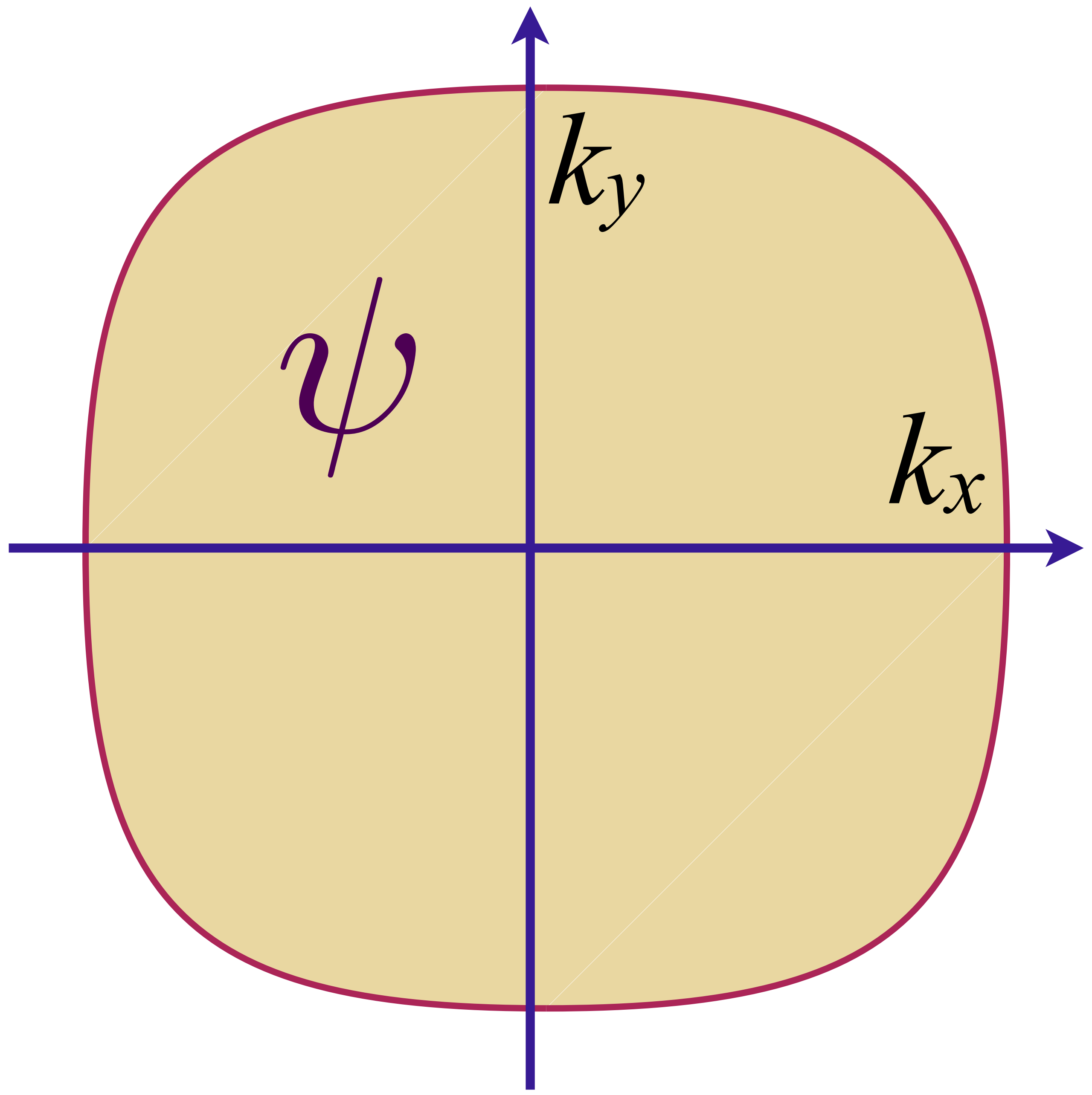
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\rm in}(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

Fermi surface + critical boson with potential disorder

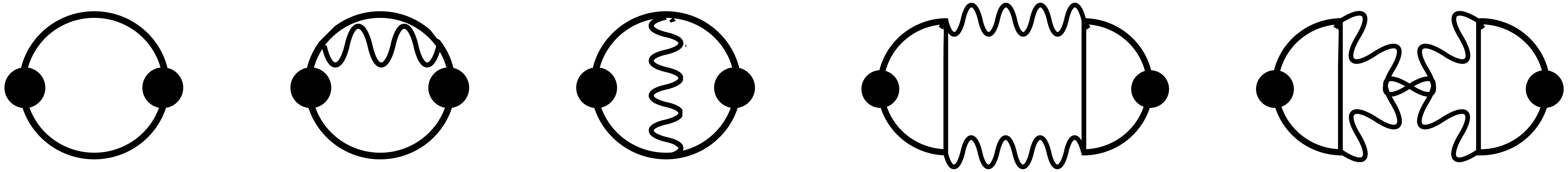
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$



Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\rm trans}} - i\omega}$; $\dfrac{1}{\tau_{\rm trans}} \sim v^2$

MFL self-energy cancels in transport.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

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A marginal Fermi liquid but NO strange metal transport

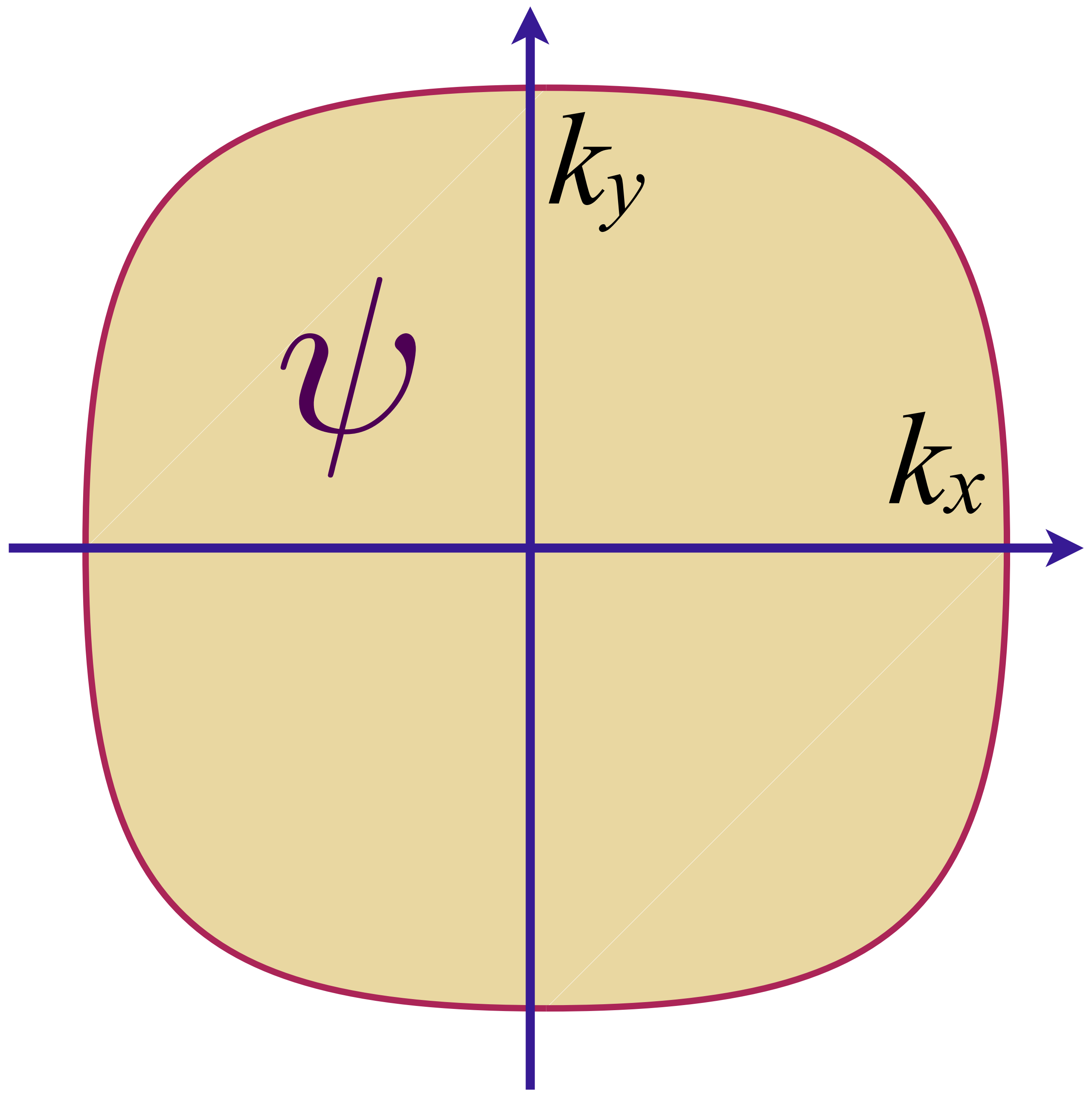
Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND strange metal transport

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ

e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

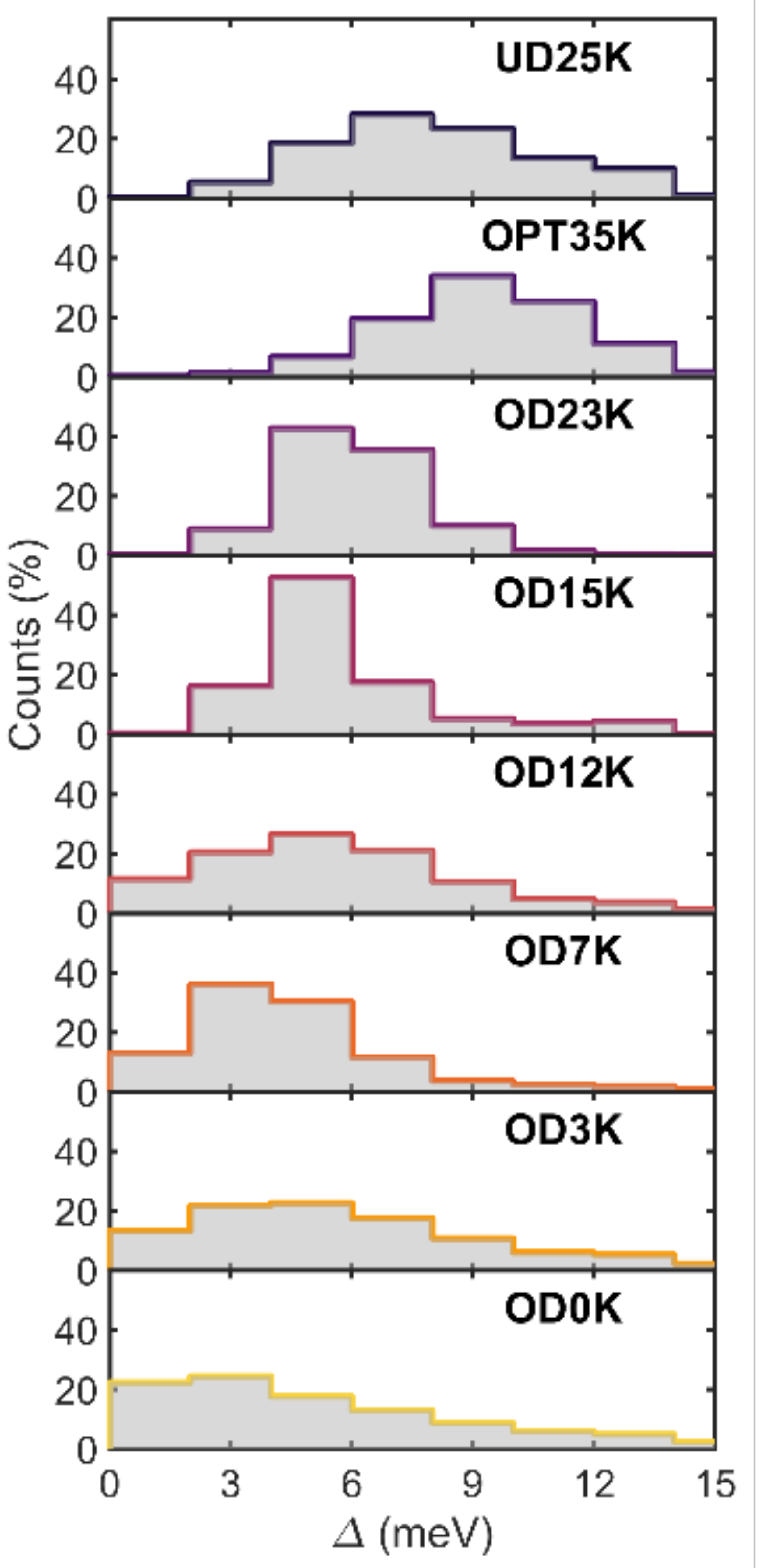
Spatially random interactions!

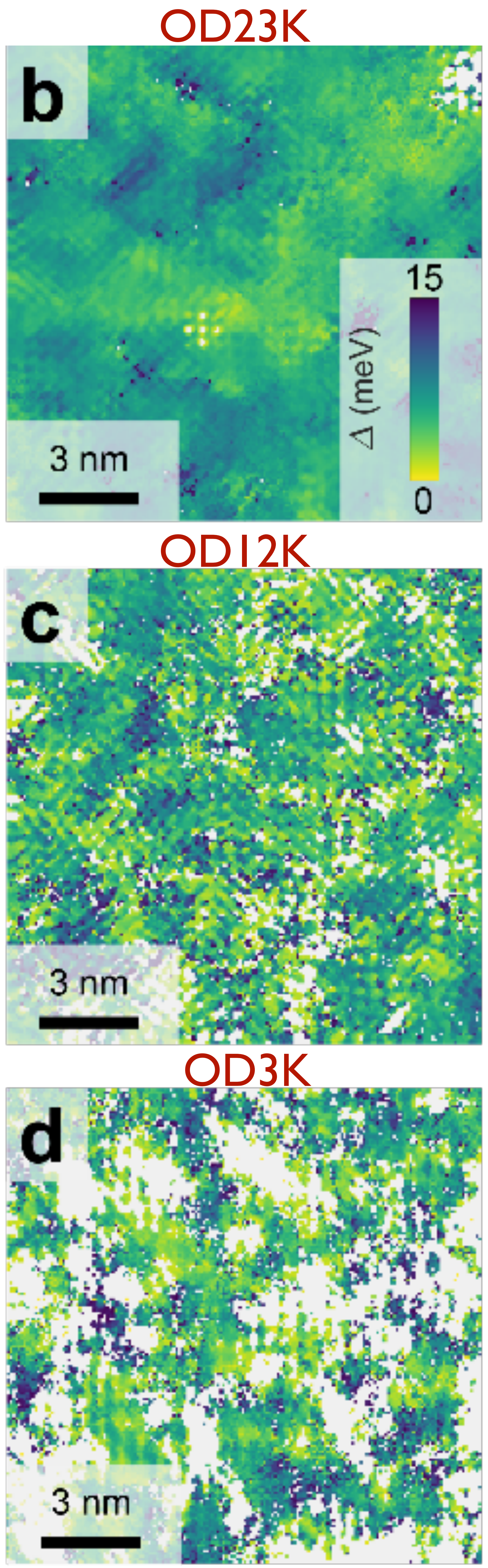
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740



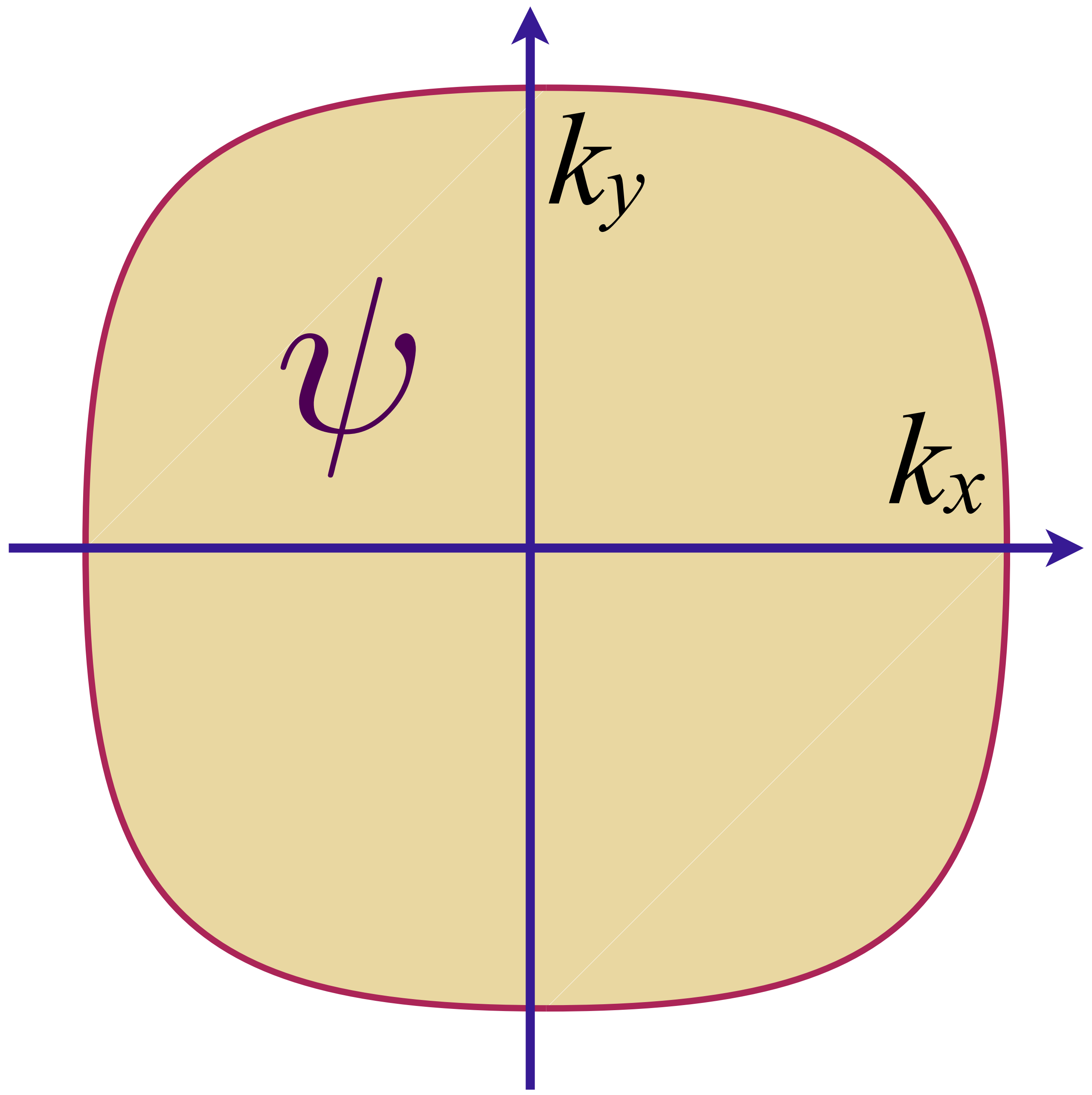


Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

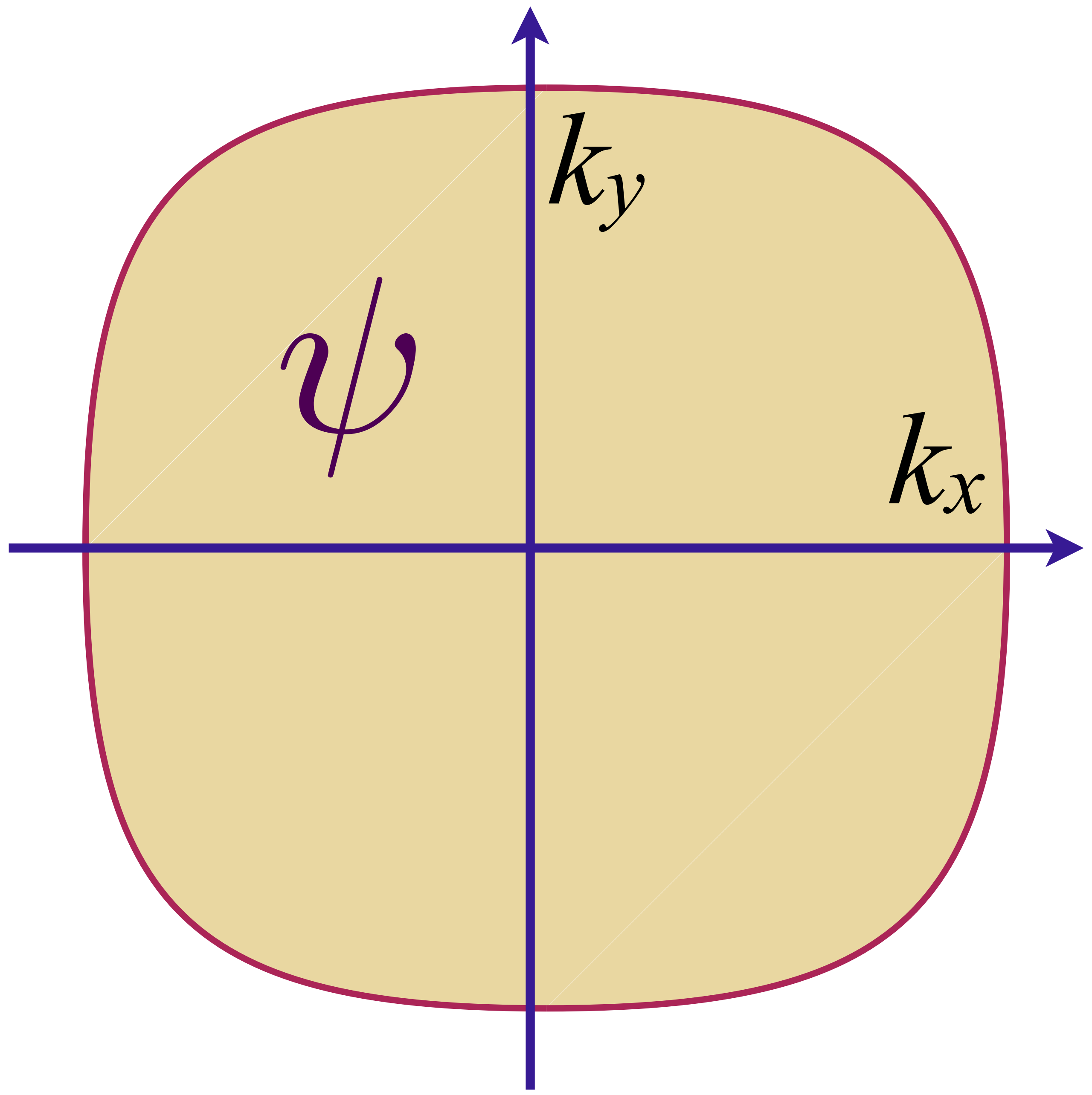
$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

2d-YSYK model: Fermi surface + critical boson with interaction disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} = & -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ & + \int d\tau d^2r \int d\tau' d^2r' \Big[-\Sigma(\tau',\mathbf{r}';\tau,\mathbf{r})G(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{1}{2}\Pi(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') \\ & + \frac{g^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{v^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \\ & + \frac{g'^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \Big] . \end{aligned}$$

2d-YSYK model: Fermi surface + critical boson with interaction disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

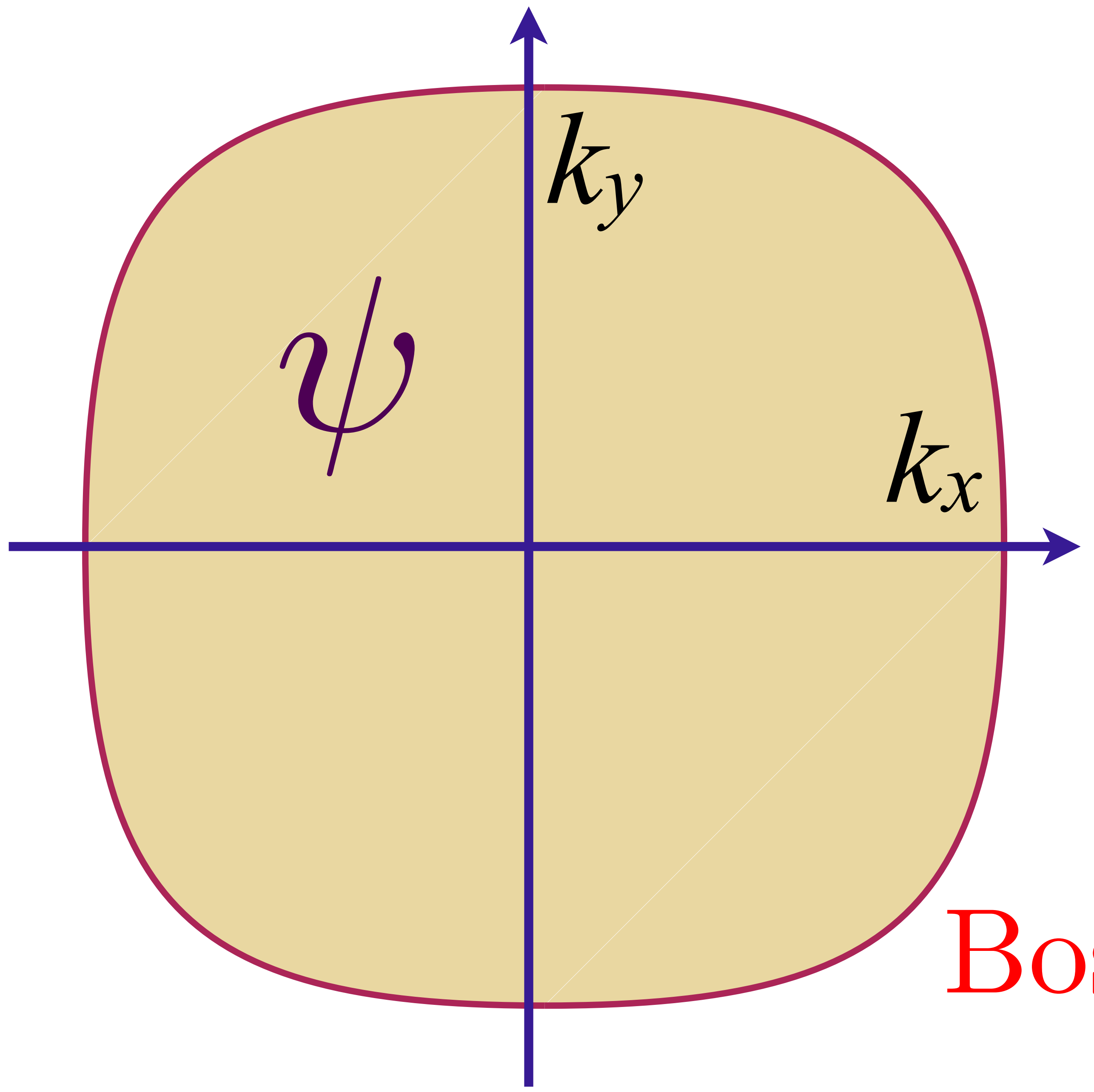
$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

$$\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\left(\frac{g^2}{v^2} + g'^2\right)\omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega|$$

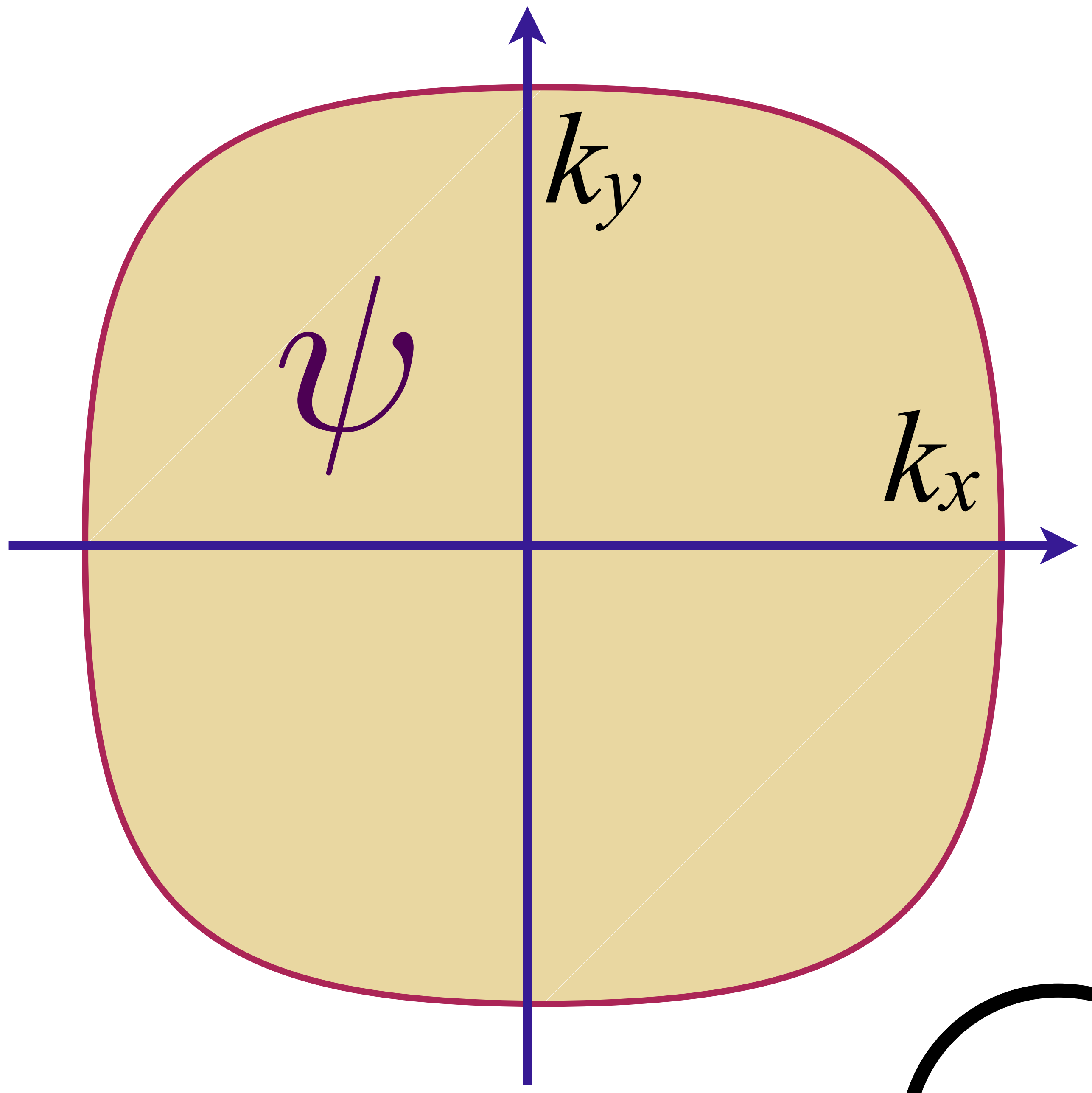
Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

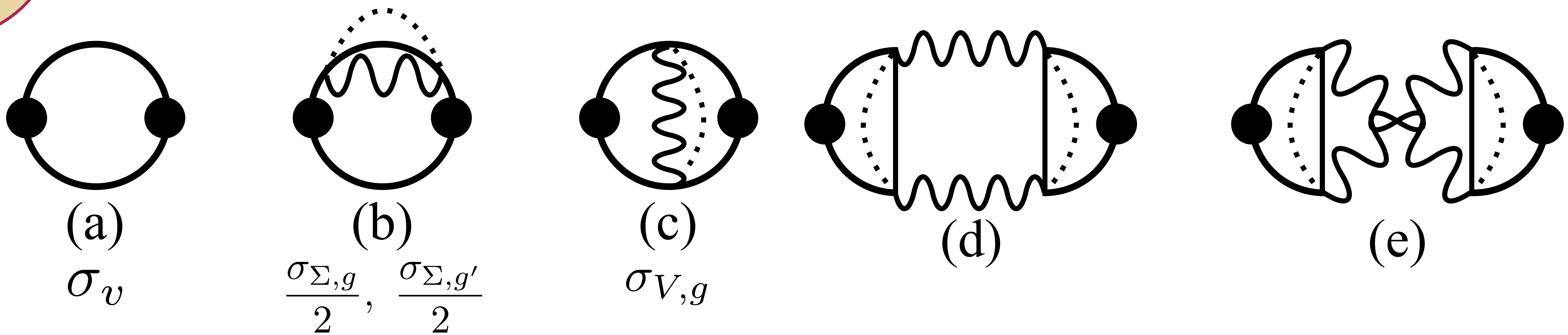
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Conductivity:



+ all ladders and bubbles…..

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)}\right)\text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND strange metal transport

Transport properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

Electronic properties of a marginal Fermi liquid:

1. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

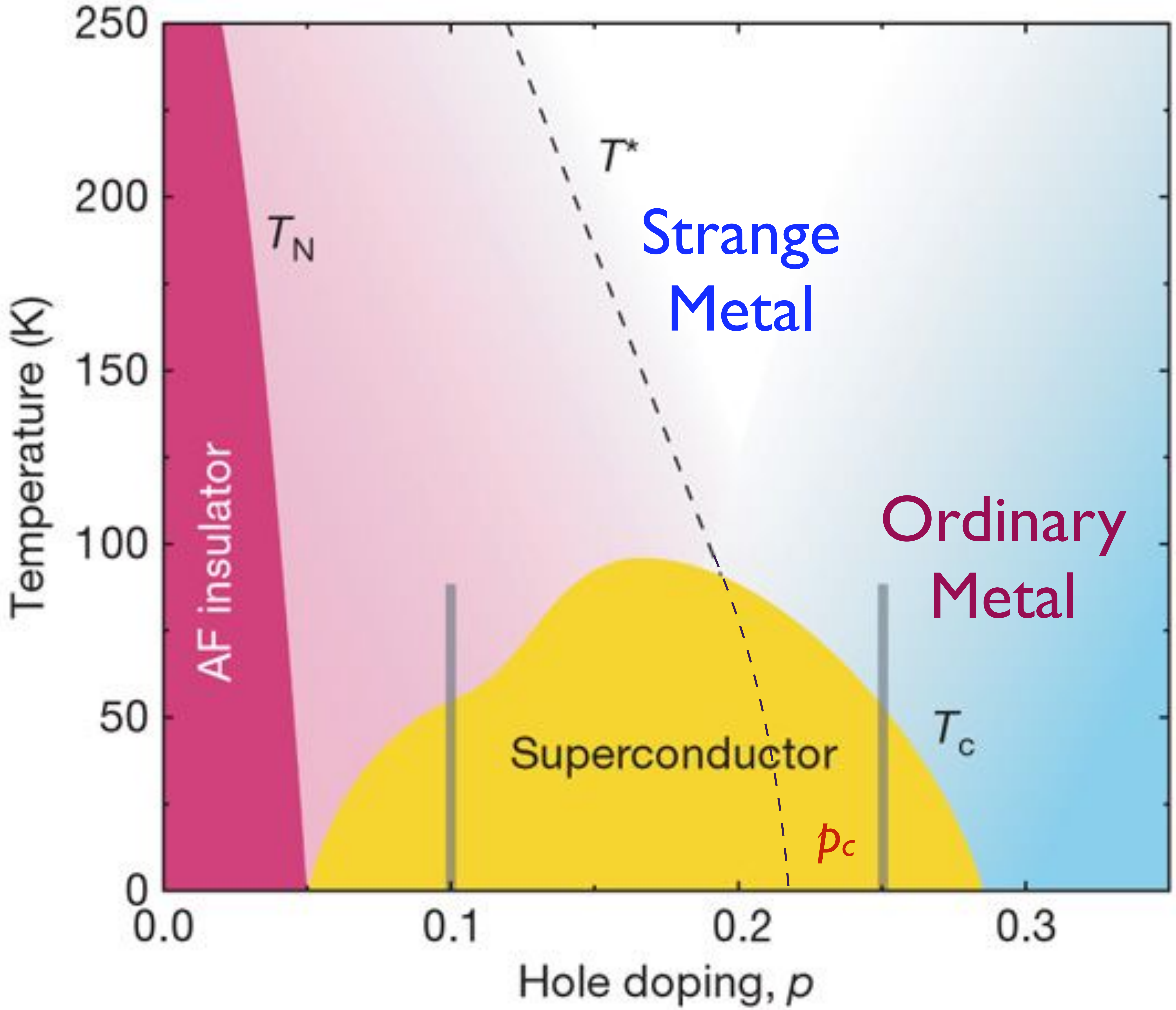
2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP (2022)

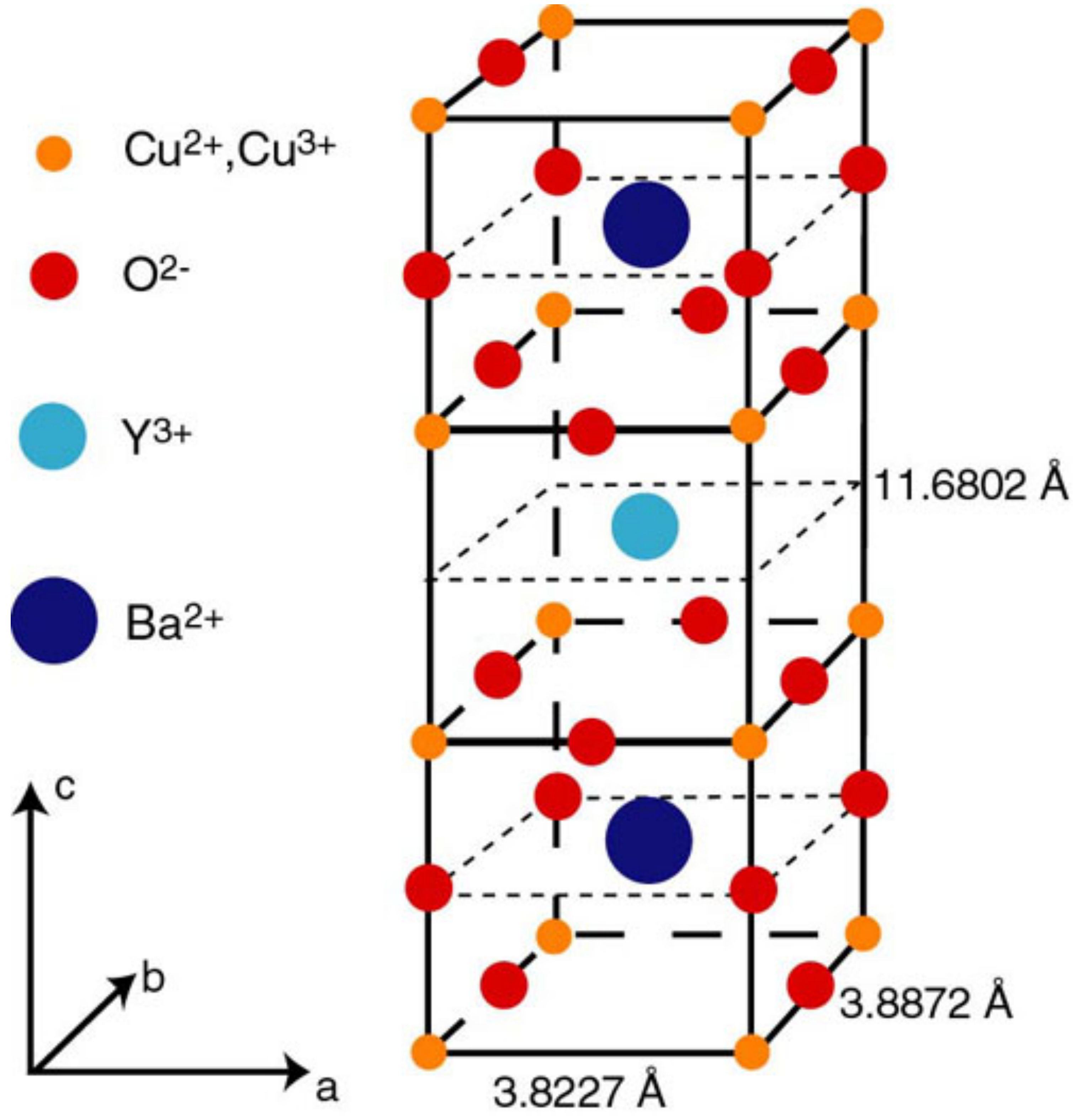
1. Ising-nematic ordering in a disordered metal

2. FL*-FL quantum-criticality in the cuprates

3. Theory of the "foot":
   quantum Griffiths SDW phase



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c

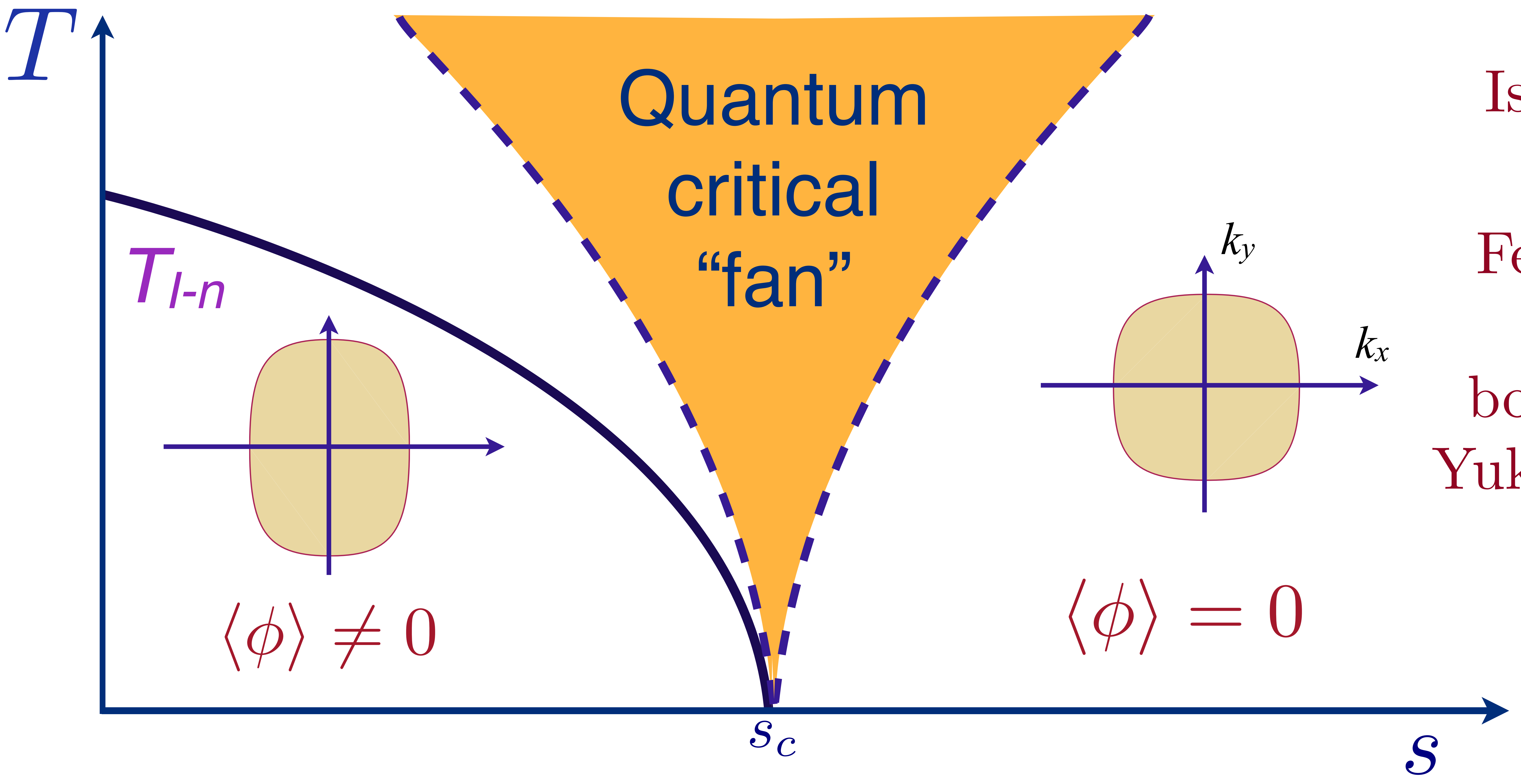


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

Quantum phase transitions in two-dimensional metals

Type I: Fermi surface deformation



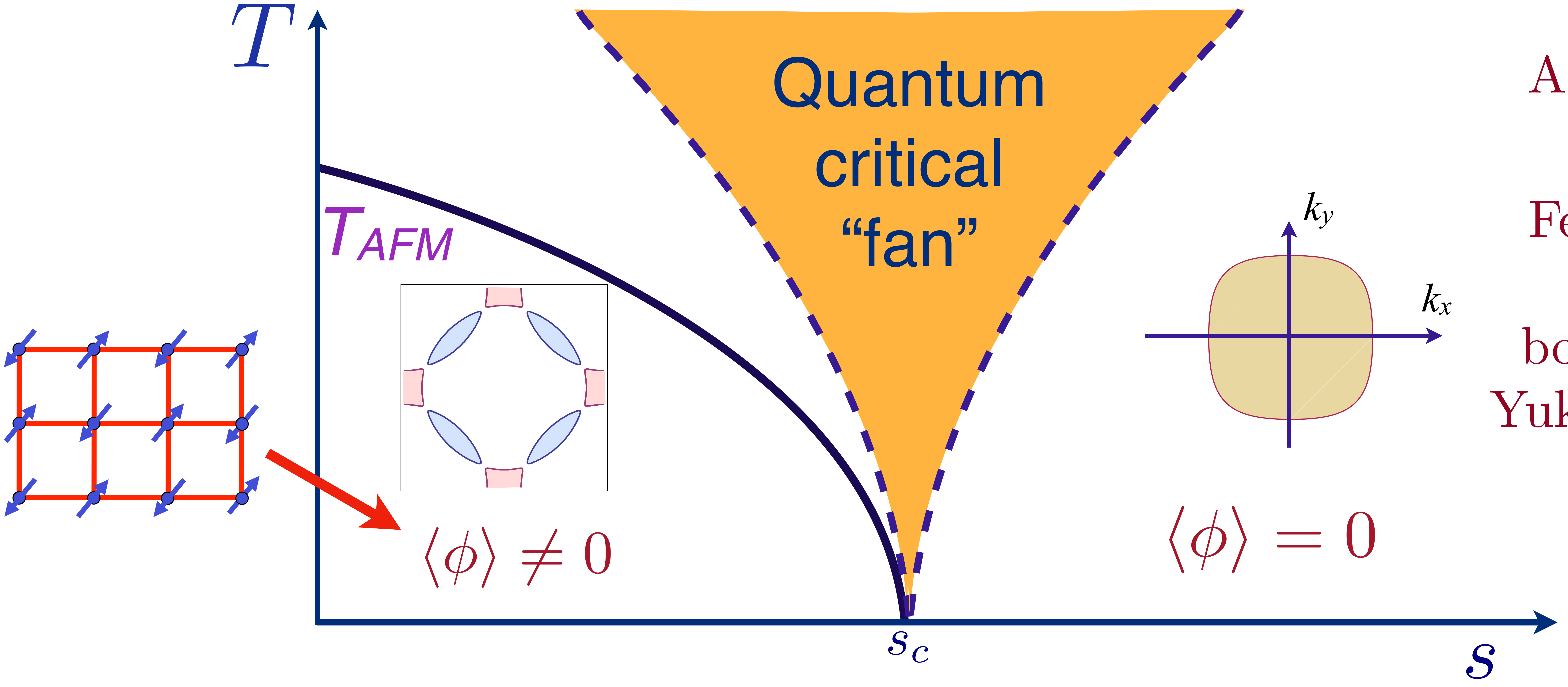
Ising field ϕ

$+$

Fermi surface with a boson-fermion Yukawa coupling

Quantum phase transitions in two-dimensional metals

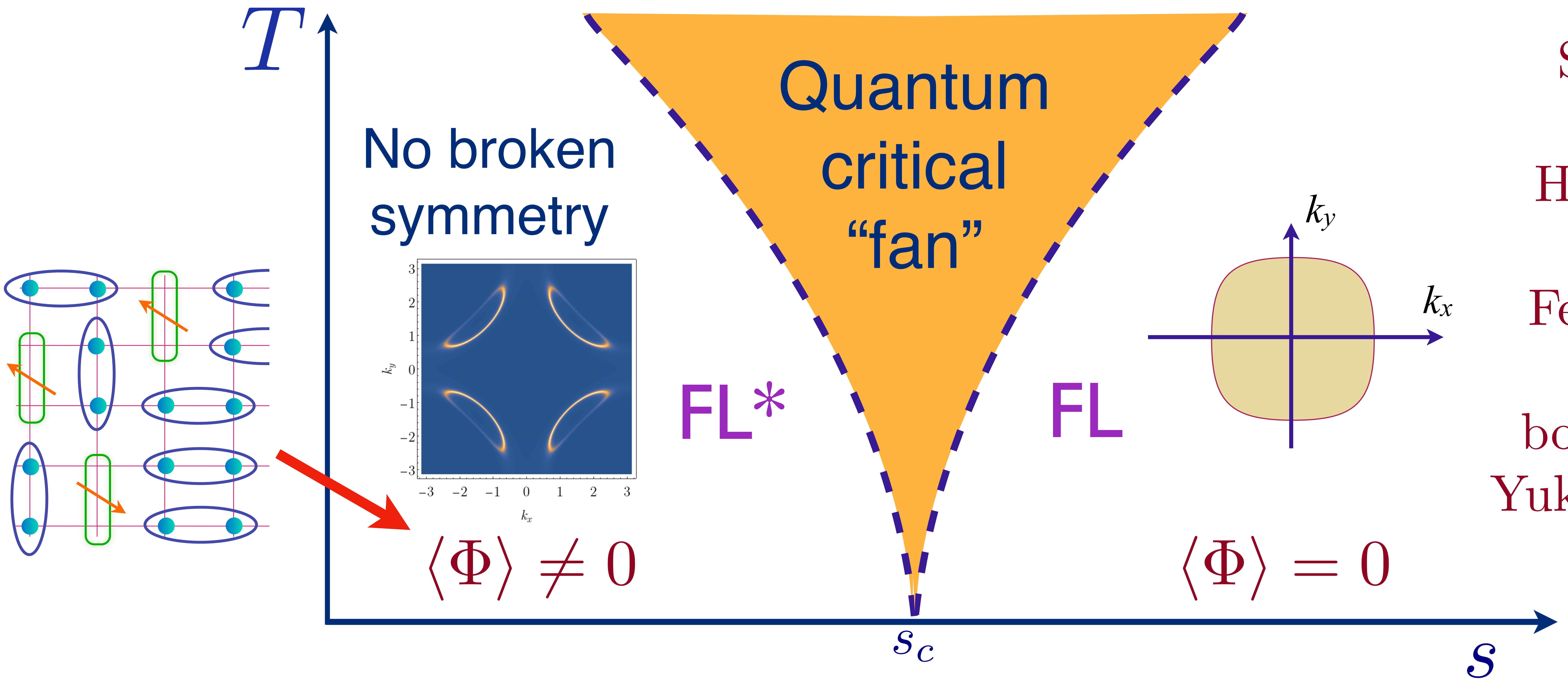
Type II: Fermi surface reconstruction



AFM field ϕ_a
+
Fermi surface
with a
boson-fermion
Yukawa coupling

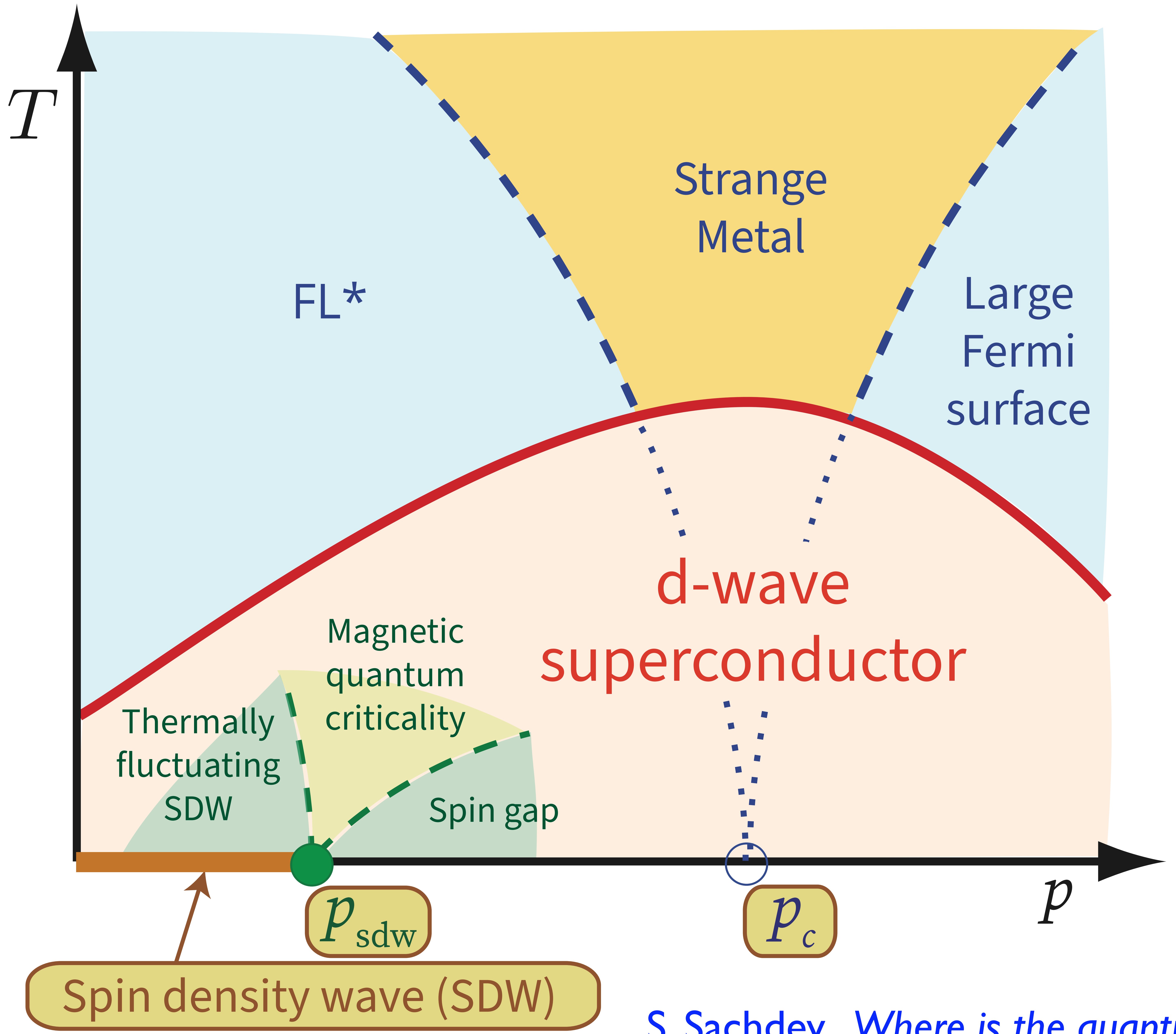
Quantum phase transitions in two-dimensional metals

Type III: Fermi surface jump with no order parameter



Spin liquid
$+$
Higgs field Φ
$+$
Fermi surface with a boson-fermion Yukawa coupling

Applies to hole-doped cuprates



Strange metal is disordered FL*-FL quantum criticality

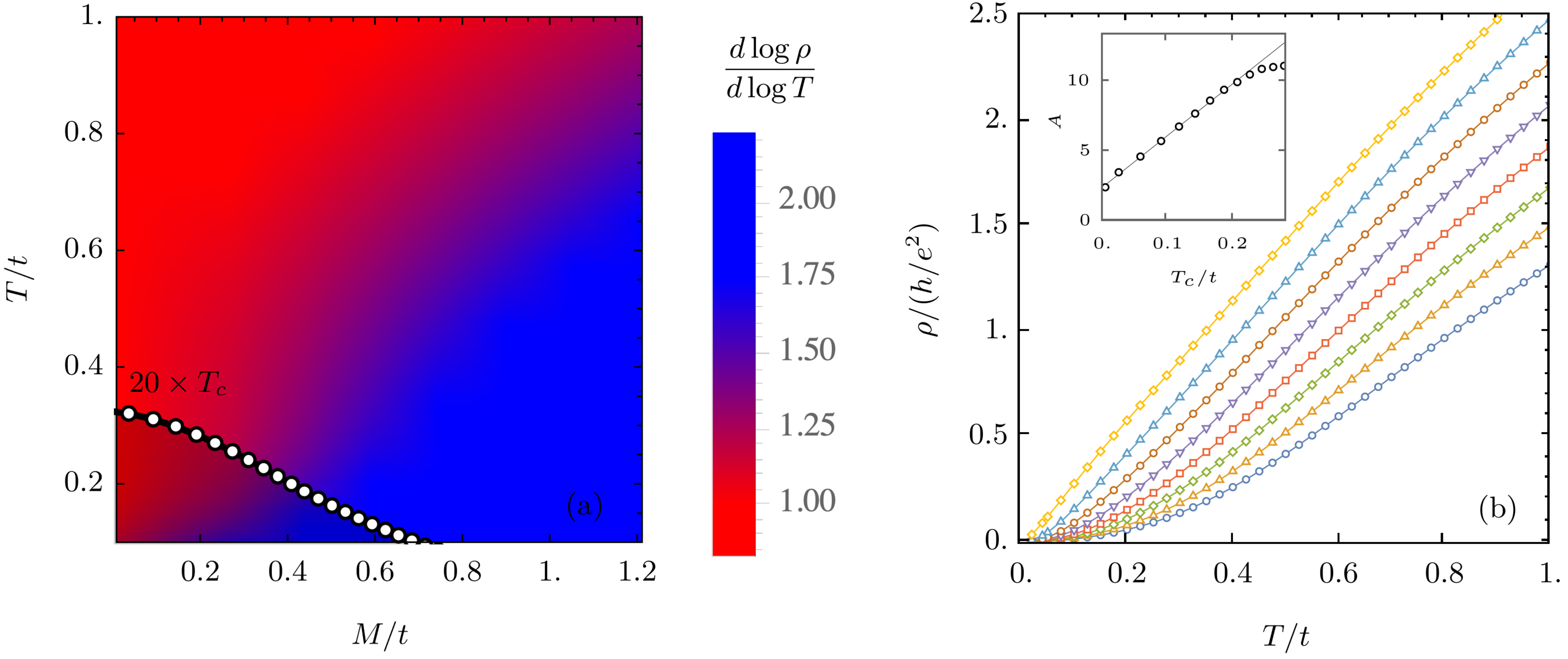
Essentially the same critical theory as for Ising-nematic ordering.

S. Sachdev, *Where is the quantum critical point in the cuprate superconductors?*, Physica Status Solidi B **247**, 537 (2010).

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

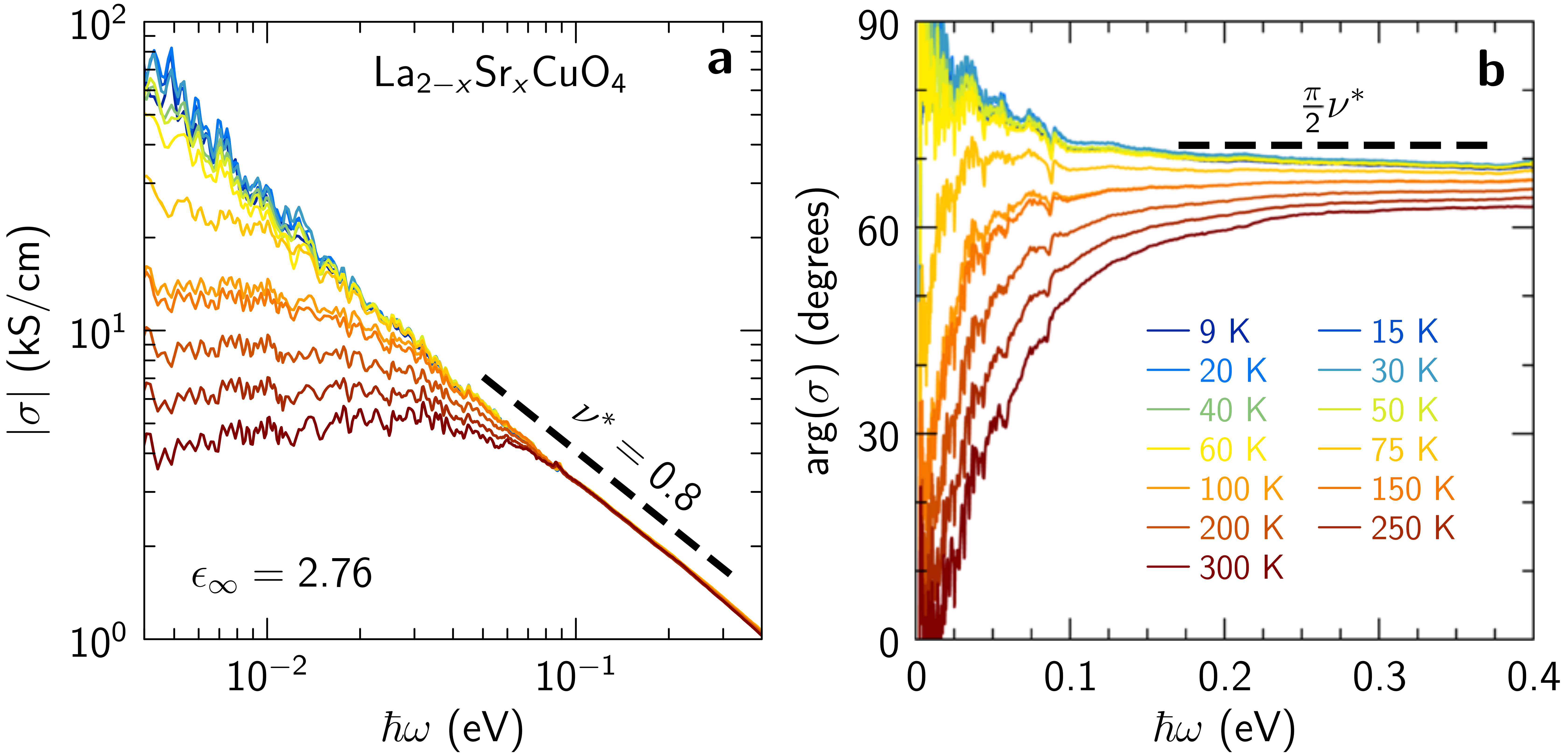


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

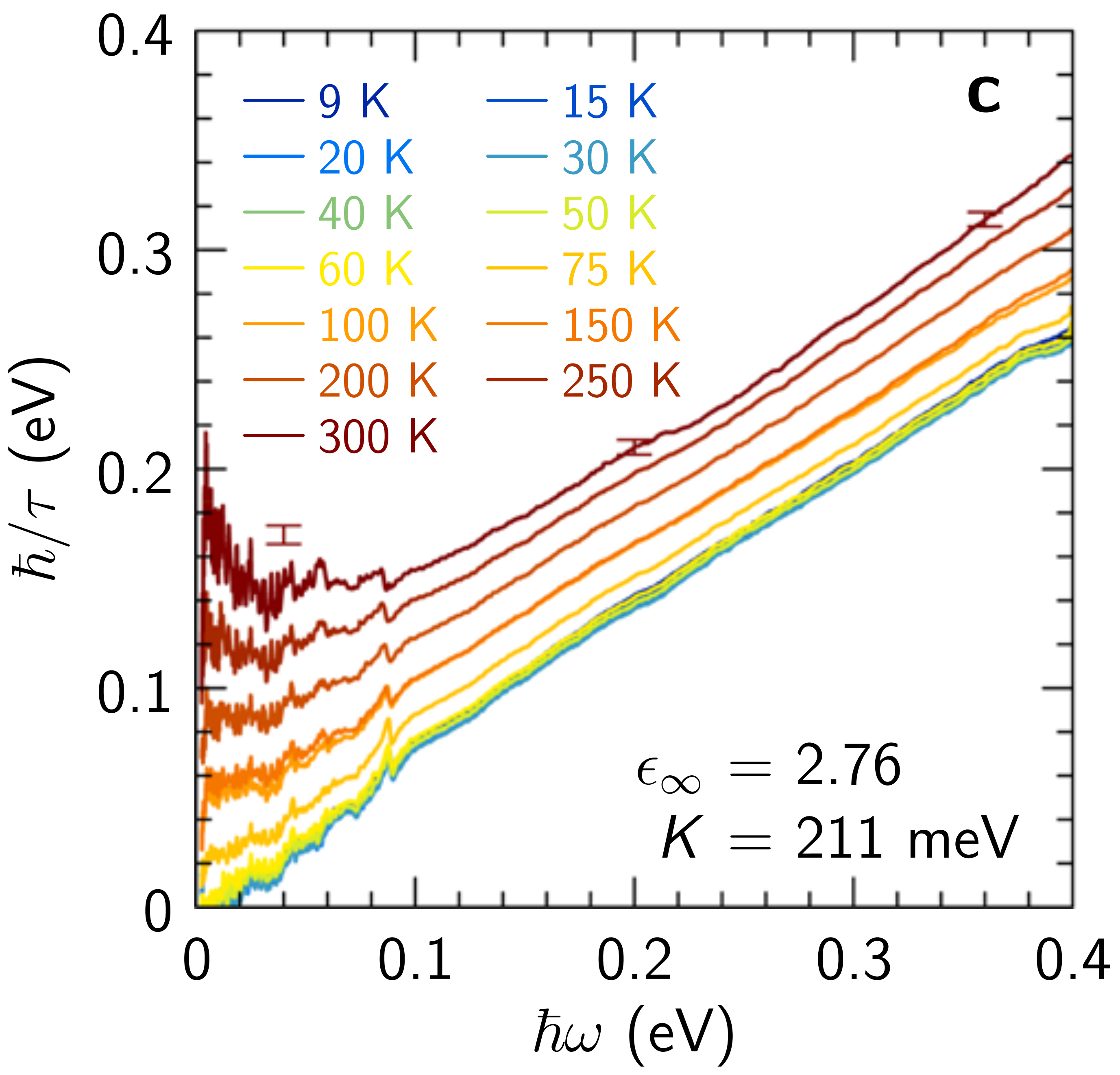
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

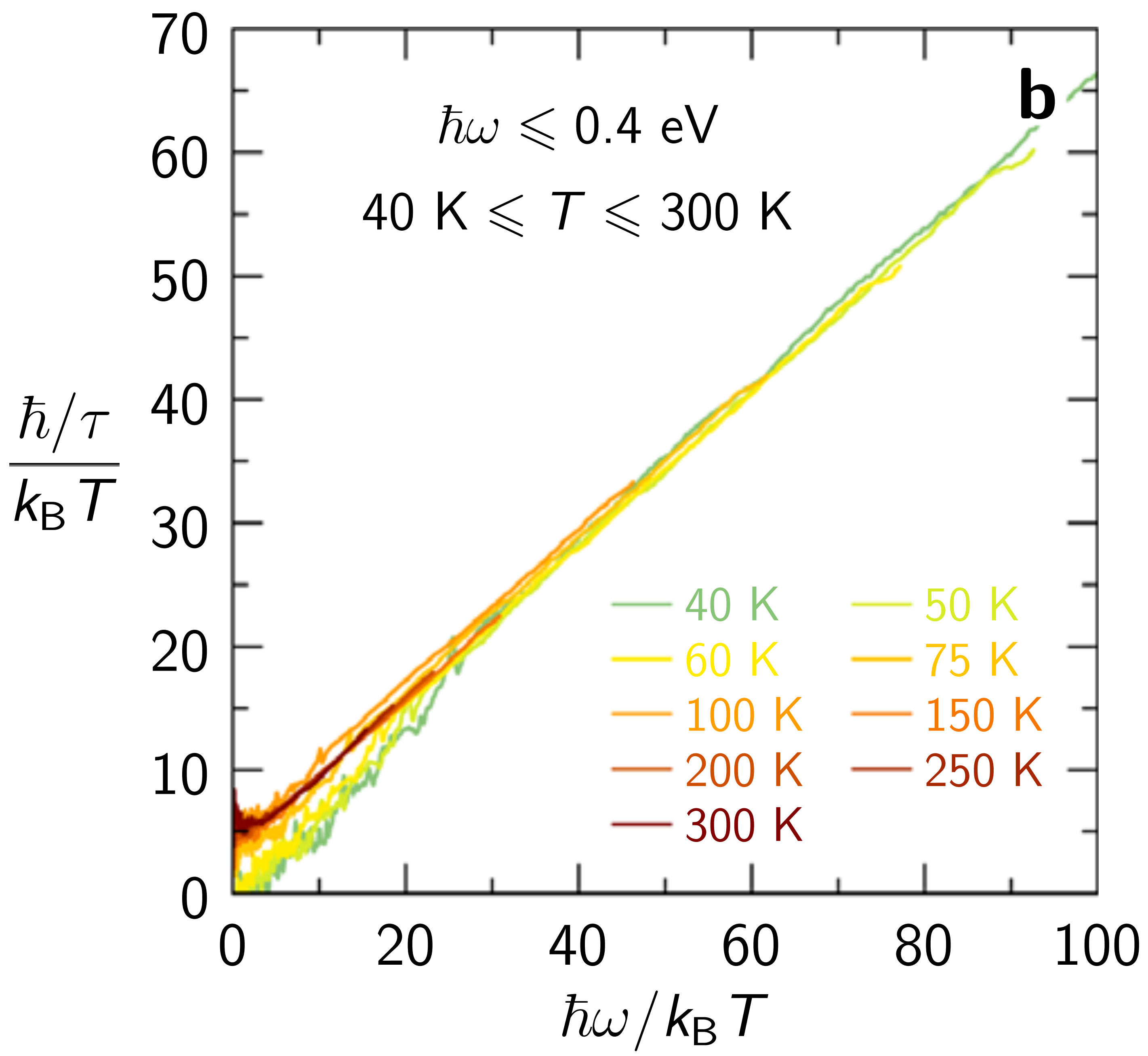
Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



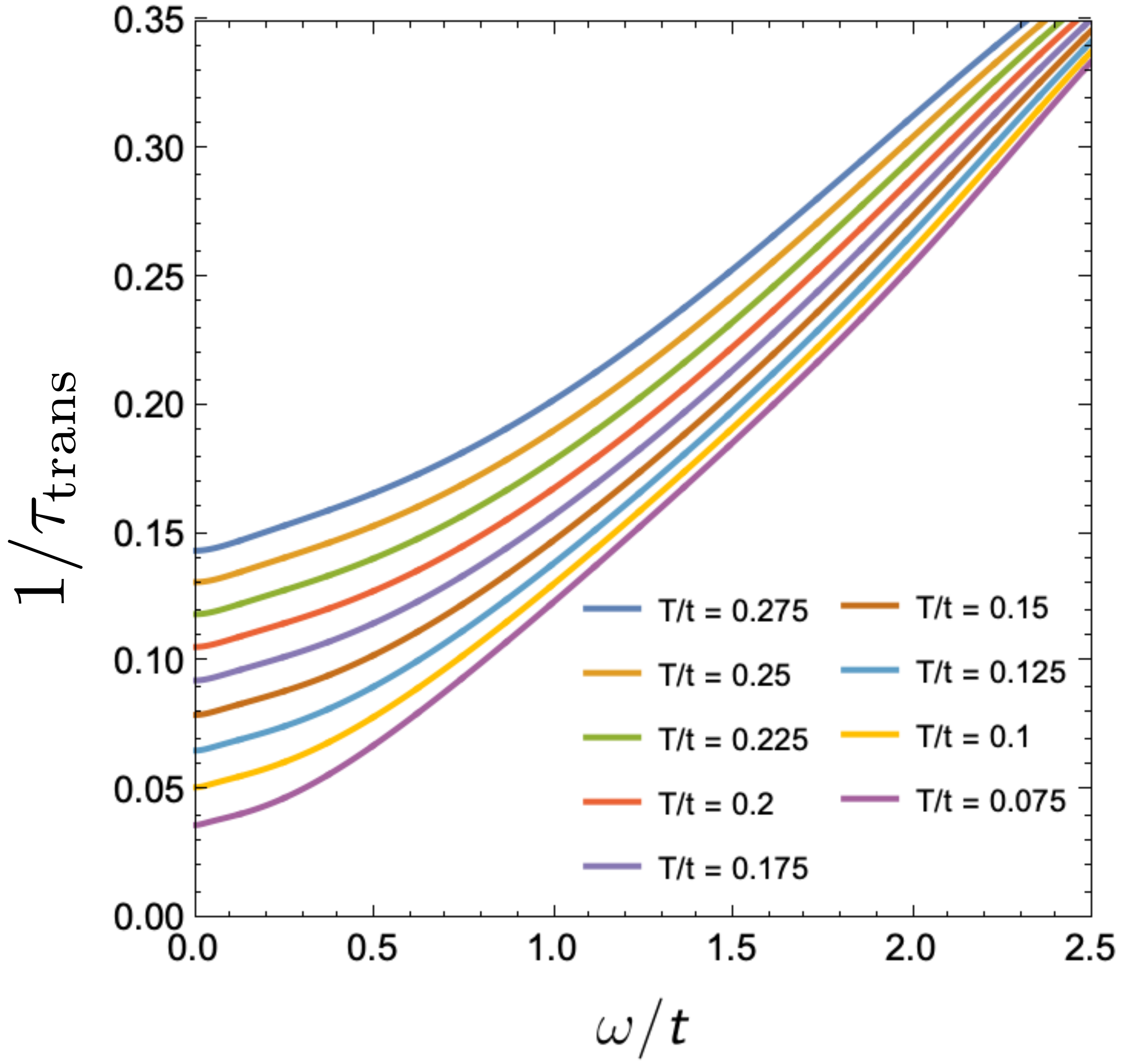
La$_{2-x}$Sr$_x$CuO$_4$
$p = 0.24$
$T_c = 19$ K

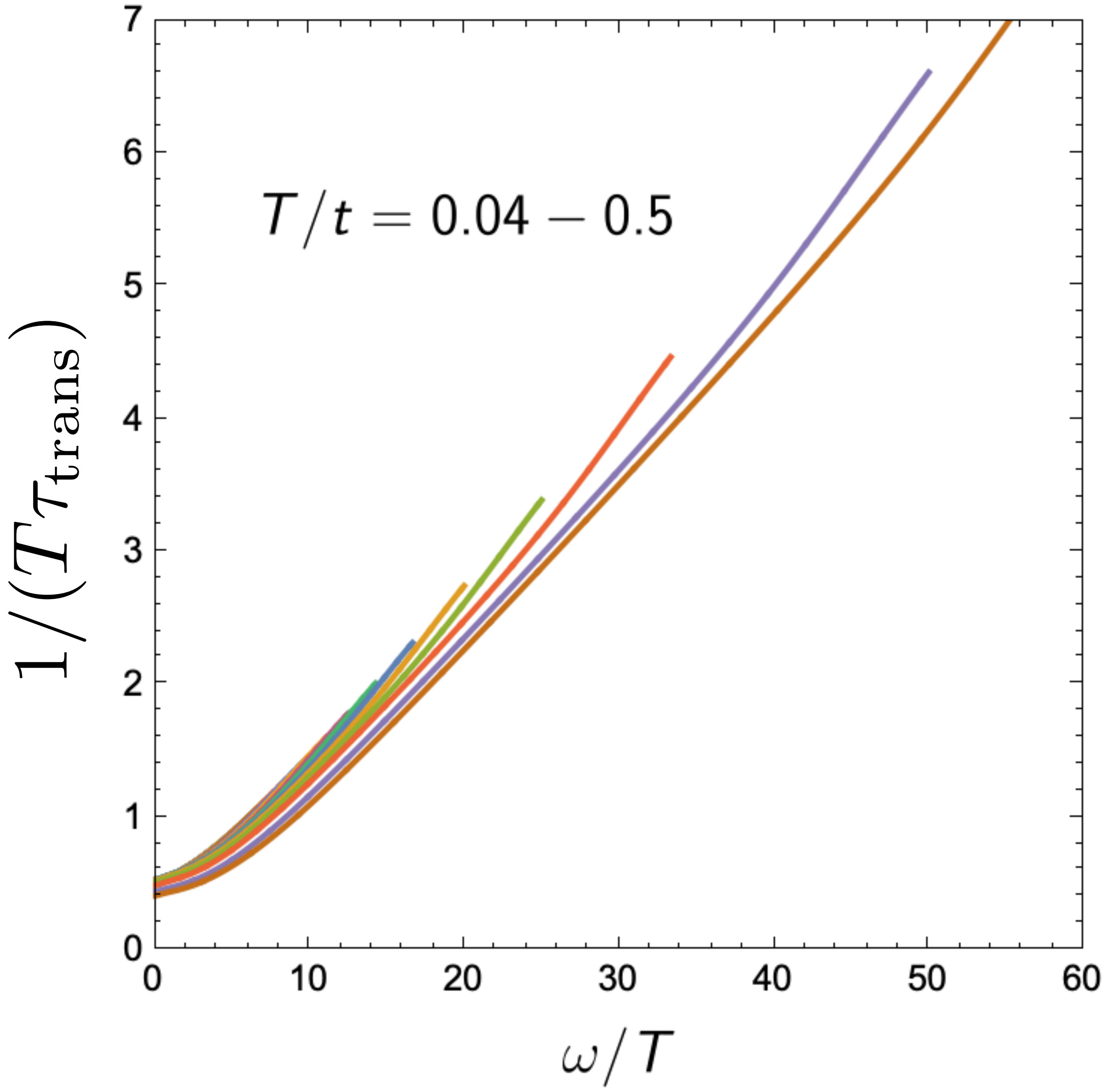




From optical conductivity data of Michon et al. (2023)

$$\frac{\hbar}{\tau} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$





2d-YSYK theory

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

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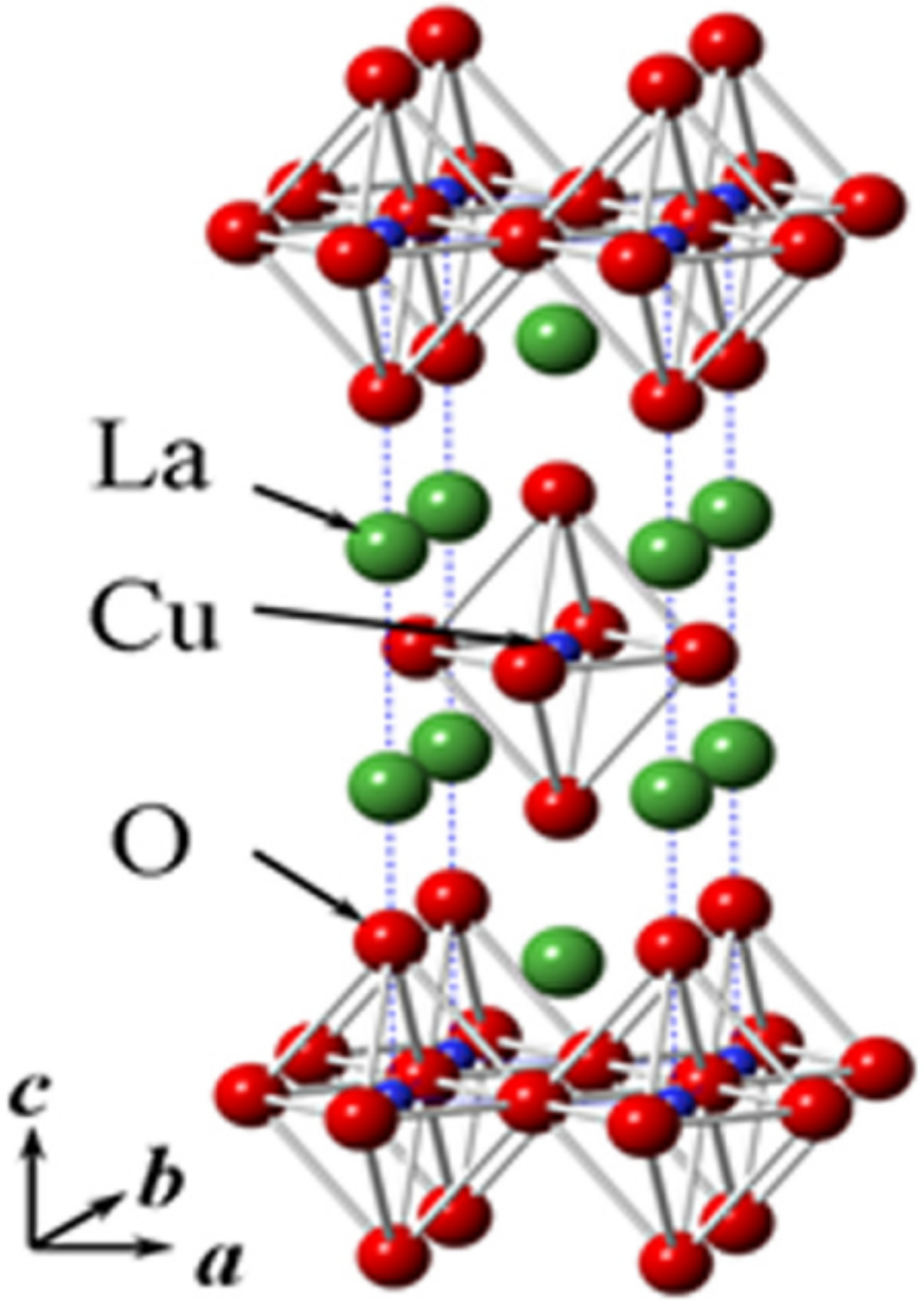
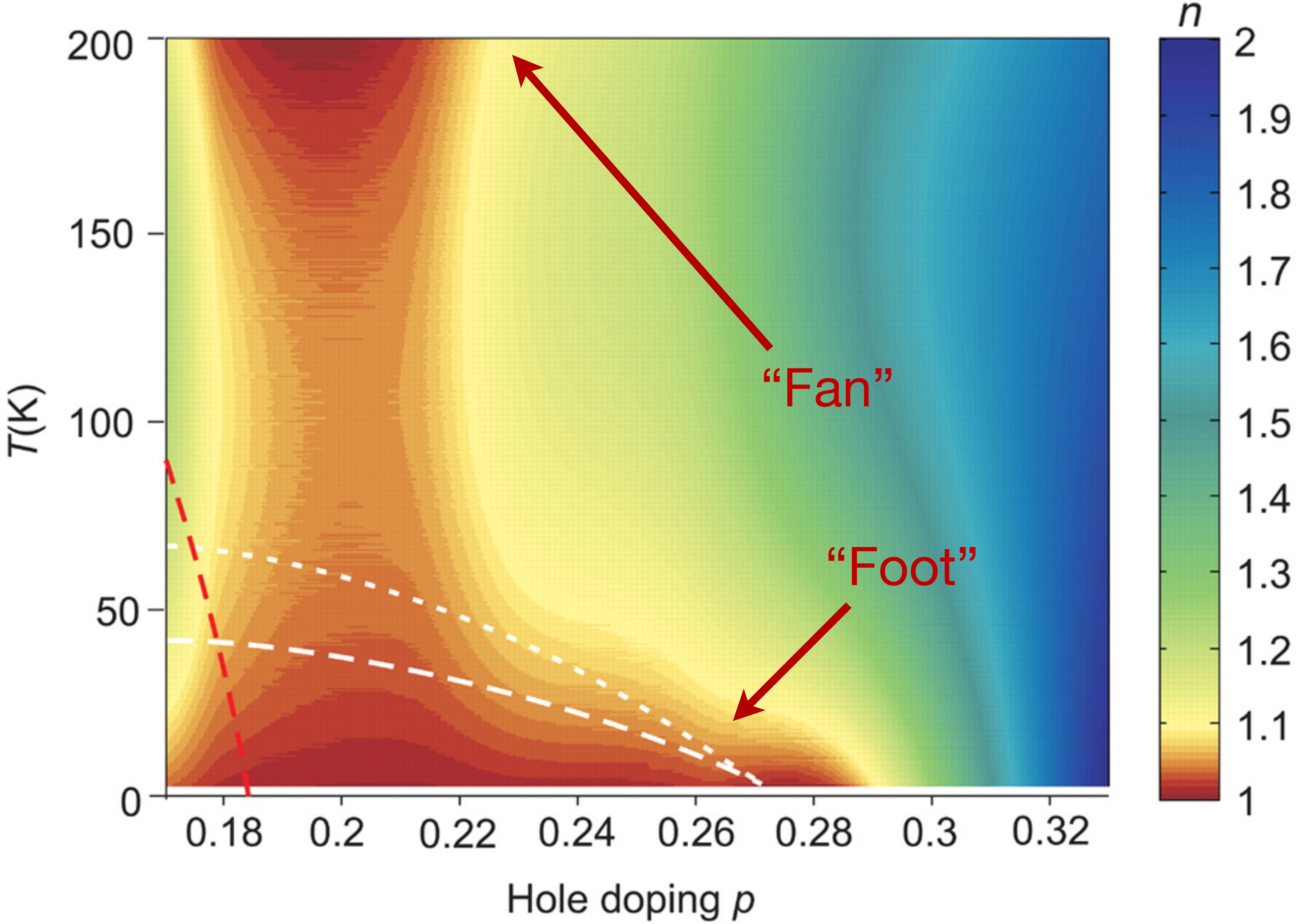
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009

Resistivity $\rho \sim T^n$





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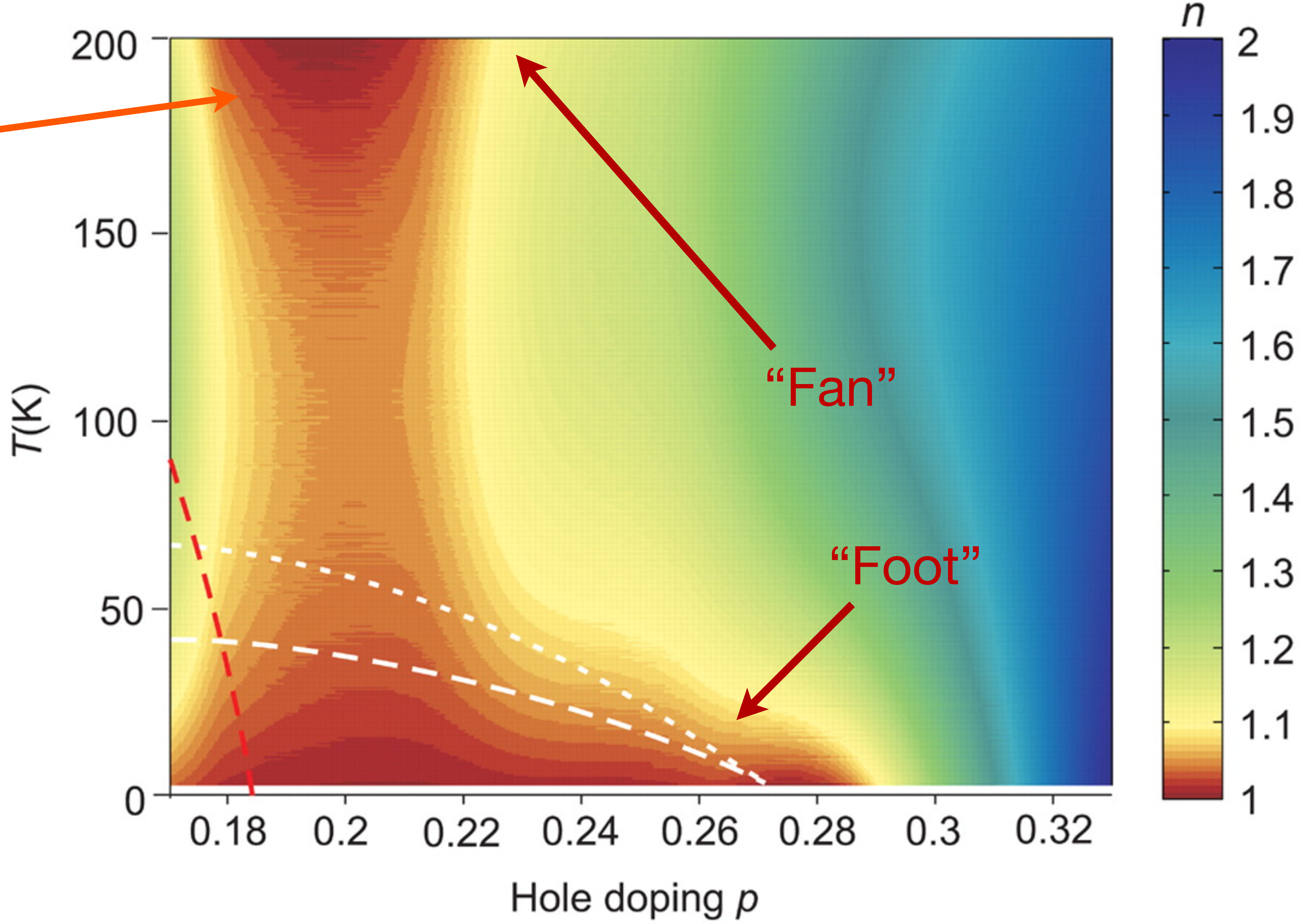
Resistivity $\rho \sim T^n$

Extended fermions and bosons: 2d-YSK theory of strange metal



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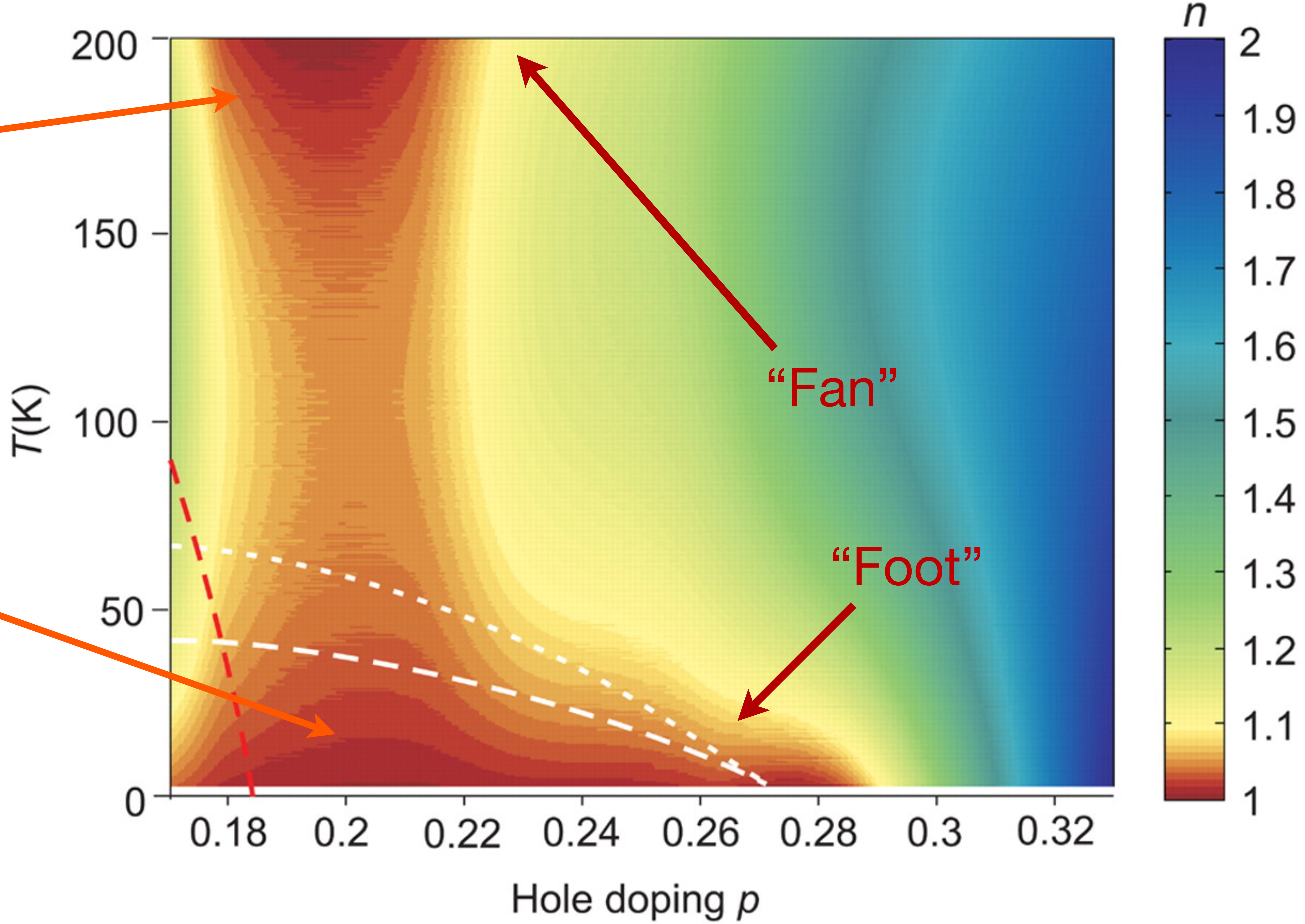
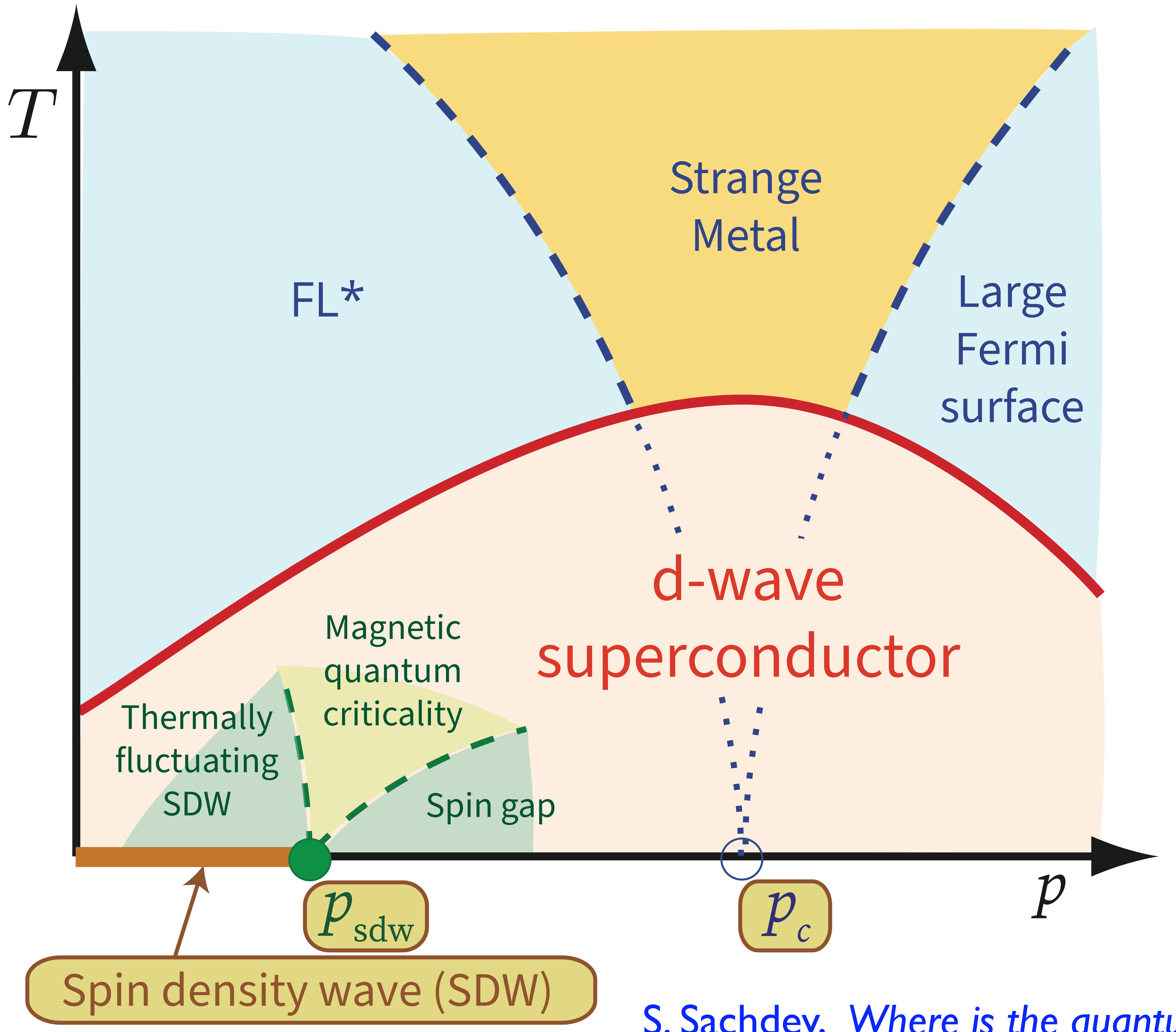
SCIENCE VOL 323 603 2009

Resistivity $\rho \sim T^n$

Extended fermions and bosons: 2*d*-YSK theory of strange metal

Localized overdamped SDW bosons, but extended fermions: Griffths strange metal





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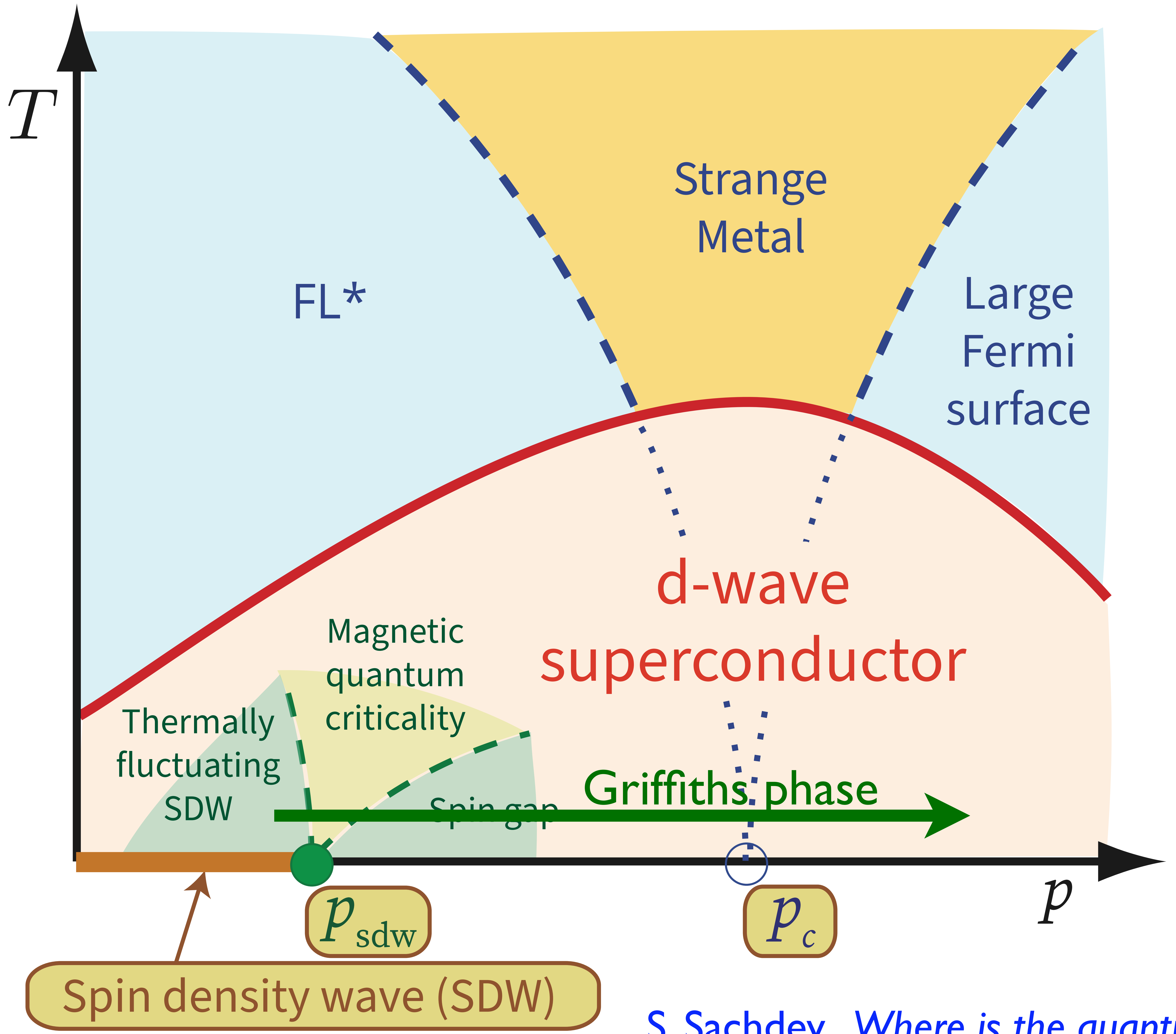

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Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_b = \int d\tau \left(-\sum_{\langle ij \rangle} J_{ij}\phi_{ia}\phi_{ja} + \sum_j \left[\frac{s_j}{2}\phi_{ja}^2 + \frac{u}{4M}\left(\phi_{ja}^2\right)^2 \right] \right) + \frac{T\gamma}{2}\sum_{\omega_n}\sum_j |\omega_n||\phi_{ja}(\omega_n)|^2$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with O(M) symmetry.

Strong disorder RG identical to that for the RTFIM (D.S. Fisher)

$$\widetilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\widetilde{s}_2 = 2\frac{s_2 s_3}{J_{23}}$$

$$H_{\mathrm{RTFIM}} = -\sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

J. A. Hoyos, Chetan Kotabage, Thomas Vojta
Phys. Rev. Lett. **99**, 230601 (2007)

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99,** 230601 (2007)

PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

- Each rare region is described by a one-dimensional classical $\mathrm{O}(M)$ model with a long-range $1/\tau^2$ interaction.
- For $M \geq 2$, the classical model has an exponentially long correlation time at weak coupling (low 'temperature') - Kosterlitz, 1976.
- This is similar to the classical Ising chain with short-range interactions.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + \delta s_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j \left(\gamma |\Omega| + \Omega^2/c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\widetilde{\delta s_j}}{2} \phi_{ja}^2 \Big]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev, Phys. Rev. Lett. **101**, 035701 (2008).

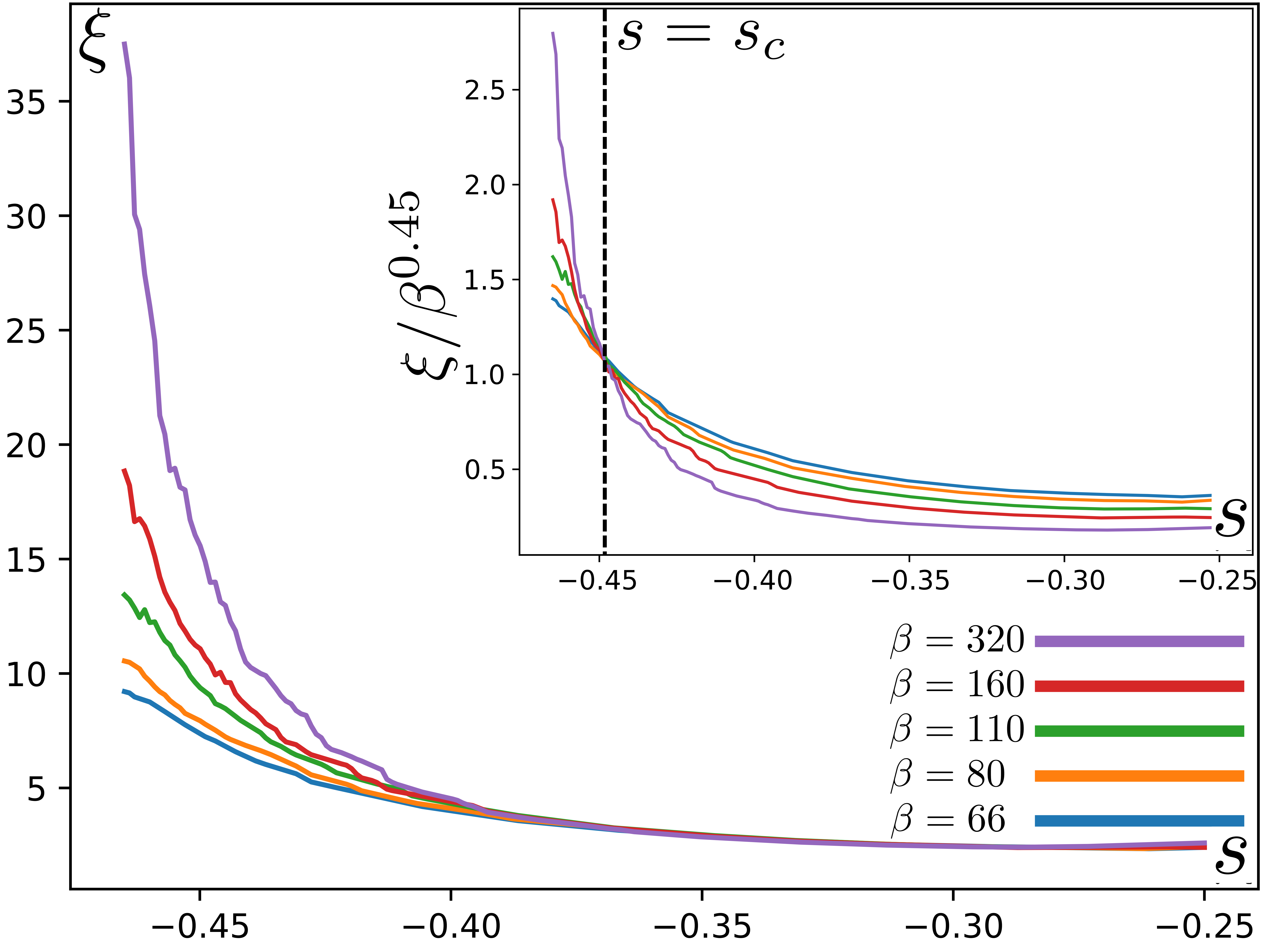
$$\widetilde{\delta s_j} = s + \delta s_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + \delta s_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2/c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

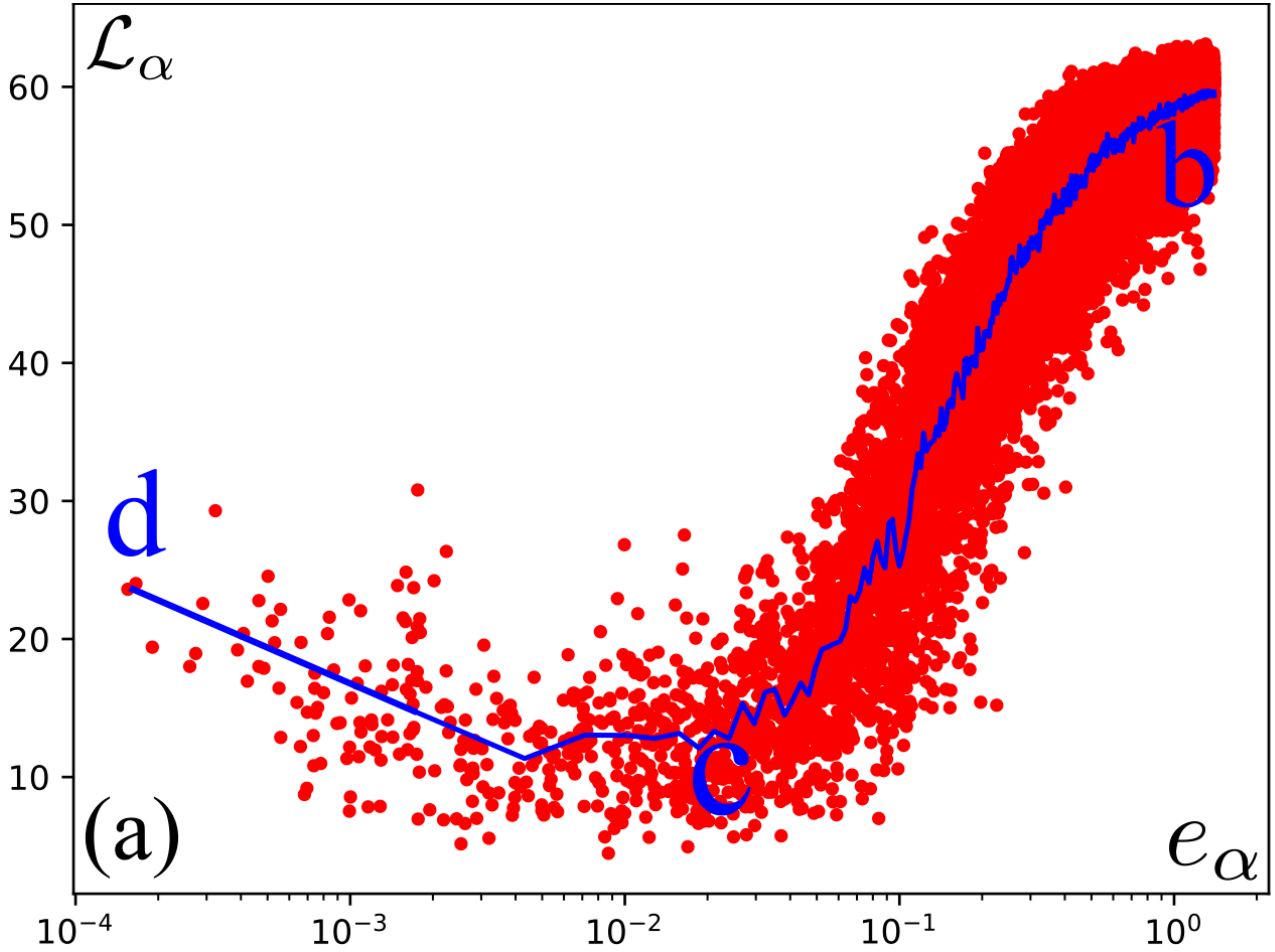
ϕ correlation length ξ



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



Aavishkar A. Patel,
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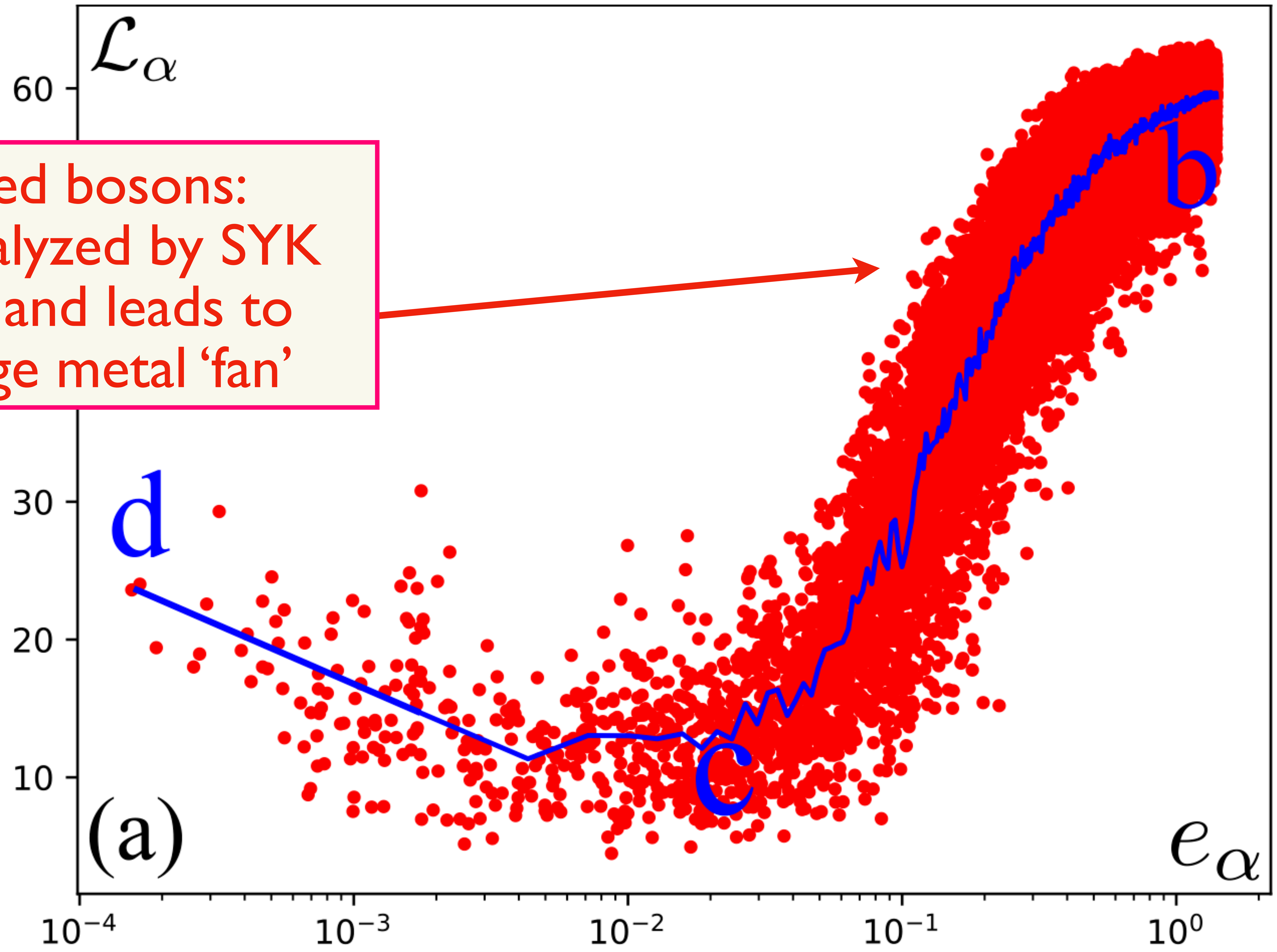
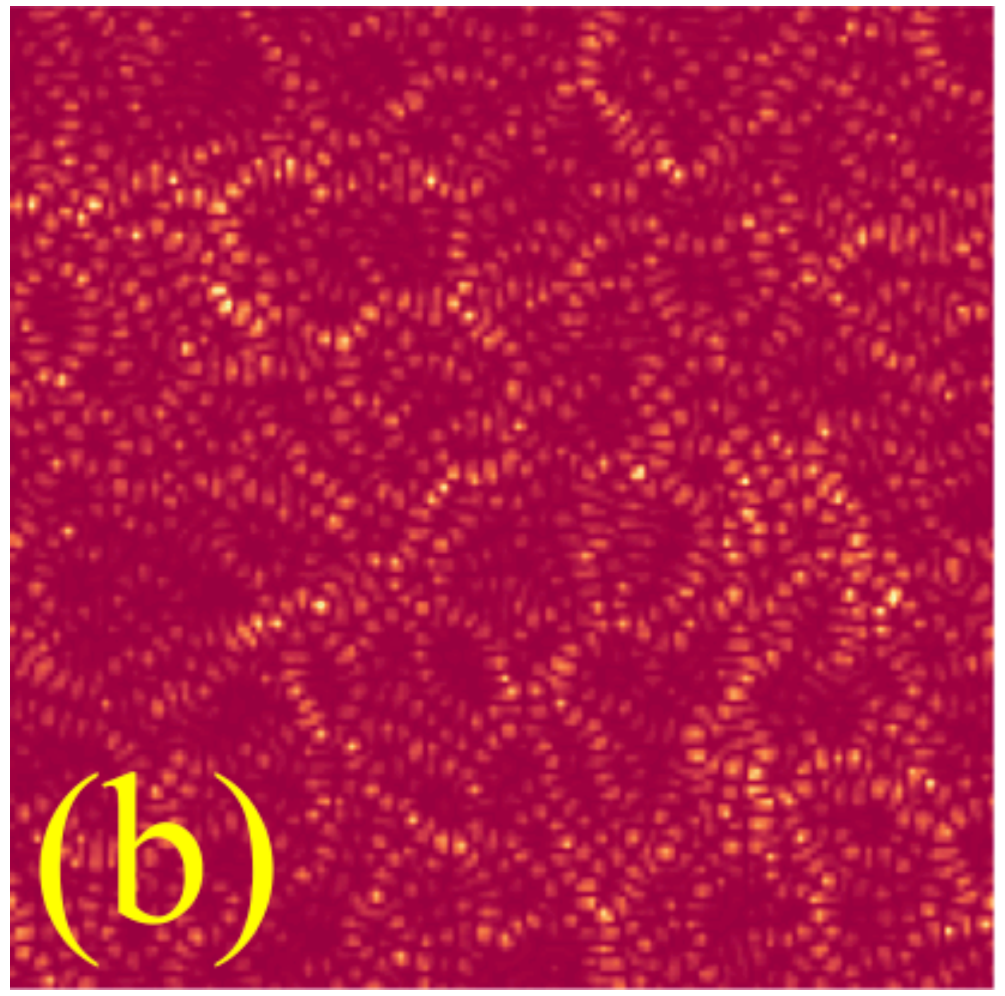
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



Extended bosons:
Can be analyzed by SYK methods, and leads to the strange metal 'fan'

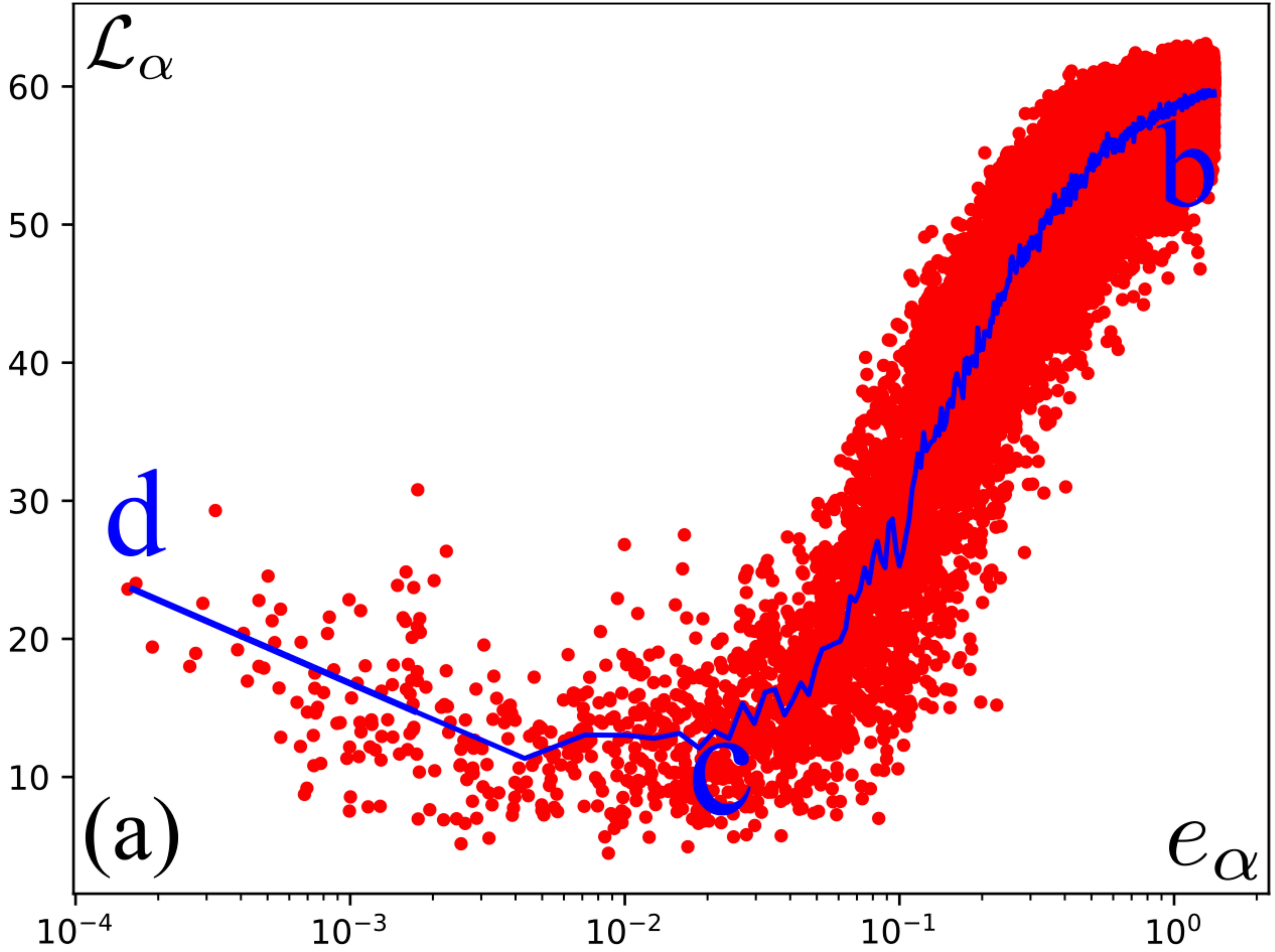


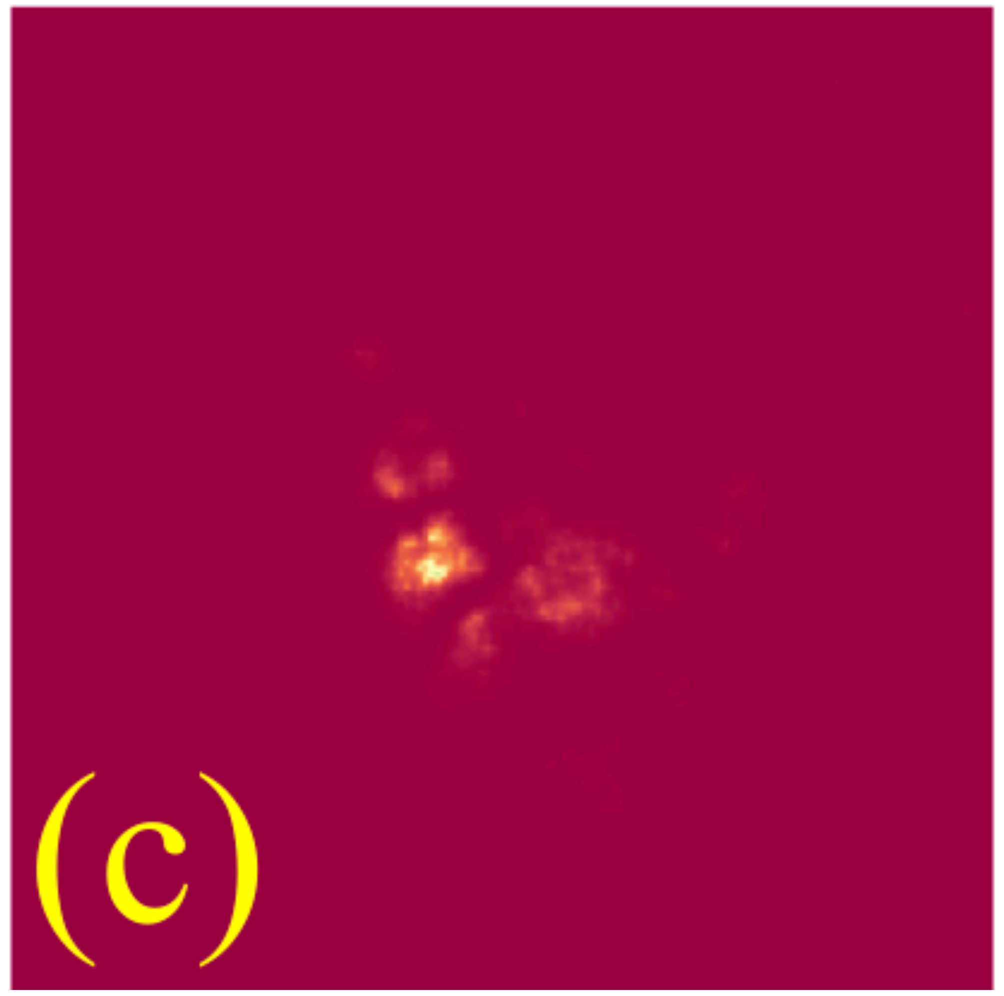
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Aavishkar A. Patel,
Haoyu Guo,
Ilya Esterlis, S. S.,
Science **381**, 790 (2023)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$





Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

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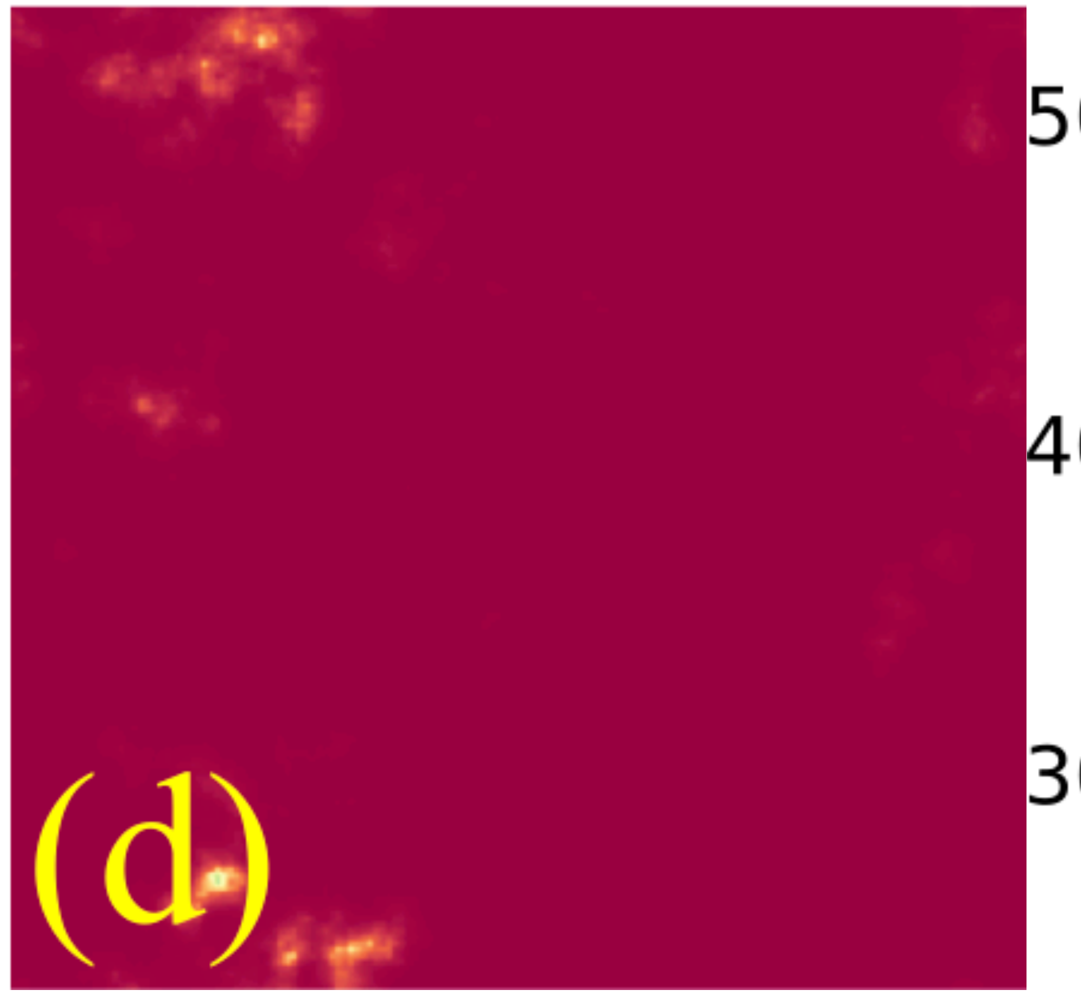
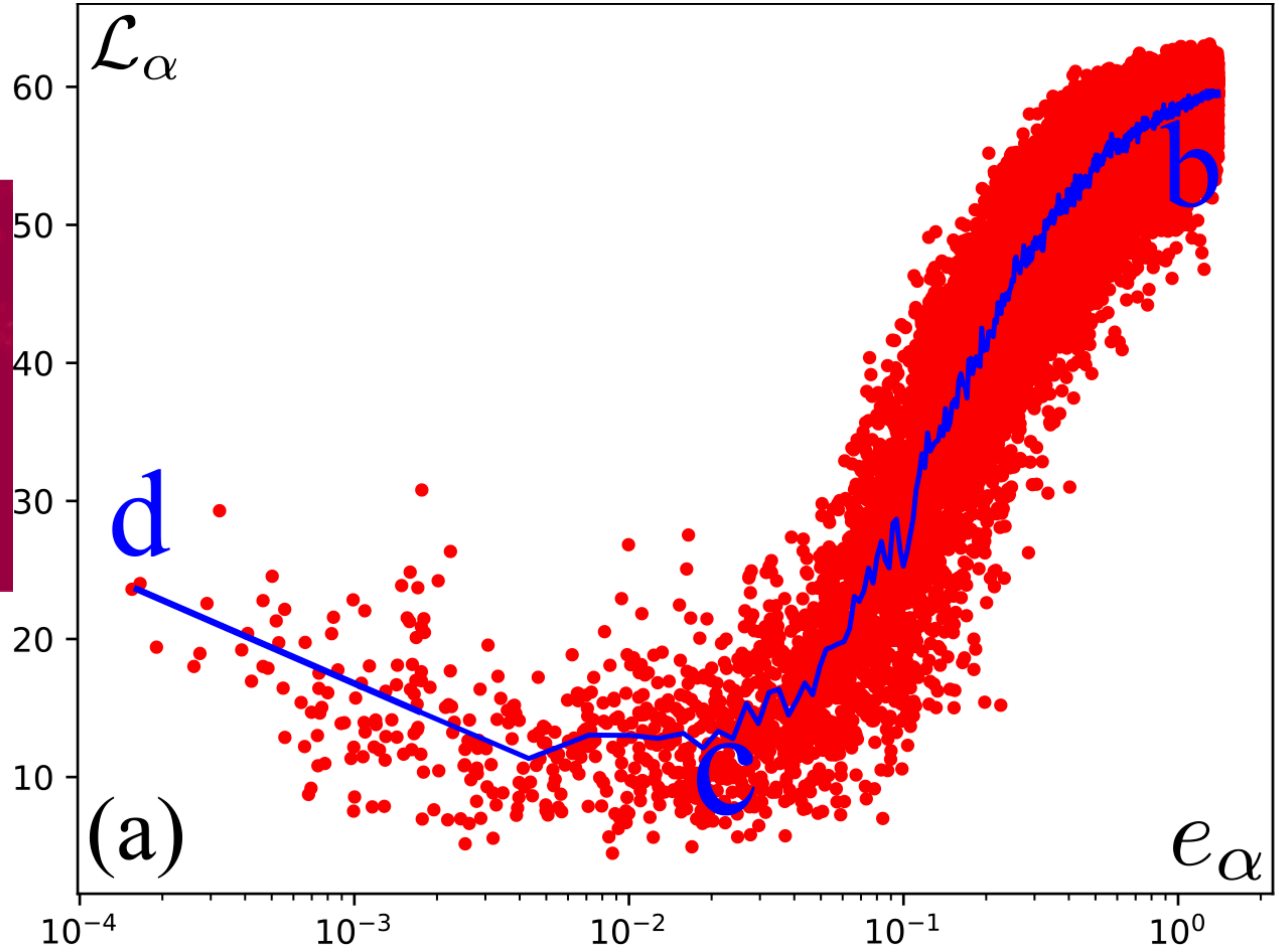
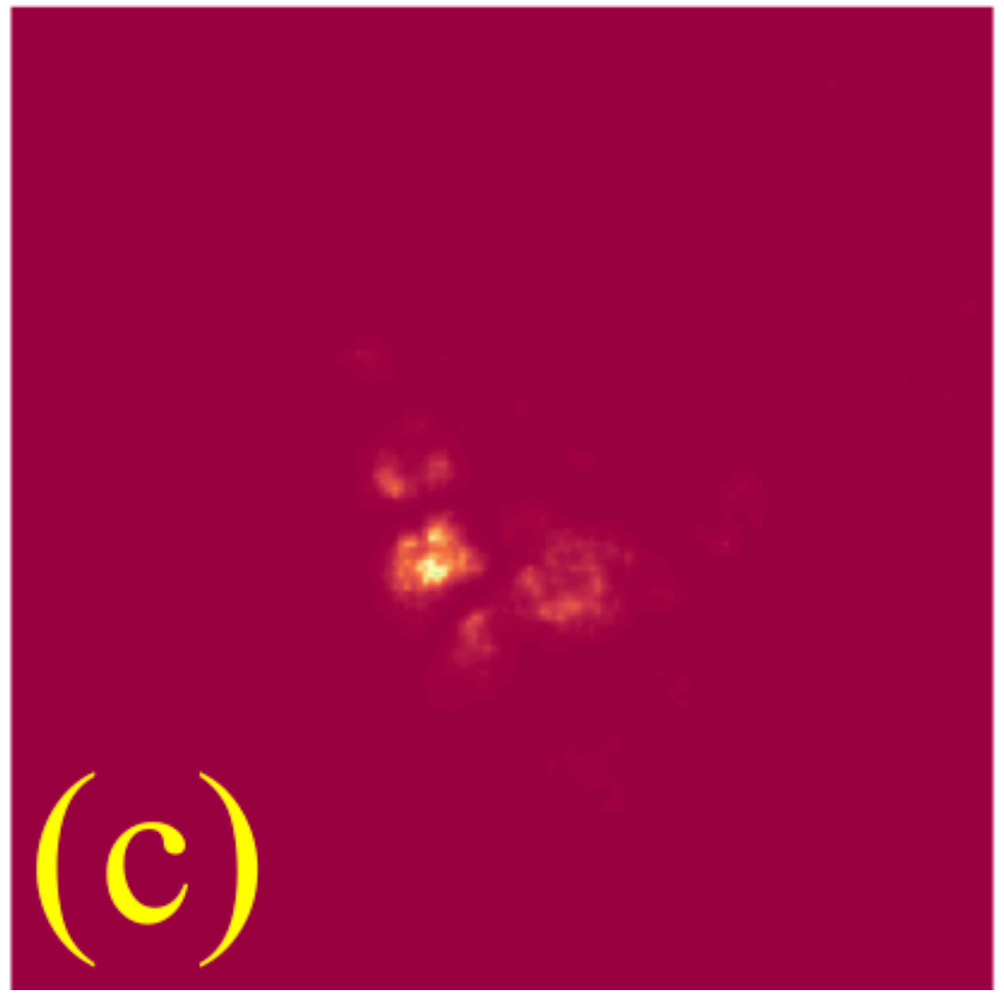


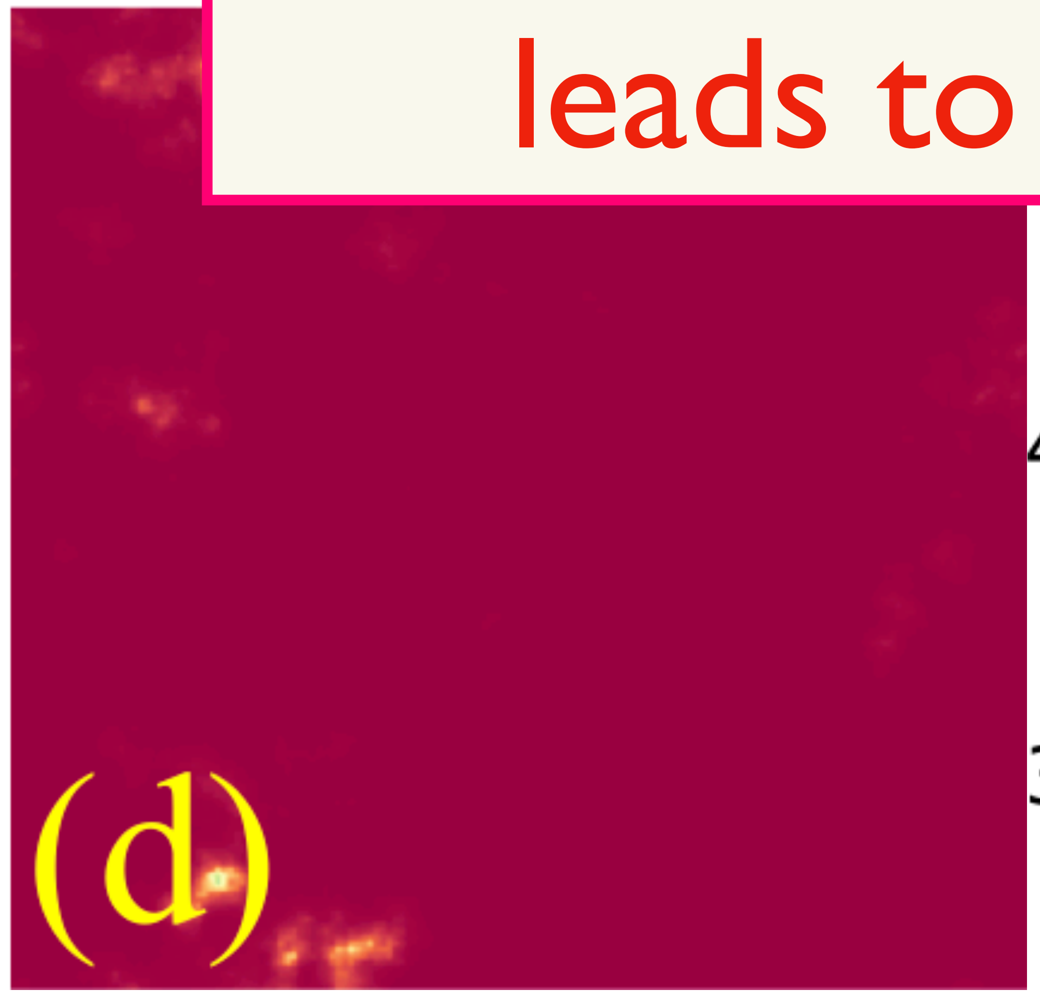


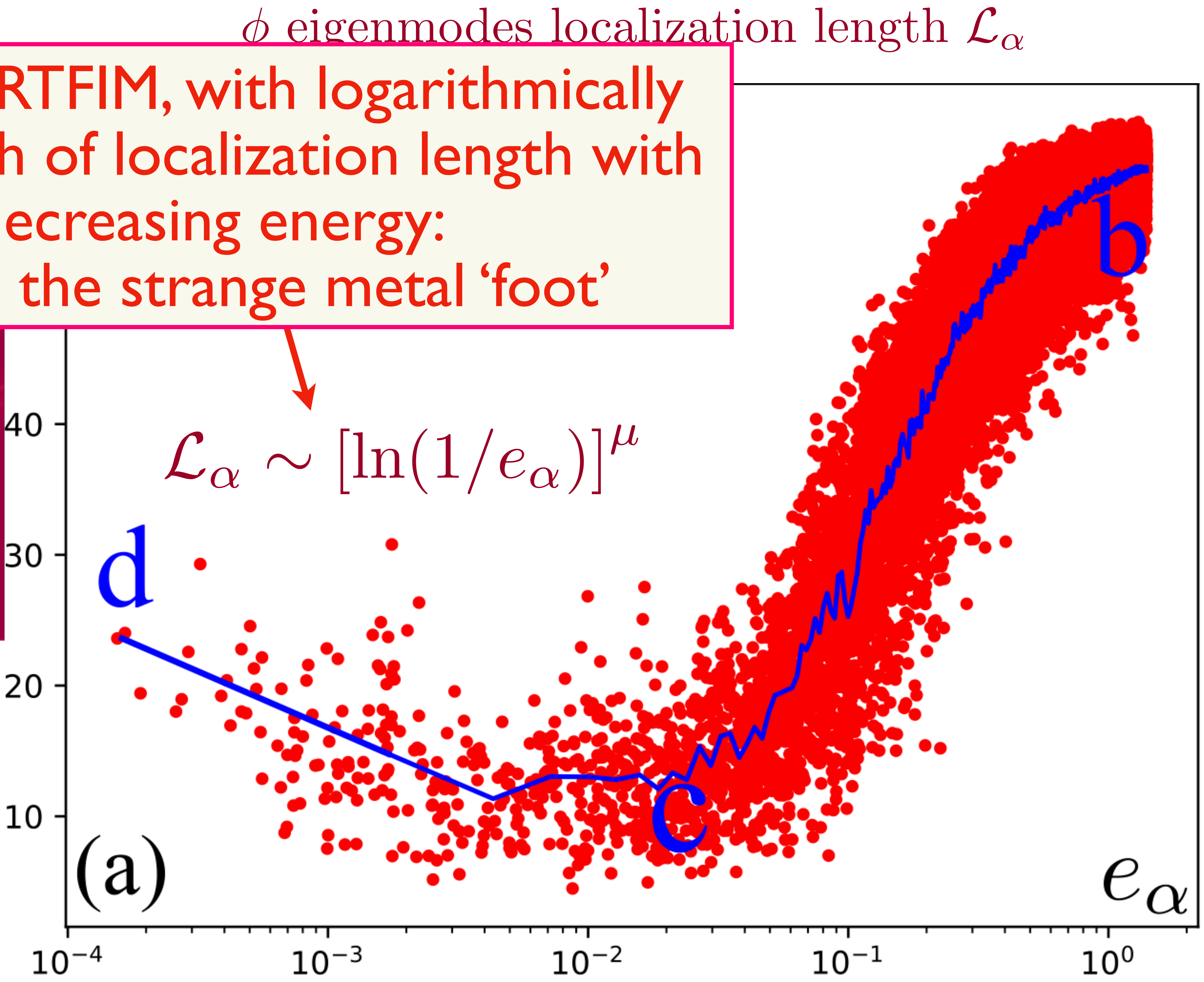


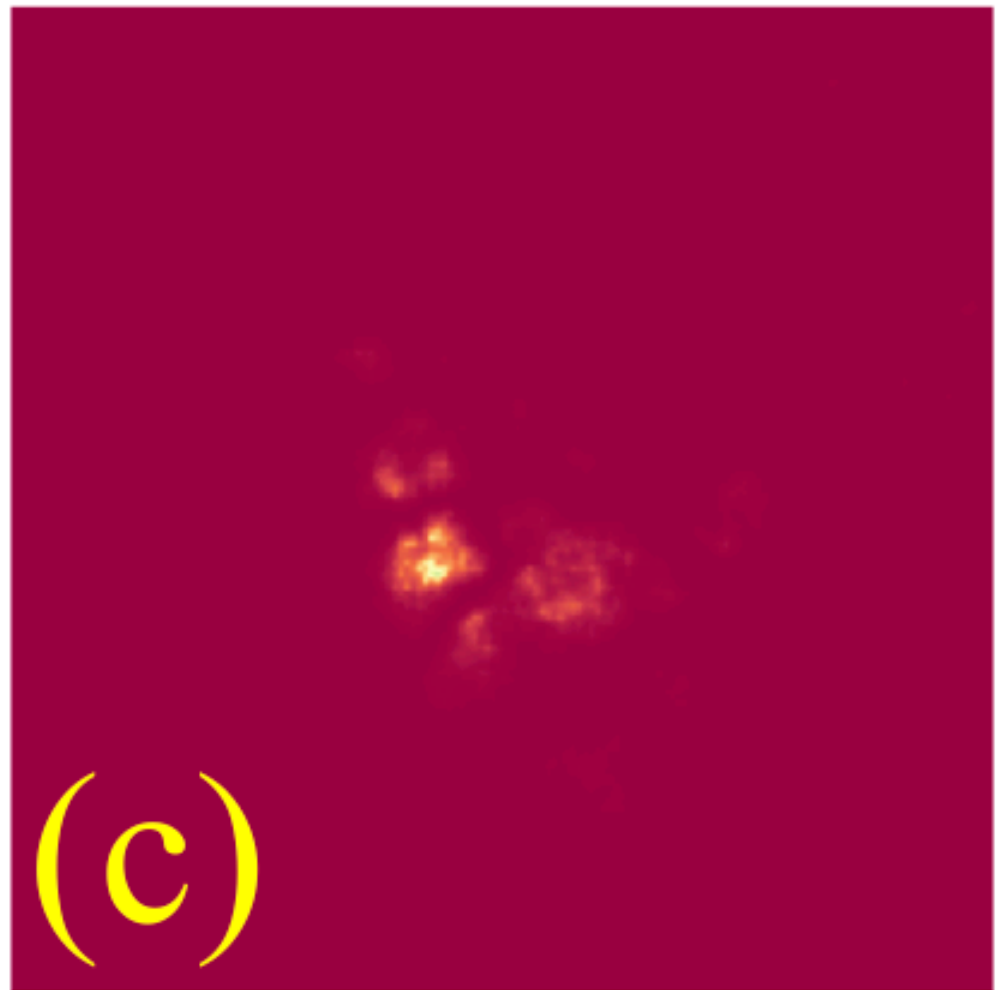
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy:
leads to the strange metal 'foot'







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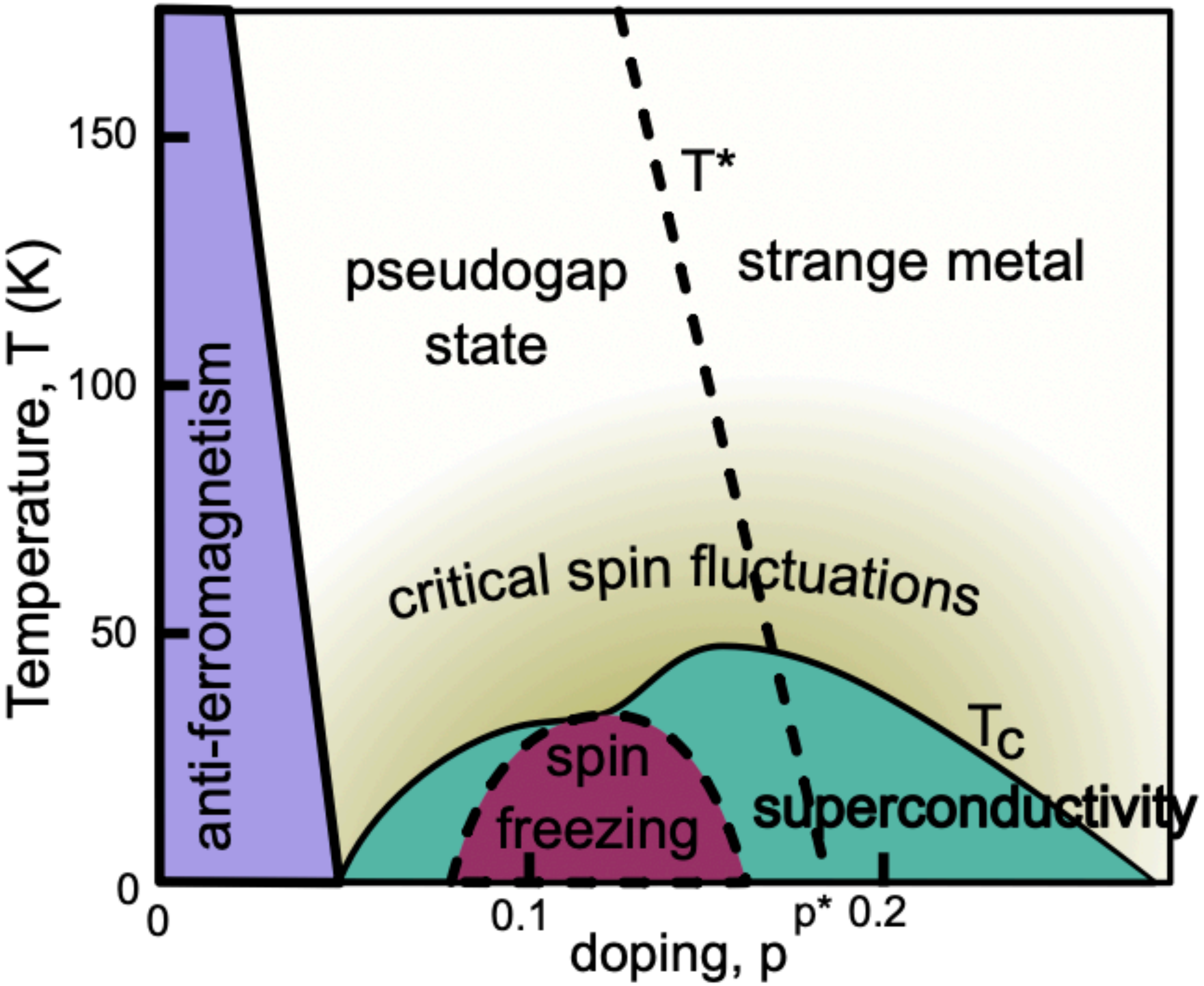
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

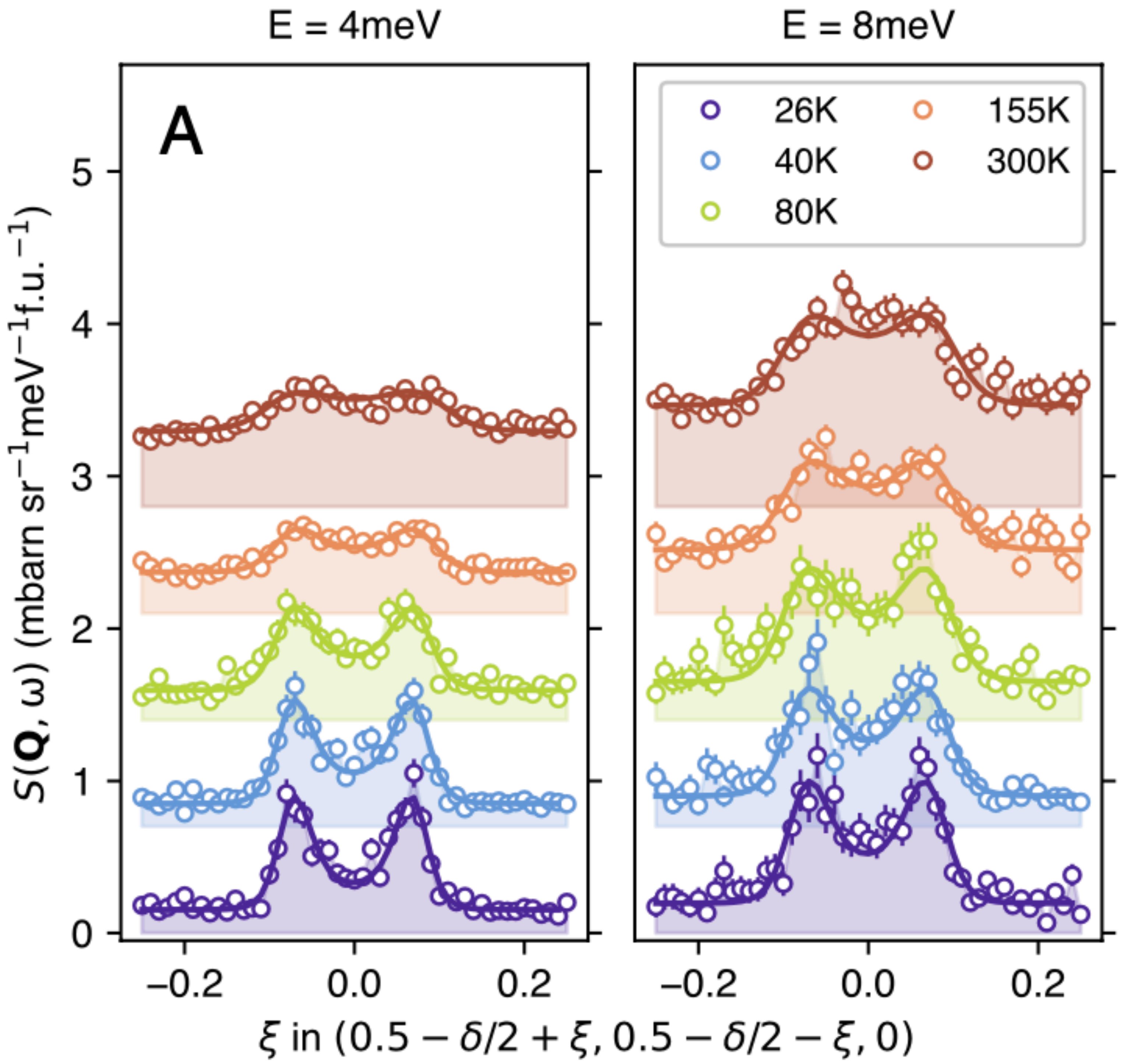
Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,

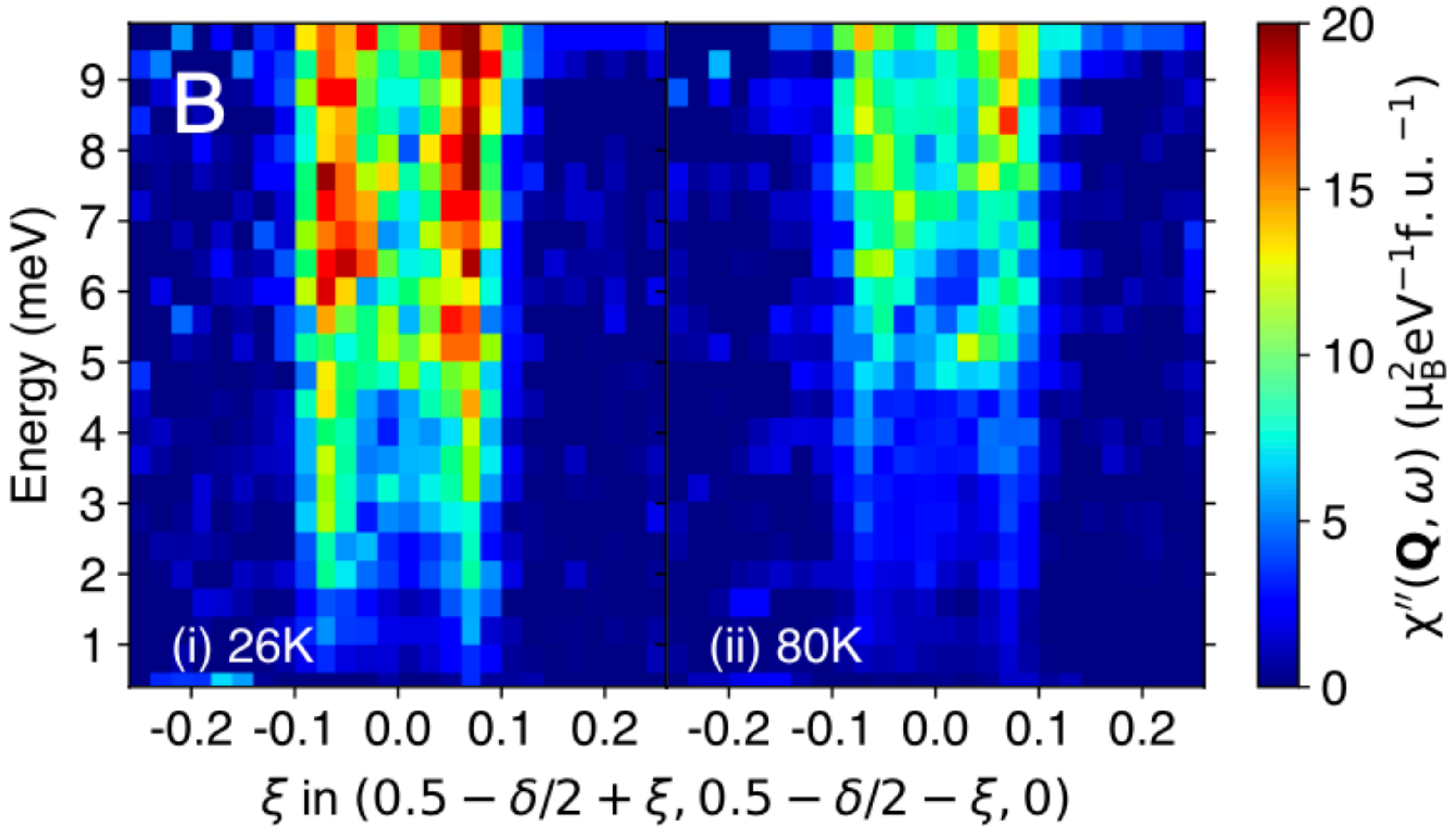
x=0.22

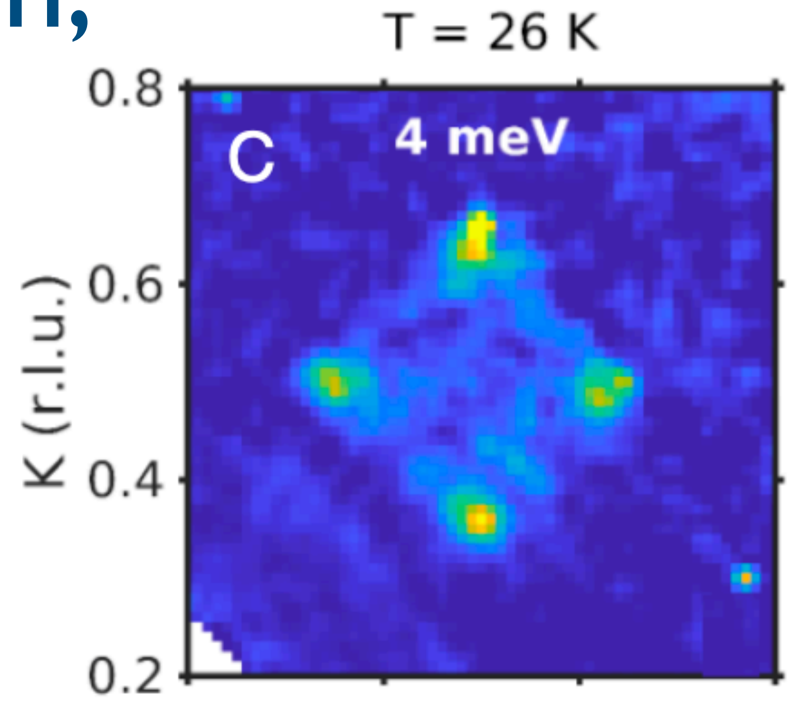
J. Radaelli, A.A. Patel, O.J. Lipscombe, Mengze Zhu, J.R. Stewart, S. S. and S.M. Hayden,

arXiv:2503.13600

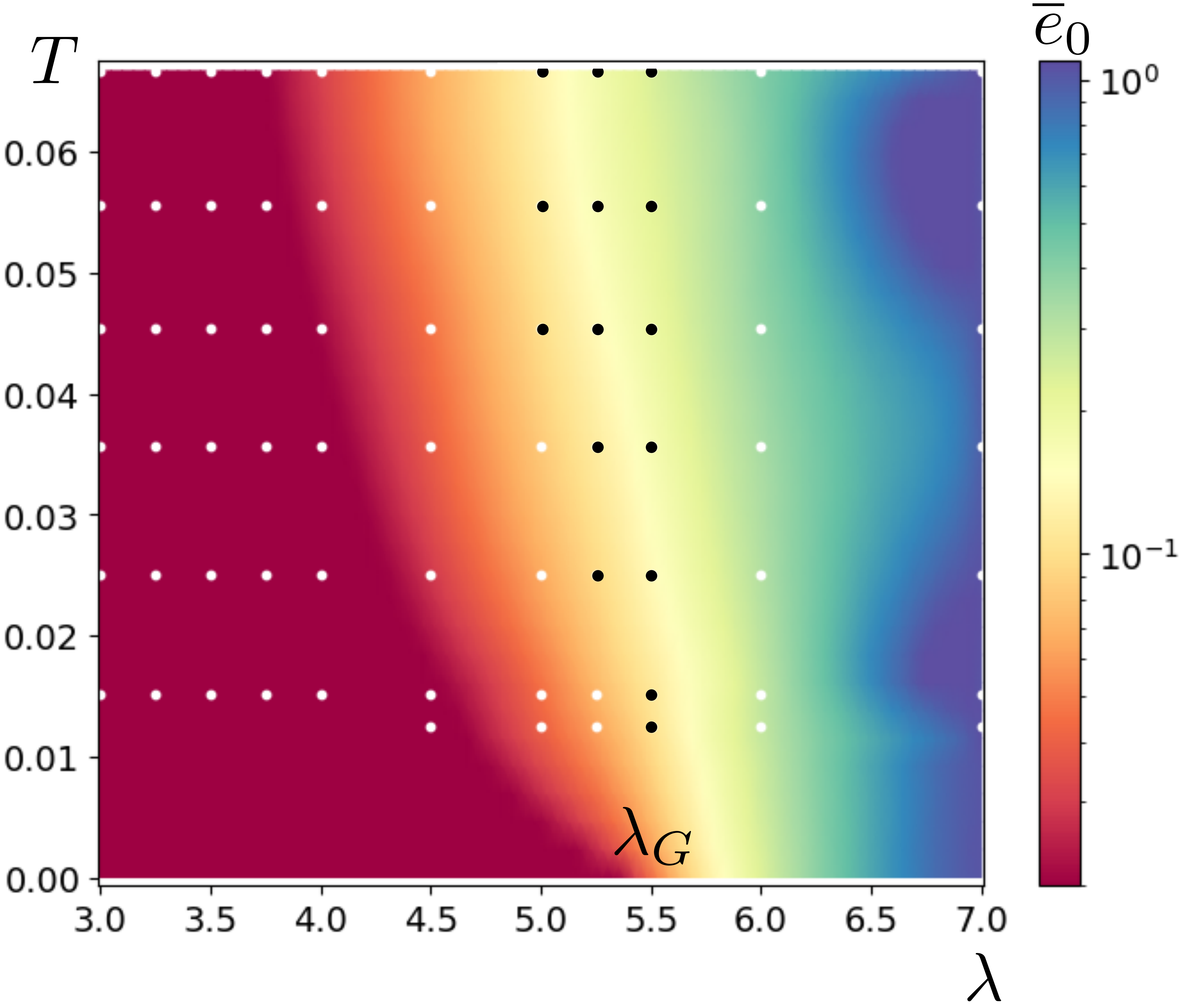








Bosonic eigenmodes in QMC of SDW bosons and electrons



Smallest eigenvalue $\overline{e}_0$ of susceptibility $\chi_{ij}(\omega_n = 0)$

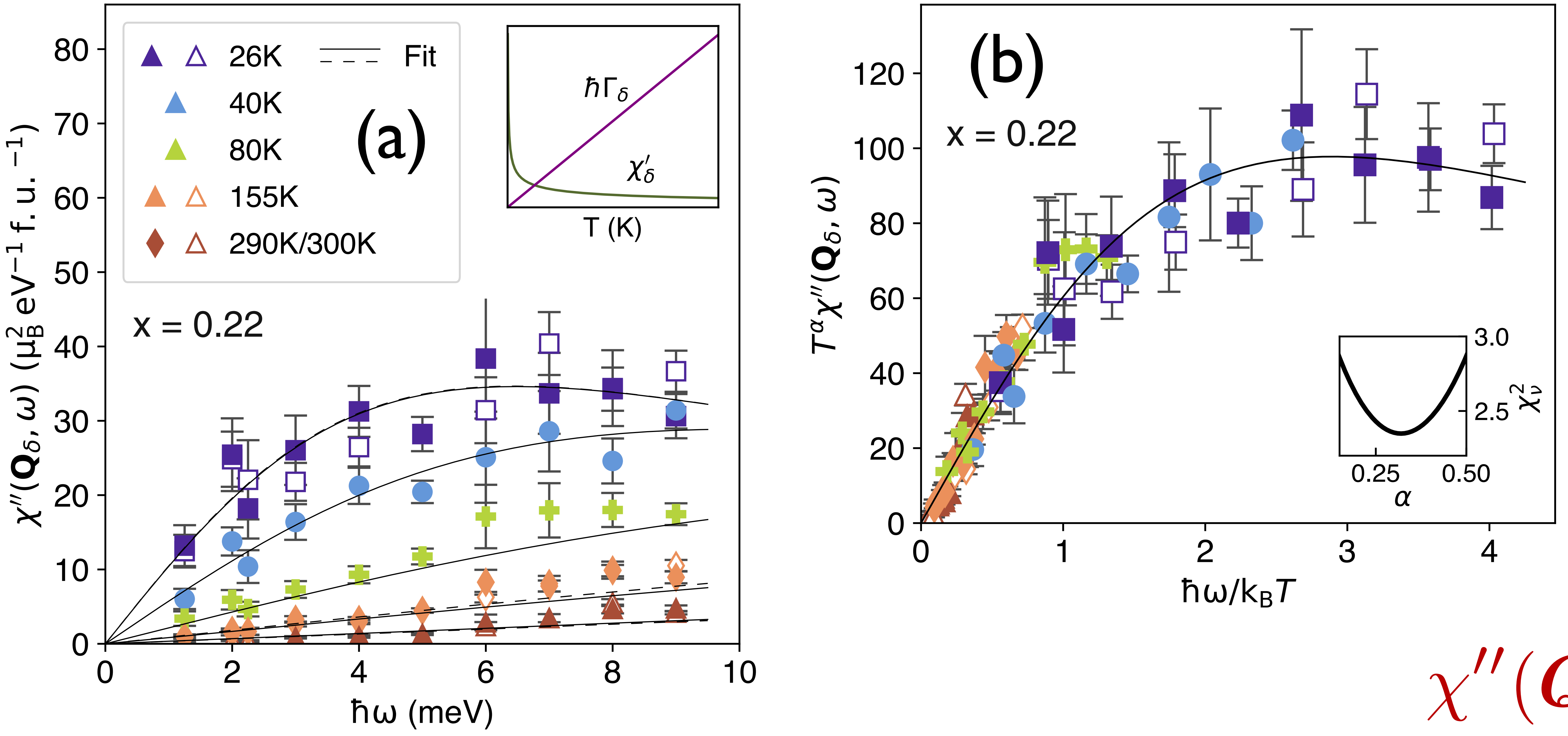
Aavishkar A. Patel,
Peter Lunts,
Michael S. Albergo

arXiv:2410.05365

Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,

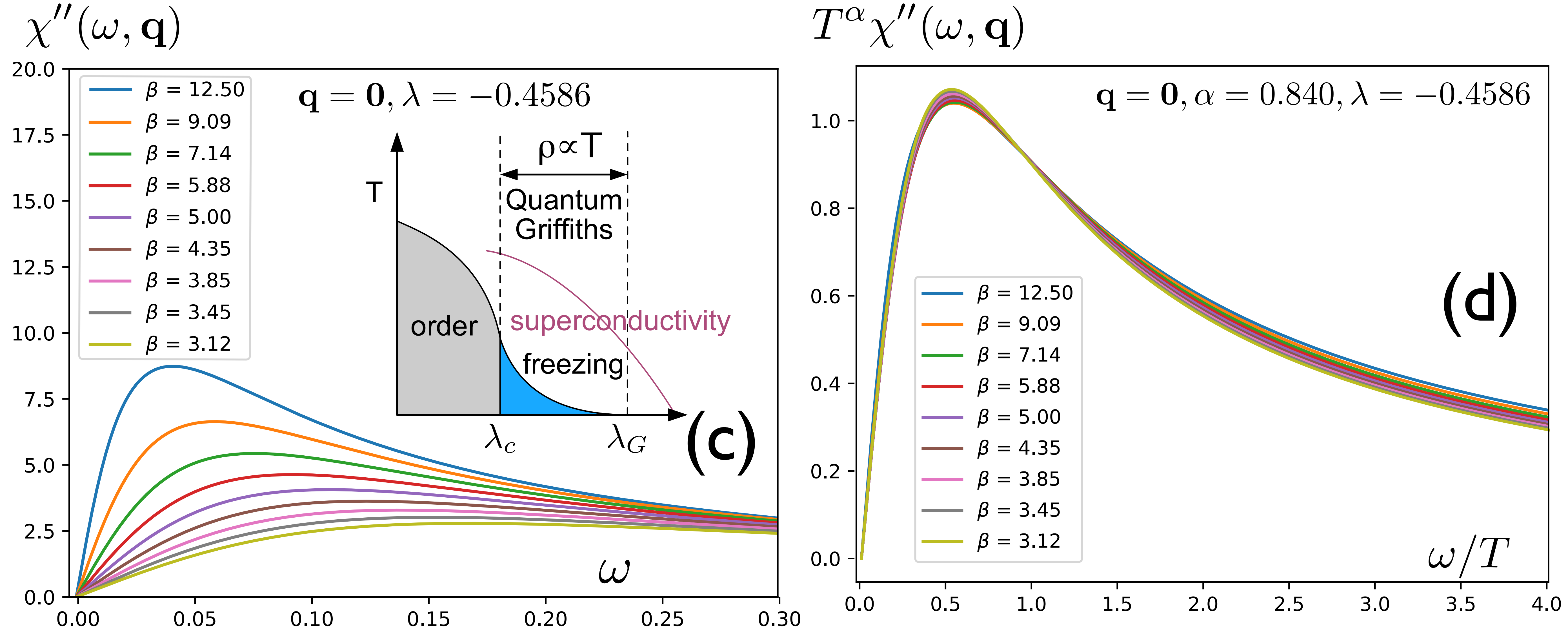
J. Radaelli, A.A. Patel, O.J. Lipscombe, Mengze Zhu, J.R. Stewart, S. S. and S.M. Hayden,

arXiv:2503.13600



Neutron scattering observations

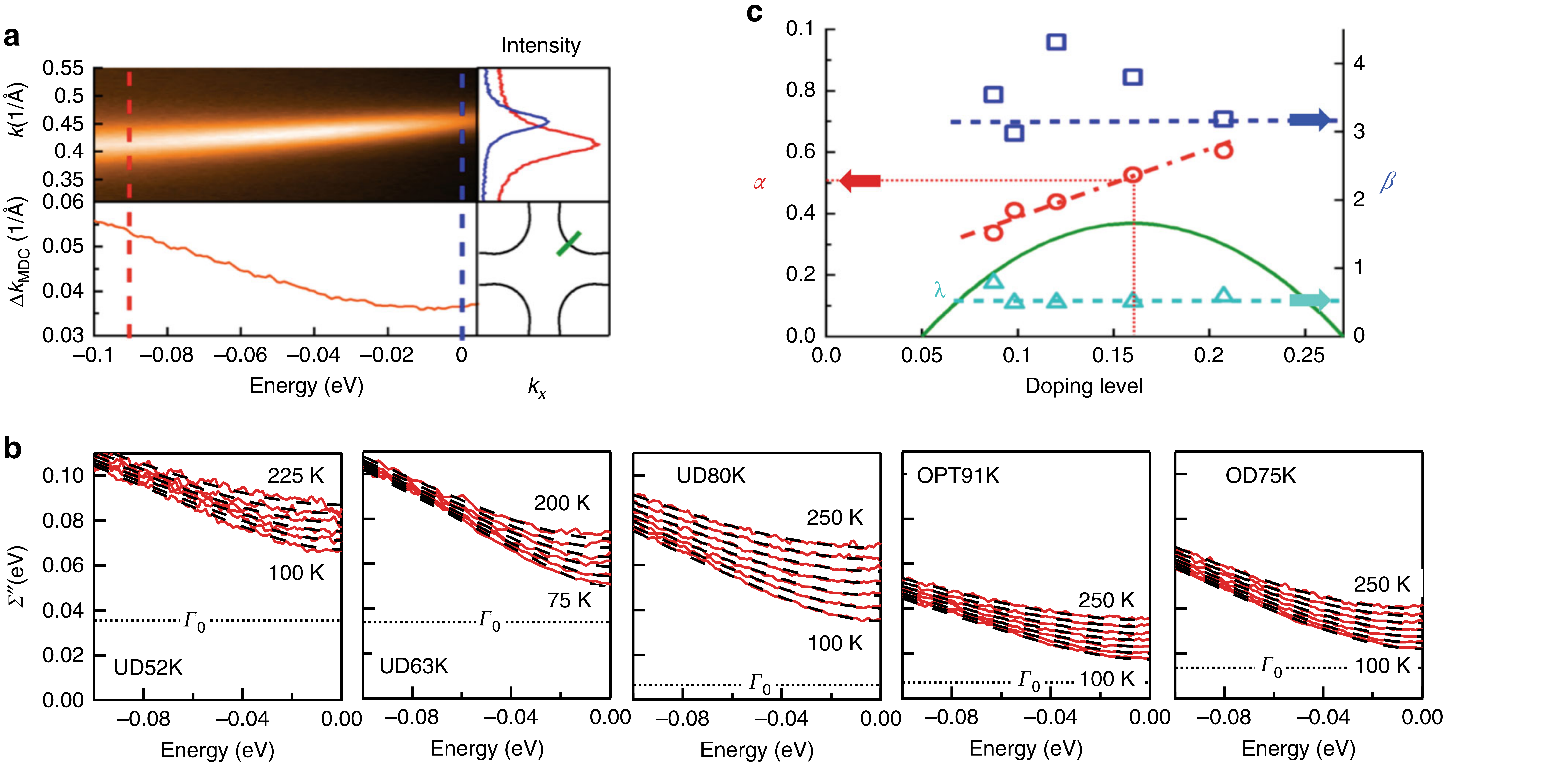
$$\chi''(\boldsymbol{Q}_\delta, \omega) \sim T^{-\alpha} \Phi_\chi \left(\frac{\hbar\omega}{k_B T} \right)$$



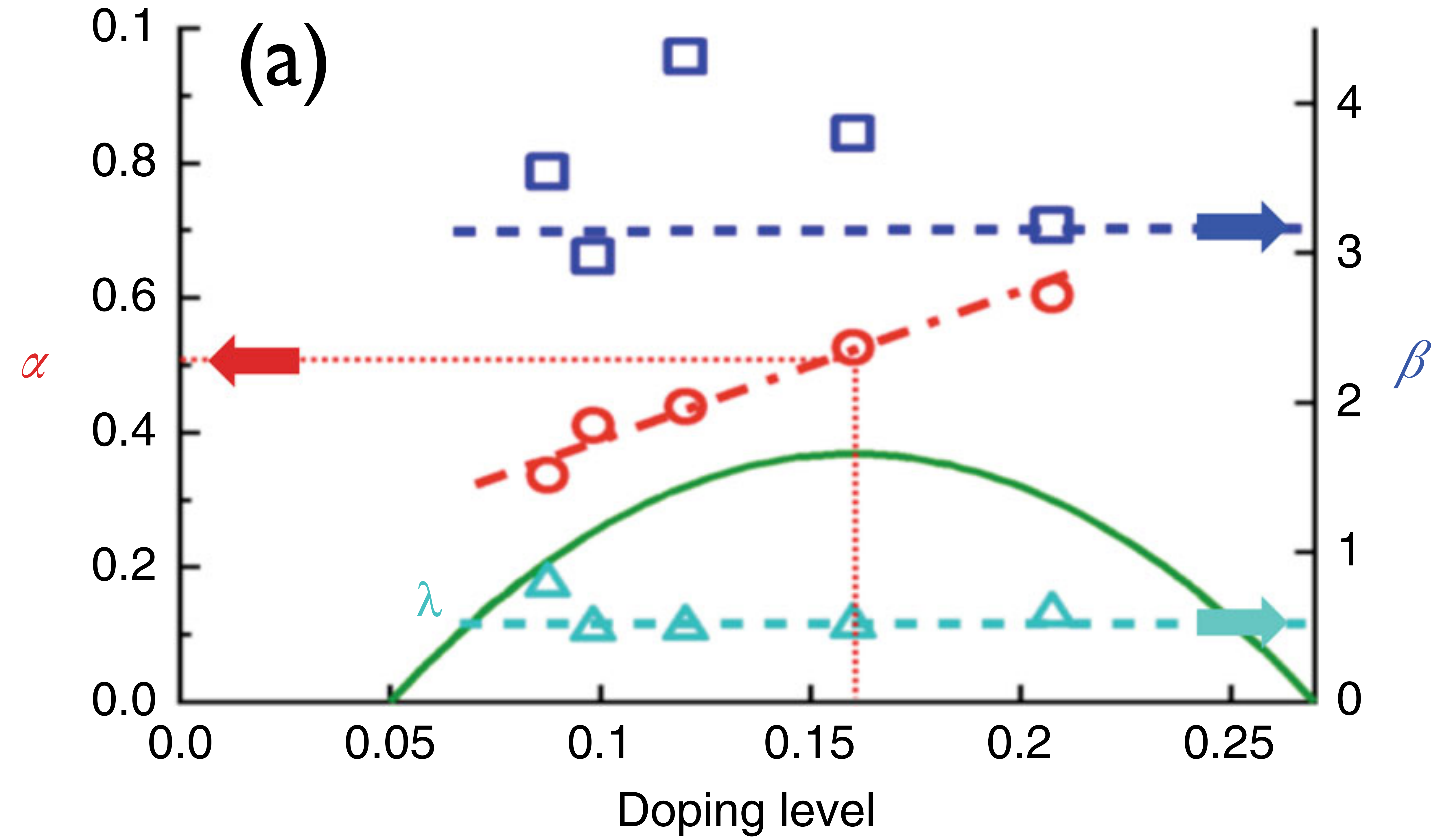
Large-*M* random “mass” SDW Hertz theory

T.J. Reber, X. Zhou, N.C. Plumb, S. Parham, J.A. Waugh, Y. Cao, Z. Sun, H. Li, Q. Wang, J.S. Wen, Z.J. Xu, G. Gu, Y. Yoshida, H. Eisaki, G.B. Arnold and D.S. Dessau, Nature Communications **10** (2019) 3447.

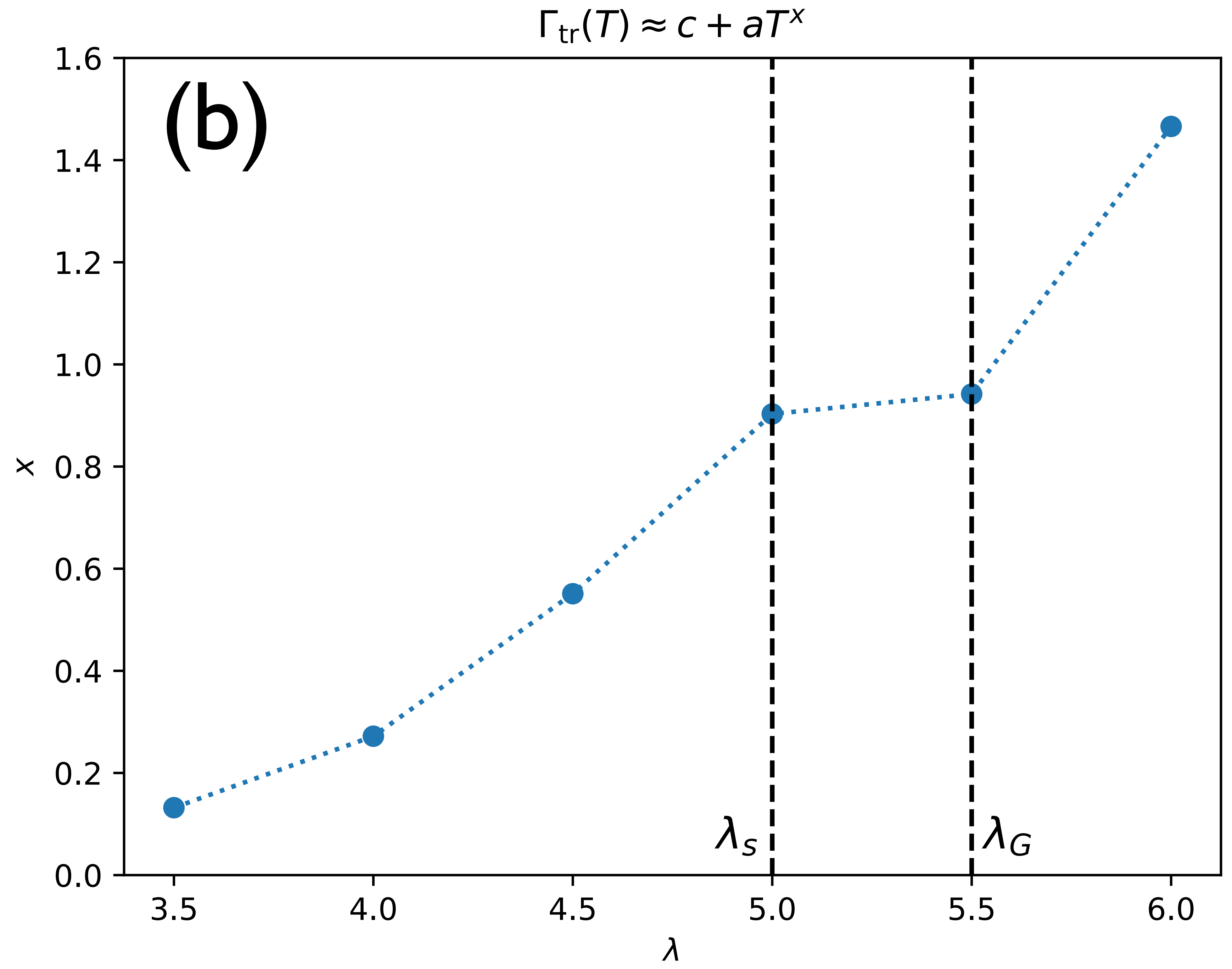
$$\Sigma''_{\rm PLL}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_{\rm B} T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$



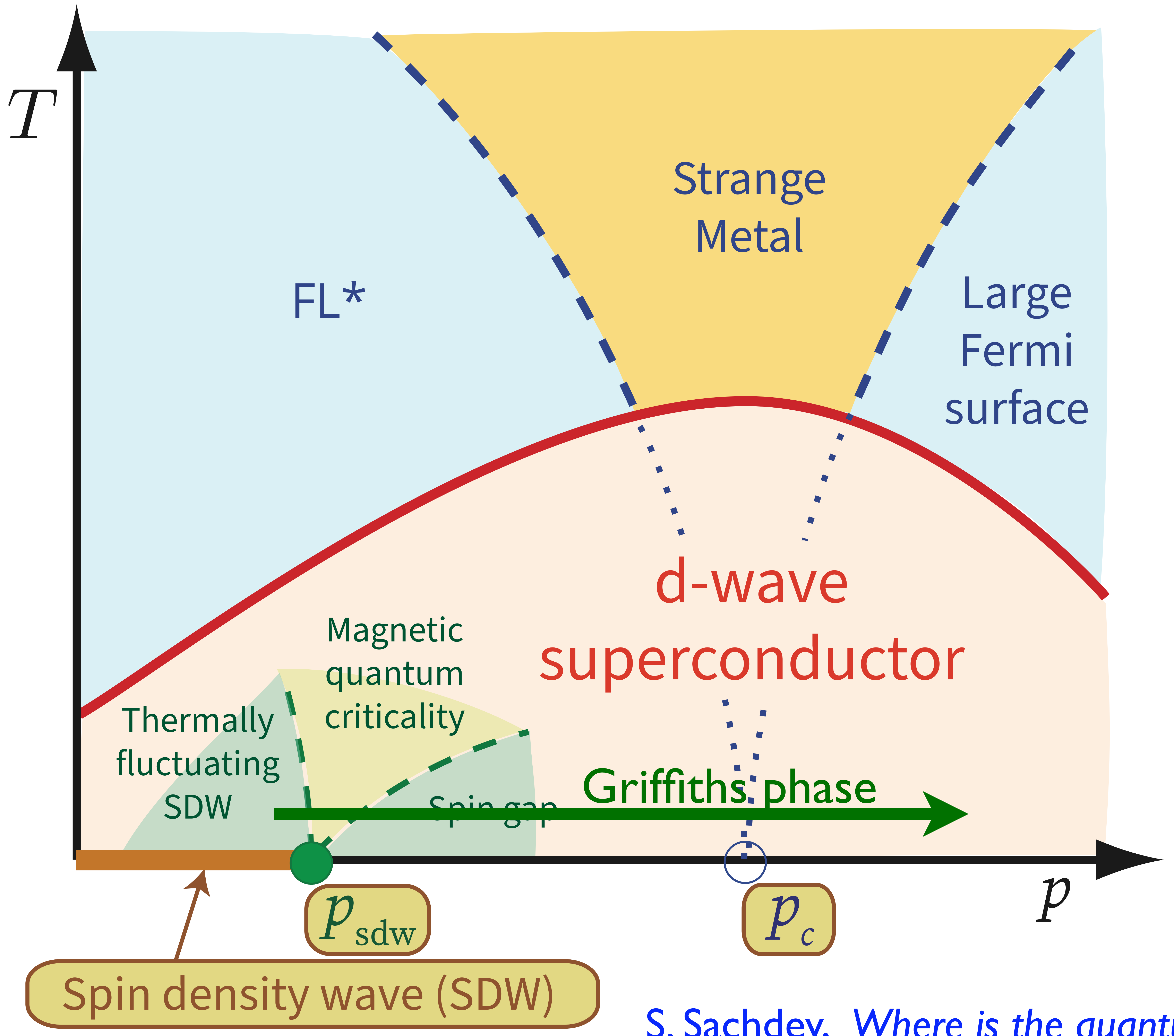
$$\Sigma''_{\mathrm{PLL}}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_{\mathrm{B}} T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$



T.J. Reber….D.S. Dessau,
Nature Communications **10** (2019) 3447.



QMC study by
Aavishkar A. Patel,
Peter Lunts,
Michael S. Albergo
arXiv:2410.05365



Strange metal is disordered FL*-FL quantum criticality

Essentially the same critical theory as for Ising-nematic ordering.

S. Sachdev, *Where is the quantum critical point in the cuprate superconductors?*, Physica Status Solidi B **247**, 537 (2010).