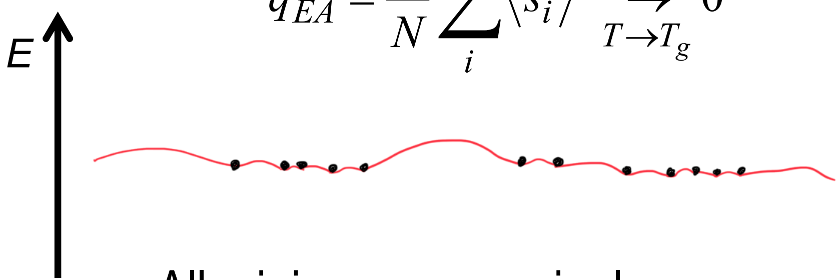
Spin glass universality classes

Two different types of (mean field) spin glasses

SK-model $H = -\sum_{i<j} J_{ij} s_i s_j$

Continuous transition

$$q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2 \underset{T\to T_g}{\to} 0$$



- All minima are marginal
- & have the same free energy density

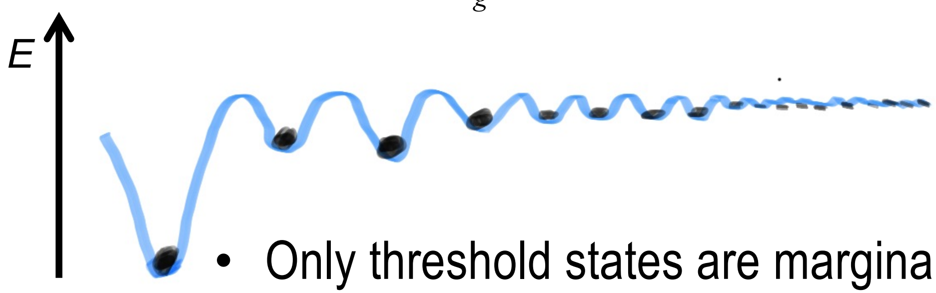
MF-Model for real spin glasses

p-spin models $H = -\sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p} s_{i_1} \cdots s_{i_p}$

$p \geq 3$

Discontinuous transition

$$q_{EA} \underset{T\to T_g}{\to} q_c > 0$$

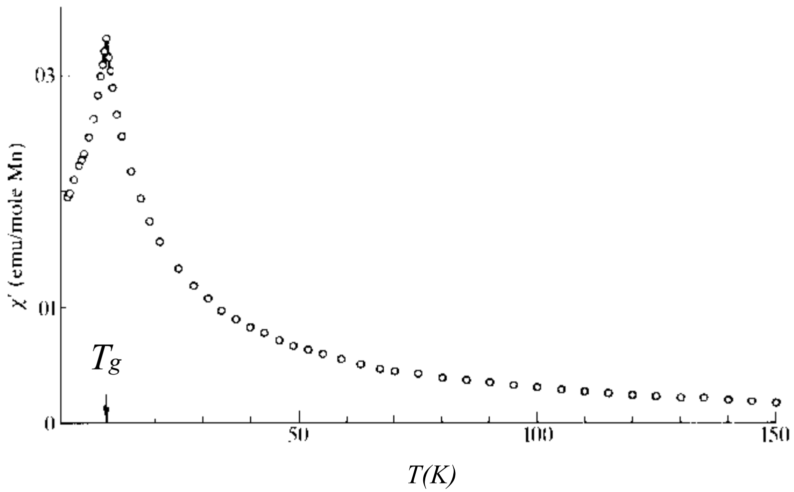


- Only threshold states are marginal
- States in extensive free energy window
- Separate dynamic (clustering) and thermodynamic (freezing) transitions

MF-analogon for structural glasses

Signatures of two different glass transitions

Signatures of the spin glass transition



AC-susceptibility in Cu-0.9%Mn

(Mulder et al., 1981, 1982)

$$\langle s_i \rangle = 0,\ T > T_g,\ \text{(paramagnet)}$$

$$\langle s_i \rangle \neq 0,\ T > T_g,\ \text{(spin glass)}$$

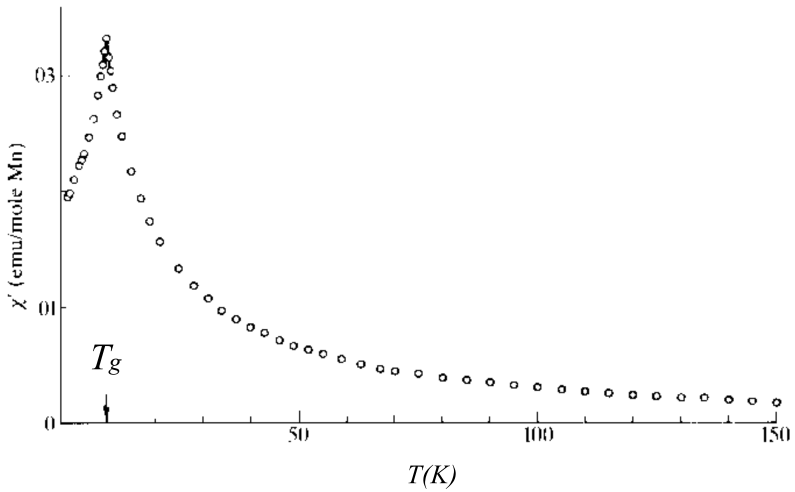
$$q_{EA} \equiv \frac{1}{N} \sum_i \langle s_i \rangle^2$$

$$\chi = \frac{T}{N} \sum_i \left(\langle s_i^2 \rangle - \langle s_i \rangle^2 \right)$$

$$\chi_{nl} = \frac{\beta^2}{N} \sum_{i,j} \langle s_i s_j \rangle^2 \xrightarrow{T \to T_g} \infty$$

Becomes critical, long ranged!

Signatures of the spin glass transition



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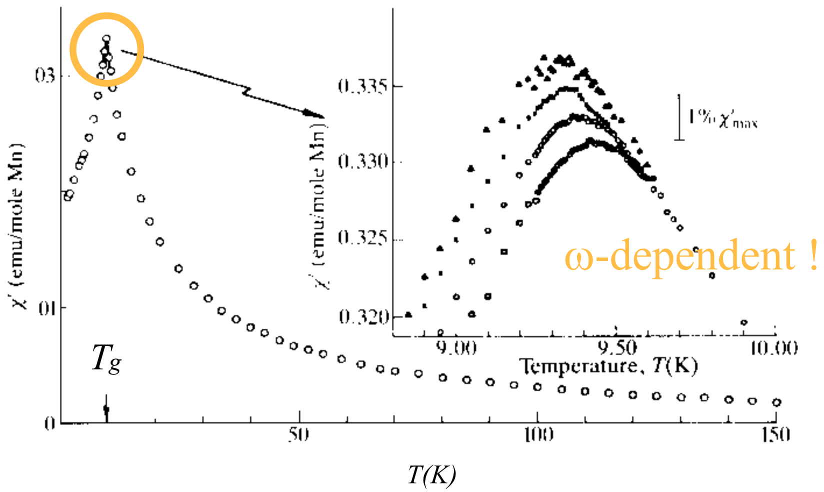
Becomes critical, long ranged!

Genuine thermodynamic transition! Clear order parameter q_{EA}.

Signatures of the spin glass transition



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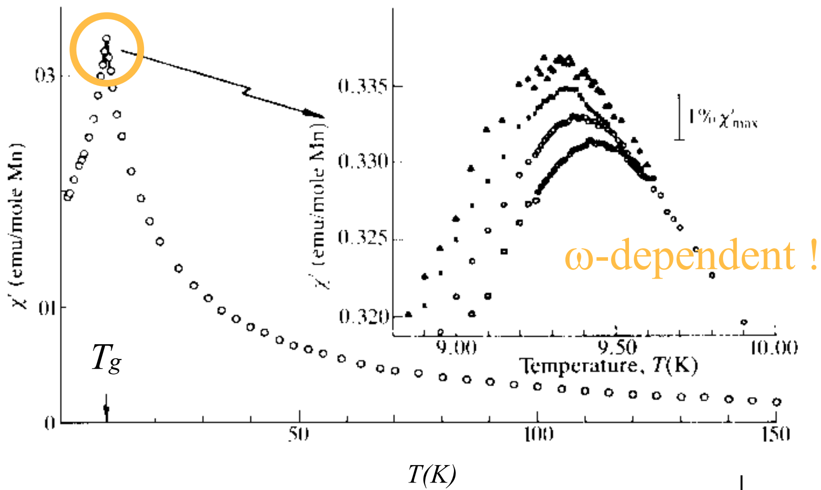
Extreme slowing down!

Probing the finite d version of interstate transitions

Signatures of the spin glass transition



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AC-susceptibility in Cu-0.9%M

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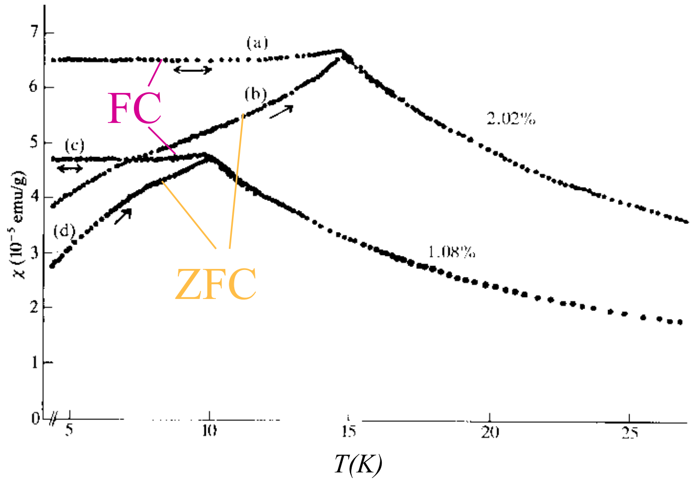

(Nagata et al., 1979)

Spin glasses: protocol dependence of susceptibility χ

$$\chi = \lim_{\mathrm{B}\to 0} \frac{M}{B}$$

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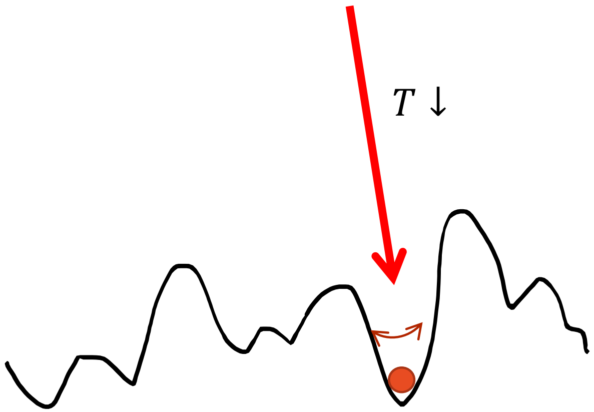

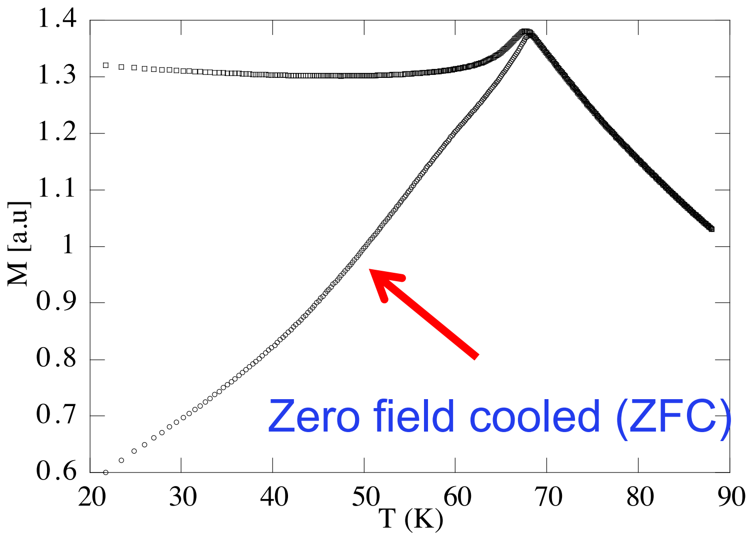

ZFC

- B=0 at $T > T_c$
- Cool to $T < T_c$
- Apply finite B

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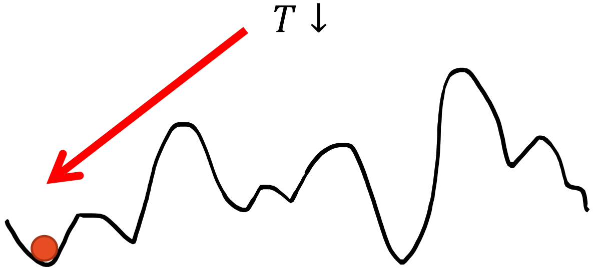

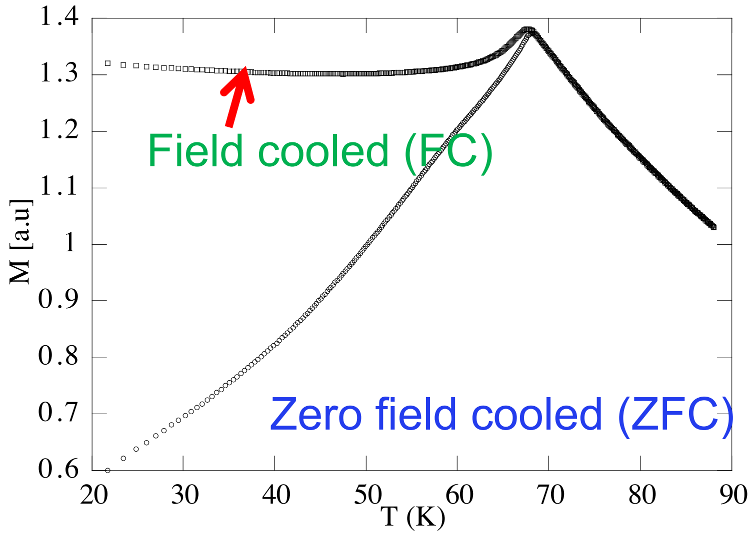

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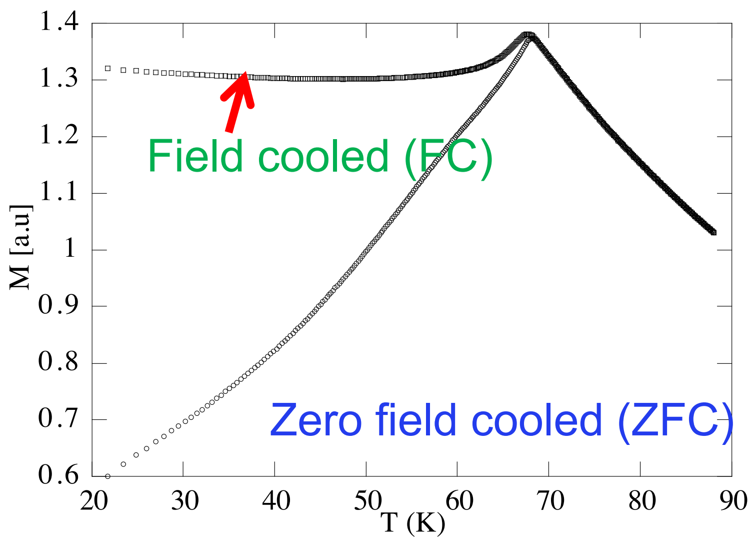

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Final state's M depends on protocol! → Out of equilibrium, ergodicity is broken!

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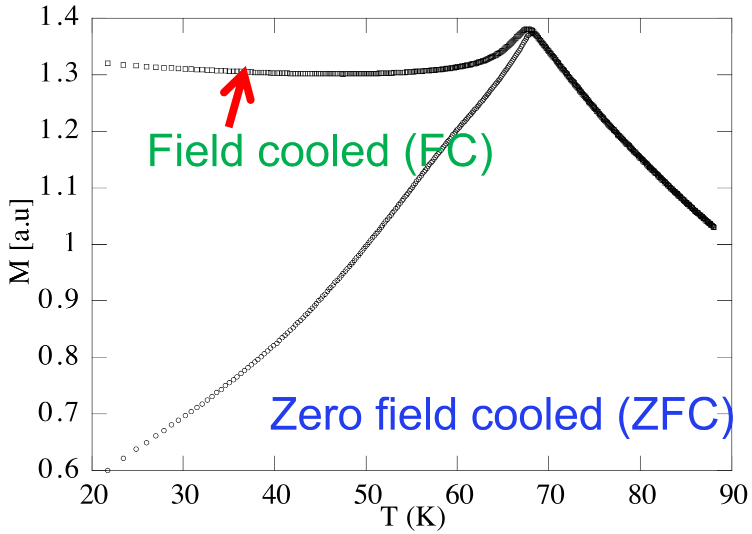

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Final state's M depends on protocol! → Out of equilibrium, ergodicity is broken!

Interesting: System remembers the past! → Store information!

Structural Glass transition: Viscosity

Supercooled liquids: (similar to p-spin models)

Liquids that fail to crystallize, and thus remain amorphous and non-rigid
but get very viscous and slow

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Empiric definition of T_g :

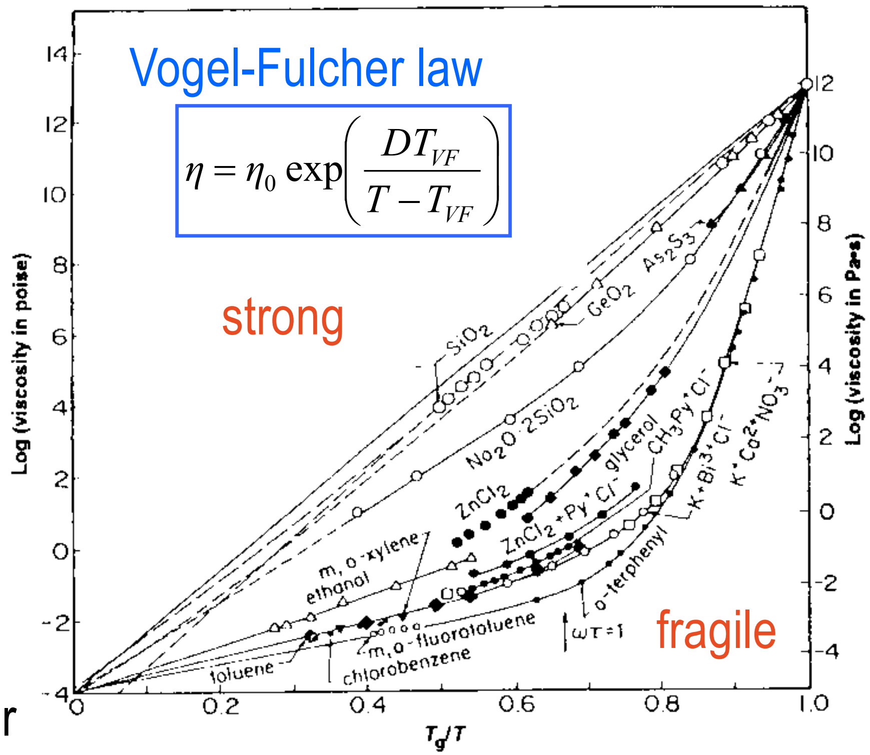
$$\eta\left(T_g\right) = 10^{14}\,\text{Poise} \;\leftrightarrow\; \tau_{rel} \approx 10^2 - 10^3\,\text{sec}$$

“Glass transition”: rather a crossover in finite d!

Mean field T_d ↔ crossover to activated behavior

Vogel-Fulcher law

$$\eta = \eta_0 \exp\left(\frac{DT_{VF}}{T - T_{VF}}\right)$$



From C. A. Angell, Science, 1995

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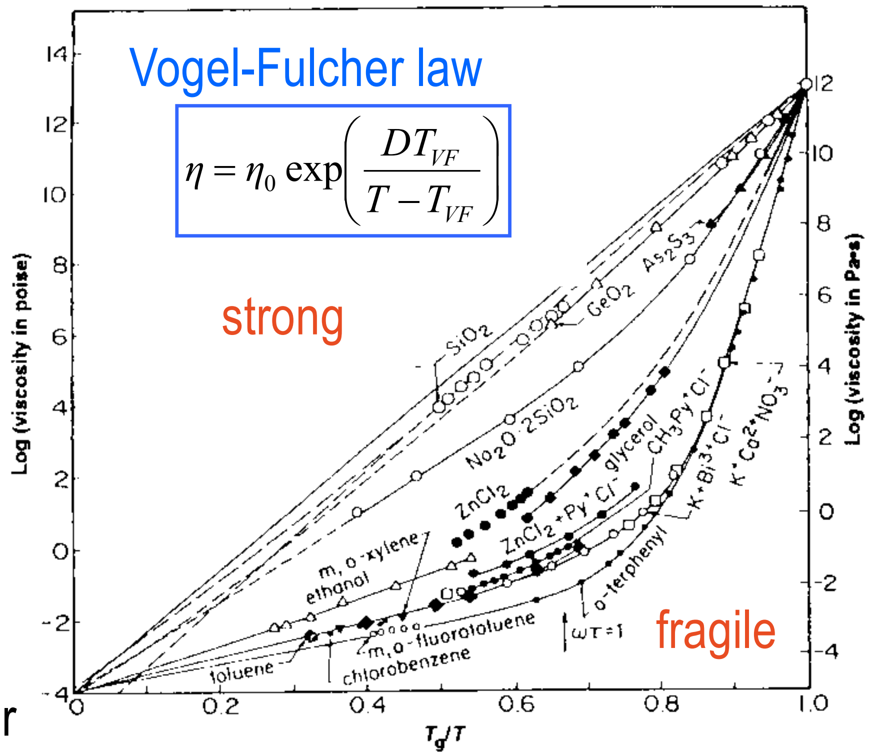
Mean field $T_d \leftrightarrow$ crossover to activated behavior

$T_{VF} \leftrightarrow T_K$?



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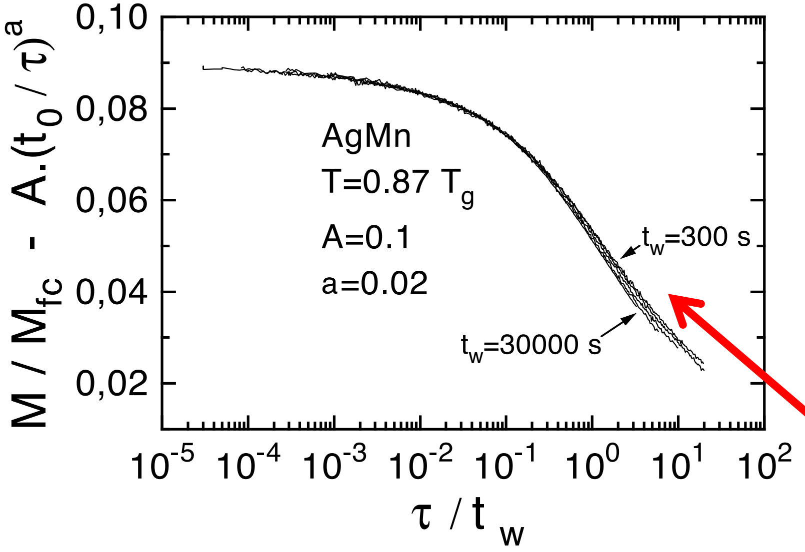
Spin glasses: Aging - Dynamics gets slower with 'age'

Protocol:

- Apply a field B at high T.
- cool to 9K = T < T_c = 10.4K
  at t = 0
- Wait for t_w
- Switch off B
- Measure the decay of M

Spin glasses: Aging - Dynamics gets slower with 'age'

$M_{slow}(\tau)$ =



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$$M(\tau) = M_{fast}(\tau) + M_{slow}(\tau)$$

$$M_{fast}(t) = A\left(\frac{t_0}{\tau}\right)^a$$

$$M_{slow}(t) = f\left(\frac{\tau}{tw}\right)$$

Spin glasses: Aging - Dynamics gets slower with 'age'

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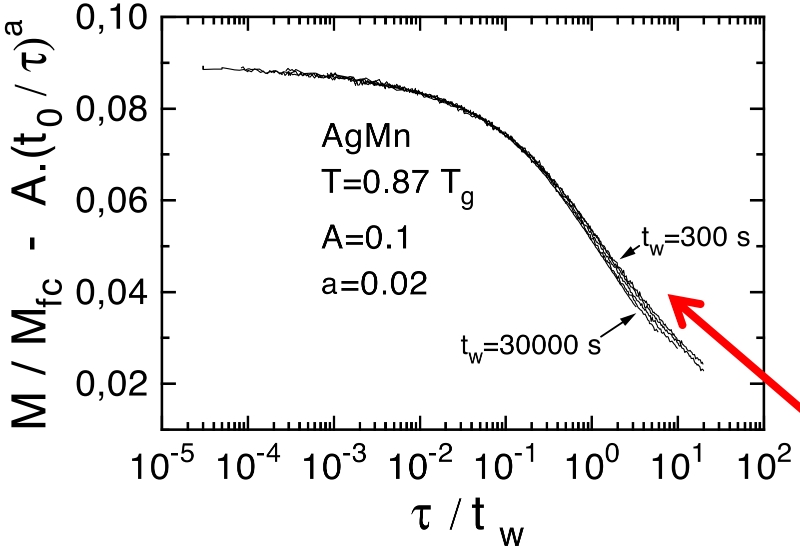

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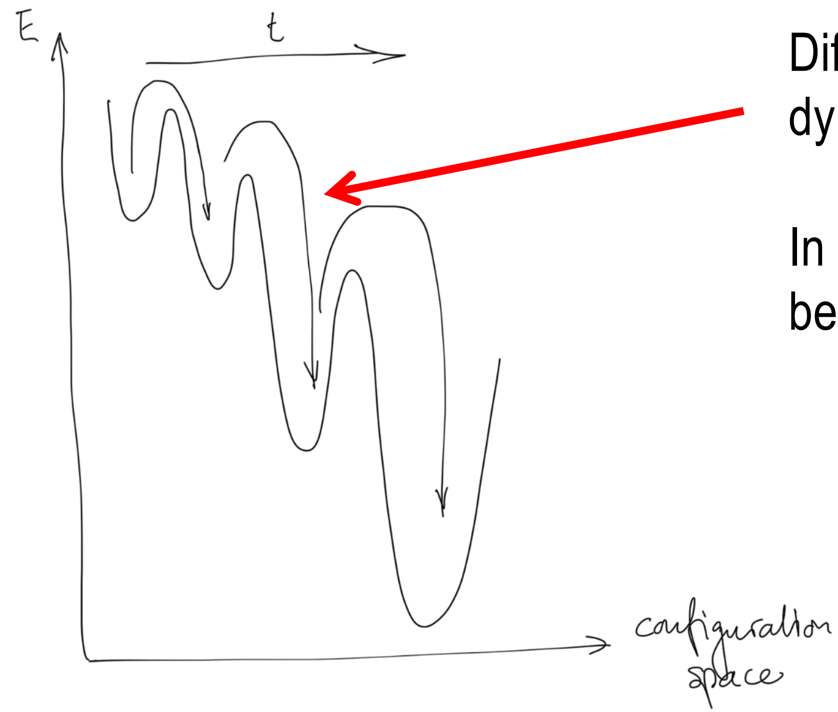
Dynamic time scale grows with t_w: the older the slower

→ the sample is not at equilibrium!

Spin glasses: Aging - Dynamics gets slower with ‘age’



Different states
dynamical escape hindered by free energy barriers

In structural glasses: At T_d escape time distribution becomes heavy-tailed, with diverging expectation

$$\langle t_{\rm esc} \rangle \propto \langle \exp(\Delta F/T) \rangle \to \infty$$

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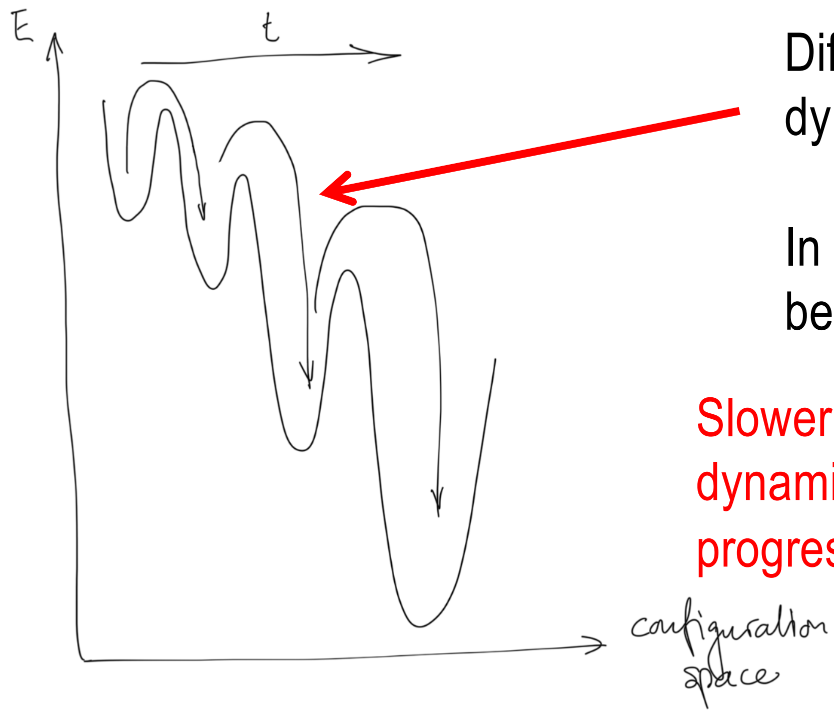

Different states
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In structural glasses: At T_d escape time distribution becomes heavy-tailed, with diverging expectation

Slower and slower dynamics as time progresses

$$\langle t_{\mathrm{esc}} \rangle \propto \langle \exp(\Delta F / T) \rangle \rightarrow \infty$$

Waiting time determines the typical time scale of dynamics and response!

Spin glass universality classes

Two different types of (mean field) spin glasses

How are they reflected in a standard mean field saddle point analysis?

1. Spherical p-spin (details)
2. SK model (sketch)

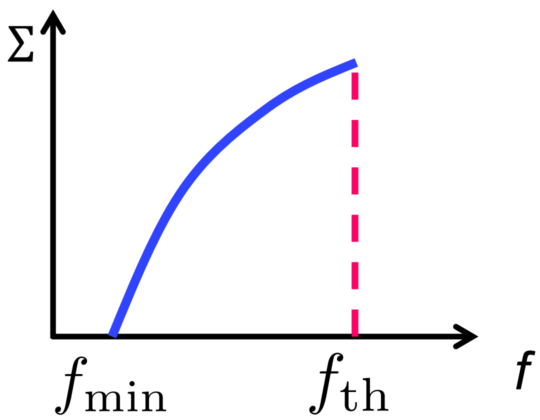
The spherical p-spin solved with replica

Aims:

- Compute the number of pure states at given free energy density f - the "complexity" $\Sigma(f)$
- Replica technique to average over disorder
- Replica symmetry breaking and its physics

Computing the complexity from cloning

Anticipate:
Many pure states in a range of free energy densities f



Computing the complexity from cloning

Clone method: couple m copies together to fall into the same state

$$Z_N^{(m)} = \int df\, e^{N\Sigma(f)} e^{-m\beta f N} \equiv e^{-\beta m \phi(m) N}$$

$$-m\beta\phi(m) = \max_{f|\Sigma(f)\geq 0} \left[\Sigma(f) - m\beta f\right]$$

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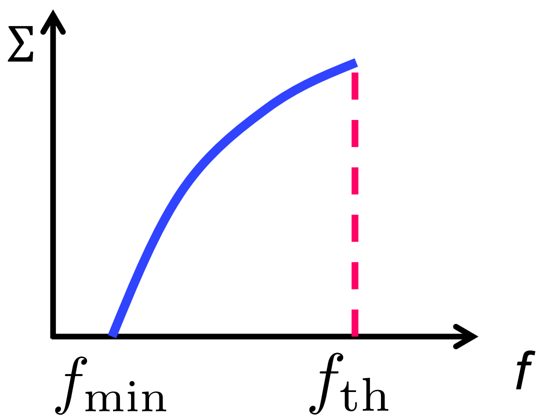

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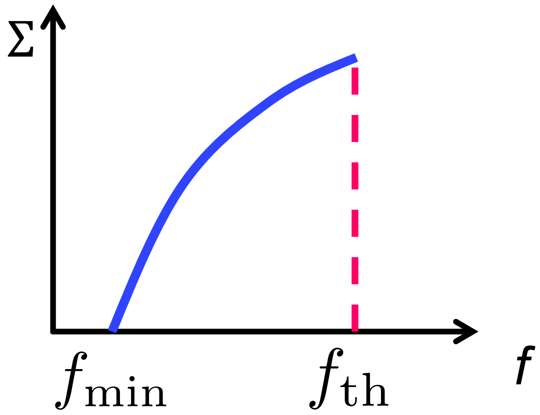

Strategy: 1. compute $Z^{(m)}$ –
2. obtain Σ(f) from Legendre transform of log($Z^{(m)}$):
reproduce quantitatively the result of landscape method

Cloned free energy of spherical p-spins with replicas

$$Z^{(m)} = \exp(-\beta N \Phi(m)) = ? \qquad \Phi(m) \equiv m\phi(m) = ?$$

$$H = H_J[\sigma_1] + \cdots + H_J[\sigma_m] - \epsilon \sum_{a,b}^{1,m} \sum_{i=1}^{N} \sigma_i^a \sigma_i^b$$

Clone forming attraction (dropped in the end)

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Disorder average?

The disorder average of a partition function Z is often dominated by rare disorder.

To obtain the information of typical samples : Average the free energy, or log(Z)!

$$\overline{\log[Z^{(m)}]} = -\beta N \Phi(m)$$

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In practice computed as

$$\overline{\log[Z^{(m)}]} = \lim_{n\to 0} \frac{\overline{(Z^{(m)})^n} - 1}{n} = \lim_{n\to 0} \left[\partial_n \overline{(Z^{(m)})^n} \right]$$

Idea: Averages of powers are easier to compute!

Cloned free energy of spherical p-spins with replicas

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Quenched average:

$$\Phi(m,T) = -\overline{\frac{T}{N} \log Z_m} = -\frac{T}{N} \overline{\log \int D\sigma_1 \cdots D\sigma_m e^{-\beta(H_J[\sigma_1]+\cdots+H_J[\sigma_m])+\beta\epsilon \sum_{a,b}^{1,m} \sum_{i=1}^{N} \sigma_i^a \sigma_i^b}} .$$

$$D\sigma = \left(\prod_i d\sigma_i\right) \delta\left(\sum_i \sigma_i^2 = N\right)$$

Cloned free energy of spherical p-spins with replicas

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Replica trick to express the log-average:

$$\Phi(m,T) = -\frac{T}{N}\lim_{n\to 0}\partial_n \overline{(Z_m)^n}$$

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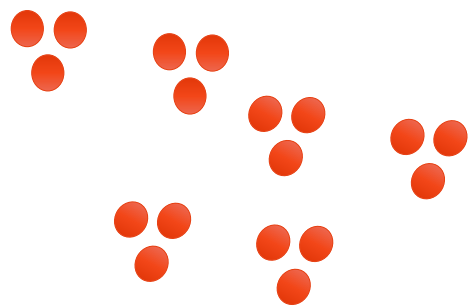
Replica trick to express the log-average:

$$\Phi(m,T) = -\frac{T}{N} \lim_{n\to 0} \partial_n \overline{(Z_m)^n}$$

For integer n:

$$\overline{(Z_m)^n} = \overline{\int D\sigma_1 \cdots D\sigma_{nm} e^{-\beta(H_J[\sigma_1]+\cdots+H_J[\sigma_{nm}])}}$$



n x m copies!

Cloned free energy of spherical p-spins with replicas

$a = 1, ..., nm$

$$\overline{(Z_m)^n} \propto \int D\sigma_i^a \prod_{i_1<\cdots<i_p} \int dJ_{i_1\cdots i_p} \ \exp\left[-J^2_{i_1\cdots i_p}\frac{N^{p-1}}{p!} + \beta J_{i_1\cdots i_p}\sum_{a=1}^{mn}\sigma^a_{i_1}\cdots\sigma^a_{i_p}\right]$$

Product over all p-tuples
(clone attraction is now not explicitly written)

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$$\propto \int D\sigma_i^a \prod_{i_1<\cdots<i_p} \exp\left[\frac{\beta^2 p!}{4N^{p-1}}\sum_{a,b}^{1,mn}\sigma^a_{i_1}\sigma^b_{i_1}\cdots\sigma^a_{i_p}\sigma^b_{i_p}\right]$$

Gaussian average over independent couplings

Get rid of disorder!

Cloned free energy of spherical p-spins with replicas

$a = 1, ..., nm$

$$\overline{(Z_m)^n} \propto \int D\sigma_i^a \prod_{i_1 < \cdots < i_p} \int dJ_{i_1 \cdots i_p} \exp\left[-J_{i_1 \cdots i_p}^2 \frac{N^{p-1}}{p!} + \beta J_{i_1 \cdots i_p} \sum_{a=1}^{mn} \sigma_{i_1}^a \cdots \sigma_{i_p}^a \right]$$

$$\propto \int D\sigma_i^a \prod_{i_1 < \cdots < i_p} \exp\left[\frac{\beta^2 p!}{4N^{p-1}} \sum_{a,b}^{1,mn} \sigma_{i_1}^a \sigma_{i_1}^b \cdots \sigma_{i_p}^a \sigma_{i_p}^b \right]$$

Gaussian average over independent couplings

Get rid of disorder!

Now the replica are coupled attractively!

Why: The information of low energy configurations (depending on J's) now hides in the attraction of replica among each other:
A low energy configuration of one copy attracts other replicas to the same configuration.

Cloned free energy of spherical p-spins with replicas

$a = 1, ..., nm$

$$\overline{(Z_m)^n} \propto \int D\sigma_i^a \prod_{i_1<\cdots<i_p} \int dJ_{i_1\cdots i_p} \exp\left[-J_{i_1\cdots i_p}^2 \frac{N^{p-1}}{p!} + \beta J_{i_1\cdots i_p} \sum_{a=1}^{mn} \sigma_{i_1}^a \cdots \sigma_{i_p}^a\right]$$

$$\propto \int D\sigma_i^a \prod_{i_1<\cdots<i_p} \exp\left[\frac{\beta^2 p!}{4N^{p-1}} \sum_{a,b}^{1,mn} \sigma_{i_1}^a \sigma_{i_1}^b \cdots \sigma_{i_p}^a \sigma_{i_p}^b\right]$$

$$= \int D\sigma_i^a \exp\left[\frac{\beta^2}{4N^{p-1}} \sum_{a,b}^{1,mn} \left(\sum_i^N \sigma_i^a \sigma_i^b\right)^p\right]$$

Crossterms with identical indices are subleading by O(1/N)

Cloned free energy of spherical p-spins with replicas

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$$= \int D\sigma_i^a \ \exp\left[\frac{\beta^2}{4N^{p-1}} \sum_{a,b}^{1,mn} \left(\sum_i^N \sigma_i^a \sigma_i^b\right)^p\right] = \int D\sigma_i^a \ \exp\left[N \frac{\beta^2}{4} \sum_{a,b}^{1,mn} \left(\frac{1}{N} \sum_i \sigma_i^a \sigma_i^b\right)^p\right]$$

Overlap (global similarity) between replica a and b :

$$Q(\sigma^a, \sigma^b) = \frac{1}{N} \sum_i \sigma_i^a \sigma_i^b$$

Cloned free energy of spherical p-spins with replicas

$$\overline{(Z_m)^n} \propto \int D\sigma_i^a \prod_{i_1<\cdots<i_p} \int dJ_{i_1\cdots i_p} \, \exp\left[-J^2_{i_1\cdots i_p}\frac{N^{p-1}}{p!} + \beta J_{i_1\cdots i_p}\sum_{a=1}^{mn}\sigma^a_{i_1}\cdots\sigma^a_{i_p}\right]$$

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$$\overline{(Z_m)^n} \propto \int D\sigma_i^a \int \prod_{a<b}^{1,mn}\left\{dQ_{ab}\,\delta\left(Q_{ab}-\frac{1}{N}\sum_i\sigma_i^a\sigma_i^b\right)\right\} \exp\left[N\frac{\beta^2}{4}\sum_{a,b}^{1,mn}Q_{ab}^p\right]$$

Hubbard-Stratonovich

Cloned free energy of spherical p-spins with replicas

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$$= \int dQ \; \exp\left[N \frac{\beta^2}{4} \sum_{a,b}^{1,mn} Q_{ab}^p\right] \int d\sigma_i^a \prod_{a\leq b}^{1,mn} \delta\left(NQ_{ab} - \sum_i \sigma_i^a \sigma_i^b\right)$$

$dQ = \prod_{a<b} dQ_{ab}$ and $Q_{aa} = 1$

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$dQ = \prod_{a<b} dQ_{ab}$ and $Q_{aa} = 1$

$$J(Q) = \int d\sigma_i^a \prod_{a\leq b}^{1,mn}\delta\left(NQ_{ab}-\sum_i\sigma_i^a\sigma_i^b\right) = \int d\vec{\sigma}^a\,\delta(NQ_{ab}-\vec{\sigma}^a\cdot\vec{\sigma}^b)$$

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Important:

Different sites have been decoupled by Hubbard-Stratonovich in this effective partition function!

Only *single-site* interactions between the replica $\sigma_i^{a=1,\ldots,mn}$

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Saddle point wrt λ_{ab} $\longrightarrow$ $Q_{ab} = (\lambda_*^{-1})_{ab}$

$$J(Q) = const \cdot \int d\sigma \exp\left(nmN - \sum_{a\leq b} Q_{ab}^{-1} \sum_{i=1}^{N} \sigma_i^a \sigma_i^b\right) = const \cdot [\det Q]^{N/2}$$

Cloned free energy of spherical p-spins with replicas

$$\overline{(Z_m)^n} \propto \int dQ_{ab}\; e^{NX(Q)}\;,$$

$$X(Q) = \frac{\beta^2}{4}\sum_{ab} Q_{ab}^p + \frac{1}{2}\log\det Q$$

Due to mean field structure:

Final integral over global replica overlaps Q_{ab} , with an action $\propto$ N

⟶ Saddle point over the "order parameter" Q_{ab} ! ?

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But recall:

$$\Phi(m,T) = -T \lim_{N\to\infty} \frac{1}{N} \lim_{n\to 0} \partial_n \int dQ_{ab}\; \exp\left[\overbrace{NX(Q)}^{\propto\, n\, N}\right]$$

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Press on as a brave physicist and find a saddle point Q^*_{ab} for any m, n!

Cloned free energy of spherical p-spins with replicas

Recall clone coupling in blocks (B) of m spins:

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$$X(Q) = \frac{\beta^2}{4}\sum_{ab} Q_{ab}^p + \frac{1}{2}\log\det Q + \beta\epsilon \sum_B \sum_{ab\in B} Q_{ab}$$

Saddle point equation for Q_{ab} is complicated: no general solution

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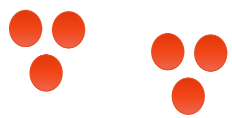
But: Physical guess of a sensible structure (confirmed by exact solution): $Q_{aa} = 1$

- Replicas of the same block are coupled in the same valley → finite overlap $Q_{a\neq b} = q$
- *a* and *b* in different blocks: uncorrelated $Q_{ab} = 0$

"One-step replica symmetry breaking structure" :

$$Q = \begin{pmatrix} \begin{pmatrix} 1 & q & q \\ q & 1 & q \\ q & q & 1 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} 1 & q & q \\ q & 1 & q \\ q & q & 1 \end{pmatrix} \end{pmatrix}$$

m=3 (clones)



n=2 (blocks of replica clones) → 0 eventually

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Evaluate with this ansatz

$$X(Q) = -\beta n m \phi_{1\mathrm{RSB}}(m, q, T)$$

$$\phi_{1\mathrm{RSB}}(m,q,T) = -\frac{1}{2\beta}\left\{\frac{\beta^2}{2}\left[1+(m-1)q^p\right] + \frac{m-1}{m}\log(1-q) + \frac{1}{m}\log\left[1+(m-1)q\right]\right\}$$

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+ continuation to $n \to 0$ and real m

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\+ continuation to $n \to 0$ and real m

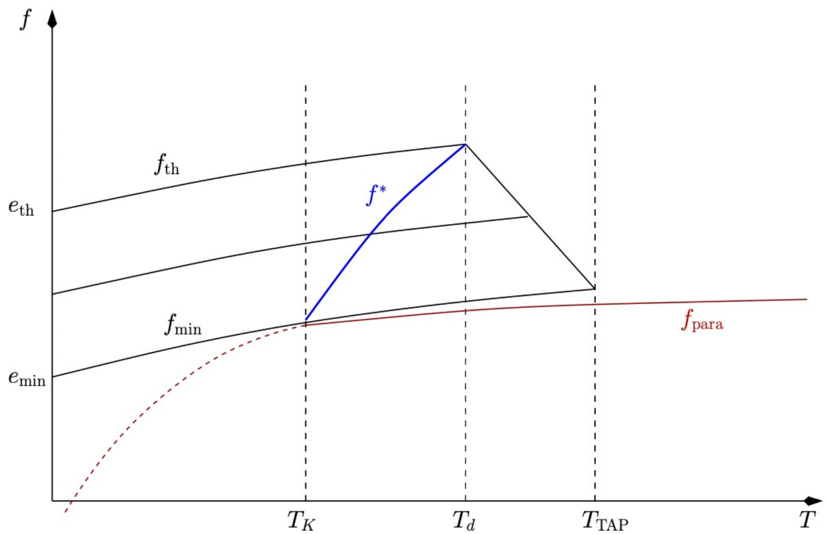
Extremize over q → q*

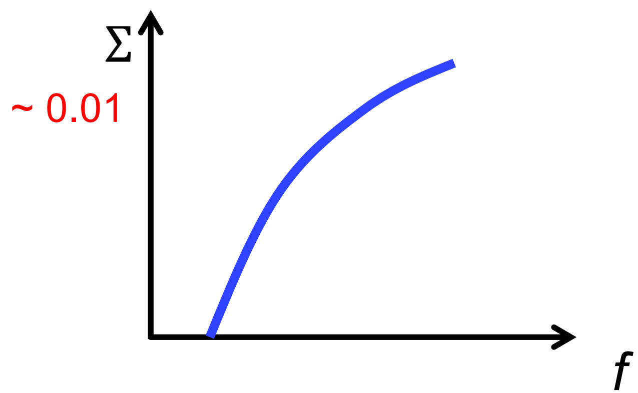
$$\Phi(m,T) = -T\partial_n X(Q^*) = m\phi_{1\mathrm{RSB}}(m,q^*,T)$$

Cloned free energy of spherical p-spins with replicas

From $\Phi(m, T)$:

Obtain the spectrum of metastable states by Legendre transform!





Confirm the structure of the landscape approach, compute $\Sigma(f)$ quantitatively!

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Note: Clone attraction explicitly breaks permutation symmetry among nm replica.

But what about computing for a single copy (with no cloning) directly?

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Parisi's equilibrium recipe: Regard *m* and *q* as variational; find stationary point!

$$f_{eq}(T) = \max_{q,0\leq m\leq 1} \phi_{1RSB}(m,q)$$

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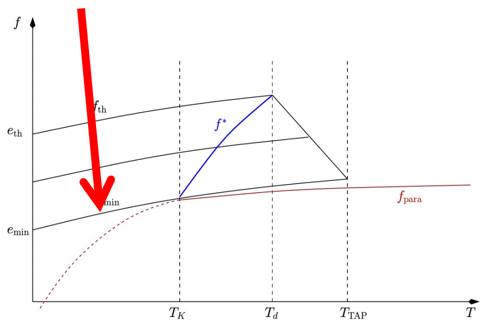

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Different replica will lie in the lowest available minima of $G(\{m_i\})$

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Two different replica (1,2) may lie in the same minimum (and have overlap Q_{12} = q)
or in different valleys (overlap Q_{12} = 0):

sum over inequivalent saddle points

$$P(Q_{12}) \equiv \overline{\delta(Q_{12} - \frac{1}{N}\sum_i \sigma_i^1 \sigma_i^2)} = \lim_{\nu \to 0} \frac{1}{\nu - 1} \sum_{b \neq 1} \delta(Q_{12} - Q_{1b})$$

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RSB reflects different pure states that contribute O(1) to the Gibbs ensemble. In high dimensions, RSB happens even with no symmetry present (e.g. no Ising symmetry)!

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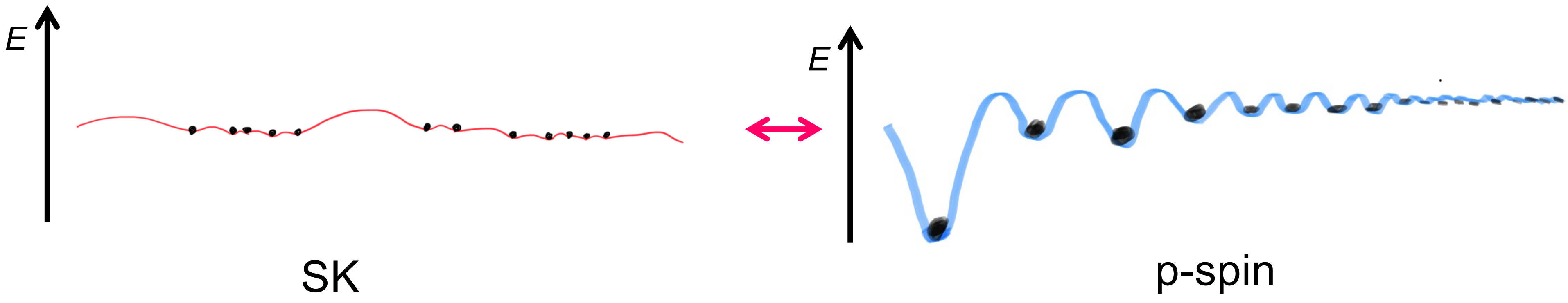
What about the other universality class: spin glasses?

How is their different physics reflected in the RSB structure of the SK spin glass?



RSB structure of the SK model

Technical steps? (no complexity anticipated: no clones)

RSB structure of the SK model

Technical steps?

- Write partition function
- Replicate *n* times
- Disorder average
- Hubbard-Stratonovich - decouple different sites by introducing an integral over the overlap *Q*
- Obtain effective action of *N* decoupled sites: extensive
- Seek saddle point Q^* (close your eyes and take large-*N* limit before $n \to 0$)
- Make Parisi's block ansatz for Q_{ab}
- Compute physical quantities and check whether they make sense.

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In SK case: 1step ansatz yields a low T entropy that becomes negative!

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How to do better?

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How to do better?

Try blocks in blocks! == “2-step RSB”:

$$Q = \begin{pmatrix} 1 & q_2 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_2 & 1 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & 1 & q_2 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & q_2 & 1 & q_0 & q_0 & q_0 & q_0 \\ q_0 & q_0 & q_0 & q_0 & 1 & q_2 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_2 & 1 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & 1 & q_2 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & q_2 & 1 \end{pmatrix}$$

m_1

m_2

A priori

$$1 < m_2 < m_1 < n$$

But as n → 0

$$1 > m_2 > m_1 > n \to 0$$

RSB structure of the SK model

2step ansatz : low T entropy is less negative, but still negative!

How to do better?

$$Q = \begin{pmatrix} 1 & q_2 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_2 & 1 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & 1 & q_2 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & q_2 & 1 & q_0 & q_0 & q_0 & q_0 \\ q_0 & q_0 & q_0 & q_0 & 1 & q_2 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_2 & 1 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & 1 & q_2 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & q_2 & 1 \end{pmatrix}$$

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How to do better?

Infinite hierarchy of blocks! == “continuous RSB” !

Parametrized by a limiting function $q(1 > x > 0)$

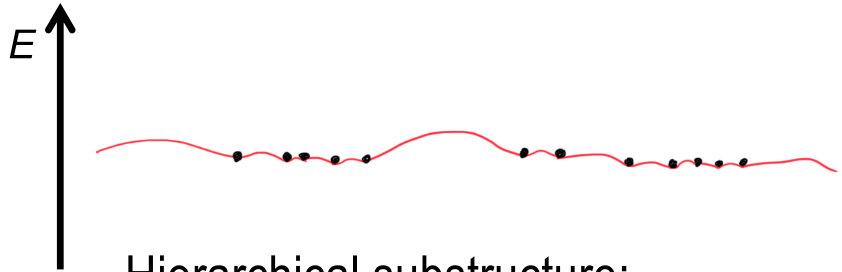
RSB structure of the SK model

2step ansatz : low T entropy is less negative, but still negative!

$$Q = \begin{pmatrix} 1 & q_2 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_2 & 1 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & 1 & q_2 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & q_2 & 1 & q_0 & q_0 & q_0 & q_0 \\ q_0 & q_0 & q_0 & q_0 & 1 & q_2 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_2 & 1 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & 1 & q_2 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & q_2 & 1 \end{pmatrix}$$

How to do better?

Infinite hierarchy of blocks! == “continuous RSB” !



Recall: overlap $Q^{ab} = \frac{1}{N} \sum_i s_i^a s_i^b$

Hierarchical substructure:
clusters of states with overlap q_2,
clustering into larger clusters of smaller overlap q_1,
forming global cluster of overlap q_0

RSB structure of the SK model

2step ansatz : low T entropy is less negative, but still negative!

$$Q = \begin{pmatrix} 1 & q_2 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_2 & 1 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & 1 & q_2 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & q_2 & 1 & q_0 & q_0 & q_0 & q_0 \\ q_0 & q_0 & q_0 & q_0 & 1 & q_2 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_2 & 1 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & 1 & q_2 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & q_2 & 1 \end{pmatrix}$$

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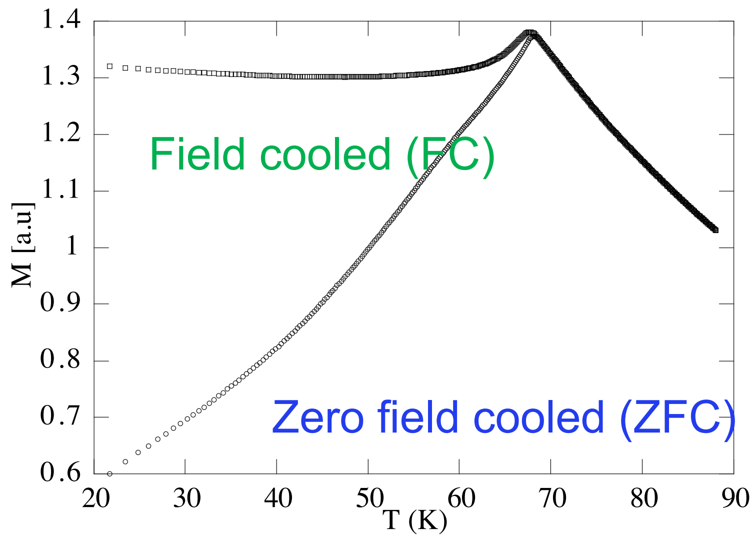
Infinite hierarchy of blocks! == “continuous RSB” !

- Entropy remains positive! ZFC/FC susceptibility sim. to experiment



RSB structure of the SK model

2step ansatz : low T entropy is less negative, but still negative!

$$Q = \begin{pmatrix} 1 & q_2 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_2 & 1 & q_1 & q_1 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & 1 & q_2 & q_0 & q_0 & q_0 & q_0 \\ q_1 & q_1 & q_2 & 1 & q_0 & q_0 & q_0 & q_0 \\ q_0 & q_0 & q_0 & q_0 & 1 & q_2 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_2 & 1 & q_1 & q_1 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & 1 & q_2 \\ q_0 & q_0 & q_0 & q_0 & q_1 & q_1 & q_2 & 1 \end{pmatrix}$$

How to do better?

Infinite hierarchy of blocks!   == “continuous RSB” !

- Entropy remains positive! ZFC/FC susceptibility sim. to experiment
- No finite complexity: always **less than exponentially many relevant states**!
- The action at the saddle point Q* is only a marginal maximum (like for threshold states in the p-spin model)
  ↔ the ***physical*** **minima** have **marginal stability**: shallow, with flat directions;
  the **whole glass phase is critical**!