Quantum effects in glasses

- Phase diagram: two universality classes
- MBL in glasses ?
- On adiabatic quantum computing
- Short time dynamics: spectral function of mean field models
- Rotor model, Ising glass and Heisenberg glass
- Long range glasses in optical cavities - localization vs glassiness

Quantum tunneling in p-spin models

$$H_p = -\sum_{(i_1 \ldots i_p)} J_{i_1 \ldots i_p} \hat{\sigma}^z_{i_1} \cdots \hat{\sigma}^z_{i_p} - \Gamma \sum_{i=1}^{N} \hat{\sigma}^x_i$$

Adding a transverse field Γ

Phase diagram?

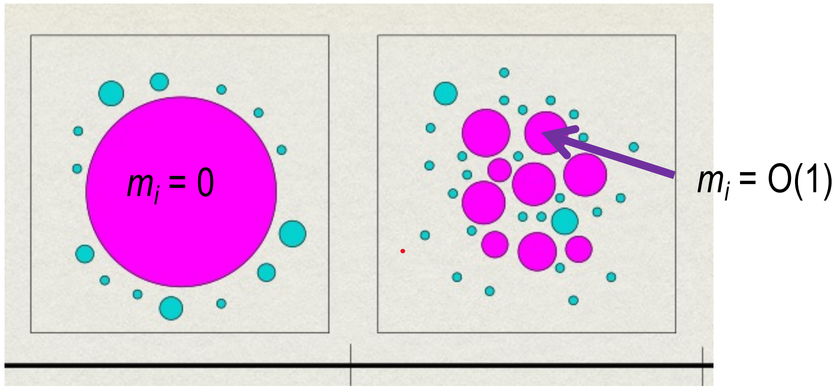
Quantum tunneling in p-spin models

Phase diagram

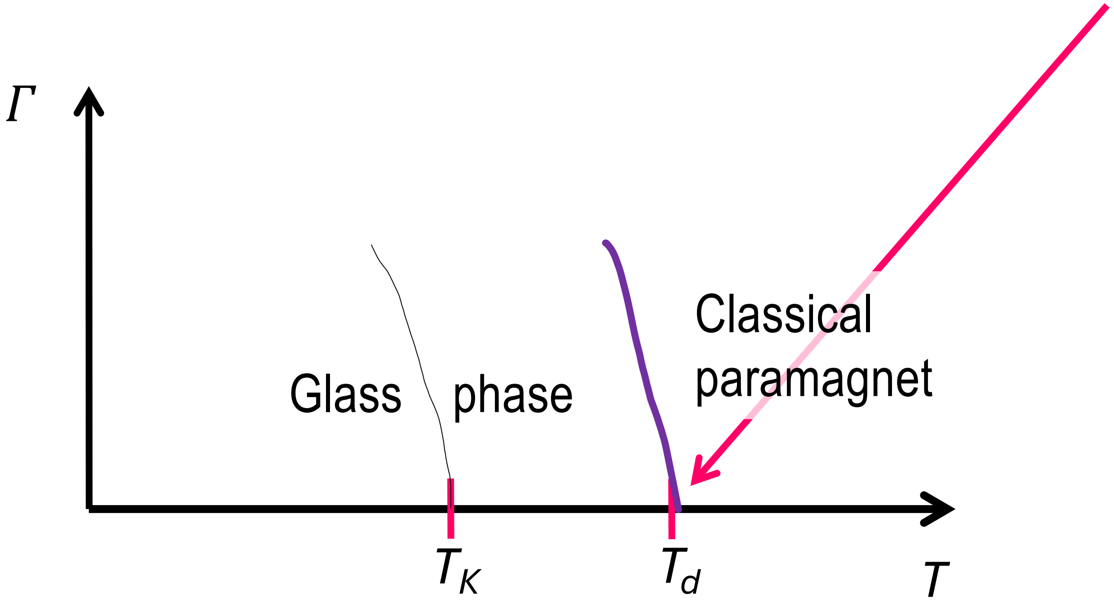
Recall: At dynamical transition at T_d
sudden jump in ordering: $m_i = O(1)$

$$p > 2, m \ll 1$$

Energy gain: $O(m^p)$ $\ll$ Entropic cost: $O(m^2)$



Breakup of paramagnet into ordered states



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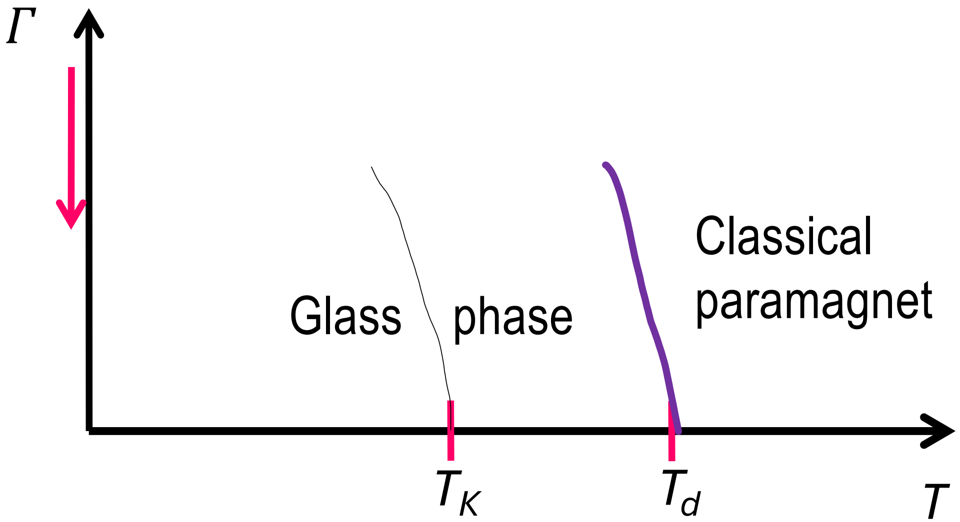

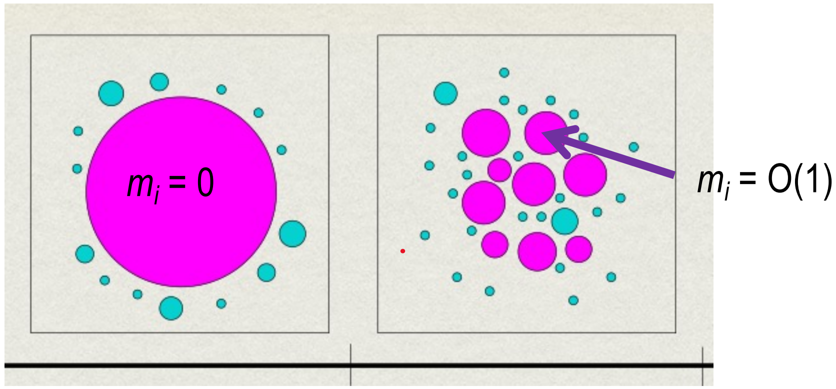

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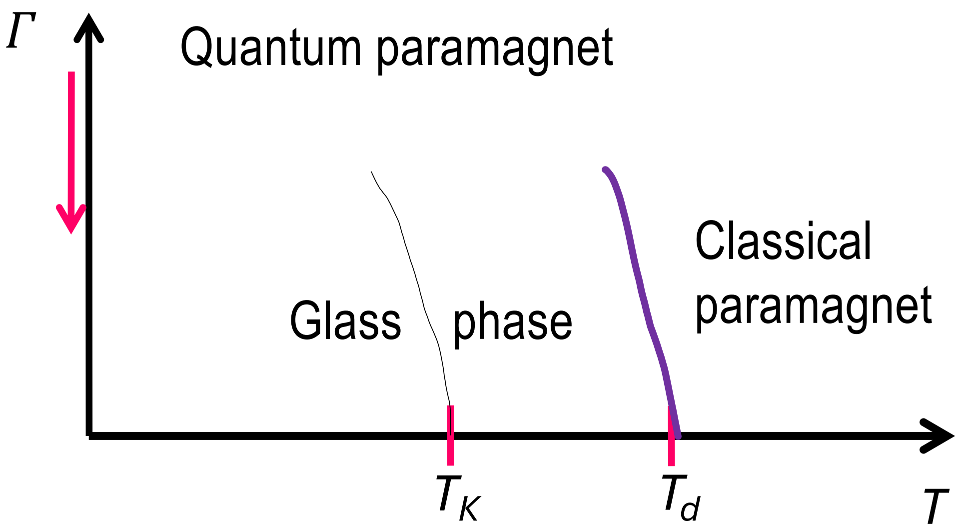

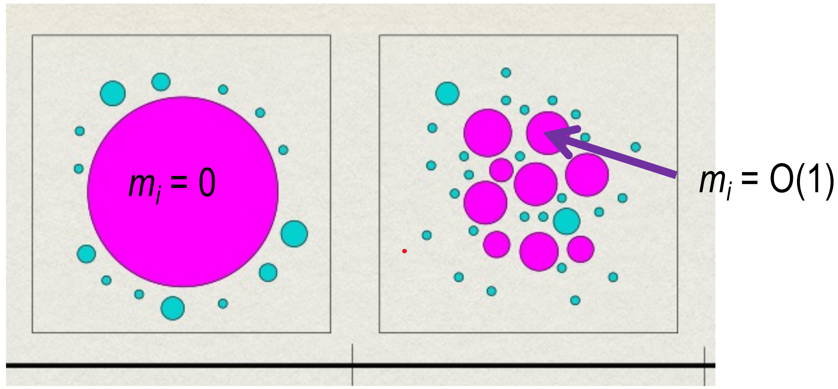

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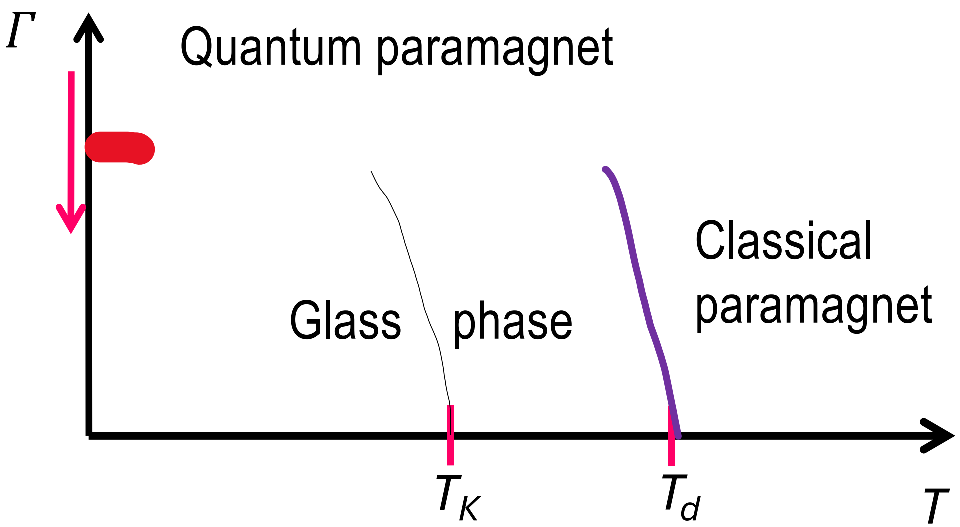

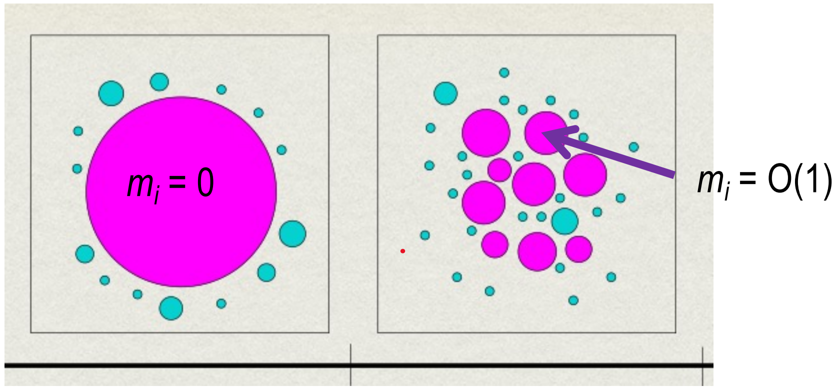

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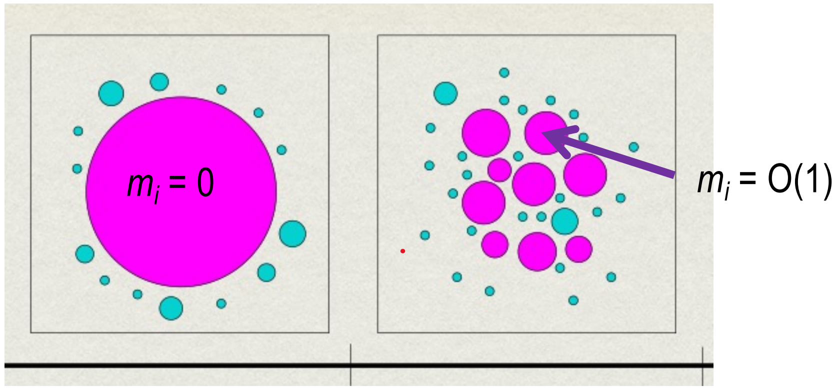

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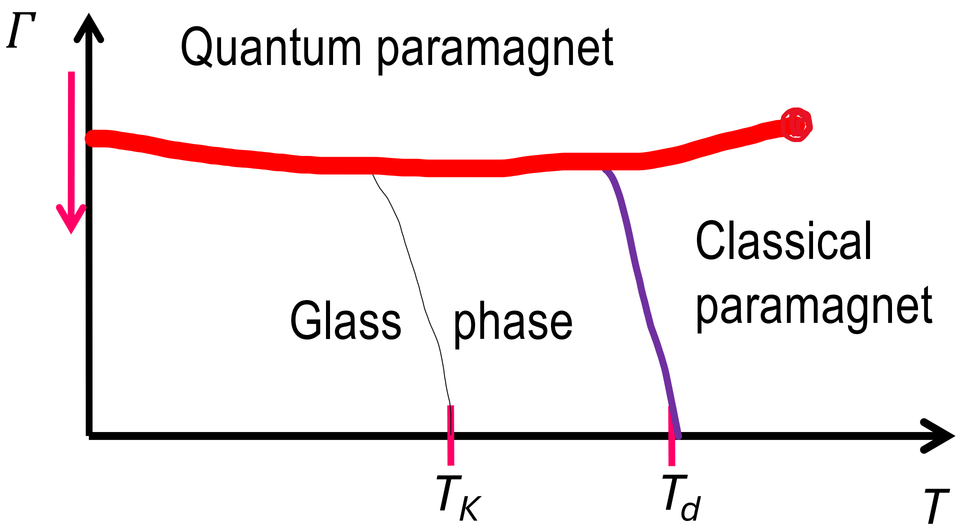

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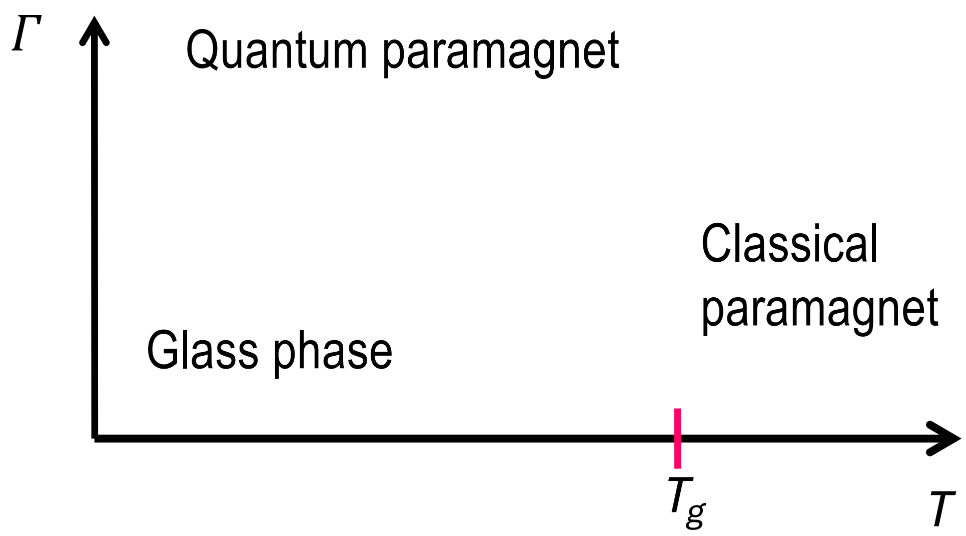

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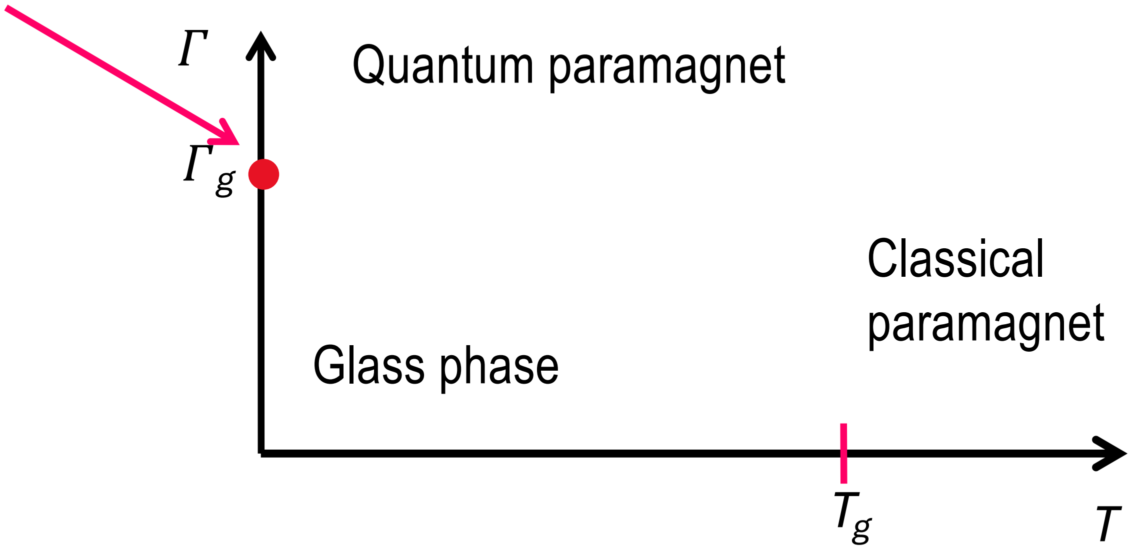

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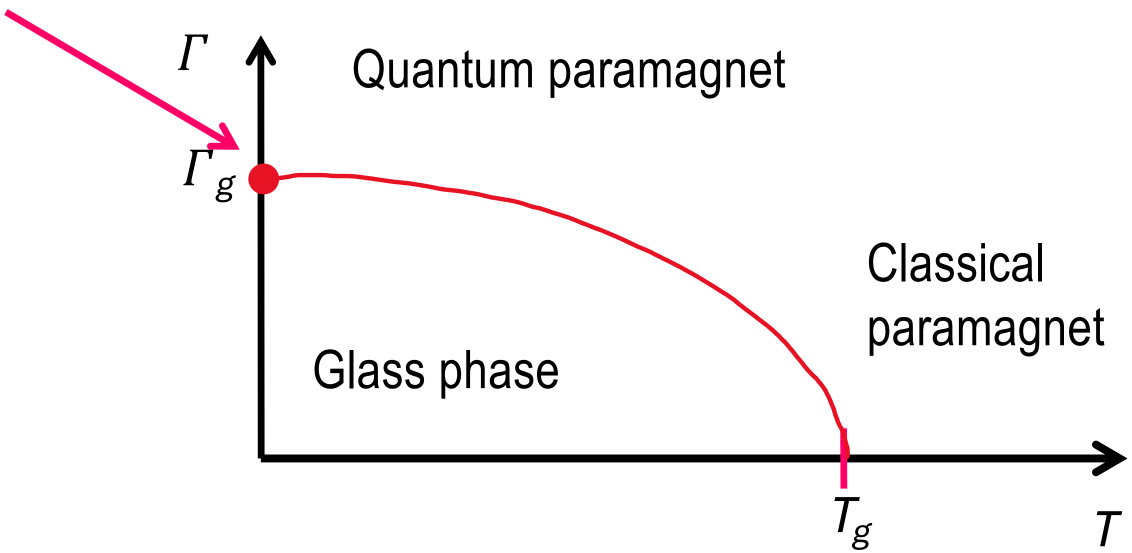

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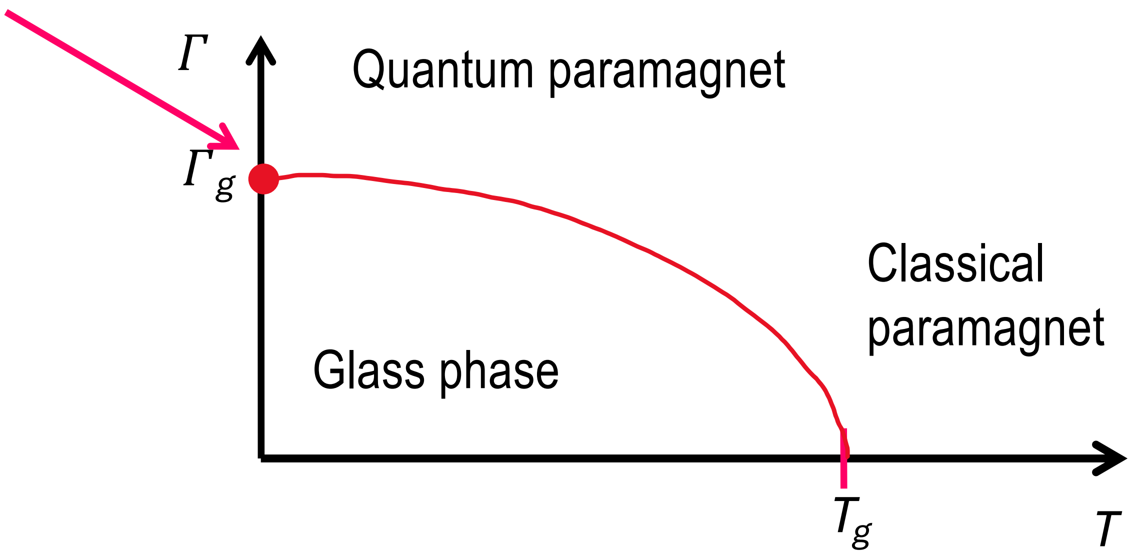
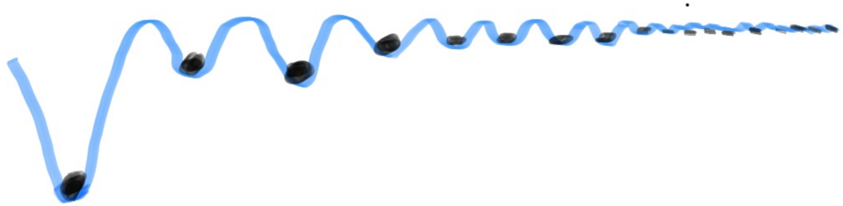


Confirmed (see below):

- Rotor model
- SK model

Localization or quantum tunneling in p-spin models

Many-body localization in states?



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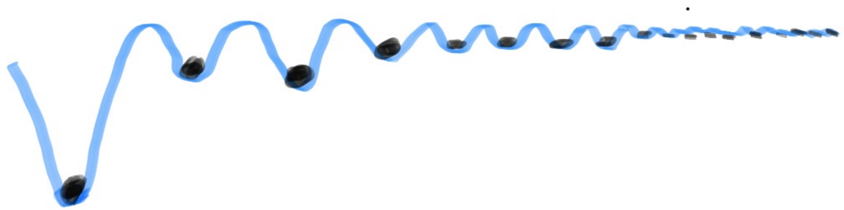
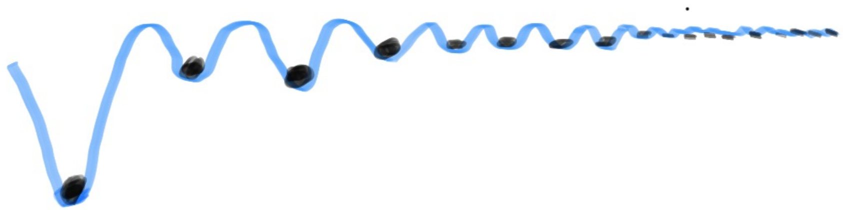


Within a state:
No reason for non-ergodic dynamics *among* the configurations forming a state (dimension is high, connectivity is large, etc)

Localization or quantum tunneling in p-spin models

Many-body localization between states?

Tunneling between two stable minima of energy E



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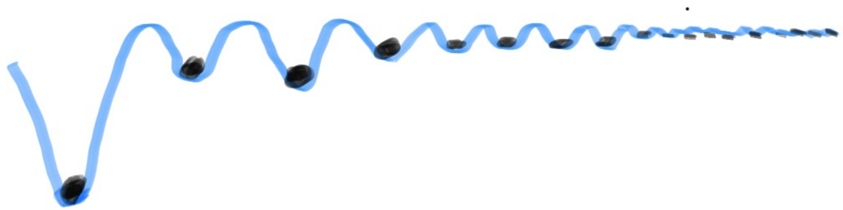

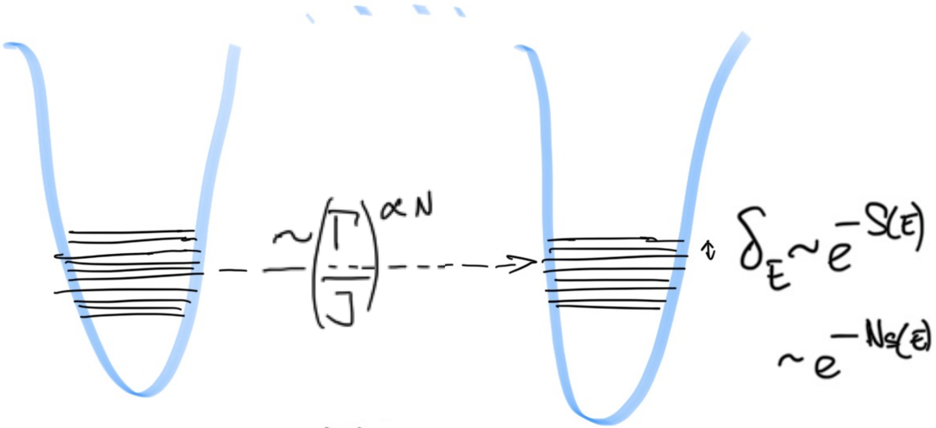

MBL: $\alpha \log\left(\frac{\Gamma}{J}\right) < s(E)$ ⟷ T^*

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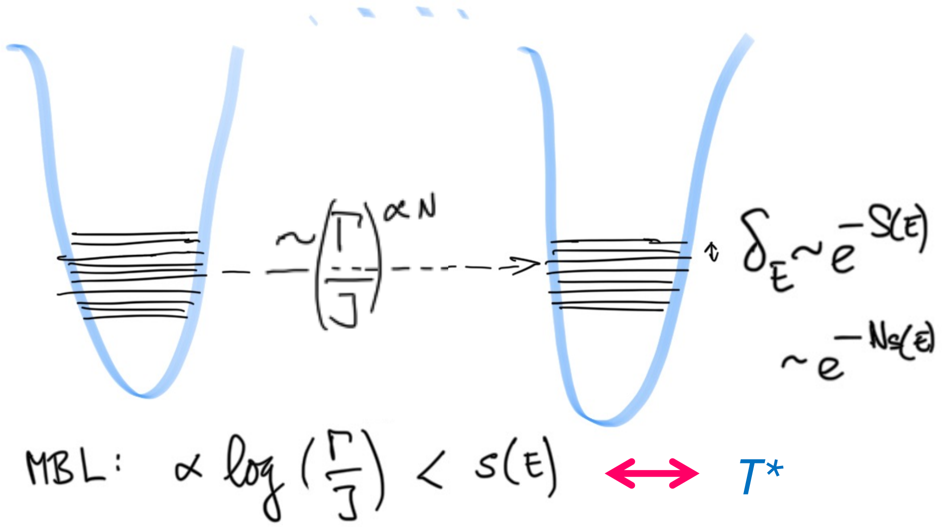
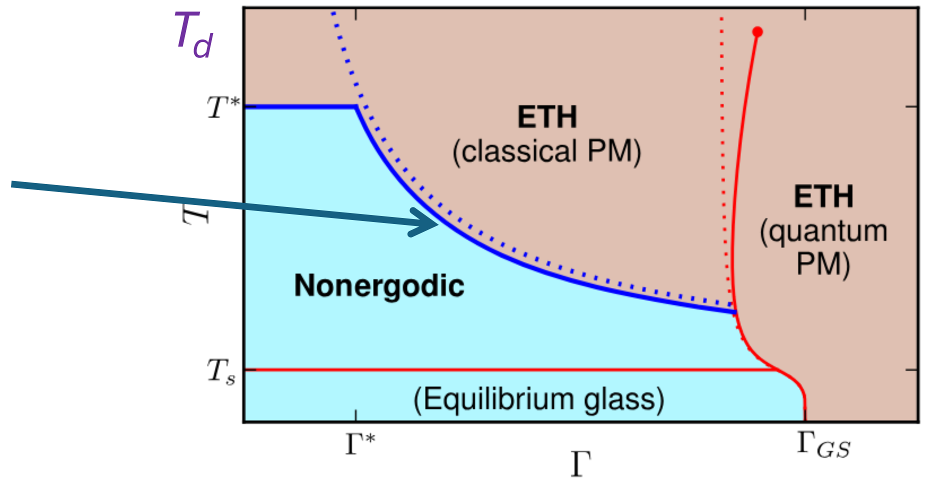
Baldwin et al. '17





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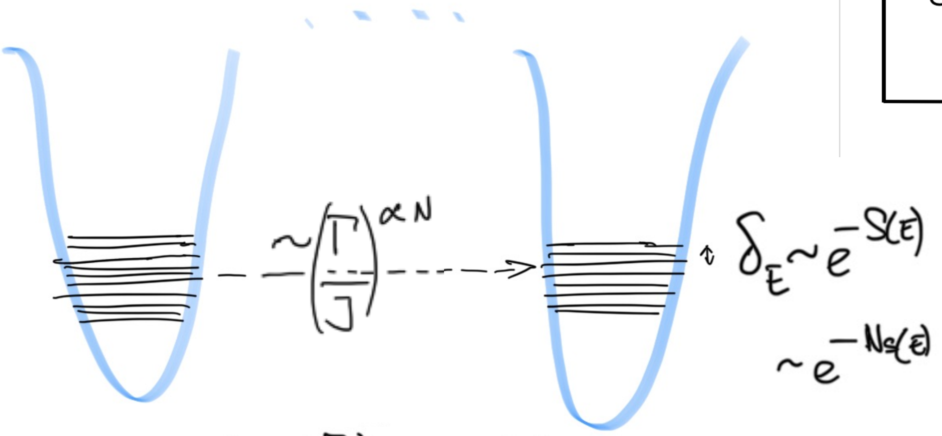

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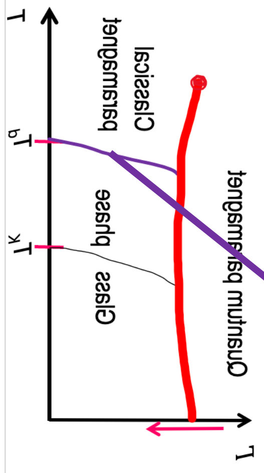

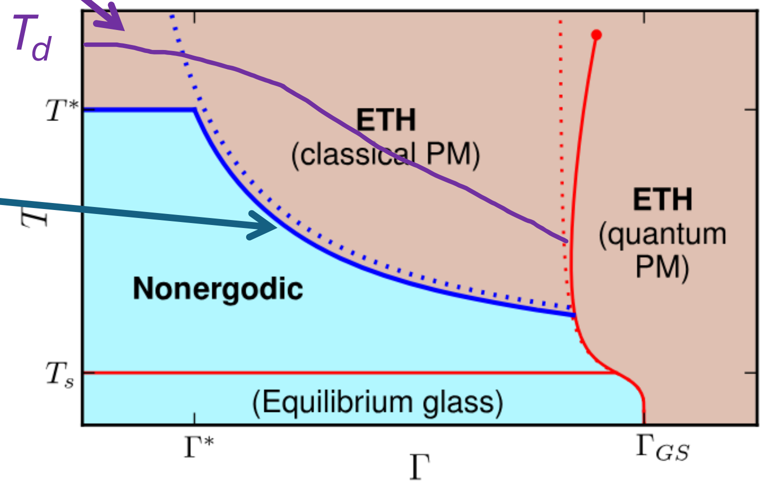

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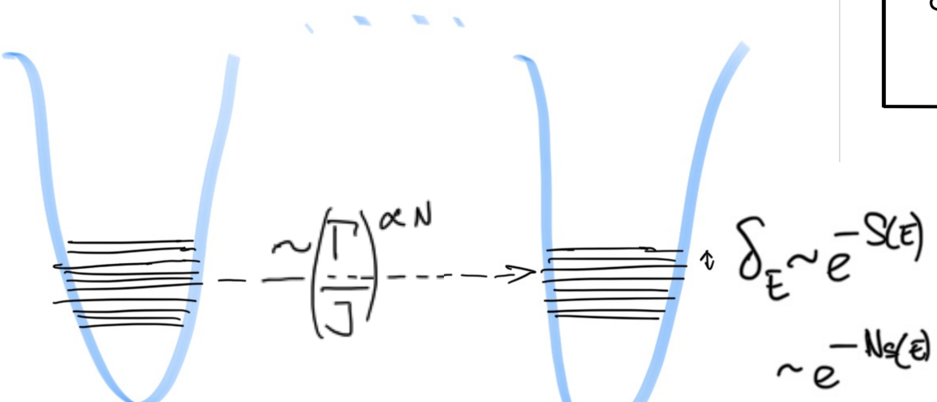
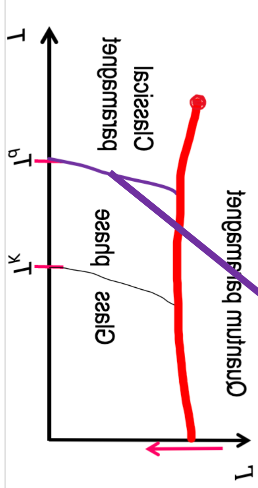
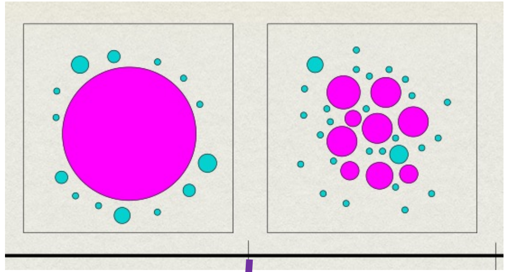


MBL: $\propto \log\left(\frac{\Gamma}{J}\right) < s(E)$ ↔ T^*





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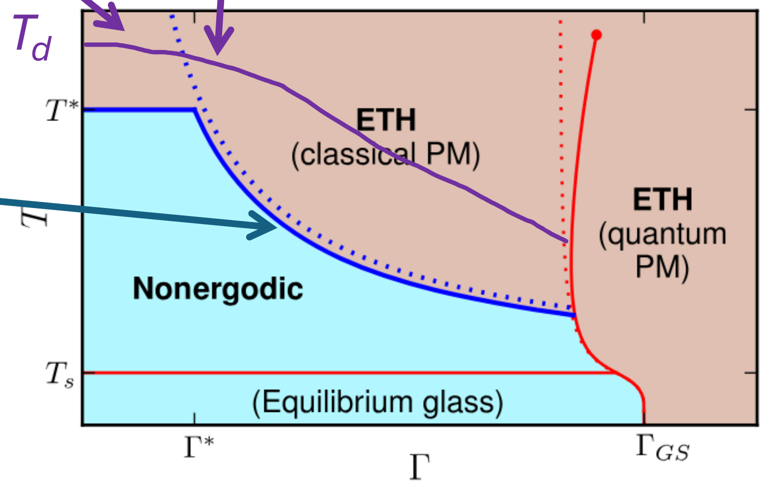

Note: this transition constitutes an energy-dependent mobility edge.
Most likely this only exists in mean field models without spatial structure!

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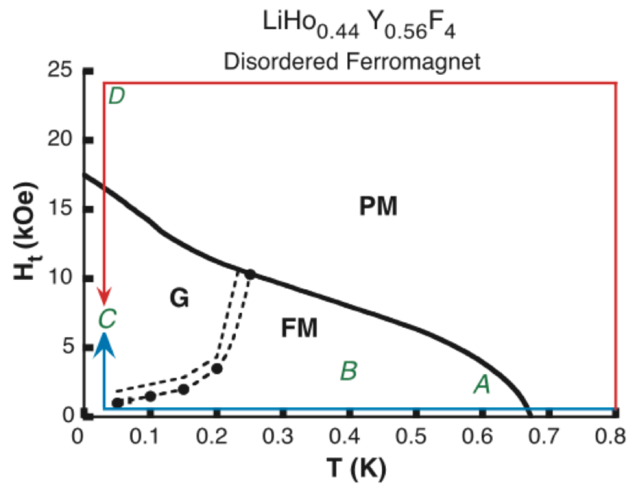
Quantum Annealing of a Disordered Magnet

J. Brooke,[1] D. Bitko,[1] T. F. Rosenbaum,[1*] G. Aeppli[2]



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Use classical «analogue computer» to solve complex problems:

1. Translate your complex problem into a spin glass and build the glass with all its couplings

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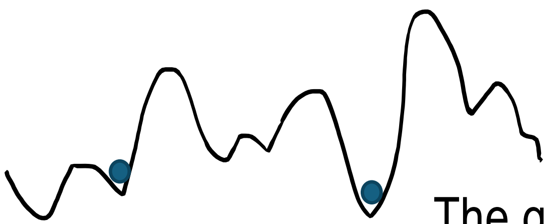

The glass gets trapped in local minima, separated by extensive barriers from the ground state → exponential relaxation times

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Kadowaki and Nishimori, 1998

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$\longrightarrow$ An elegant way to find the ground state !?

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$$h_x \to h_x + \delta h_x$$

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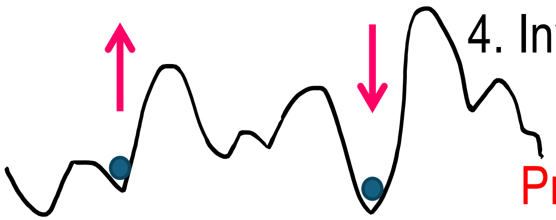

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Problem: As bottoms of low states cross the gap becomes exponentially small (nearly no level repulsion)!

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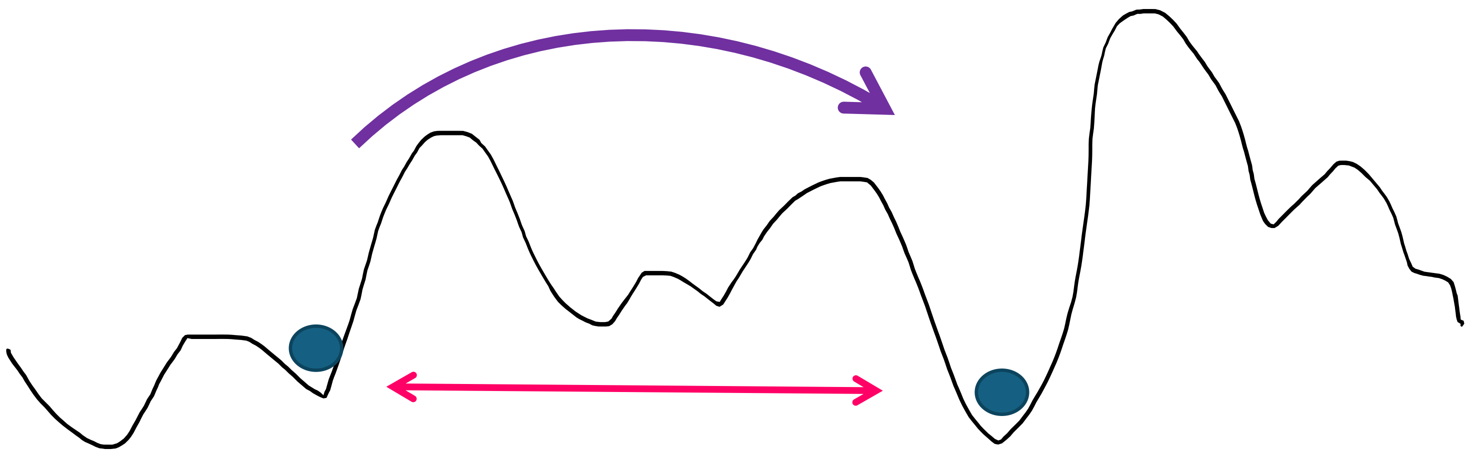
Conclusion

In the glass phase : High barriers between minima.

Thermal activation and quantum tunneling are both **exponentially slow.**



Short time quantum dynamics

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Approaches:

- Landscape approach: Self-consistent *dynamical* mean field equations for

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$$\chi_i(\tau) = \langle S_i(\tau' + \tau) S_i(\tau') \rangle - m_i^2$$

G. Biroli, L. Cugliandolo '02

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→ • Path integral over imaginary time for $S_{i,a}(0 \leq \tau \leq \beta)$ *e.g. A. Bray, M. Moore '80's*

Replica approach + disorder average, saddle point method

Saddle point properties in quantum glasses

Order parameter

$$Q_{ab}(\tau,\tau') = \overline{\langle S_a(\tau) S_b(\tau')\rangle} = \frac{1}{N}\sum_i \overline{\langle S_{ia}(\tau) S_{ib}(\tau')\rangle}$$

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Replica off-diagonal is time-independent!

Dynamics in quantum glasses

- Short time quantum dynamics: encoded in $Q_{aa}(\tau - \tau')$

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[see later:

Analytic continuation to real time yields dynamic susceptibility and the spectral function = information on collective excitations]

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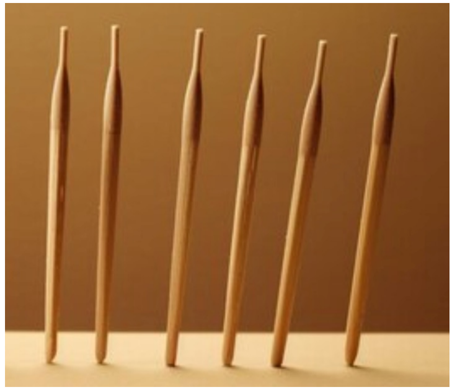
- Long time dynamics (slow floating over the landscape; aging) occurs well after quantum coherence is lost → identical to classical dynamics
- Replica symmetry breaking structure – Q_{ab} and P(q) and landscape – : mostly insensitive to quantum fluctuations (they just reduce the amplitude of possible q's).

The simplest quantum 'spin' glass: Mean field rotor model

Ye, Sachdev, Read '93



Rigid rods, described by *M*-component unit vectors $\hat{n}_i$

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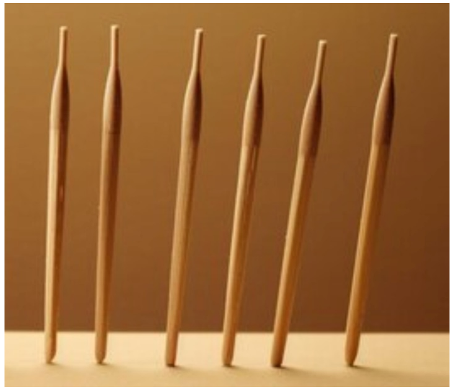

$$H = \frac{g}{2M} \sum_i \hat{L}^2 + \frac{M}{\sqrt{N}} \sum_{i<j} J_{ij} \hat{n}_i \cdot \hat{n}_j$$

$$\hat{n}_i^2 = 1$$

Factors of *M* and *N* chosen such that *H* ~ *O(MN)*

$$[n_{i\mu}, n_{j\nu}] = 0$$

Commuting components - unlike quantum spins

$$P(J_{ij}) \sim \exp(-J_{ij}^2/(2J^2))$$

Gaussian all-to-all couplings

The simplest quantum 'spin' glass: Mean field rotor model

Ye, Sachdev, Read '93

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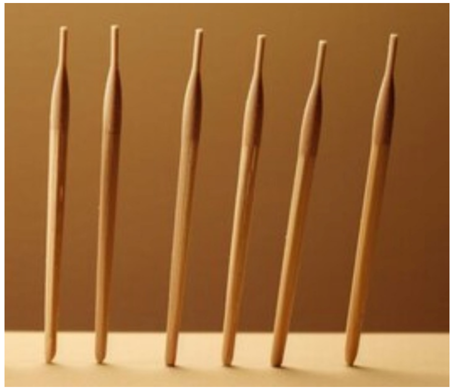

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Commuting components - unlike quantum spins

$$L_{i\mu\nu} = -i \left(n_{i\mu} \frac{\partial}{\partial n_{i\nu}} - n_{i\nu} \frac{\partial}{\partial n_{i\mu}} \right)$$

$$1 \leq \mu < \nu \leq M$$

Generators of rotations in $\mu\nu$ -plane of rotor *i*

g: generates quantum fluctuations

$$P(J_{ij}) \sim \exp(-J_{ij}^2/(2J^2))$$

Gaussian all-to-all couplings

Self-consistent single-site problem

Path integral representation in Matsubara time.
Replicate *n* times, disorder average.
Take a saddle point, assuming *O(M)* invariance of saddle-point

$$\frac{1}{N}\sum_i \langle \hat{n}^a_{i\alpha}(\tau)\hat{n}^b_{i\beta}(\tau')\rangle = Q^{ab}_{\alpha\beta}(\tau,\tau')$$
$$= \frac{\delta_{\alpha\beta}}{M} Q^{ab}(\tau-\tau')$$

→ $$Z_0 = \int \mathcal{D}\hat{n}^a(\tau)\delta\left(\hat{n}^{a2}(\tau)-1\right)\exp\left(-\frac{M}{2g}\int_0^\beta d\tau(\partial_\tau\hat{n}^a)^2 + \frac{MJ^2}{2}\int_0^\beta d\tau d\tau' Q^{ab}(\tau-\tau')\hat{n}^a(\tau)\cdot\hat{n}^b(\tau')\right)$$

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$$Q^{ab}(\tau-\tau') = \langle \hat{n}^a(\tau)\cdot\hat{n}^b(\tau')\rangle_{Z_0}$$

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$$Q(i\omega_n) = g\left(\omega_n^2 + \lambda - gJ^2 Q(i\omega_n)\right)^{-1}$$

RHS: Inverse of the replica (n x n) matrix!

Paramagnetic phase

Paramagnetic phase: $Q^{a\neq b} = 0$

Solve quadratic equation for $Q^{aa}(i\omega)$: $Q^{aa}(\omega = i\omega_n) = \dfrac{\omega_n^2 + \lambda - \sqrt{(\omega_n^2+\lambda)^2 - (2gJ)^2}}{2gJ^2}$

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$$\chi(\omega) = \frac{1}{N}\sum_{i=1}^{N}\sum_{\alpha=1}^{M}\int_0^\infty dt\chi_{ii}^{\alpha\alpha}(t)e^{i(\omega+i\eta)t}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{\alpha=1}^{M}\int_0^\infty dt\langle -i[\hat{n}_i^\alpha(t), \hat{n}_i^\alpha(0)]\rangle e^{i(\omega+i\eta)t}$$

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Insert a sum over eigenstates ψ_m. Take imaginary part and obtain the spectral function (here at *T* = *0*)

$$\mathrm{Im}[\chi(\omega)] \equiv \chi''(\omega) = \frac{\pi}{N}\sum_i\sum_m |\langle\psi_m|\hat{n}_i^\alpha|\psi_0\rangle|^2\left[\delta(\omega - E_m + E_0) - \delta(\omega + E_m - E_0)\right] = \pi\rho(\omega),$$

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Information about strength and energy of excitations created by acting with $\hat{n}_i^\alpha$!

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$$\chi''(\omega) = \mathrm{sgn}(\omega) \frac{[(\omega^2 - \lambda + 2Jg)(\lambda + 2Jg - \omega^2)]^{1/2}}{2J^2 g}$$

for $\lambda - 2Jg < \omega^2 < \lambda + 2Jg$

$\chi'' = 0$ elsewhere

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Determine $\lambda(g,T)$ from equal time constraint $\hat{n}^{a2} = 1$ $\leftrightarrow$ $\displaystyle\int_0^\infty \frac{d\omega}{\pi}\chi''(\omega)\coth(\beta\omega/2) = 1$

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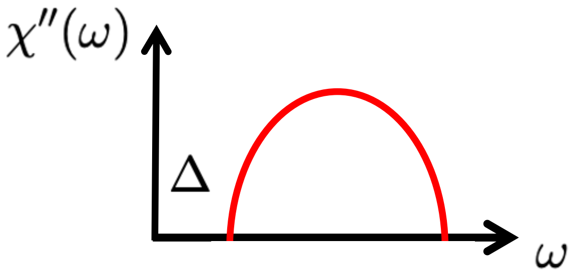


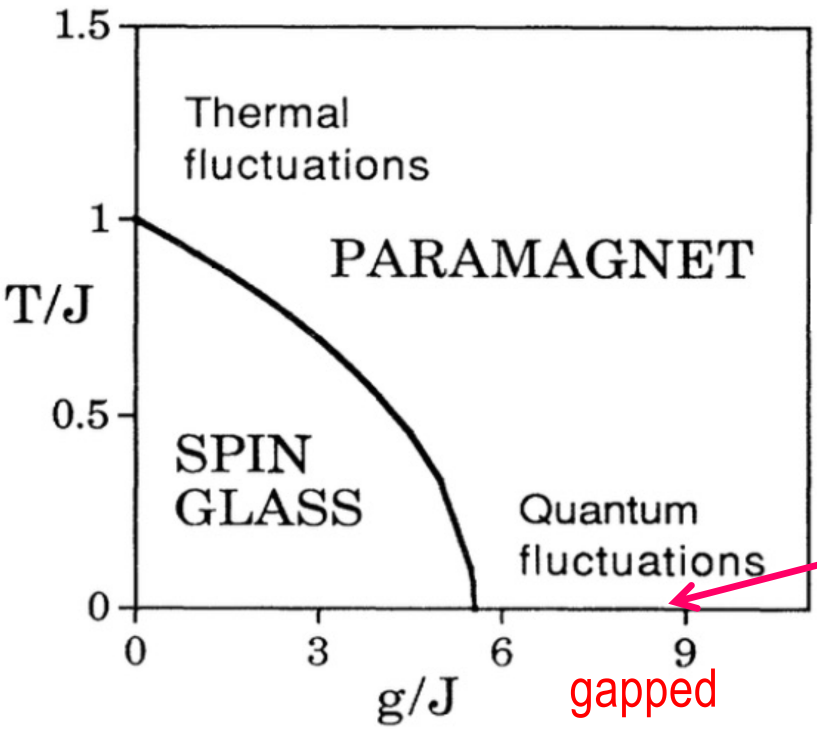
Spectral gap:

$$\Delta = \sqrt{\lambda - 2Jg} \geq 0$$

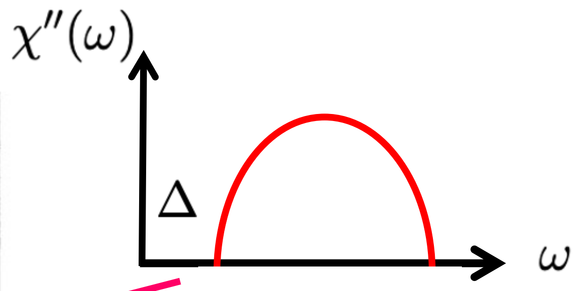
Determine $\lambda(g,T)$ from equal time constraint $\hat{n}^{a2} = 1 \leftrightarrow \int_0^\infty \frac{d\omega}{\pi}\chi''(\omega)\coth(\beta\omega/2) = 1$

Phase diagram

J. Ye, S. Sachdev, and N. Read, PRL 1993



Mean field quantum rotor model with $M = \infty$ components.



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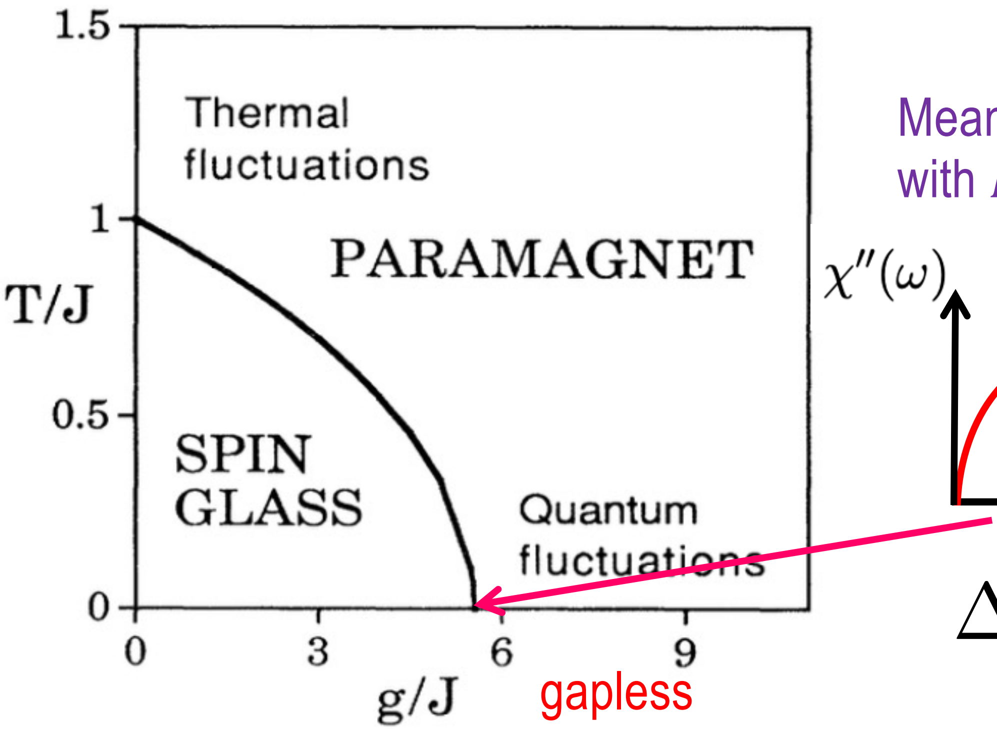

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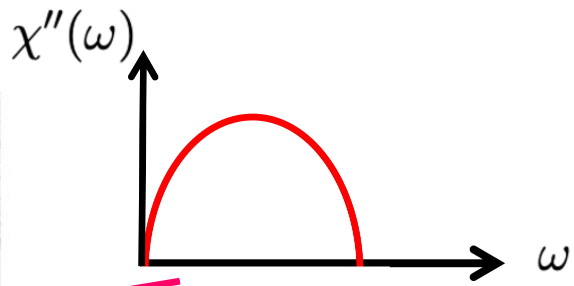

$\Delta \rightarrow 0$ Quantum glass transition!

Phase diagram

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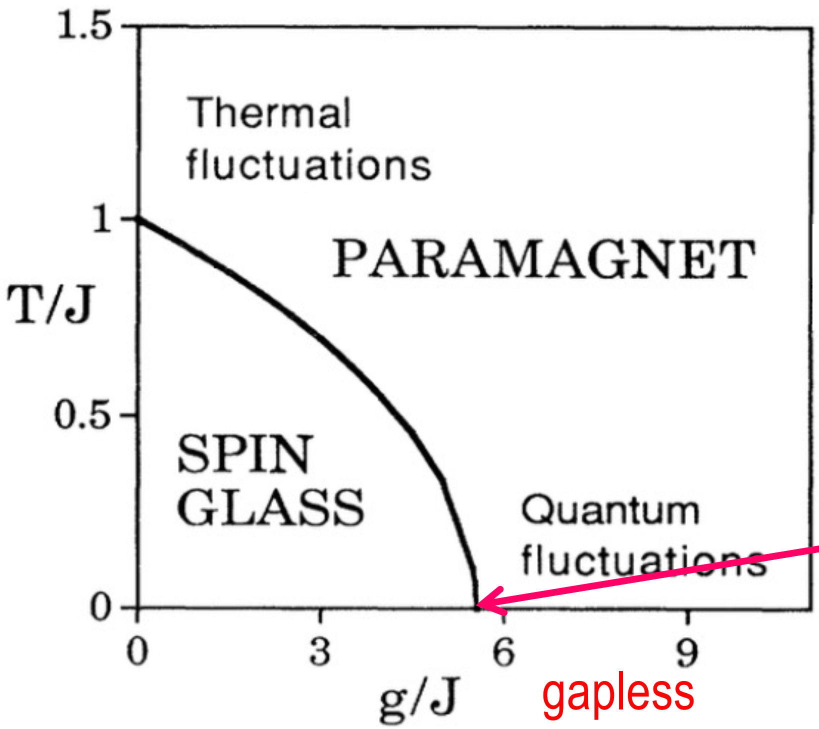

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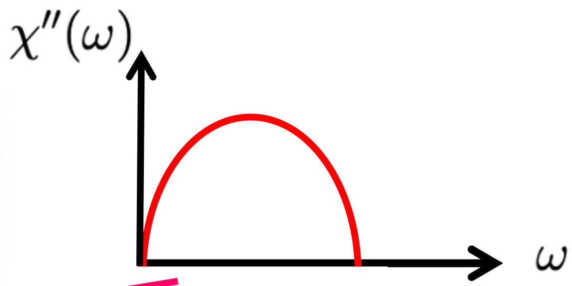

$\Delta \to 0$ Quantum glass transition!

What happens in the glass phase?

Glass phase

Glass phase: $Q^{\mathrm{a}\neq b} = q_{EA}$

Find: off-diagonal is constant (replica symmetric, no RSB)
(peculiarity of large *M* limit – similar to *p* = *2* spherical spins)

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In Matsubara space:

$$Q^{ab}(\omega_n) = \beta q_{\mathrm{EA}} \delta_{\omega_n,0} + \delta^{ab} Q_{\mathrm{reg}}(\omega_n)$$

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Q_{reg} at finite frequency satisfies the same equation as Q^{aa} in the paramagnetic phase

$$\chi''_{\mathrm{reg}}(\omega) = \mathrm{sgn}(\omega) \frac{\left[(\omega^2 - \lambda + 2Jg)(\lambda + 2Jg - \omega^2)\right]^{1/2}}{2J^2 g}$$

but now with different λ!

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\+ Self-consistency of replica – off-diagonal part imposes marginal stability → spectral gap remains closed! → $\Delta = \sqrt{\lambda - 2Jg} \overset{!}{=} 0$

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$$\chi''_{\rm reg}(\omega) = {\rm sgn}(\omega)\frac{\left[(\omega^2-\lambda+2Jg)(\lambda+2Jg-\omega^2)\right]^{1/2}}{2J^2 g}$$

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\+ Equal time constraint $\hat{n}^{a2} = 1$ ↔ $q_{\rm EA} + \int_0^\infty \frac{d\omega}{\pi}\chi''_{\rm reg}(\omega)\coth(\beta\omega/2) = 1$

Marginality condition from replica saddle point

$$Q^{ab} = A \cdot \begin{pmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{pmatrix} + B \cdot \begin{pmatrix} 1 & 1 & ... & 1 \\ 1 & 1 & ... & 1 \\ ... & ... & ... & ... \\ 1 & 1 & ... & 1 \end{pmatrix}$$

$$Q^{ab}(i\omega_n = 0) \equiv Q_0^{ab}$$

$$A = Q_{\rm reg}(i\omega_n = 0) \equiv Q_{\rm reg,0}$$
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Saddle point (matrix equation!) : $f(Q_0) \equiv Q_0(\lambda - gJ^2 Q_0) - g = 0$

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General replica matrix algebra:

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 1 \end{pmatrix}^2 = n \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 1 \end{pmatrix} \to 0$$

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$\to f(Q_{\mathrm{reg},0}) = f'(Q_{\mathrm{reg},0}) = 0$

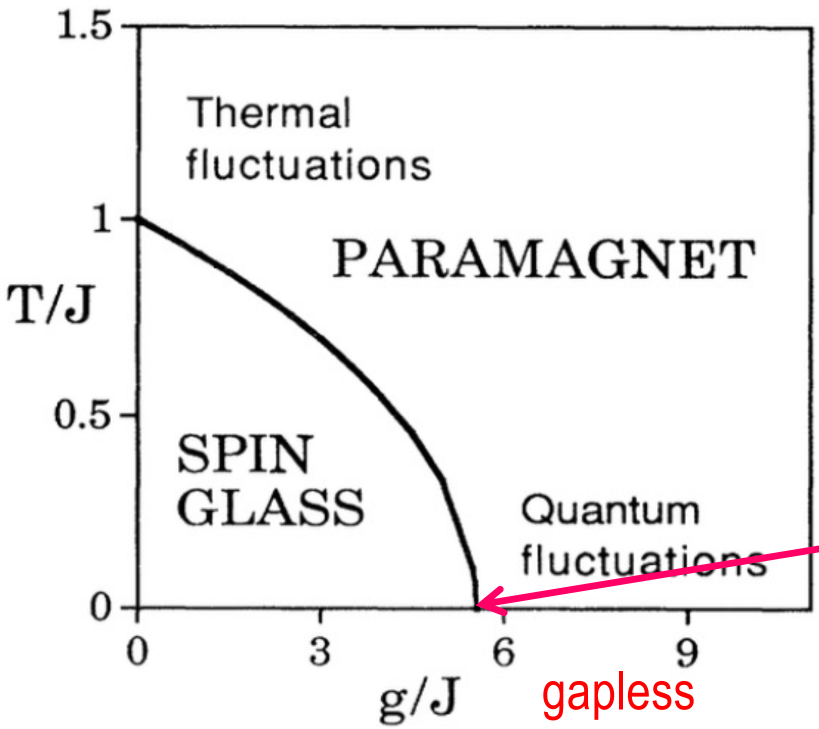
$Q_{\mathrm{reg},0}$ is a double zero of $f(Q_0)$ (→ critical!)

→ $Q_{\mathrm{reg}}(\omega = 0^+)$ immediately has imaginary part

→ finite spectral weight → gapless state

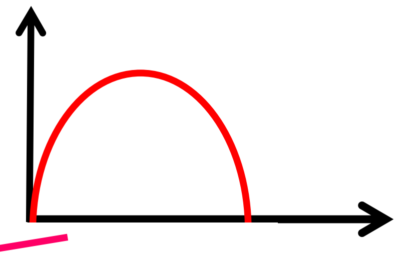
Phase diagram

J. Ye, S. Sachdev, and N. Read, PRL 1993



gapless

Mean field quantum rotor model with $M = \infty$ components.

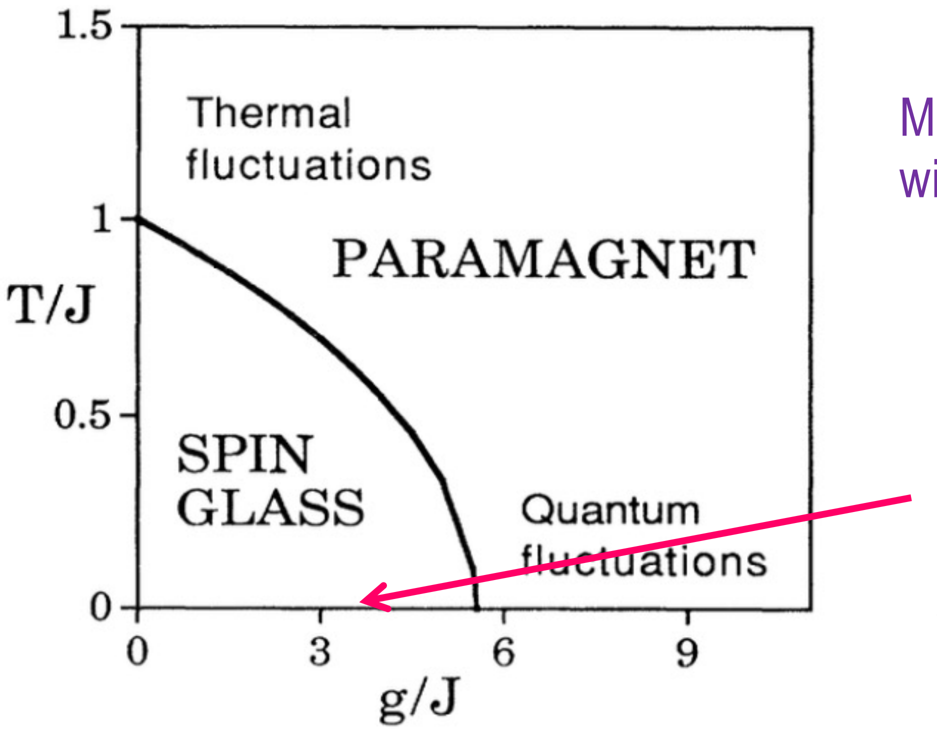


$\Delta \to 0$ Quantum glass transition!

What happens in the glass phase?

Phase diagram

J. Ye, S. Sachdev, and N. Read, PRL 1993



Glass remains gapless : Marginal stability!

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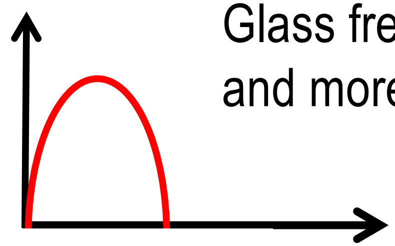

Glass freezes more and more

$$\Delta = 0$$

Phase diagram

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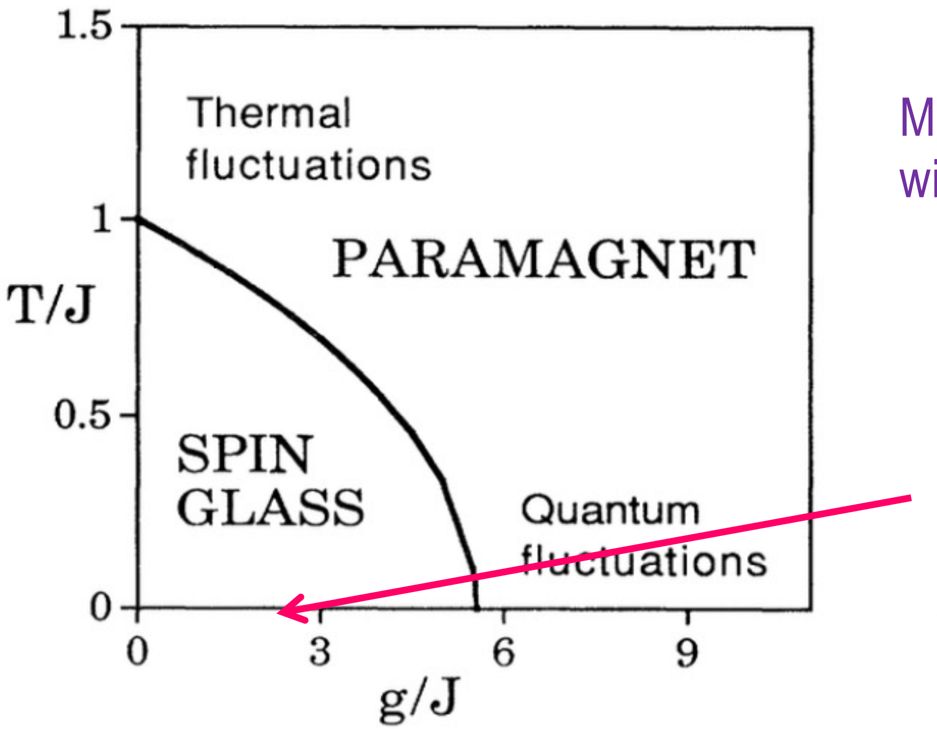

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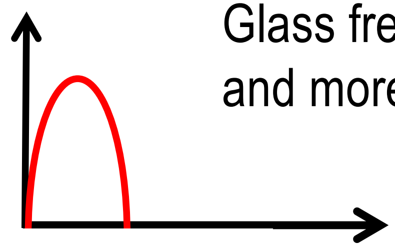

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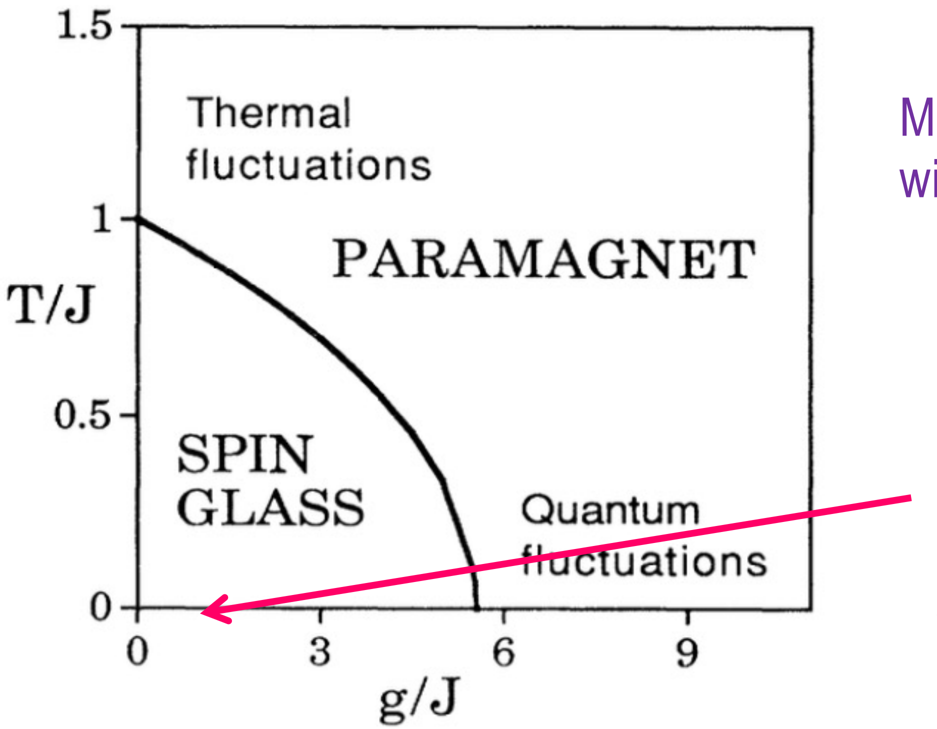

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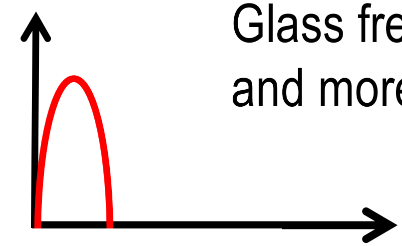

Glass freezes more and more

$$\Delta = 0$$

Glass remains gapless : Reflects marginal stability of replica saddle point and landscape.

Beyond rotors: real spins?

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Solved mean field models:

- Transverse field Ising glass (SK model)

$$H_{\text{Ising}} = -\Gamma \sum_i s_i^x + \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} s_i^z s_j^z$$

↕

- Heisenberg glass:

$$H_{\text{Hb}} = \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Difference:

Ising: all interaction terms commute

Heisenberg: interactions do not commute

How much does this matter?

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TFSK model, N =17 (exact diagonalization)

Spectral function at T = 0

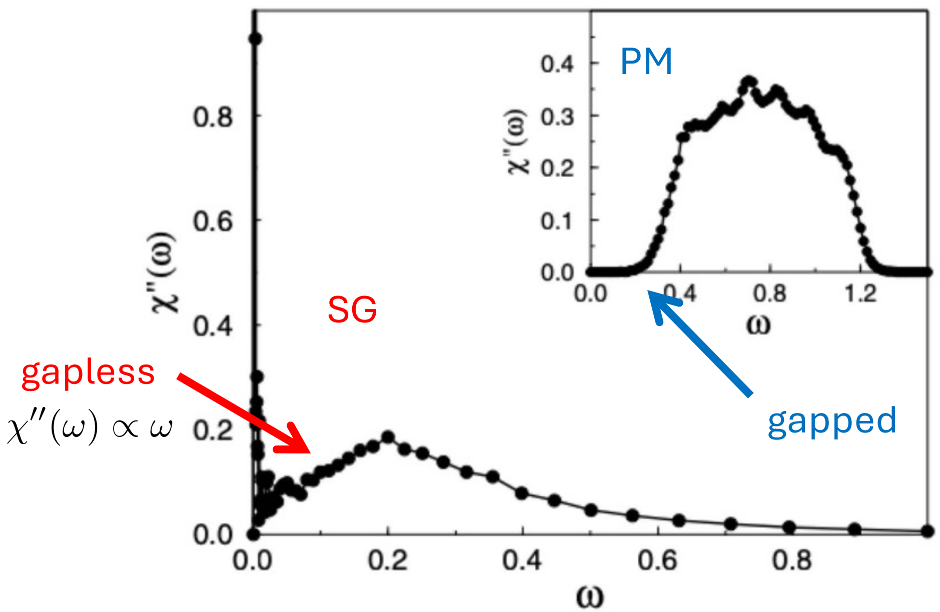


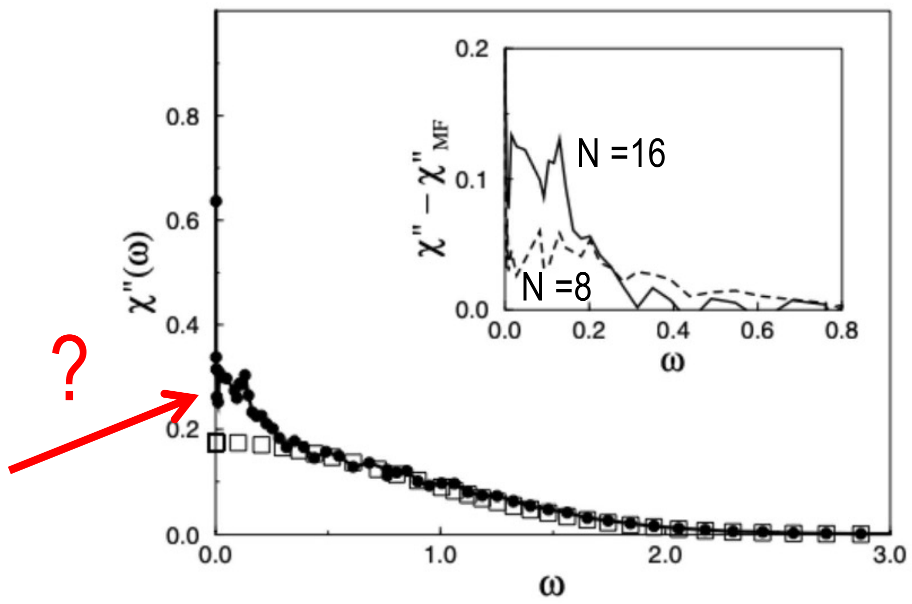
FIG. 1. The dynamical response $\chi''(\omega)$ of the RITF model at $\Gamma = 0.2$ for $N = 17$. The spectral function shows a $\delta(\omega)$ part plus a regular contribution at finite frequencies with a maximum at $\omega \approx \Gamma$. Inset: gapped $\chi''(\omega)$ in the paramagnetic phase at $\Gamma = 0.8 > \Gamma_c$.

L. Arrachea and M. J. Rozenberg '01

$$\chi''(\omega) \propto \omega/J^2$$

Heisenberg model, N =16 (exact diagonalization)

Spectral function at $T = 0$

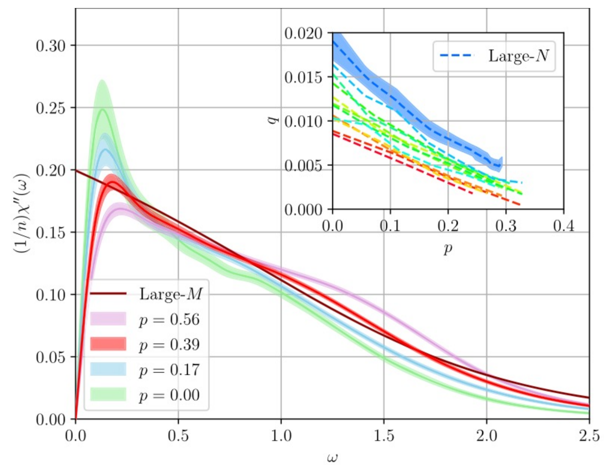


L. Arrachea and M. J. Rozenberg '01

$\chi''(\omega) \propto \mathrm{sign}(\omega)$?? as in SYK ??

Unclear!
Finite-size broadened $\delta(\omega)$
blurs low frequency behavior

$$H = \frac{1}{\sqrt{N}} \sum_{i \neq j=1}^{N} t_{ij} P c_{i\alpha}^{\dagger} c_{j\alpha} P + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



Shackleton, Wietek, Georges, Sachdev '21

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- Heisenberg glass:

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Numerics suggested a significant difference

but the truth is different!

Beyond rotors: real spins?

A. Andreanov, MM ‘11
A. Kiss, G.Zarand, I. Lovas, ‘24

Solved mean field models:

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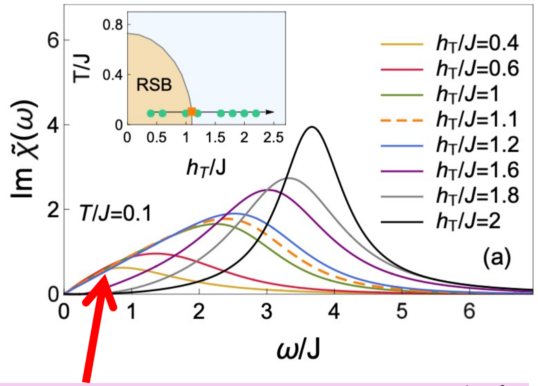
Same universality class as rotors in the $M \to 1$ limit
But: full continuous RSB:
Many states, all marginal, gapless



$$\chi''_{\rm Ising}(\omega) \approx 0.5 \frac{\omega}{J^2}$$

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  - SU(2) spins (Kavokine, MM, Parcollet Georges, '24)

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

SY-pre-K model(s)

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Different representations of SU(M) = different models / loc Hilbert space

	Schwinger bosons		Abrikosov fermions
	$S_{\alpha\beta} = b^\dagger_\alpha b_\beta - S\,\delta_{\alpha\beta}$	$\leftrightarrow$	$S_{\alpha\beta} = f^\dagger_\alpha f_\beta$
+ Constraint:	$\Sigma_\alpha b^\dagger_\alpha b_\alpha = SM \quad (0 \leqslant S)$		$\Sigma_\alpha f^\dagger_\alpha f_\alpha = q_0 M\,(0 \leqslant q_0 \leqslant 1)$

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High T: Famous SY(-K) physics (partons are no quasiparticles) $\chi''(\omega) \propto \mathrm{sign}(\omega)$

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Low T: Bosons condense discontinuously, like p=4 spins (structural-glass like),
in threshold states: $\chi''(\omega) \propto \omega / J^2$

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Low T: Fermions require finite M to order: continuous spin-glass transition $T_g \sim \exp[-c\sqrt{M}]$

Full RSB - but again: $\chi''(\omega) \propto \omega / J^2$

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RSB solution + continuous time QMC

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Again:

$$\chi''_{\rm Hb}(\omega) \approx 3.5 \frac{\omega}{J^2} \quad \leftrightarrow \quad \chi''_{\rm Ising}(\omega) \approx 0.5 \frac{\omega}{J^2}$$

Heisenberg glasses have lower T_c and more soft excitation spectrum than Ising systems with the same coupling. But the spectral density has the same linear frequency scaling.

Interpretation?

A. Andreanov, MM ‘11
L. Cugliandolo, MM ’23 (Review on Quantum glasses)

Quantum SK model

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Physical interpretation: [applies to ALL insulating meanfield glasses]

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Marginally stable energy landscape $G(\{m_i\})$
Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

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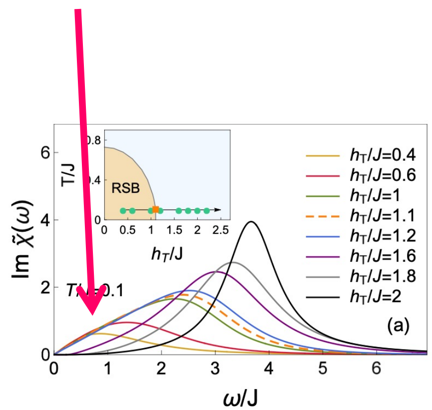

Gapless spectral function

$$\chi''(\omega) \sim x_\omega^2 \rho(\omega) \sim \frac{\Gamma}{\omega} \frac{\omega^2}{\Gamma J^2} \sim \frac{\omega}{J^2}$$

Non-trivial check: Independent of transverse field Γ!

Quantum glasses beyond mean field?

L. Vitteriti, ..., G. Carleo, A. Scardicchio,
arXiv:2507.05073

Promising prospect: Numerics on Heisenberg glass (spin 1/2) in 2d

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Promising prospect: Numerics on Heisenberg glass (spin 1/2) in 2d

Neural network variational wavefunctions that are adapted to arbitrary disorder
→ Efficient numerics

Results:

- There **is** a glass at $T = 0$, despite strong quantum fluctuations: Random ordering of spins
- Large S analysis allows to study low frequency spectrum and spatial mode properties (localization of spin waves)

Back to mean field: Metallic glasses

Is there any escape from the super-universal spectral function? $\chi''(\omega) \propto \omega / J^2$

Back to mean field: Metallic glasses

Is there any escape from the super-universal spectral function? $\chi''(\omega) \propto \omega/J^2$

Yes: If the spins interact with a gapless bath (e.g. conduction electrons)
$\rightarrow$ The collective oscillators (landscape normal modes) are overdamped

$\rightarrow$ yet slower modes
$\rightarrow$ more spectral weight at low frequency, $\chi''(\omega) \propto \omega^\alpha$ $\alpha = 0.5$

Coupling to an Ohmic bath

Sengupta, Georges; Read, Sachdev;

or even $\alpha = 0$

Glass in a doped Mott insulator

Kavokine et al., Sachdev et al

Interplay of glassiness and localization

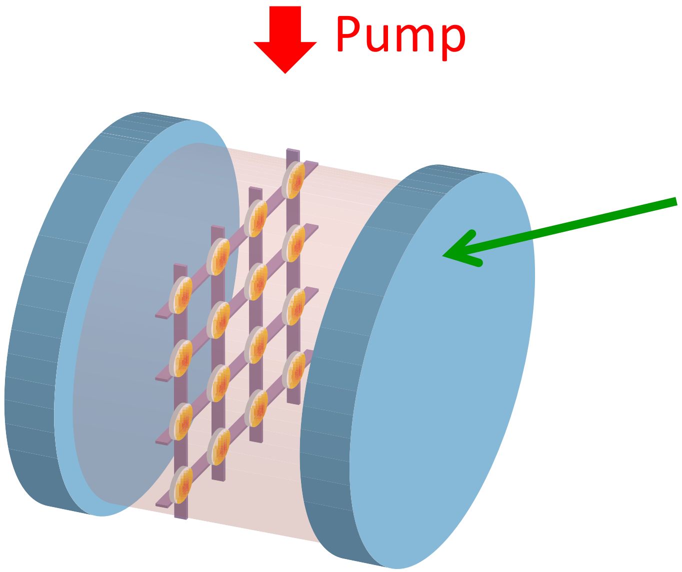
Long range frustrated quantum glasses?

Infinite range quantum glasses = a theorists' toy fantasy?

Interesting case: Long range glass with short range hopping: the quantum Fermi glass in optical cavities

MM, P. Strack, S. Sachdev '12

Lattice fermions in laser cavity



Three building blocks:

1) Fermionic atoms in optical lattice
2) Laser cavity with multiple photon modes
3) Classical pump laser, driving transitions between fermion ground and excited state (sufficiently off resonance)

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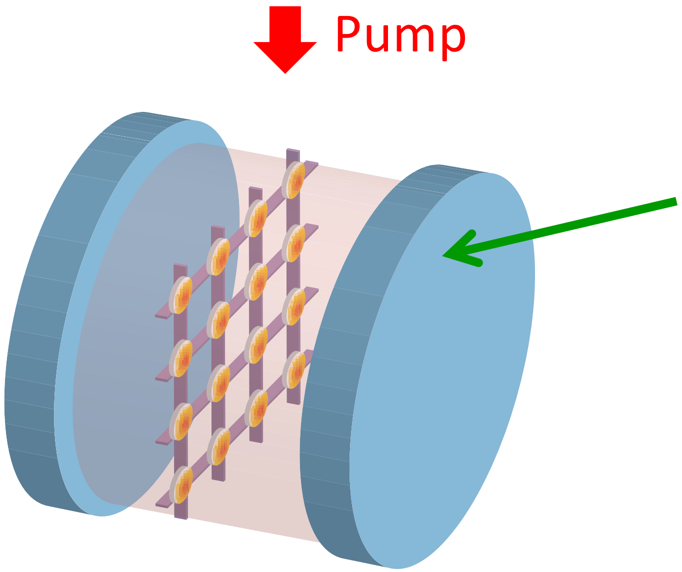

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Integrating out pump and cavity photons:

$$H\left[c^{\dagger},c\right] = -t\sum_{\langle i,j\rangle}\left(c_i^{\dagger}c_j + h.c.\right) + \sum_{i=1}^{N}(\varepsilon_i-\mu)\,n_i - \frac{1}{2}\sum_{i,j=1}^{N} V_{ij}n_in_j$$

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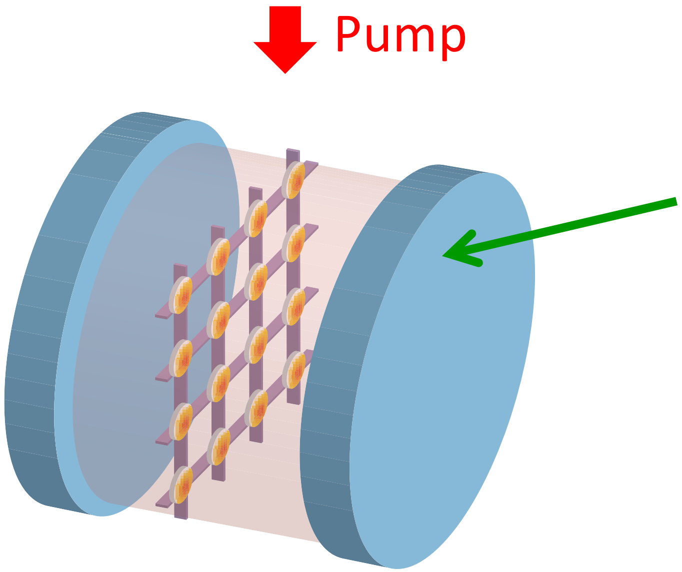

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Random couplings

$$V_{ij}(\Omega) = 2\sum_{\ell=1}^{M} \frac{g_{i\ell} g_{j\ell} h_i h_j}{\Delta^2} \frac{\omega_\ell}{\Omega^2 + \omega_\ell^2}$$

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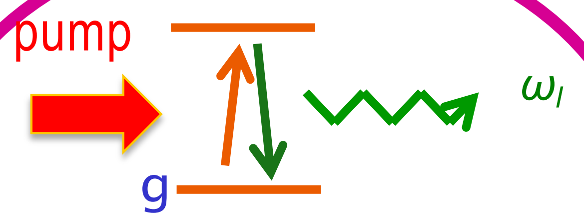
Pumped optical cavities create mean field quantum Fermi glasses



$$\sum_{\ell=1}^{M} \frac{g_{i\ell} h_i}{\Delta} n_{i,g}(a_\ell + a_\ell^\dagger)$$

$$H_{f-ph} = -t \sum_{\langle i,j \rangle} \left(c_i^\dagger c_j + h.c. \right) + \sum_{i=1}^{N} (\epsilon_i - \mu) n_i + \sum_{\ell=1}^{M} \omega_\ell a_\ell^\dagger a_\ell + \sum_{i=1}^{N} \sum_{\ell=1}^{M} \frac{g_{i\ell} h_i}{\Delta} n_i \left(a_\ell + a_\ell^\dagger \right)$$

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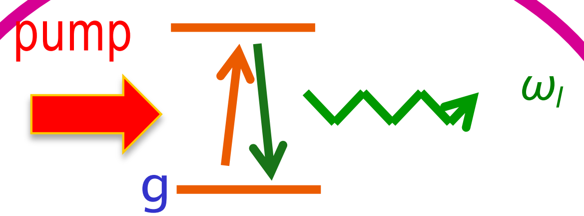

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Integrating out the cavity photons:

$$\rightarrow \quad H\left[c^\dagger, c\right] = -t \sum_{\langle i,j \rangle} \left(c_i^\dagger c_j + h.c. \right) + \sum_{i=1}^{N} (\varepsilon_i - \mu) n_i - \frac{1}{2} \sum_{i,j=1}^{N} V_{ij} n_i n_j$$

Long range, frustrated interactions + short range hopping!

$$V_{ij}(\Omega) = 2 \sum_{\ell=1}^{M} \frac{g_{i\ell} g_{j\ell} h_i h_j}{\Delta^2} \frac{\omega_\ell}{\Omega^2 + \omega_\ell^2}$$

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Interesting case: Long range glass with short range hopping: the quantum Fermi glass in optical cavities

MM, P. Strack, S. Sachdev '12

Basic mechanisms:

can be made weak

$$H\left[c^{\dagger},c\right] = -t\sum_{\langle i,j\rangle}\left(c_i^{\dagger}c_j + h.c.\right) + \sum_{i=1}^{N}(\varepsilon_i-\mu)\,n_i - \frac{1}{2}\sum_{i,j=1}^{N}V_{ij}n_in_j$$

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Short range hopping ↔ Long range interaction

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Glassy density order $\longrightarrow$ effective, selfgenerated disorder potential

$\longrightarrow$ possibly Anderson localization of single fermion modes

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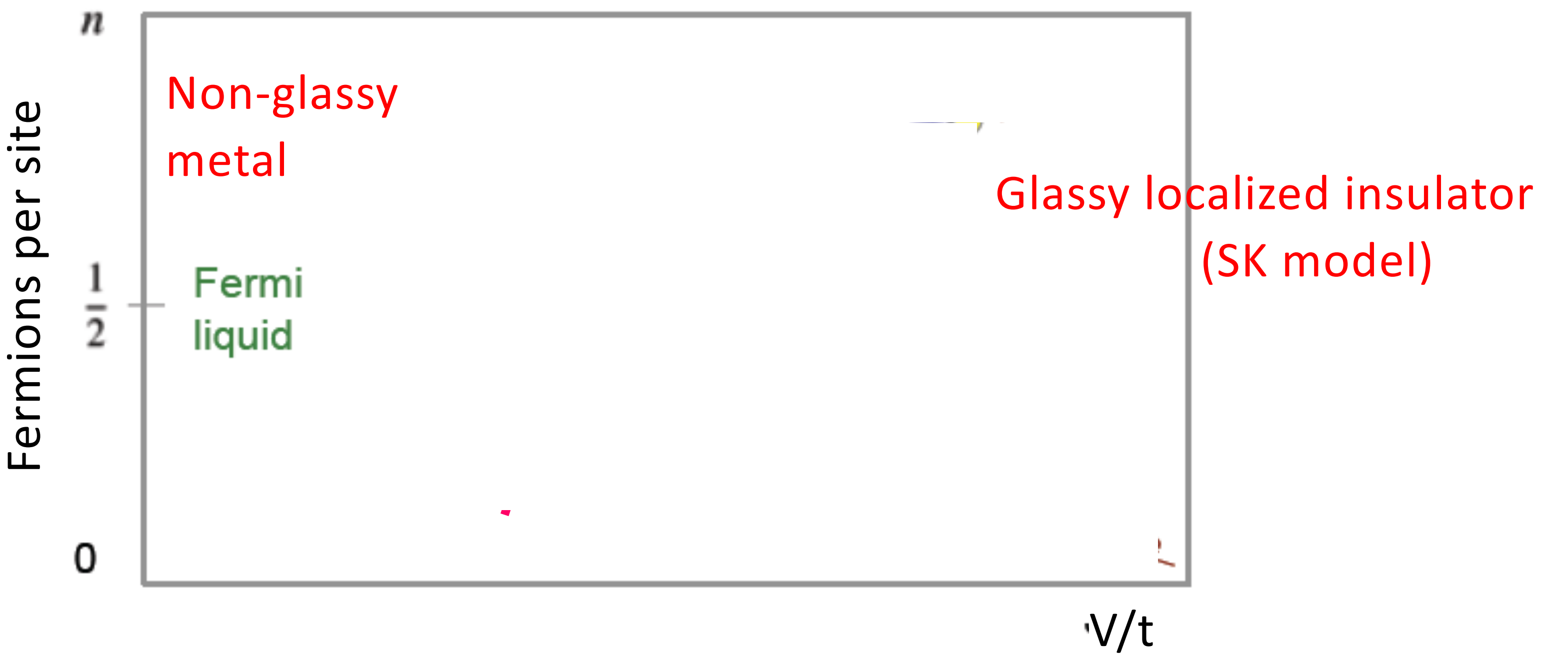
Phase diagram:



Interesting case: Long range glass with short range hopping: the quantum Fermi glass in optical cavities

MM, P. Strack, S. Sachdev '12

Basic mechanisms:

can be made weak

$$H\left[c^{\dagger},c\right] = -t\sum_{\langle i,j\rangle}\left(c_i^{\dagger}c_j + h.c.\right) + \sum_{i=1}^{N}(\varepsilon_i - \mu)\,n_i - \frac{1}{2}\sum_{i,j=1}^{N} V_{ij}n_i n_j$$

Glassy density order → effective, selfgenerated disorder potential

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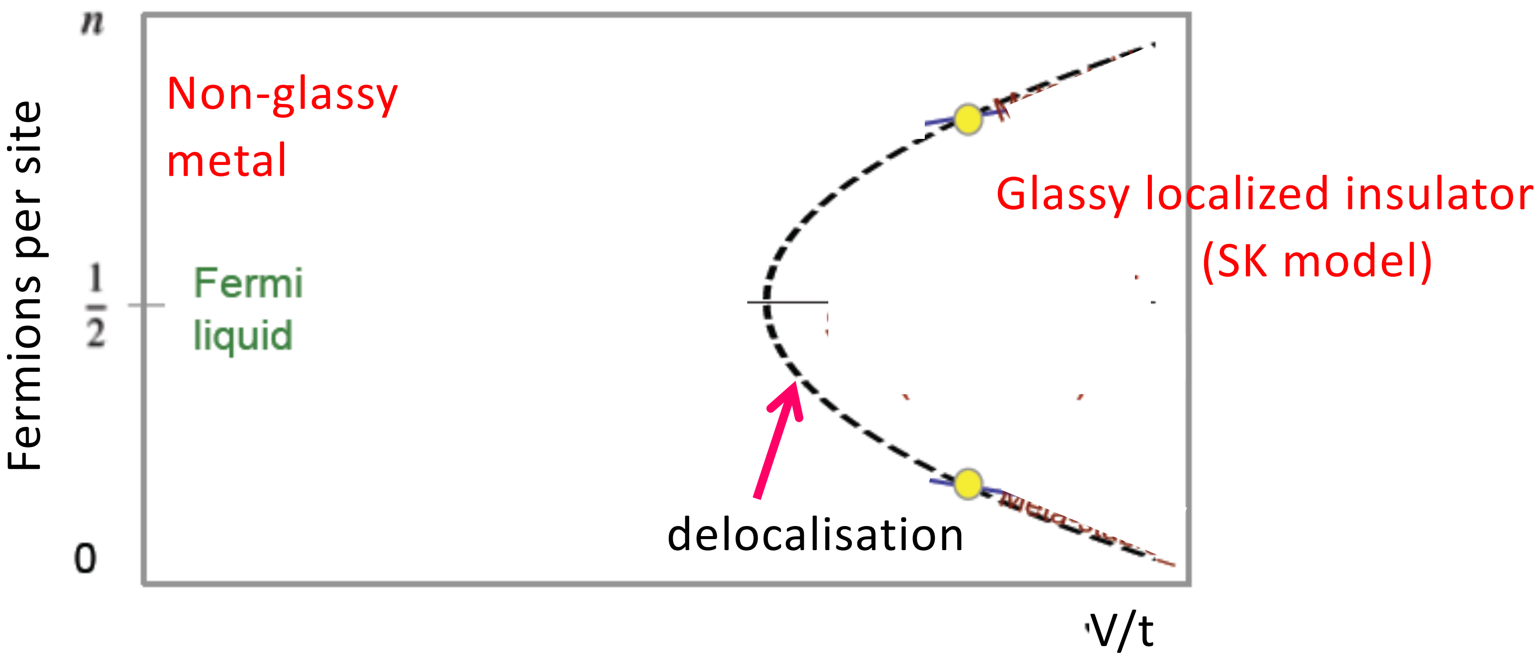

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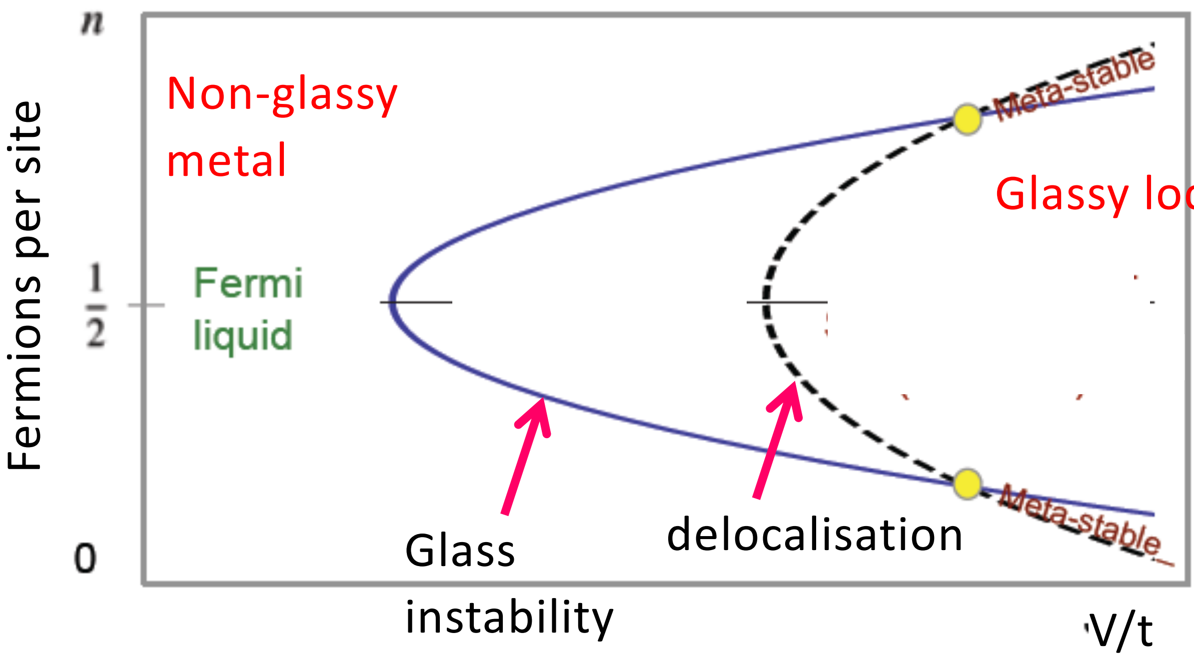

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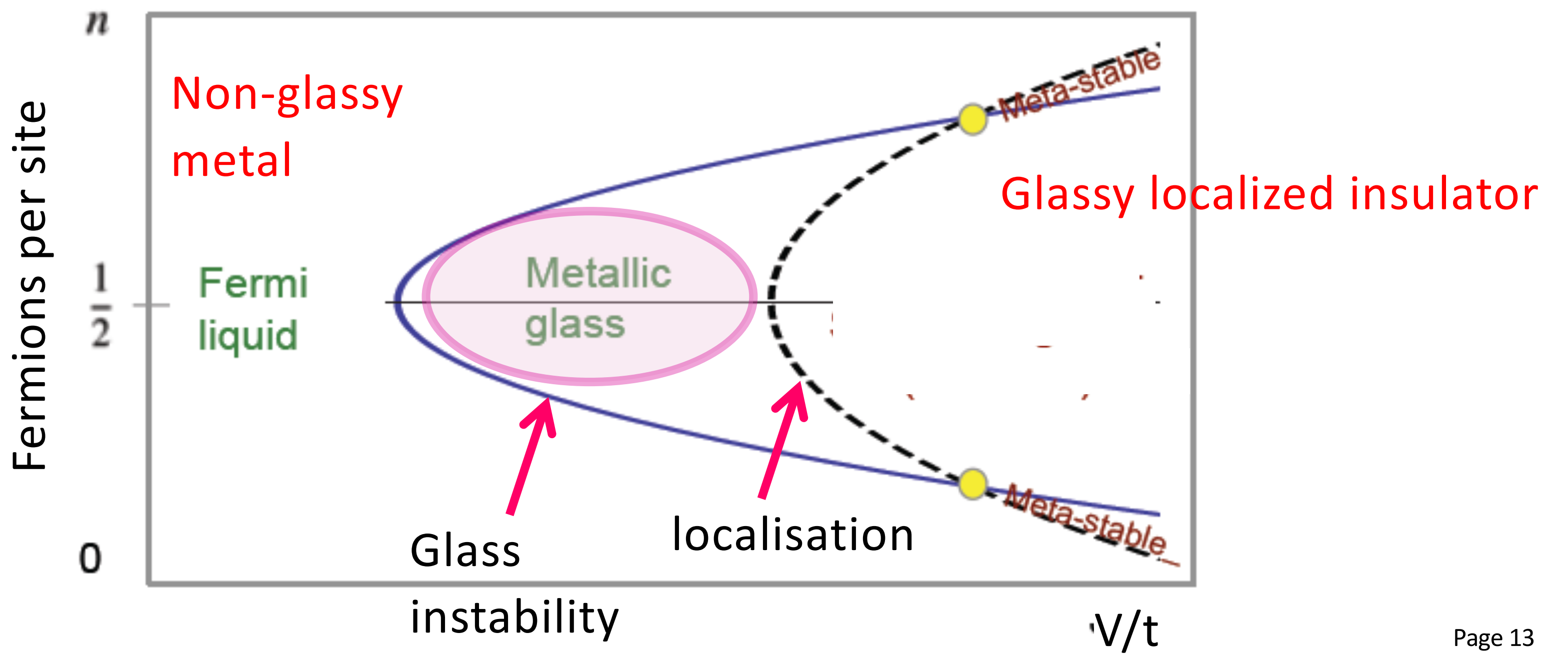
n ~ ½ → Intermediate phase: both glassy & delocalized!

Phase diagram:



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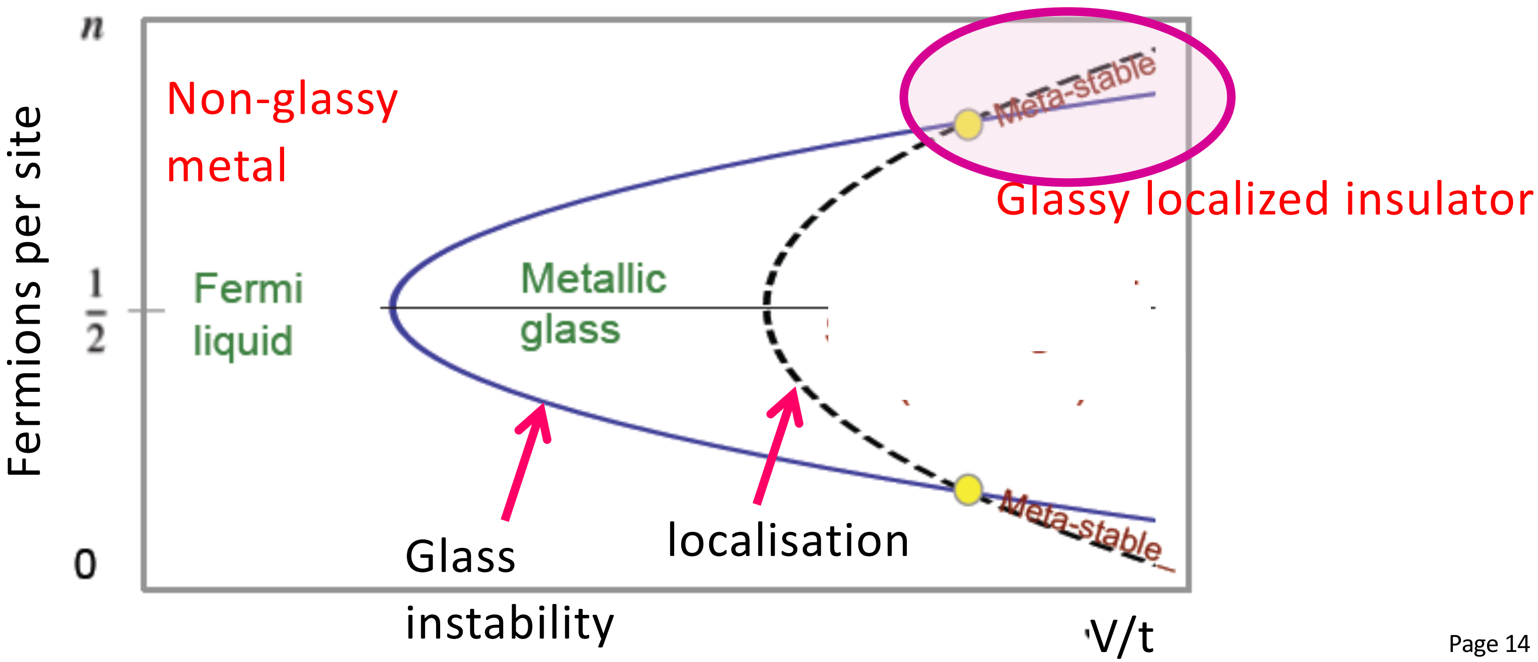
n → 0,1 → Instabilities cross: → 1st order transition, metastability!

Phase diagram:



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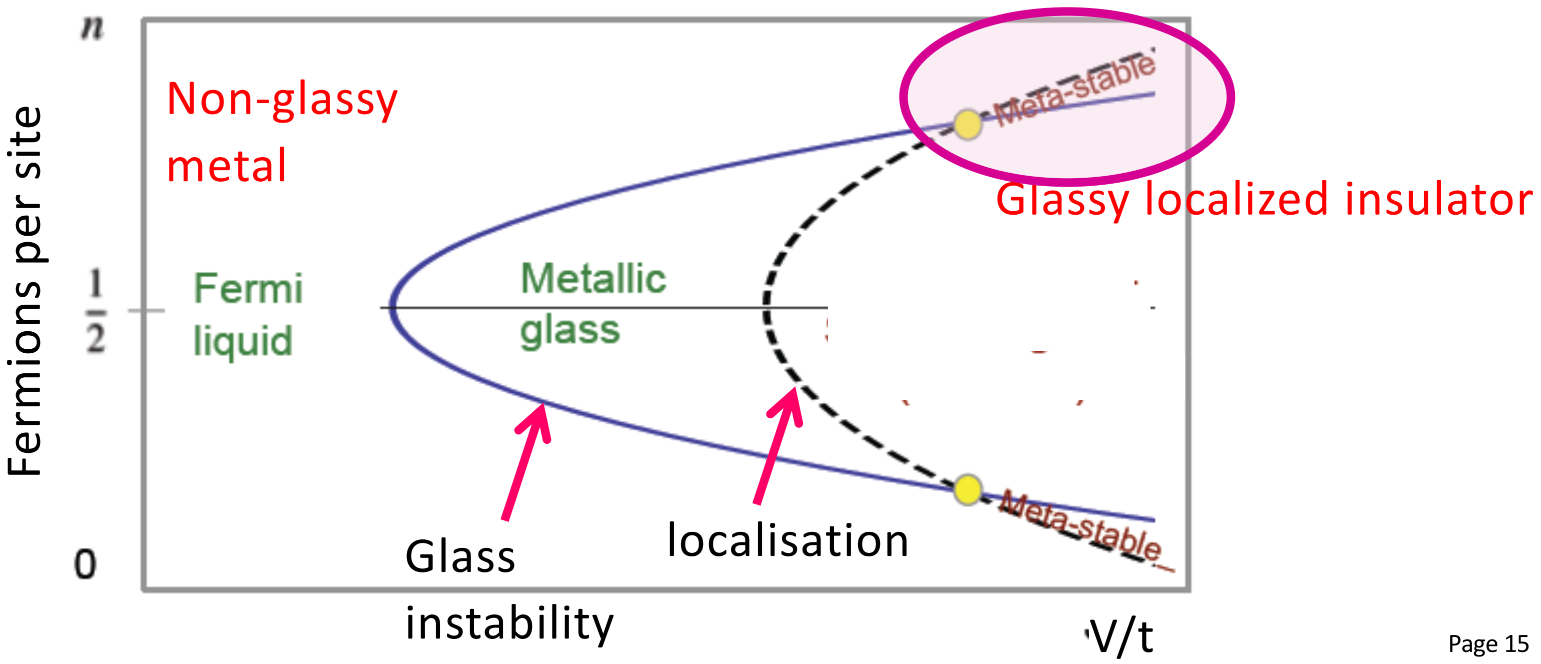
Dynamics across the transition? Nucleation of delocalised phase?

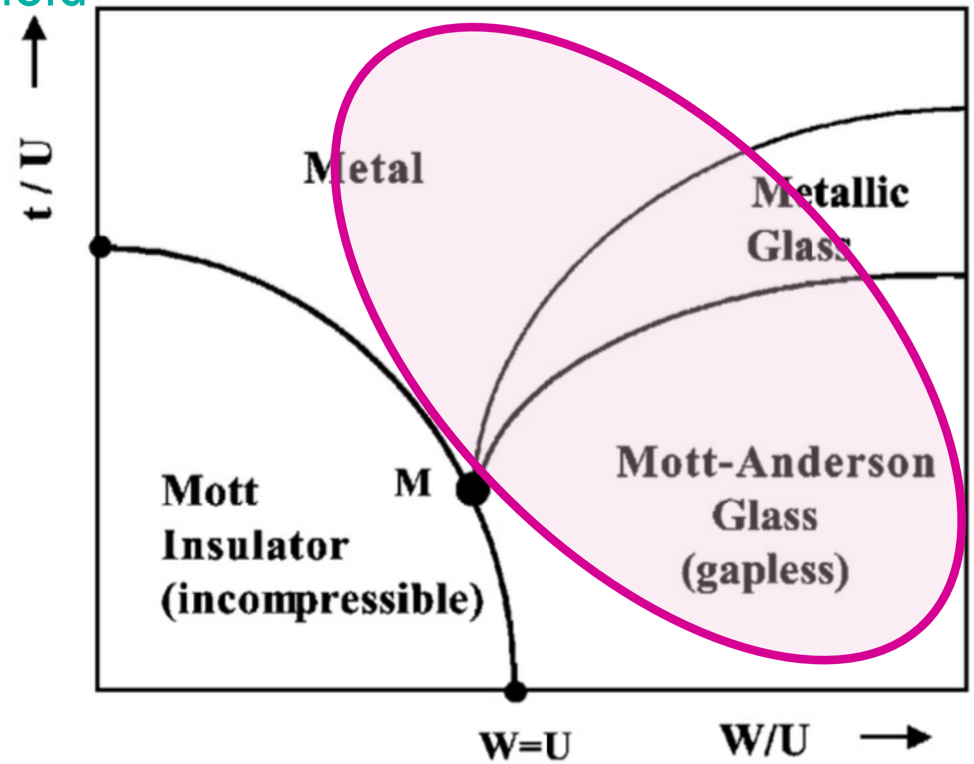
Phase diagram:



Adding on-site disorder: Perturbing the Anderson transition

Dobrosavljevic, Tanaskovic, Pastor '03

Analogous phase diagram proposed in electron glasses (d=2,3) within mean field



$$V_{ij} = \frac{e^2}{r_{ij}}$$

$$H_{\text{eff}} = -t \sum_{\langle i,j \rangle} \left(c_i^\dagger c_j + h.c. \right) + \sum_{i=1}^{N} (\varepsilon_i - \mu) n_i - \frac{1}{2} \sum_{i,j=1}^{N} V_{ij} n_i n_j$$