Topological states and non-reciprocity in active matter

Lecture 2: Non-reciprocal active solids

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ICTP School on Quantum Dynamics of Matter, Light and Information

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UNIVERSITY OF CAMBRIDGE
Cavendish Laboratory

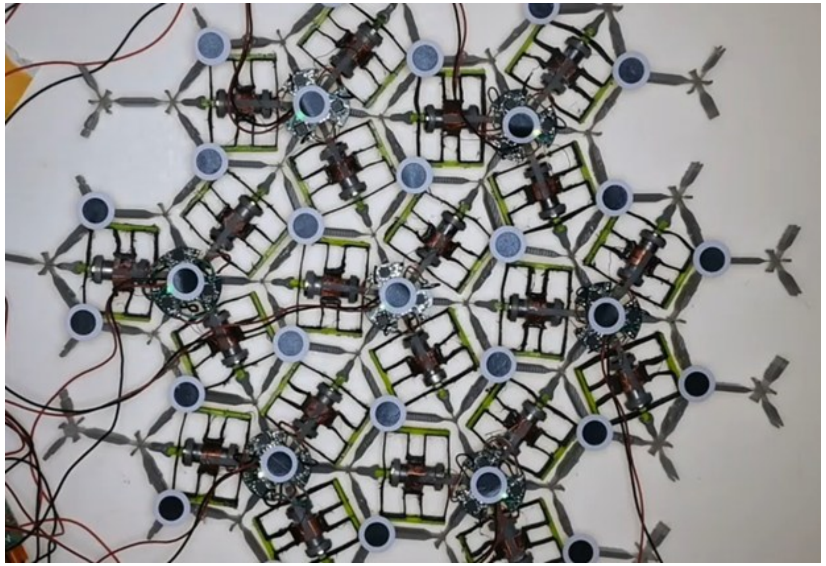
Questions to ask

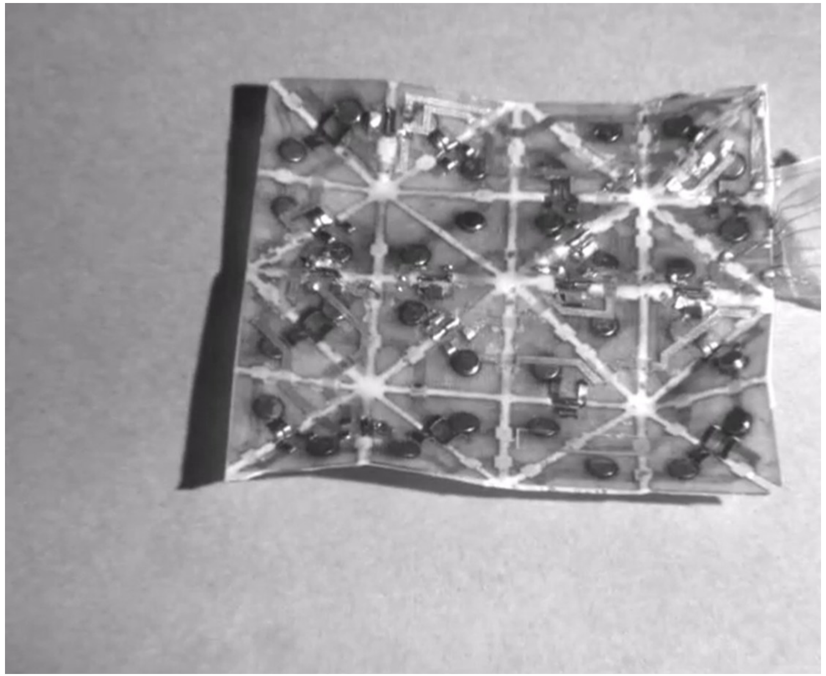
How do we **describe** and **classify** *active matter* based on symmetries and conservation laws?

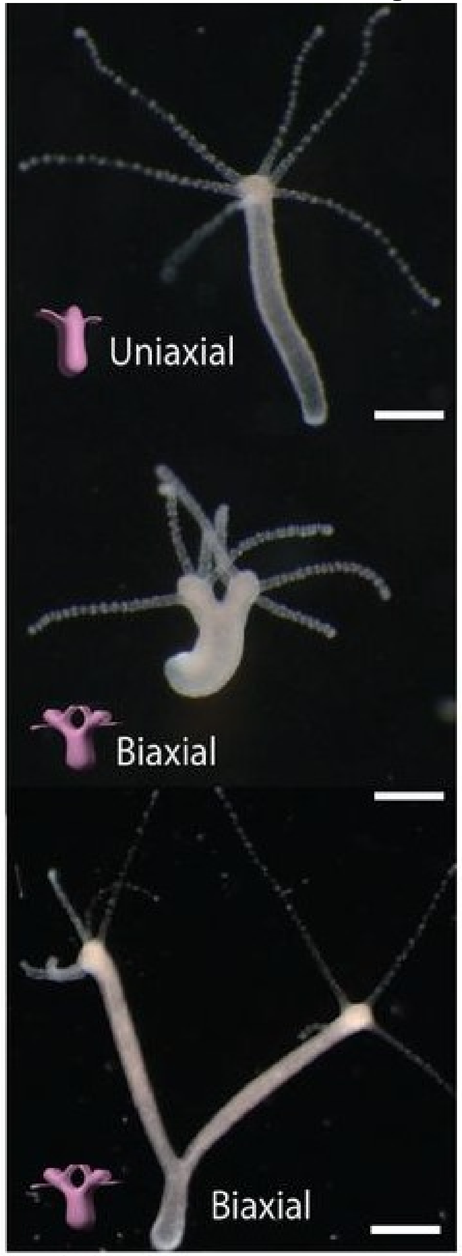
What features of *active matter* are **universal** and independent of microscopic detail?

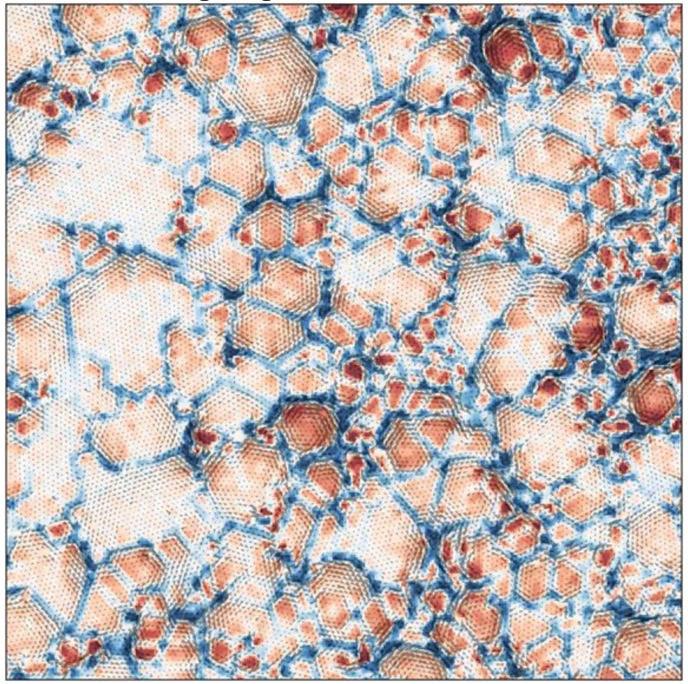
How can we design active materials with *mechanical* properties which are unusual or do not occur naturally?

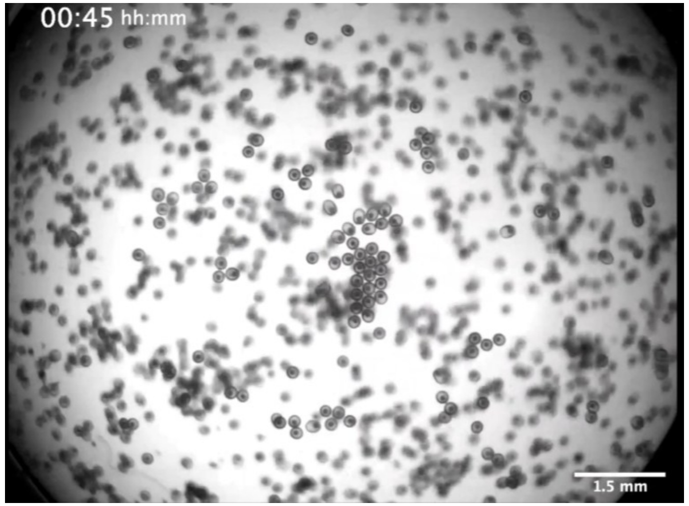
Part 2.1: Active Solids: examples & applications



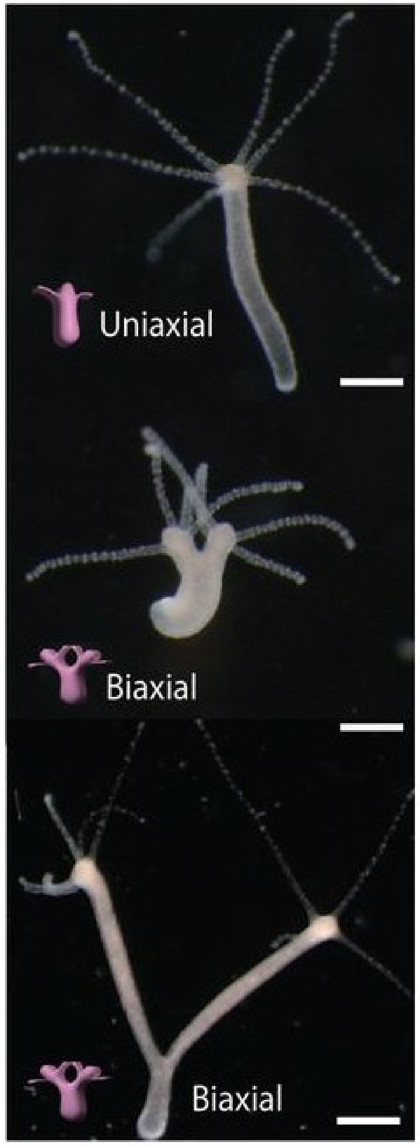


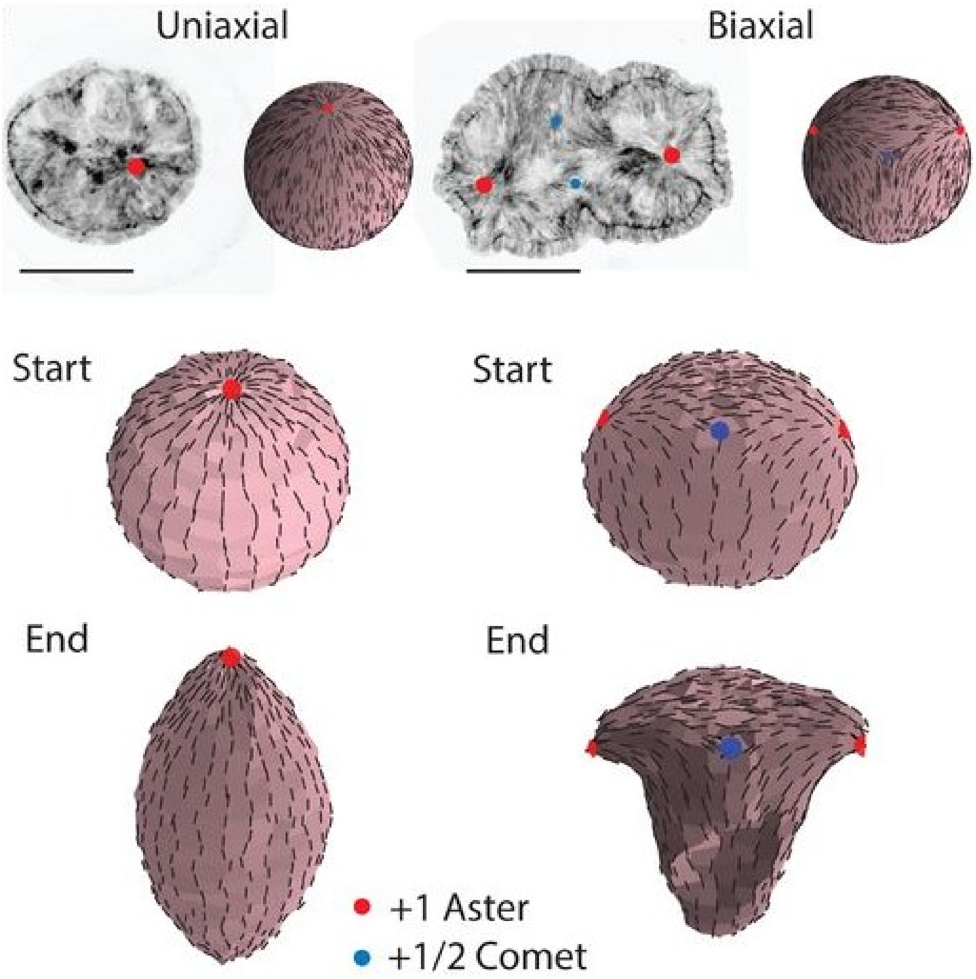






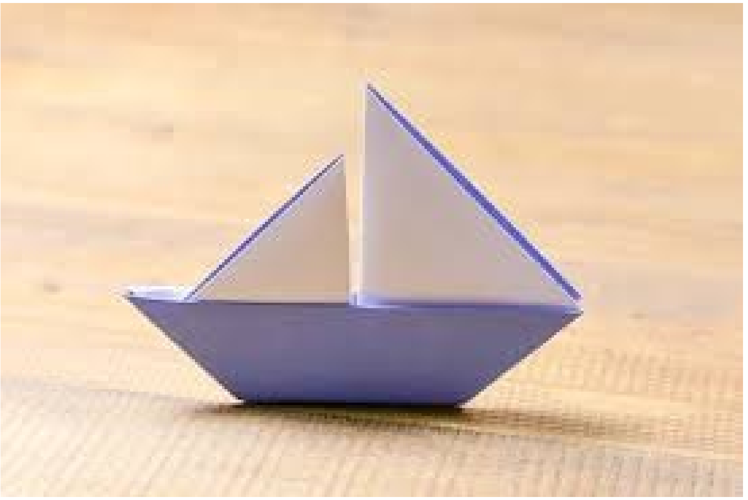
Biological systems: *Hydra* development

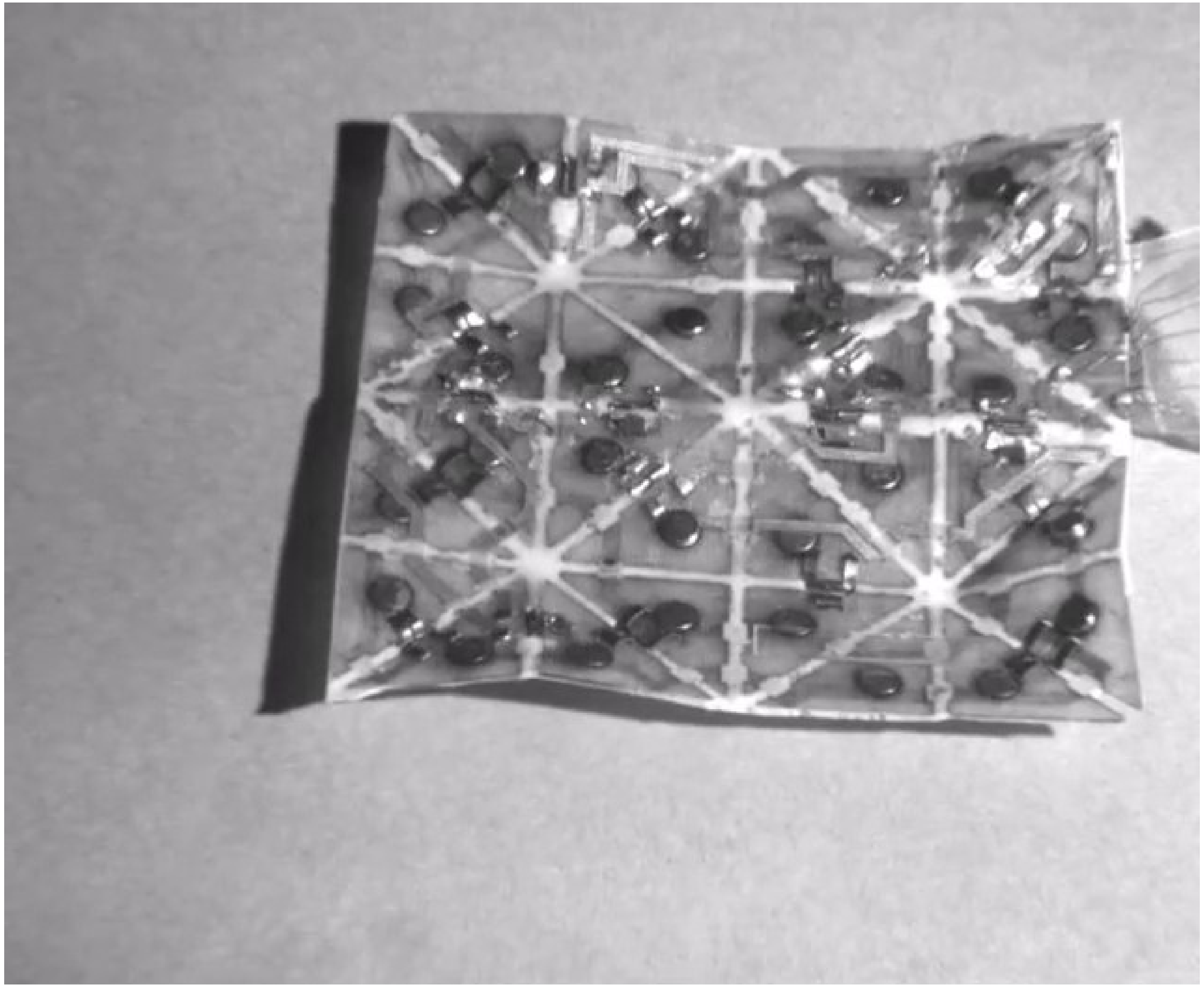




Hawkes et al. *PNAS* (2010)

Active origami





Why active elasticity?

Does not refer to only the solids in which active particles are embedded, “elastic interactions.”

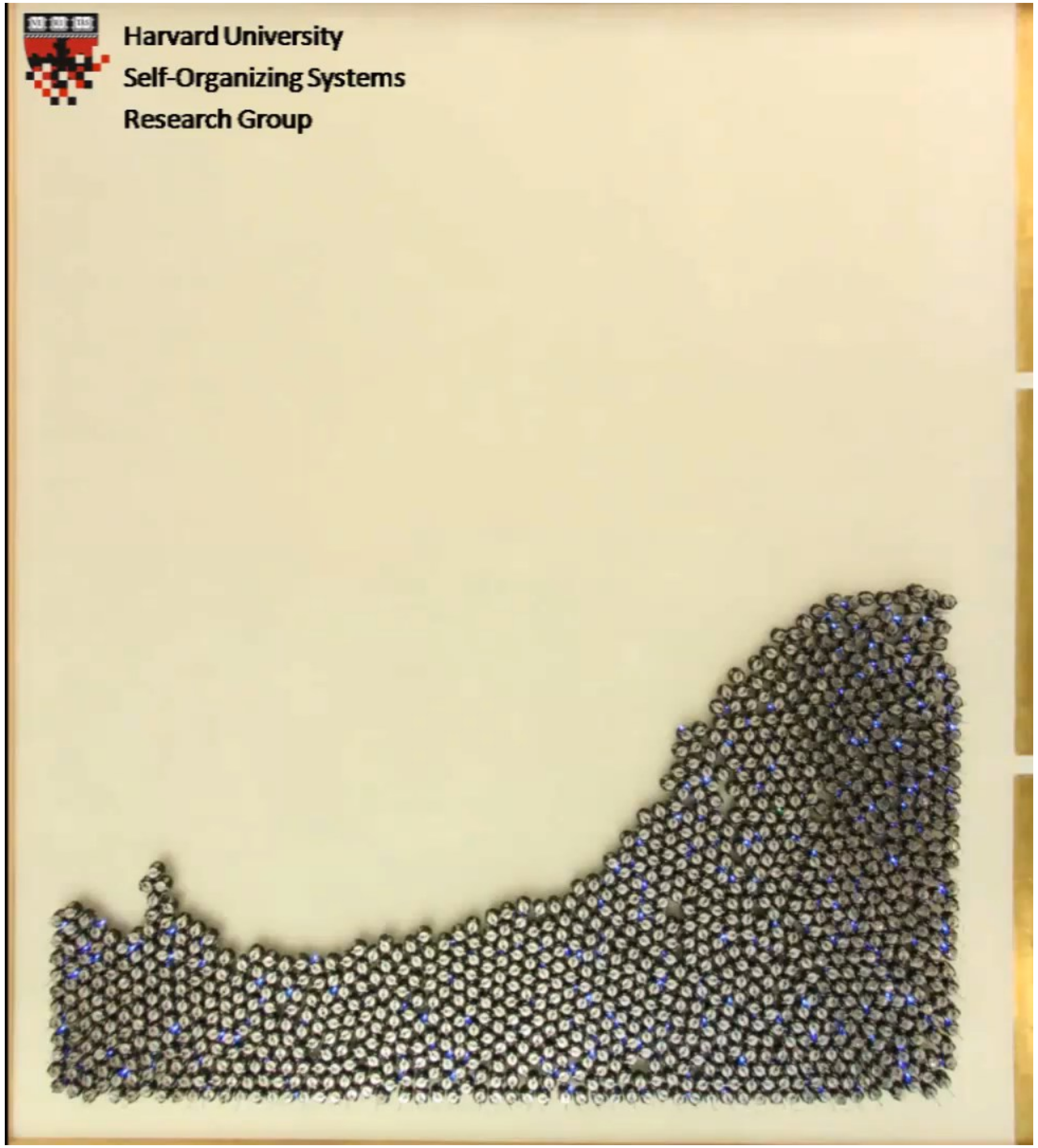
Describes large-lengthscale, slow-timescale phenomena associated with coherent collections of active particles.

Well-developed applications across both biological systems and synthetic materials, but most questions are unexpoled.

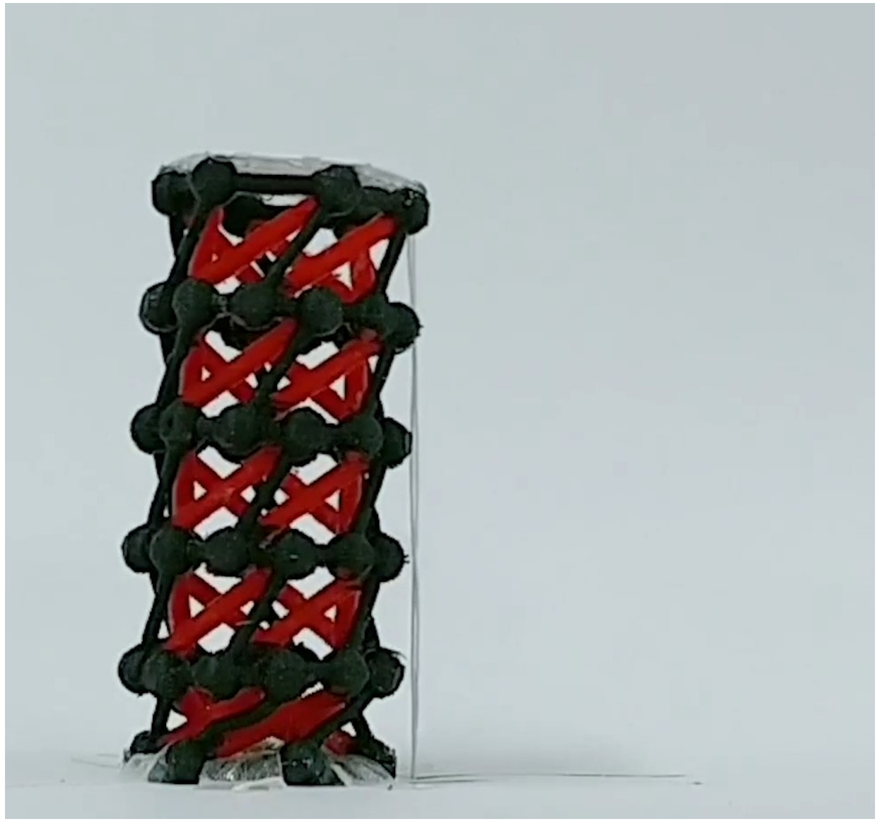
Applications: deployable structures

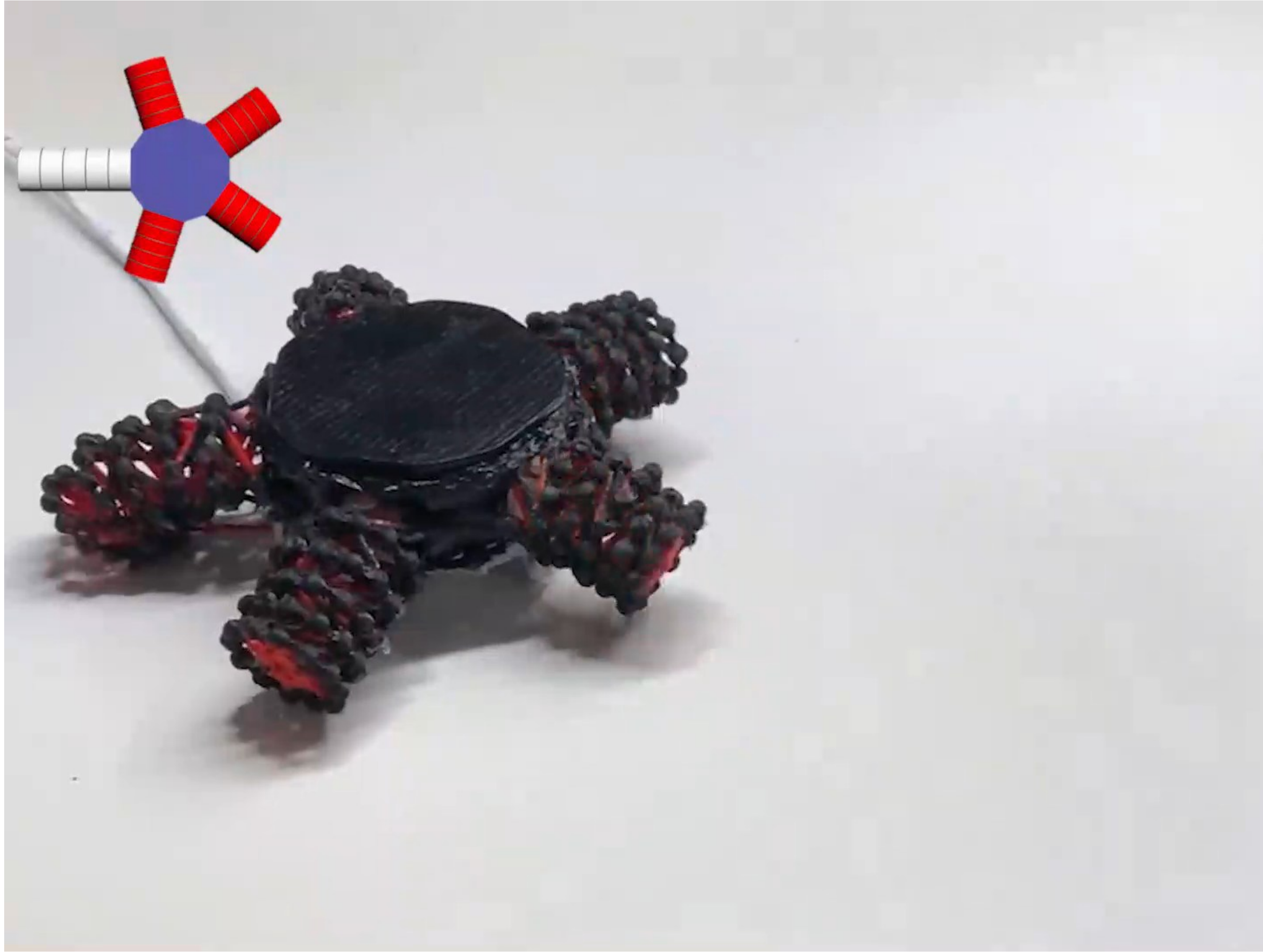
Rubenstein, Alejandro, Nagpal.
Science (2014)

Applications: Robotic swarms



Applications: Soft robotics



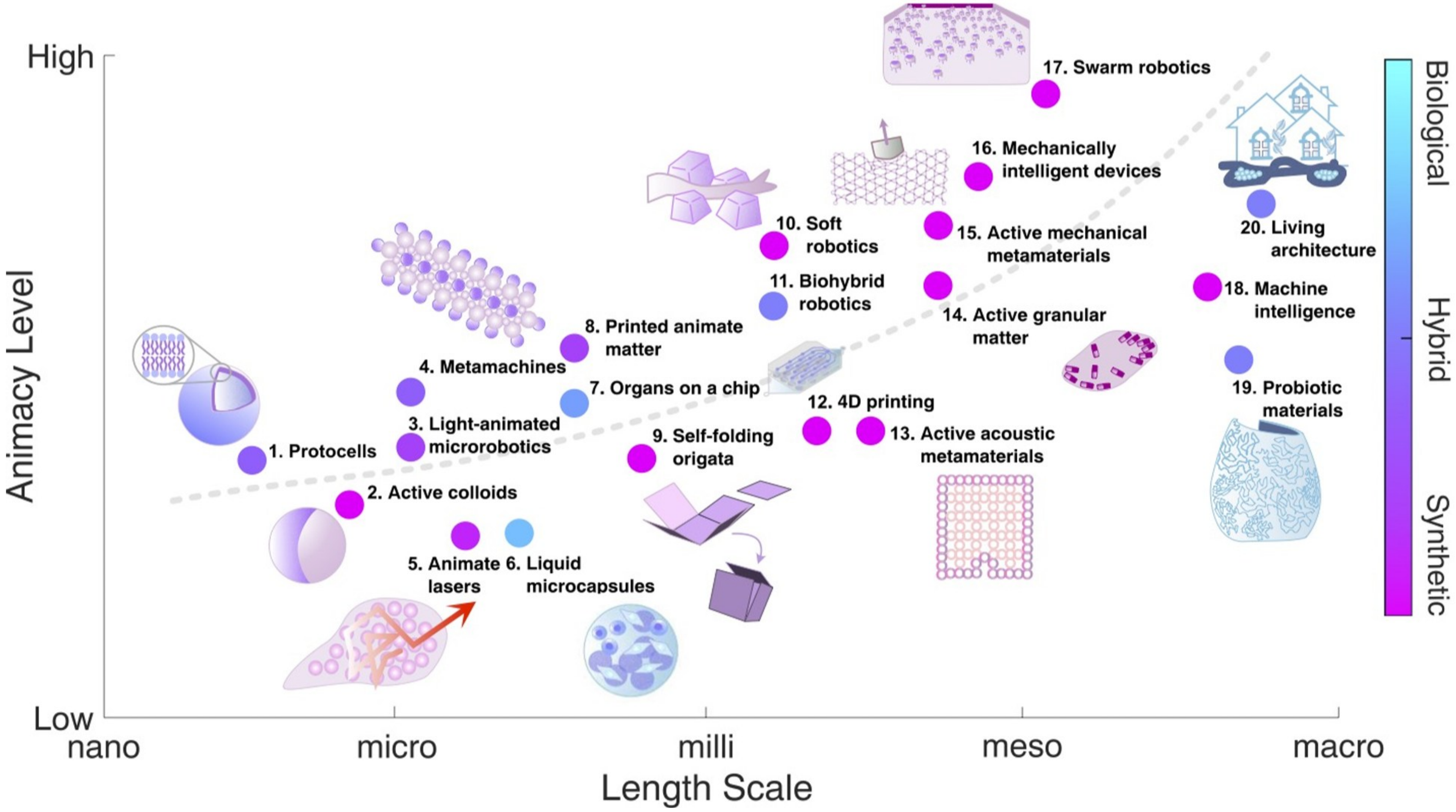


Applications: animate matter

ACTIVE

AUTONOMOUS

ADAPTIVE



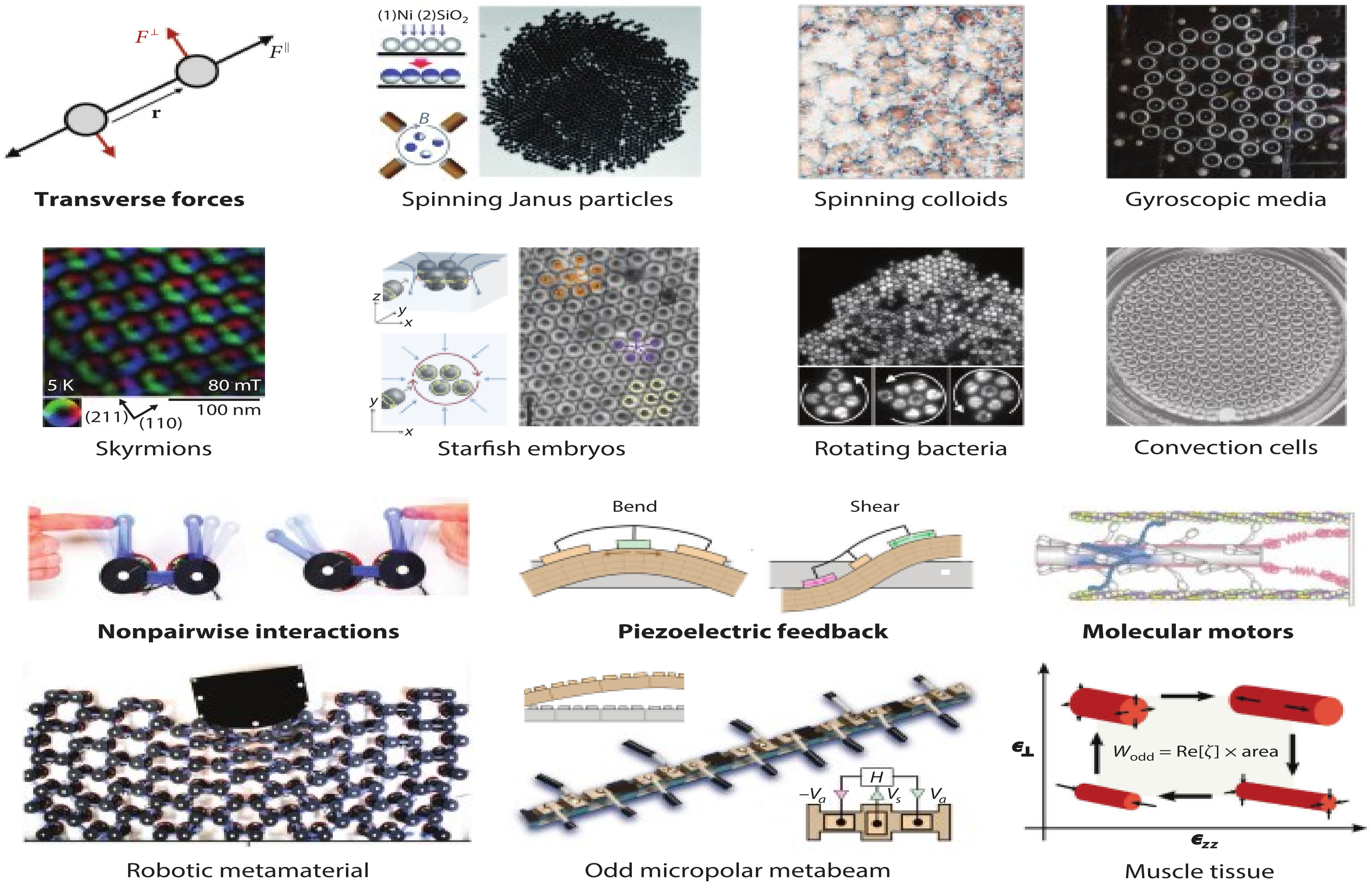
Roadmap for Animate Matter.
G. Volpe *et al.*

Labs:
Granick, Irvine,

Part 2.2: Non-reciprocal mechanics

Pfleiderer, Fakri,
Libchaber, Pallas,

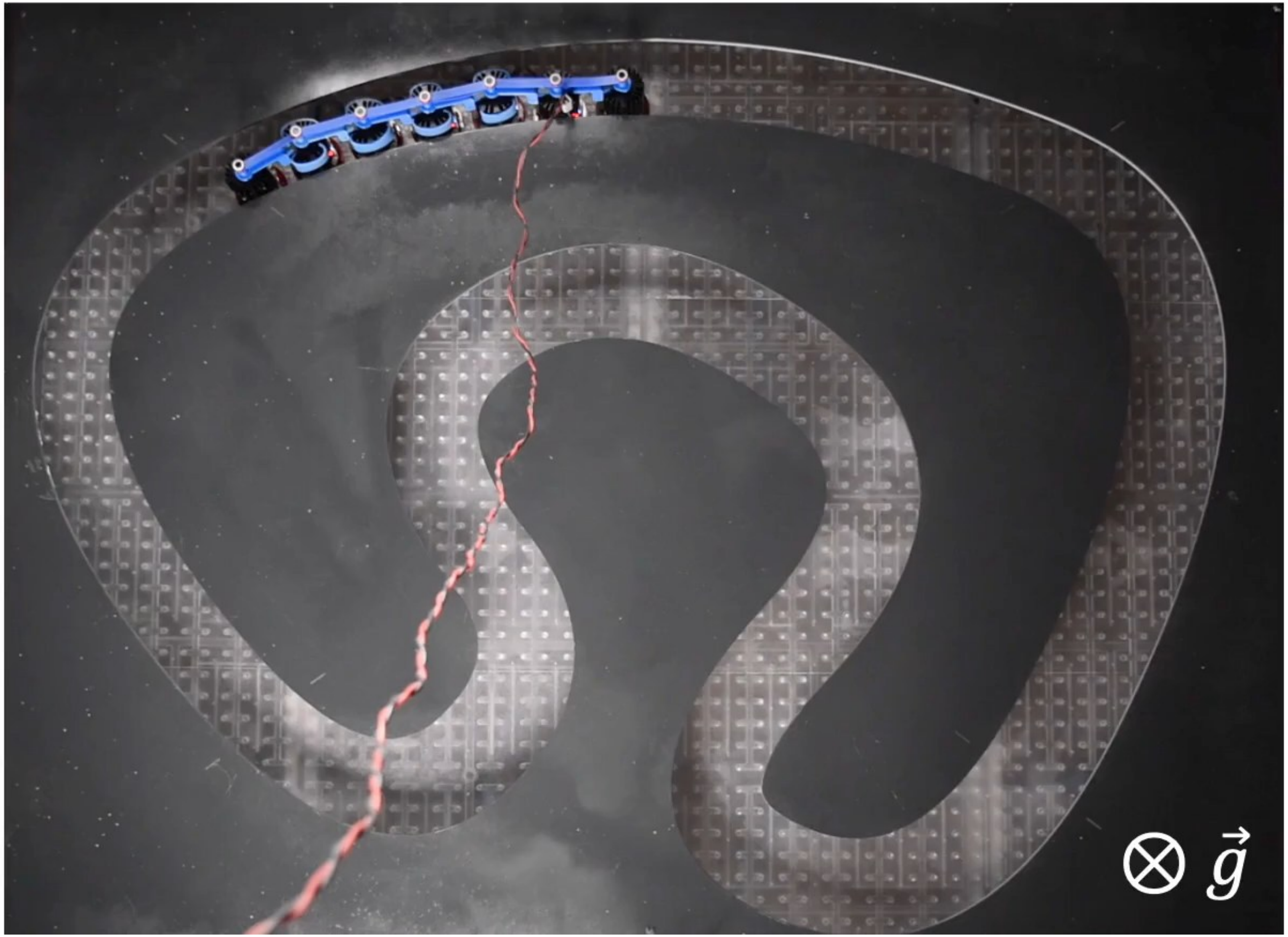
Coulais, Huang,
Mahadevan



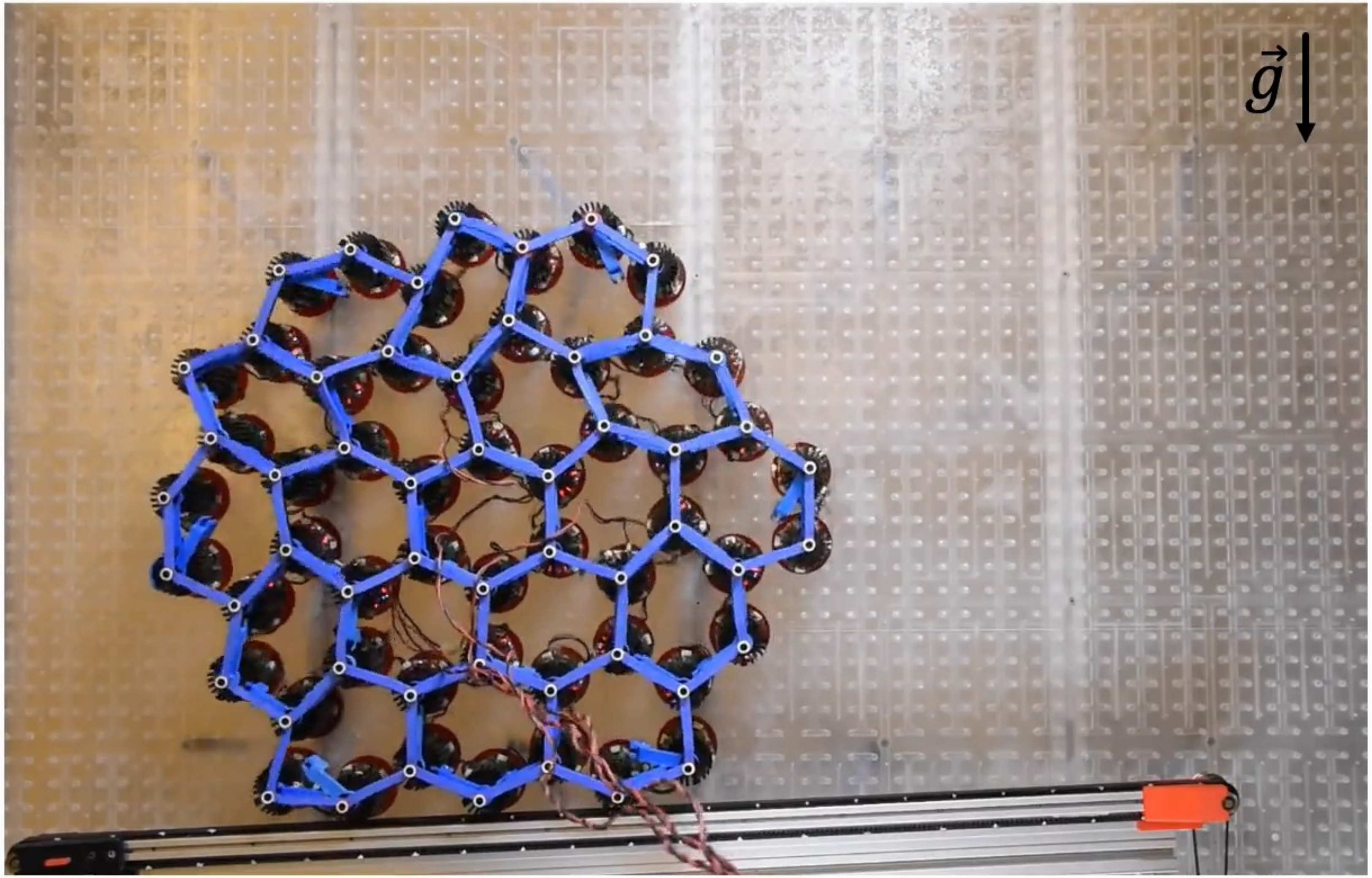
M. Fruchart , C. Scheibner, V. Vitelli. "Odd viscosity and odd elasticity." Annual Review of Condensed Matter Physics 14, 471 (2023)

Veenstra, … , AS, Vitelli, Coulais. *Nature* (2025)

Non-reciprocal interactions lead to locomotion



Functionality: Robust locomotion

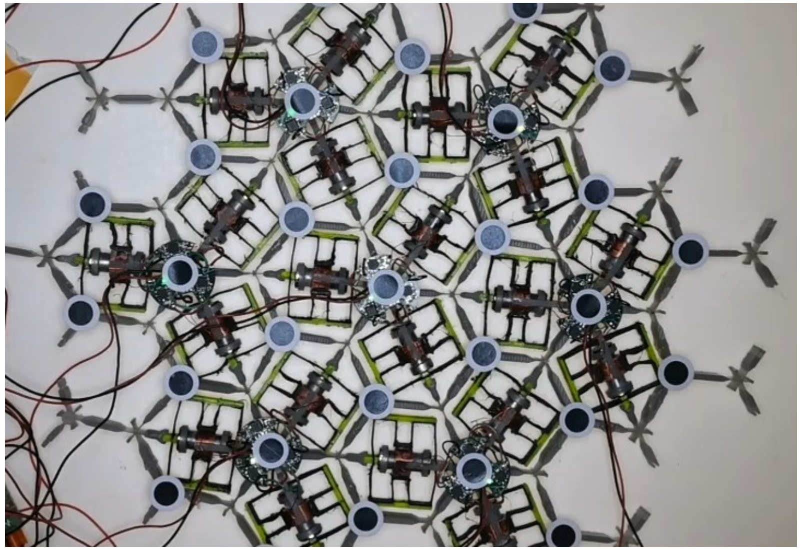


The active solid autonomously adapts its locomotion pattern.

Veenstra, … , AS, Vitelli, Coulais. *Nature* (2025)

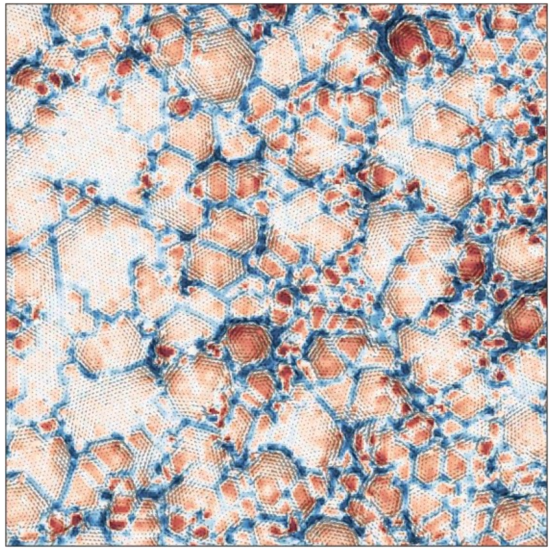
Odd elasticity: experiment

Robotic metamaterials



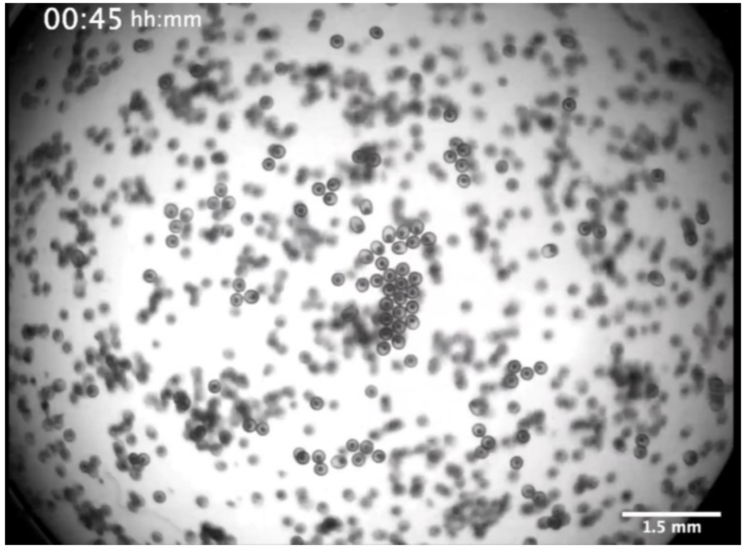
Coulais lab, University of Amsterdam

Rotating colloids



Bililign et al. *Nature Physics* (2022)

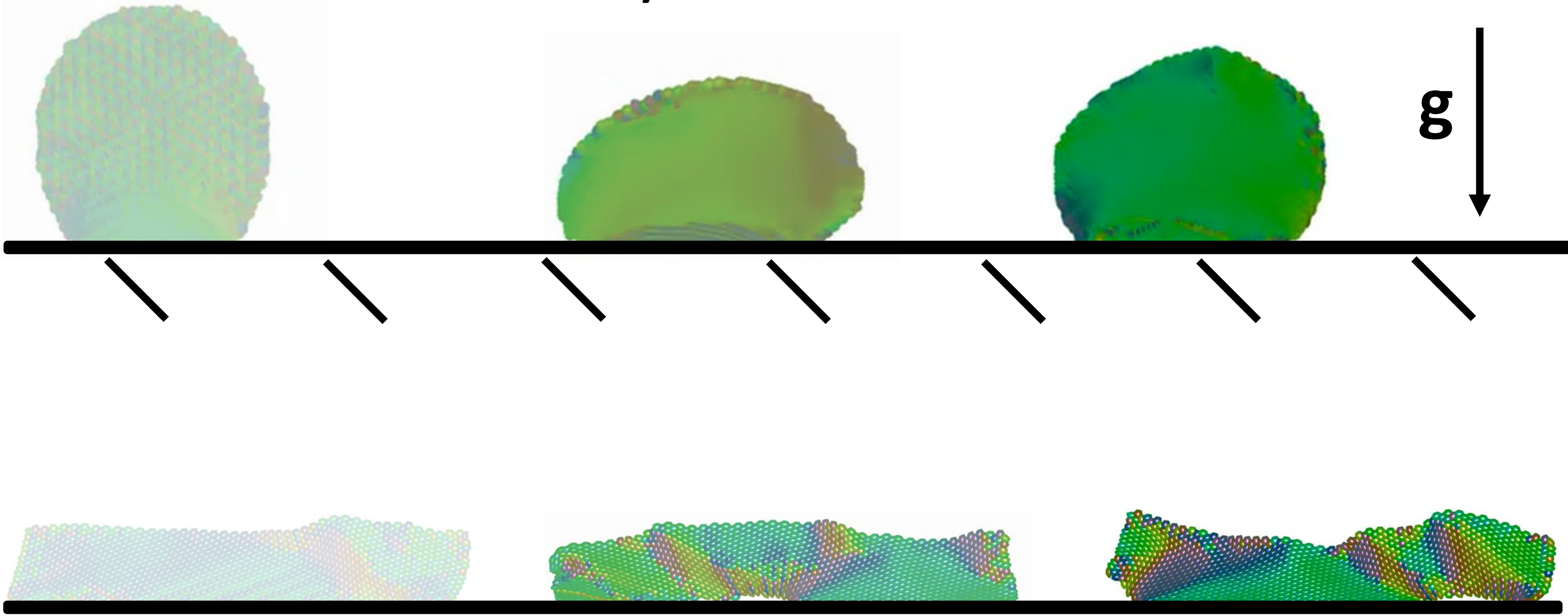
Starfish embryos



Tan et al. *Nature* (2022)

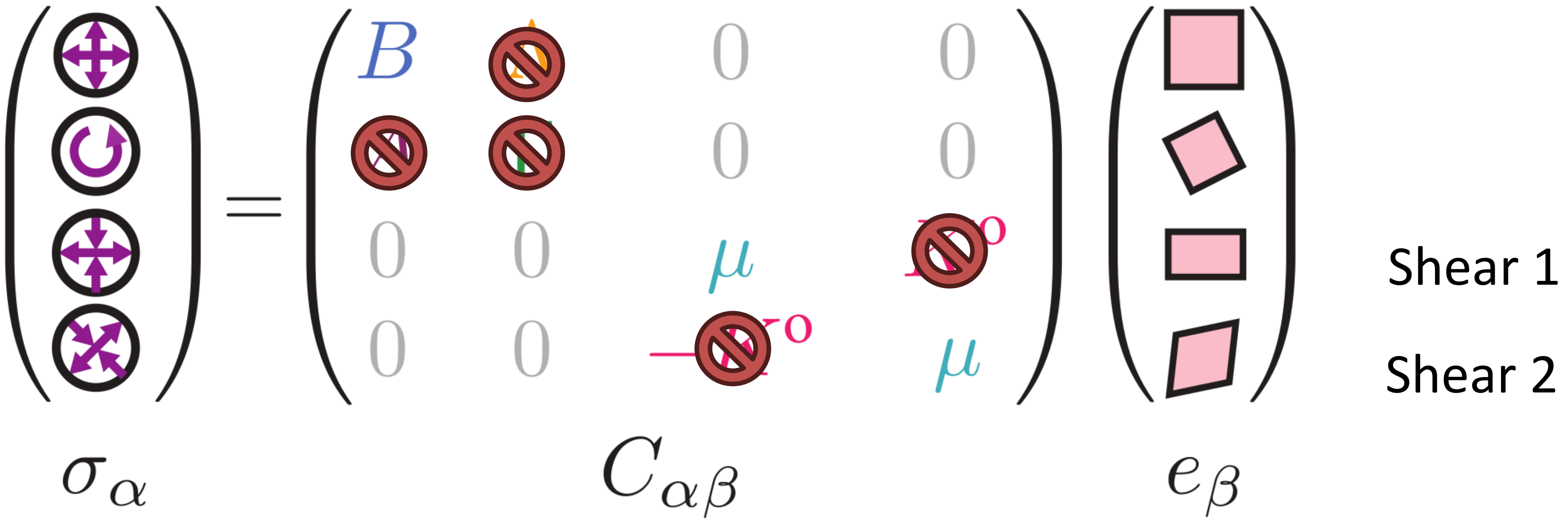
Non-reciprocal interactions in active crystals lead to anomalous mechanical response

Functionality: Robust locomotion



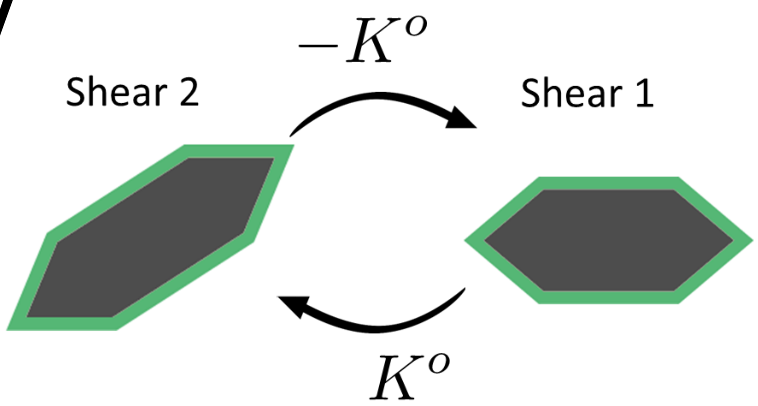
Scheibner, AS, et al. *Nature Physics* (2020)
Shankar et al. *Nat Rev Phys* (2022)

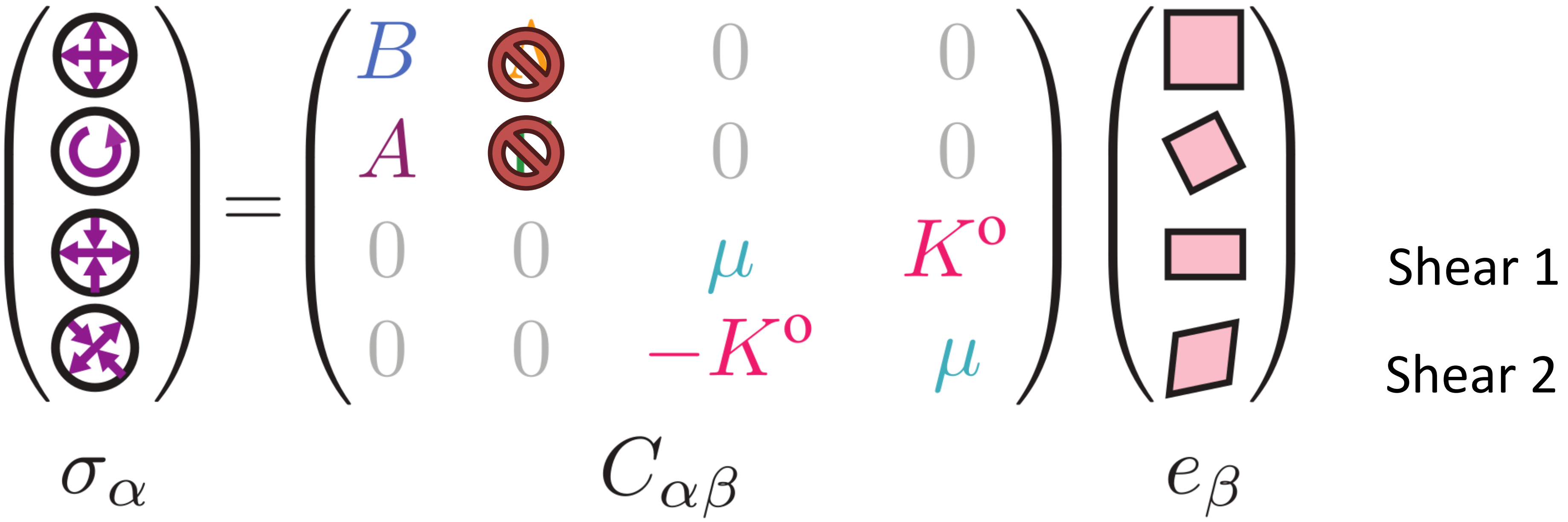
Passive elasticity: theory



Scheibner, AS, et al. *Nature Physics* (2020)
Shankar et al. *Nat Rev Phys* (2022)

Odd elasticity: theory

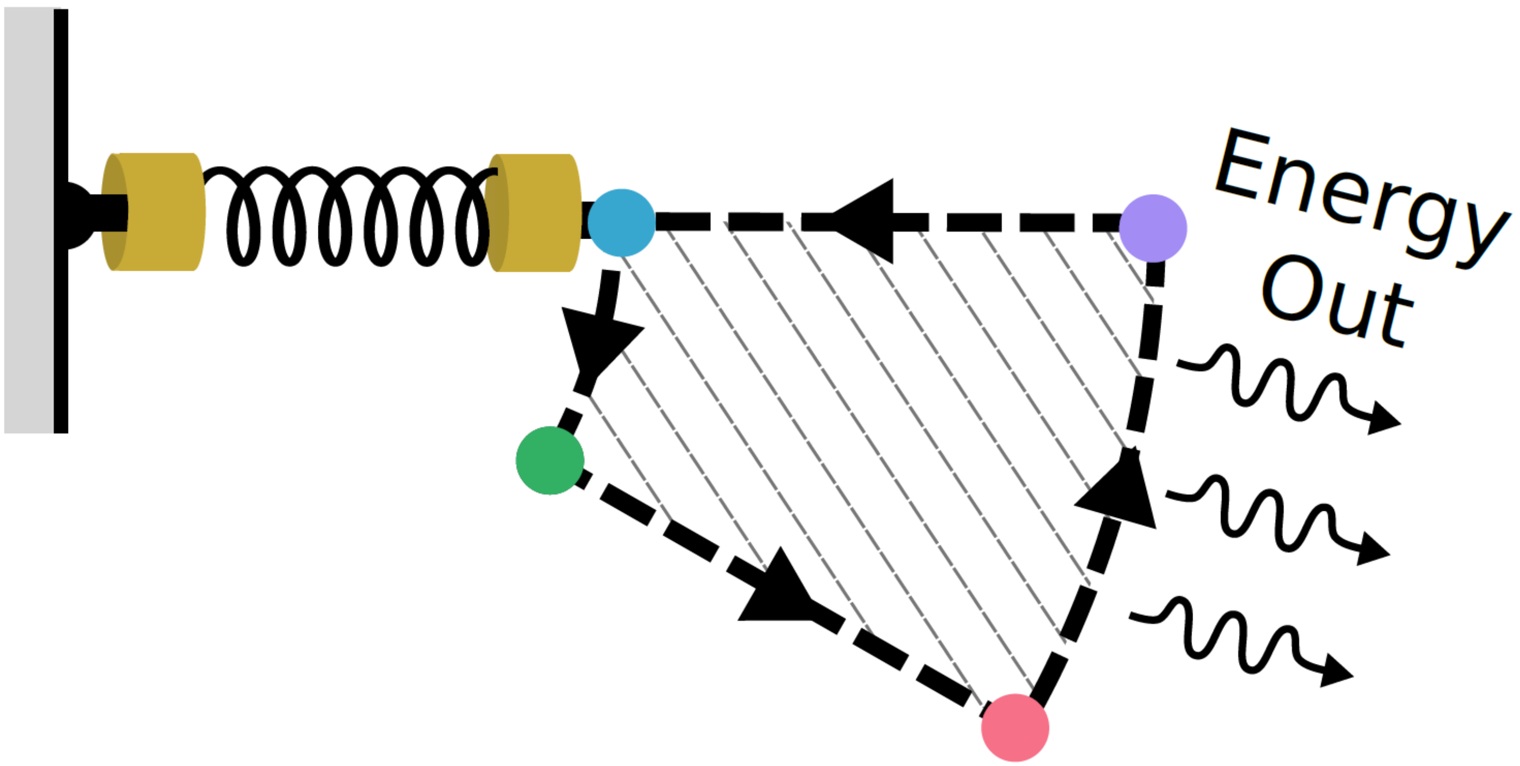




Anti-symmetric components of the stress-strain relation arise in materials with active springs

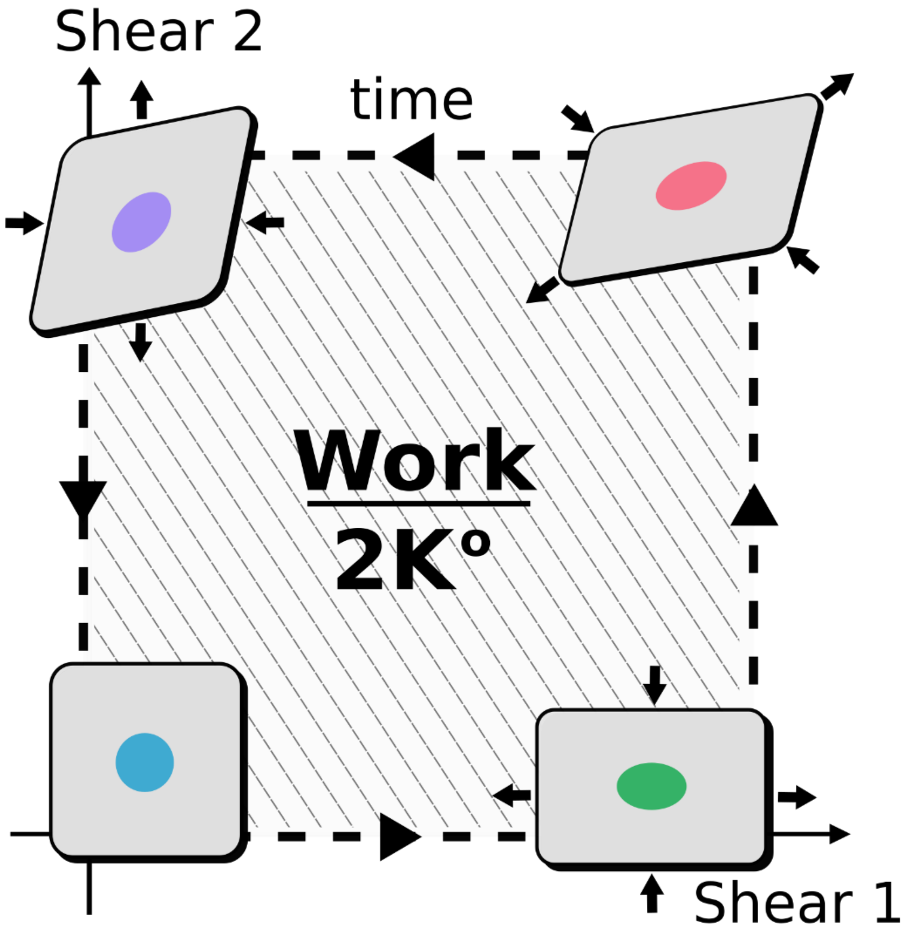
Scheibner, AS, et al. *Nature Physics* (2020)
Shankar et al. *Nat Rev Phys* (2022)

Cycles of active springs



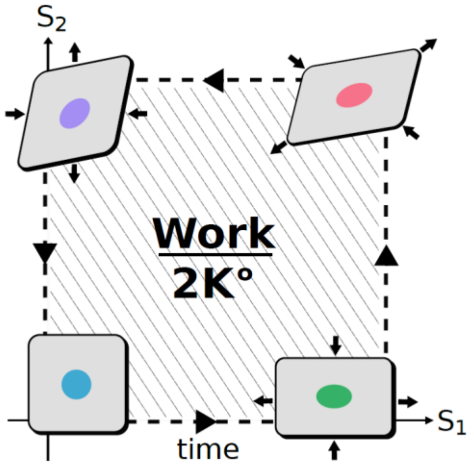
Path-dependent extraction of energy without introducing extra degrees of freedom

Abstraction: Elastic engine cycle



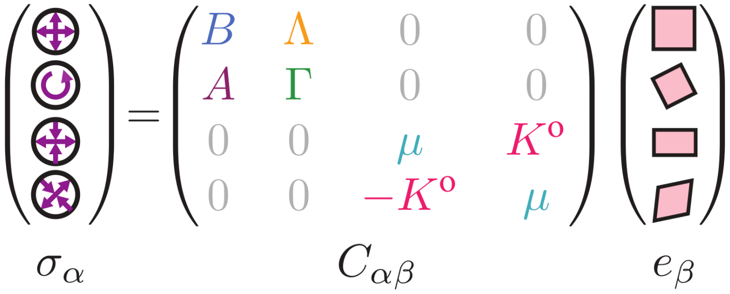
Path-dependent extraction of energy

Scheibner, AS, et al. *Nature Physics* (2020)
Shankar et al. *Nat Rev Phys* (2022)



Work extraction formula

$$W = \oint \sigma_{ij}\mathrm{d}u_{ij} = \iint \mathrm{d}\sigma_{ij} \wedge \mathrm{d}u_{ij} = -\iint \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}u_{kl}} \mathrm{d}u_{ij} \wedge \mathrm{d}u_{kl}$$

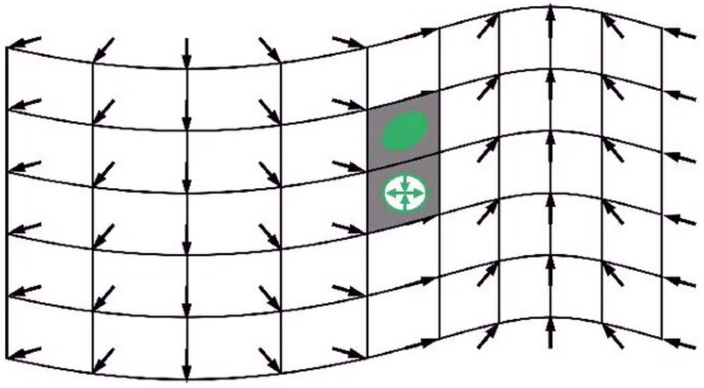


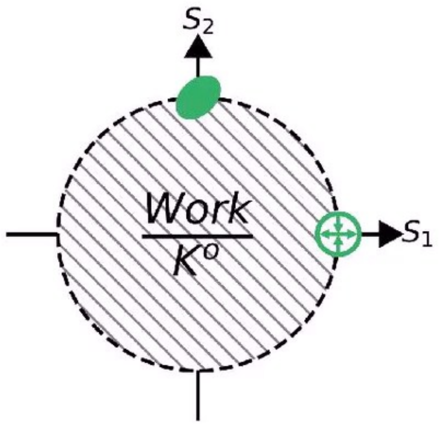
Only odd part of the elastic tensor contributes to active work

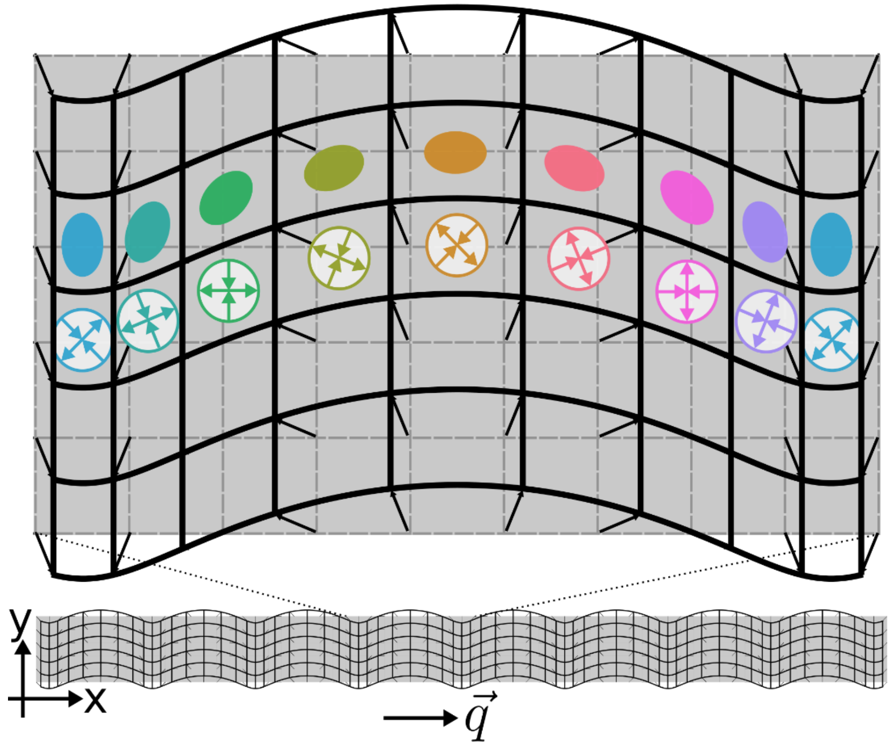
Scheibner, AS, et al. *Nature Physics* (2020)

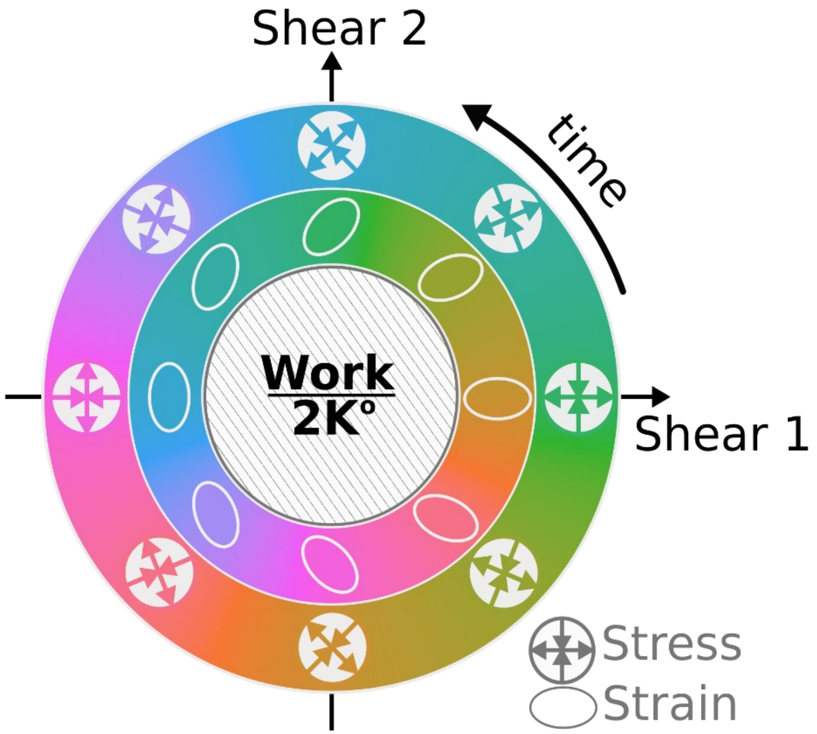
$$\eta \dot{u}_j = \partial_i \sigma_{ij}$$

Cycles can self-sustain









Elasto-dynamic waves

Scheibner, AS, et al. *Nature Physics* (2020)

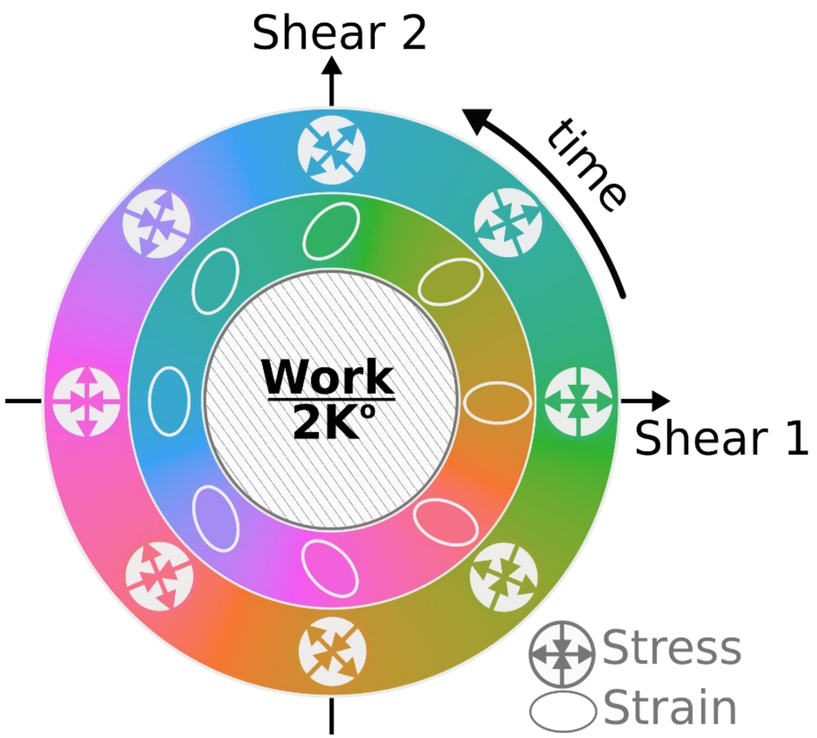
$$\eta \dot{u}_j = \partial_i \sigma_{ij}$$

Wave dispersion

Power injected (per unit area): $K^o q^2$

Power dissipated (per unit area): $\eta\omega$

$$\omega = \frac{K^o q^2}{\eta}$$



Dispersion from balancing energy in and out

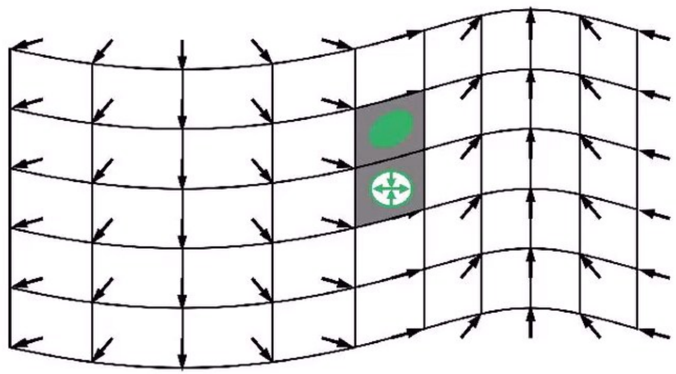
Scheibner, AS, et al. *Nature Physics* (2020)

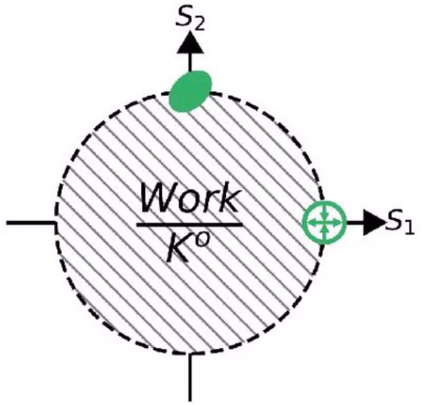
Scheibner, AS, et al. *Nature Physics* (2020)
Shankar et al. *Nat Rev Phys* (2022)

Tan et al. *Nature* (2022)

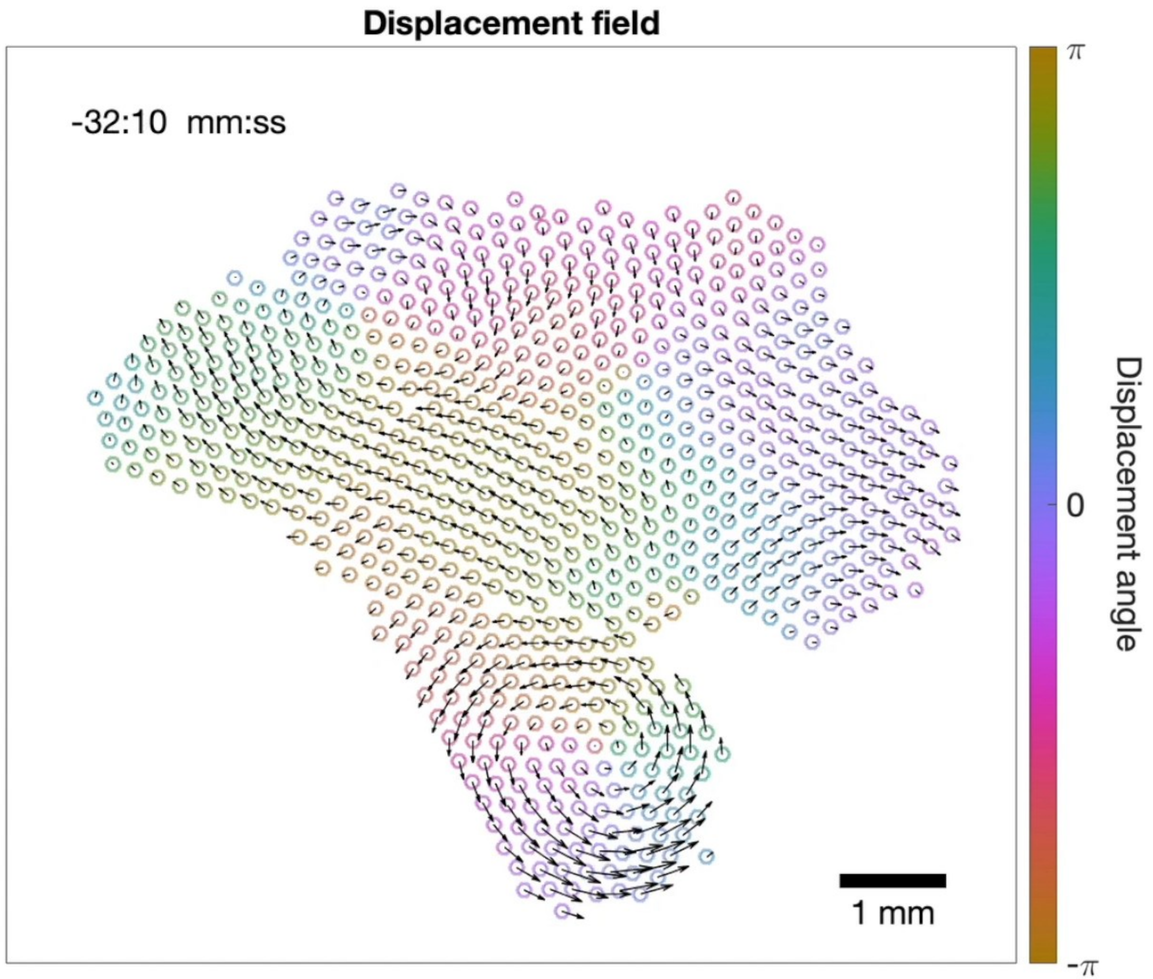
Odd elastic waves

Theory



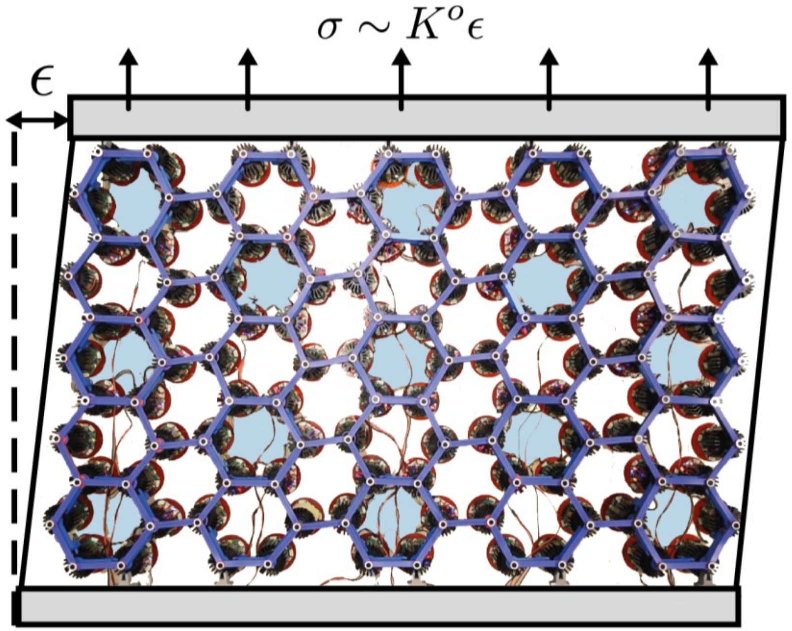


Starfish embryos

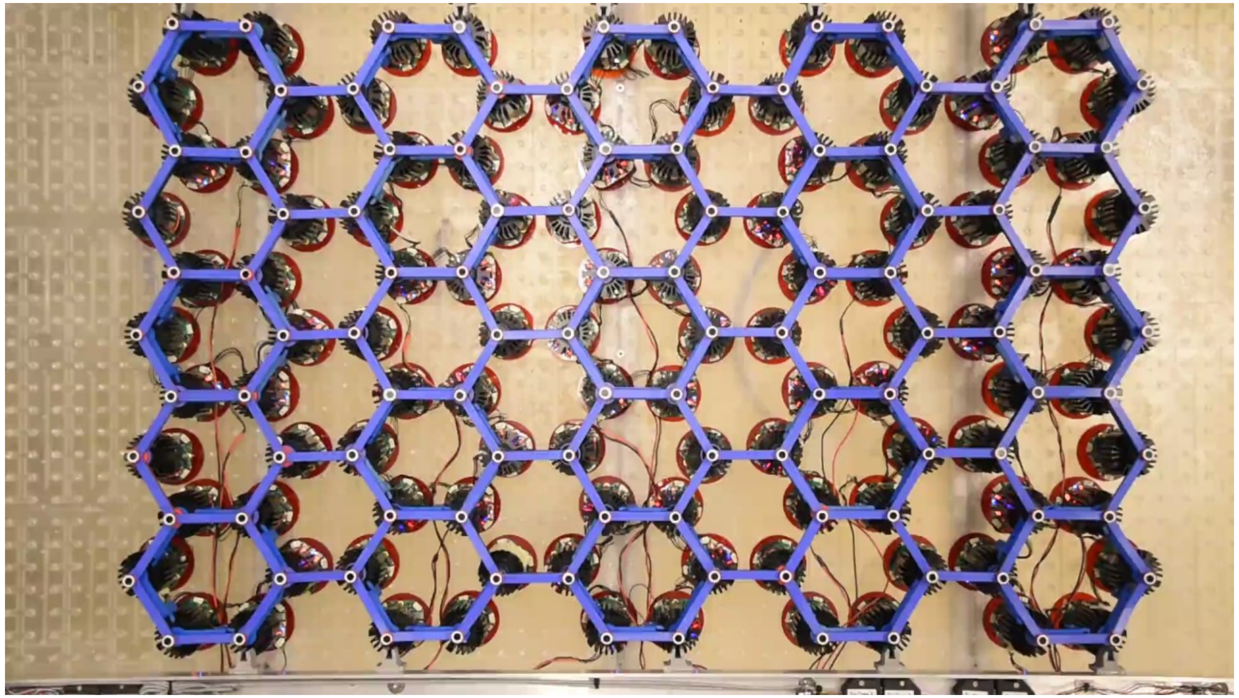


Energy injected at microscale can be extracted at the macroscale through work cycles

Part 2.3: Current topics: Active percolation, pattern formation



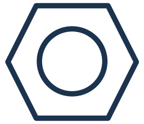
Active Plaquettes

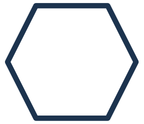


10x Real Time

Directly measure odd modulus from normal force:

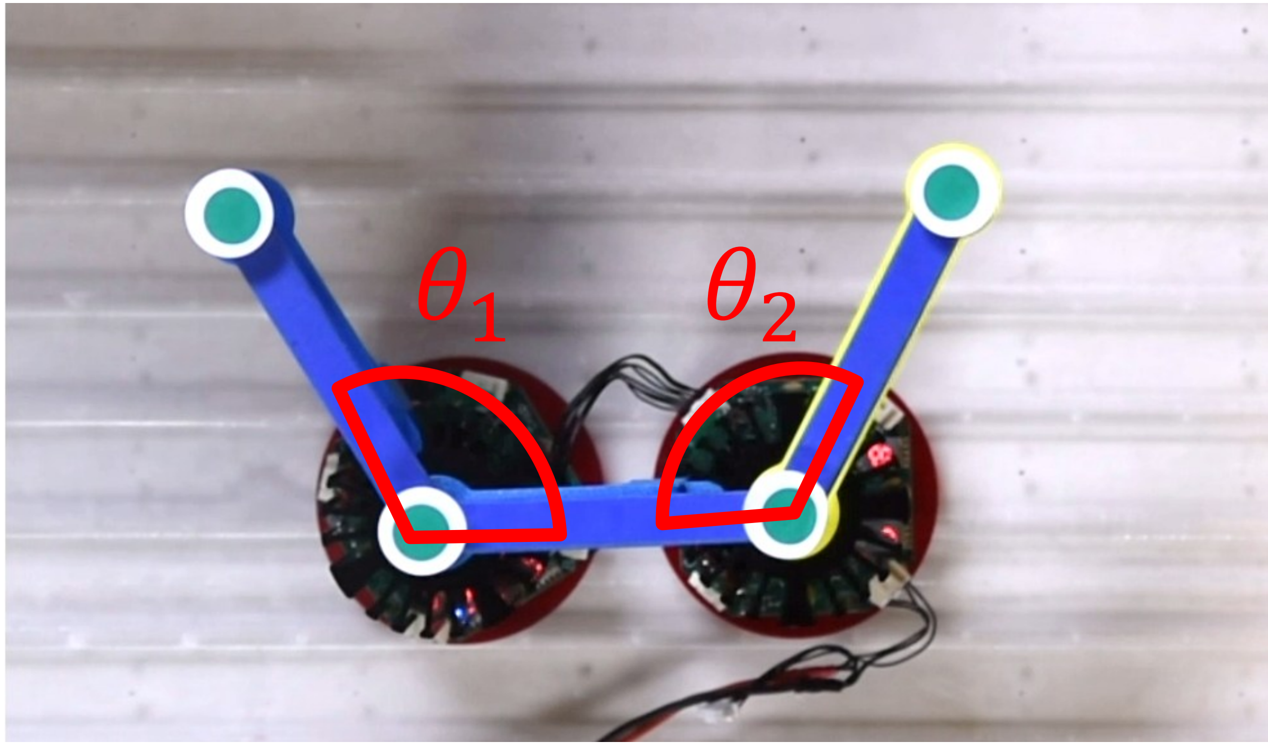
$$K^o = \sigma / \epsilon$$

 Active

 Passive (elastic)

Binysh et al arXiv:2504.18362

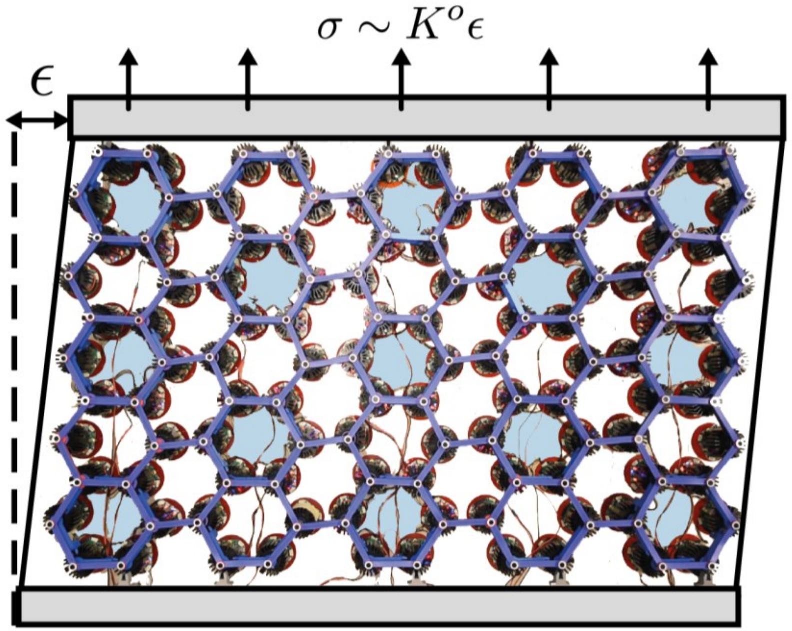
Non-reciprocal Active Solids



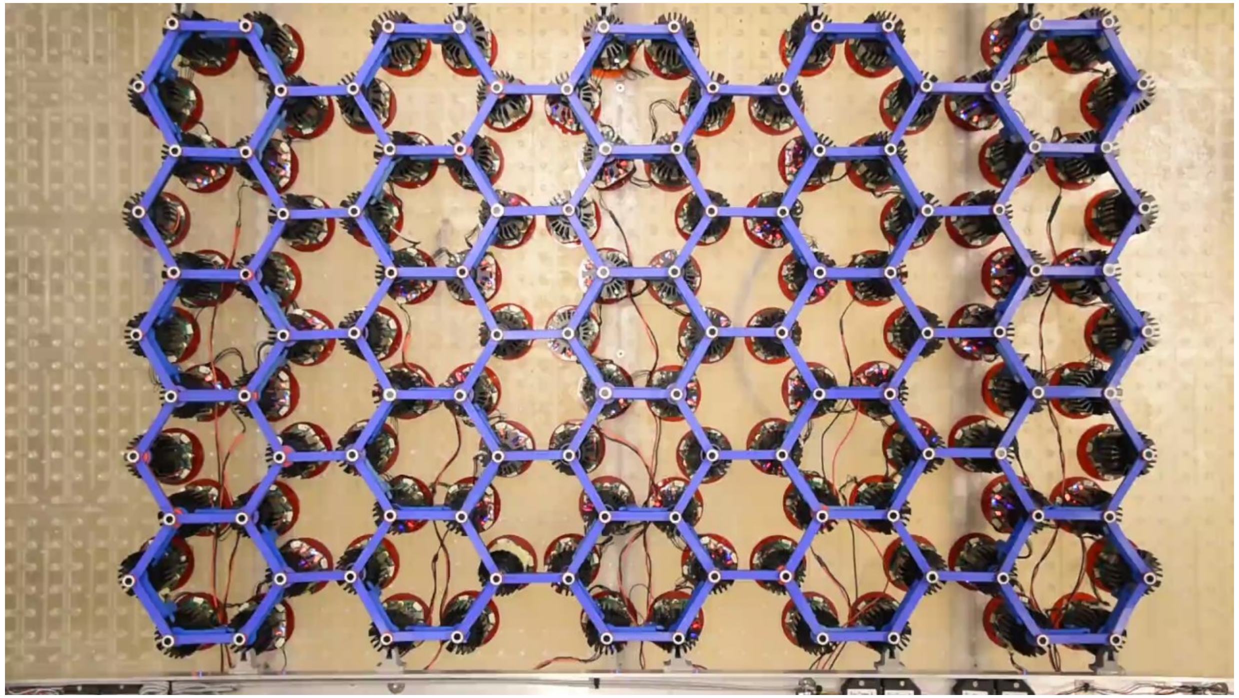
$$\tau_i = \kappa^a(\delta\theta_{i+1} - \delta\theta_{i-1})$$

Active

Coulais lab, University of Amsterdam



Active Plaquettes



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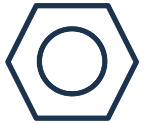
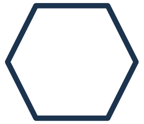


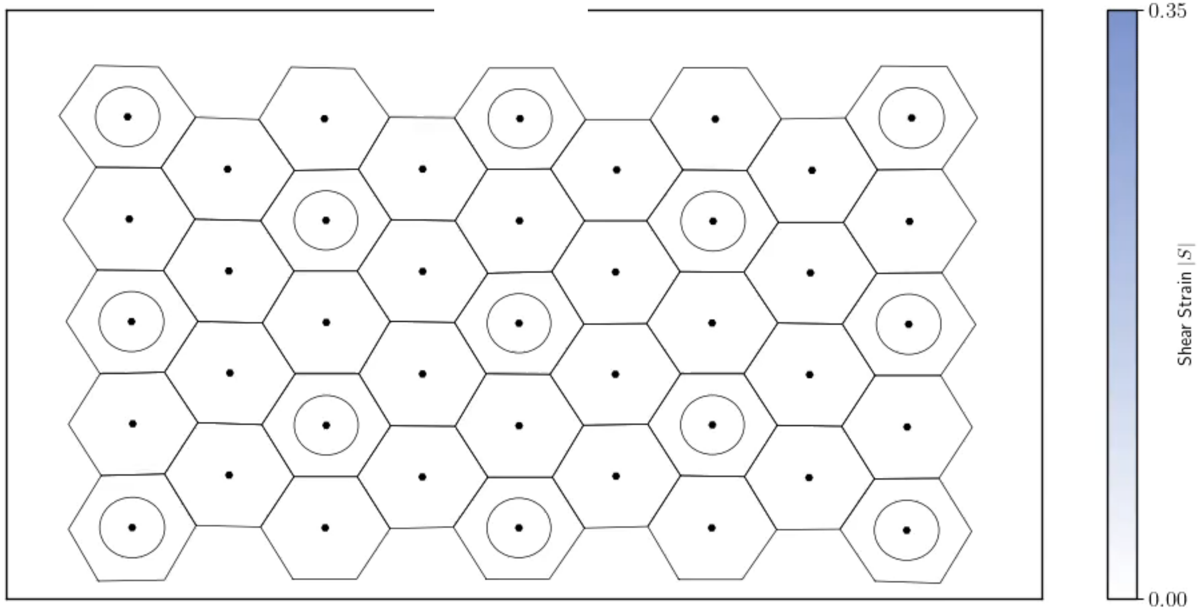
Active

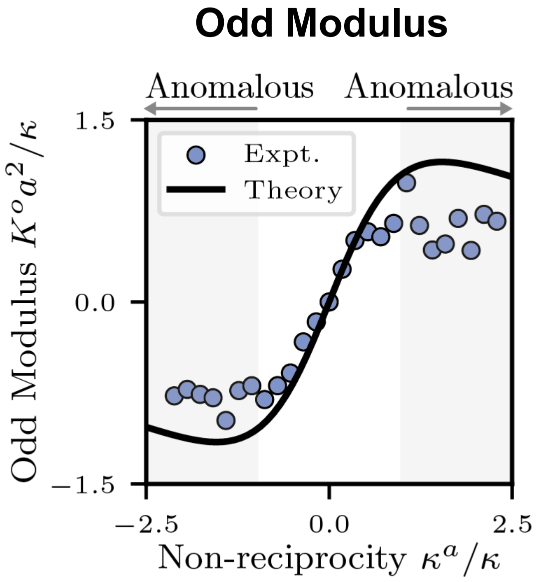


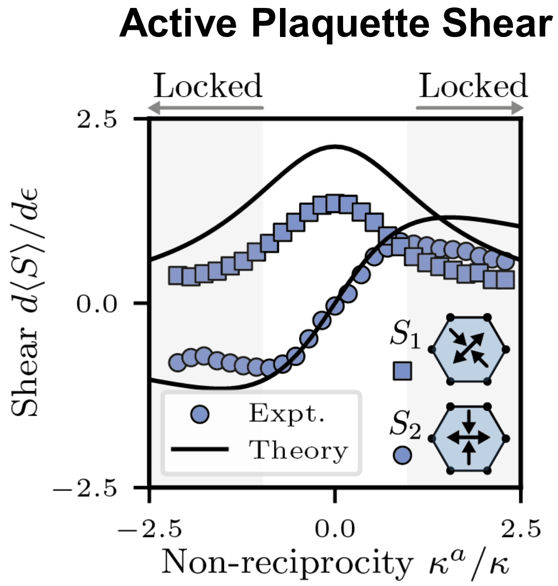
Passive (elastic)

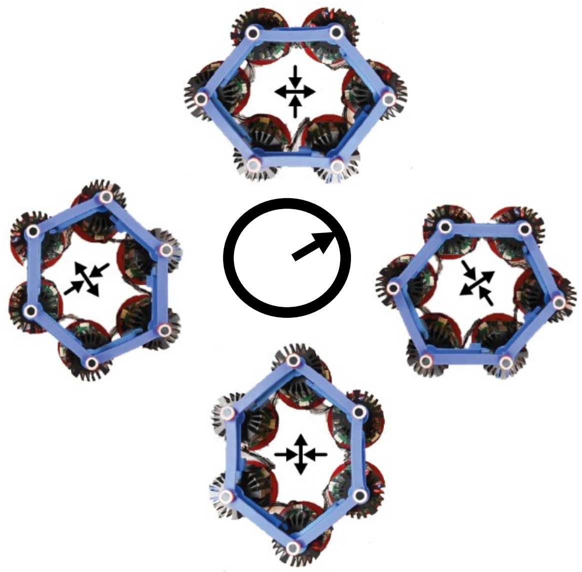
Binysh et al arXiv:2504.18362

Anomalous odd response in non-reciprocal materials









$$S_1 + iS_2$$

High activity

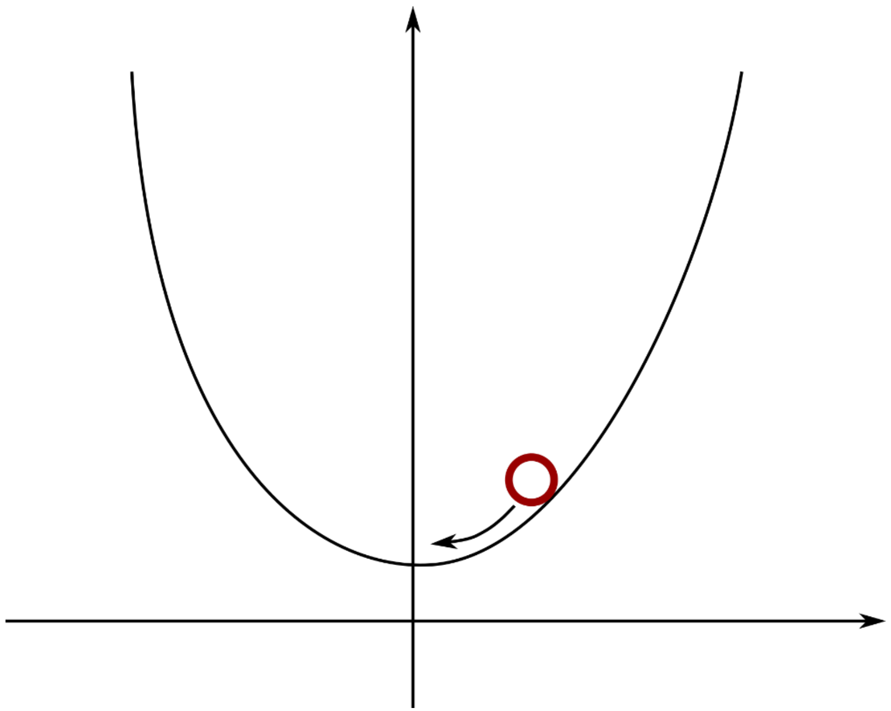
Binysh et al arXiv:2504.18362

Binysh, Baardink, Veenstra, Coulais, Souslov
arXiv:2504.18362

More is less in unpercolated active solids

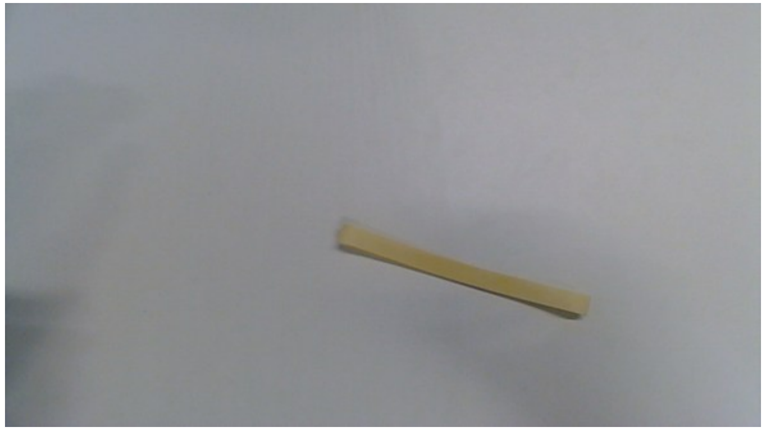
Generalised Le Chatelier's principle

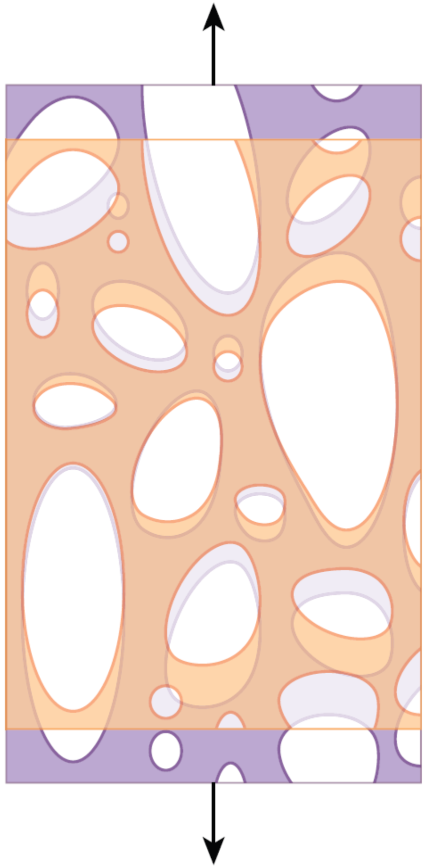
*Short definition: A system at (**thermodynamic or mechanical**) equilibrium shifts to counteract an external stimulus.*



Broad consequence of equilibrium at potential minimum

Example: Elastic micro-macro relations





Young’s modulus: Rubber, Foams etc…

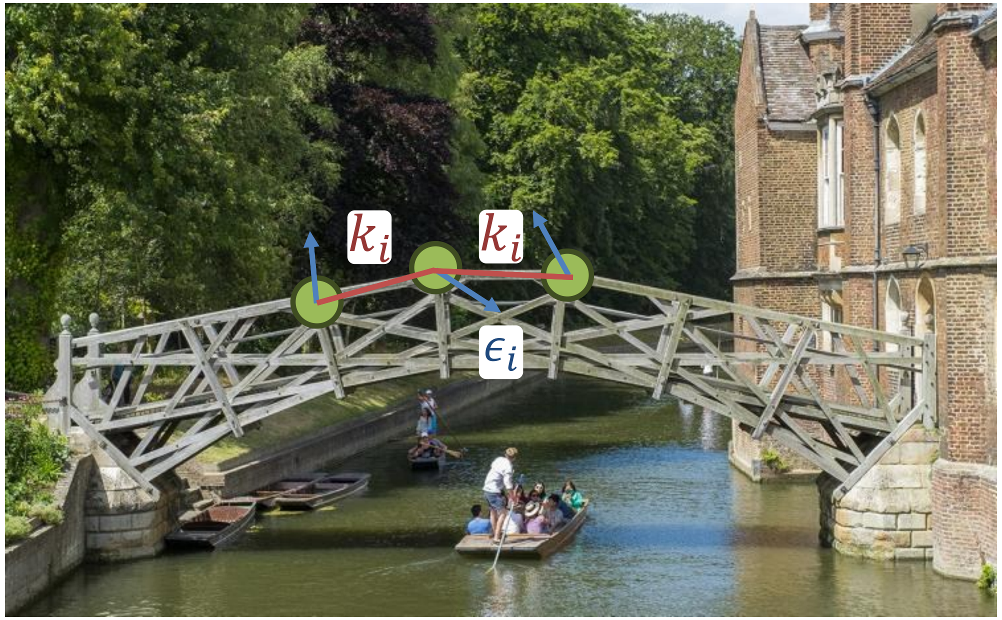
Structures: truss bridges, etc…

Consequence: Increasing one spring constant increases overall stiffness

Example: Elastic micro-macro relations

Consequence: Increasing one spring constant increases overall stiffness

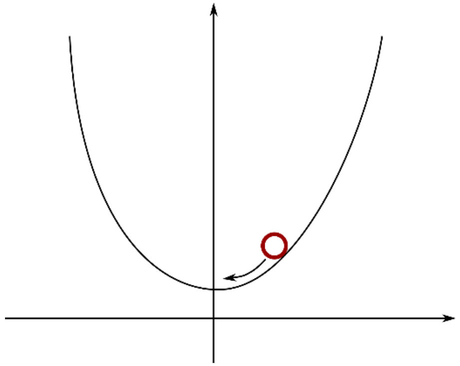
Potential energy: $V = \frac{1}{2}\epsilon_i \, K_{ij} \, \epsilon_j$



$$K_{ij} = \begin{pmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_N \end{pmatrix}$$

spring constants, $k_i > 0$

ϵ_i *particle displacements*

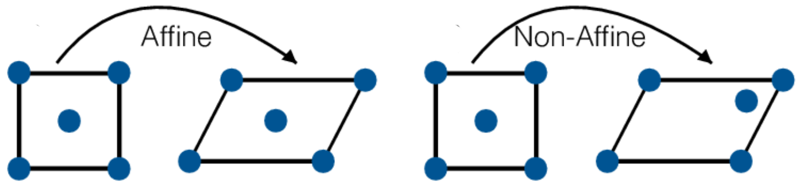


Simple proof

Binysh et al arXiv:2504.18362

Example: Elastic micro-macro relations

Consequence: Increasing one spring constant increases overall stiffness



Potential energy: $V = \frac{1}{2}\epsilon_i K_{ij} \epsilon_j = \frac{1}{2} u_a C_{ab} u_b$ + *non-affine*

(minimize energy over non-affine displacements to obtain C_{ab})

$K_{ij} = \begin{pmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_N \end{pmatrix}$ *spring constants, $k_i > 0$*

ϵ_i *particle displacements*

$C_{ab} = \begin{pmatrix} C_{11} & \cdots & C_{61} \\ \vdots & \ddots & \vdots \\ C_{16} & \cdots & C_{66} \end{pmatrix}$ *elastic tensor*

u_a *strains*

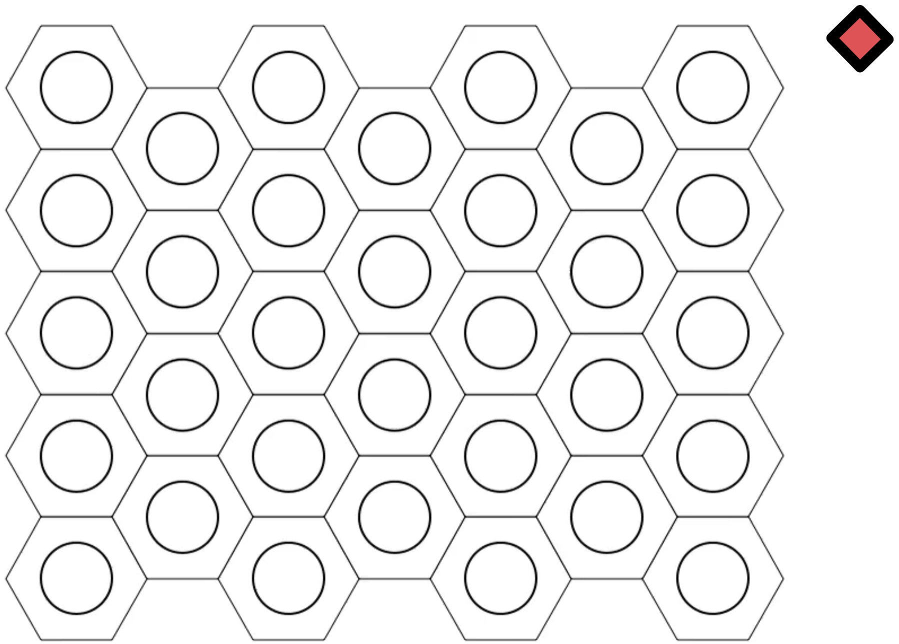
Simple proof: Eigenvalues of C_{ab} increase monotonically in k_i

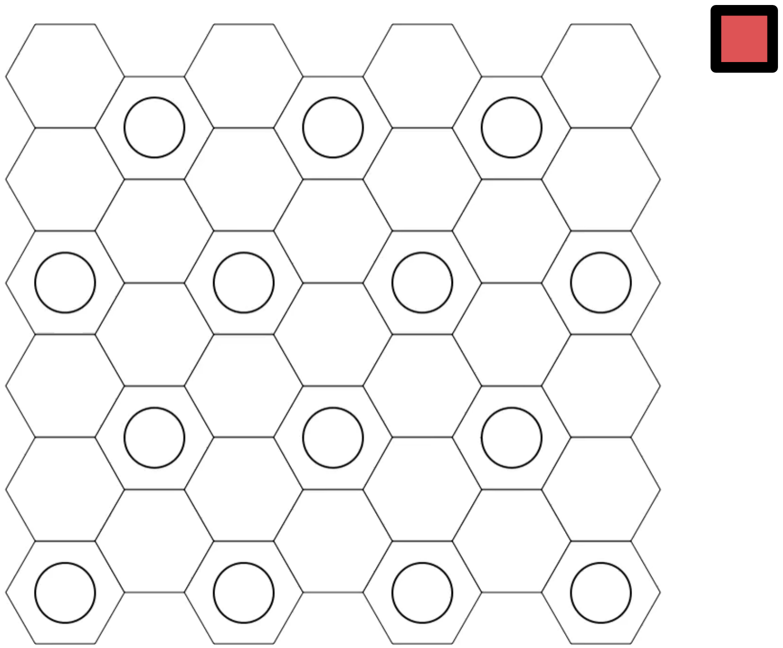
Binysh et al arXiv:2504.18362

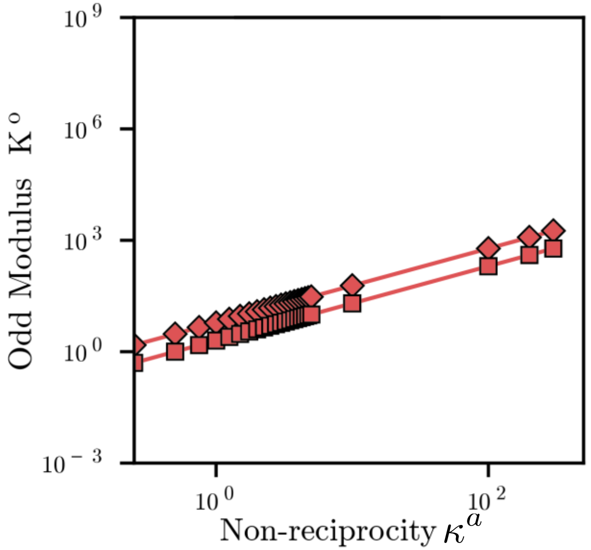
How can Le Chatelier's principle be broken?

Out of equilibrium: no longer a thermodynamic potential to minimize

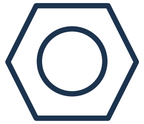
Dense active lattices

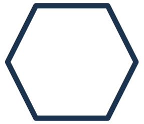






Monotonic micro-macro in dense lattices

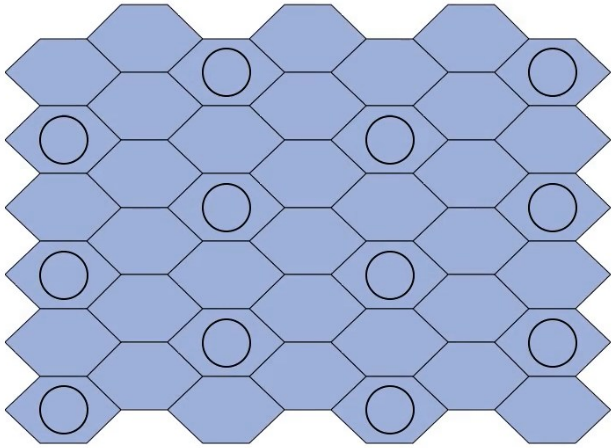
 Active

 Passive (elastic)

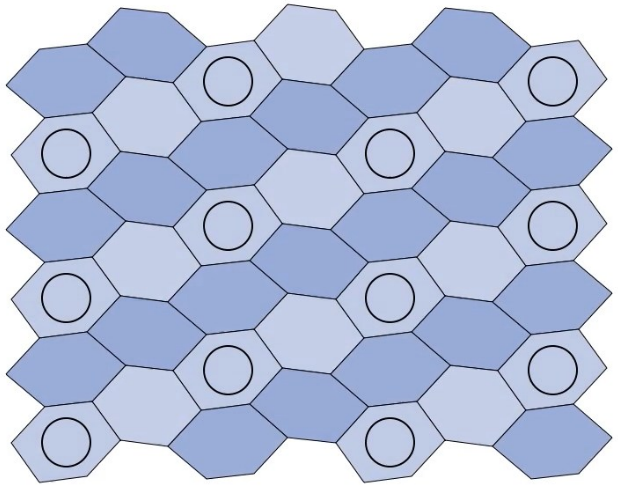
Binysh et al arXiv:2504.18362

Dilute lattices lose odd response

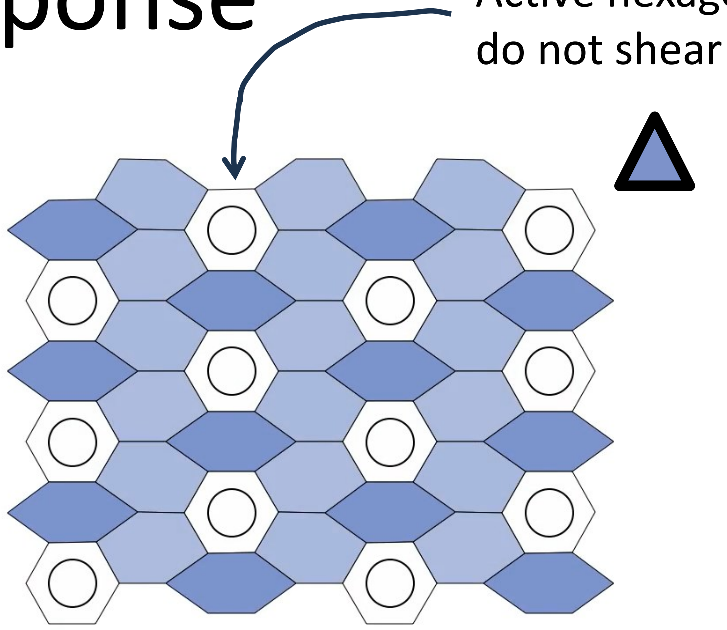
Active hexagons do not shear



$\kappa^a/\kappa \sim 0$

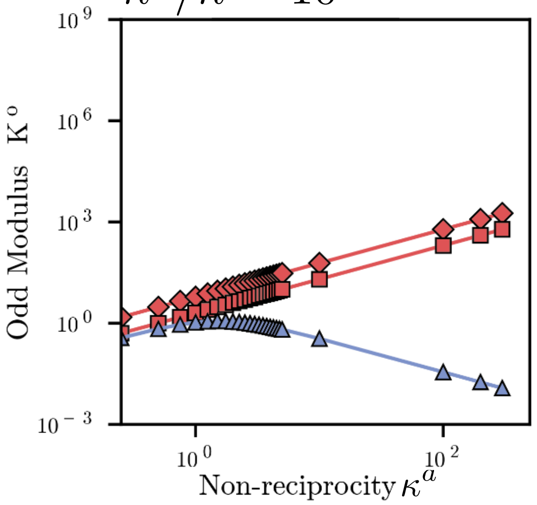


$\kappa^a/\kappa \sim 1$



$\kappa^a/\kappa \sim 10$

Shear Strain $|S|$



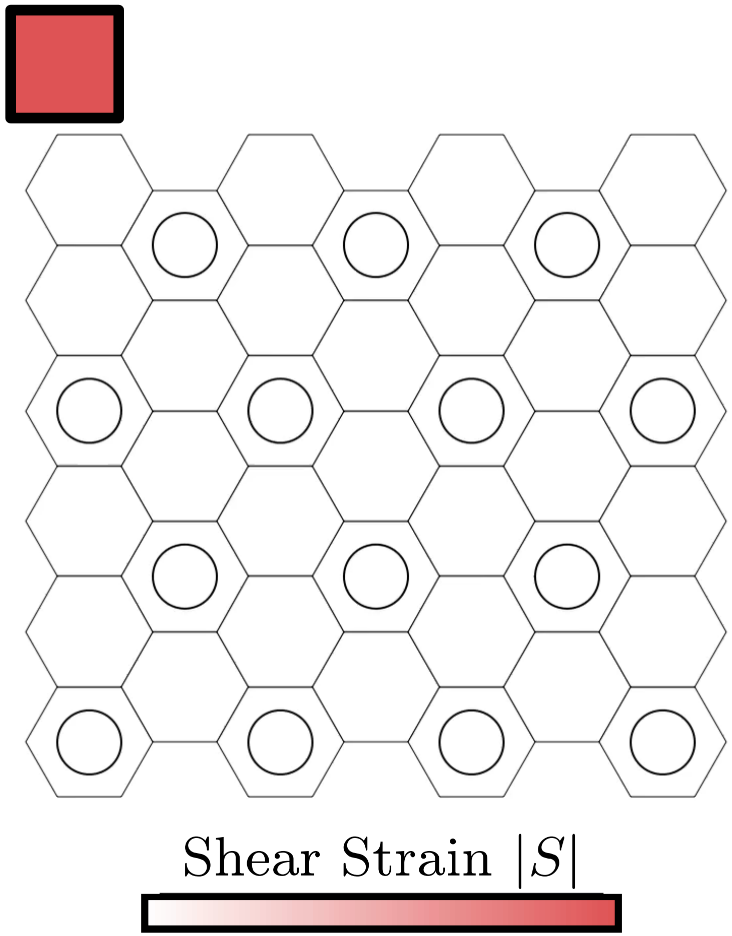
Non-monotonic micro-macro in dilute lattices

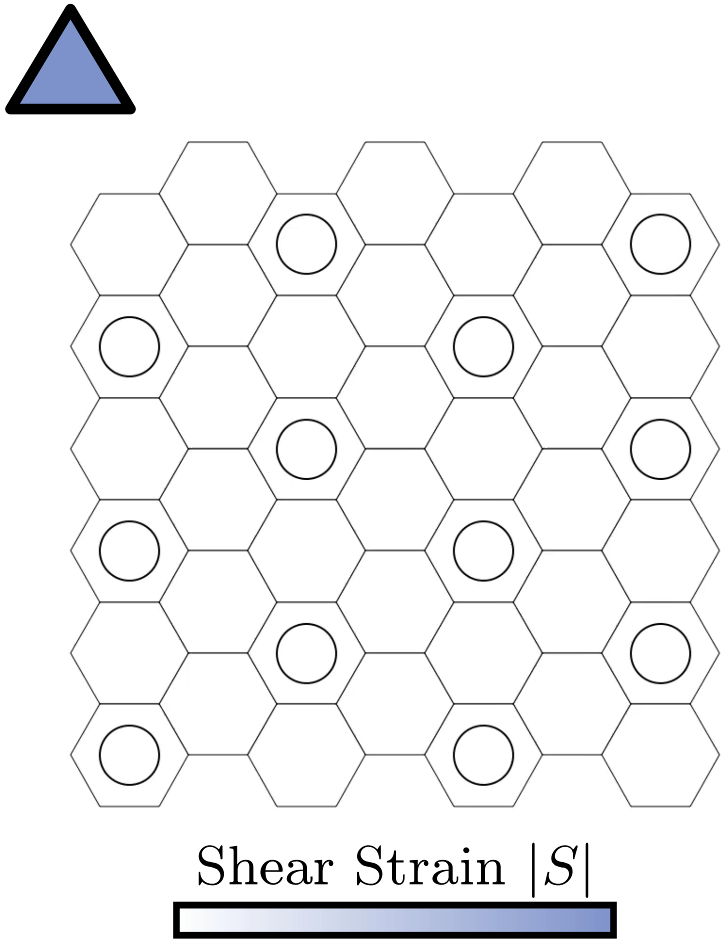
Active

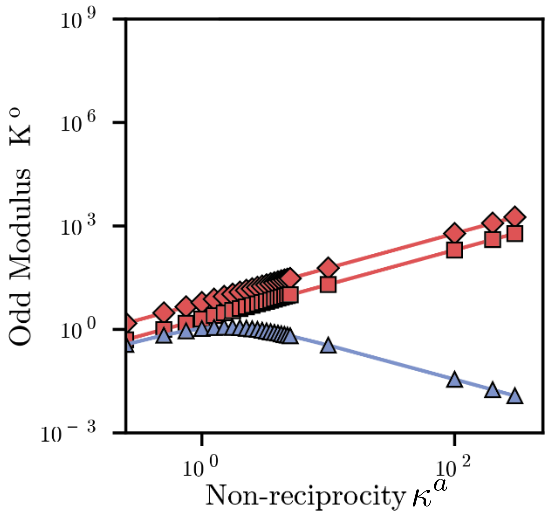
Passive (elastic)

Binysh et al arXiv:2504.18362

Active percolation

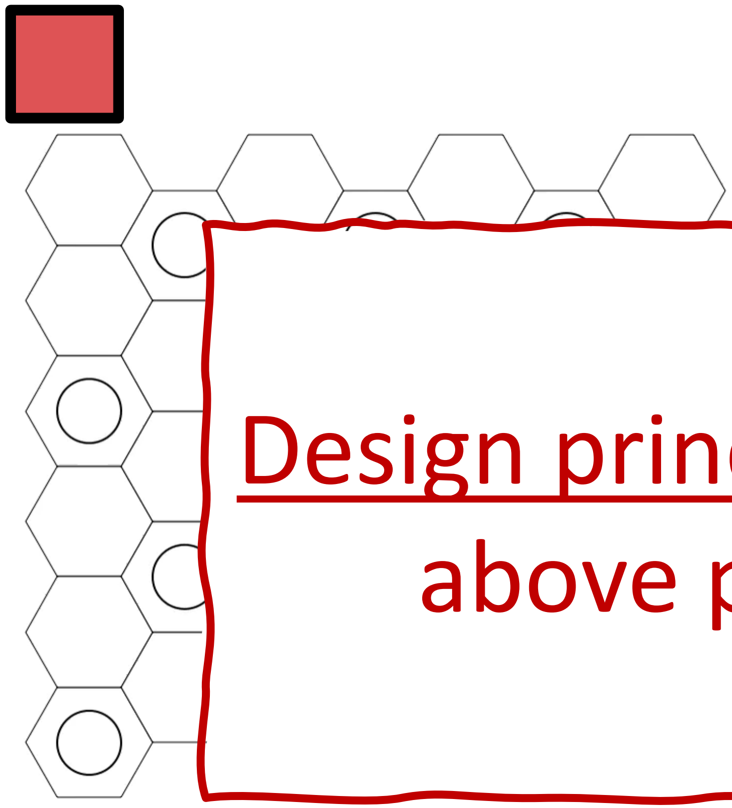


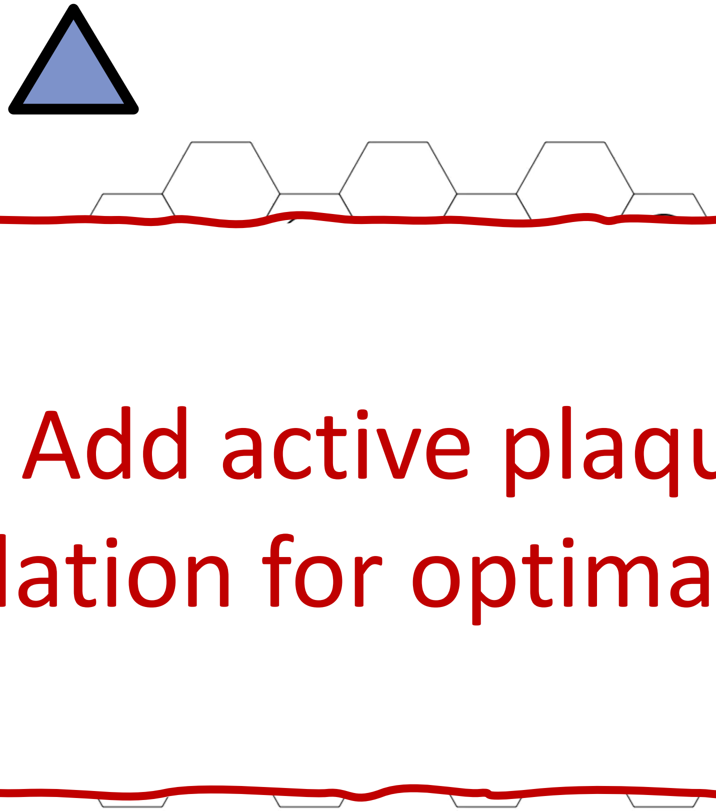


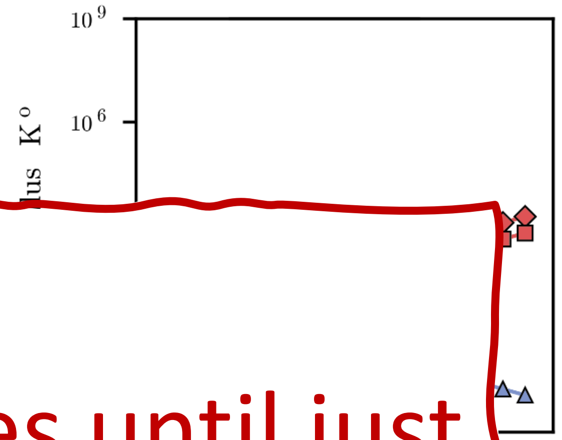


Binysh et al arXiv:2504.18362

Active percolation



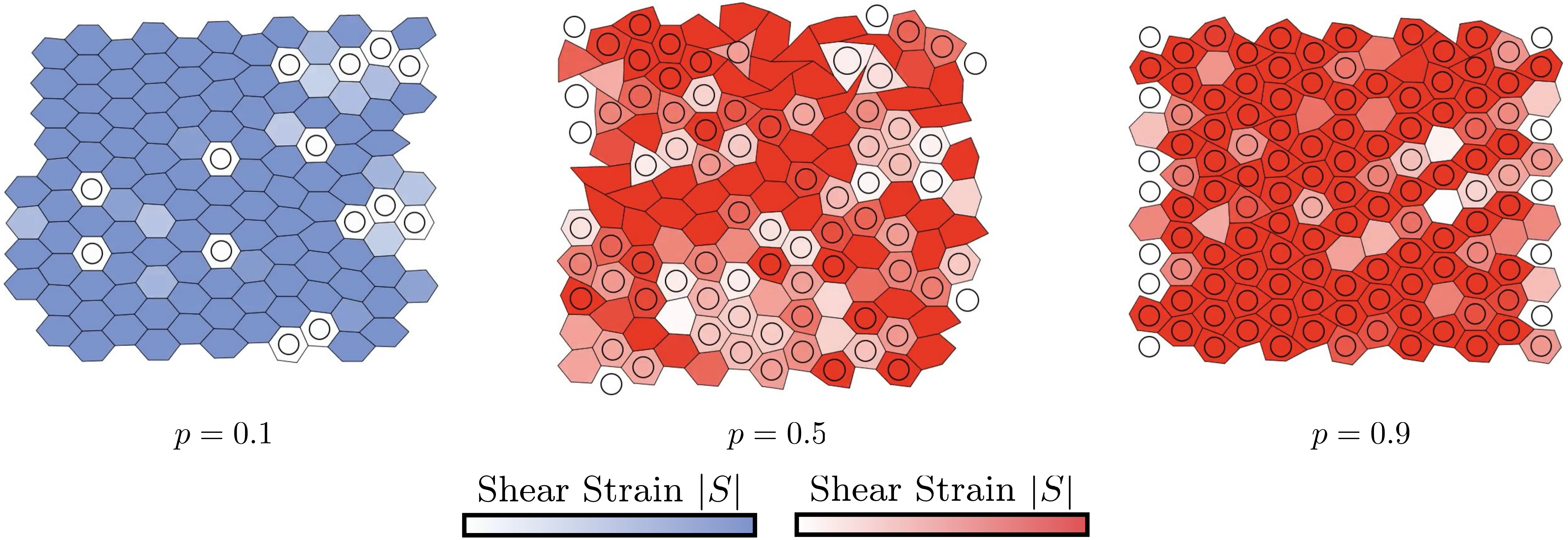




Design principle: Add active plaquettes until just above percolation for optimal efficiency

Binysh et al arXiv:2504.18362

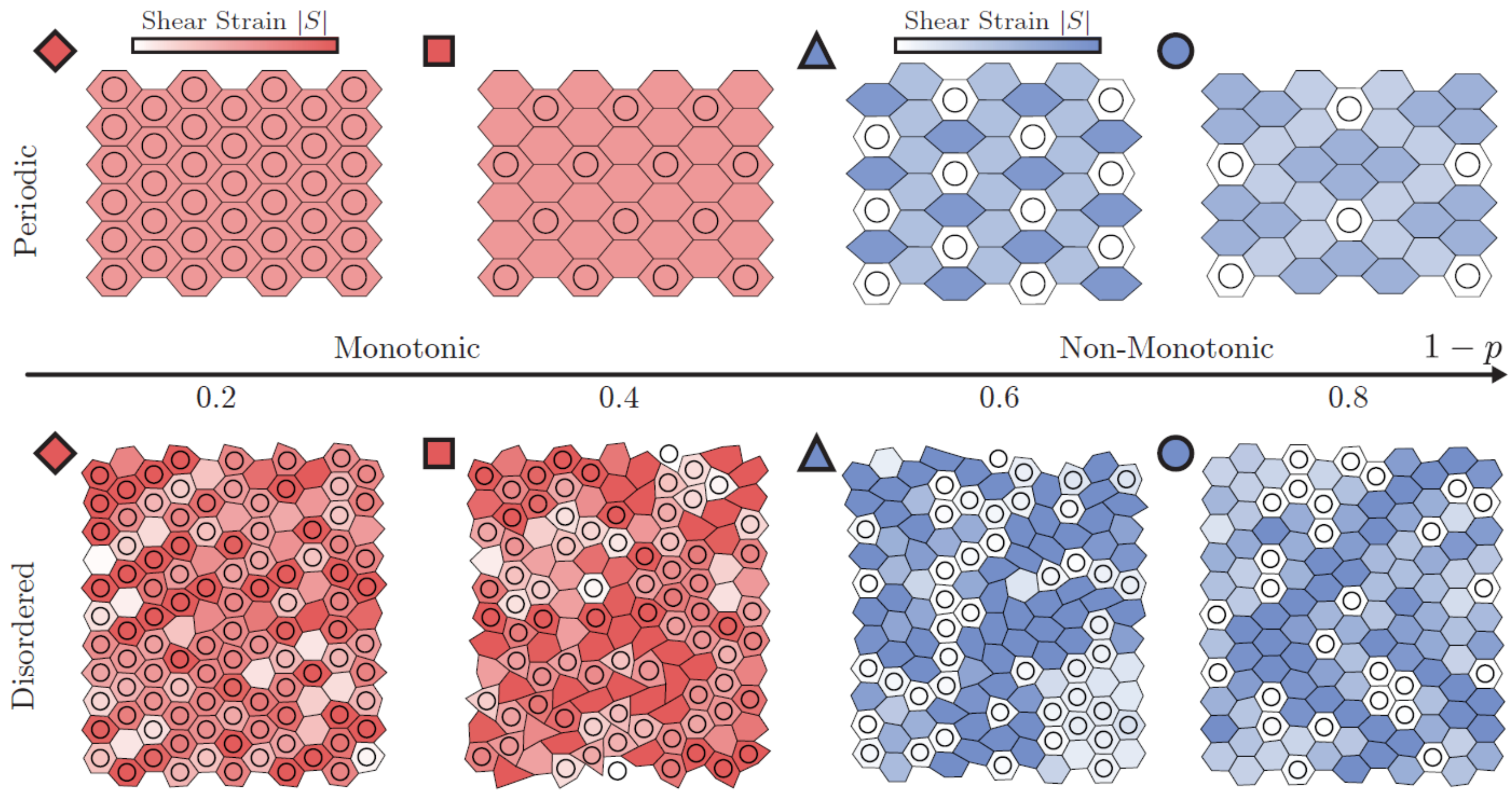
Dilute lattices lose odd response

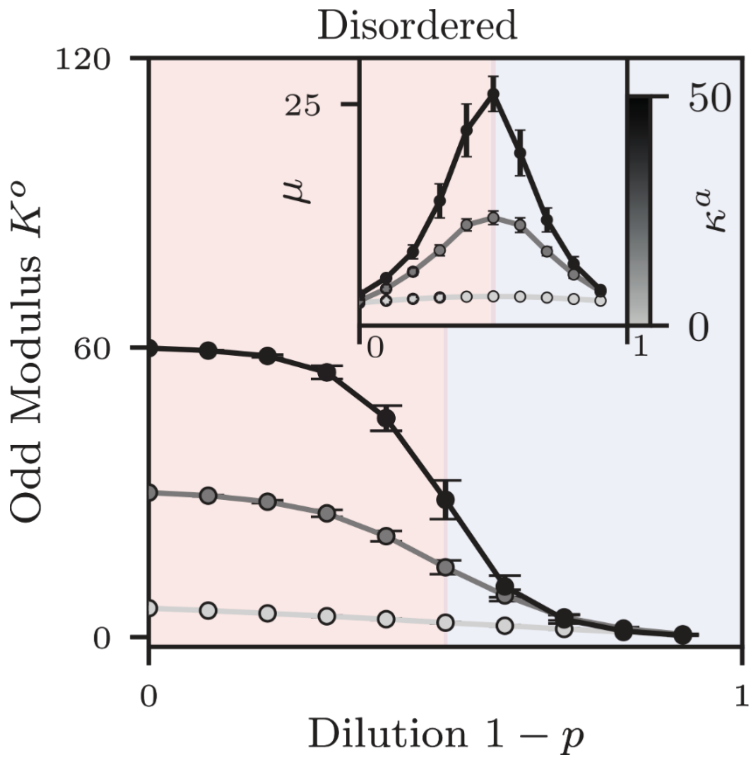


In disordered lattices, change the fraction p of active plaquettes to tune through an active percolation transition

Binysh et al arXiv:2504.18362

Dilute lattices lose odd response

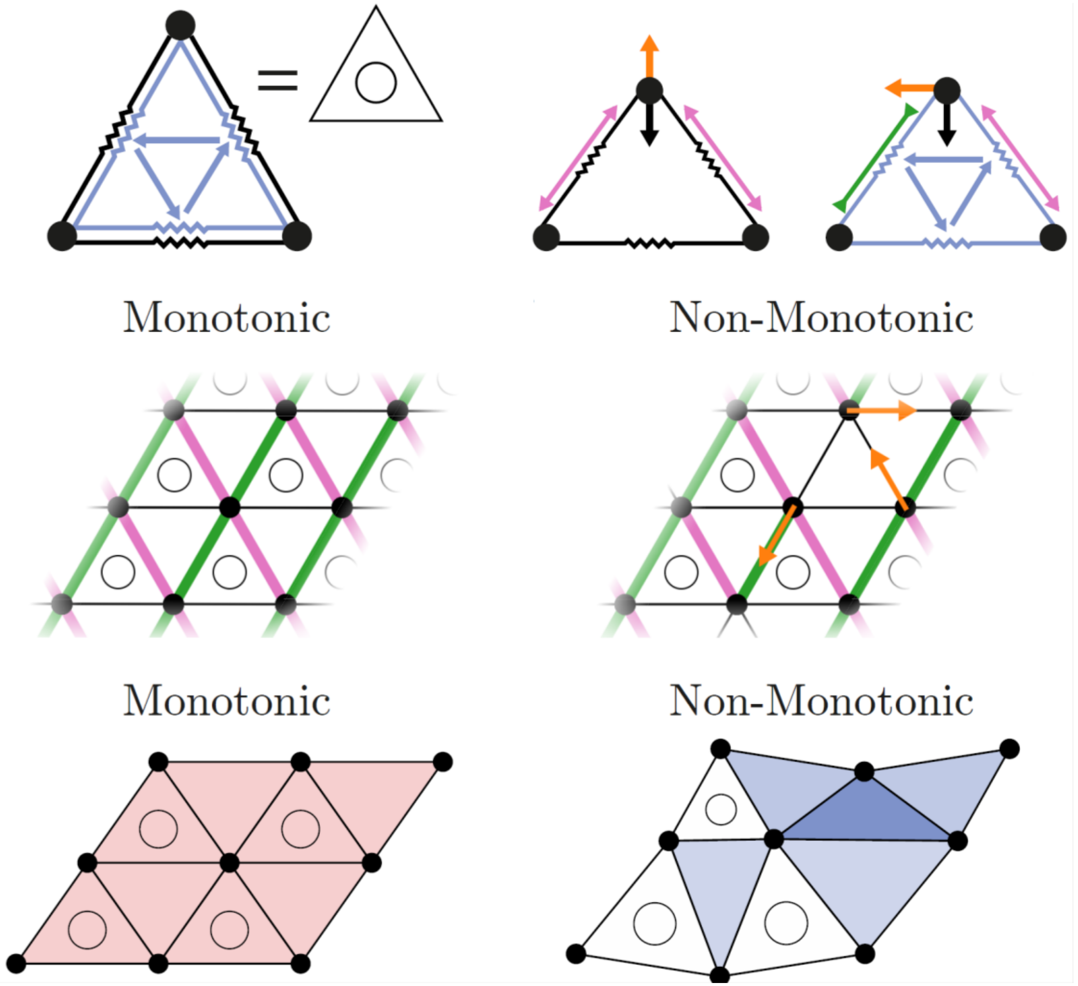


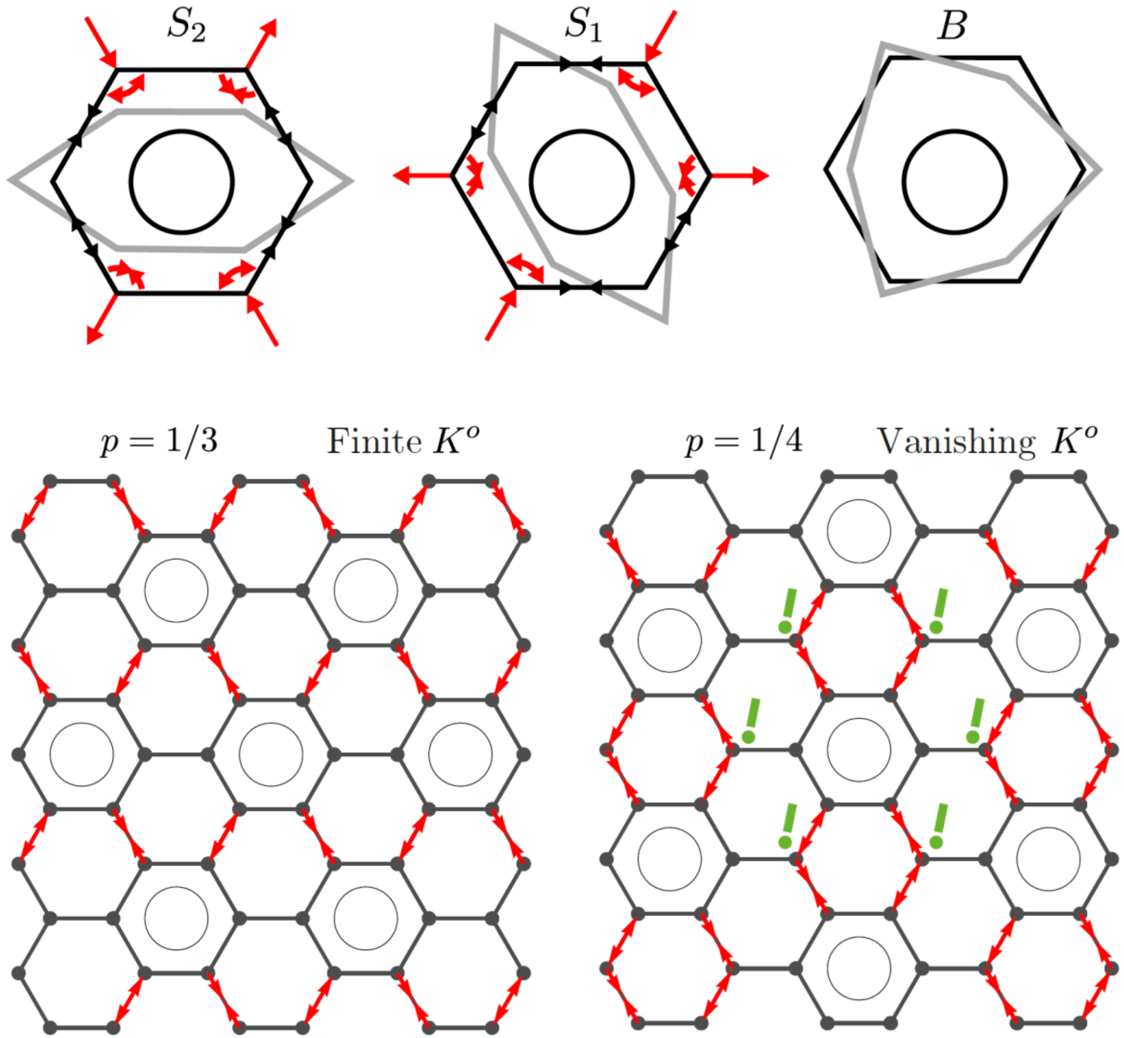


In disordered lattices, change the fraction p of active plaquettes to tune through an active percolation transition

Binysh et al arXiv:2504.18362

Anatomy of vanishing response

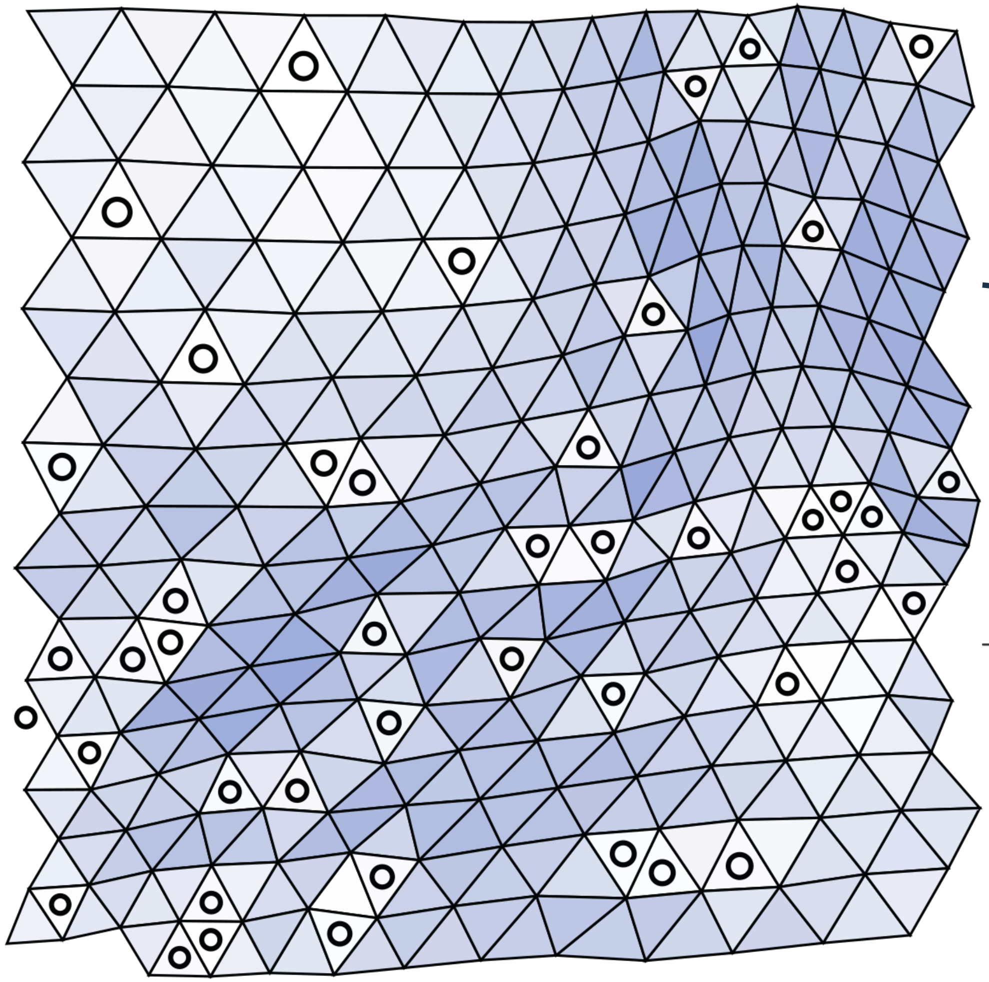


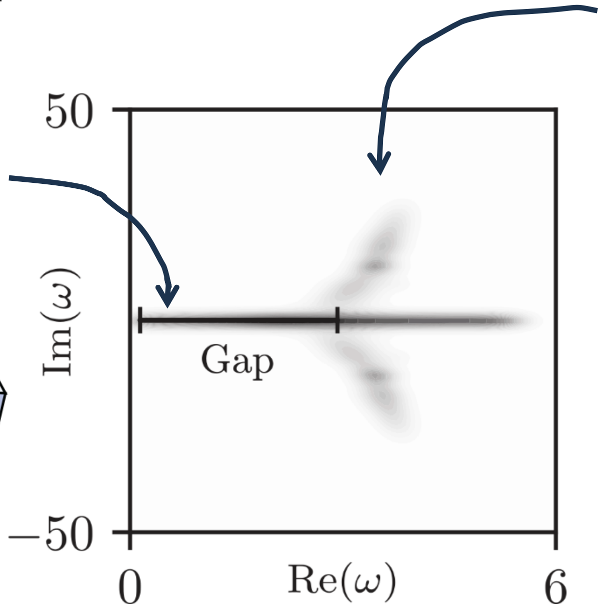


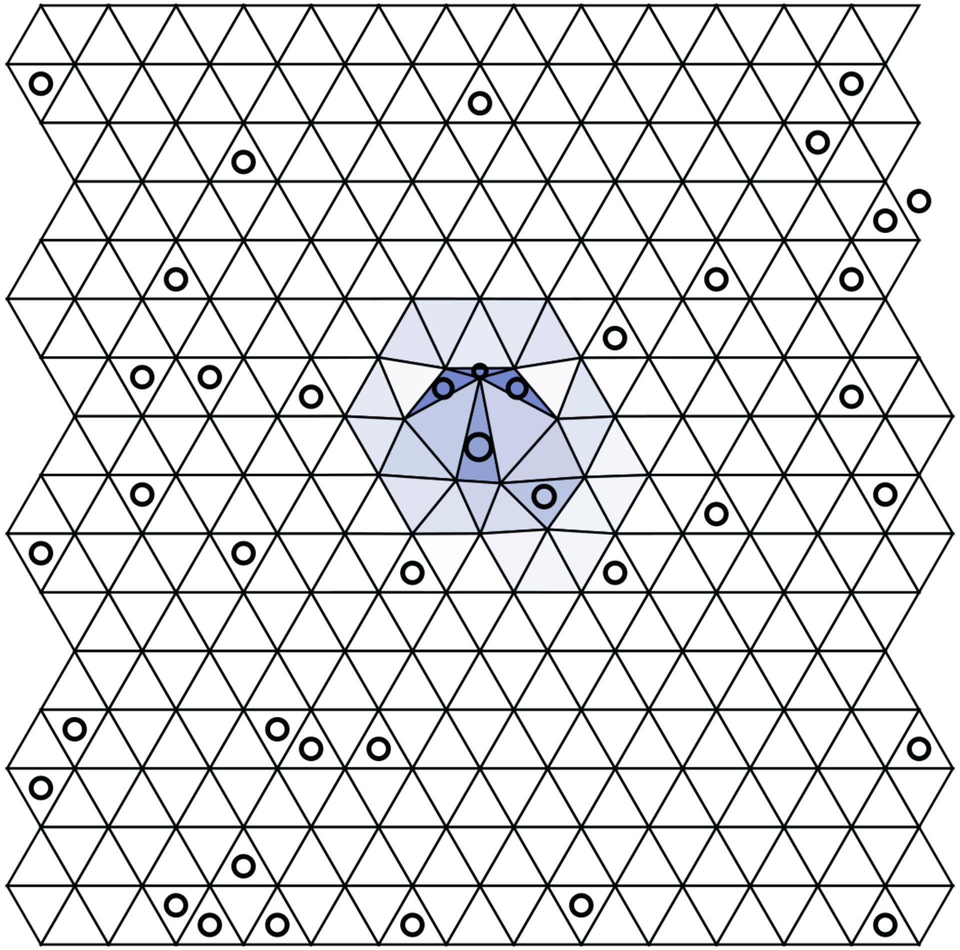
Binysh et al arXiv:2504.18362

Binysh et al arXiv:2504.18362

Are unpercolated lattices passive?



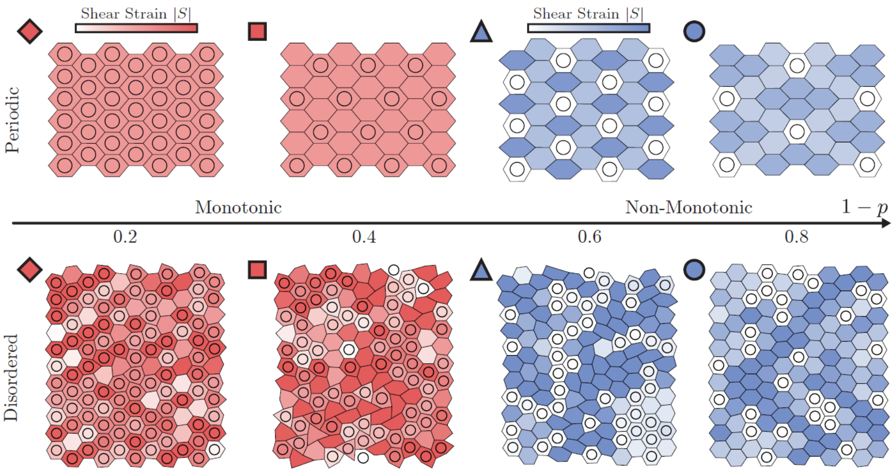




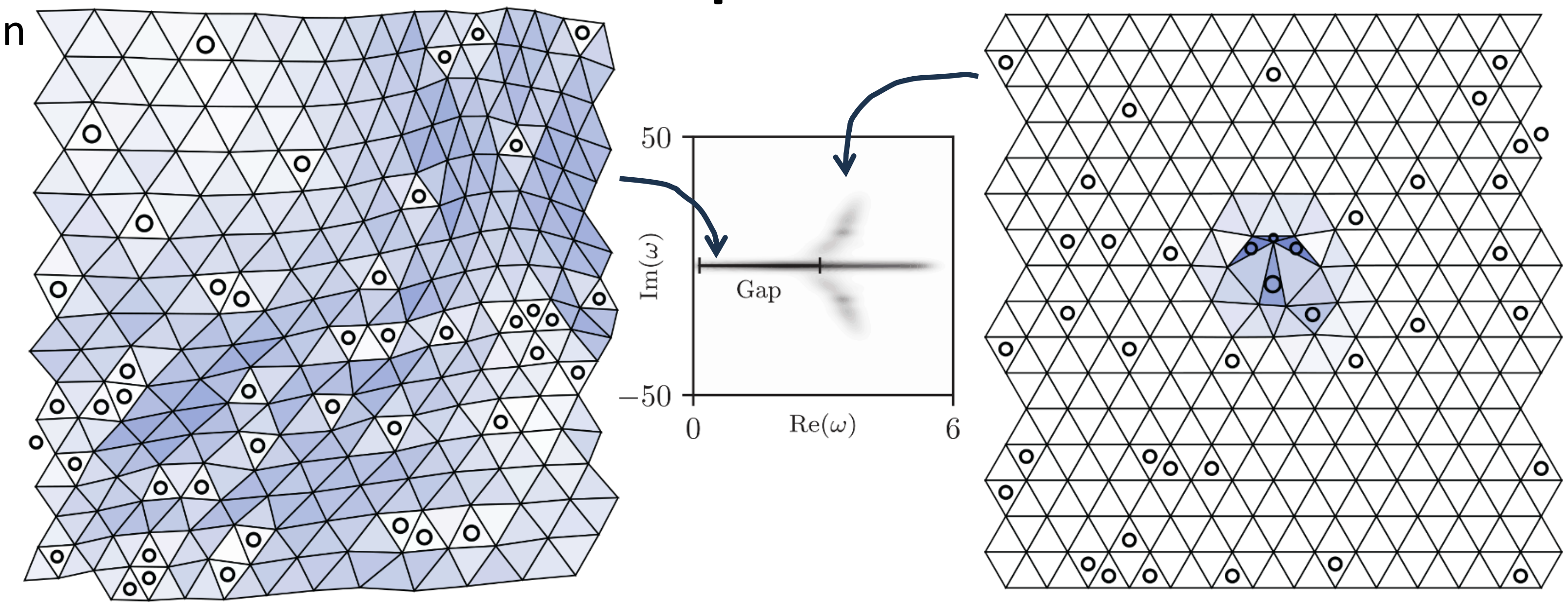
Signatures of activity remain in the high-frequency spectrum:
Localized oscillations despite overdamped dynamics

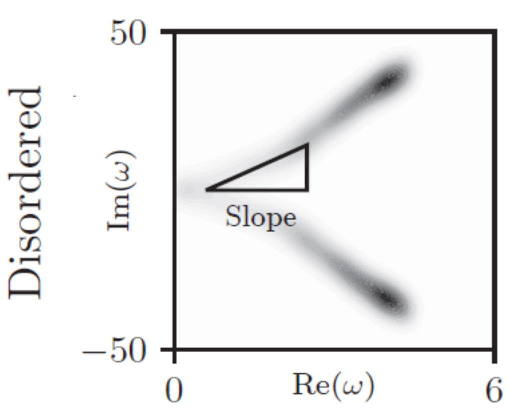
Binysh et al arXiv:2504.18362

Are unpercolated lattices passive?

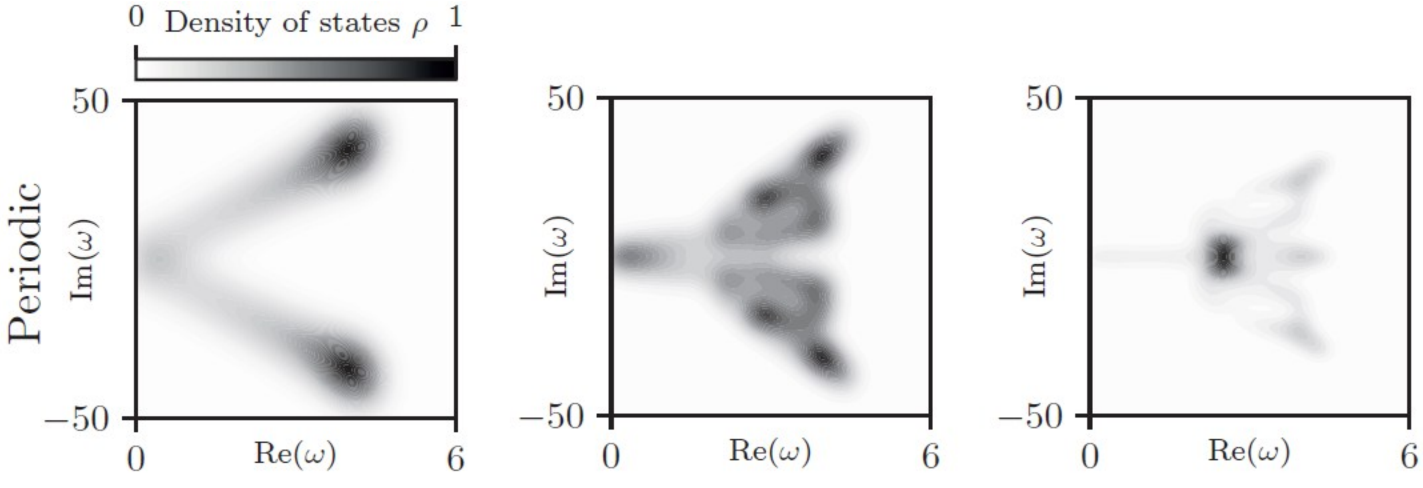


Below percolation





Above percolation



Above percolation

Below percolation

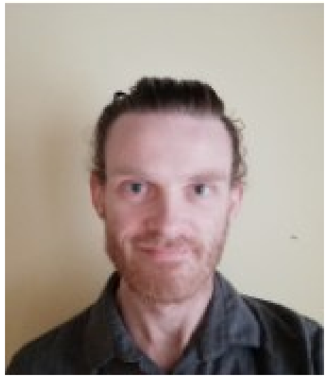
More is less in unpercolated active solids *arXiv:2504.18362*

Former lab members:

Guido Baardink, PhD
Since: Consulting (Netherlands)

Dr Jack Binysh
Since: Marie Curie fellow, University of Amsterdam

Collaborators:



Jonas Veenstra
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Corentin Coulais
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Current lab members:
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Zory Davoyan

Dawid Dopierala

Dr Aditya Jha

Ian Tan

University of Bath:

Jamie Mclauchlan

Brook Salter

EPSRC
Engineering and Physical Sciences Research Council

TCM

UNIVERSITY OF CAMBRIDGE
Cavendish Laboratory

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Odd viscosity

Banerjee, AS, Abanov, and Vitelli. Nat. Commun. (2017)

Han, Fruchart, Scheibner, Vaikuntanathan, de Pablo, Vitelli. Nature Physics 17, 1260 (2021)

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AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

Baardink, Cassella, Neville, Milewski, AS. Phys. Rev. E (2021)

Soni*, Bililign*, Magkiriadou*,…, Irvine. Nature Physics (2019)

Odd elasticity

Scheibner, AS, et al. Nature Physics (2020)

Bililign et al. Nature Physics (2022)

Odd viscoelasticity

Fodor, AS Phys. Rev. E 104, L062602 (2021)

Banerjee, Surowka, Vitelli, Julicher Phys. Rev. Lett. 126, 138001 (2021)

Reviews

Fruchart , Scheibner, Vitelli. "Odd viscosity and odd elasticity." Annual Review of Condensed Matter Physics 14, 471 (2023)

Shankar, AS, Bowick, Marchetti, Vitelli. “Topological active matter” Nat. Rev. Phys. (2022)

About these lectures

Lecture 1. Topological active matter
Part 1.1:
Overview; Definition of active matter
Part 1.2:
Classification of active fluids
Part 1.3:
Topological active matter

Lecture 2. Non-reciprocal active solids
Part 2.1:
Introduction to active solids
Part 2.2:
Non-reciprocal mechanics and odd elasticity
Part 2.3:
Current topics: active percolation, pattern formation

Review articles on active matter:
Shankar et al Topological active matter
Nature Reviews Physics (2022)

Fruchart, Scheibner, Vitelli. Odd viscosity and odd elasticity.
Annual Review of Condensed Matter Physics 14, 471 (2023)

Marchetti et al Hydrodynamics of soft active matter
Reviews of Modern Physics 85, 1143 (2013)

Background textbook:
P. M. Chaikin and T. C. Lubensky (1995) Ch. 6-10
Principles of Condensed Matter Physics

Topology:
David Mermin *Rev Mod Phys* (1979)
The topological theory of defects in ordered media

This lecture:
Introduction to active solids and recent work