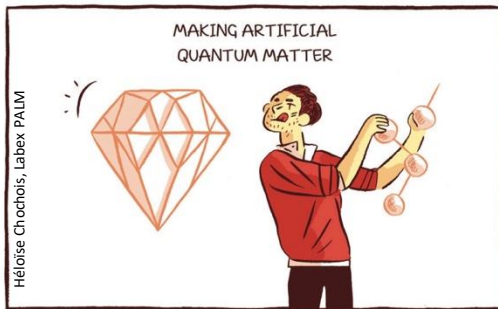


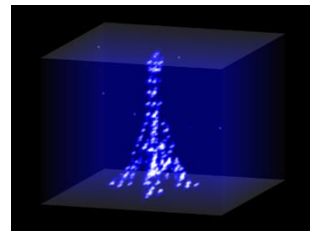
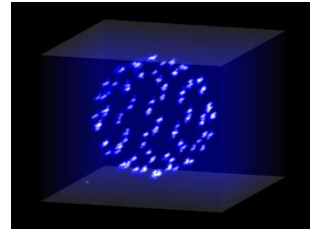
Many-body physics with arrays of Rydberg atoms (I)



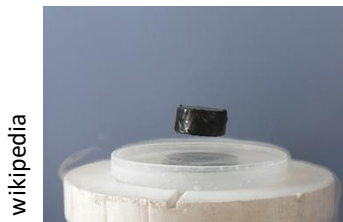
Antoine Browaeys

*Laboratoire Charles Fabry,
Institut d'Optique, CNRS, FRANCE*

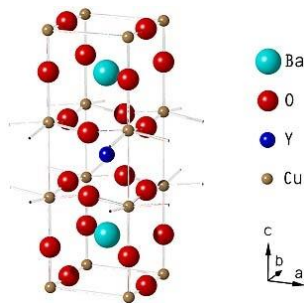
ICTP Summer School, august 21st, 2025



The many-body problem: the art of modelling...



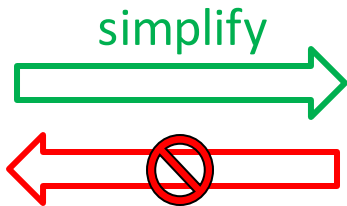
YBaCuO



Observe unexpected effects
Ex: high- T_c superconductivity

Microscopic understanding?

Experiment on
“real” system



Too hard to calculate...

Cook up a model

$$H_{\text{model}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$

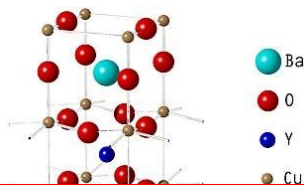
Problem: exponential complexity

2 d. of freedom (spin...) $\psi_i = \begin{pmatrix} a \\ b \end{pmatrix}$

Many-body wavefunction: $\Psi = \Psi(1, 2, \dots, N) \Rightarrow \Psi$ requires 2^N numbers

Record *ab-initio* calculation (2025) $N \sim 50 \Rightarrow 2^{50} \sim 10^{15} = 1000 \text{ Tb RAM} !!$

The many-body problem: the art of modelling...



Observe unexpected effects

Ex: high- T_c superconductivity

Approximations possible!!

mean-field, perturbation theory, Monte-Carlo,
variational methods: DFT, MPS, Neural Quantum States...

But... can be poorly controlled or not valid
when *interactions dominate*

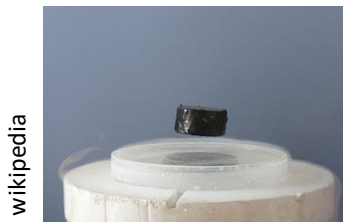
= **Strongly correlated** systems

$$\sum_i n_{i\downarrow} n_{i\uparrow}$$

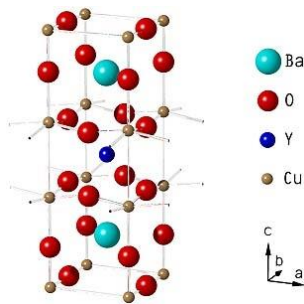
$$\begin{pmatrix} a \\ b \end{pmatrix}$$

Record *ab-initio* calculation (2025) $N \sim 50 \Rightarrow 2^{50} \sim 10^{15} = \text{1000 Tb RAM !!}$

One approach: build a synthetic many-body quantum systems



YBaCuO



Observe unexpected effects
Ex: high- T_c superconductivity

Microscopic understanding?

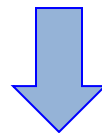
Experiment on
“real” system

simplify



Cook up a model

$$H_{\text{model}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$



Lab...

Measure on system:
Supercond. or not?



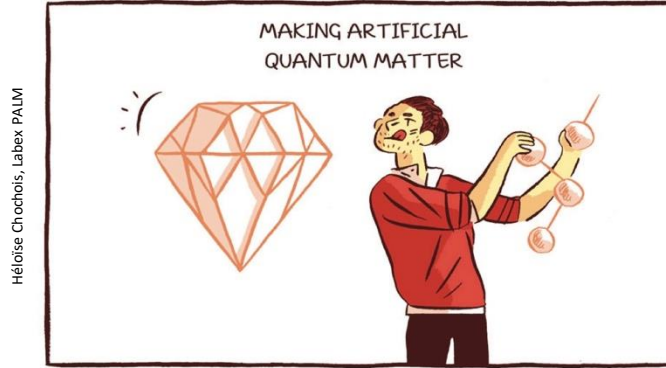
Engineer a system
ruled by H_{model}

Many-body physics with synthetic quantum systems



R.P. Feynman

Int. J. Theo. Phys. **21** (1982)



Quantum simulation

Georgescu, Rev. Mod. Phys. (2014)

Well-controlled quantum systems implementing **many-body Hamiltonians**
= quantum simulator

Larger tunability than “real” systems (geometry, interactions...)

+

New types of probe & methods (e.g. out-of-equilibrium)

A new way to look at many-body using quantum information concepts
(entanglement...)

Analog versus digital quantum simulation

Analog

The platform implements
directly H_{model}

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \int_0^t H_{\text{mod}}(t') dt'\right) |\psi(0)\rangle$$

e.g.: Fermi Hubbard, spin models,
electrons in B-fields...

Non-universal

Digital

H_{model} synthesized digitally

$$H_{\text{mod}} = \sum_{n=1}^N H_n$$

↑

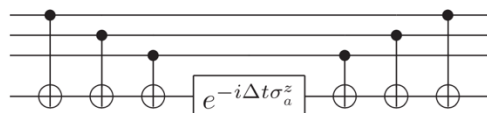
e.g. single & 2-qbit operations

$$e^{-iH_{\text{mod}}t} \approx$$

$$\left(e^{-iH_1t/n} e^{-iH_2t/n} \dots e^{-iH_3t/n}\right)^n$$

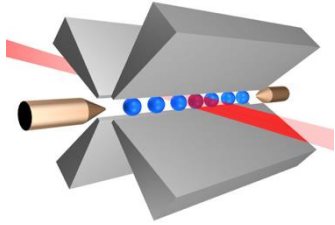
= “universal” quantum simulation

Ex:

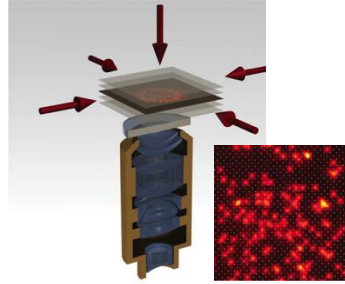


$$H_{\text{mod}} = \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$$

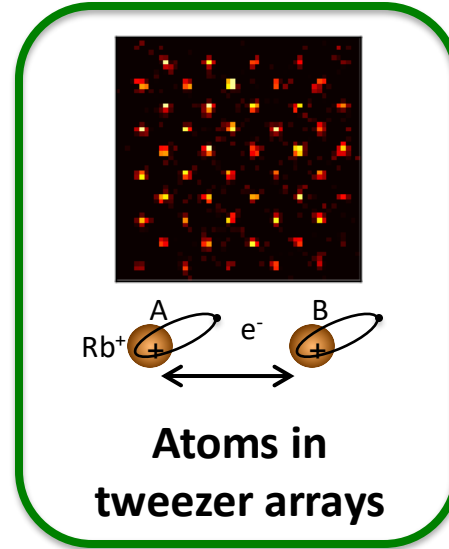
Engineering with individual quantum systems (examples)



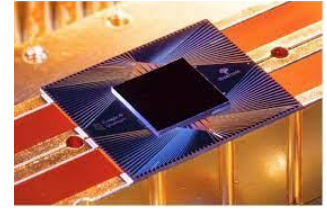
Trapped ions



Atoms in
optical lattices



Atoms in
tweezer arrays



Supercond.
Circuits
IBM, Google...

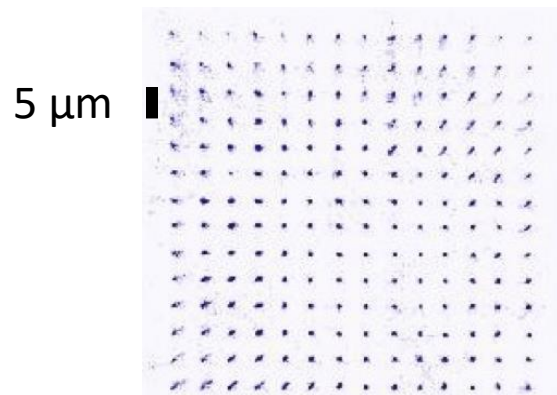
Scalable: beyond 100 particles ; potential > 1000

Addressability: local manipulations and measurement

$$\langle \sigma_i^\alpha \rangle, \langle \sigma_i^\alpha \sigma_j^\beta \rangle, \dots$$

Programmable: controlled geometry, interactions...

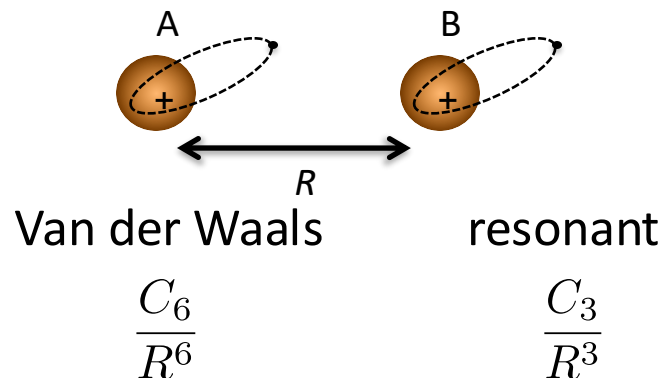
These lectures: combining arrays of atoms and Rydberg interactions



Addressable!!

+

Rydberg interactions



Quantum simulation (mainly spin models)

Quantum information processing

The program

Lecture 1: Arrays of atoms & “Rydbergology”
Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 2: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism

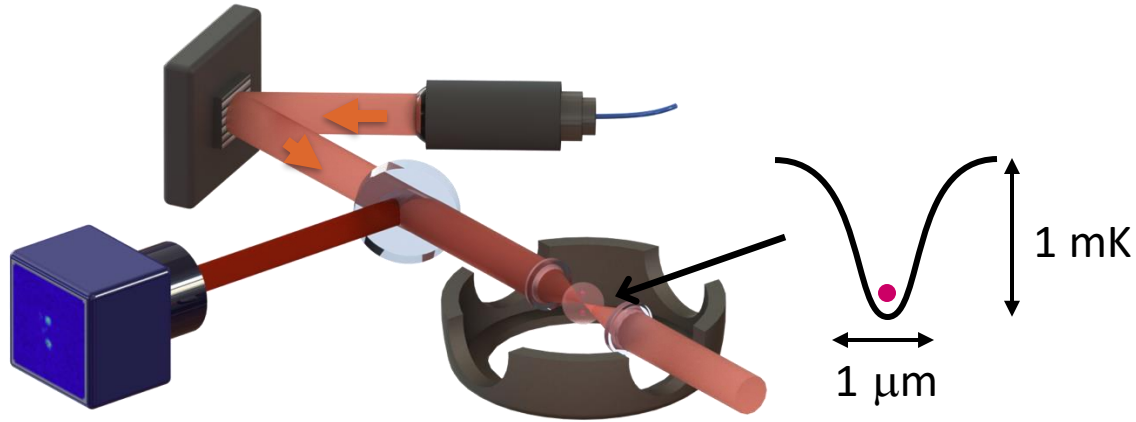
Outline – Lecture 1

1. Arrays of individual atoms in optical tweezers
2. Basics of Rydberg physics and their interaction
3. Interaction between Rydberg atoms and spin models
 - “Natural”: Ising and XY Hamiltonians
 - Hard-core bosons and $t - J$ model
 - Floquet engineering of XYZ models

Outline – Lecture 1

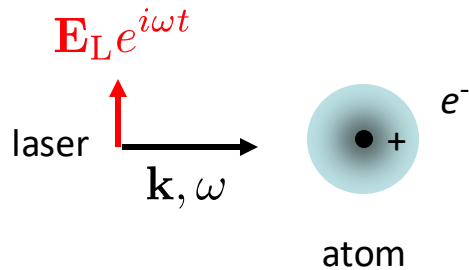
1. Arrays of individual atoms in optical tweezers
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A single Rb atom in an optical tweezer

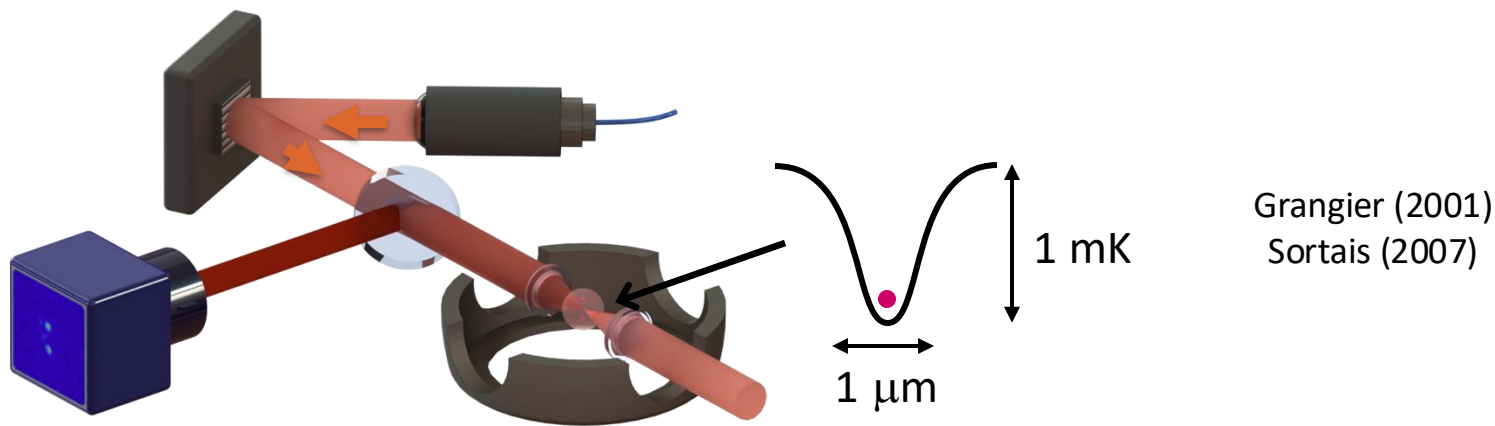


Grangier (2001)
Sortais (2007)

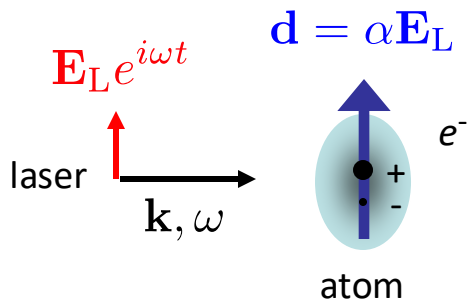
Dipole force



A single Rb atom in an optical tweezer

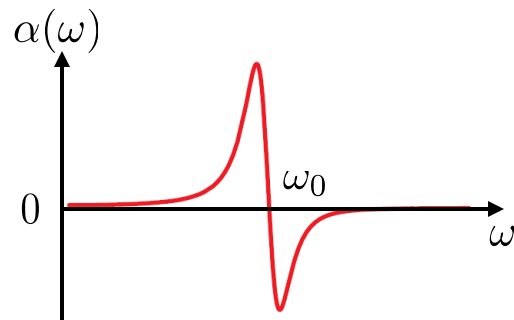


Dipole force



$$U = -\frac{1}{2} \langle \mathbf{d} \cdot \mathbf{E}_L \rangle$$

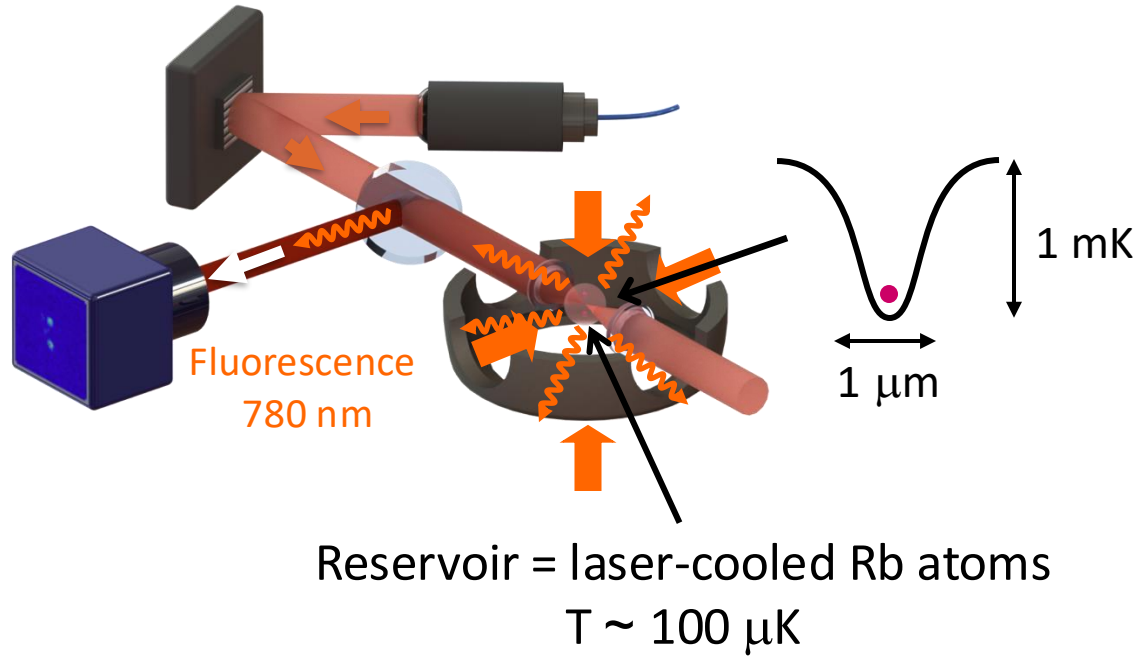
$$= -\frac{1}{2} \alpha \langle \mathbf{E}_L^2 \rangle$$



$\omega < \omega_0 \Rightarrow$ high-intensity seeker

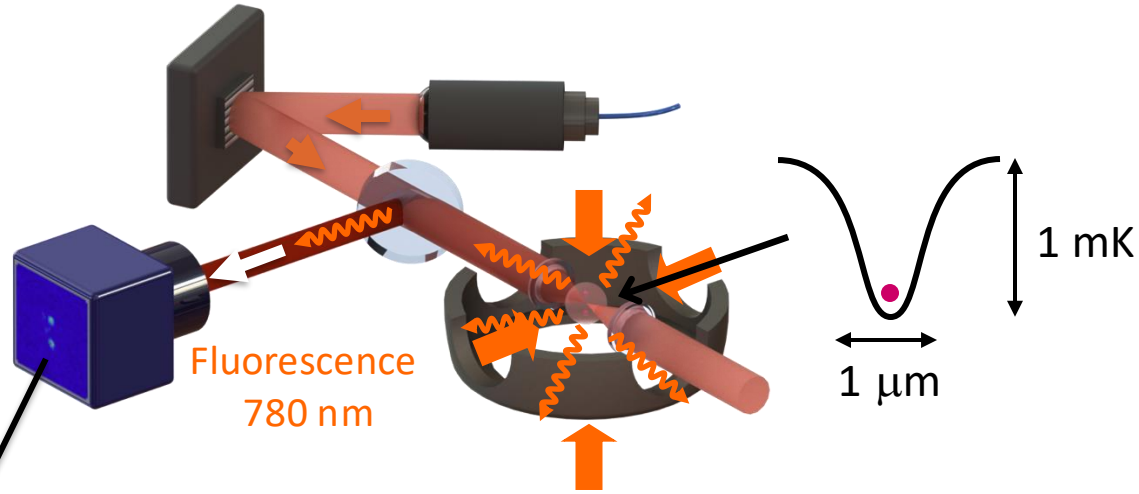
Ex: 1 mW on 1 μm $\Rightarrow$ **Trap depth = 1 mK** $\Rightarrow$ Laser cooled atoms...

A single Rb atom in an optical tweezer

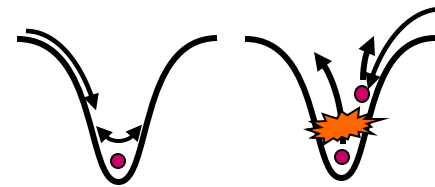
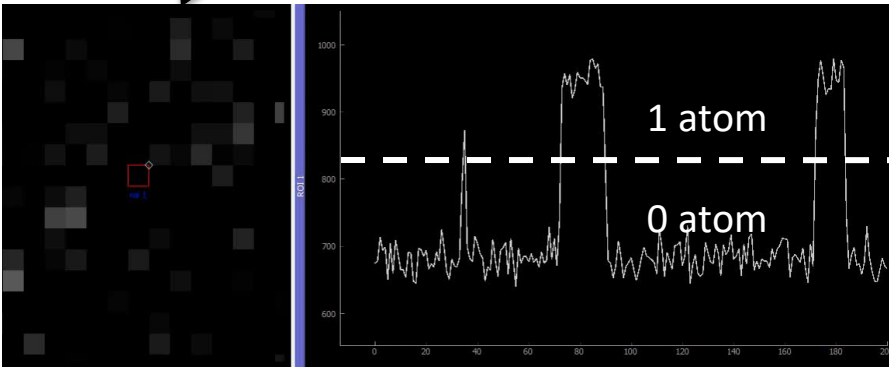


Grangier (2001)
Sortais (2007)

A single Rb atom in an optical tweezer



Grangier (2001)
Sortais (2007)



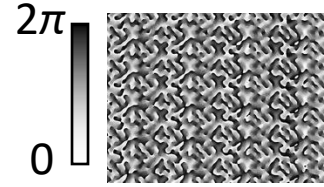
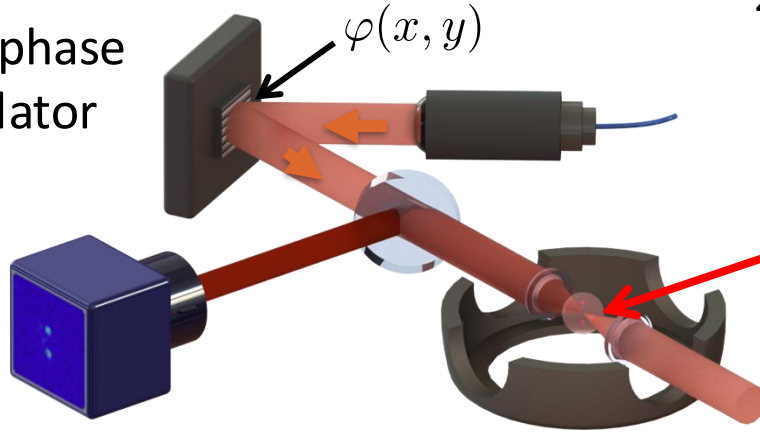
Non-deterministic
single-atom source

- Laser cooled
- Single atom in tweezer

sciencenotes.org

Atoms in arrays of optical tweezers

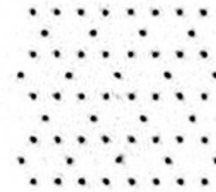
Spatial phase
modulator



Phase mask

Nogrette, PRX (2014)

$$\left| \text{FT}[e^{i\varphi(x,y)}] \right|^2$$

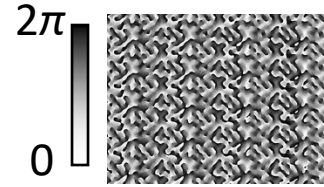
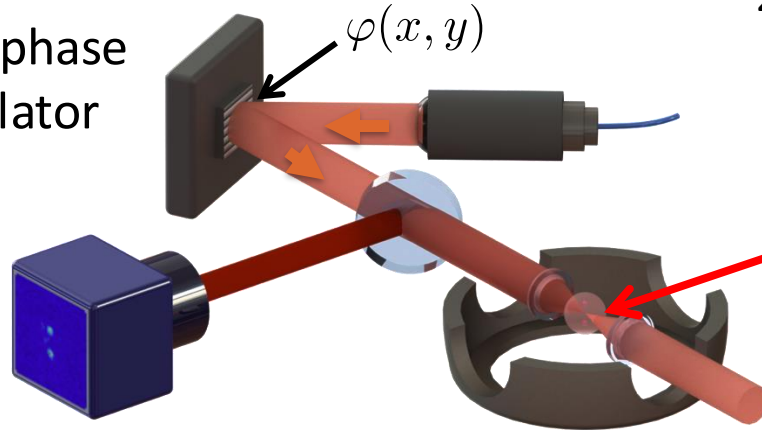


10 μm



Atoms in arrays of optical tweezers

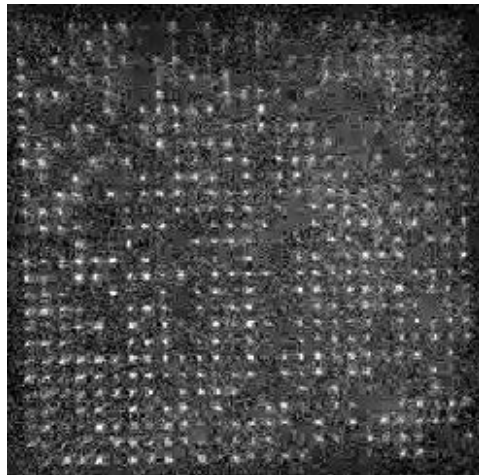
Spatial phase
modulator



Phase mask

Nogrette, PRX (2014)

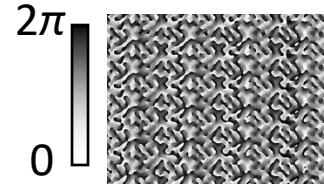
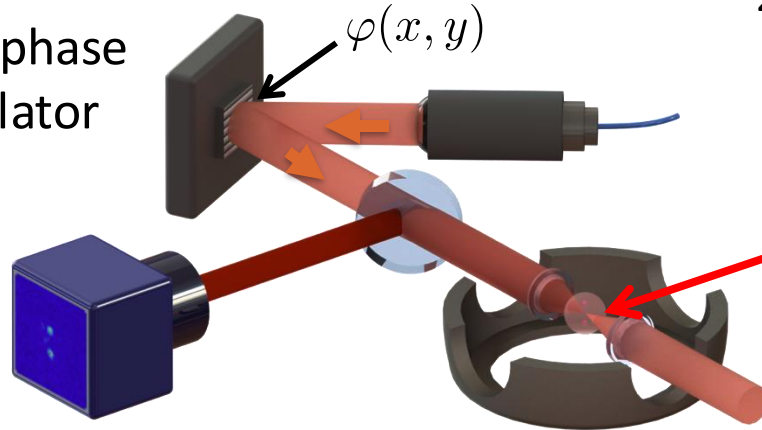
$$\left| \text{FT}[e^{i\varphi(x,y)}] \right|^2$$



Fluorescence (729 traps)

Atoms in arrays of optical tweezers

Spatial phase
modulator



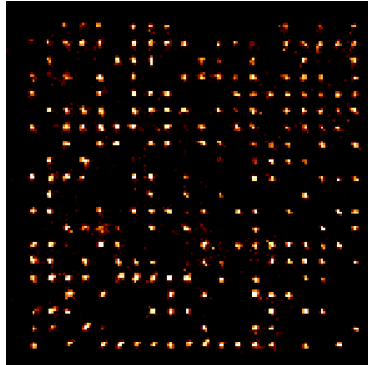
Phase mask

Nogrette, PRX (2014)

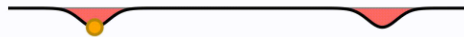
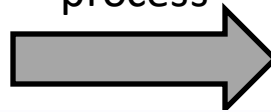
$$\left| \text{FT}[e^{i\varphi(x,y)}] \right|^2$$

First demo (1D): Meschede, Nature (2006);
Beugnon, Nat. Phys. (2007)

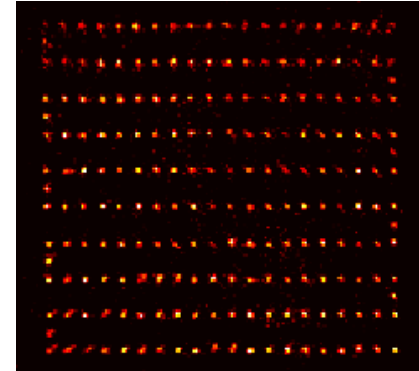
Initial configuration



Assembling
process

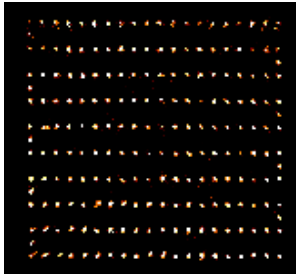


Assembled configuration



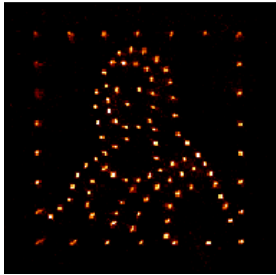
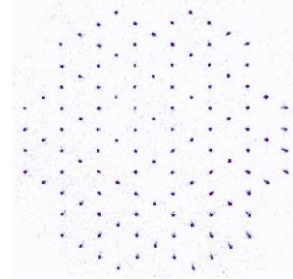
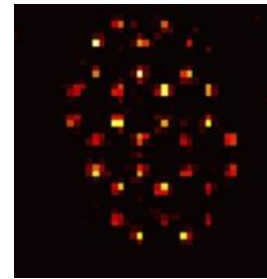
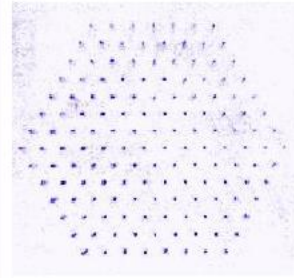
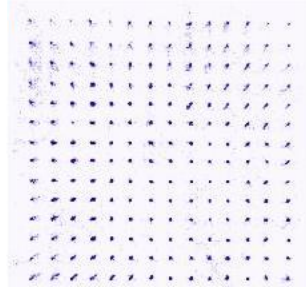
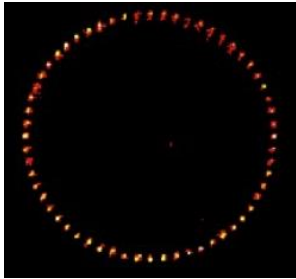
Atoms in arrays of optical tweezers (single-shot images)

1D



~100 μm

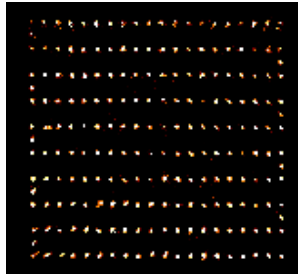
2D



L. da Vinci

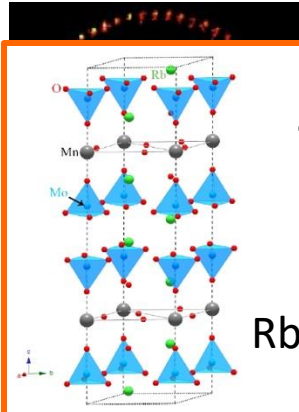
Atoms in arrays of optical tweezers (single-shot images)

1D



~100 μm

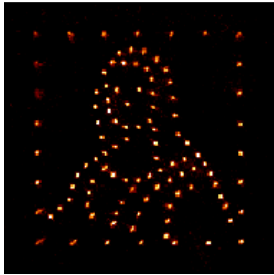
2D



Triangular

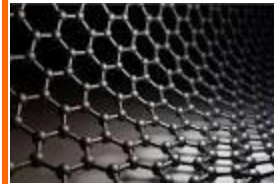
Mn^{2+}

$\text{Rb}_4\text{Mn}(\text{MoO}_4)_3$

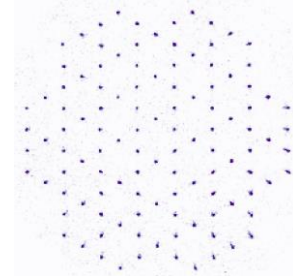
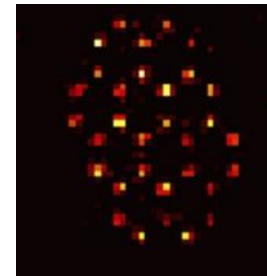
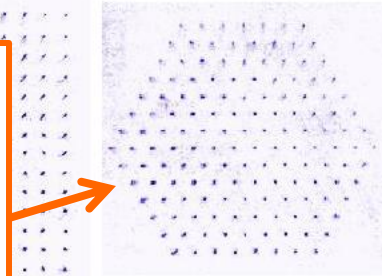


L. da Vinci

Hexagonal

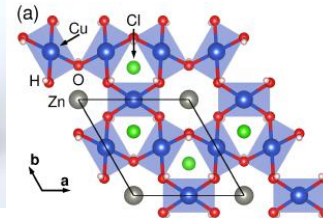


graphene



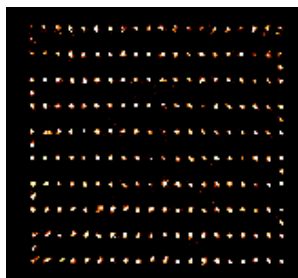
Kagome: Herbertsmithite

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$



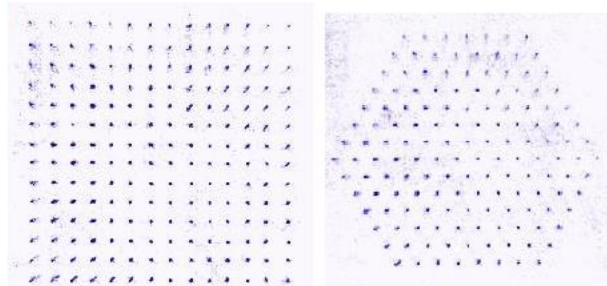
Atoms in arrays of optical tweezers (single-shot images)

1D

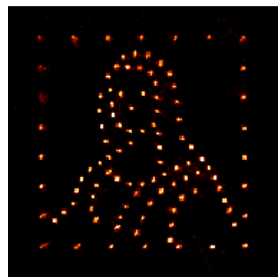
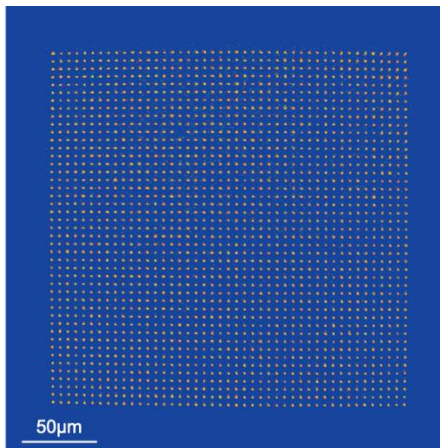


~100 μm

2D



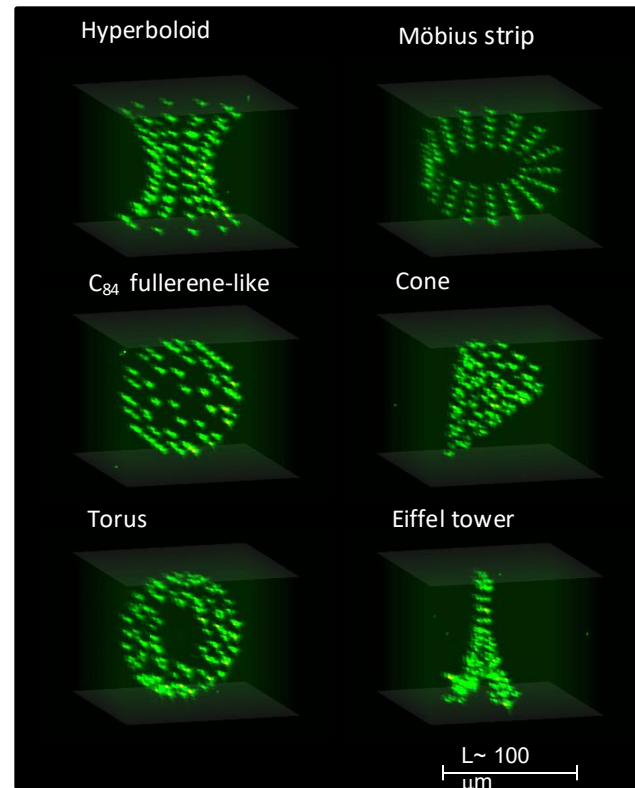
2024 atoms (AI + fast SLM)



L. da Vinci

3D

Barredo, Nature (2018)



Outline – Lecture 1

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Rydberg atoms: the discovery

1814 Joseph von Fraunhofer

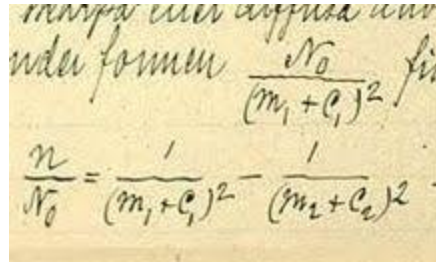


observation of dark lines in spectrum of the sun

1888 “Rydberg formula”



Johannes Rydberg
1854-1919

A photograph of a handwritten note on aged paper. The text includes the Rydberg formula:
$$\frac{n}{N_0} = \frac{1}{(m_1 + c_1)^2} - \frac{1}{(m_2 + c_2)^2}$$
 and some other handwritten text in German.

$$\frac{1}{\lambda_{nm}} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Idea of an infinite series
 $\Rightarrow$ **highly excited** states

Examples: alkali atoms

Alkali: 1 external electron

$$1s^2 2s^2 \dots (n - 1)p^6 ns$$

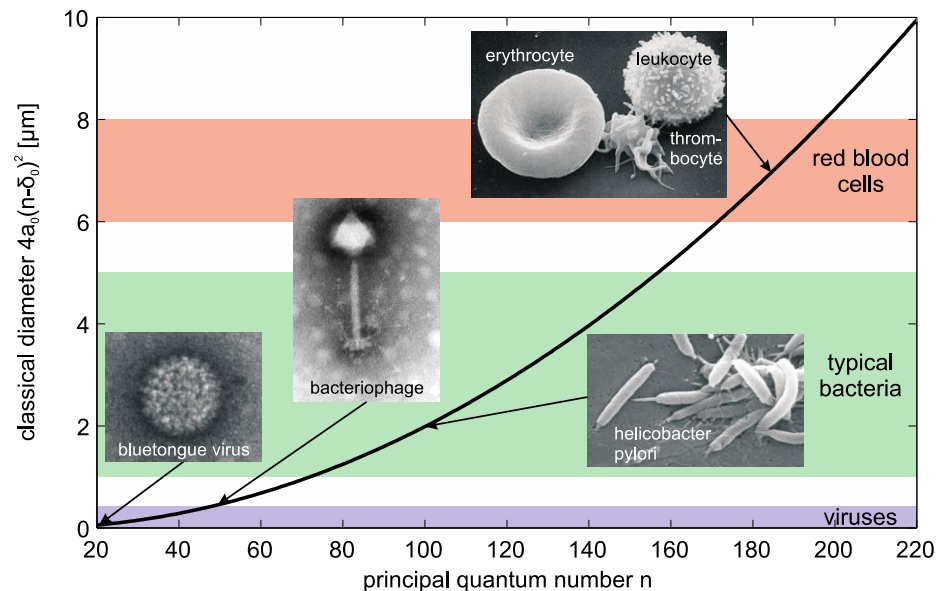
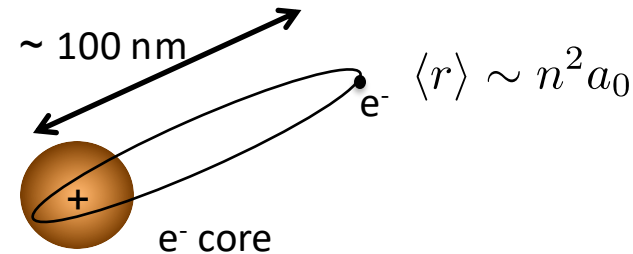
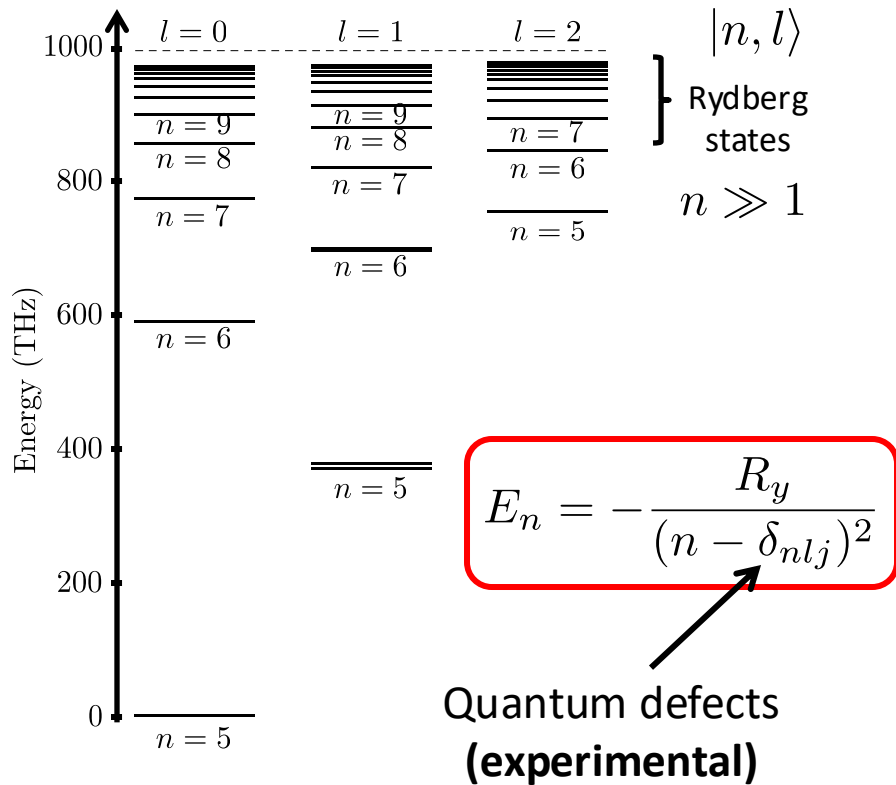
1 IA H Hydrogen 1.008	2 IIA Be Beryllium 9.012																	18 VIIIA He Helium 4.003															
3 Li Lithium 6.941	4 B Boron 10.811	5 C Carbon 12.011	6 N Nitrogen 14.007	7 O Oxygen 15.999	8 F Fluorine 18.998	9 Ne Neon 20.180																											
11 Na Sodium 22.990	12 Mg Magnesium 24.305	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948																										
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.933	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.972	35 Br Bromine 79.904	36 Kr Krypton 84.80																
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.29																
55 Cs Cesium 132.905	56 Ba Barium 137.327	57-71	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.85	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [209]	85 At Astatine [210]	86 Rn Radon [222]																
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium [278]	114 Fl Flerovium [289]	115 Uup Ununpentium [289]	116 Lv Livermorium [293]	117 Uus Ununseptium [294]	118 Uuo Ununoctium [294]																

Lanthanide Series

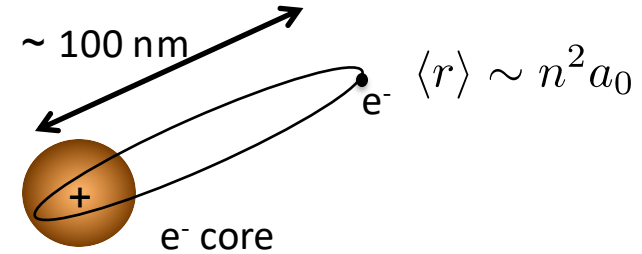
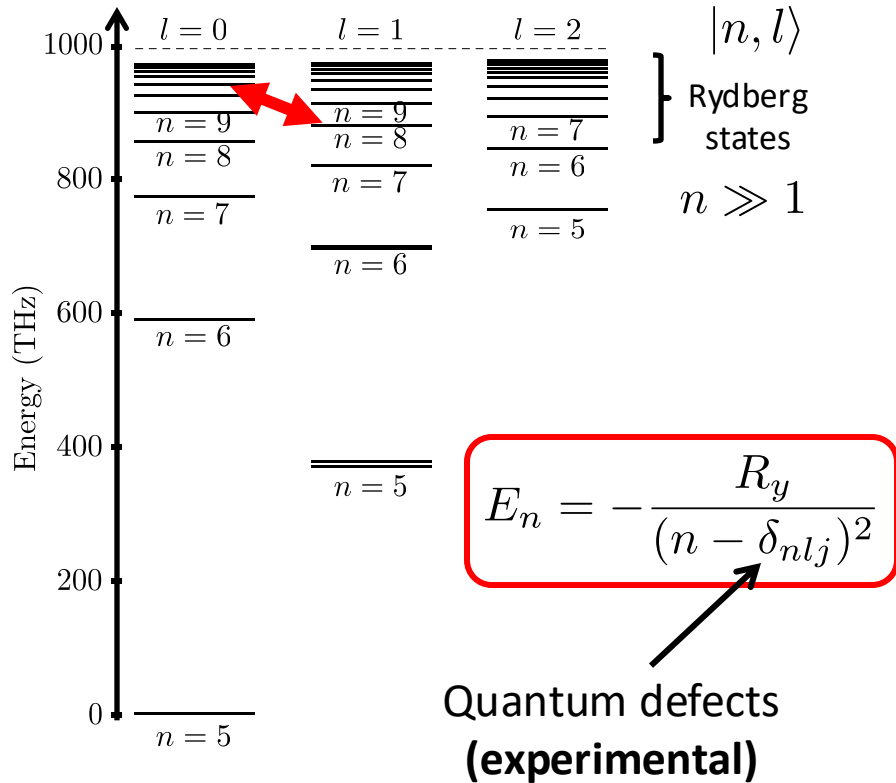
Actinide Series

57 La Lanthanum 138.906	58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.966	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.50	67 Ho Holmium 164.930	68 Er Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967
89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]

“Rydberg atom” = a highly excited atom (e.g. Rb)



“Rydberg atom” = a highly excited atom (e.g. Rb)



Long lifetime: $\tau \sim n^3$
 $\Rightarrow n > 60, \tau > 100 \mu\text{s}$

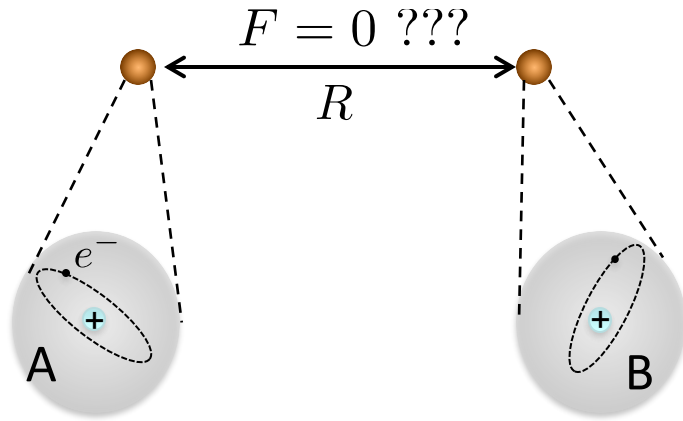
Large transition dipole:
 $\langle n, l | \hat{D} | n, l \pm 1 \rangle \sim n^2 e a_0$

Large polarizability: $\alpha \sim n^7$

$\Rightarrow$ **Exaggerated properties:**

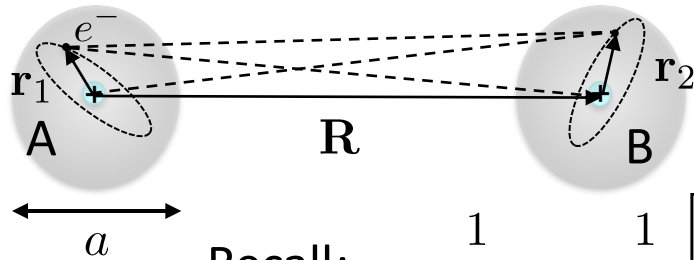
- strong interaction
- strong coupling to fields (DC, MW)

Dipolar Interaction between atoms



Dipolar Interaction between atoms

$$e^2 = \frac{q^2}{4\pi\epsilon_0}$$



$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2} - \frac{e^2}{|\mathbf{R} - \mathbf{r}_1|} - \frac{e^2}{|\mathbf{R} + \mathbf{r}_2|} + \frac{e^2}{|\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|} + \frac{e^2}{R}$$

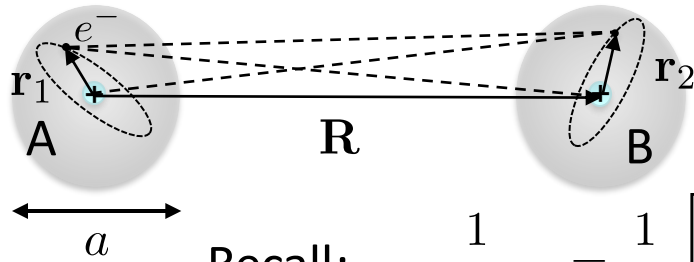
Recall: $\frac{1}{|\mathbf{R} - \mathbf{r}|} = \frac{1}{R} \left[1 - \frac{\mathbf{r} \cdot \mathbf{R}}{2R^2} - \frac{r^2}{2R^2} + \frac{3}{2} \left(\frac{\mathbf{r} \cdot \mathbf{R}}{R^2} \right)^2 \right] + \mathcal{O} \left(\frac{r^4}{R^4} \right)$

Dipole-dipole interaction: $H_{\text{dd}} = \frac{1}{4\pi\epsilon_0 R^3} [\mathbf{d}_1 \cdot \mathbf{d}_2 - 3(\mathbf{d}_1 \cdot \mathbf{u})(\mathbf{d}_2 \cdot \mathbf{u})] \quad , \quad \mathbf{u} = \frac{\mathbf{R}}{R}$
 $a \ll R$

with dipoles: $\mathbf{d}_{1,2} = q \mathbf{r}_{1,2}$

Dipolar Interaction between atoms

$$e^2 = \frac{q^2}{4\pi\epsilon_0}$$



$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2} - \frac{e^2}{|\mathbf{R} - \mathbf{r}_1|} - \frac{e^2}{|\mathbf{R} + \mathbf{r}_2|} + \frac{e^2}{|\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|} + \frac{e^2}{R}$$

Recall: $\frac{1}{|\mathbf{R} - \mathbf{r}|} = \frac{1}{R} \left[1 - \frac{\mathbf{r} \cdot \mathbf{R}}{2R^2} - \frac{r^2}{2R^2} + \frac{3}{2} \left(\frac{\mathbf{r} \cdot \mathbf{R}}{R^2} \right)^2 \right] + \mathcal{O} \left(\frac{r^4}{R^4} \right)$

Dipole-dipole interaction: $\hat{H}_{\text{dd}} = \frac{1}{4\pi\epsilon_0 R^3} \left[\hat{\mathbf{d}}_1 \cdot \hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1 \cdot \mathbf{u})(\hat{\mathbf{d}}_2 \cdot \mathbf{u}) \right], \quad \mathbf{u} = \frac{\mathbf{R}}{R}$
 $a \ll R$

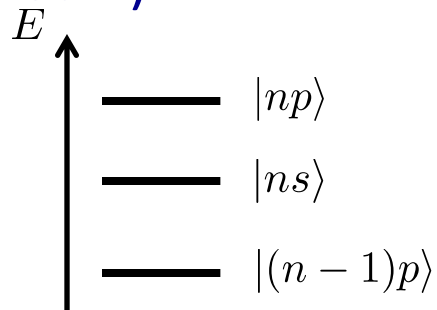
with dipoles: $\hat{\mathbf{d}}_{1,2} = q \hat{\mathbf{r}}_{1,2}$

2 atom basis: $\{|n, l, m\rangle \otimes |n', l', m'\rangle\}$

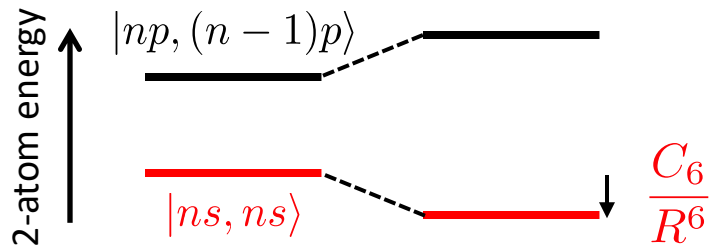
Interactions between Rydberg atoms (simplified...)



$$\hat{H}_{\text{dd}} = \frac{1}{4\pi\epsilon_0} \frac{\hat{d}_{Az}\hat{d}_{Bz}}{R^3}$$



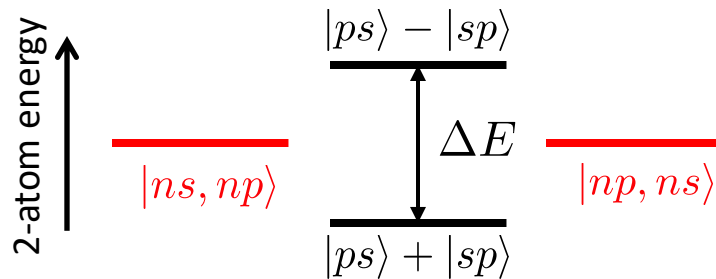
van der Waals regime



$$\Delta E_{ss}^{(2)} \approx \frac{|\langle np, (n-1)p | \hat{H}_{\text{dd}} | ns, ns \rangle|^2}{E_{ss} - E_{pp}}$$

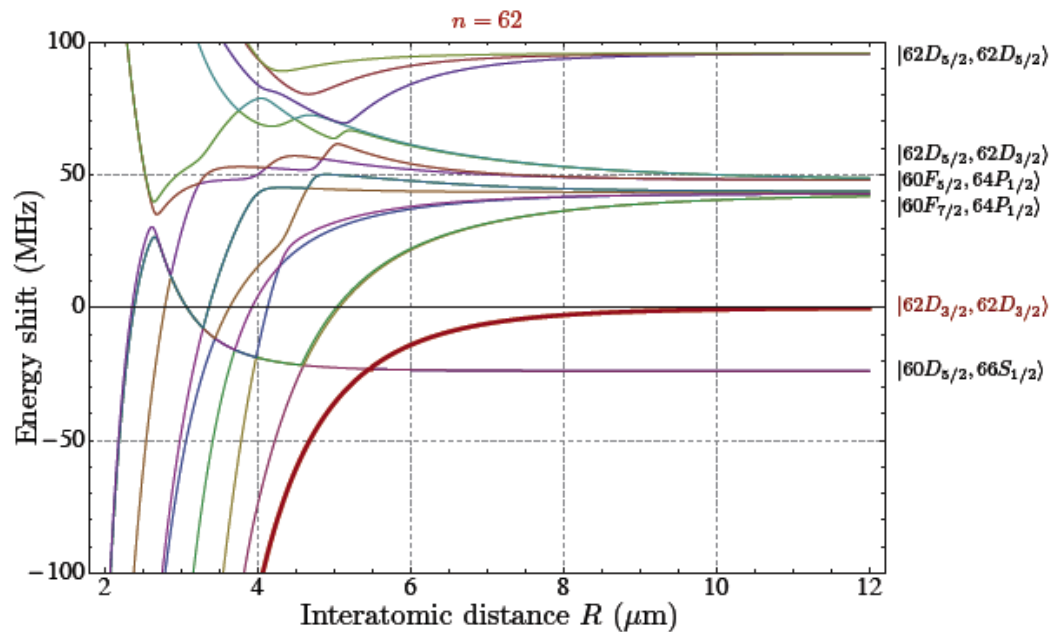
$$\propto \frac{d_{sp}^4}{E_{ss} - E_{pp}} \frac{1}{R^6} = \frac{C_6}{R^6} \quad C_6 \propto n^{11}$$

Resonant regime



$$\Delta E \propto \langle sp | \hat{H}_{\text{dd}} | ps \rangle = \frac{d_{sp}^2}{R^3} \propto n^4$$

Interactions between “real” Rydberg atoms

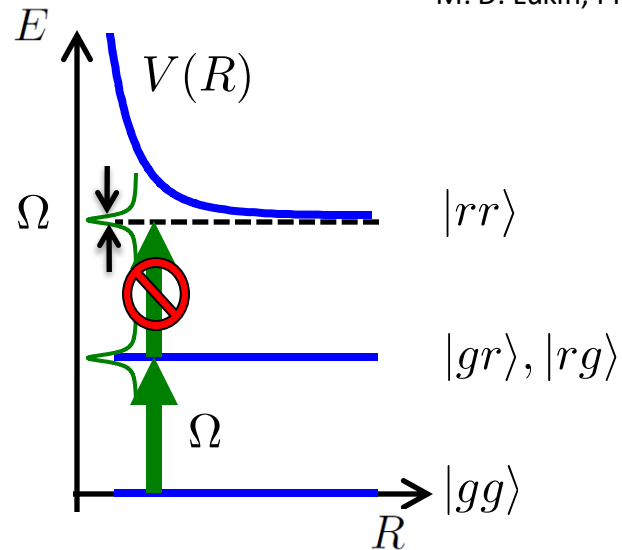
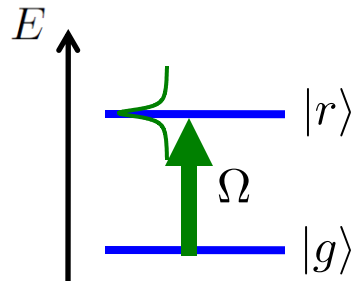
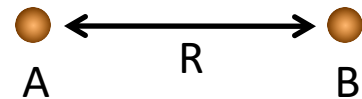


$$R = 10 \mu\text{m} \Rightarrow V_{\text{int}}/h \sim 1 - 10 \text{ MHz} \Rightarrow \text{timescales} < \mu\text{sec}$$

A fruitful idea: the Rydberg blockade

D. Jaksch, PRL **85**, 2208 (2000)

M. D. Lukin, PRL **87**, 037901 (2001)

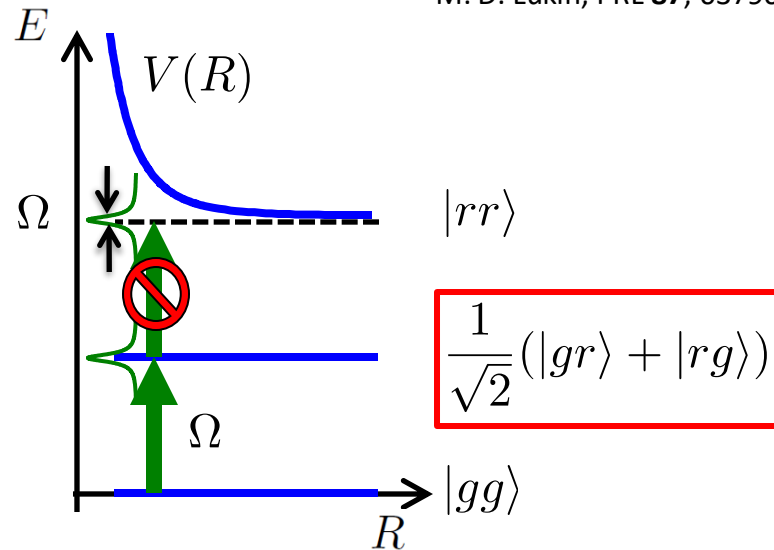
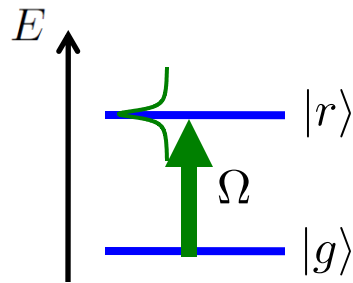
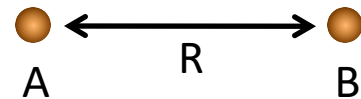


If $\hbar\Omega \ll V(R)$: no excitation of $|rr\rangle \Rightarrow$ **blockage**

A fruitful idea: the Rydberg blockade

D. Jaksch, PRL **85**, 2208 (2000)

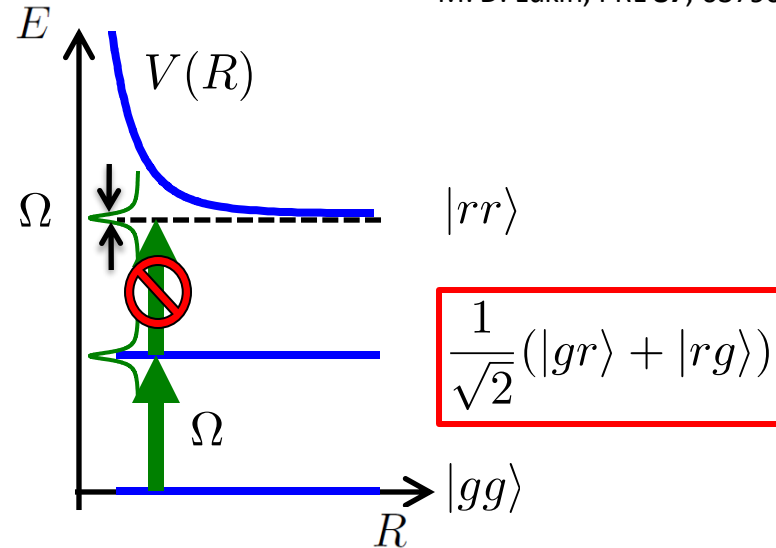
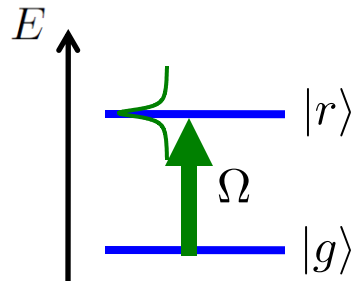
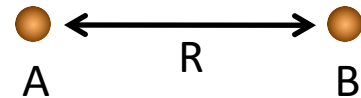
M. D. Lukin, PRL **87**, 037901 (2001)



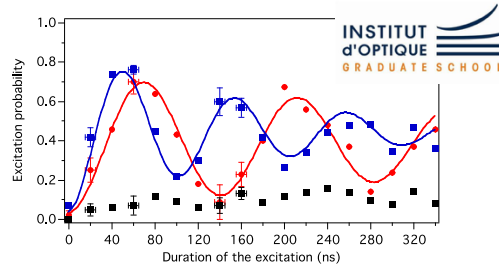
Blockade $\Rightarrow$ **entanglement and gates!!**

A fruitful idea: the Rydberg blockade

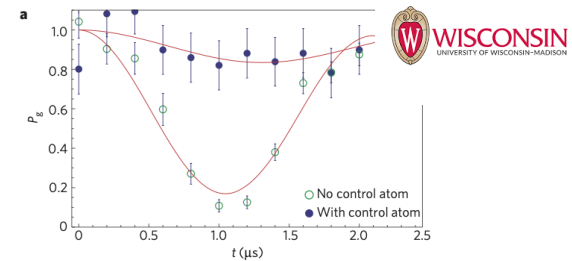
D. Jaksch, PRL **85**, 2208 (2000)
M. D. Lukin, PRL **87**, 037901 (2001)



1st demonstrations of controlled Rydberg interactions

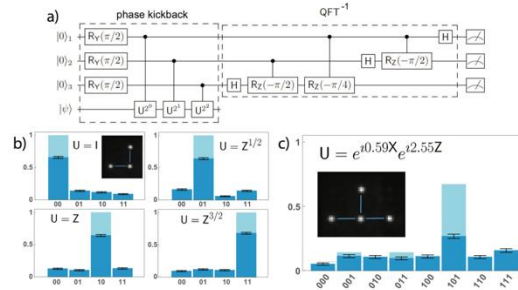


Nat. Phys. 2009

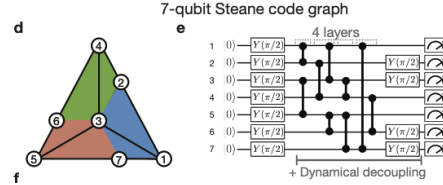


And now (2025)... a few examples

QIP: entanglement and gates



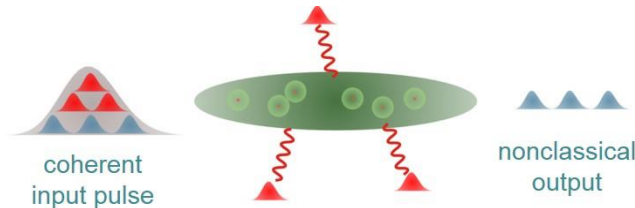
Saffman, Nature 2022



Lukin, Nature 2022-25

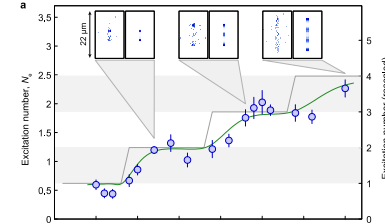
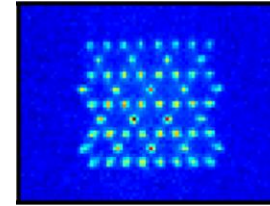
Saffman, RMP **82**, 2313 (2010)
Henriet, Quantum (2020)
Whitlock, AVS Quantum Science (2021)

Non-linear classical & quantum optics



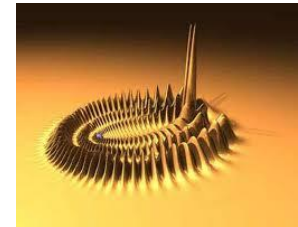
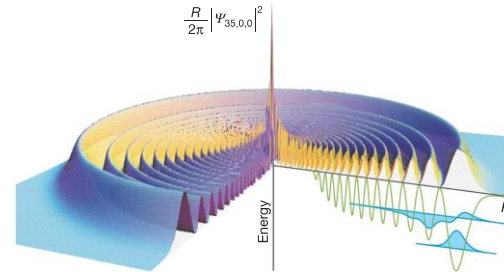
Firstenberg, J Phys. B (2016)

Many-body physics Quantum simulation



Schauss Quantum Sci. Techno 2018
Browaeys, Lahaye Nat. Phys. 2020

Exotic long-range molecules

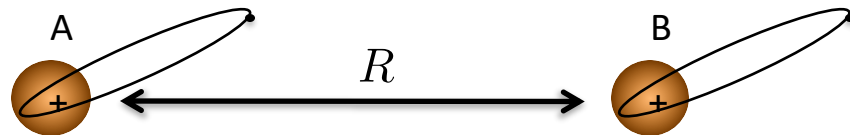


Pfau, Nature 2009

Outline – Lecture 1

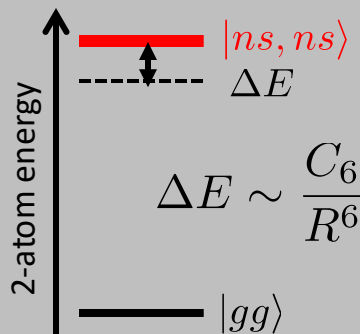
1. Arrays of individual atoms in optical tweezers
2. Basics of Rydberg physics and their interaction
3. Interaction between Rydberg atoms and spin models
 - “Natural”: Ising and XY Hamiltonians
 - Hard-core bosons and $t - J$ model
 - Floquet engineering of XYZ models

Interactions between Rydberg atoms and spin models

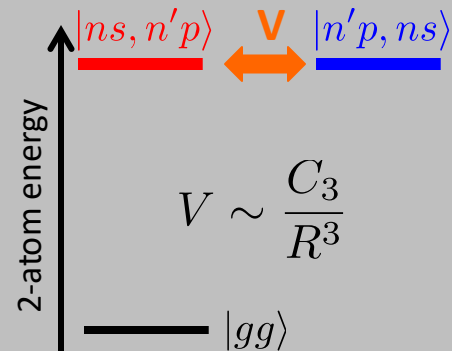


Browaeys & Lahaye, Nat.Phys. (2020)

van der Waals

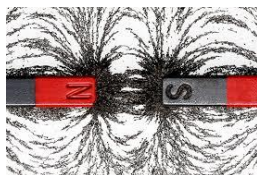
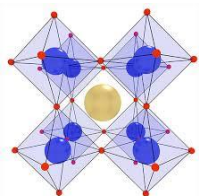


Resonant dipole

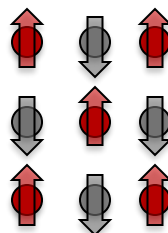


Ising model

$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j$$

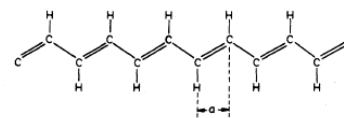


Spin 1/2

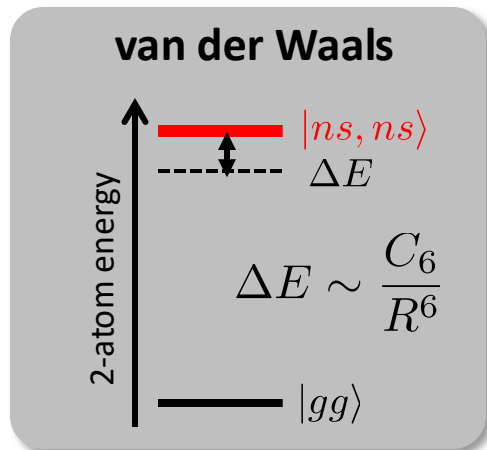


XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$



From van der Waals interaction to spin models...



$C_6 \propto n^{11} \Rightarrow$ switchable interaction

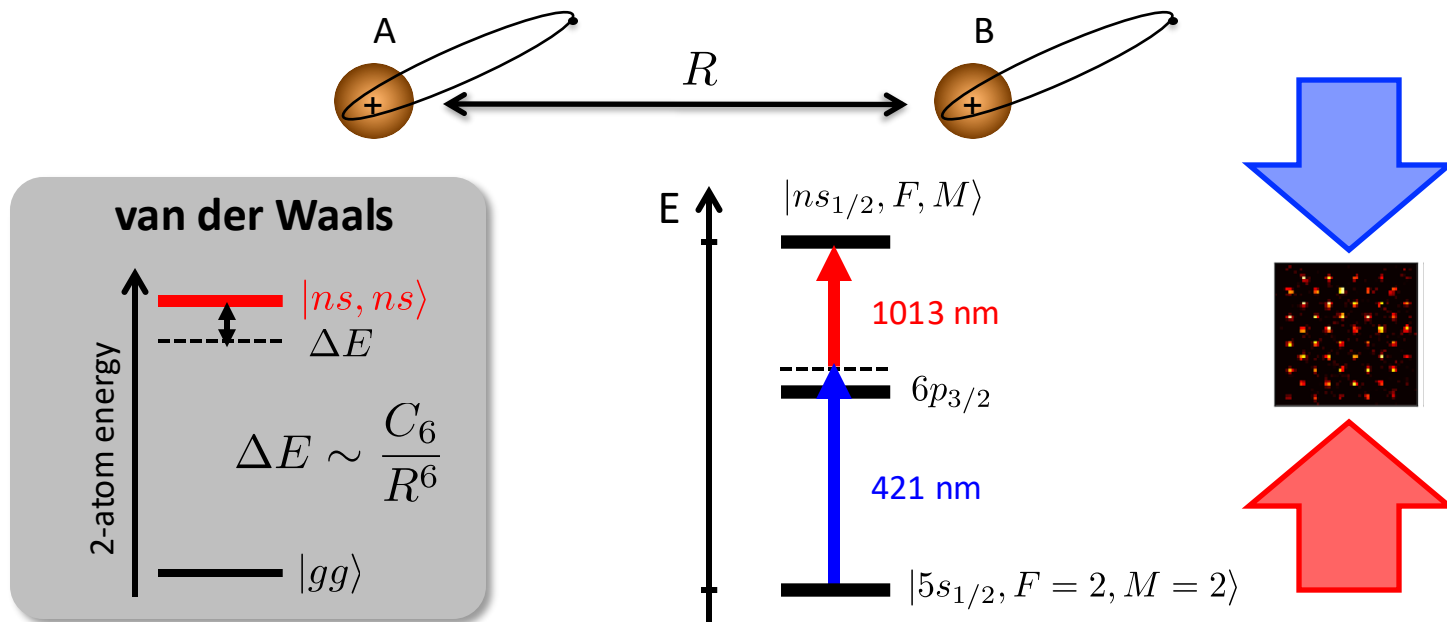
Ground state: $n = 5$
 Rydberg: $n = 50$ $\times 10^{11}$

Ising - like!!

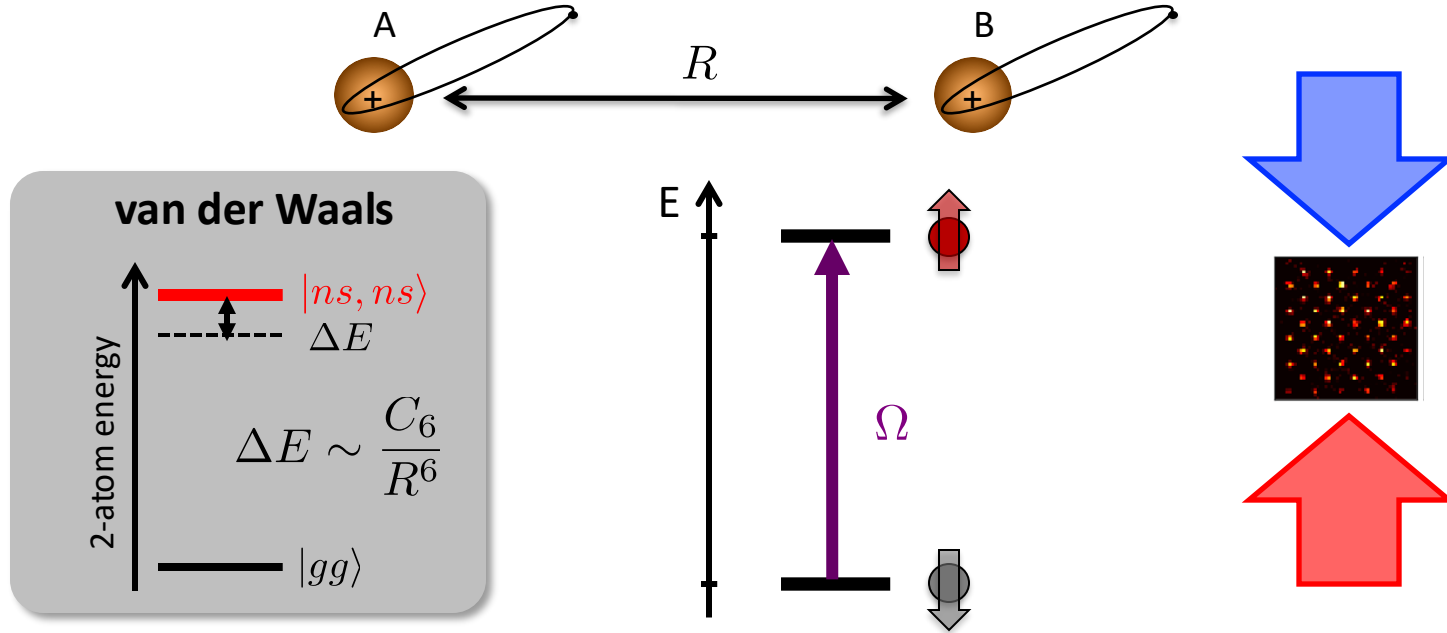
$$\hat{H}_{\text{int}} = \frac{C_6}{R^6} \hat{n}_1 \hat{n}_2 \sim J \hat{\sigma}_1^z \hat{\sigma}_2^z$$

Rydberg $n_{1,2} = 1$
 Ground state $n_{1,2} = 0$

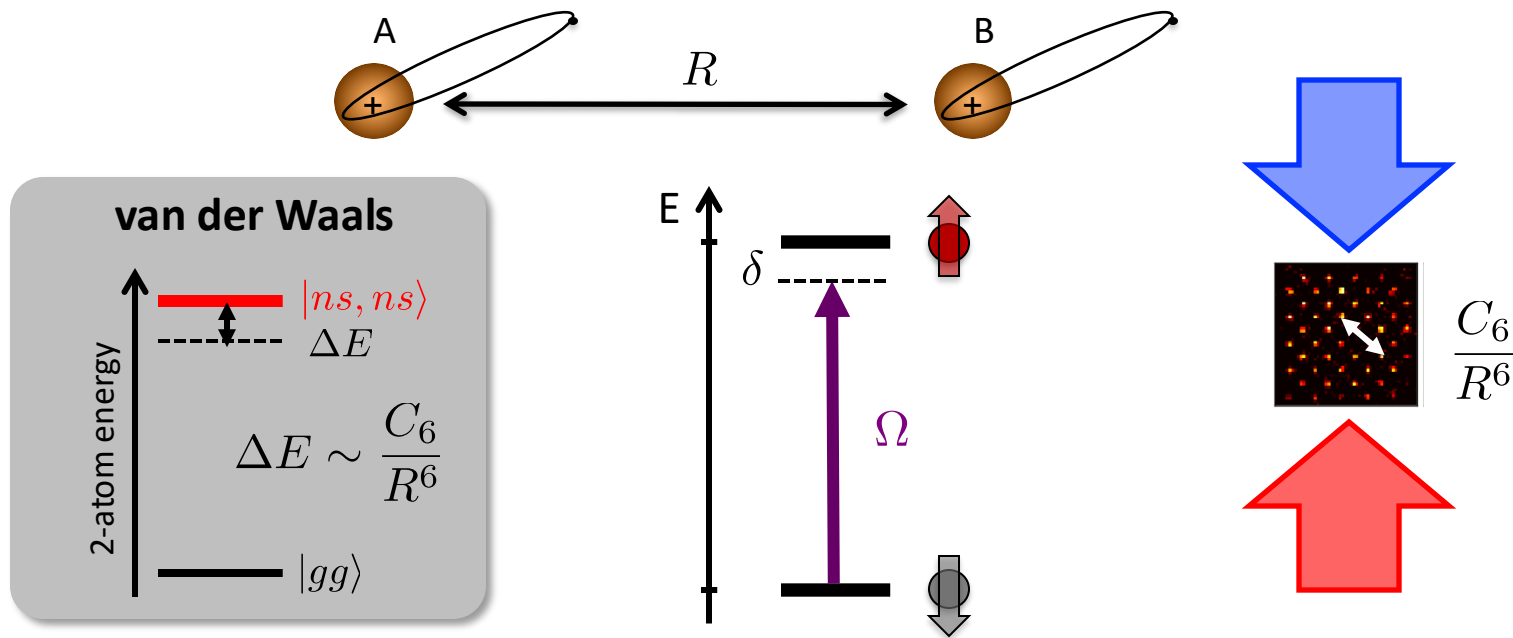
From van der Waals interaction to spin models...



From van der Waals interaction to spin models...

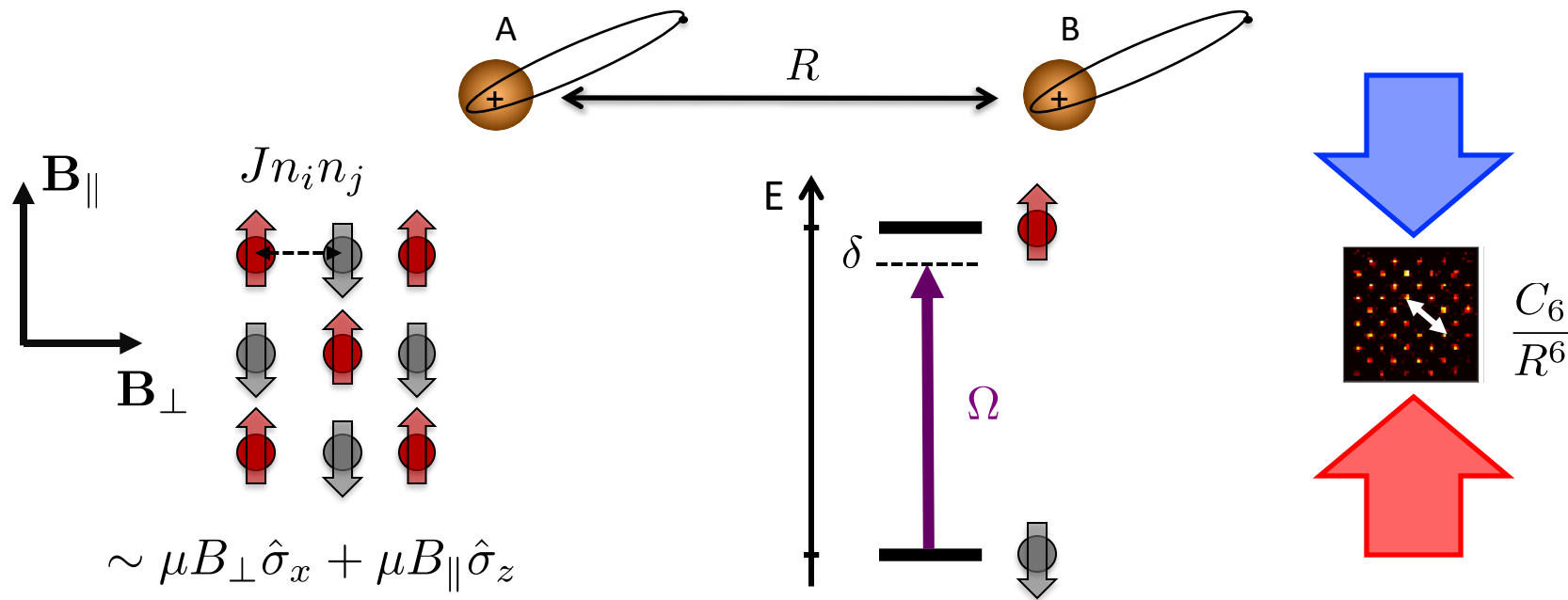


From van der Waals interaction to spin models...



$$H = \frac{\hbar\Omega}{2} \sum_i \hat{\sigma}_x^i + \hbar\delta \sum_i \hat{\sigma}_z^i + \sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

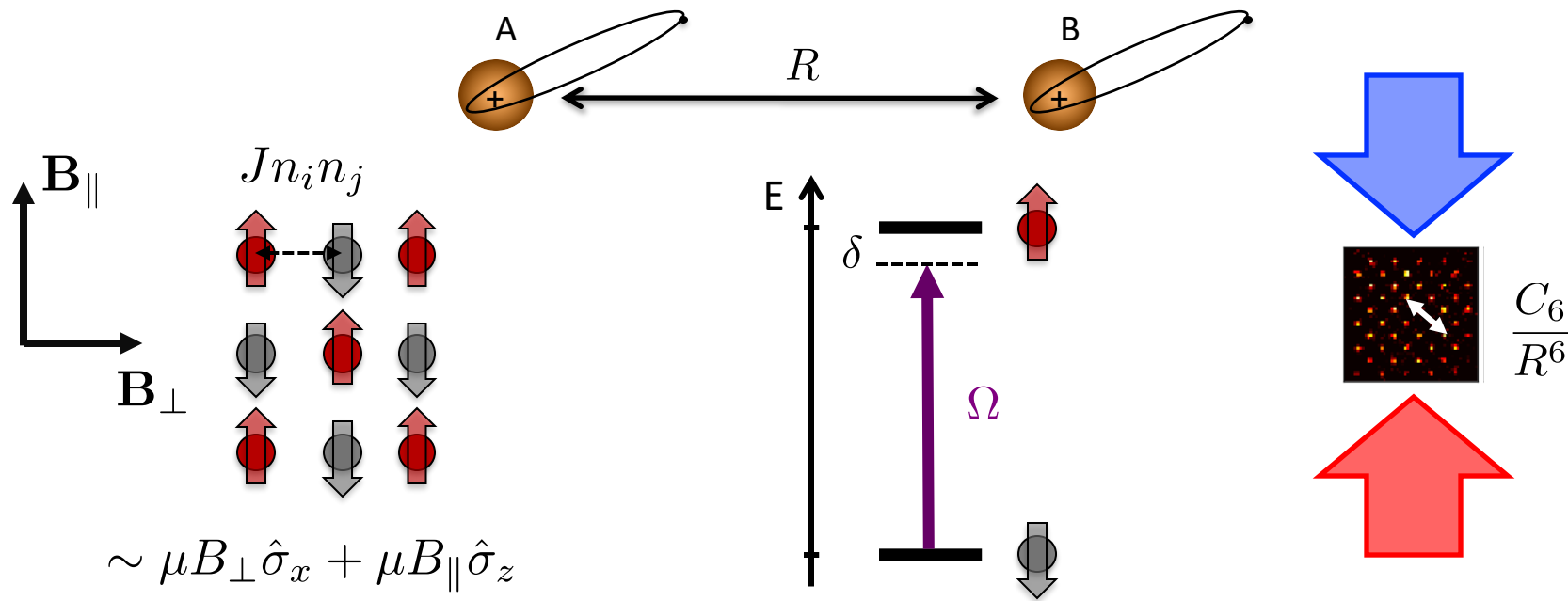
From van der Waals interaction to spin models...



Transverse Field Ising model:

$$H = \underbrace{\frac{\hbar\Omega}{2} \sum_i \hat{\sigma}_x^i}_{\text{Laser: } B_{\perp}} + \underbrace{\hbar\delta \sum_i \hat{\sigma}_z^i}_{B_{\parallel}} + \underbrace{\sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j}_{\text{spin-spin interactions}}$$

From van der Waals interaction to spin models...



$$\sim \mu B_{\perp} \hat{\sigma}_x + \mu B_{\parallel} \hat{\sigma}_z$$

Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2} \sum_i \hat{\sigma}_x^i + \hbar\delta \sum_i \hat{\sigma}_z^i + \sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

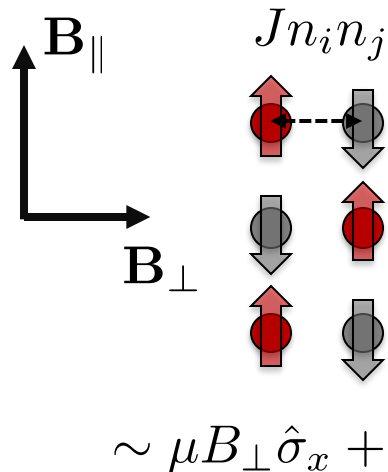
Laser: $B_{\perp}$

$B_{\parallel}$

spin-spin interactions

Controlled parameters:
From negligible to dominant
interactions

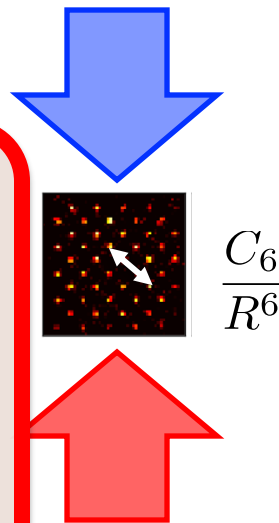
From van der Waals interaction to spin models...



Quantum simulation:

Emulate a system by another one

Similar equations lead to same solutions!!



Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2} \sum_i \hat{\sigma}_x^i + \hbar\delta \sum_i \hat{\sigma}_z^i + \sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Laser:

$B_{\perp}$

$B_{\parallel}$

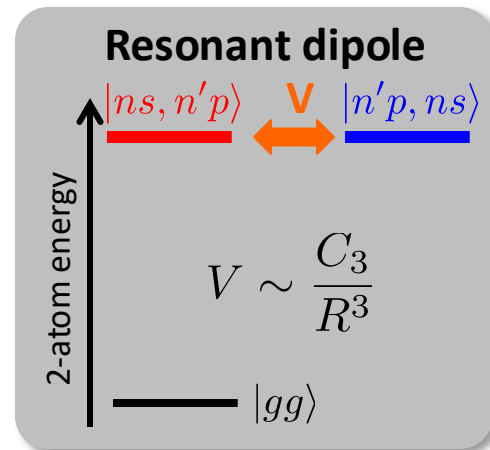
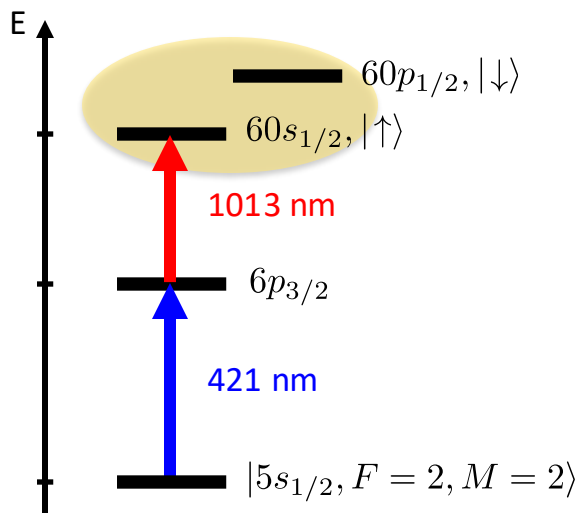
spin-spin interactions

Controlled parameters:

From negligible to dominant interactions

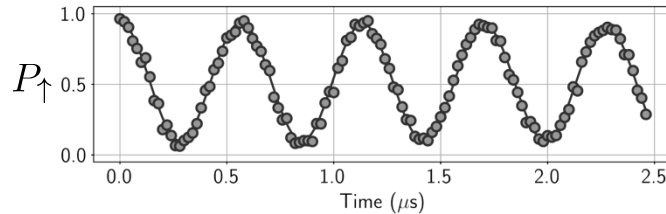
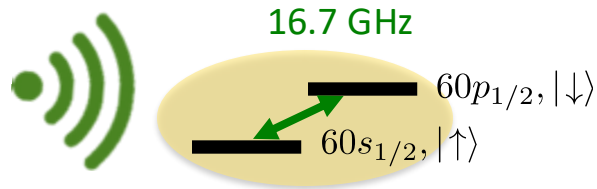
Resonant interaction between Rydbergs and XY spin model

Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)



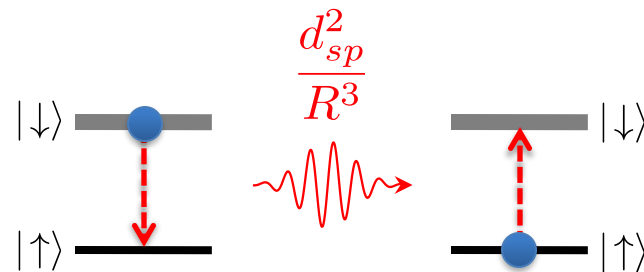
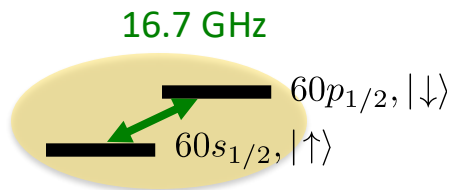
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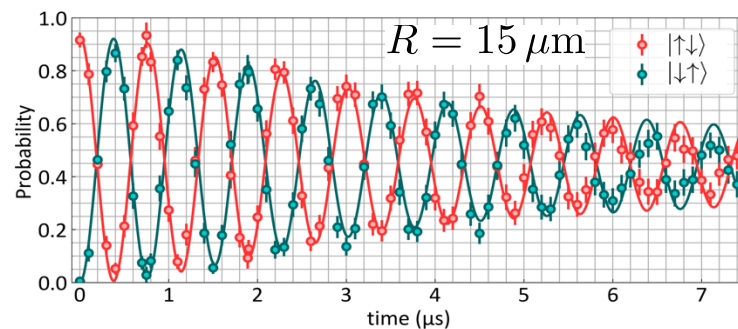


Resonant interaction between Rydbergs and XY spin model

Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)



Non radiative “exchange” of excitation

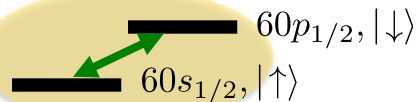


$$\begin{aligned}\hat{H}_{\text{XY}} &= \frac{C_3}{R_{ij}^3} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+) \\ &= \frac{C_3}{2R_{ij}^3} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)\end{aligned}$$

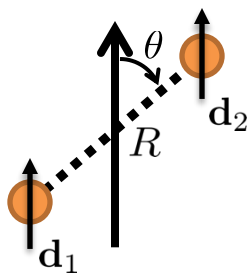
Resonant interaction between Rydbergs and XY spin model

Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)

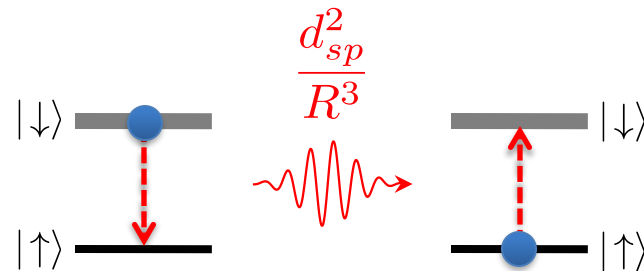
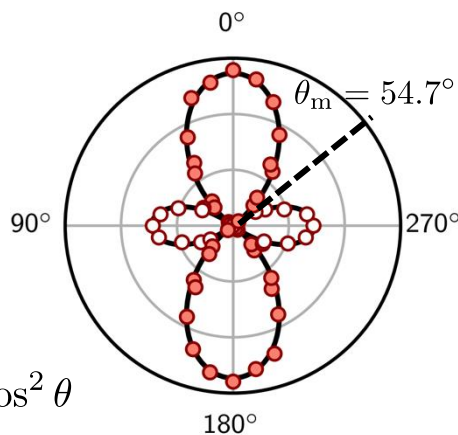
16.7 GHz



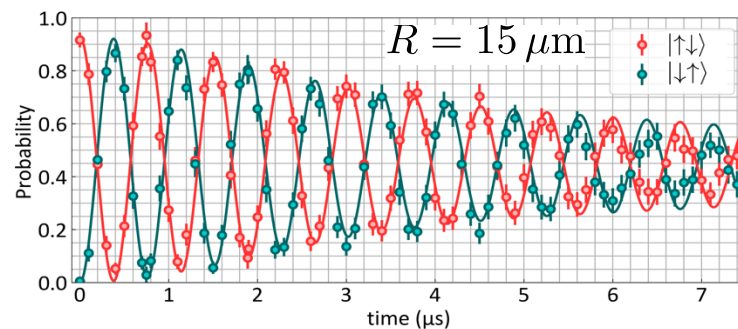
Quantization axis (B)



$$C_3(\theta) \propto 1 - 3 \cos^2 \theta$$

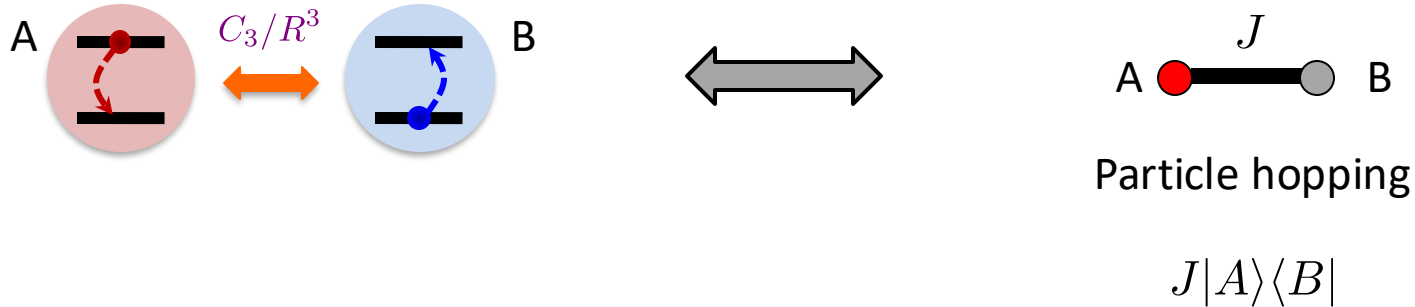
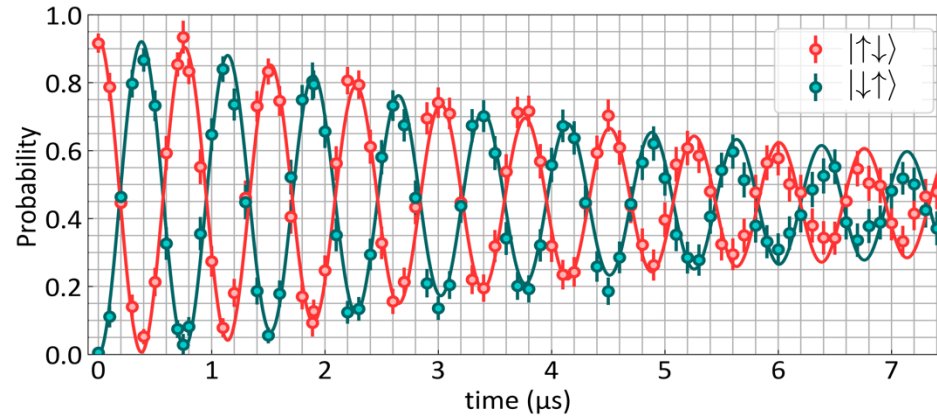


Non radiative “exchange” of excitation



$$\begin{aligned} \hat{H}_{XY} &= \frac{C_3}{R_{ij}^3} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+) \\ &= \frac{C_3}{2R_{ij}^3} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y) \end{aligned}$$

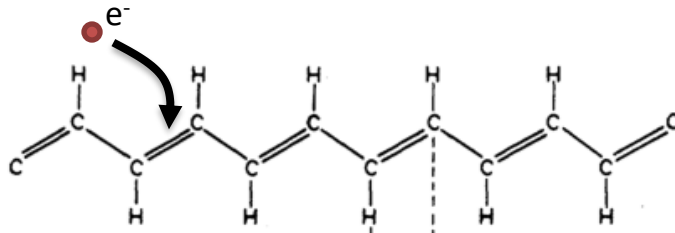
XY spin model and transport of excitations



Outline – Lecture 1

1. Arrays of individual atoms in optical tweezers
2. Basics of Rydberg physics and their interaction
3. Interaction between Rydberg atoms and spin models
 - “Natural”: Ising and XY Hamiltonians
 - Hard-core bosons and $t - J$ model
 - Floquet engineering of XYZ models

The Su-Schrieffer-Heeger model

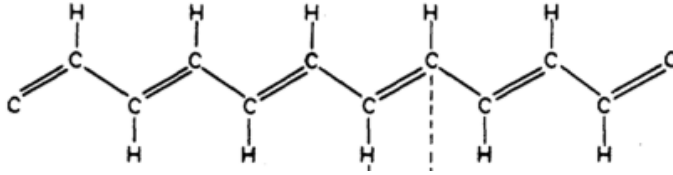


Electronic transport in
polyacetylene

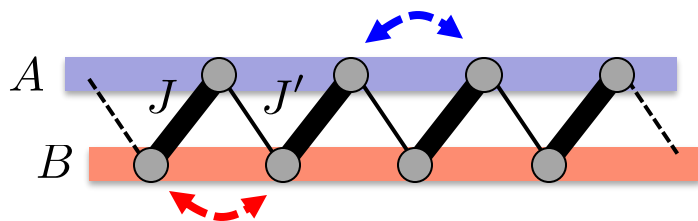
PRL **42**, 1698 (1979)

Now, considered as simplest example of **topological** model

The Su-Schrieffer-Heeger model



The Su-Schrieffer-Heeger model

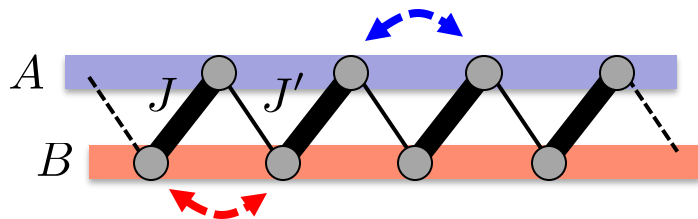


Model: tight-binding
dimerization: $J > J'$

$J'' = 0$: chiral symmetry $\Rightarrow$ symmetric **single particle** spectrum

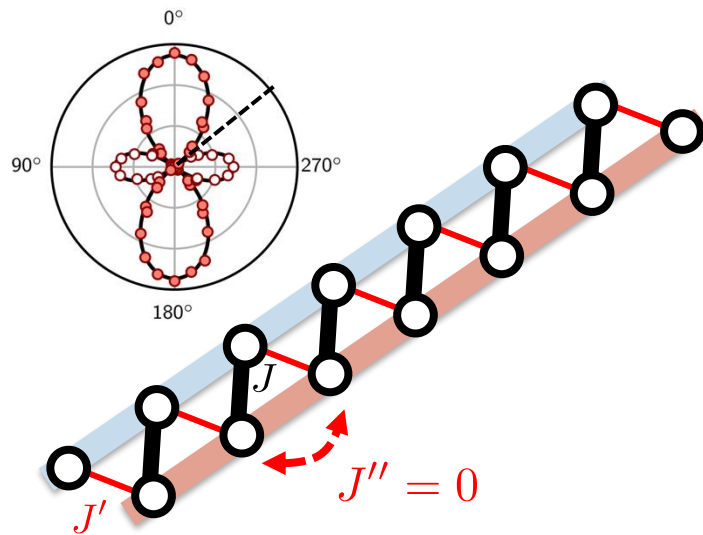
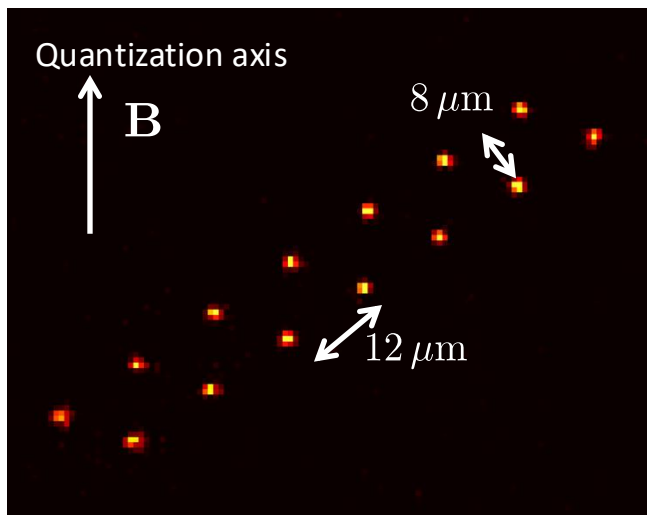
Implementation of SSH spin chain with Rydberg atoms

Déléseleuc, Science **365**, 775 (2019)



$J'' = 0$: chiral symmetry $\Rightarrow$ symmetric **single particle** spectrum

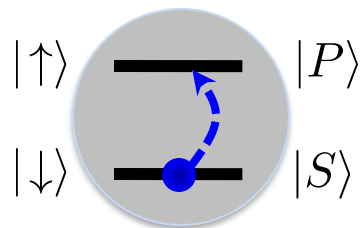
Model: tight-binding
dimerization: $J > J'$



Asboth, [arXiv:1509.02295](https://arxiv.org/abs/1509.02295)
Cooper, [arXiv:1803.00249](https://arxiv.org/abs/1803.00249)

Spin excitations interact: hard core bosons

Spin excitation = “particle”

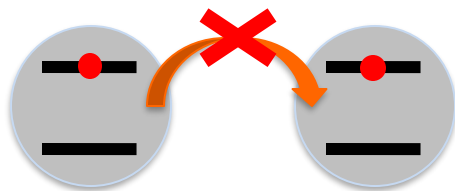


$$\hat{\sigma}^+ \rightarrow \hat{b}^\dagger, \quad b^\dagger|0\rangle = |1\rangle$$

$$\hat{\sigma}^- \rightarrow \hat{b}, \quad b|1\rangle = |0\rangle$$

$$[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$$

Atom cannot carry 2 excitations $\Rightarrow$ excitations = **hard-core bosons**



On-site interaction $U \rightarrow \infty$

$$H_B = \sum_{i,j} J_{ij} (b_i^\dagger b_j + b_i b_j^\dagger), \quad b_i^{\dagger 2} = 0$$



$\Rightarrow$ The first **symmetry protected topological** phase...

Predicted in **2012**

The program

Lecture 1: Arrays of atoms & “Rydbergology”
Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 2: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism