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Non-ergodicity and glassiness in quantum systems

School on Quantum Dynamics of Matter, Light and Information

ICTP Trieste, Aug 18 – Sep 5, 2025.

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Quantum glasses

Review:

L. Cugliandolo and M. Müller
arXiv:2208.05417

Review on Quantum Glasses

Chapter 18 in:
Spin Glass Theory & Far Beyond –
40 years of Replica Symmetry Breaking,
1st ed. World Scientific. (2023)

Ergodicity and thermalization

General tenet of statistical physics: Interacting many body systems establish equilibrium and are ergodic

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General tenet of statistical physics: Interacting many body systems establish equilibrium and are ergodic

If that happens:

- Unique state described by Gibbs ensemble
- No dependence on history, no memory
- Usually fast thermalization on microscopic timescales, even in closed systems (cf. ETH hypothesis)

Convenient to calculate, reliably reproducible - but also a bit dull.

Non-Ergodicity and non-thermalization

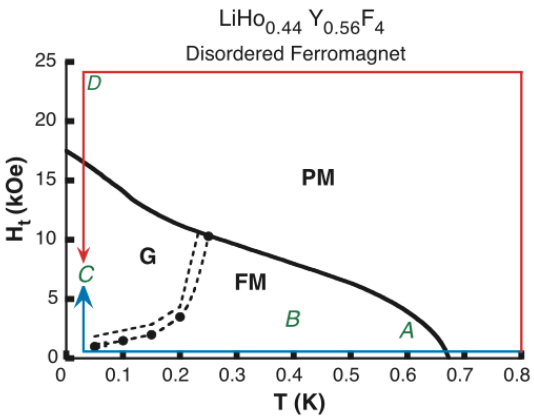
An example where it does not happen:

Quantum glasses:
Intriguing history dependence

Science 1999

Quantum Annealing of a Disordered Magnet

J. Brooke,[1] D. Bitko,[1] T. F. Rosenbaum,[1*] G. Aeppli[2]



$LiHo_xY_{1-x}F_4$: Dipolar Ising spin glass in transverse fie

$$\mathcal{H} = -\sum_{i,j}^{N} J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_{i}^{N}\sigma_i^x$$

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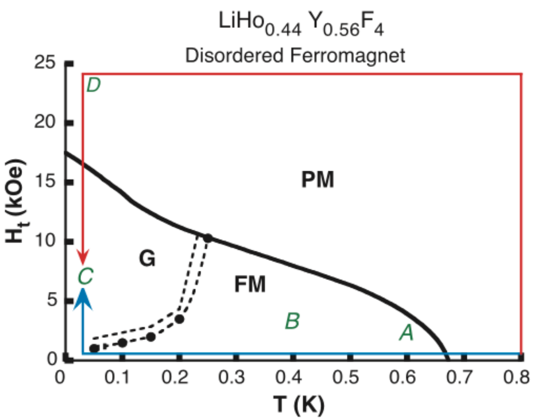

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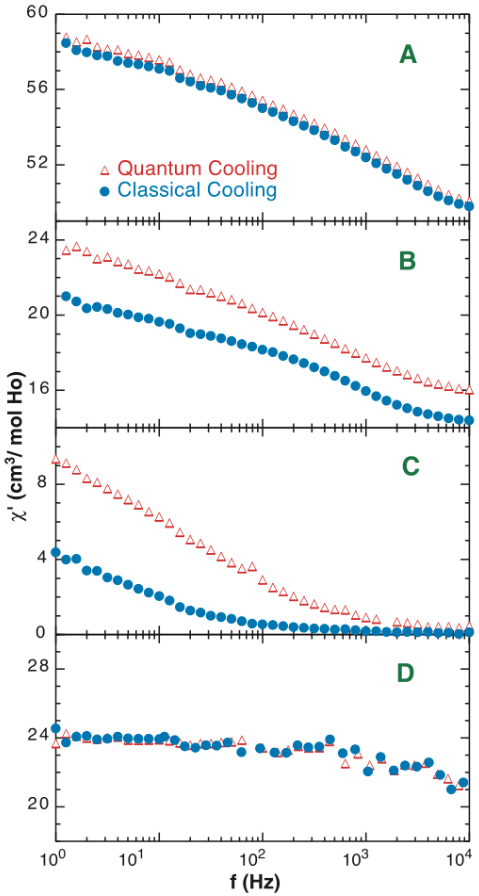
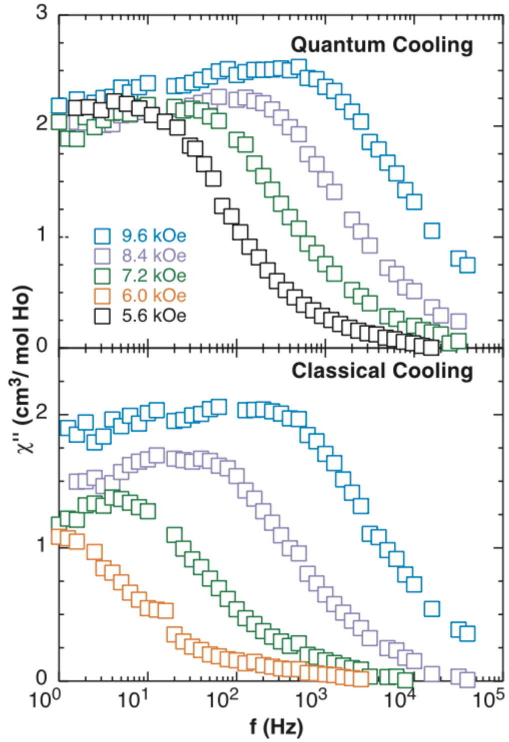
Response χ'



Absorption χ''



Utterly different response under *same* conditions (H_t,T)!

Long-lived non-equilibrium & history dependence

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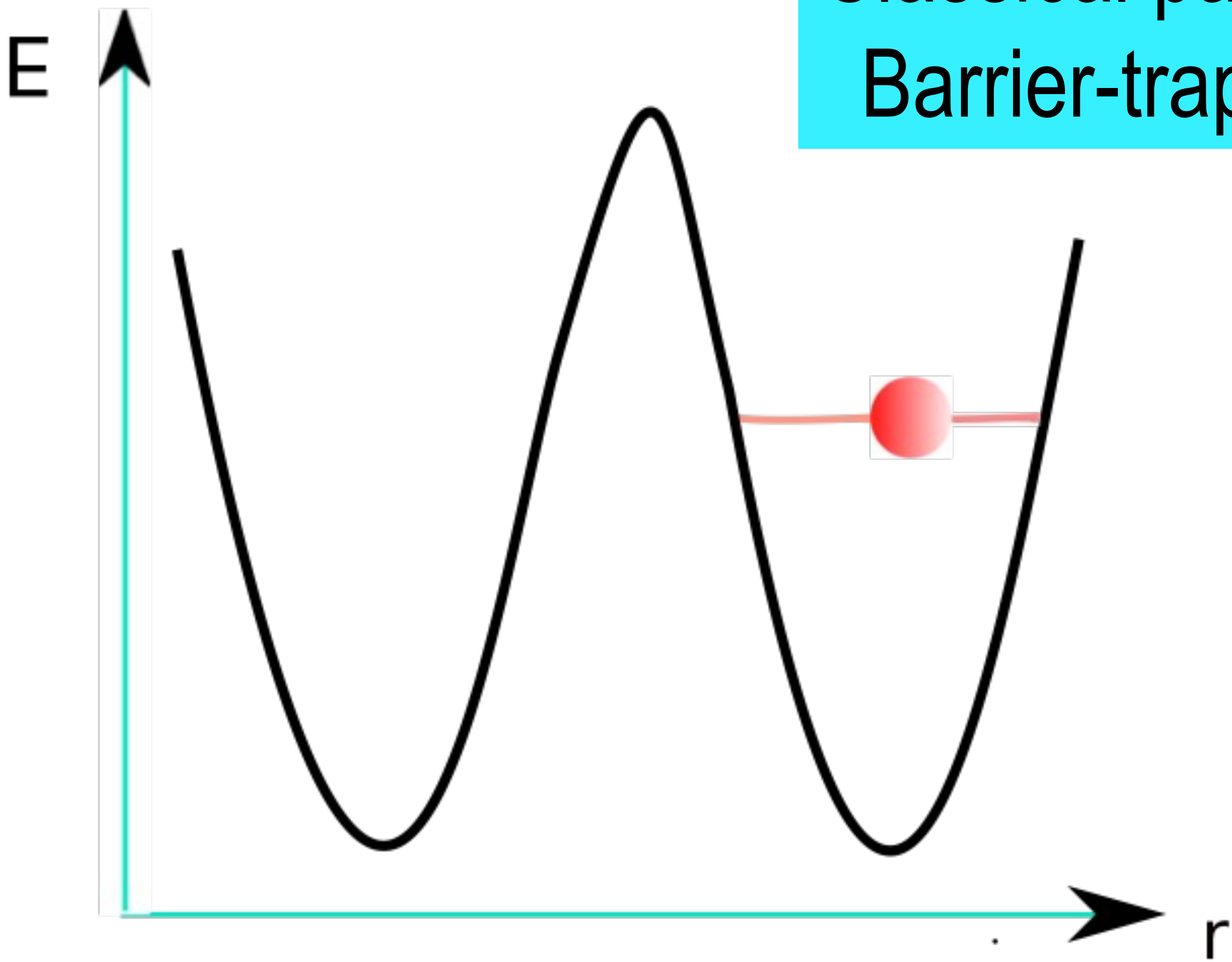
Routes to non-ergodicity

What underlies the belief of ubiquitous ergodicity?

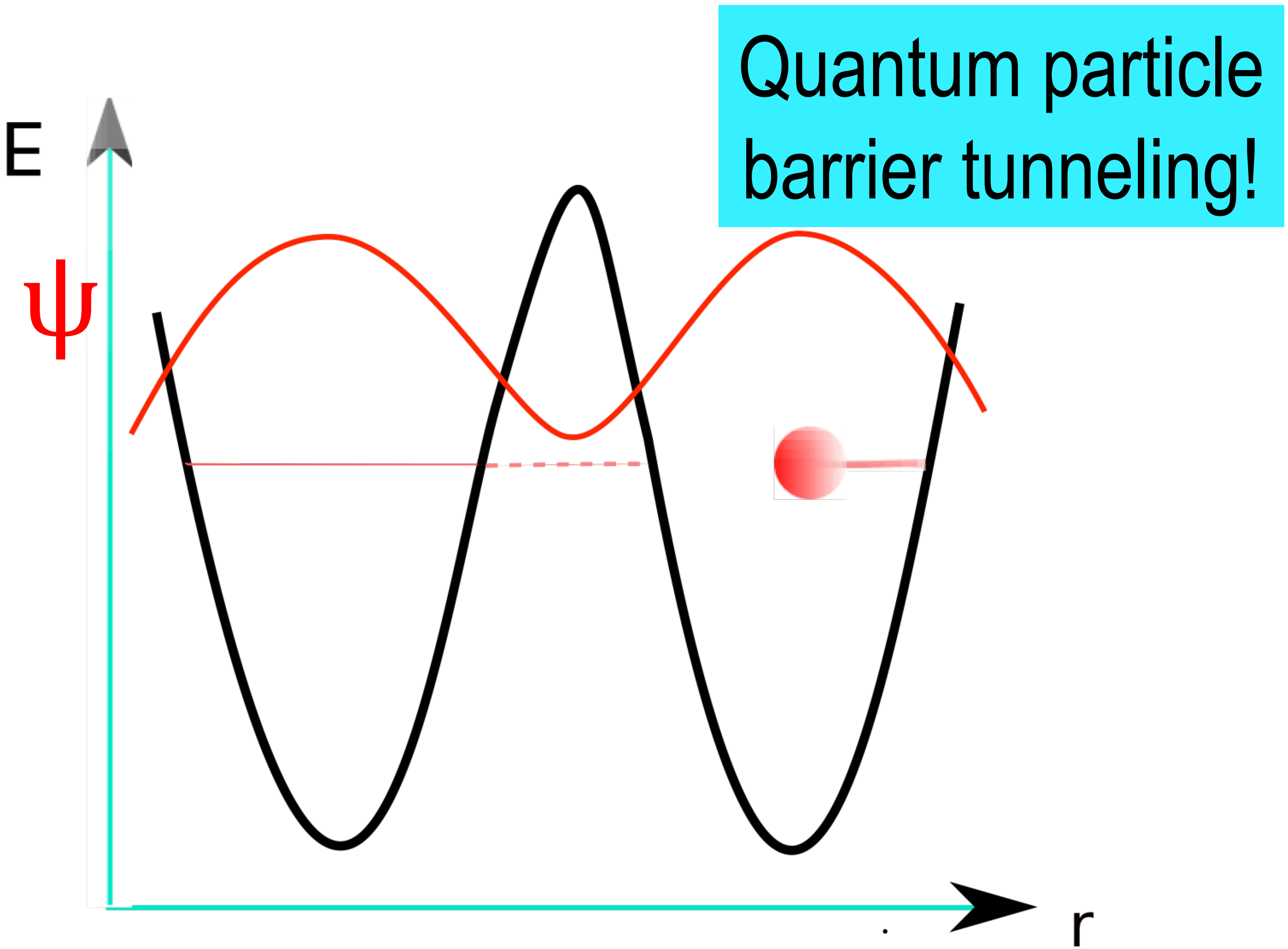
And how can one escape from it?

Quantum localization

Classical particle:
Barrier-trapped

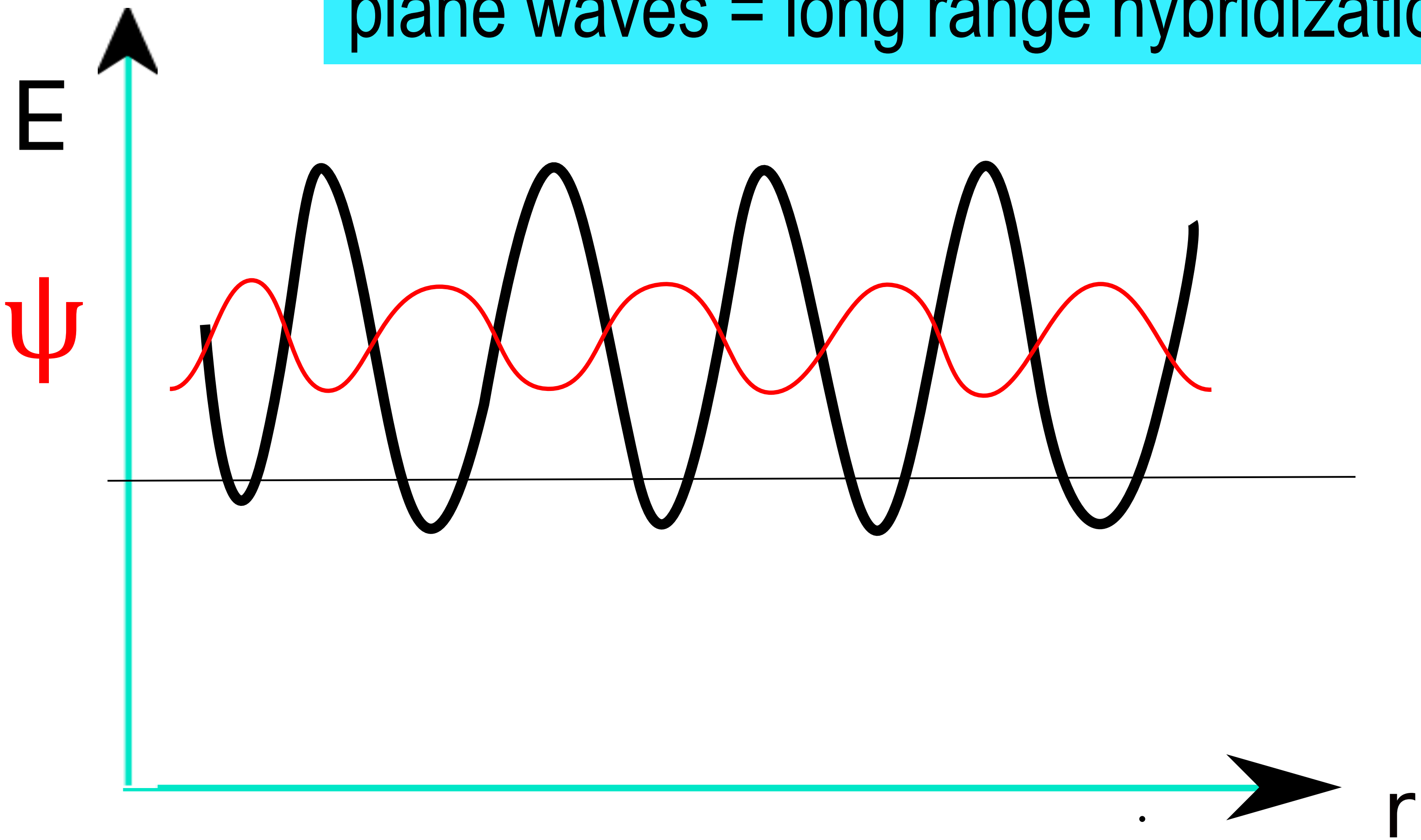


Quantum localization



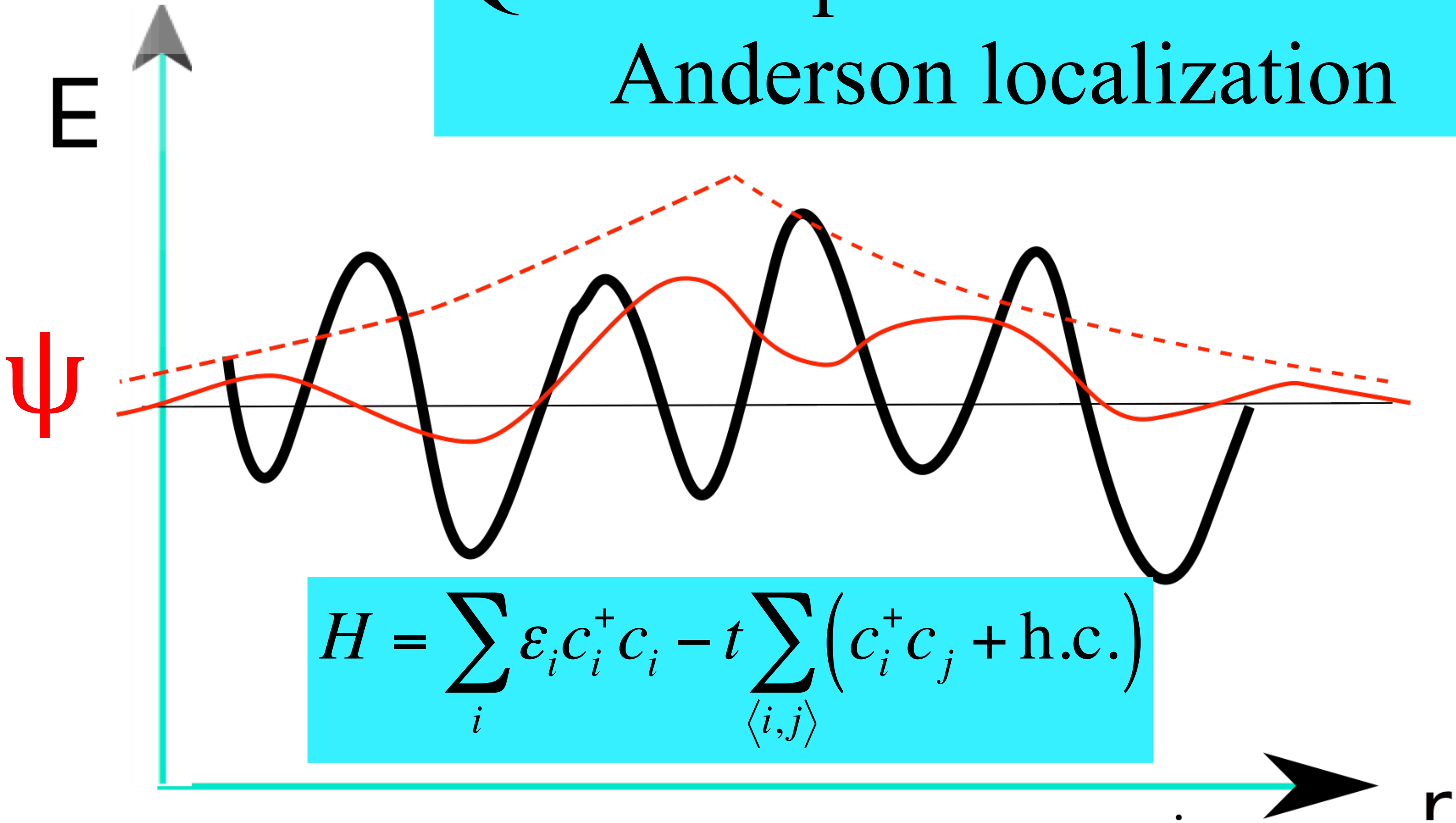
Quantum localization

Quantum particle – no disorder
plane waves = long range hybridization!



Quantum localization

Quantum particles in disorder: Anderson localization



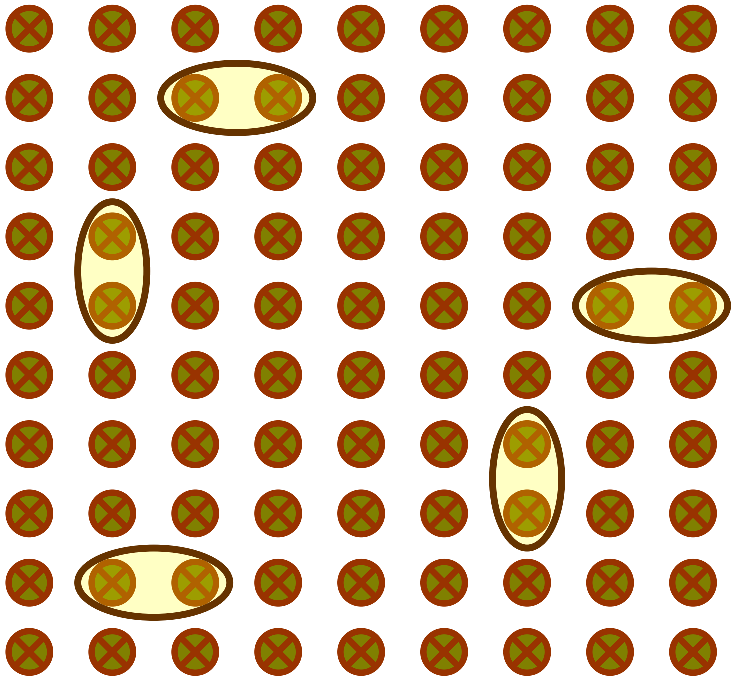
$$H = \sum_i \varepsilon_i c_i^+ c_i - t \sum_{\langle i,j \rangle} \left(c_i^+ c_j + \text{h.c.} \right)$$

If $|\epsilon_i - \epsilon_j| \gg t$: no hybridization, wavefunction localizes on i or j

If $|\epsilon_i - \epsilon_j| \lesssim t$: “resonance”, wavefunction spreads

Quantum localization: no-percoation of resonances!

Anderson 1958

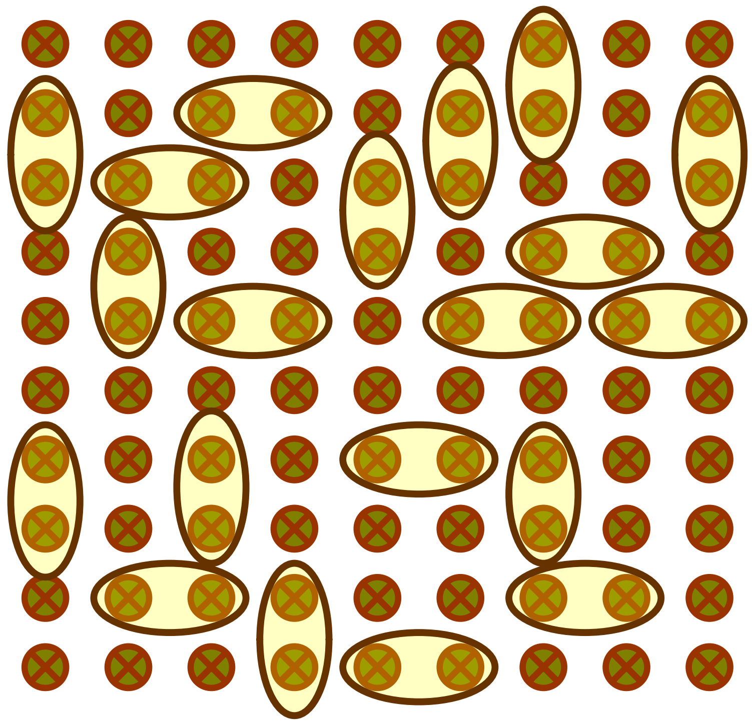


Anderson insulator

Few isolated resonances

No diffusion!

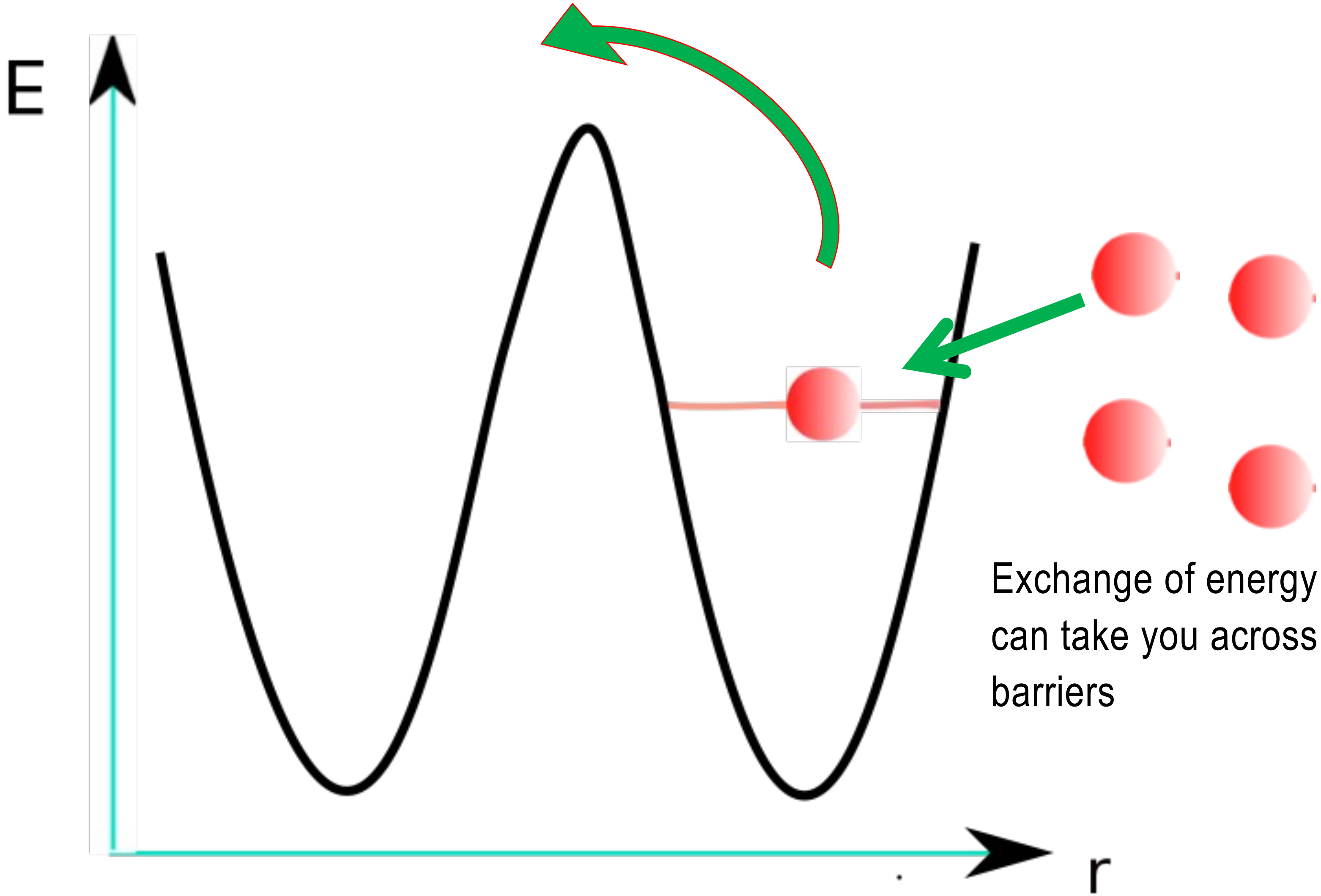
No ergodicity!



Anderson metal

There are many resonances and they overlap

Now add interactions!



Exchange of energy can take you across barriers

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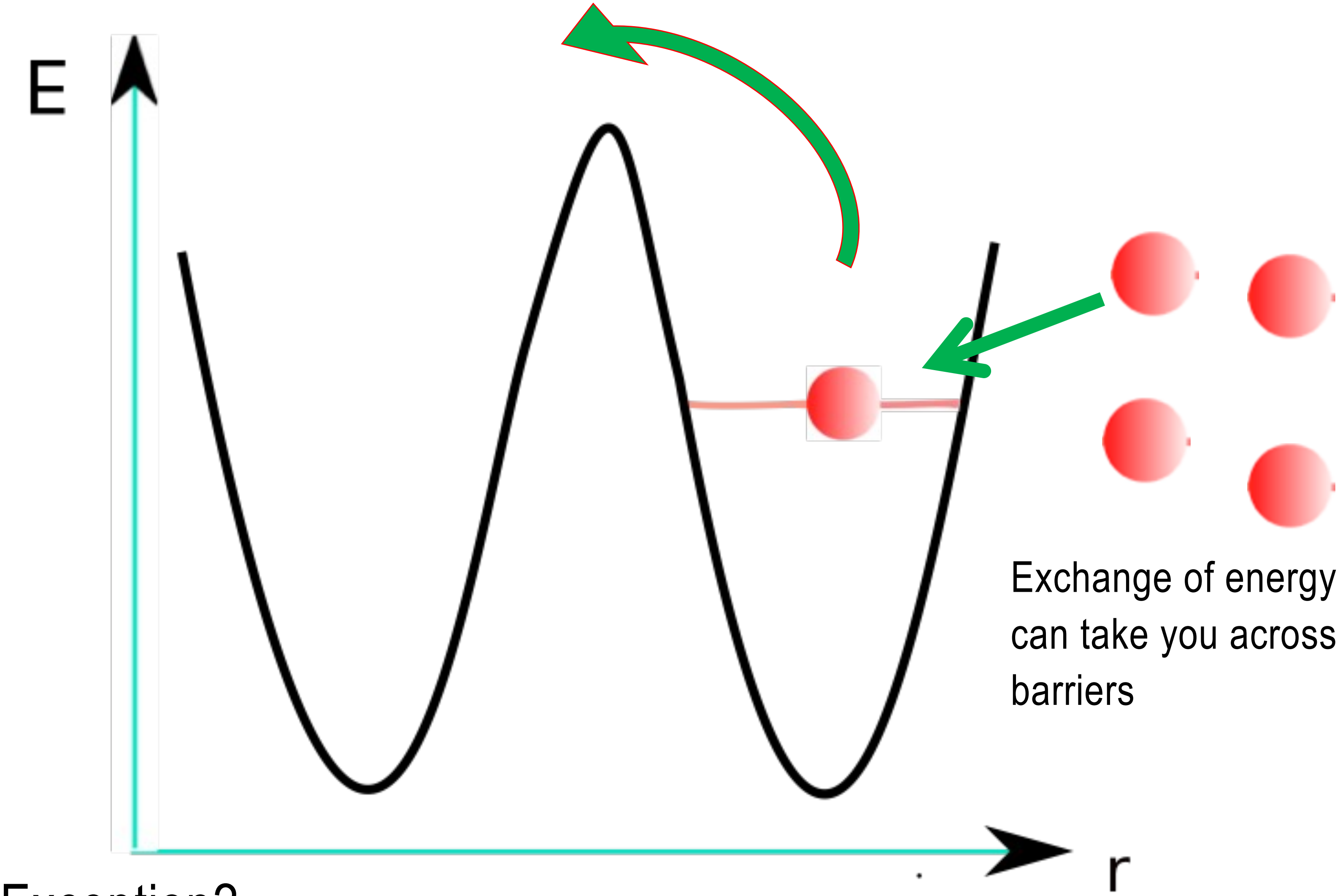

Exchange of energy can take you across barriers

Exception?

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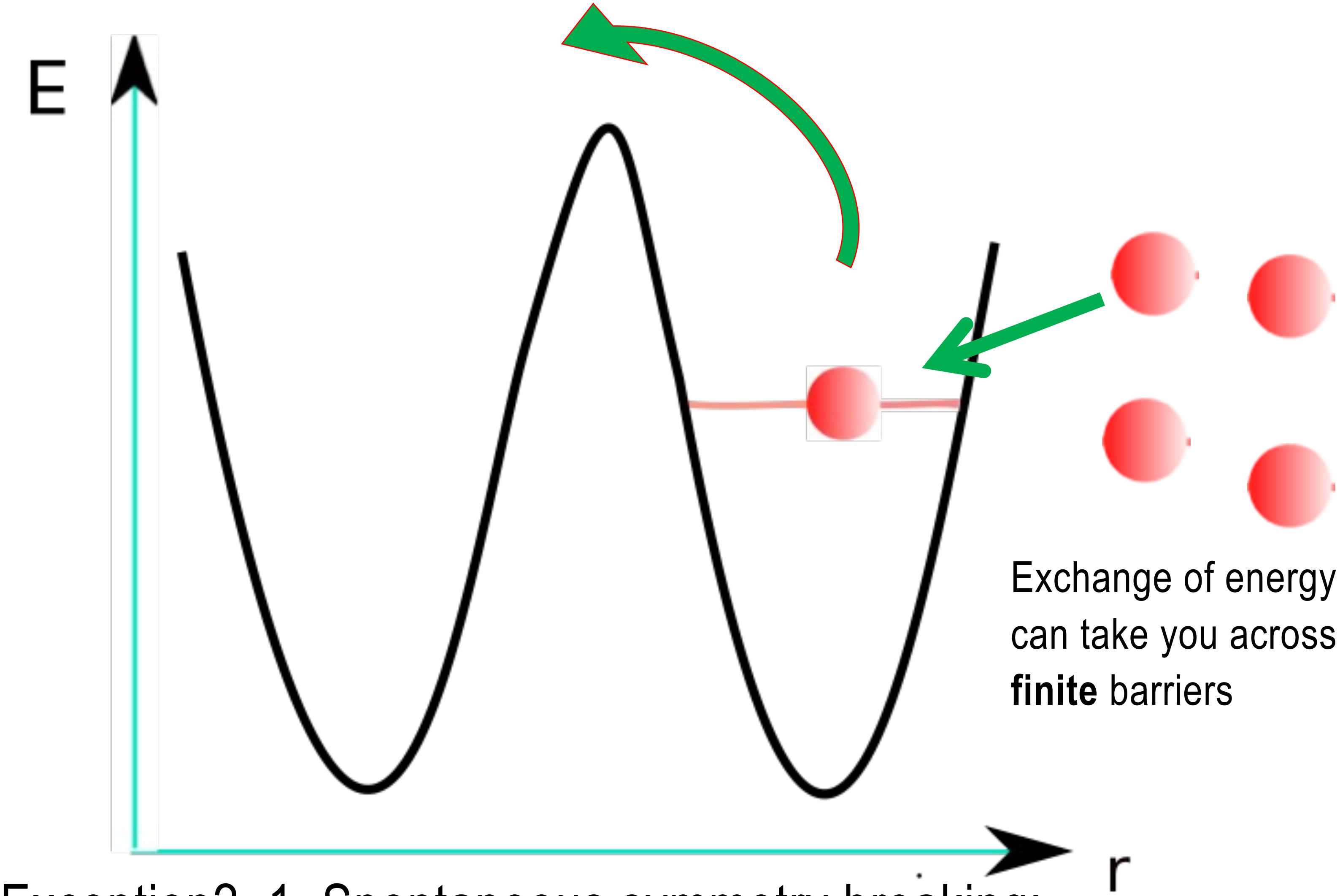
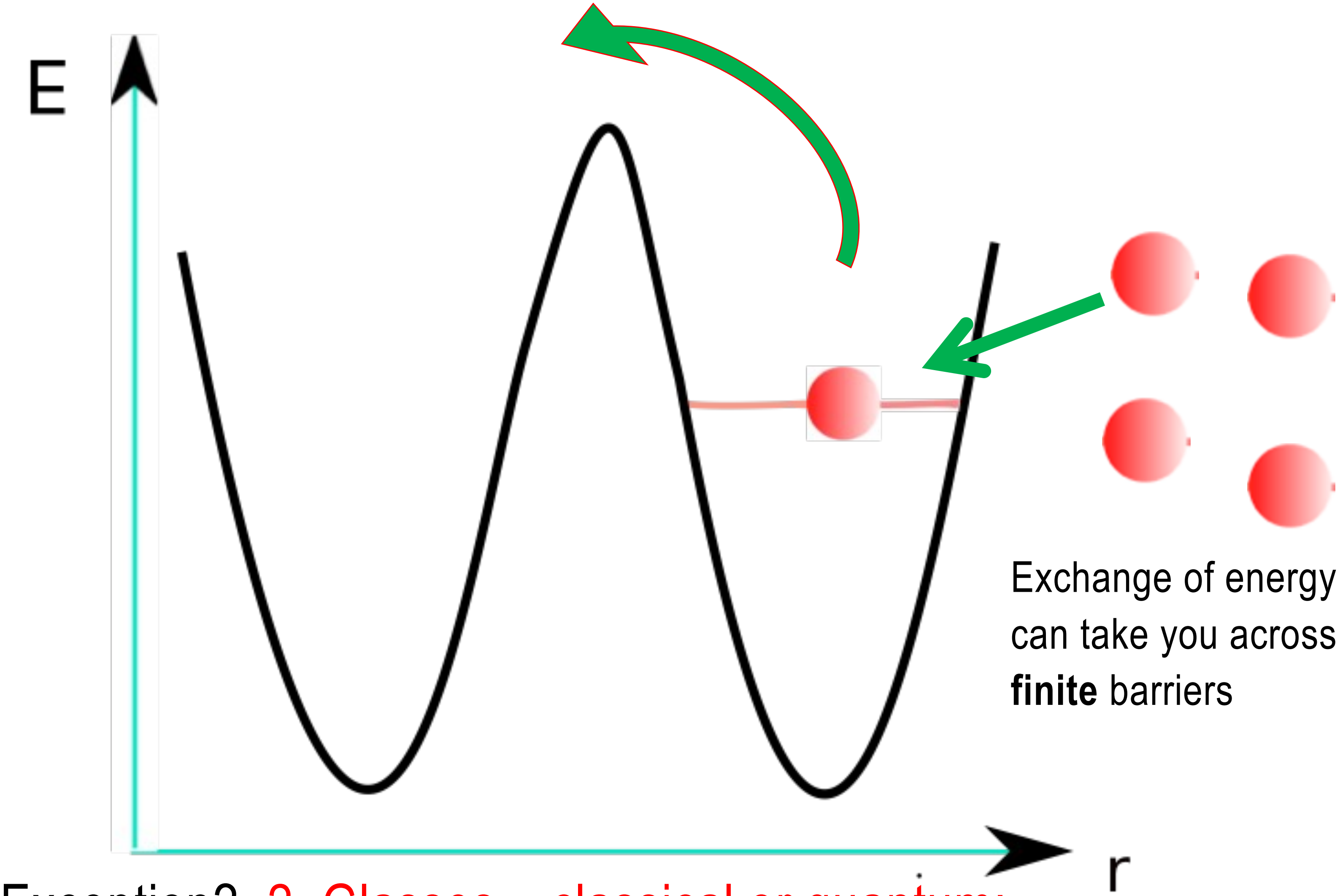


Exchange of energy can take you across **finite** barriers

Exception? 1. Spontaneous symmetry breaking: energy barriers between different ordered states diverge

Now add interactions!



Exchange of energy can take you across **finite** barriers

Exception? 2. Glasses – classical or quantum:

Many collective states separated by barriers (Lectures I+II)

What about adding interactions to Anderson localization?

Anderson, Fleischman 1979
Basko Aleiner, Altshuler 2006
Gornyi, Mirlin, Polyakov 2005
...

"Many-body-localization"?

$$H = \sum \epsilon_i c_i^\dagger c_i - t \sum_{i,j} (c_i^\dagger c_j + \text{h.c.})$$

$$\longrightarrow$$

$$H = \sum \epsilon_i c_i^\dagger c_i - t \sum_{i,j} (c_i^\dagger c_j + \text{h.c.}) + \sum_{ij} J_{ij} c_i^\dagger c_i c_j^\dagger c_j$$

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Can energy mismatch that localizes single particles be bridged by exchange of energy with other particles, forming a bath ("dephasing") ?

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Can energy mismatch that localizes single particles be bridged by exchange of energy with other particles, forming a bath (“dephasing”) ? No, not always!

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Rewritten in single-particle localized basis:

$$H = \sum \epsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_{\alpha,\beta,\gamma,\delta} J_{\alpha\beta,\gamma\delta} c_\alpha^\dagger c_\beta c_\gamma^\dagger c_\delta$$

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MBL $\leftrightarrow$ Non-percolation of resonances in many-body space!

$$J_{\alpha\beta,\gamma\delta} \leftrightarrow \epsilon_\alpha - \epsilon_\beta + \epsilon_\gamma - \epsilon_\delta$$

The surrounding particles, being localized themselves do not form a continuous bath!

Glass physics $\neq$ Manybody localization

Two ways to break ergodicity

Spin/structural glasses ↔ Quantum localization

Obstruction: Big mountains

Obstruction: Bad tunnels

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“Many-body-localization”

↔

Glasses

Are these ergodicity breaking mechanisms related?

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Two types of ergodicity breakers

Frustrated disordered systems	Quantum localized systems
Spin/structural glass	Anderson insulator (Fermi glass)

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Neither implies the other &
Neither excludes the other!

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Interplay of glassiness and quantum dynamics/ localization?

A very rich playground! (see lecture III)

Other ways to avoid ergodicity and thermalization?

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Fine-tuned!

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Exotic ergodicity breaking
Related neither to glasses
nor to MBL

Interplay of glassiness and (many-body) localization

Interesting questions: (see lecture III)

- How does glassy order affect localization & loc. transitions?

- Can glassy order coexist with delocalized quantum modes:
  Bose condensates and/or metallic, delocalized fermions?

Phenomenology in long range, frustrated quantum glasses:

- Long-range-coupled cold atoms
- Quantum Coulomb glass and the metal insulator transition:
  A very rich phase transition

Many interesting open questions

Glasses

Glasses : = Ergodicity breakers with a large number of *amorphously ordered states* that are separated by high barriers.

Multitude of states and their organization in phase space entail interesting properties, also with regard to quantum dynamics.

Frustration

Typical spin glass Hamiltonian:

$$H = \sum_{\langle i,j \rangle} J_{ij} S_i S_j$$

Randomly signed couplings
(due to random doping/exchange paths,
RKKY or dipolar couplings, etc)

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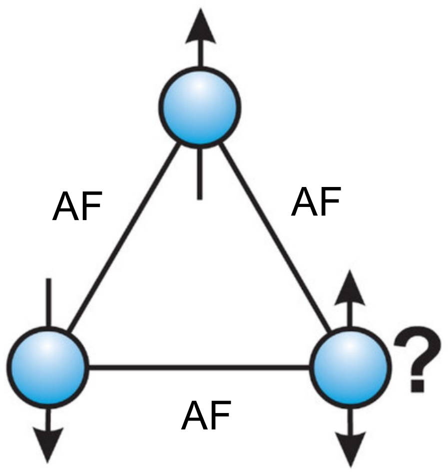

Lots of plaquettes with at least one unhappy bond

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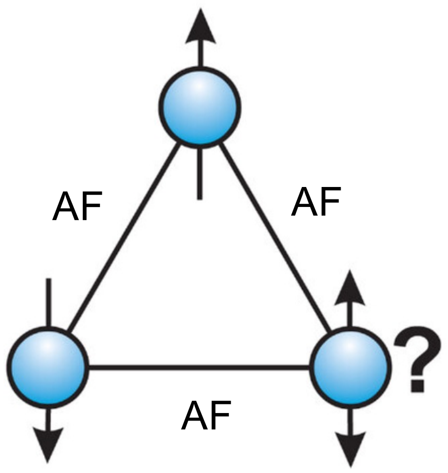

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Spin glass: percolating magnetic unhappiness.
Many different ways to minimize the unhappiness!

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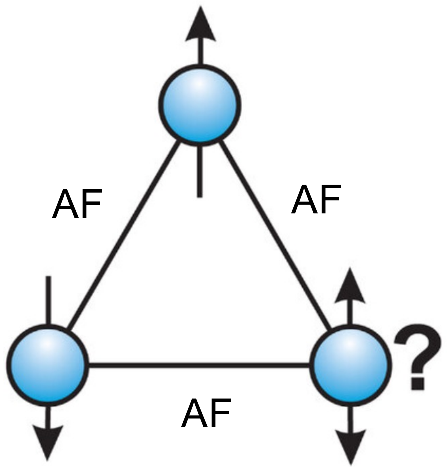

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Spin glass: percolating magnetic unhappiness.
Many different ways to minimize the unhappiness!

One of many NP-hard optimization problems

Glasses: systems with many states

Anticipate:

Unlike simple magnets, glasses can order in many amorphous patterns

Fundamental questions

- Is there a phase transition? – What is the order parameter?
- How to deal with disorder?
- How many ordering patterns are there, and how are they organized?
- What are quantum dynamics and excitations in such states?
  Impact on quantum phenomena like localization, Bose condensation etc?

Free energy landscape:

Ordering patterns as minima of a free energy functional?

Free energy landscape: mean field Ising ferromagnet

Warm-up: mean-field, all-to-all Ising ferromagnet

$$S_i = \pm 1$$

$$H = -\frac{J}{2N} \sum_{i,j=1}^{N} S_i S_j - B \sum_{i=1} S_i$$

$$= -\frac{NJ}{2} m^2 - NBm \equiv Ne(m)$$

$$m := \frac{1}{N} \sum_i S_i;$$

Average magnetization per spin

Free energy landscape: mean field Ising ferromagnet

Entropy of configurations of magnetization m:

$$S(m) = N s_0(m)$$

$$s_0(m) = -\frac{1+m}{2}\log\frac{1+m}{2} - \frac{1-m}{2}\log\frac{1-m}{2}$$

Free energy constrained to have magnetization m:

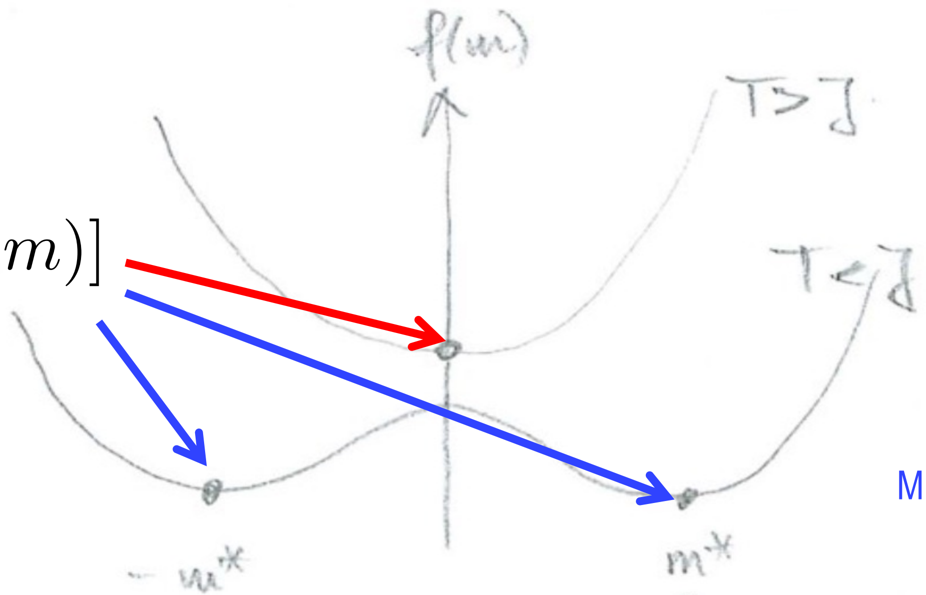
$$F(m) = E(m) - TS(m) = N(e(m) - Ts_0(m))$$

Free energy landscape: mean field Ising ferromagnet

(B = 0)

Full free energy:

$$F = \min_{-1 \leq m \leq 1} [F(m)]$$



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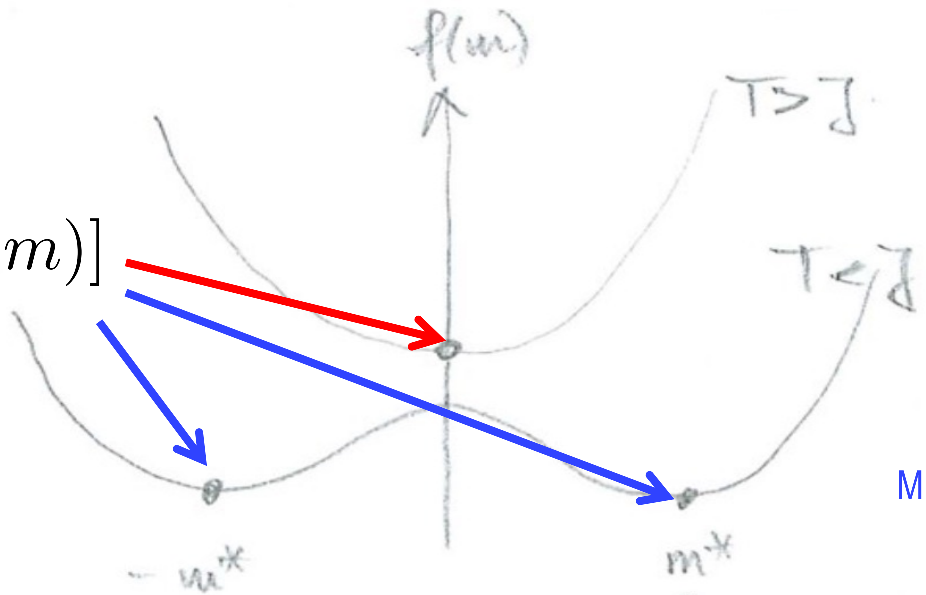

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Spontaneous symmetry breaking:

$$\lim_{B \uparrow 0^-} \lim_{N \to \infty} P_{\text{Gibbs}}(\{S\}; B) \neq \lim_{B \downarrow 0^+} \lim_{N \to \infty} P_{\text{Gibbs}}(\{S\}; B)$$

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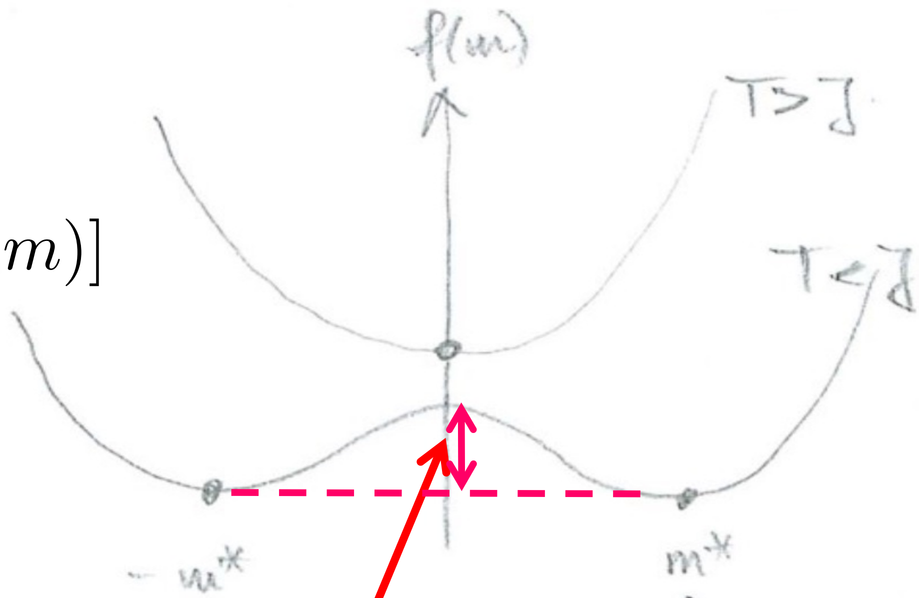

$$F(m) = E(m) - TS(m) = N(e(m) - Ts_0(m))$$

Extensive free energy barrier: spontaneous symmetry breaking: the two pure states are infinitely long-lived in the limit $N \to \infty$

Free energy landscape: mean field Ising ferromagnet

Full free energy:

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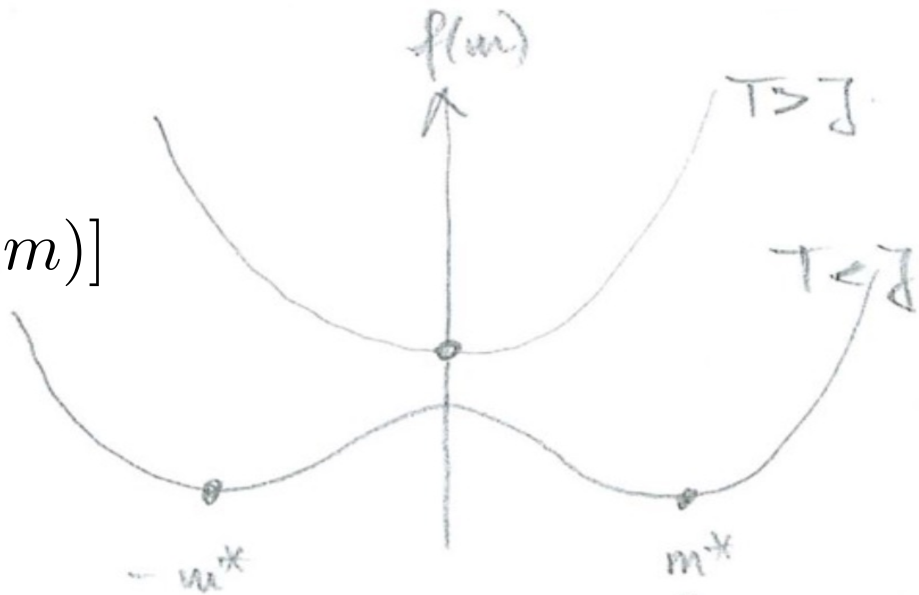

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Important: Only full *F* = *F(T)* is non-analytic due to the minimization over m, which bifurcates at T_c.
But **"energy landscape"** ***F(m,T)*** **is analytic** in both *m* and *T*. It can be obtained from a high *T*-expansion!

Energy landscape in glasses?

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Typical spin glass Hamiltonian:

$$H = \sum_{\langle i,j \rangle} J_{ij} S_i S_j$$

Randomly signed couplings

Difficulties:

- No obvious symmetry breaking / ordering pattern
- Order parameter (analogue of m)?
- How many pure states are there? - What are their properties?

Energy landscape in glasses?

Consider mean field glasses (random all to all interactions):

- energy landscape can be construct unambiguously;
- saddle point methods can be used (replica approach)

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Two universality classes of glasses, with very different phenomenology!

1) Mean field version of spin glasses: Pairwise interacting spins $H = -\frac{1}{2}\sum_{i,j} J_{ij} S_i S_j$

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1) Mean field version of spin glasses: Pairwise interacting spins $H = -\frac{1}{2} \sum_{i,j} J_{ij} S_i S_j$

2) Toy model for structural glasses (super-cooled liquids): p-tuple interactions (e.g. p = 3) $H = -\frac{1}{6} \sum_{i,j,k} J_{ijk} S_i S_j S_k$

Believed to capture amorphous glasses in high dimensions:
Dynamical equations are structurally identical to those of mode coupling theory of liquids

1. The Ising mean field glass: Sherrington-Kirkpatrick (SK) model

Hamiltonian

$$H_{\mathrm{SK}}[S] = \sum_{i<j=1}^{N} J_{ij} S_i S_j$$

Gaussian disorder J_{ij} with zero mean and variance:

$$\overline{J_{ij}^2} = \frac{1}{N}$$

ensures O(1) local field b_i on a given spin s_i, and thus O(N) total energy.

$$b_i = \frac{\partial H}{\partial S_i} = \sum_{j(\neq i)} J_{ij} S_j$$

2. The spherical p-spin model

Hamiltonian

$$H[\sigma] = -\frac{1}{p!}\sum_{i_1\cdots i_p} J_{i_1\cdots i_p}\sigma_{i_1}\cdots\sigma_{i_p} = -\sum_{i_1<i_2<\cdots<i_p} J_{i_1\cdots i_p}\sigma_{i_1}\cdots\sigma_{i_p}$$

Spherical constraint only (easy to compute - but for p=2 trivializes the model)

$$\sum_i \sigma_i^2 = N \qquad \sum_{\{s_i\}} \to \int \prod_i d\sigma_i \delta\left(\sum_i \sigma_i^2 - N\right)$$

Gaussian disorder with zero mean and variance:

$$\overline{J^2_{i_1\cdots i_p}} = \frac{p!}{2N^{p-1}}$$

ensures O(1) local fields on a given spin, and thus O(N) total energy.

Free energy landscape

Free energy functional

Spins are not equivalent → construct free energy landscape $G(\{m_i\}_{i=1,\dots,N})$
= Gibbs free energy of system constrained such that spin S_i has magnetization m_i

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Computation *(Georges & Yedidia, J. Phys. A 1991)*

- At any T, apply local fields h_i that impose magnetizations m_i
- Perform Legendre transform

$$F(\{h_i\}_{i=1,\ldots,N}) \stackrel{L.T.}{\rightarrow} G(\{m_i\}_{i=1,\ldots,N})$$

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- Order by order in a high T (or small J) expansion:
  In the limit $N \rightarrow \infty$ expansion terminates after second term!

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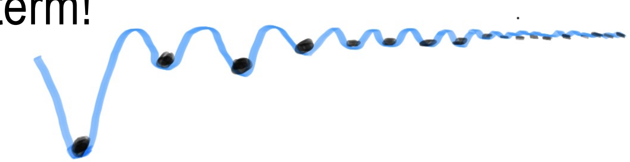

- → Functional of all m_i - convex at high T
  But: develops lots of local minima at low $T < T_c$! ↔ ordering patterns

Free energy landscape

SK model: *(Thouless-Anderson Palmer, 1975)*

$$G\{m_i\} = -T\sum_i s_0(m_i) - \sum_{i<j} m_i J_{ij} m_j - \frac{1}{2}\sum_{i<j}(1-m_i^2)J_{ij}^2\beta(1-m_j^2) + O(\beta^2)$$

Standard mean field energy

Van der Waals-like interaction:
"Onsager back reaction"

$$s_0(m) = -\frac{1+m}{2}\log\frac{1+m}{2} - \frac{1-m}{2}\log\frac{1-m}{2}$$

Note: local susceptibility of Ising spins $\chi_j = \frac{dm_j}{dh_j} = \frac{d\tanh(\beta h_j)}{dh_j} = \beta(1-m_j^2)$

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P-spin model:

$$\frac{G(\{m_i\})}{N} = -\frac{1}{2\beta}\log(1-q) - \frac{1}{p!N}\sum_{i_1\cdots i_p} J_{i_1\cdots i_p} m_{i_1}\cdots m_{i_p} - \frac{\beta}{4}[1-pq^{p-1}+q^p(p-1)]$$

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Pure states = Minima of G !

$$P_{\rm Gibbs}[S] = \sum_\alpha w_\alpha P_\alpha[S]$$

Clustering property in mean field models implies:

$$P_\alpha[S] = \prod_i \left[\frac{1+m_i^{(\alpha)}}{2}\delta_{1,S_i} + \frac{1-m_i^{(\alpha)}}{2}\delta_{-1,S_i}\right] = \prod_i \frac{1+m_i^{(\alpha)}S_i}{2}$$

Free energy landscape

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Free energy of minimum $Nf_\alpha = G(\{m_i^\alpha\})$

Weight of **pure state** in the full Gibbs measure $w_\alpha \propto \exp(-\beta N f_\alpha)$

Free energy landscape

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Can show: metastable states capture the essence of phase space: $\overline{\log(\sum_\alpha w_\alpha)} = \overline{\log(Z_{\text{full}})}$

Free energy landscape

The two universality classes of glasses have very different landscapes

- Number and nature of minima
- The way they appear at low T
- Organization in energy and configuration space

TAP Equations: SK-model

$$G\{m_i\} = -T\sum_i s_0(m_i) - \sum_{i<j} m_i J_{ij} m_j - \frac{1}{2}\sum_{i<j}(1-m_i^2)J_{ij}^2\beta(1-m_j^2) + O(\beta^2)$$

Minima: $$\frac{\partial G}{\partial m_i} = 0 \leftrightarrow$$

$$m_i = \tanh\left[\beta\sum_{j\neq i} J_{ij}\left(m_j - m_i J_{ij}\beta(1-m_j^2)\right)\right] \leftrightarrow$$

$$m_i = \tanh\left[\beta\sum_{j\neq i} J_{ij} m_j^{(i)}\right]$$

Magnetization *in absence of spin i:*

$$m_j^{(i)} = m_j - m_i J_{ij}\beta(1-m_j^2)$$

Onsager reaction to m_i

Standard mean field - diminished by polarization response of environment

TAP states: SK-model

$$m_i = \tanh\left[\beta \sum_{j \neq i} J_{ij}\left(m_j - m_i J_{ij} \beta (1 - m_j^2)\right)\right]$$

Linearize

$$m_i = \beta \sum_{j \neq i} J_{ij} m_j - m_i \beta^2 J^2 \quad + O(m^3, 1/N)$$

High T : only solution is $m_i = 0$. When/how do ordered minima with $m_i \neq 0$ occur?

TAP states: SK-model

$$m_i = \tanh\left[\beta \sum_{j\neq i} J_{ij}\left(m_j - m_i J_{ij}\beta(1-m_j^2)\right)\right]$$

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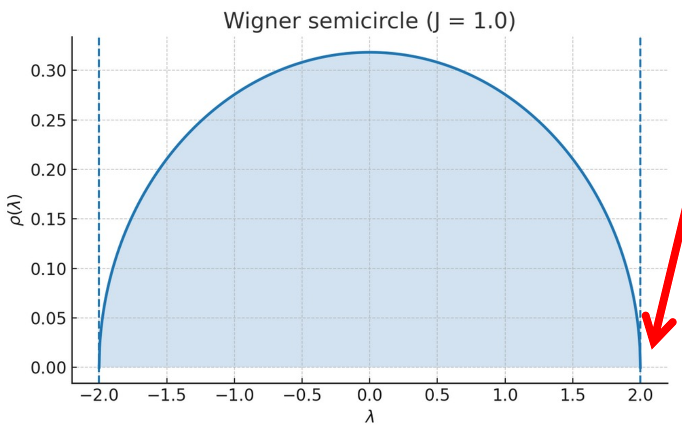
$$m_i = \beta \sum_{j\neq i} J_{ij} m_j - m_i \beta^2 J^2 \quad + O(m^3, 1/N)$$

Instability when first mode of the Gaussian random matrix J_{ij} goes soft

Spectrum of J_{ij}



Glass transition (spin freezing $m_i \neq 0$) at $T_c = J$

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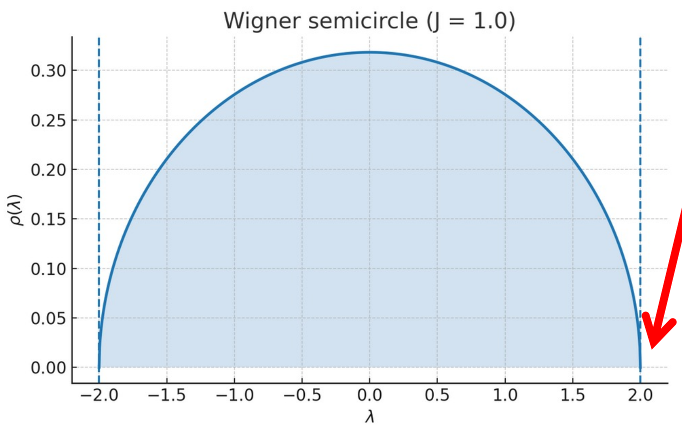


Glass transition (spin freezing $m_i \neq 0$)

at $T_c = J$

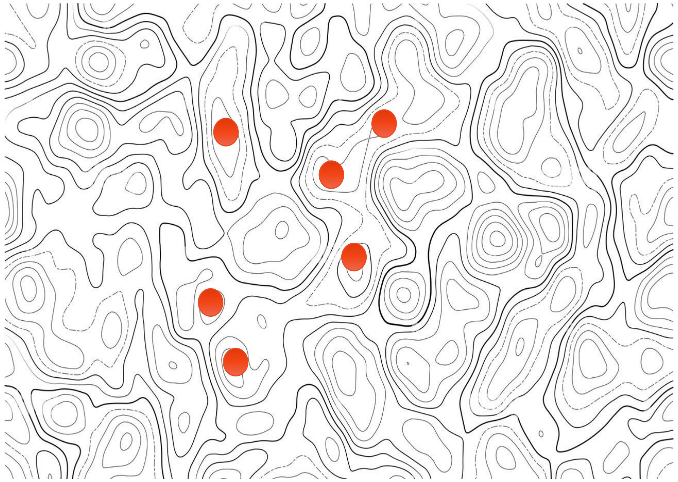
But: many modes become soft almost simultaneously: competing condensates

→ multiple minima!

TAP states: SK-model

A priori: exponentially many minima (in N) for $T < T_c$

But: only those at lowest free energy density are physically relevant
minima at higher energy are pathologically fragile



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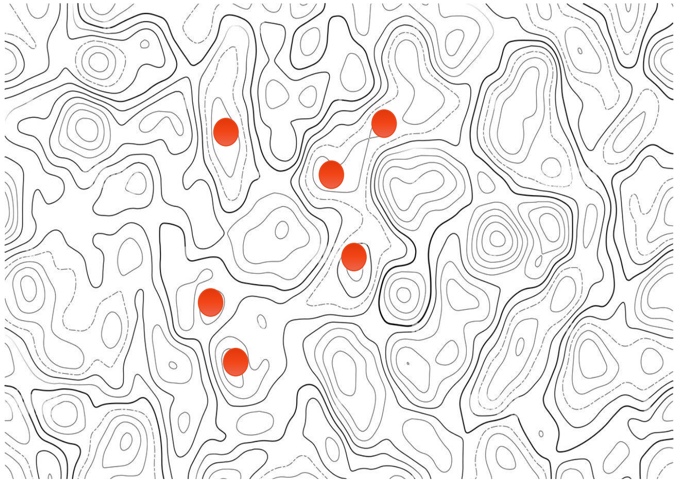
Intriguing aspect: At *all T* the relevant minima have a *gapless* Hessian $\frac{\partial^2 G}{\partial m_i \partial m_j}$

→ Minima are marginally stable, reflecting vicinity of lots of competing states



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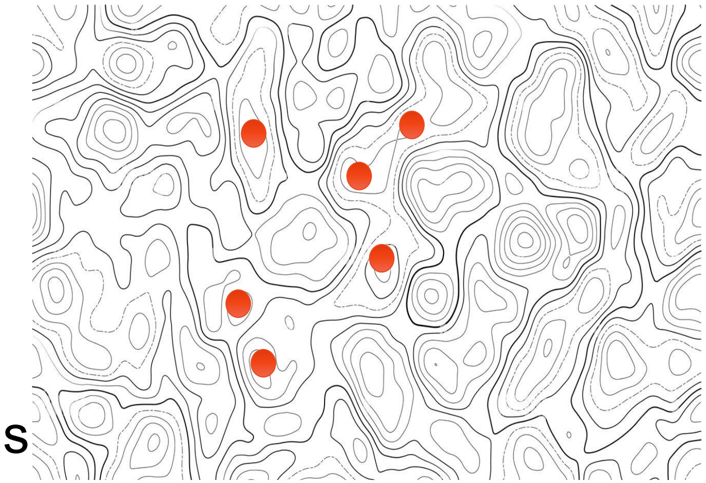

→ many soft collective excitations

→ very sensitive to external parameters
→ critical spin flip avalanches upon applying fields

Minima in the spherical p-spin model

$$\frac{G(\{m_i\})}{N} = -\frac{1}{2\beta}\log(1-q) - \frac{1}{p!N}\sum_{i_1\cdots i_p} J_{i_1\dots i_p} m_{i_1}\cdots m_{i_p} - \frac{\beta}{4}[1 - pq^{p-1} + q^p(p-1)]$$

Pure states = Minima of G !

Write $m_i = \sqrt{q} n_i \quad \sum n_i^2 = N$

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$$= e(\{n_i\})q^{p/2}$$

Pure states = Minima of G !

Write $m_i = \sqrt{q}n_i$ $\sum n_i^2 = N$

Peculiarity of spherical model:

- Minimization of *G* w.r.t. n_i is independent of *T* !
- Minima have constant "angular" texture n_i.

- Only *q* = *q(T)* changes with *T* - impose minimum $\partial G/\partial q = 0$!
  until instability occurs at T*(e) where minimum merges with saddle and evaporates

The spherical p-spin model

Solutions to the angular equations:

T = 0 :

- minima exist with energies $e \in [e_{\min}, e_{\mathrm{th}}]$ $\quad$ $e = -\frac{1}{p!N} \sum_{i_1 \cdots i_p} J_{i_1 \cdots i_p} m_{i_1} \cdots m_{i_p}$

$m_i = \pm 1$

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e> e_{th}: energy landscape dominated by saddles

Threshold
Marginal states

Ground state

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Threshold Marginal states

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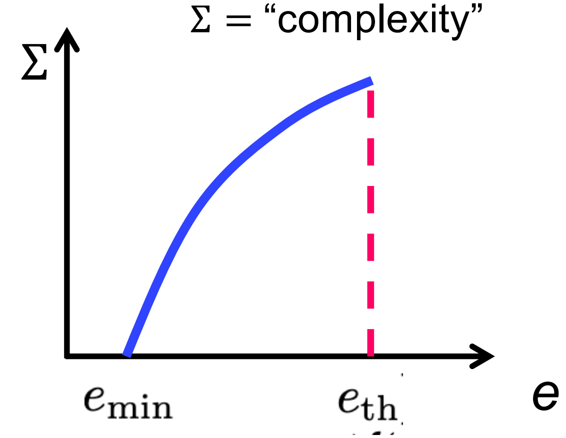
Σ = “complexity”



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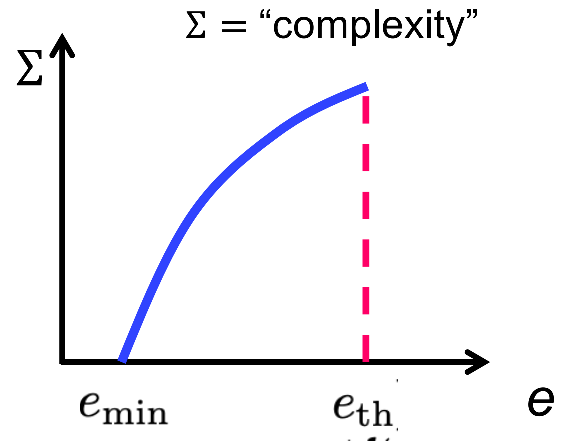
Threshold Marginal states

Ground state (gapped Hessian)



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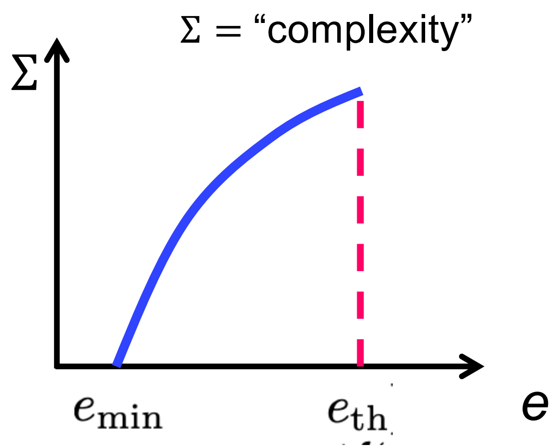
Ground state (gapped Hessian)



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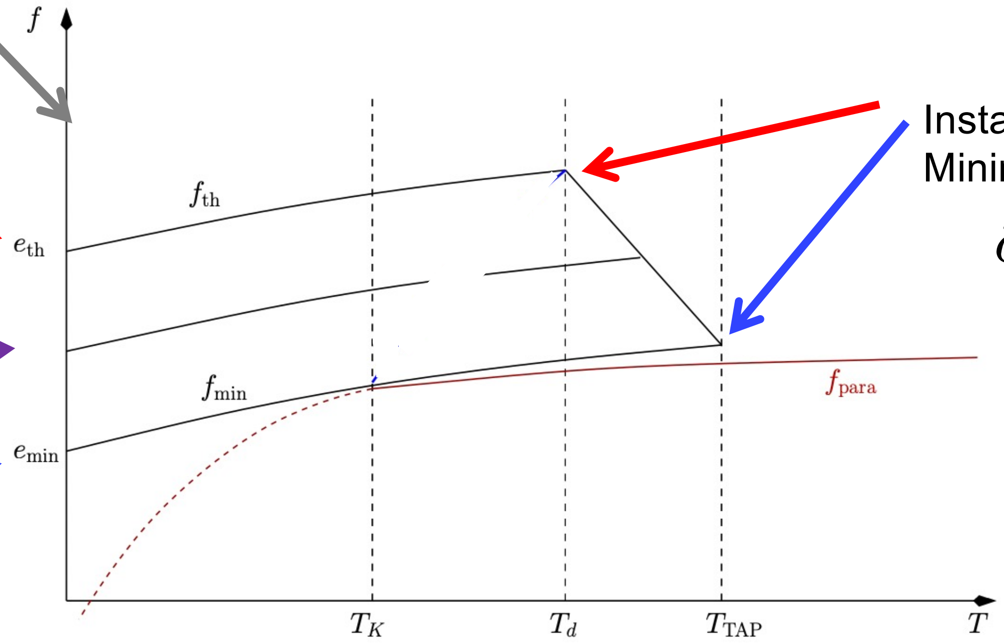

Instability: *T*=T*(e)*
Minima merge with saddles

$$\partial^2 G/\partial q^2 = 0$$

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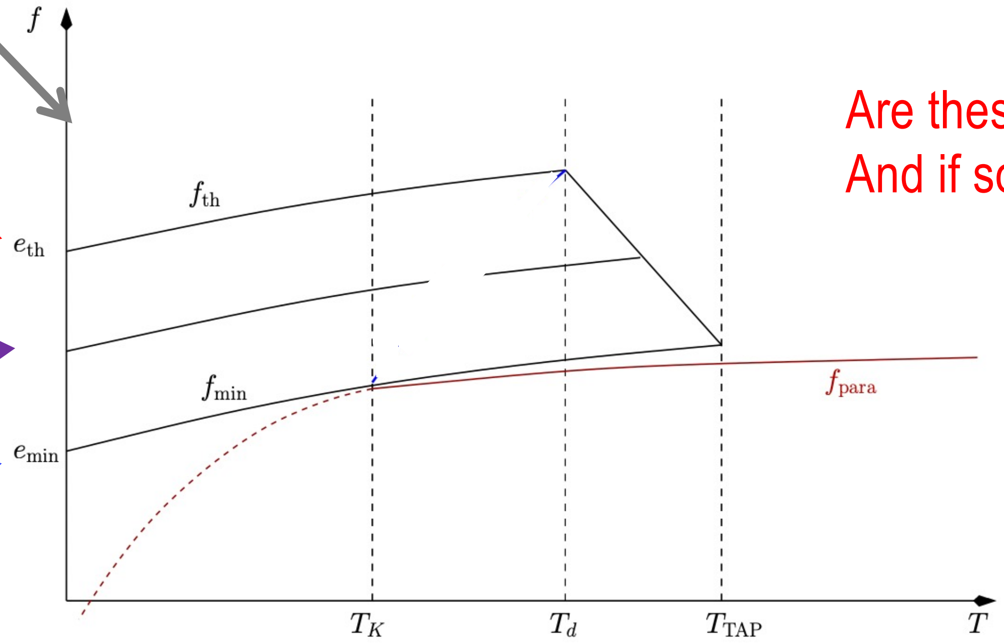

Are these minima relevant?
And if so, which ones?

The spherical p-spin model

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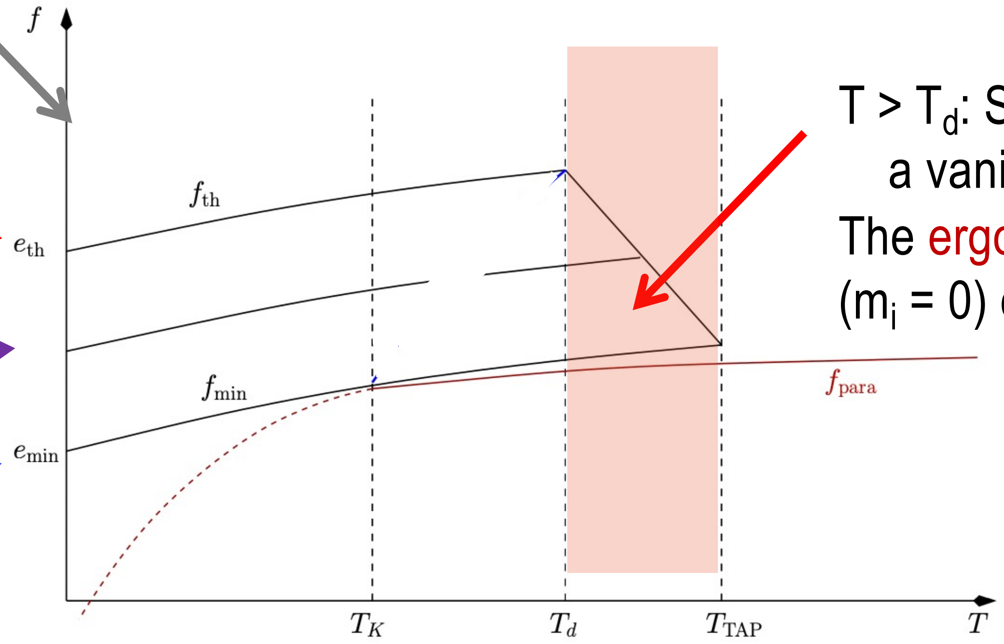

T > T_d: Some minima exist but occupy a vanishing fraction of phase space.

The ergodic paramagnetic minimum (m_i = 0) dominates

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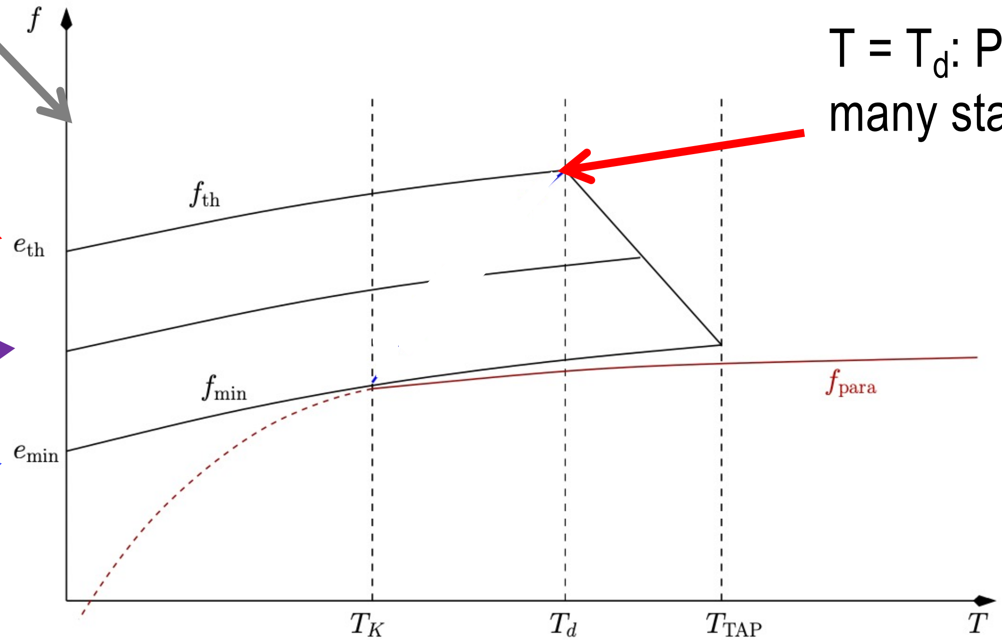
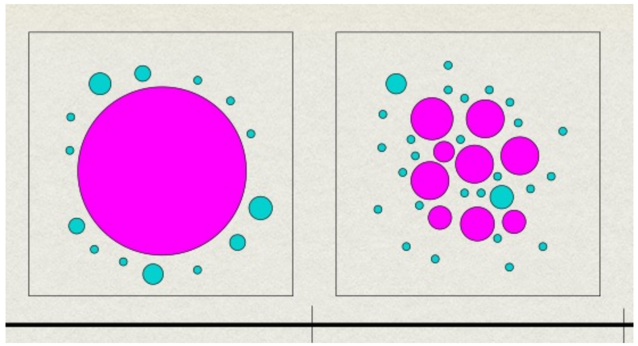


T = T_d: Paramagnetic state fractures into many states



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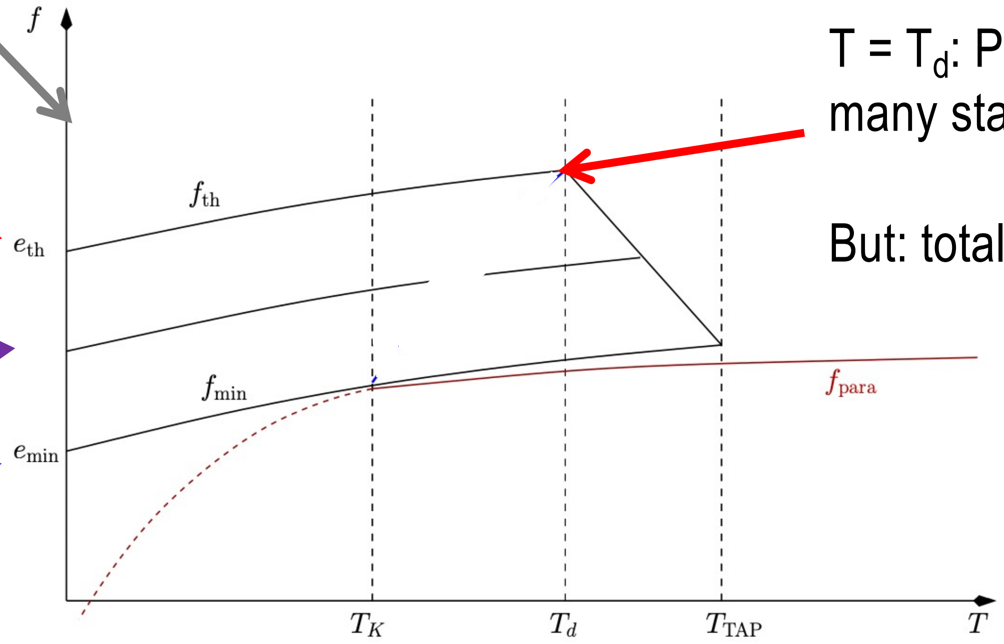

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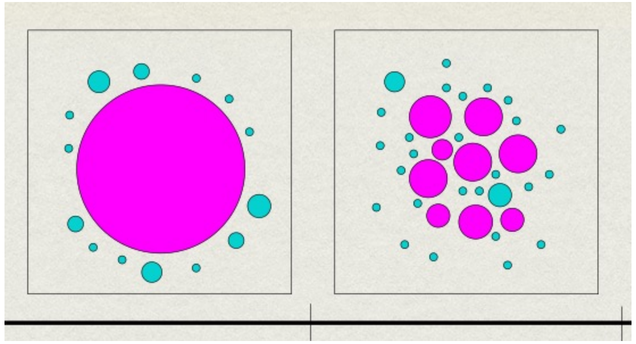

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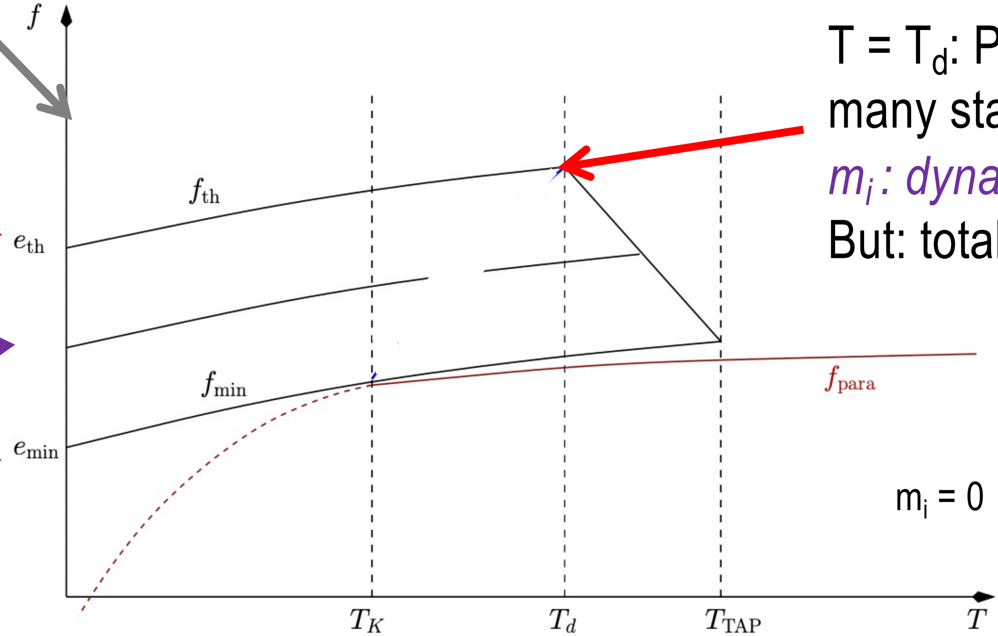
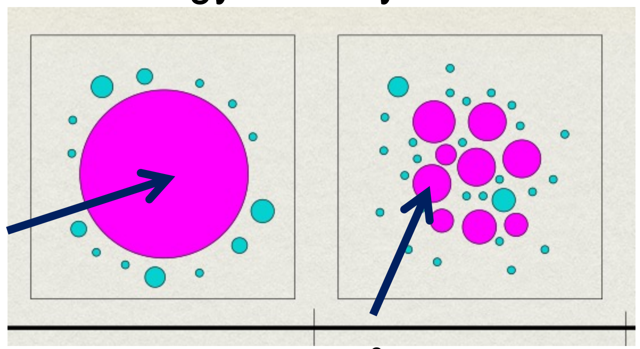


T = T_d: Paramagnetic state fractures into many states with sudden, finite freezing

m_i: dynamic clustering transition

But: total free energy is analytic!



$m_i = 0$

$m_i \neq 0$

The spherical p-spin model

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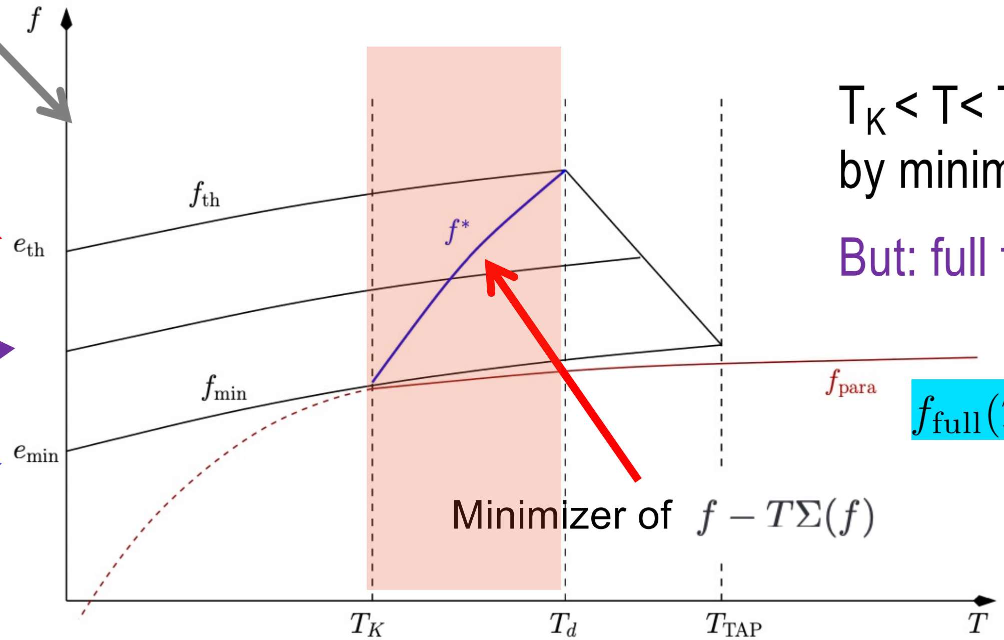

T_K < T< T_d: Gibbs weight is dominated by minima with internal free energy f*

But: full free energy remains analytic:

$$f_{\text{full}}(T) = f^* - T\Sigma(f^*) \stackrel{!}{=} f_{\text{para}}(T)$$

The spherical p-spin model

Solutions to the angular equations:

T = 0 :

- minima exist with energies $e \in [e_{\min}, e_{\text{th}}]$
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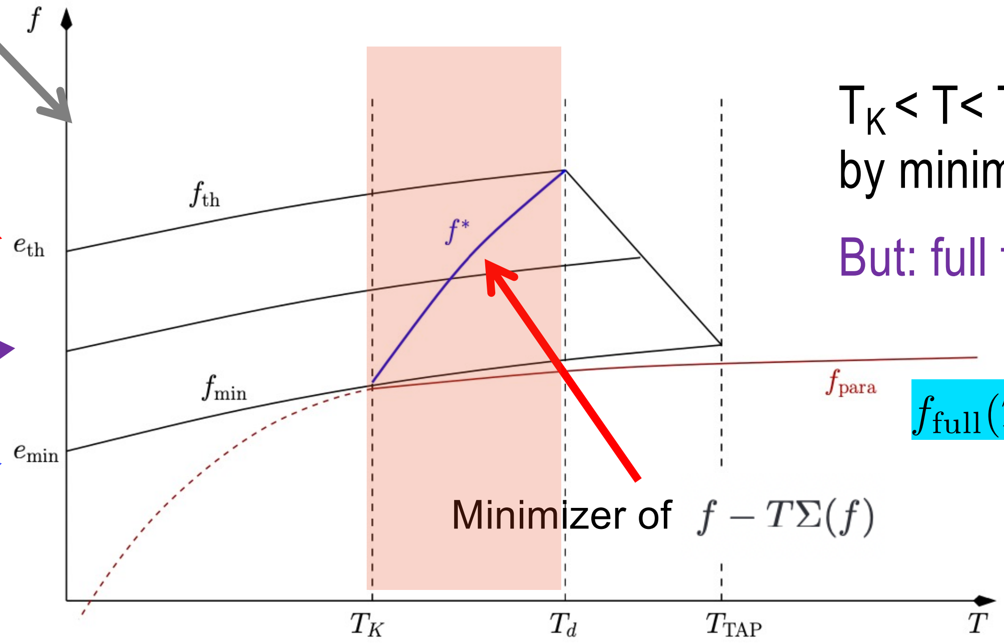
- $\Sigma(e_{min}) = 0$, and Σ is concave, increases with e

e> e_{th}: energy landscape dominated by saddles

Threshold Marginal states (like in SK)

Stable higher energy states (gapped Hessian)

Ground state (gapped Hessian)



$T_K < T < T_d$: Gibbs weight is dominated by minima with internal free energy f*

But: full free energy remains analytic:

$$f_{\text{full}}(T) = f^* - T\Sigma(f^*) \stackrel{!}{=} f_{\text{para}}(T)$$

Dynamics get stuck in threshold states → non-ergodic below T_d

The spherical p-spin model

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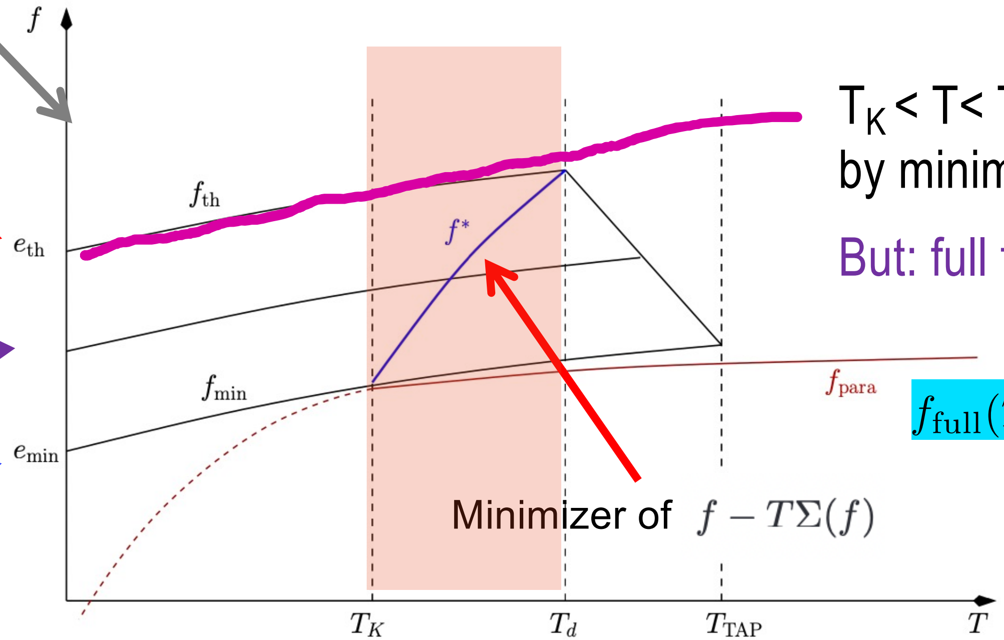

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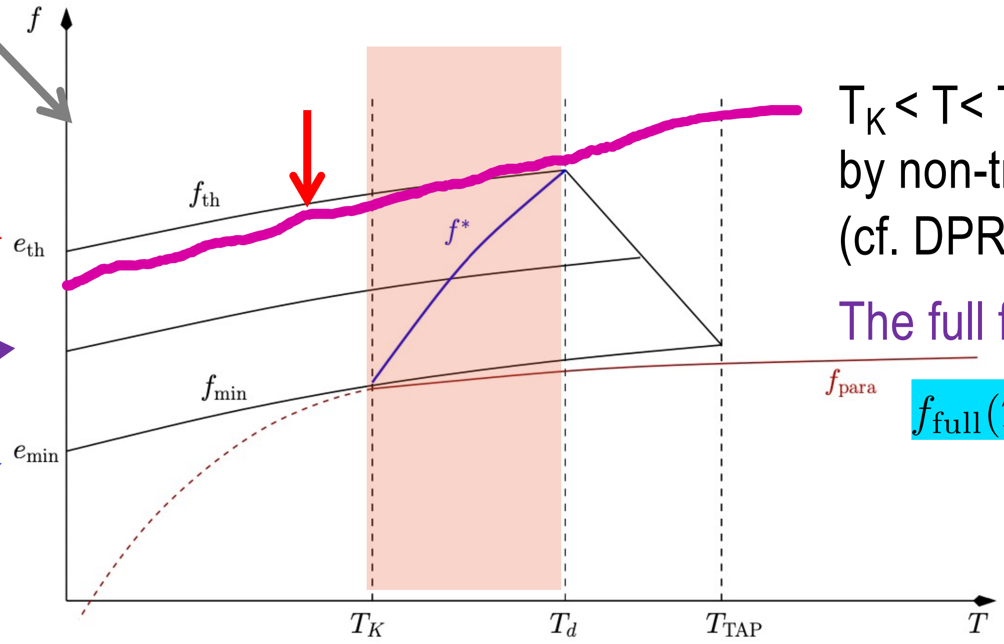

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Beyond mean field: onset of activated dynamics across (finite) barriers

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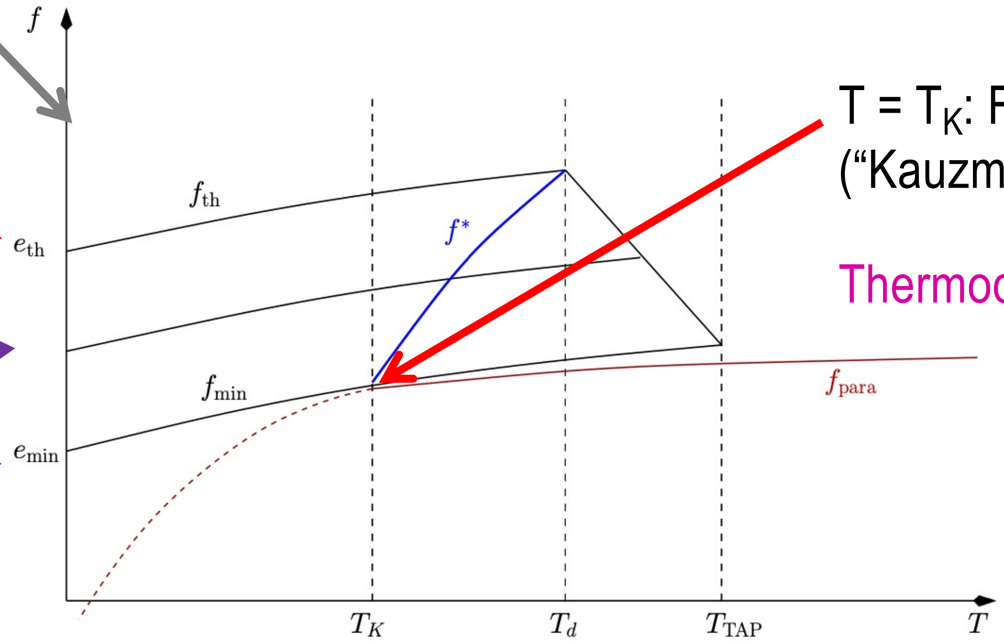

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Thermodynamic transition!

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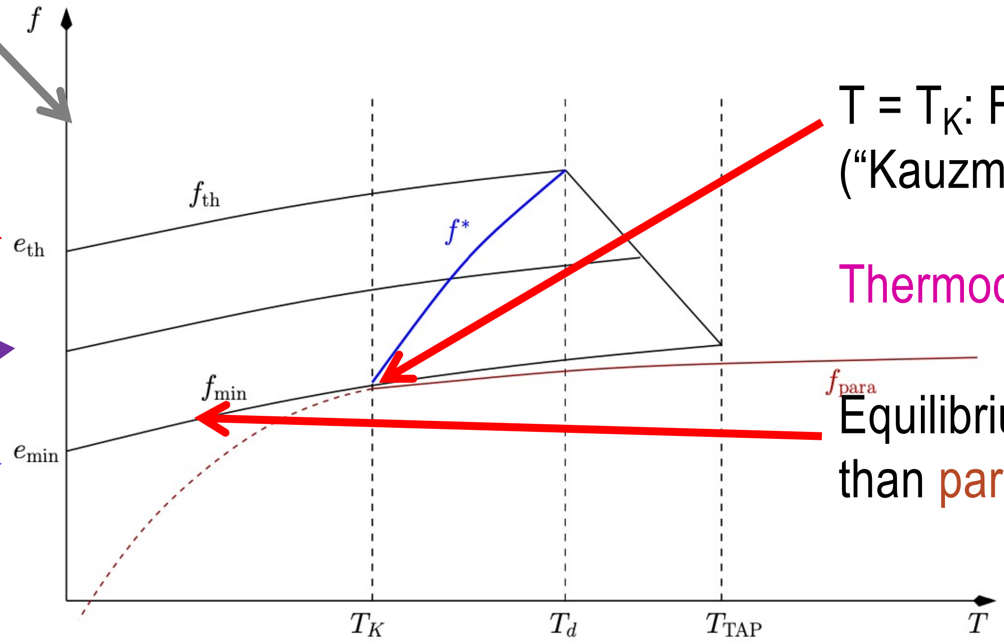

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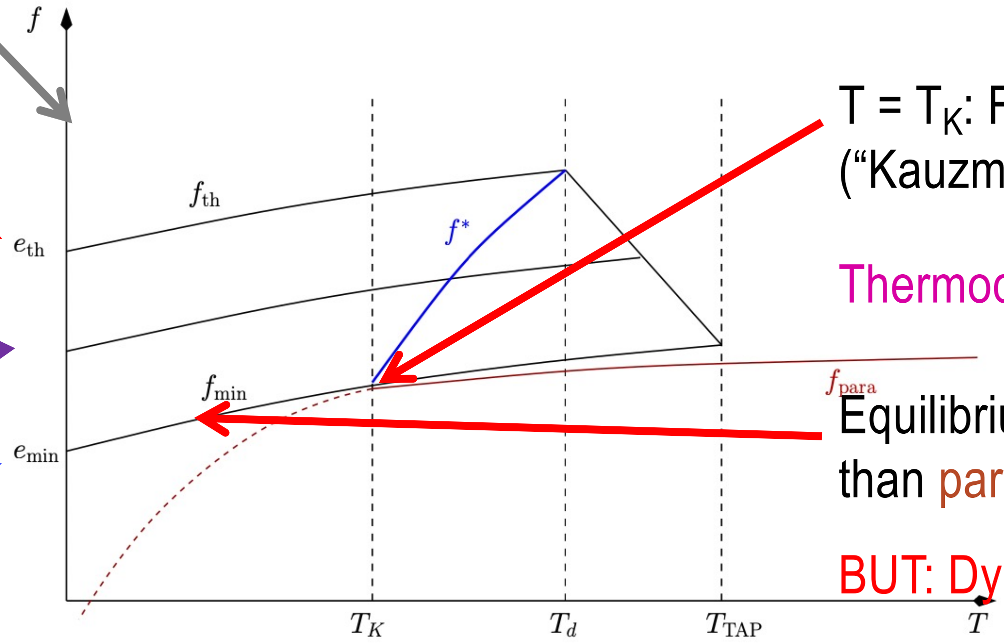

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Thermodynamic transition!

Equilibrium free energy: Higher than paramagnetic continuation!

BUT: Dynamically inaccessible!

First order nature of the dynamic transition

Important difference to p=2 spin glasses (cf. SK model):

Paramagnetic state m = 0 has no instability!

$$\frac{G(\{m_i\})}{N} = -\frac{1}{2\beta}\log(1-q) - \frac{1}{p!N}\sum_{i_1\cdots i_p} J_{i_1\ldots i_p} m_{i_1}\cdots m_{i_p} - \frac{\beta}{4}[1 - pq^{p-1} + q^p(p-1)]$$

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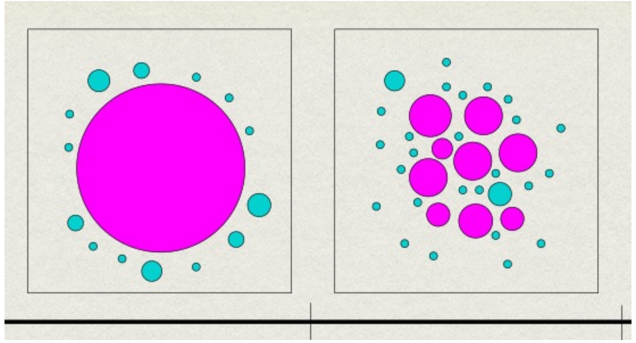
$p > 2$: Order parameter q **jumps** to finite value in minima at T_d!

Discontinuous (first-order-like) onset magnetization

(due to clustering and dynamic arrest)



Spin glass universality classes

Two different types of (mean field) spin glasses

SK-model $H = -\sum_{i<j} J_{ij} s_i s_j$

Continuous transition

$$q_{EA} = \frac{1}{N} \sum_i \langle s_i \rangle^2 \underset{T \to T_g}{\to} 0$$

MF-Model for real spin glasses

p-spin models $H = -\sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p} s_{i_1} \cdots s_{i_p}$

$p \geq 3$

Discontinuous transition

$$q_{EA} \underset{T \to T_g}{\to} q_c > 0$$

MF-analogon for structural glasses

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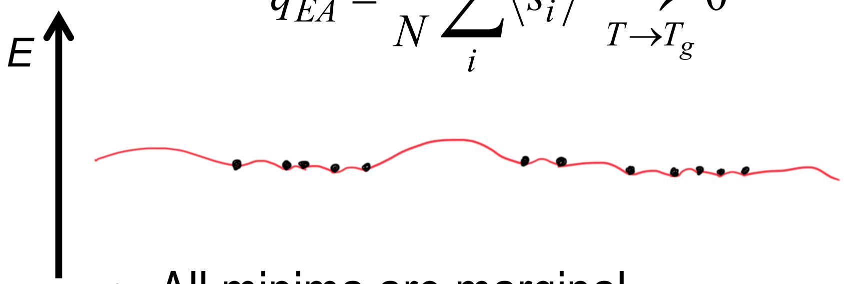

- All minima are marginal
- & have the same free energy density

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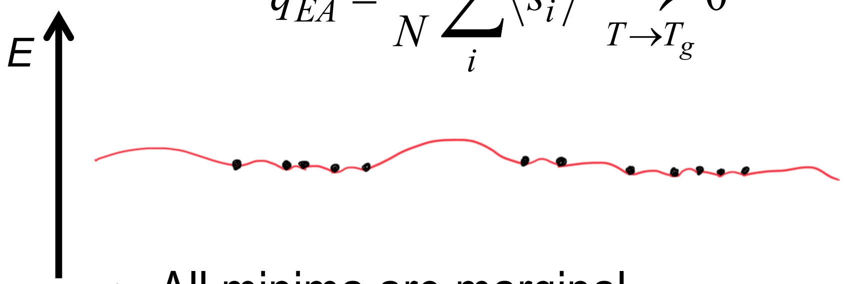

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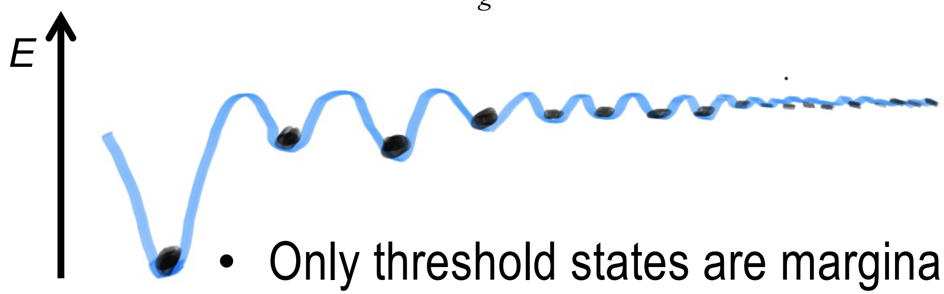

- Only threshold states are marginal
- States in extensive free energy window
- Separate dynamic (clustering) and thermodynamic (freezing) transitions

MF-analogon for structural glasses