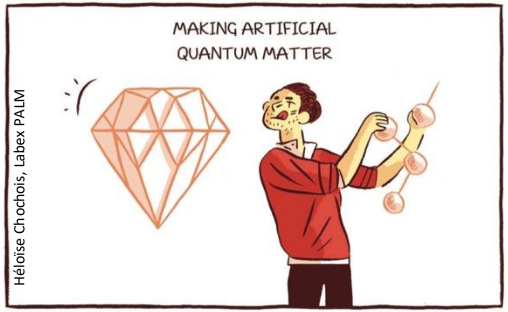
Many-body physics

with arrays of Rydberg atoms (II)

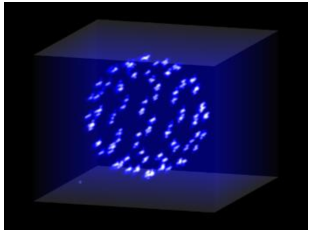


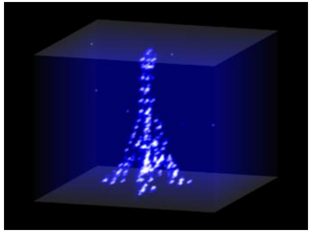
Héloïse Chochois, Labex PALM

Antoine Browaeys

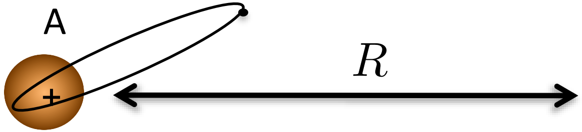
Laboratoire Charles Fabry,
Institut d'Optique, CNRS, FRANCE

ICTP Summer School, august 22nd, 2025

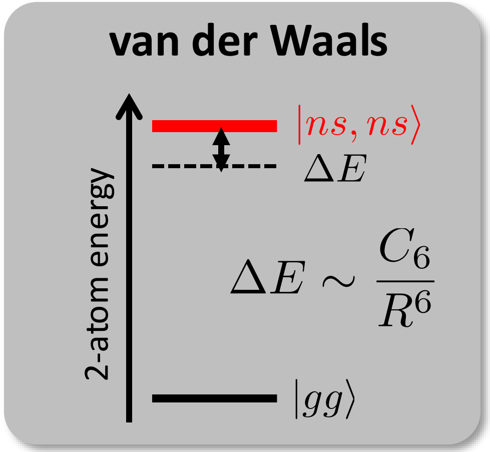


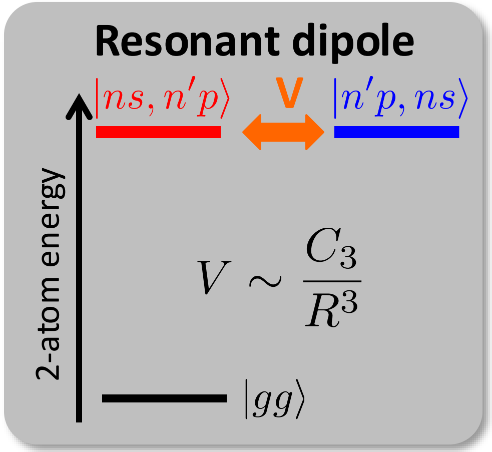


Interactions between Rydberg atoms and spin models

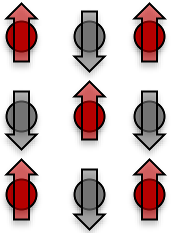


Browaeys & Lahaye, Nat.Phys. (2020)



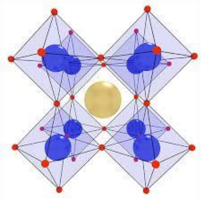


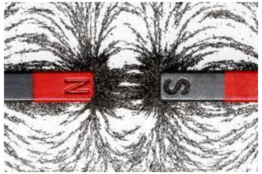
Spin 1/2



Ising model

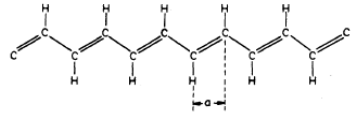
$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j$$





XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$



Outline – Lecture 1

1. Arrays of individual atoms in optical tweezers

2. Basics of Rydberg physics and their interaction

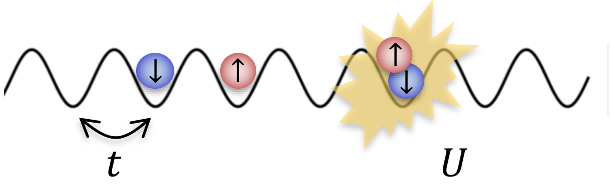
3. Interaction between Rydberg atoms and spin models

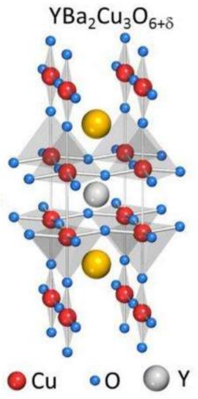
   - "Natural": Ising and XY Hamiltonians
   - Hard-core bosons and $t-J$ model
   - Floquet engineering of XYZ models

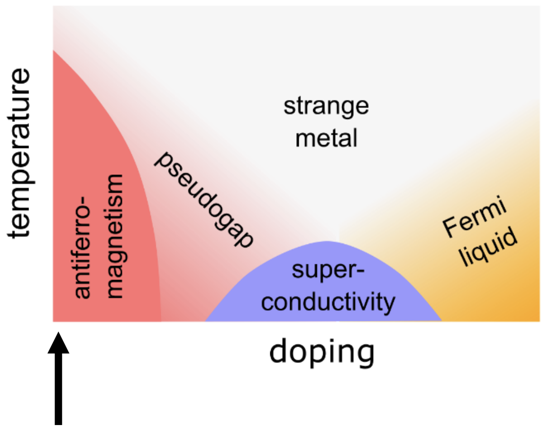
Doped magnets and $t - J$ model

Hubbard model

$$H_{\mathrm{FH}} = -t \sum_{\langle i,j \rangle, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\downarrow} n_{i\uparrow}$$





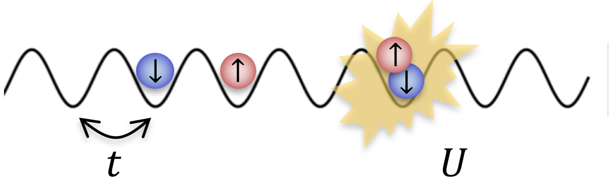


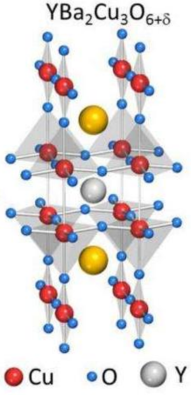
Doping = 0 + $U \gg t \Rightarrow H_{\mathrm{FH}} = \frac{4t^2}{U} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$

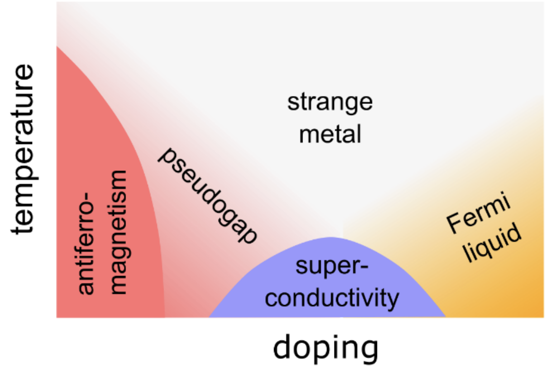
Doped magnets and $t-J$ model

Hubbard model

$$H_{\mathrm{FH}} = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\downarrow} n_{i\uparrow}$$







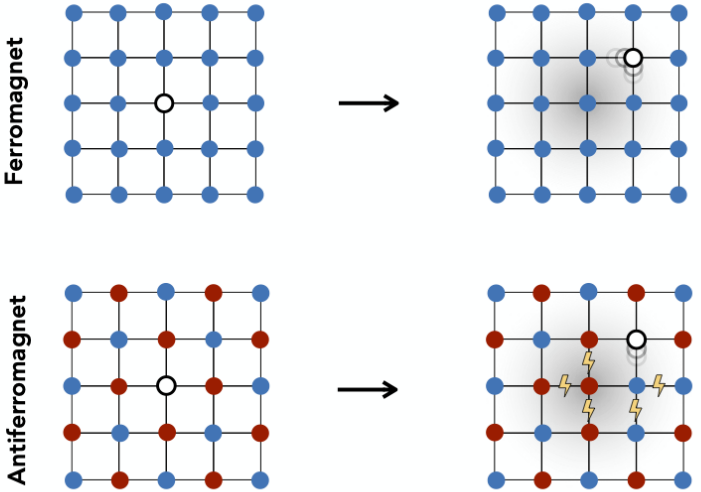
Doping = 0 + $U \gg t \Rightarrow H_{\mathrm{FH}} = \frac{4t^2}{U} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$

Doping ≠ 0: hole motion coupled to magnetic background

$t-J$ model

Auerbach, Wiley 1994

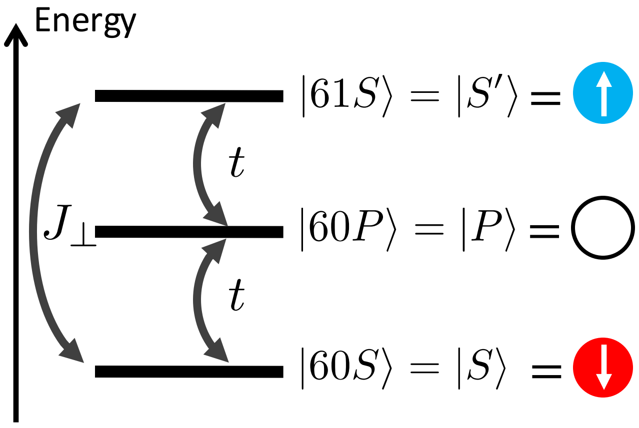
$$H_{\mathrm{FH}} = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + J \sum_{\langle i,j \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{n_i n_j}{4} \right) + \mathcal{O}(t^3/U^2)$$



L. Homeier, thesis

$t - J - V$ model using 3 Rydberg states: 2 spins and 1 hole

Idea: Homeier *et al.*, PRL 2024

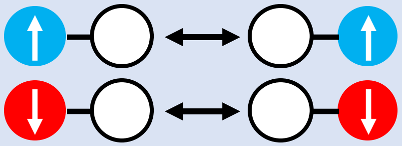


Qiao *et al.* Nature (2025)

direct dipolar exchange

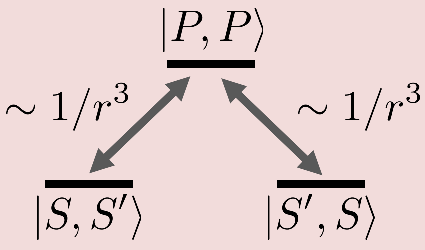
$|S, P\rangle \leftrightarrow |P, S\rangle$

$t \propto 1/r^3$

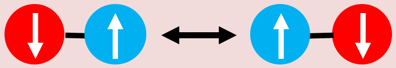


Dipolar tunneling

Indirect dipolar exchange



$J_\perp, J_z \propto 1/r^6$

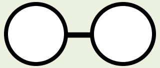


spin exchange

van der Waals shift

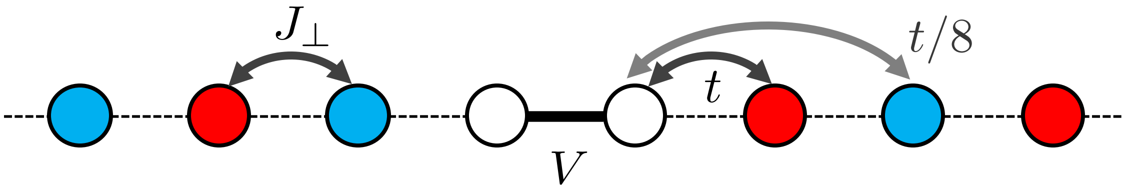
$|S, S\rangle, |P, P\rangle \ldots$

$V \propto 1/r^6$



hole-hole repulsion

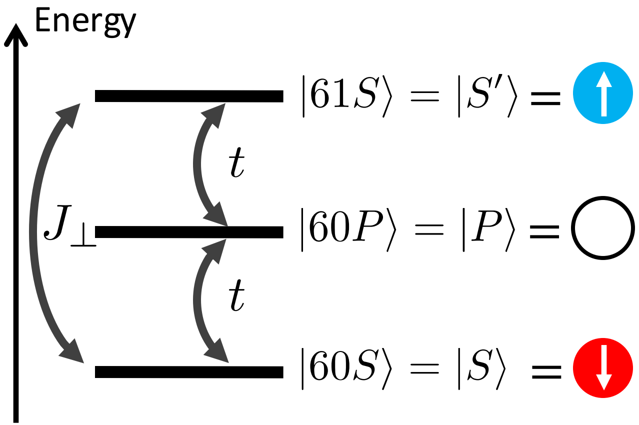
Bosonic *t*-*J*-*V* model:



Long-range hopping…

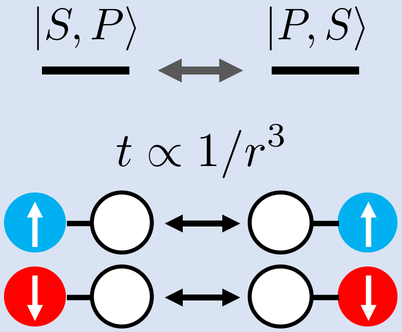
$t-J-V$ model using 3 Rydberg states: 2 spins and 1 hole

Idea: Homeier *et al.*, PRL 2024



Qiao *et al.* Nature (2025)

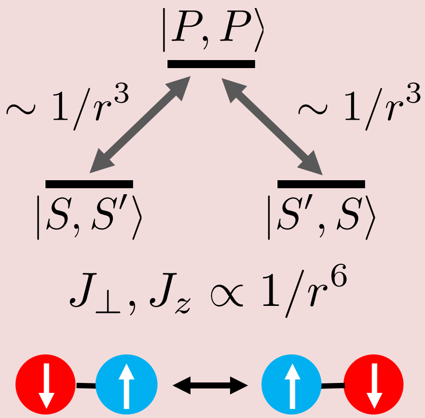
direct dipolar exchange



$t \propto 1/r^3$

Dipolar tunneling

Indirect dipolar exchange



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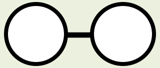
spin exchange

van der Waals shift

$|S,S\rangle, |P,P\rangle \ldots$

$V \propto 1/r^6$



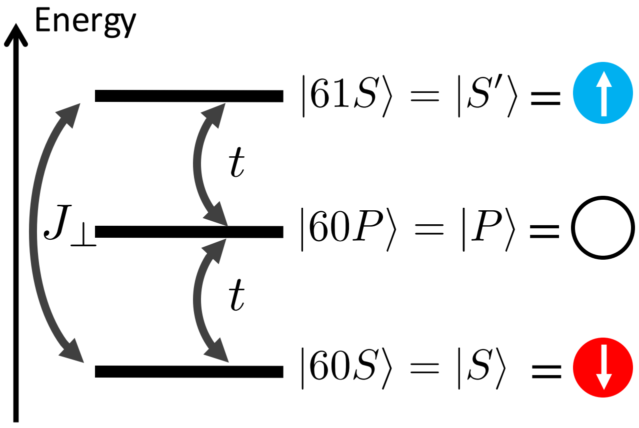
hole-hole repulsion

Bosonic *t*-*J*-*V* model:

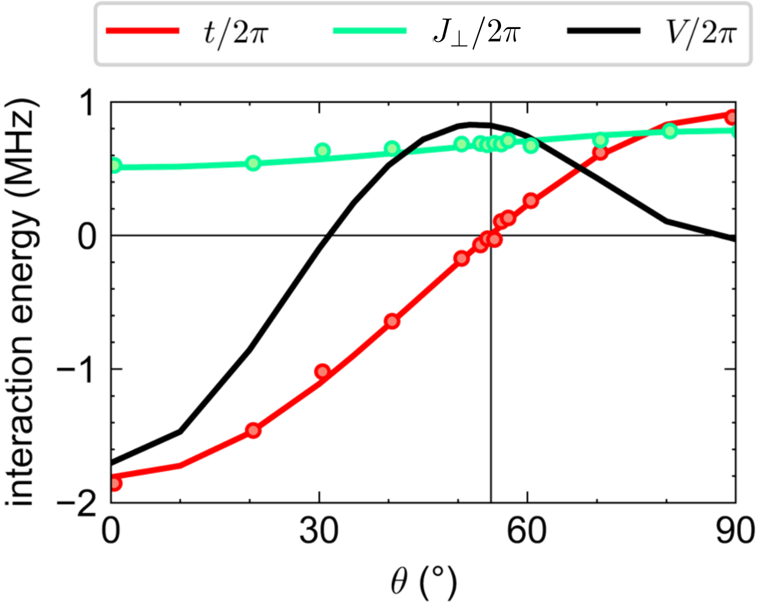
$$\hat{H}_{tJV} = -\sum_{i\neq j}\sum_{\sigma=\downarrow,\uparrow}\frac{t_\sigma}{r_{ij}^3}\left(\hat{a}^\dagger_{i,\sigma}\hat{a}^\dagger_{j,h}\hat{a}_{i,h}\hat{a}_{j,\sigma}\right) + \sum_{i\neq j}\frac{1}{r_{ij}^6}\left[J^z\hat{S}^z_i\hat{S}^z_j + \frac{J_\perp}{2}\left(\hat{S}^+_i\hat{S}^-_j\right)\right] + \sum_{i\neq j}\frac{V}{r_{ij}^6}\hat{n}^h_i\hat{n}^h_j$$

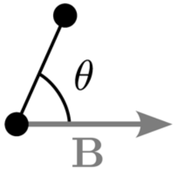
$t-J-V$ model using 3 Rydberg states: 2 spins and 1 hole

Idea: Homeier *et al.*, PRL 2024



Qiao *et al.* Nature (2025)





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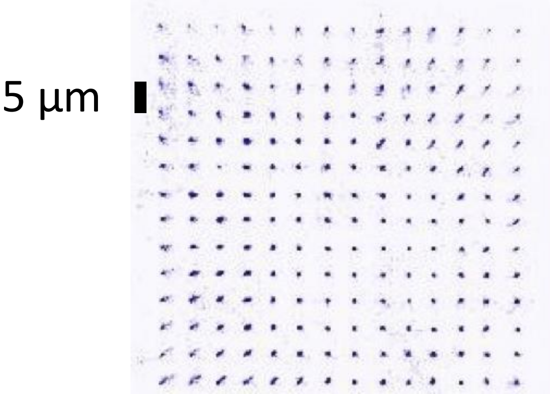
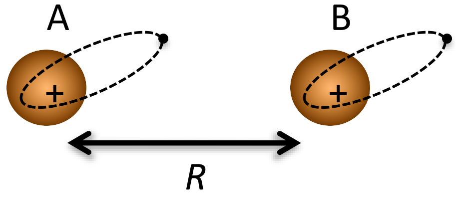
Combining arrays of atoms and Rydberg interactions



+

Rydberg interactions



Van der Waals $\frac{C_6}{R^6}$

resonant $\frac{C_3}{R^3}$

Quantum Ising
s = 1/2

Hardcore boson

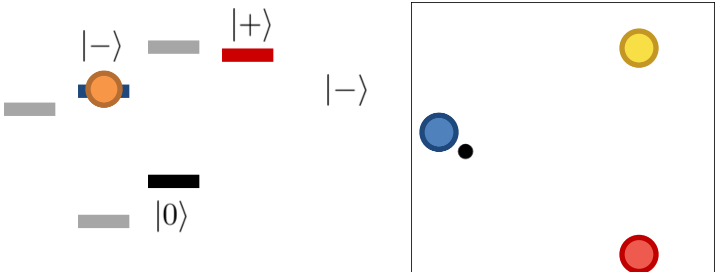
Bosons/ Fermions
Softcore potential

XY, s = 1/2
$\frac{1}{R^3}, \frac{1}{R^6}$

XYZ
Heisenberg
s = 1/2
Floquet

t- J model

Spin-orbit coupling



Lienhard, PRX 2021

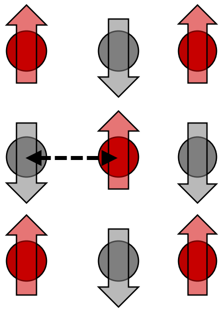
The program

Lecture 1: Arrays of atoms & “Rydbergology”
Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 2: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism

Spin models: one of the "simplest" many-body systems

Interacting spin ½ particles on a lattice:

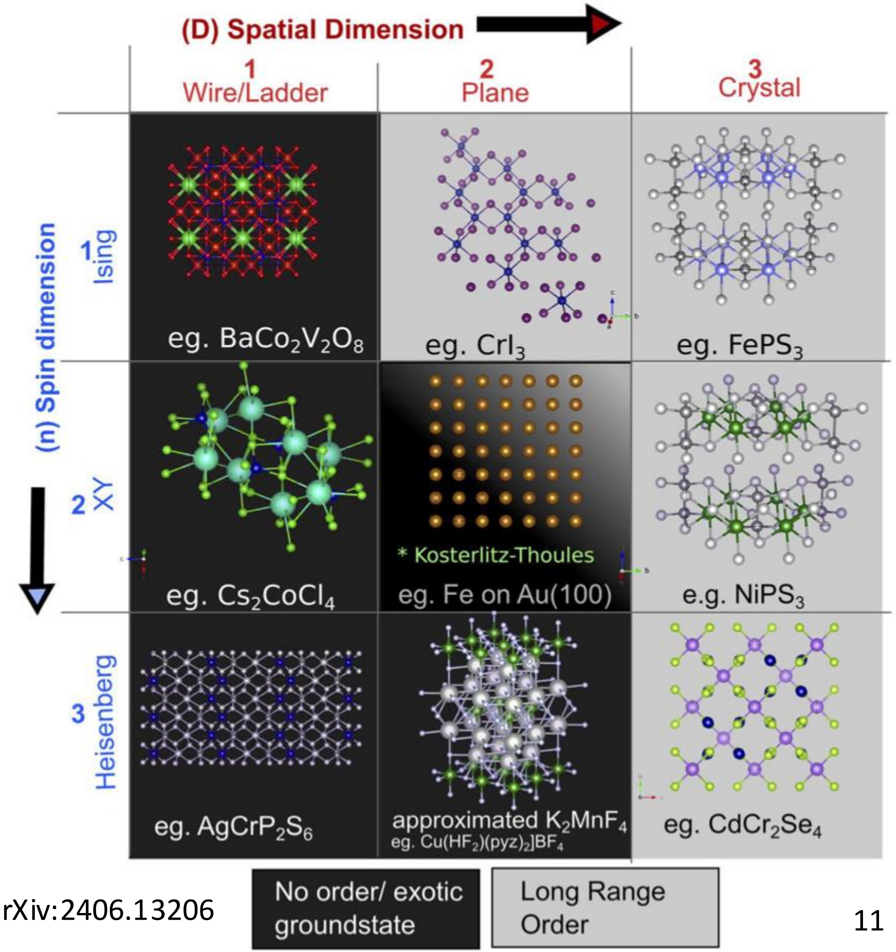


$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Heisenberg

Ising $$\hat{H}_{\text{Ising}} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

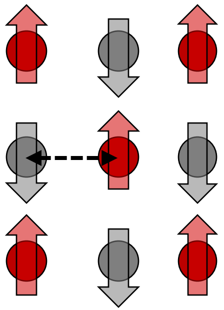
XY model $$\hat{H}_{\text{XY}} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$



arXiv:2406.13206

Spin models: one of the “simplest” many-body systems

Interacting spin ½ particles on a lattice:



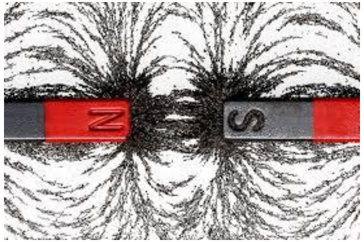
$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Heisenberg

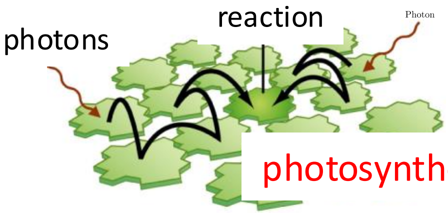
Ising $$\hat{H}_{\rm Ising} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

XY model $$\hat{H}_{\rm XY} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^{+} \hat{\sigma}_j^{-} + \hat{\sigma}_i^{-} \hat{\sigma}_j^{+} \right)$$

Magnetism



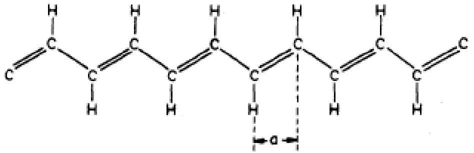
Transport of excitations



photosynthesis

Light scattering

excitons

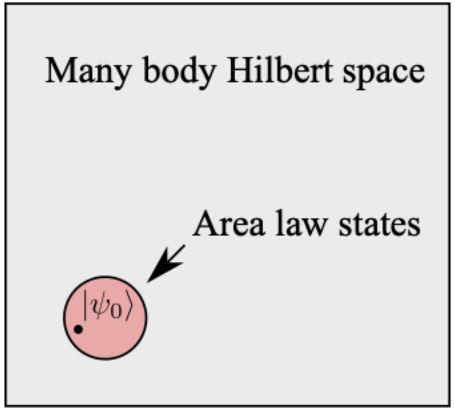


Spin models = generic systems to study many-body questions:

Quantum phase transition, out-of equilibrium, topology, entanglement…

How to study a many-body system?

Ground-state



g.s. of *gapped* many-body system
= weakly entangled (area laws)

"Easy" to simulate: Monte Carlo, MPS
(when no frustration, away from QPT)

Out-of-equilibrium
Dynamics following "quench"

Involve ~ *all states* of Hilbert space

Very hard to simulate:
Massive generation of entanglement

(Volume law)

Hauschild & Pollman, SciPost 2016

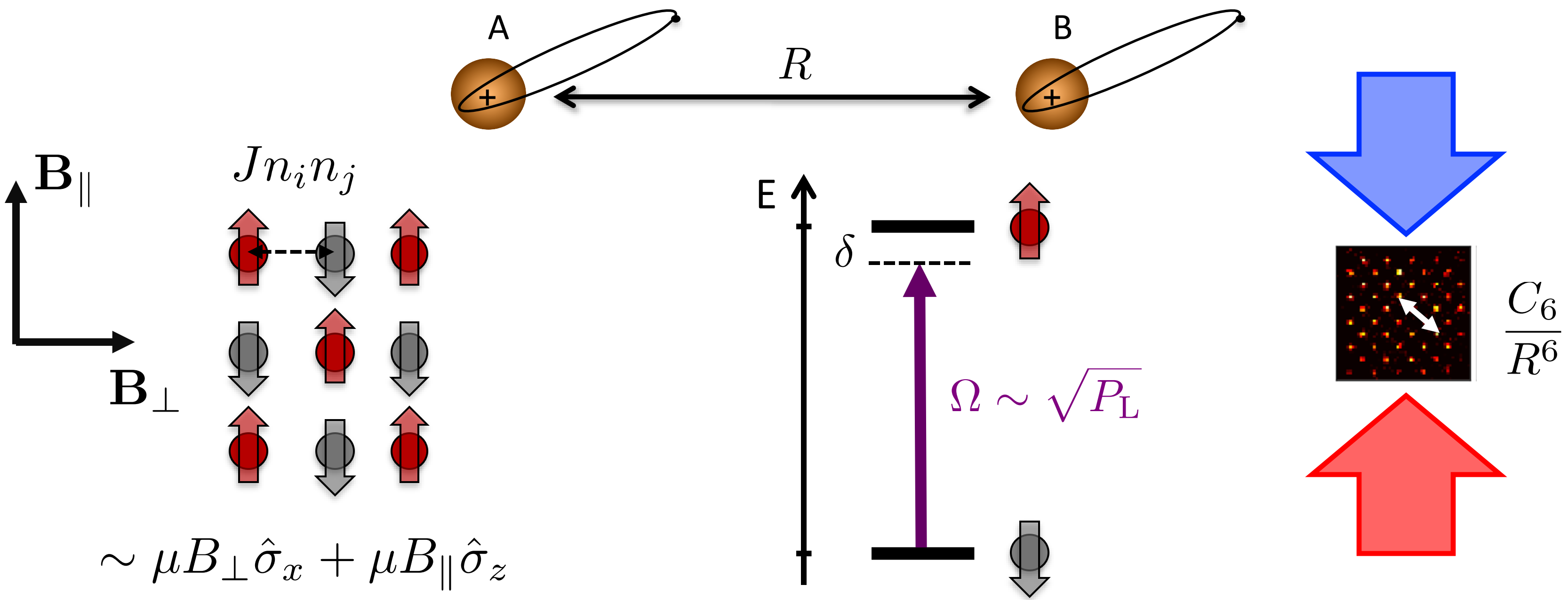
Outline – Lecture 2

1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

2. Out-of-equilibrium dynamics
    - Quench dynamics in Ising model: thermalization or not…
    - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… & beyond

From van der Waals interaction to spin models...



Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta\sum_i \hat{\sigma}_z^i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

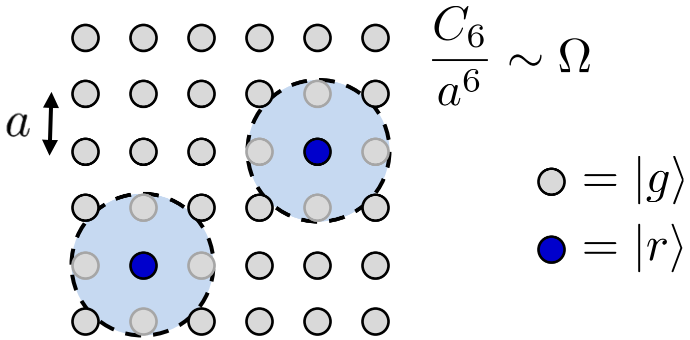
Laser: $B_{\perp}$ $B_{\|}$ spin-spin interactions

Controlled parameters:

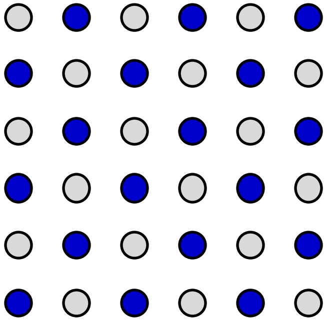
From negligible to dominant interactions

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

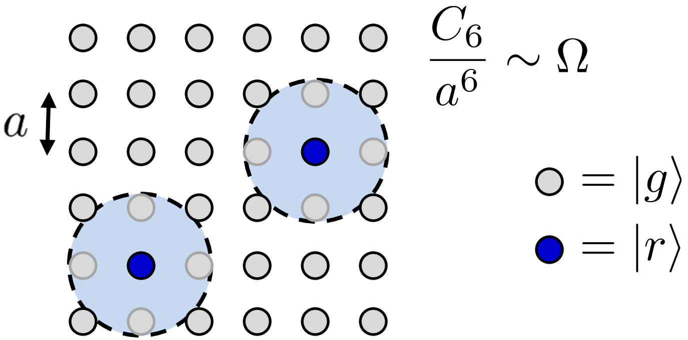


Anti-ferromagnetic ground state

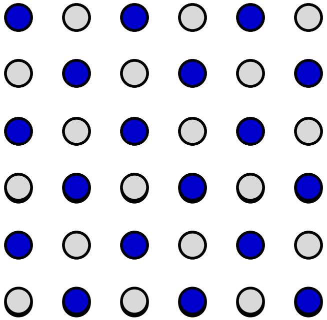


2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

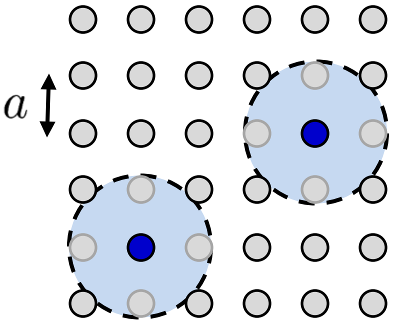


Anti-ferromagnetic ground state



2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

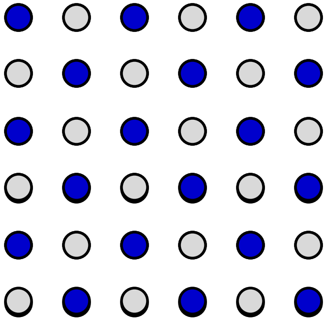


$$\frac{C_6}{a^6} \sim \Omega$$

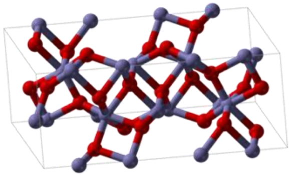
$\bigcirc = |g\rangle = |\downarrow\rangle$

$\bullet = |r\rangle = |\uparrow\rangle$

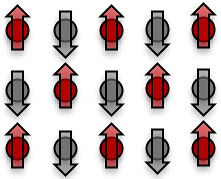
Anti-ferromagnetic ground state



Ex of antiferromagnets:
MnO, FeO, CoO, NiO, $FeCl_2$...

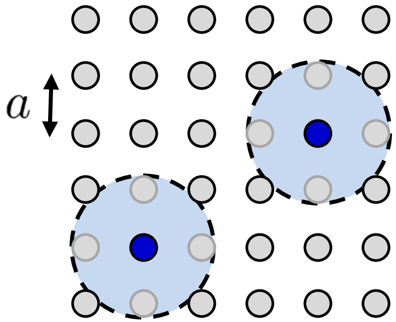


AFM (Néel) ordering (Z_2 phase)



2D Ising anti-ferromagnet on a square

Nearest neighb. interaction



$$\frac{C_6}{a^6} \sim \Omega$$

○ $= |g\rangle = |\downarrow\rangle$

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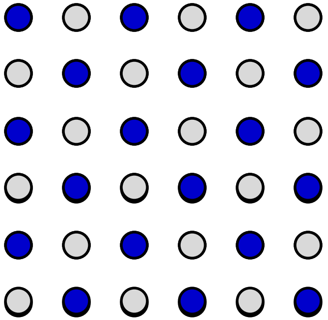
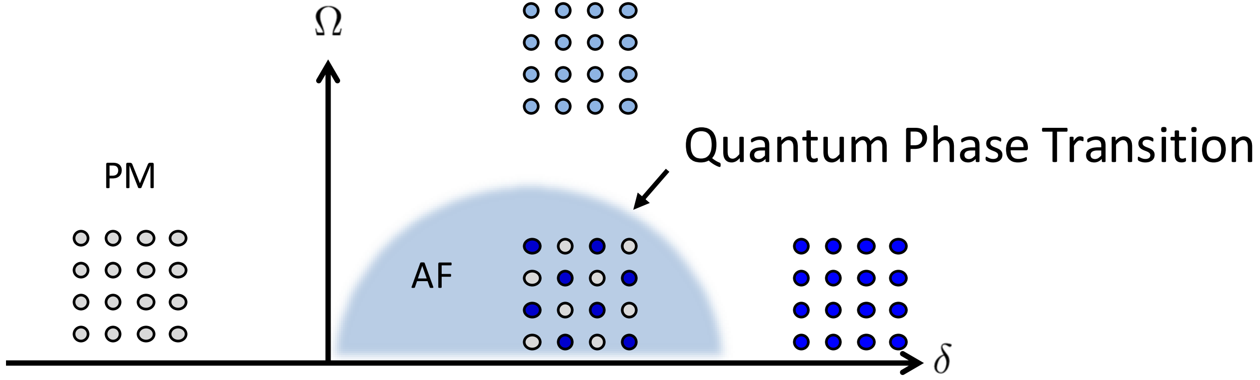
Anti-ferromagnetic ground state



2D phase diagram
(1970)

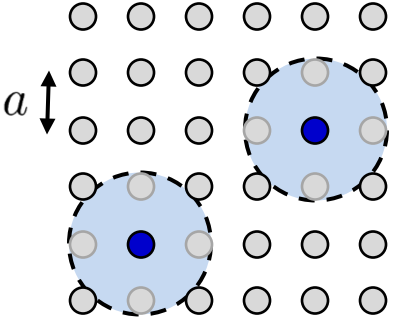


Known by Quantum Monte-Carlo

Never implemented and measured in 2D… (approximation in material)

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

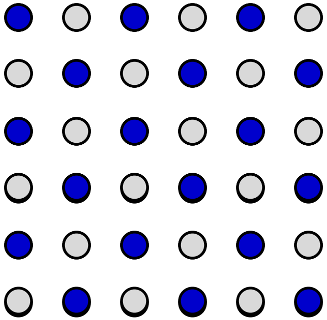


$$\frac{C_6}{a^6} \sim \Omega$$

$$\bigcirc = |g\rangle = |\downarrow\rangle$$

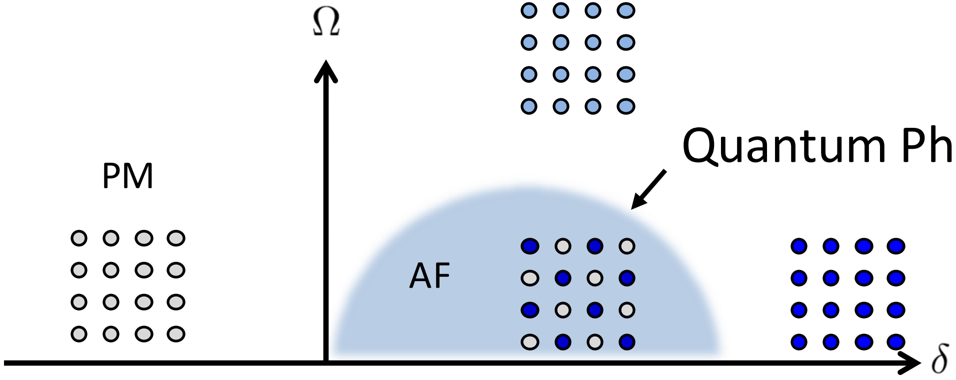
$$\bullet = |r\rangle = |\uparrow\rangle$$

Anti-ferromagnetic ground state



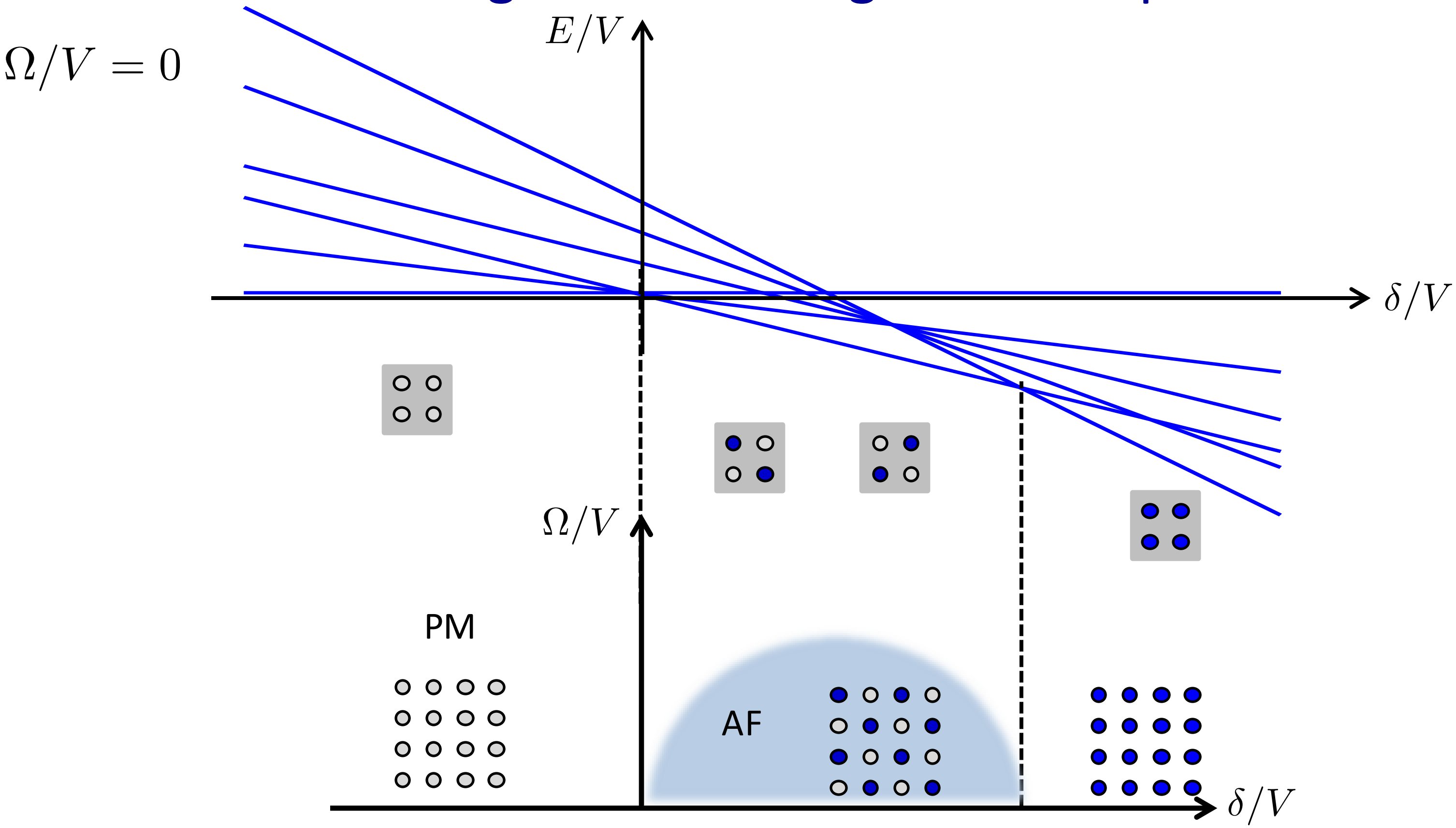
2D phase diagram

(1970)



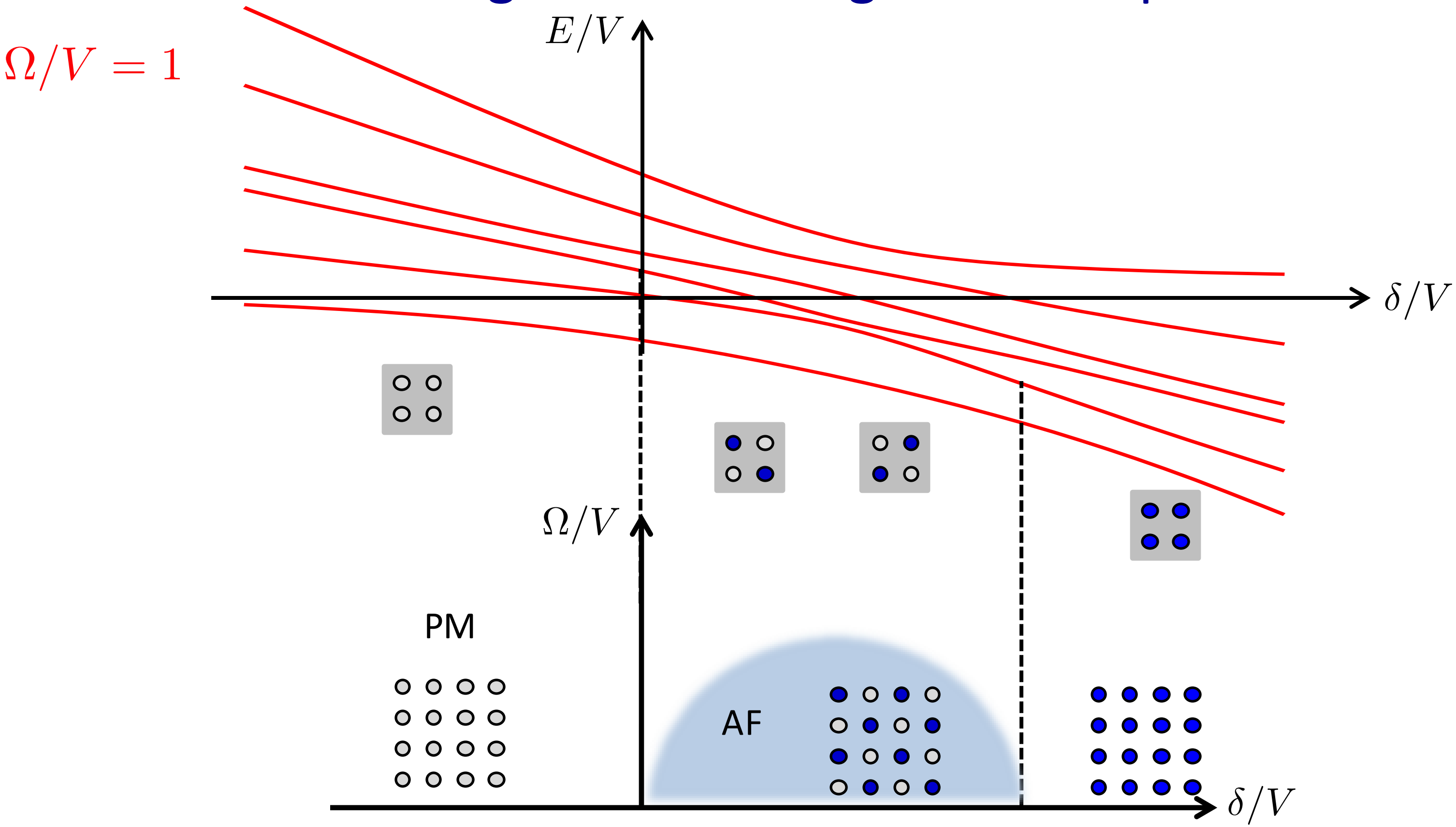
$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

2D Ising anti-ferromagnet on a square



$$H = -\hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

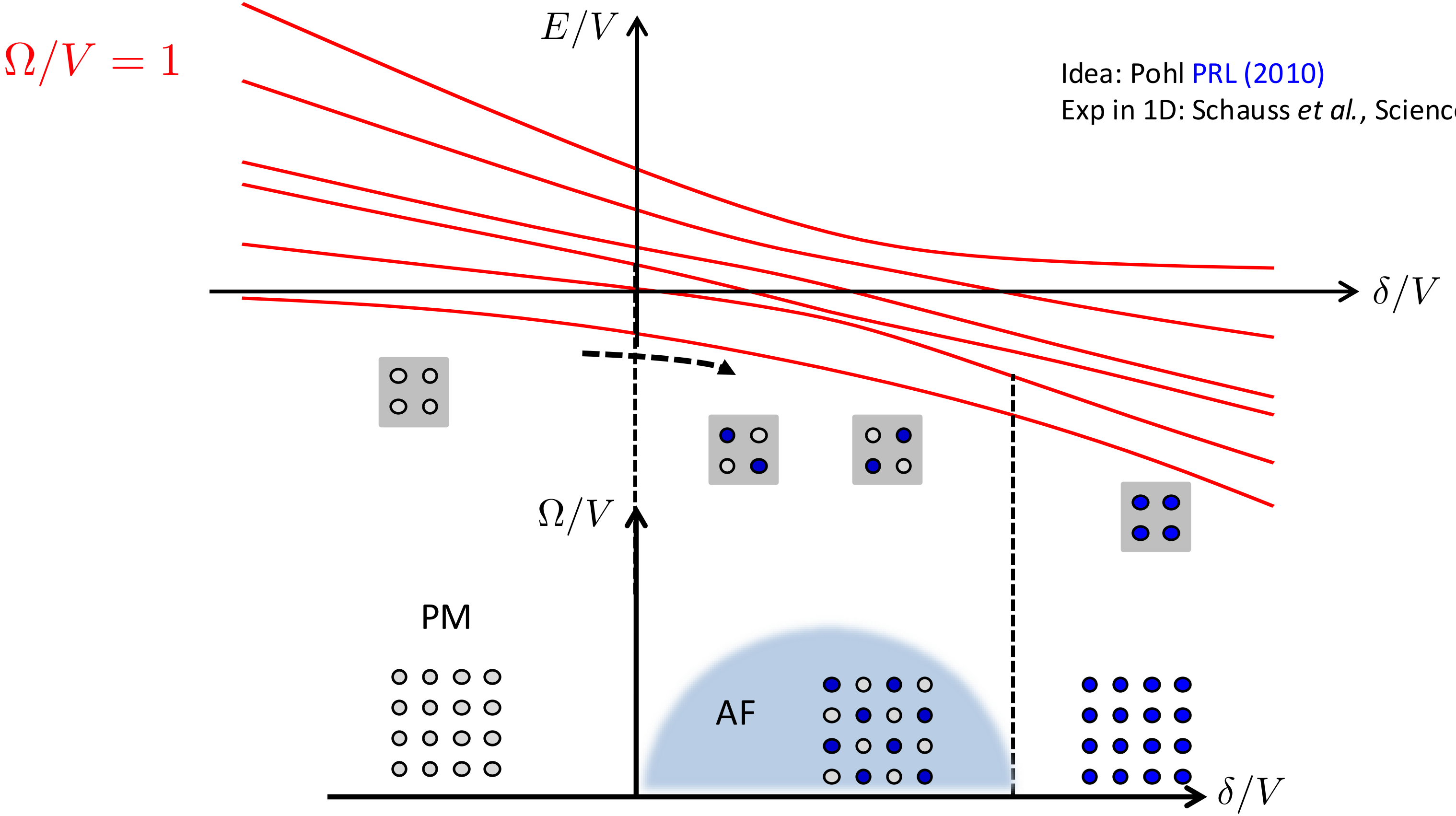
2D Ising anti-ferromagnet on a square



$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

Adiabatic preparation of a 2D Ising anti-ferromagnet

Idea: Pohl PRL (2010)
Exp in 1D: Schauss *et al.*, Science (2015)



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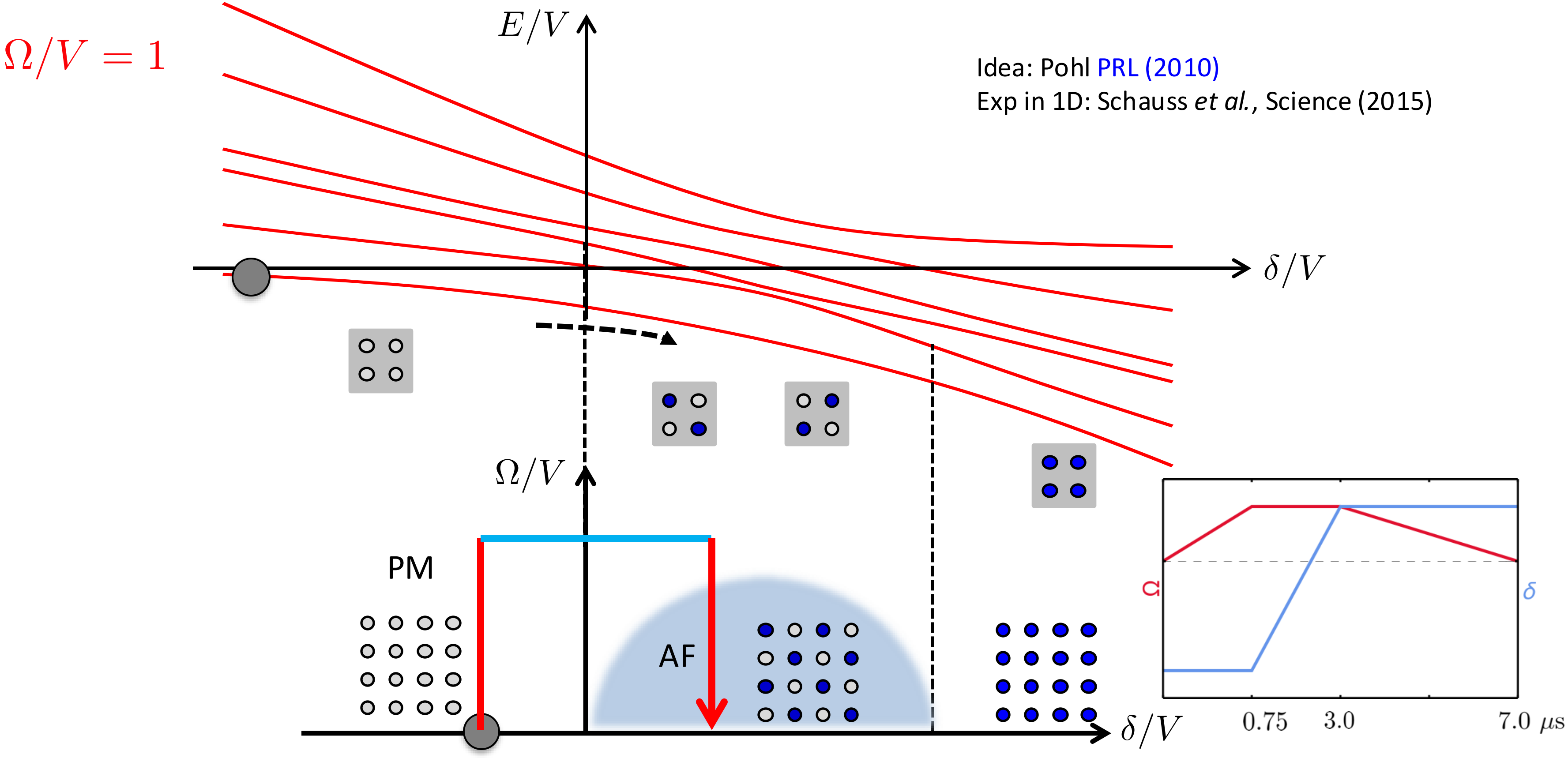
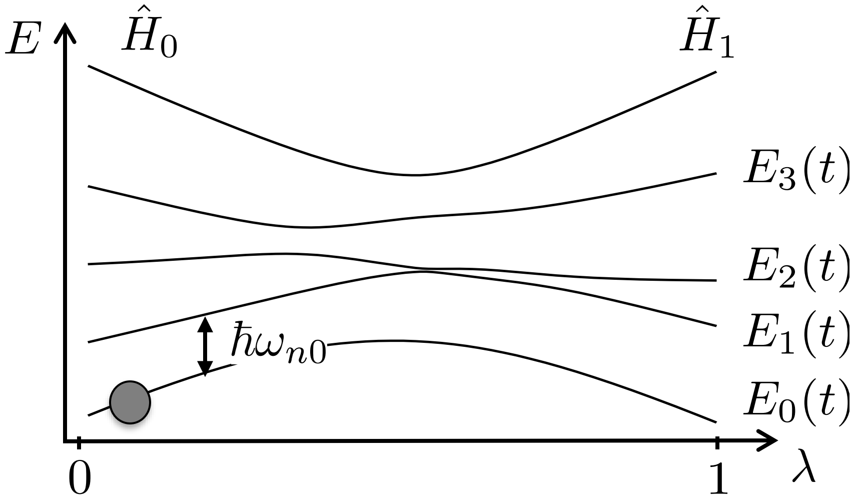


$$H = \sum_i \left(\frac{\hbar\Omega(t)}{2} \sigma_x^i - \hbar\delta(t) \hat{n}_i \right) + \sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Adiabatic preparation of a ground state: quantum annealing



Sakurai, Quantum Mechanics & Wikipedia

$$\hat{H}(t) = (1-\lambda(t))\hat{H}_0 + \lambda(t)\hat{H}_1$$

Instantaneous eigenstates:

$$\hat{H}(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle$$

Solve: $i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$ with $|\psi(t)\rangle = \sum_n a_n(t)|\phi_n(t)\rangle,\ a_n(0) = \delta_{n,0}$

$$\Rightarrow |a_n(t)| \sim \frac{|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2}{\hbar\omega_{n0}^2}$$

Adiabatic following: $|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2 \ll \hbar\omega_{n0}^2$

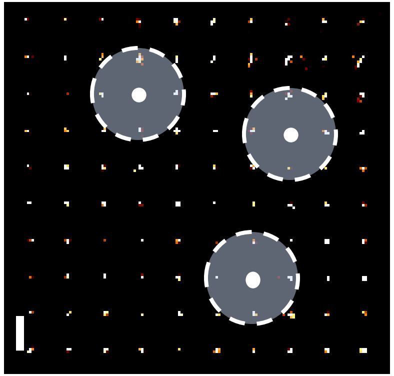
Rate of change of *H* slow with respect to the energy gaps…

Adiabatic preparation of an antiferromagnet on a square array

$$\frac{C_6}{a^6} \sim \Omega$$

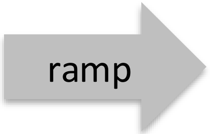
Scholl et *al.* Nature (2021)

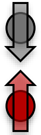
10 × 10



10 μm

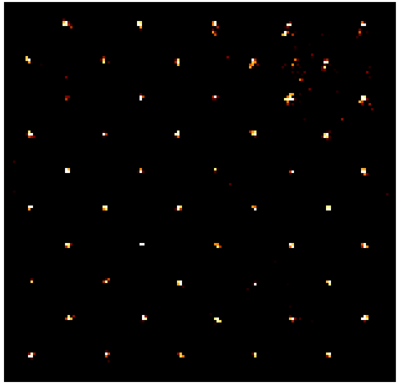
$\Omega(t), \delta(t)$





$= |f\rangle$ "bright"

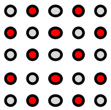
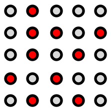
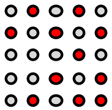
$= |r\rangle$ "dark"



1D: Pohl PRL 2010; Bloch Science 2015; Lukin Nature 2017, 2019;
2D: Lienhard PRX 2018, Bakr PRX 2018; Lukin Nature 2021

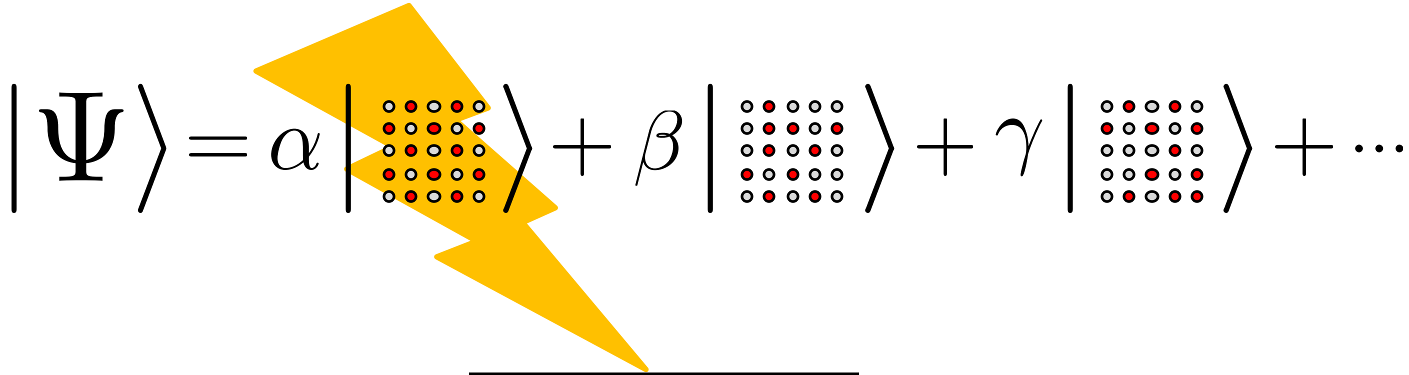
Seeing the many-body wavefunction…

At the end of experiment:

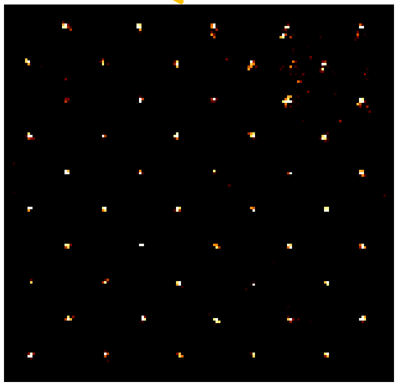
$|\Psi\rangle = \alpha \,|$  $\rangle + \beta \,|$  $\rangle + \gamma \,|$  $\rangle + \cdots$

Seeing the many-body wavefunction…

At the end of experiment:



$$|\Psi_f\rangle =$$



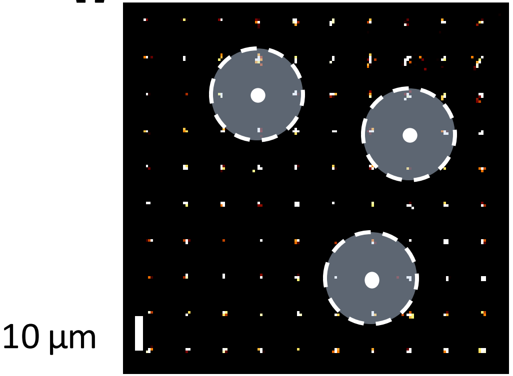
probability $|\alpha|^2$

Adiabatic preparation of an antiferromagnet on a square array

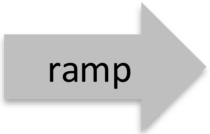
$$\frac{C_6}{a^6} \sim \Omega$$

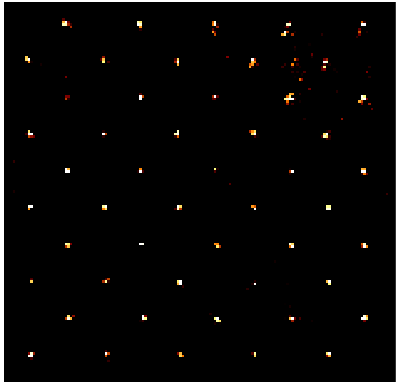
Scholl et *al.* Nature (2021)

10×10



$\Omega(t), \delta(t)$

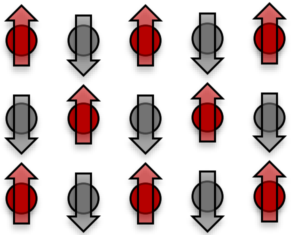




$= |f\rangle$ “bright”

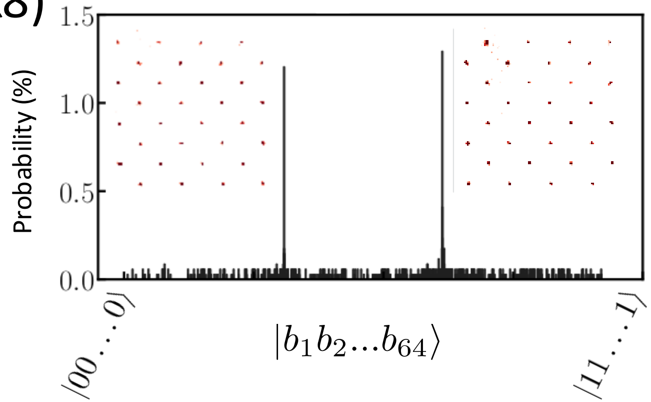
$= |r\rangle$ “dark”

Perfect AF (Néel) ordering!
(proba < 1%)



(8x8)

2^{64} states!!!

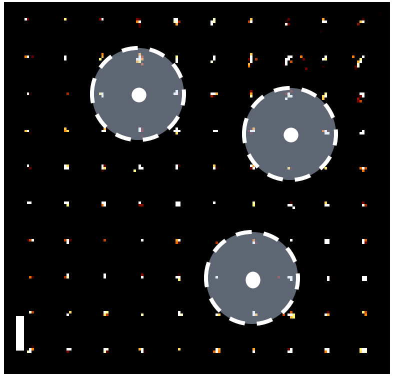


Adiabatic preparation of an antiferromagnet on a square array

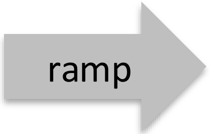
$$\frac{C_6}{a^6} \sim \Omega$$

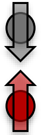
10 × 10

Scholl et *al.* Nature (2021)



$$\Omega(t), \delta(t)$$





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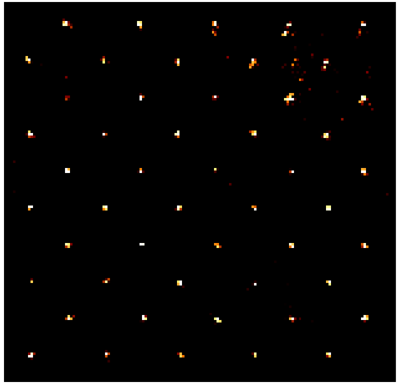
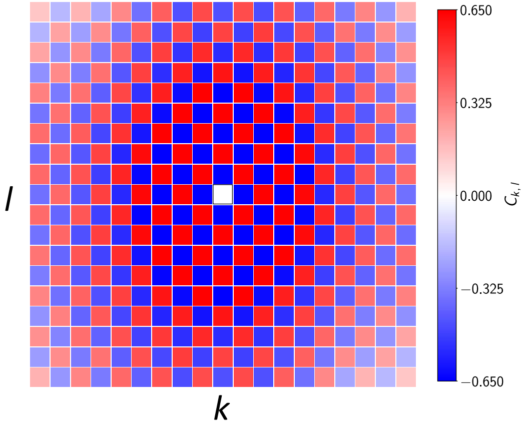
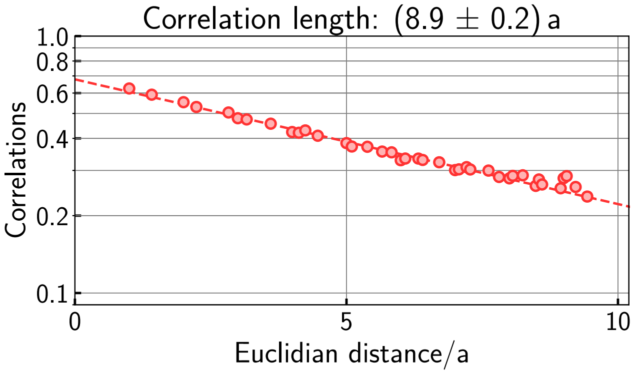


$$C_{kl} \sim \langle n_k n_l \rangle - \langle n_k \rangle \langle n_l \rangle$$



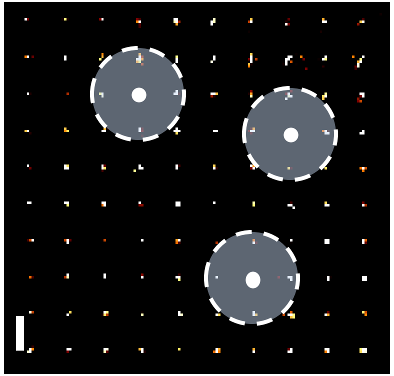


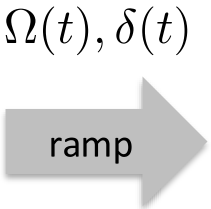
Also: Lukin Nature 2021

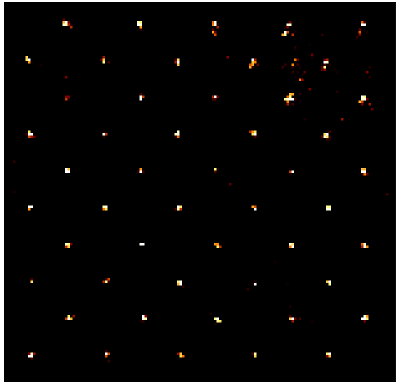
Classical simulation of the preparation

Scholl et *al.* Nature (2021)

$$\frac{C_6}{a^6} \sim \Omega$$







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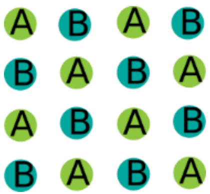
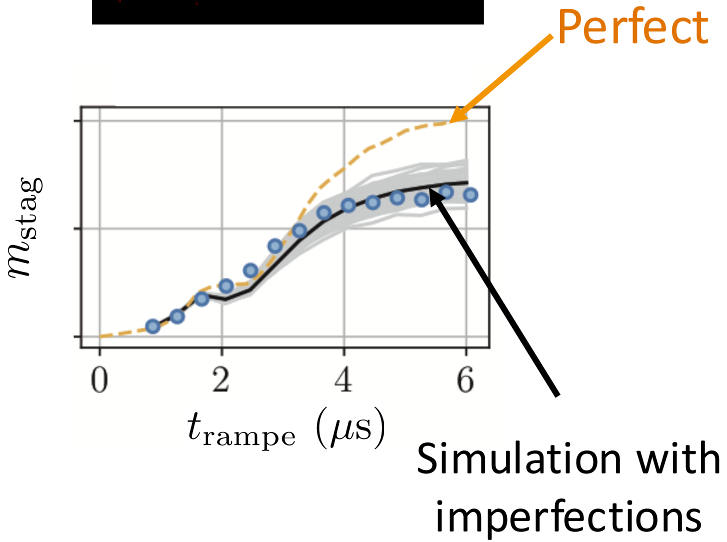


Dynamics of magnetization

$$m_{\text{stag}} = \langle |n_A - n_B| \rangle$$

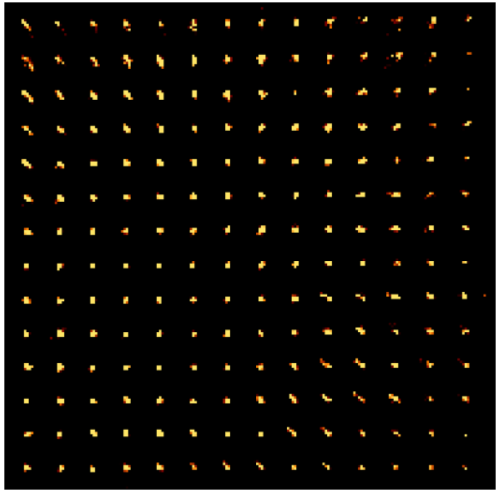


State of-the-art simulation (2021):
MPS limited to 10 x 10
(14 days on super supercomputer!!)

But we can push the atom number… by a lot…

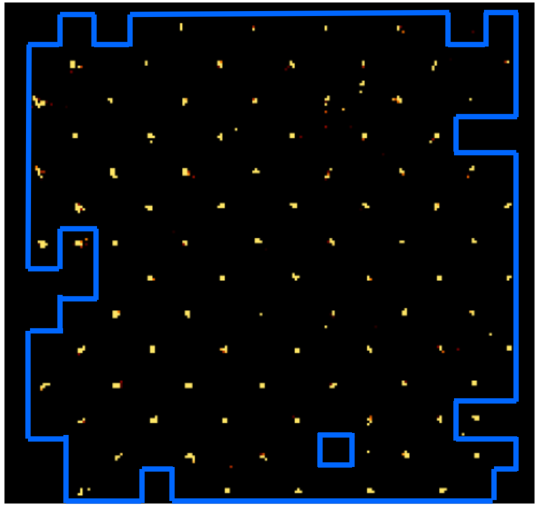
Scholl et *al.* Nature (2021)

14x14



$$\Omega(t), \delta(t)$$

ramp

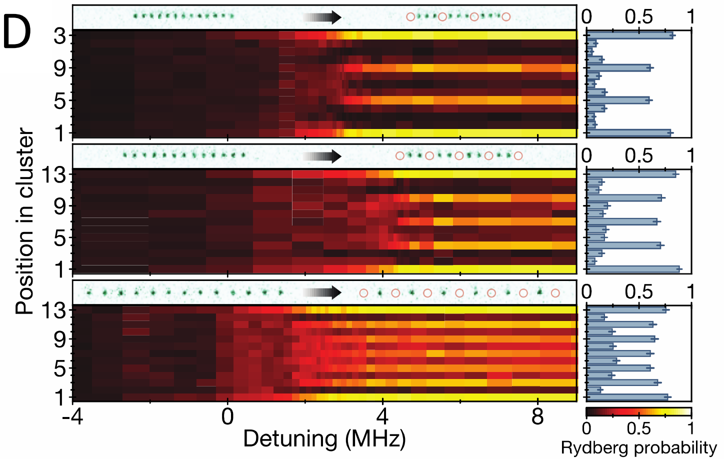


Antiferromagnetic cluster:
182 atomes

Since 2022: more elaborate numerical methods …!!

Ising model in other geometries

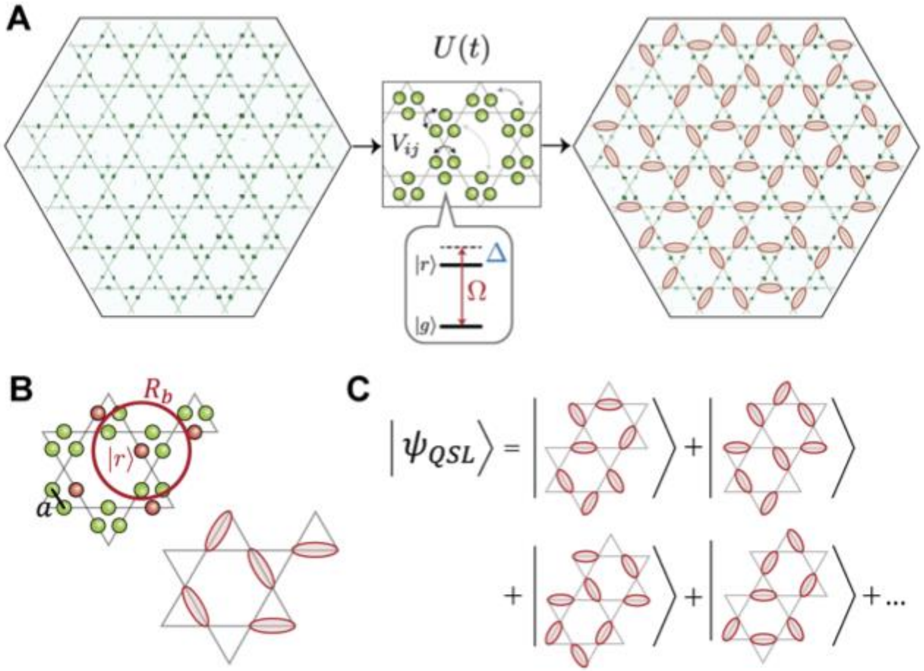
1D



Bernien, Nature 2017

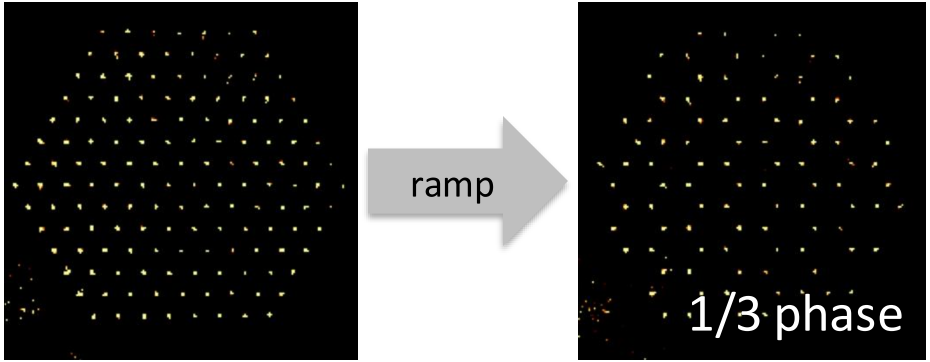
Ruby lattice: spin liquid?

Lukin, Science 2021



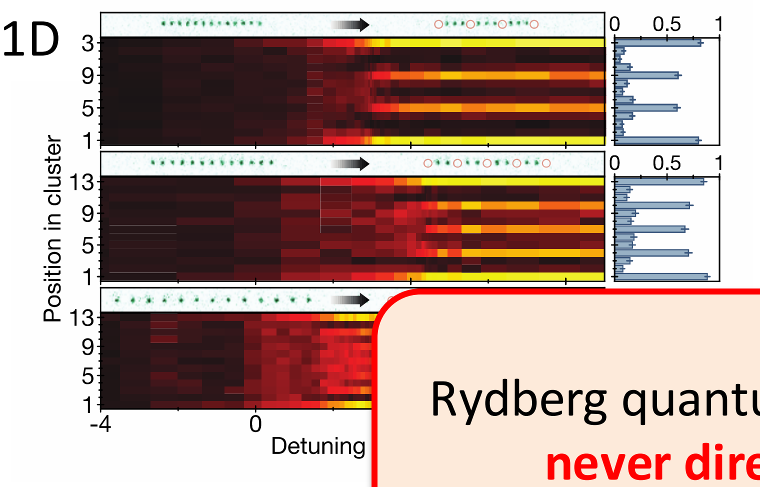
Triangle (frustration)

Scholl et *al.* Nature (2021)



Ising model in other geometries

1D



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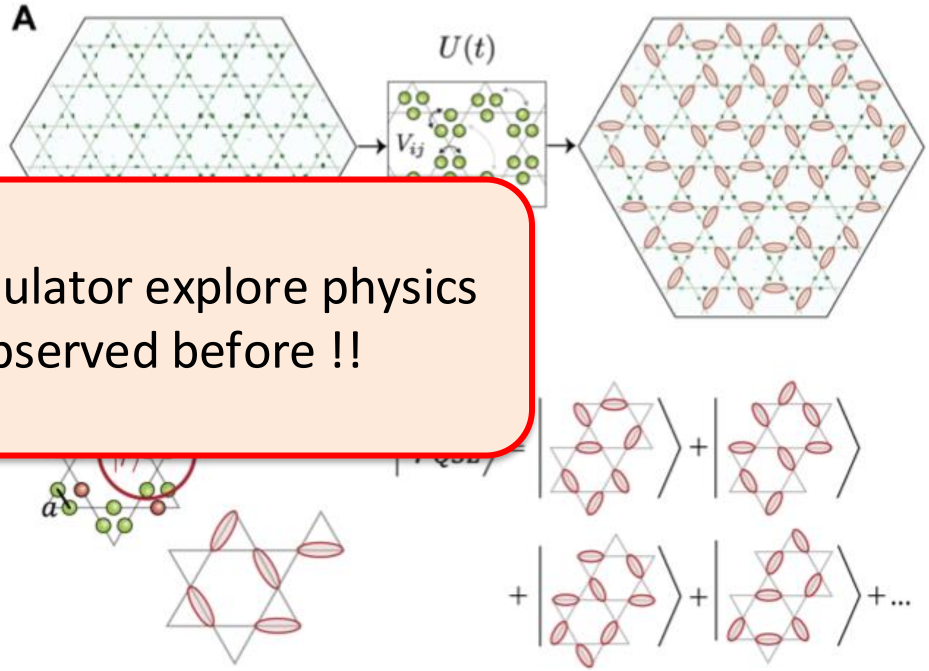
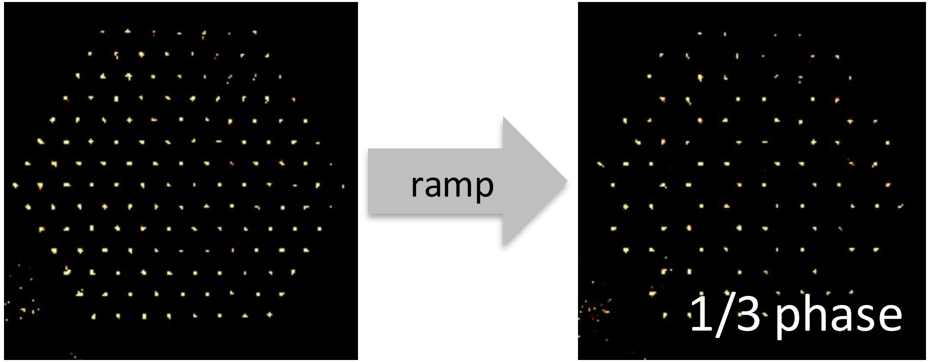
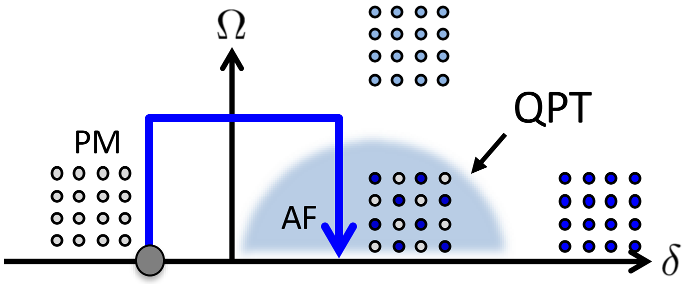


Rydberg quantum simulator explore physics **never directly** observed before !!

Triangle (frustratio



Use failure of adiabaticity to study quantum phase transition



Adiabaticity criteria:

$$H(t) = (1 - \lambda(t))H_0 + \lambda(t)H_{\mathrm{MB}}$$

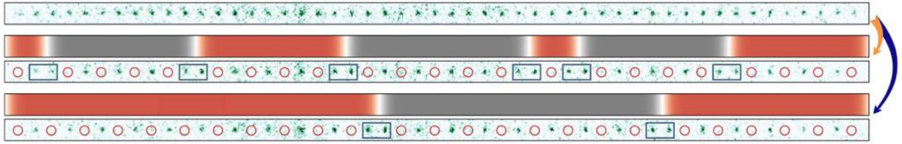
$$|\langle\psi_1(t)|\frac{dH}{dt}|\psi_0(t)\rangle| \ll \frac{\Delta E(t)^2}{\hbar}$$

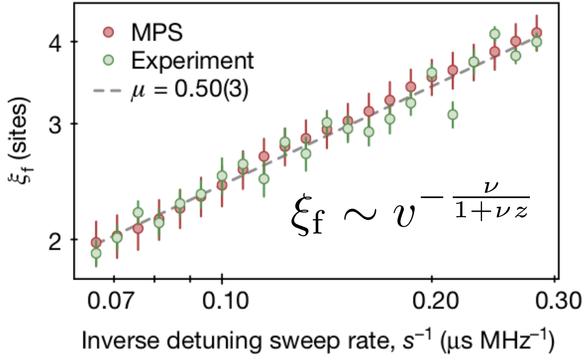
But...gaps close at the QPT!!

Sweeping too fast ⇒ create defects

1D: Keesling, Nature (2019), 2D: arXiv.2012.12281

$R_b \sim a$ 51 atoms





Kibble-Zurek mechanism:

statistics of defects ⇒ critical exponent

ν_{1D} = 0.50(3) (ν_{MF} = 1/3)

$\nu_{2D,square}$ = 0.62(4) (ν_{MF} = 1/2)

Outline – Lecture 3

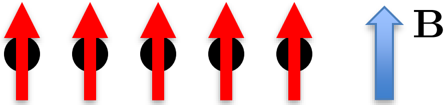
1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

2. Out-of-equilibrium dynamics
    - Quench dynamics in Ising model: thermalization or not…
    - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… & beyond

Quench in Ising Hamiltonian with Rydberg simulator

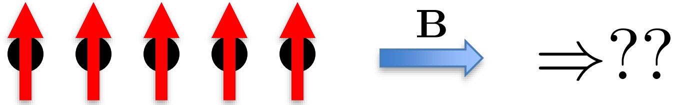
Labuhn *et al.*, Nature 2016

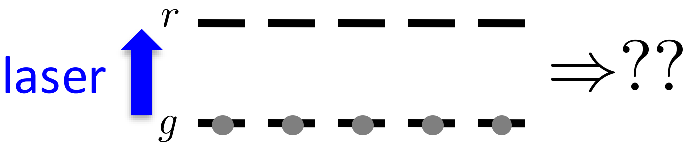


$\Rightarrow ??$

Quench in Ising Hamiltonian with Rydberg simulator

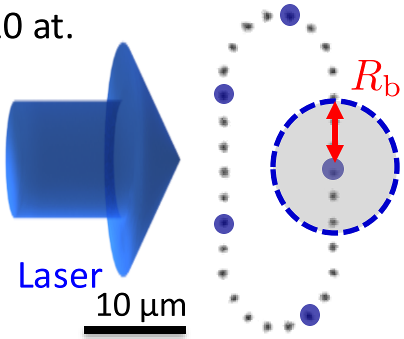
Labuhn *et al.*, Nature 2016





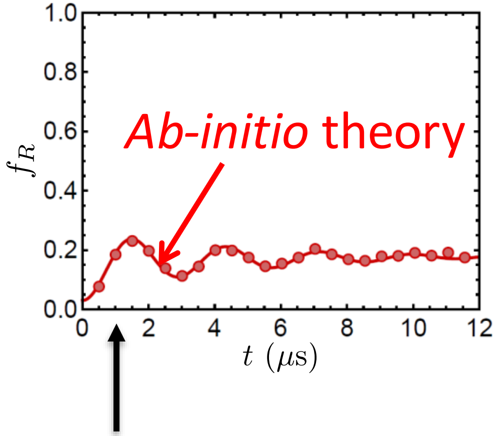
1D with periodic boundaries

20 at.



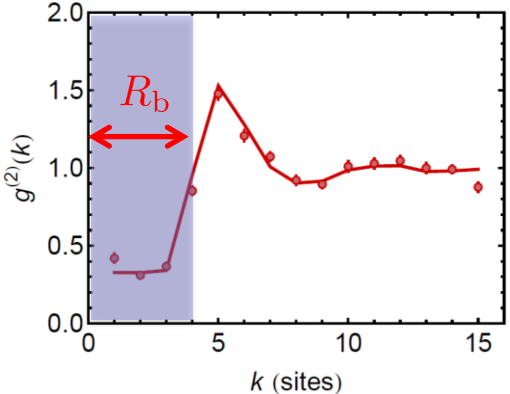
"Magnetization"

$$f_r = \frac{\langle N_r \rangle}{N}$$



Spin-spin correlation

$$\sim \langle n_j n_{j+k} \rangle$$

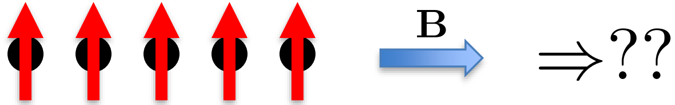


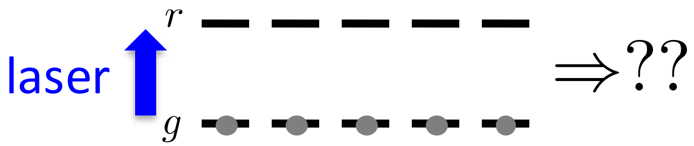
1 Rydberg atom
= hard sphere R_b

Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

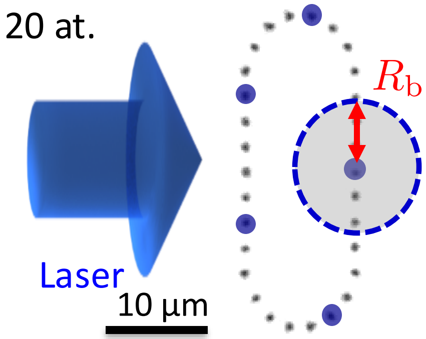
Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016





1D with periodic boundaries



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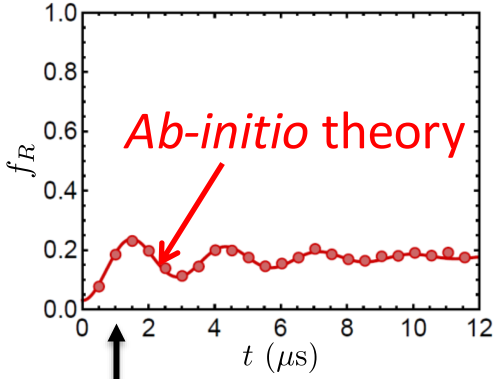

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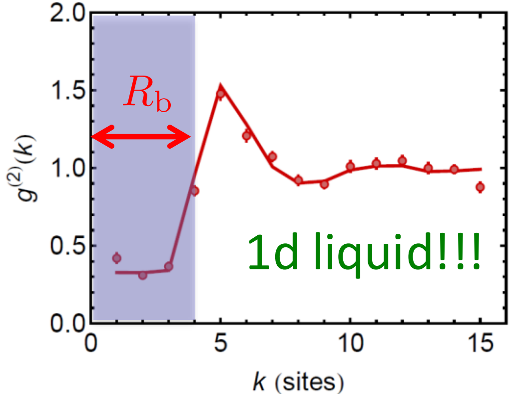
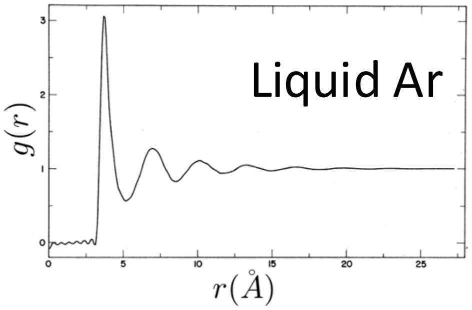
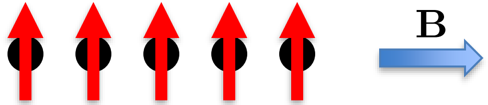




Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

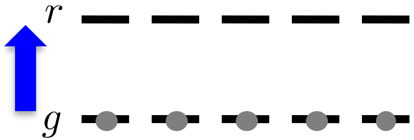
Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016



$\Rightarrow ??$

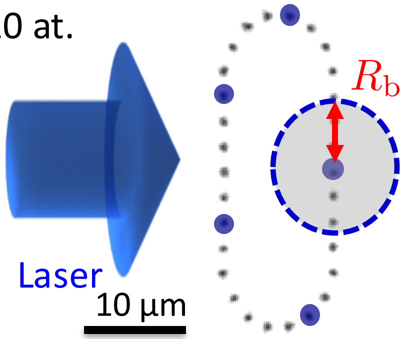
laser



$\Rightarrow ??$

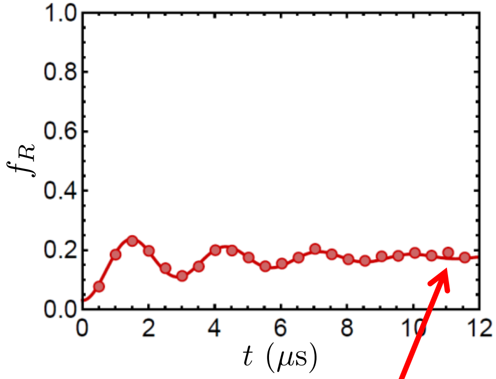
1D with periodic boundaries

20 at.



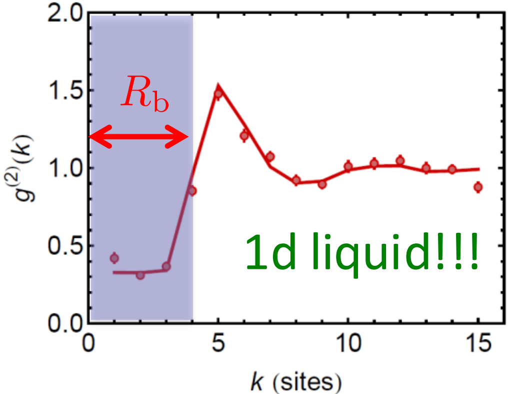
"Magnetization"

$$f_\mathrm{r} = \frac{\langle N_\mathrm{r} \rangle}{N}$$



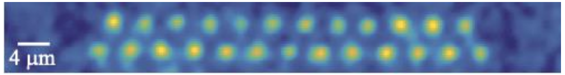
Spin-spin correlation

$$\sim \langle n_j n_{j+k} \rangle$$



Thermalization??

Kim,… Ahn, PRL 2018



Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

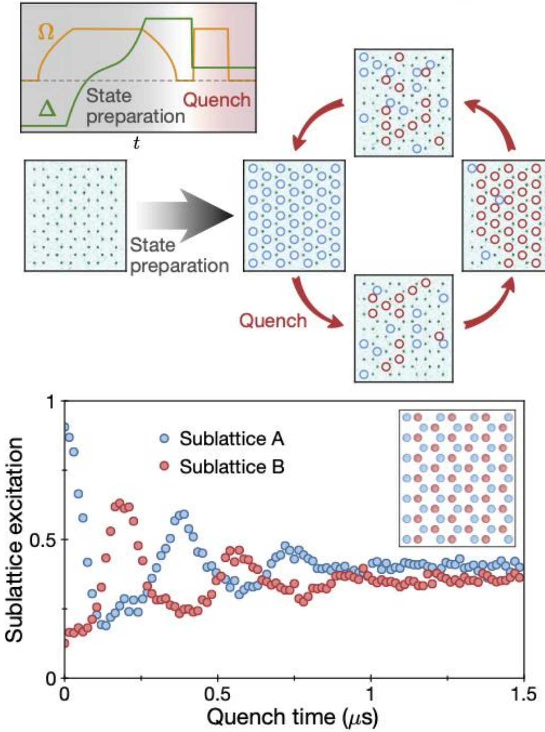
Thermalization of closed Many-Body systems

Question: do closed systems always reach equilibrium?

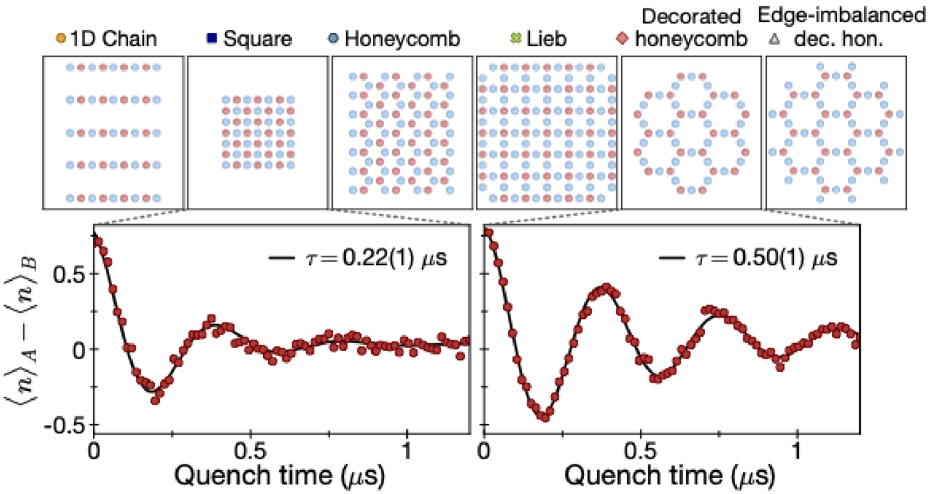
Answer: it depends… ETH, many-body localization and Quantum Scars

Quantum scarrs in 2D (**1D:** Lukin Nature 2019)

Bluvstein…Lukin, Science 2021



Scarrs depends on geometry



Blockade constraint breaks ergodicity

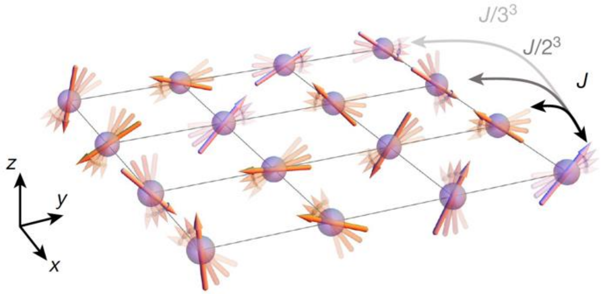
Outline – Lecture 3

1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

2. Out-of-equilibrium dynamics
    - Quench dynamics in Ising model: thermalization or not…
    - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… & beyond

Dipolar interactions leads to frustration…



XY model

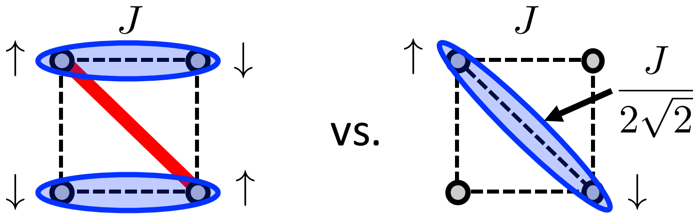
$$\hat{H} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$

Dipolar interactions

$$J_{ij} = C_3 / R_{ij}^3$$

$1/R^3 \Rightarrow$ frustration!!

AFM: $J < 0$



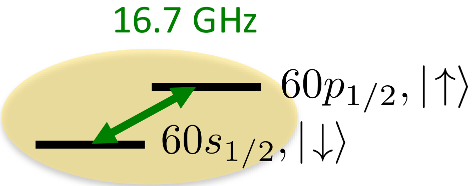
Hard to calculate

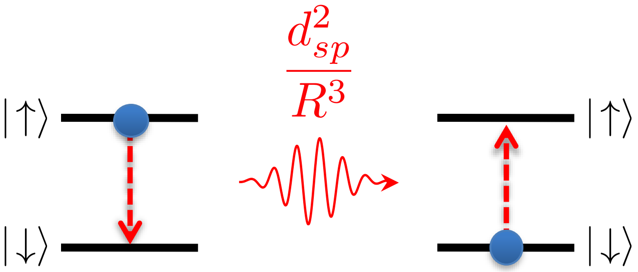
N > 100 !!

Out-of-equilibrium properties with “long-range”?

Resonant interaction between Rydbergs and XY spin model

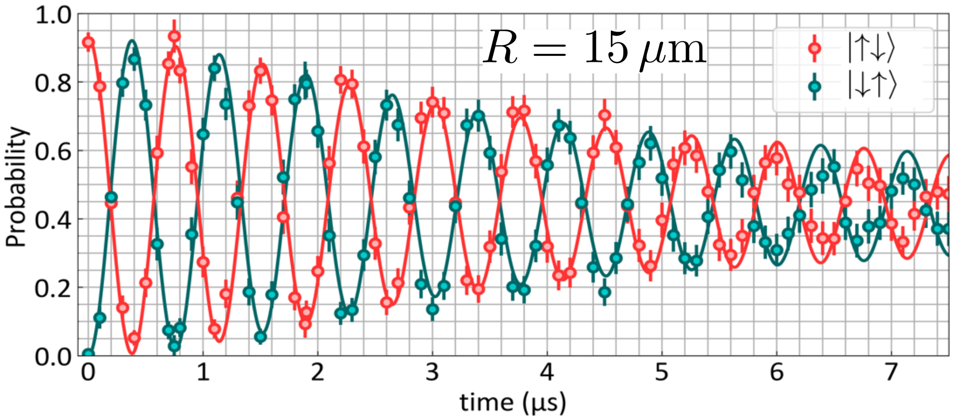
Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)
Emperauger PRA (2025)



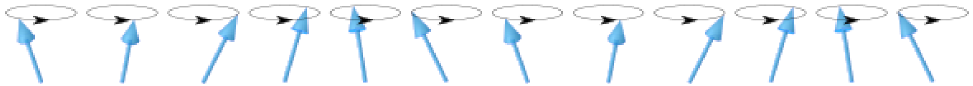


$$\hat{H}_{\mathrm{XY}} = \frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+) = \frac{C_3}{2R_{ij}^3}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y)$$

Non radiative “exchange” of excitation



Elementary excitations of *dipolar* XY model: spin waves



Wikipedia

Büchler *et al.*, PRL 109, 025303 (2012)
Roscilde *et al.*, arXiv:2303.00380

Linear spin wave theory + Bogolubov:

$$H_{\mathrm{XY}} = -J \sum_{i<j} \frac{a^3}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$$

$\sigma^+ \to \hat{b}_{\mathbf{q}}^\dagger$

$\sigma^- \to \hat{b}_{\mathbf{q}}$

Ground state — Spin wave

$$H_{\mathrm{XY}} \approx E_0 + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}}\, a_{\mathbf{q}}^\dagger a_{\mathbf{q}}$$

$$\omega_{\mathbf{q}} = J_0 \sqrt{1 - J_{\mathrm{q}}/J_0} \quad \text{with} \quad J_{\mathbf{q}} = \frac{J a^\alpha}{N} \sum_{i \neq j} (\pm 1)^{|i-j|} \frac{e^{i\mathbf{q}\cdot\mathbf{r}_{ij}}}{r_{ij}^\alpha}$$

FM (+), AFM (−)

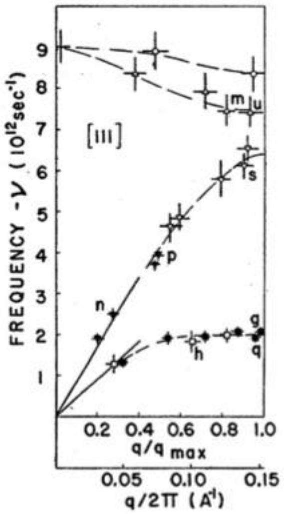
		N.N.	Dipolar
Ferro	$J > 0$	$\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$	$\omega(\mathbf{q}) \propto \sqrt{\lvert\mathbf{q}\rvert}$
Antiferro	$J < 0$	$\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$	$\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$

Manousakis RMP 1991

“Quench spectroscopy” of dipolar XY magnets

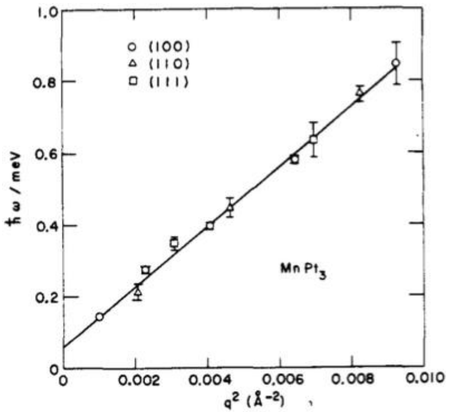
Usually: Excite system above ground state & measure dynamics

Neutron scattering



phonons

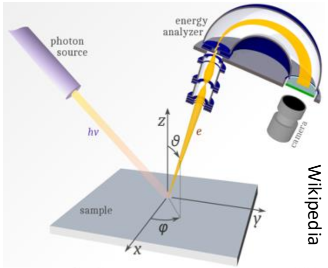
Brockhouse PR 1958



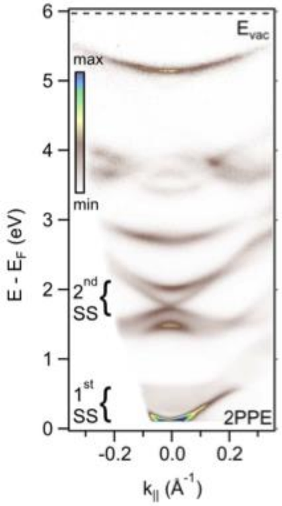
Magnons

Antonini Sol. St. Comm. 1972

ARPES

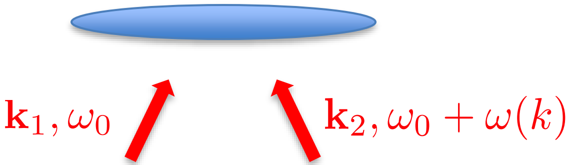


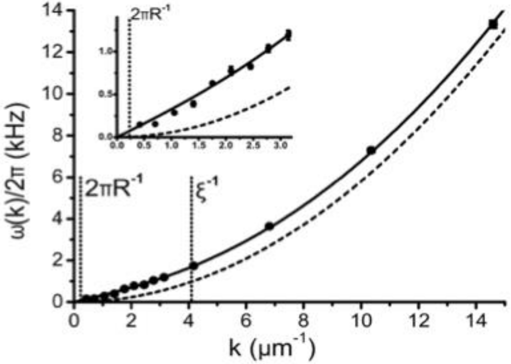
Wikipedia



Bragg spectroscopy

$\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$





Steinhauer PRL 2002

Phonons in BEC

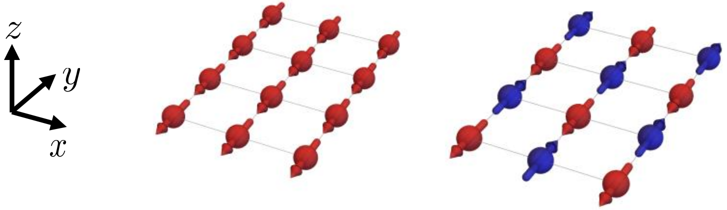
"Quench spectroscopy" of dipolar XY magnets

Usually: Excite system above ground state & measure dynamics

Here: Measure dynamics after quench preparing MEAN-FIELD ground state

"Mean-field ground state = true ground state + spin waves"

Villa, Despres & Sanchez-Palencia, PRA 2019
Roscilde *et al.*, PRB 2018, arXiv:2303.00380



Classical FM / AFM in (xy)
= MEAN-FIELD ground state
= easy to prepare state (product state)

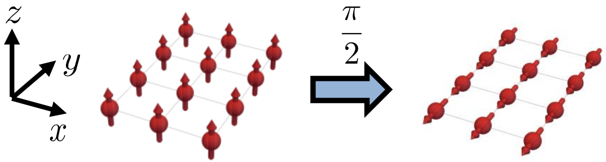
***t*-Structure factor**:
$$S_{zz}(\mathbf{q},t) \propto \sum_{i,j} \langle \sigma_i^z \sigma_j^z \rangle(t)\ e^{i\mathbf{q}\cdot(\mathbf{r}_j - \mathbf{r}_i)}$$
$$= 1 - \frac{J_\mathbf{q}}{2J_0} + \frac{J_\mathbf{q}}{2J_0}\cos(2\omega_\mathbf{q} t)$$
LSW theory

Related work: ultra-cold atoms in lattices, ions, superconducting circuits…

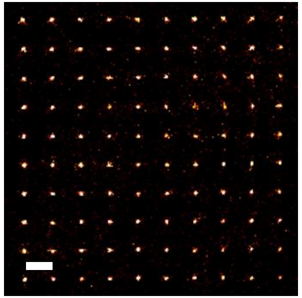
Quench spectroscopy: measuring the “dispersion relation” FM / AFM

C. Cheng *et al.*, arXiv:2311.11726

FM



10 x 10

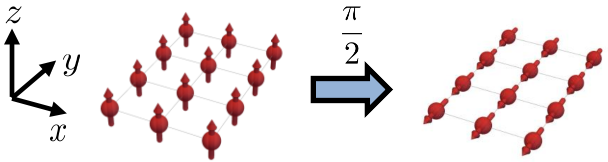


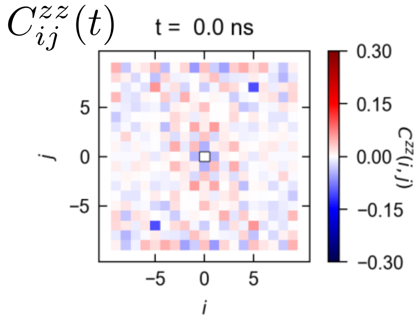
15 µm

Quench spectroscopy: measuring the “dispersion relation” FM / AFM

C. Cheng *et al.*, arXiv:2311.11726

FM





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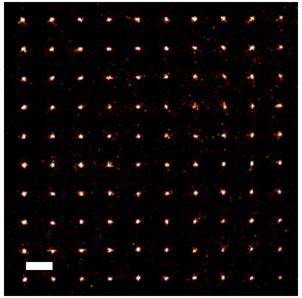

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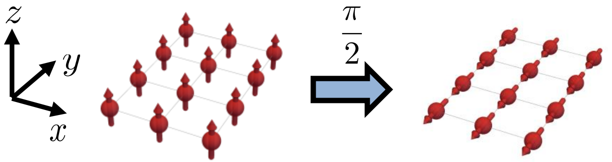
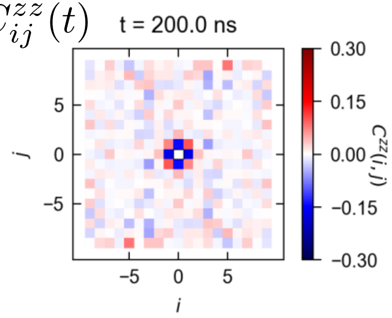


$C_{ij}^{zz}(t)$



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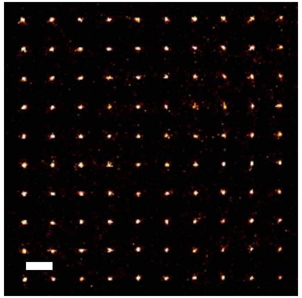

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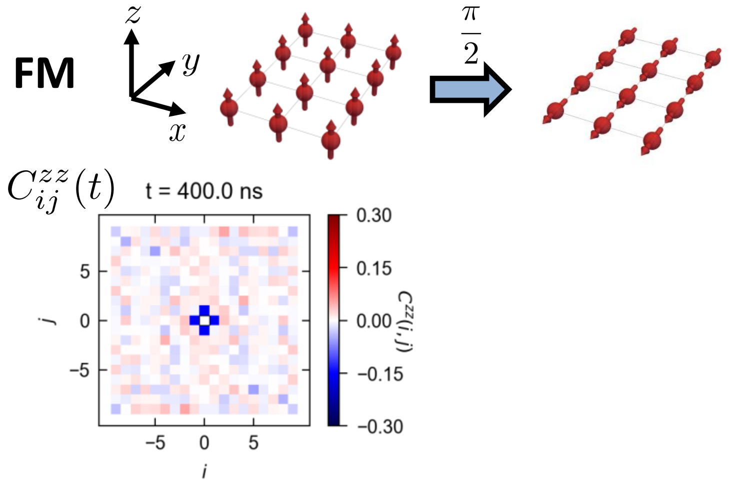
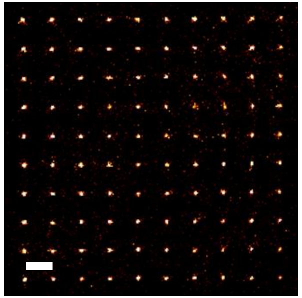
FM



10 x 10



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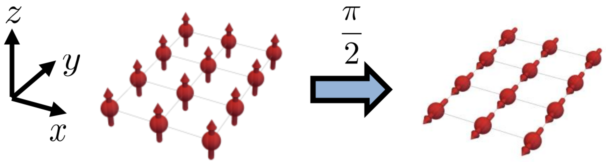

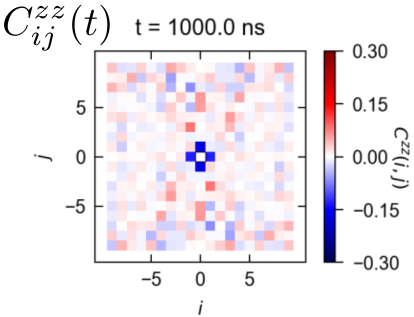

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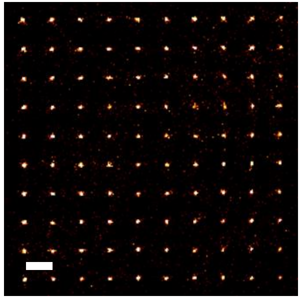

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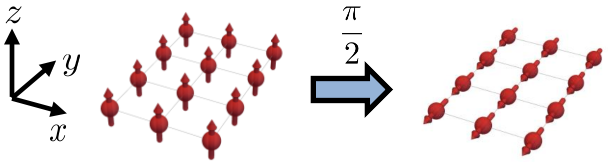
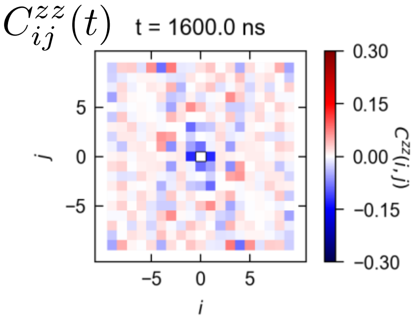
FM





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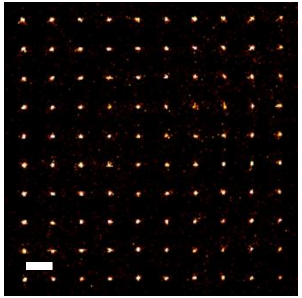

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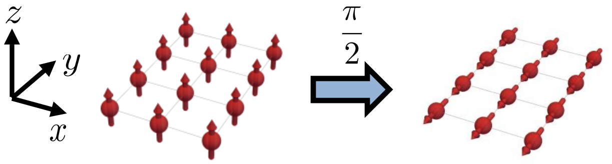

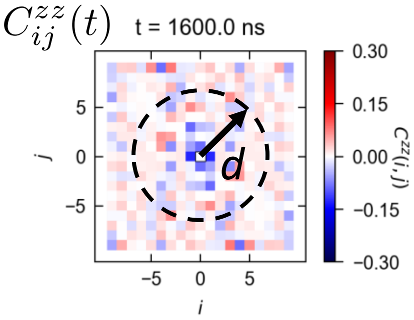

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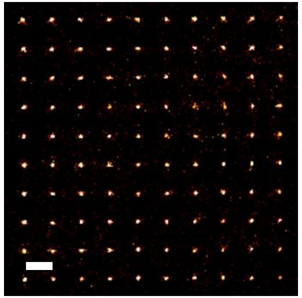

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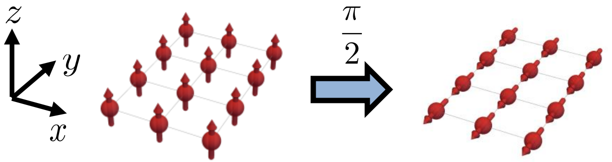
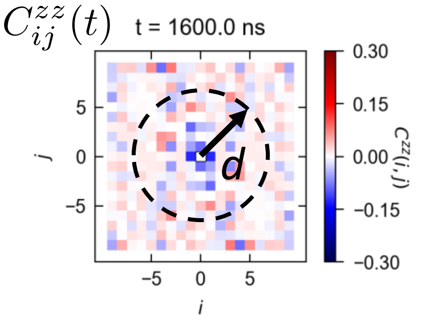
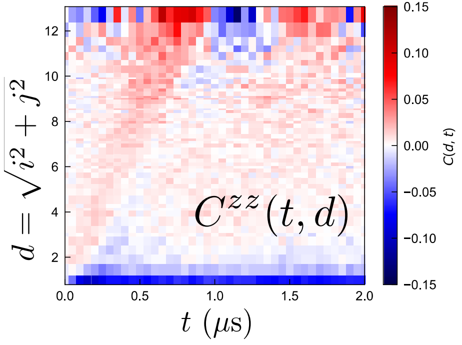
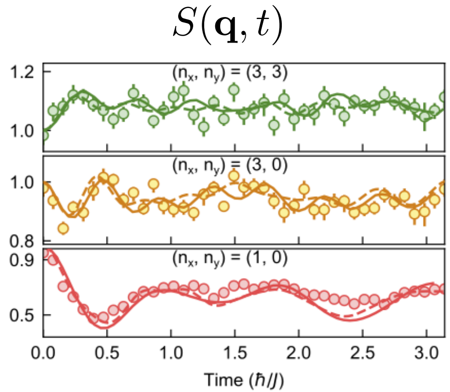
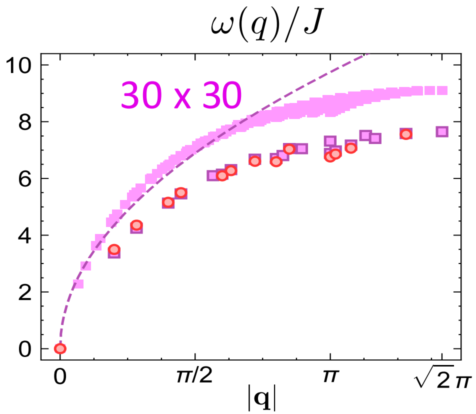






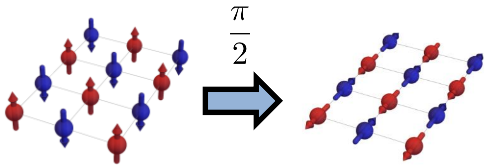


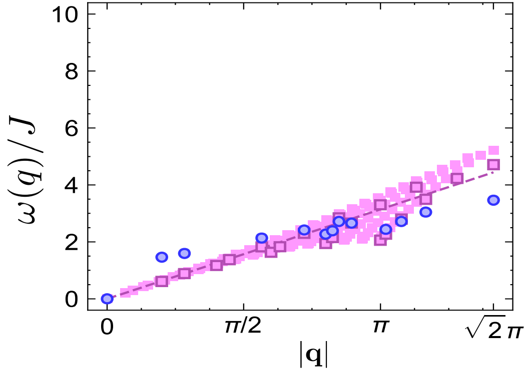


AFM

$J_{60S-60P} > 0$

FM coupling





Highest excited state $H_{\rm XY}$ = **ground state** $-H_{\rm XY}$

Same dynamics $-H_{\rm XY}/H_{\rm XY}$

$1/r^3$ Interaction modifies dispersion!!

Outline – Lecture 3

1. Studying the ground state of quantum magnets
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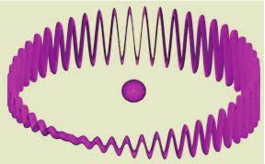
    - Dipolar XY model in 2D

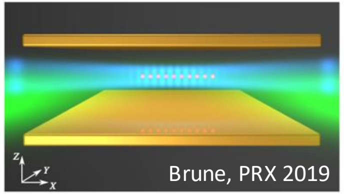
2. Out-of-equilibrium dynamics
    - Quench dynamics in Ising model: thermalization or not…
    - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… & beyond

Outlook: what we did not discuss…

New developments: circular Rydberg states ⇒ lifetimes > 50 s…





Brune (Paris): arXiv:2407.04109, Nat. Phys. 2022
Covey (Urbana Champaign)
Thompson (Princeton)
Meinert & Pfau (Stuttgart)…

overhead:
Rydberg trapping

High precision quantum simulation: validation of the simulation

Article

Benchmarking highly entangled states on a 60-atom analogue quantum simulator

https://doi.org/10.1038/s41586-024-07173-x
Received: 18 August 2023

Adam L. Shaw[1,5✉], Zhuo Chen[2,3,5], Joonhee Choi[1,4,5], Daniel K. Mark[2,5], Pascal Scholl[1], Ran Finkelstein[1], Andreas Elben[1], Soonwon Choi[2✉] & Manuel Endres[1✉]

First attempt of digital quantum simulation (and hybrid analog-digital)

Variational simulation of the Lipkin-Meshkov-Glick model on a neutral atom quantum computer

R. Chinnarasu,[1] C. Poole,[1] L. Phuttitarn,[1] A. Noori,[1,2] T. M. Graham,[1] S. N. Coppersmith,[3,1] A. B. Balantekin,[1] and M. Saffman[1,4]

arXiv:2501.06097

Probing topological matter and fermion dynamics on a neutral-atom quantum computer

Simon J. Evered[1,*], Marcin Kalinowski[1,*], Alexandra A. Geim[1], Tom Manovitz[1], Dolev Bluvstein[1], Sophie H. Li[1], Nishad Maskara[1], Hengyun Zhou[1,2], Sepehr Ebadi[1,3], Muqing Xu[1], Joseph Campo[2], Madelyn Cain[1], Stefan Ostermann[1], Susanne F. Yelin[1], Subir Sachdev[1], Markus Greiner[1], Vladan Vuletić[4], and Mikhail D. Lukin[1,†]

arXiv:2501.18554

Digital quantum simulation: resource estimates…

Quantum Science and Techno. **7**, 045025 (2022)

Number of *perfect* gates to reproduce current *imperfect* analog simulation

Gate	Gate Count	Depth
CNOT	1.7×10^5	8.4×10^3
$R_Z(\theta)$	6.8×10^4	6.7×10^2

TABLE I. Gate count and depth estimates for digital quantum simulation of the Hubbard model with $J\tau = 2.7$, $M = 100$ and $tJ = 10$.

M sites

Gate	Gate Count	Depth
CNOT	1.6×10^3	5.5×10^2
$R_Z(\theta)$	2.1×10^4	3.5×10^2

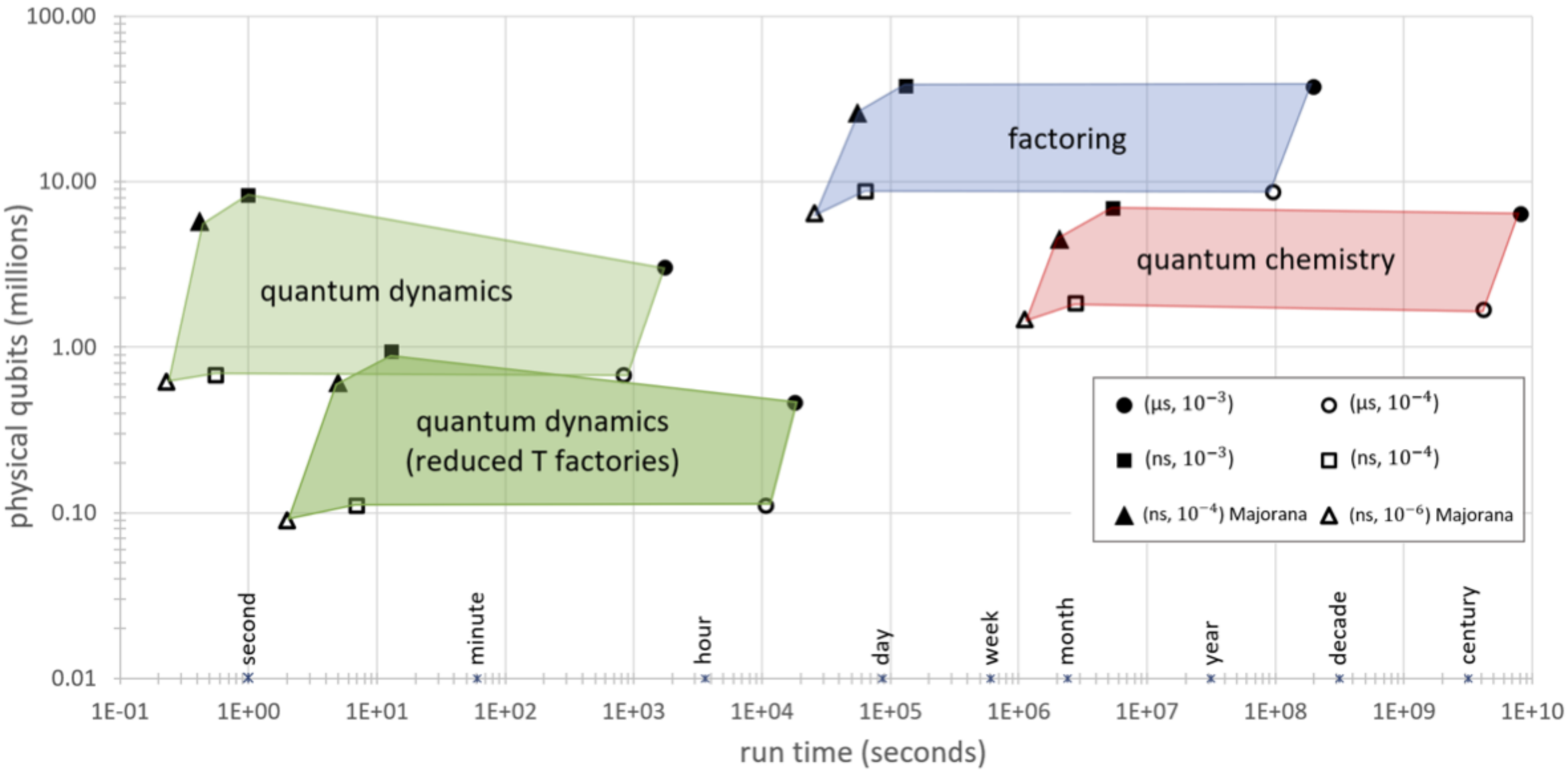
TABLE III. Gate count and depth estimates for digital quantum simulation of the nearest neighbour Ising model with $J\tau = 2.6$, $M = 100$, $tJ = 10$.

Gate	Gate Count	Depth
CNOT	6.9×10^5	1.4×10^4
$R_Z(\theta)$	3.5×10^5	7.0×10^3

TABLE II. Gate count and depth estimates for digital quantum simulation of the long-range Ising model with $J\tau = 2.6$, $M = 100$ and $tJ = 10$.

Numbers explode when analog errors → 0

Digital quantum simulation: resource estimates…



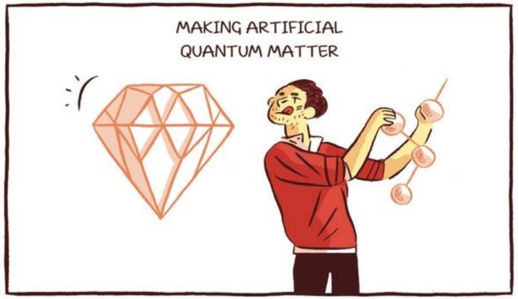
arXiv:2211.07629v1

Many-body physics with synthetic systems or Quantum Simulation?

Experiments are imperfect… ⇒ **Not a pristine quantum simulation** of a model…

Study the noisy many-body system for itself…

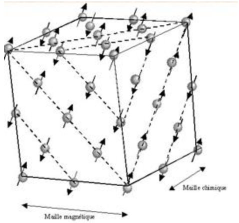
Héloïse Chochois, Labex PALM



Use "toy many-body systems" to

- Develop intuition ("simple to complex", noise…)
- Trigger new theoretical methods
- Generate "interesting" quantum states (squeezed…)

Understand better "real" systems?

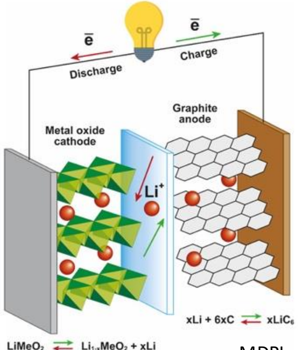


Strongly correlated matter

Develop applications?

Fraunhofer

Material



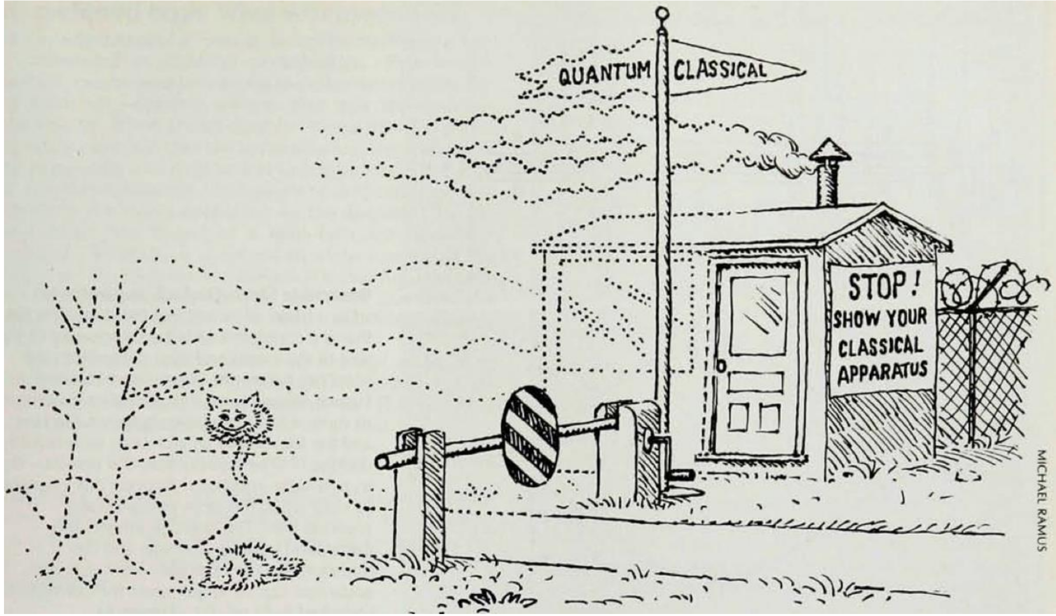
MDPI

Chemistry

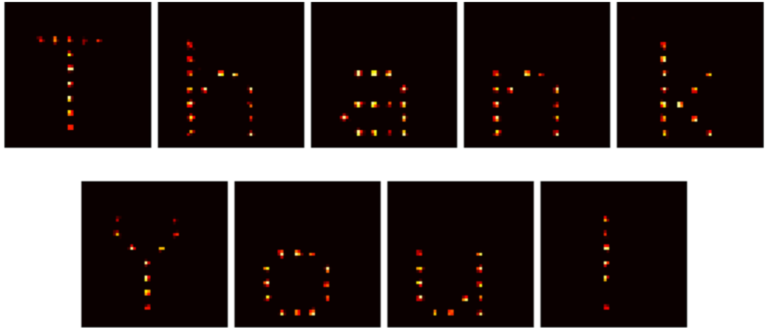
Catalysis

…

How large can a quantum system be?



Zurek, Physics Today 1991



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