Topological states and non-reciprocity in active matter

Lecture 1: Topological Active Matter

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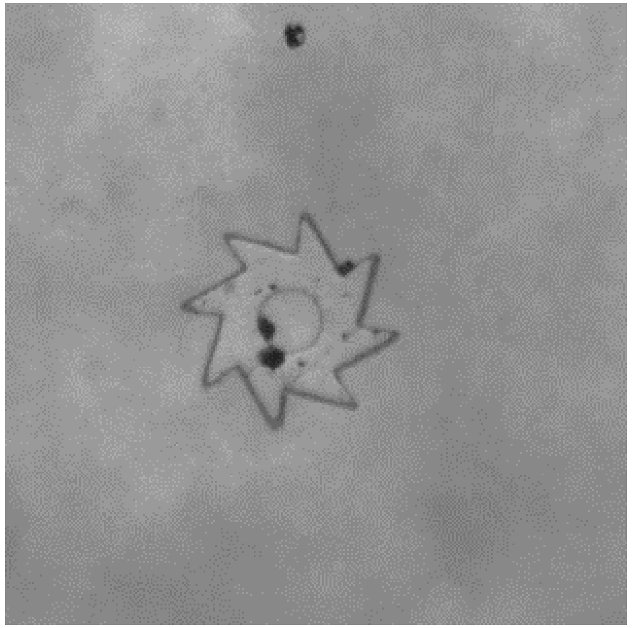
ICTP School on Quantum Dynamics of Matter, Light and Information

28 August 2025

UNIVERSITY OF CAMBRIDGE
Cavendish Laboratory

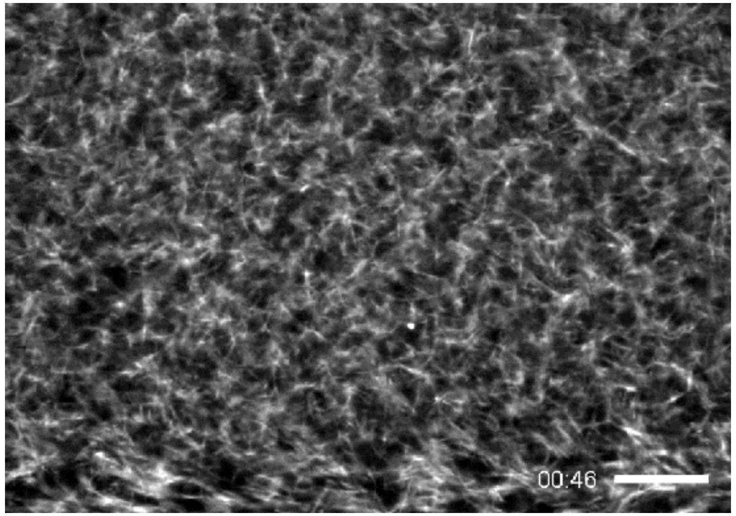
What is active matter?

Bacteria



Sokolov et al. PNAS (2010)

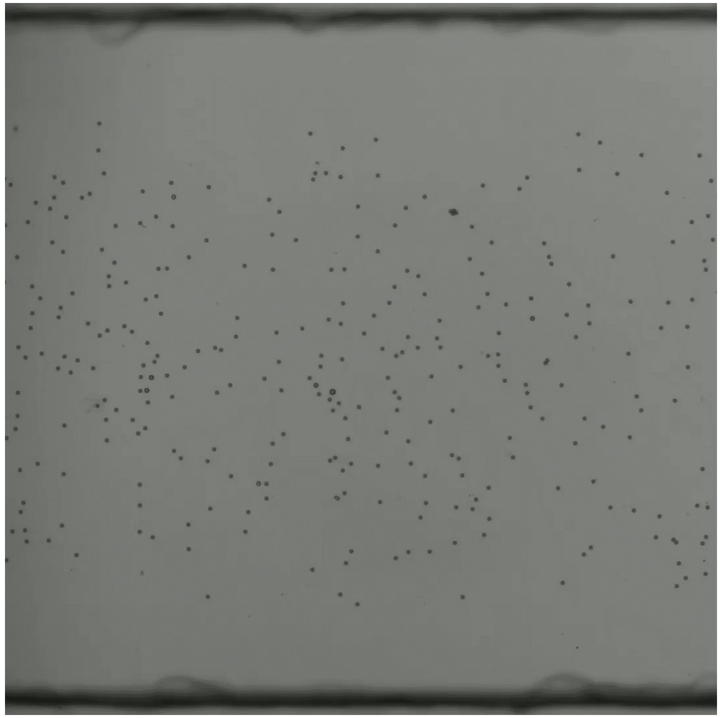
Isolated biological motors



Dogic lab. *Nature* (2012)

Flows, Work

Colloids in an electric field



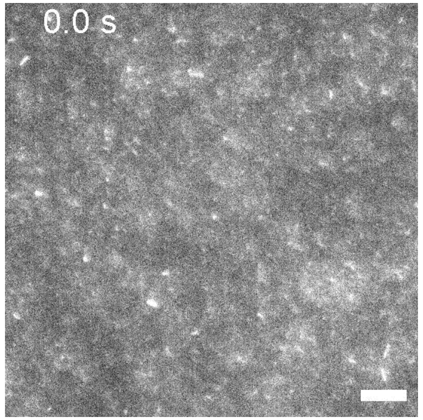
Bartolo lab. *Nature* (2013)

Energy consumed at the **microscale**
leads to emergent phenomena on the **macroscale**

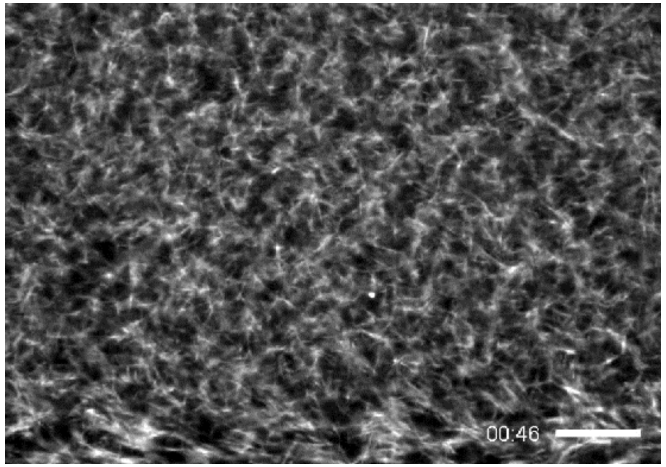
Active fluids vs active solids

Xu et al., *Nature Physics* (2023)

Bacterial biofilms

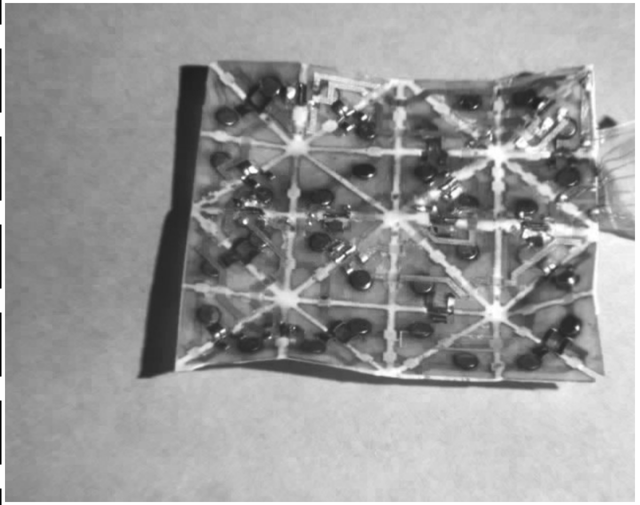


Active fluids



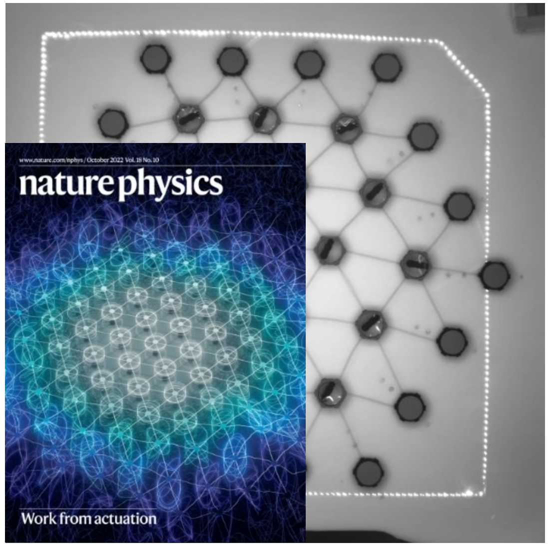
Dogic lab. *Nature* (2012)

Active solids: self-folding origami



Hawkes et al *PNAS* (2010)

Caged robots



Baconnier et al., *Nature Physics* (2022)

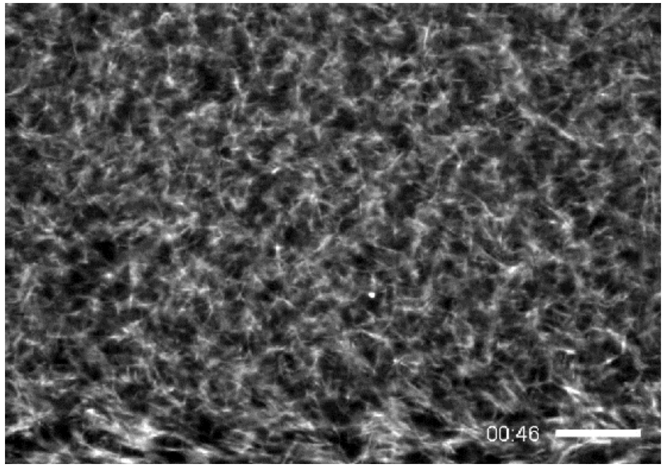
Actuation, mechanical work

Reconfigurable shape

Energy consumed at the microscale
leads to **elastic** phenomena on the macroscale

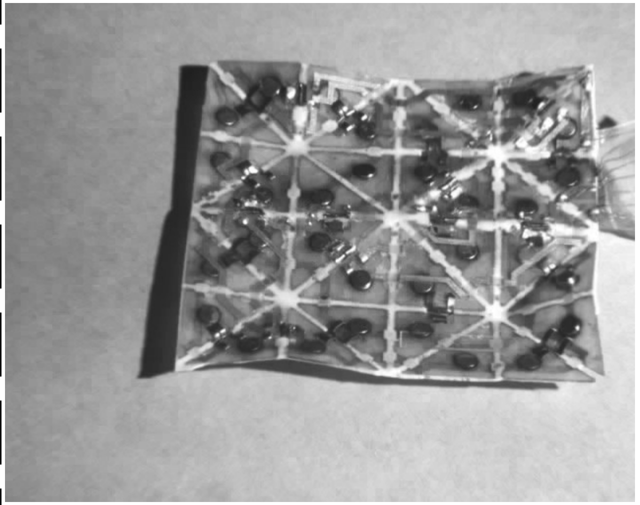
Active fluids vs active solids

Active fluids



Dogic lab. *Nature* (2012)

Active solids: self-folding origami



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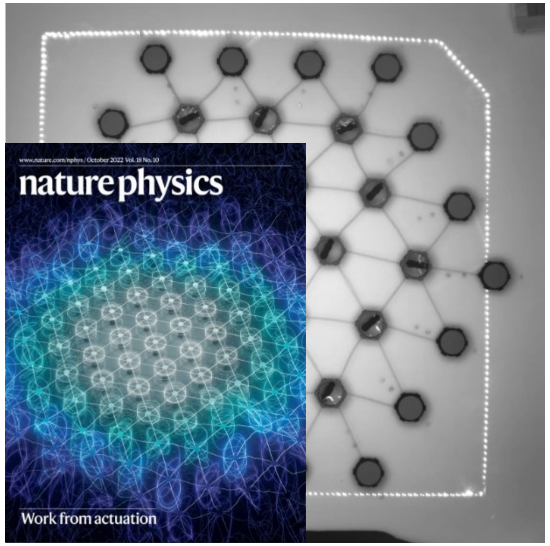
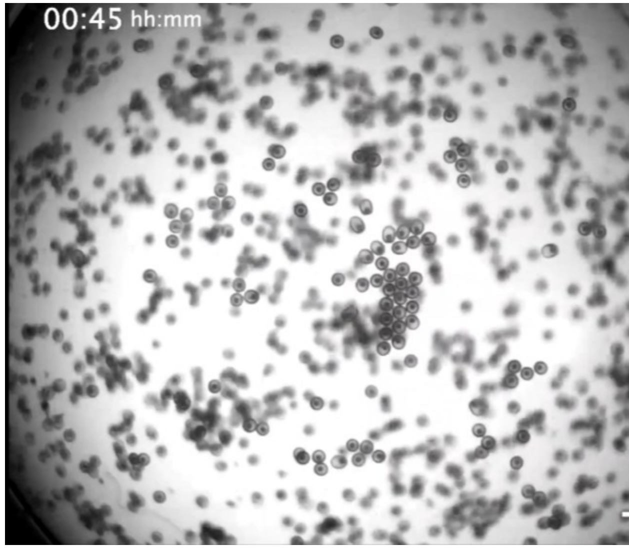


Baconnier et al., *Nature Physics* (2022)

Starfish embryos



Tan et al. *Nature* (2022)

Actuation, mechanical work   Reconfigurable shape

Energy consumed at the microscale
leads to **elastic** phenomena on the macroscale

Questions to ask

How do we **describe** and **classify** *active matter* based on symmetries and conservation laws?

What features of *active matter* are **universal** and independent of microscopic detail?

How can we design active materials with *mechanical* properties which are unusual or do not occur naturally?

About these lectures

Lecture 1. Topological active matter
Part 1.1:
Overview; Definition of active matter
Part 1.2:
Classification of active fluids
Part 1.3:
Topological active matter

Lecture 2. Non-reciprocal active solids
Part 2.1:
Introduction to active solids
Part 2.2:
Odd elasticity
Part 2.3:
Current topics: active percolation, pattern formation

Review articles on active matter:
Shankar et al Topological active matter
Nature Reviews Physics (2022)

Fruchart, Scheibner, Vitelli. Odd viscosity and odd elasticity.
Annual Review of Condensed Matter Physics 14, 471 (2023)

Marchetti et al Hydrodynamics of soft active matter
Reviews of Modern Physics 85, 1143 (2013)

Background textbook:
P. M. Chaikin and T. C. Lubensky (1995) Ch. 6-10
Principles of Condensed Matter Physics

Topology:
David Mermin *Rev Mod Phys* (1979)
The topological theory of defects in ordered media

This lecture:

Broad introduction to active matter and connections to topology

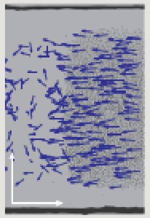
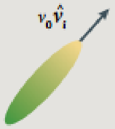
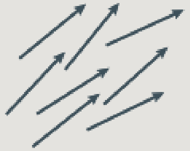
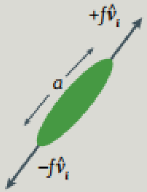
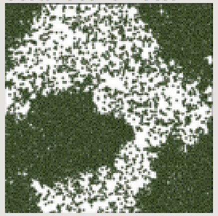
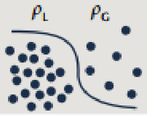
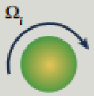
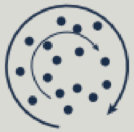
Why active hydrodynamics?

Does not refer to only the fluid in which active particles are embedded, “hydrodynamic interactions.”

Describes large-lengthscale, slow-timescale phenomena associated with coherent collections of active particles.

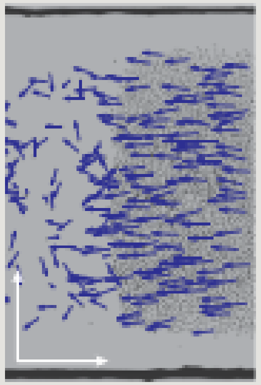
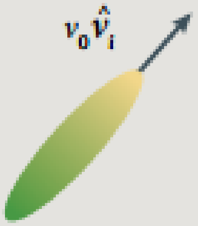
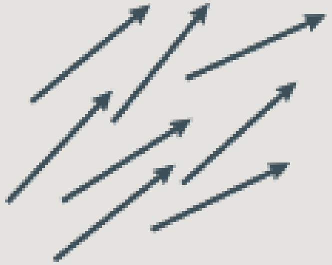
Well-developed applications across both biological systems and synthetic materials, but many questions are current research topics.

Part 1.2: Classify active fluids based on symmetries

System	Particles	Order parameter	Example model	Model notes
Active polar fluids, such as polar fluid composed of colloidal rollers 	Polar: self-propulsion speed v_0 	Collective polar order: vector order parameter $\mathbf{P} = \langle \sum_i \hat{\nu}_i \delta(\mathbf{r}-\mathbf{r}_i) \rangle$ 	Toner–Tu equations[52,53]: $\partial_t \rho + v_0 \nabla \cdot (\rho \mathbf{P}) = 0$, $\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \nabla \mathbf{P} = [a_2 - a_4 \lvert \mathbf{P} \rvert^2] \mathbf{P}$ $+ K \nabla^2 \mathbf{P} - \nabla \Pi$	v_0 and λ are active parameters capturing advection; a_2 and a_4 control the ordering transition; K is an elastic constant; and $\Pi(\rho)$ a density-dependent pressure
Active nematic fluids, such as a microtubule–kinesin film 	Apolar: exerts force dipole $\alpha \sim f\ell$ 	Nematic order: tensor order parameter (in d dimensions) $\mathbf{Q} = \langle \sum_i (\hat{\nu}_i \hat{\nu}_i - \frac{1}{d}) \delta(\mathbf{r}-\mathbf{r}_i) \rangle$ 	Incompressible hydrodynamics of nematic order (**Q**) coupled with flow (**u**) driven by an active stress ($\sigma_a = \alpha \mathbf{Q}$)[39]: $\partial_t \mathbf{Q} + \mathbf{u} \cdot \nabla \mathbf{Q} + [\omega, \mathbf{Q}] =$ $\lambda \mathbf{E} + [a_2 - a_4 S^2] \mathbf{Q} + K \nabla^2 \mathbf{Q}$, $\eta \nabla^2 \mathbf{u} - \Gamma \mathbf{u} + \nabla \cdot \sigma_a - \nabla \Pi = \mathbf{0}$, $\nabla \cdot \mathbf{u} = 0$	Force balance involves friction (Γ), viscosity (η) and pressure (Π); **E** and $\boldsymbol{\omega}$ are the symmetric and antisymmetric parts, respectively, of the strain rate tensor ($\nabla \mathbf{u}$); λ is the flow alignment parameter; nematic ordering $[S^2 = \mathrm{tr}(\mathbf{Q}^2) d/(d-1)]$ is controlled by a_2 and a_4; and K is the elastic stiffness
Scalar active matter 	Scalar active particle with no alignment 	Phase separation: scalar order parameter, the density difference between liquid (ρ_L) and gas (ρ_G) phases $\rho = \langle \sum_i \delta(\mathbf{r}-\mathbf{r}_i) \rangle$, $\phi = \frac{(2\rho - \rho_L - \rho_G)}{(\rho_L - \rho_G)}$ 	Motility-induced phase separation described by Cahn–Hilliard dynamics involving the density (ρ)[253]: $\partial_t \rho = \nabla \cdot [D(\rho) \nabla \mu]$, $\mu = \ln[\rho v(\rho)] + \kappa(\rho) \nabla^2 \rho$	The effective chemical potential μ includes the density suppression of motility $v(\rho)$ and nonintegrable gradient terms ($\kappa'(\rho) \neq 0$); density also suppresses the diffusion constant ($D \propto [v(\rho)]^2$)
Chiral active fluids, such as colloidal spinning magnets 	Chiral active particle self-spinning at rate Ω_i in 2D 	Collective chirality: scalar field, the intrinsic rotation frequency $\Omega = \langle \sum_i \Omega_i \delta(\mathbf{r}-\mathbf{r}_i) \rangle$ 	Hydrodynamics of an isotropic chiral active fluid in 2D, including density (ρ), flow (**u**) and the internal spin density (Ω)[168,175]: $\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$, $\partial_t \mathbf{u} = \eta \nabla^2 \mathbf{u} - \Gamma \mathbf{u}$ $+ \eta_R \nabla_\perp (2\Omega - \omega) + \eta_o \nabla^2 \mathbf{u}_\perp - \nabla \Pi$, $\partial_t \Omega = \tau_0 - \Gamma_\Omega \Omega - 2\eta_R (2\Omega - \omega)$ $+ D_\Omega \nabla^2 \Omega$	Besides regular viscosity (η) and friction (Γ), odd viscosity (η_o) and rotational viscosity (η_R) are also present, the latter in the antisymmetric stress; chirality enters through terms involving $\mathbf{u}_\perp = \hat{\mathbf{z}} \times \mathbf{u}$ and the vorticity $\omega = \hat{\mathbf{z}} \cdot (\nabla \times \mathbf{u})$; the active torque ($\tau_0$) injects spin into the fluid, which is damped by spin friction (Γ_Ω) and diffusion (D_Ω)

Shankar et al Topological active matter *Nature Reviews Physics* (2022)

Part 1.2: Examples. Active polar fluids

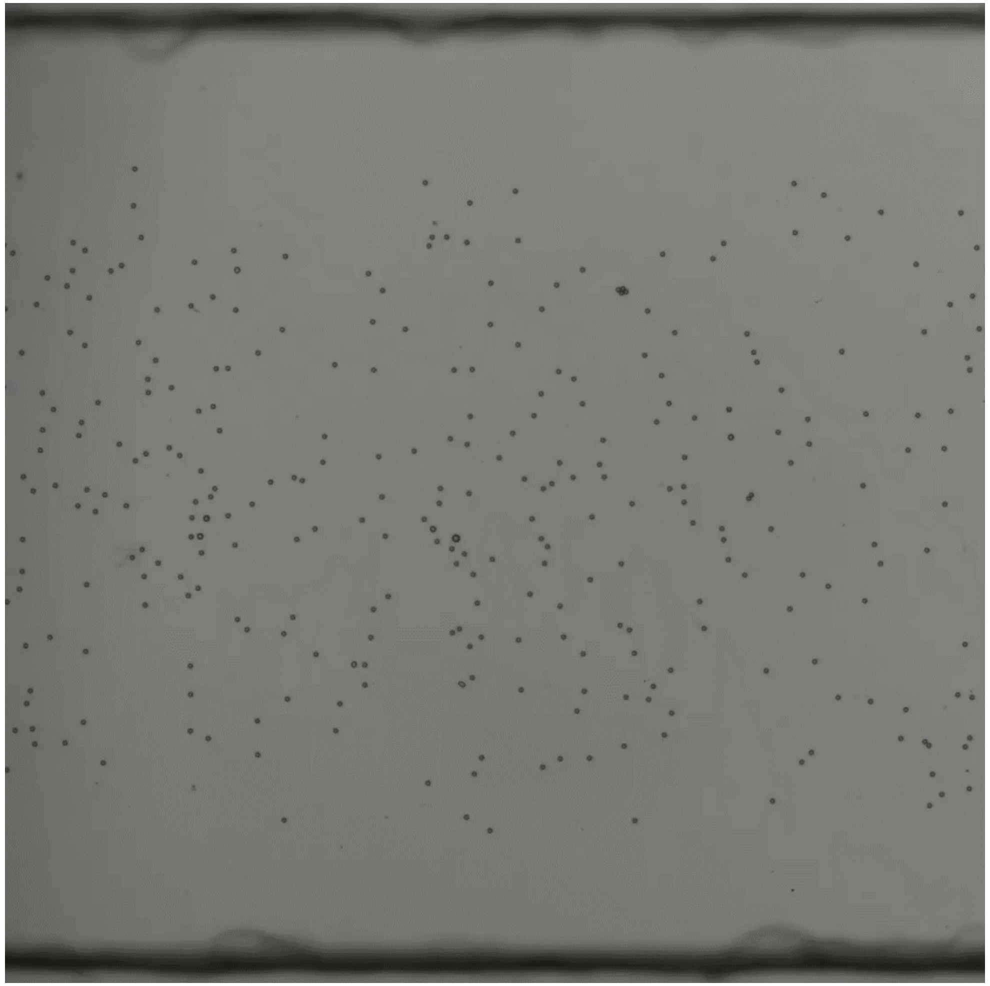
System	Particles	Order parameter	Example model	Model notes
Active polar fluids, such as polar fluid composed of colloidal rollers 	Polar: self-propulsion speed v_0 	Collective polar order: vector order parameter $\mathbf{P} = \langle \sum_i \hat{\nu}_i \delta(\mathbf{r} - \mathbf{r}_i) \rangle$ 	Toner–Tu equations[52,53]: $\partial_t \rho + v_0 \nabla \cdot (\rho \mathbf{P}) = 0$, $\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \nabla \mathbf{P} = [a_2 - a_4 \lvert \mathbf{P} \rvert^2] \mathbf{P} + K \nabla^2 \mathbf{P} - \nabla \Pi$	v_0 and λ are active parameters capturing advection; a_2 and a_4 control the ordering transition; K is an elastic constant; and $\Pi(\rho)$ a density-dependent pressure

$$\partial_t \rho + v_0 \boldsymbol{\nabla} \cdot (\rho \mathbf{P}) = 0$$

$$\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \boldsymbol{\nabla} \mathbf{P} = [a_2 - a_4 |\mathbf{P}|^2] \mathbf{P} + K \nabla^2 \mathbf{P} - \boldsymbol{\nabla} \Pi$$

Shankar et al <u>Topological active matter</u>
Nature Reviews Physics (2022)

Active polar fluids



Colloids in an electric field
Bartolo lab. *Nature* (2013)

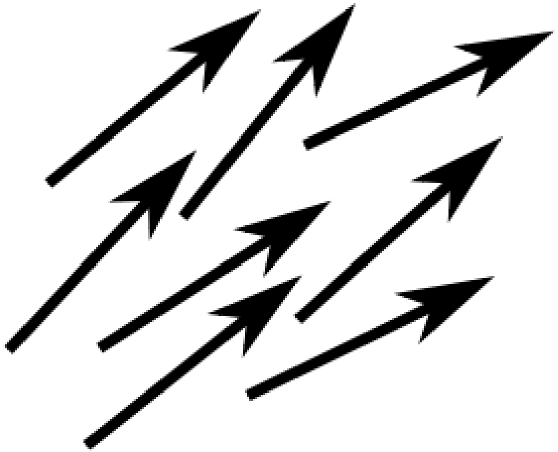
$$\partial_t \rho + v_0 \boldsymbol{\nabla} \cdot (\rho \mathbf{P}) = 0$$

$$\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \boldsymbol{\nabla} \mathbf{P} = [a_2 - a_4 |\mathbf{P}|^2] \mathbf{P} + K \nabla^2 \mathbf{P} - \boldsymbol{\nabla} \Pi$$

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active polar fluids

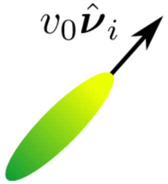
Polar active particle with self-propulsion speed v_0.



Collective polar order is captured by a vector order parameter.

$$\mathbf{P} = \langle \sum_i \hat{\boldsymbol{\nu}}_i \delta(\mathbf{r} - \mathbf{r}_i) \rangle$$

Shankar et al Topological active matter
Nature Reviews Physics (2022)

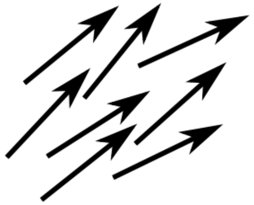


Polar active particle with self-propulsion speed v_0.

Toner-Tu equations

Density ρ

Pressure $\Pi(\rho)$



Collective polar order is captured by a vector order parameter.

$\mathbf{P} = \langle \sum_i \hat{\boldsymbol{\nu}}_i \delta(\mathbf{r} - \mathbf{r}_i) \rangle$

$$\partial_t \rho + v_0 \boldsymbol{\nabla} \cdot (\rho \mathbf{P}) = 0$$ Continuity equation

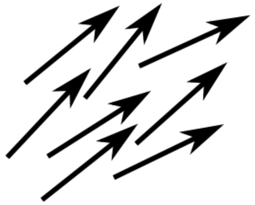
$$\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \boldsymbol{\nabla} \mathbf{P} = [a_2 - a_4 |\mathbf{P}|^2]\mathbf{P} + K \nabla^2 \mathbf{P} - \boldsymbol{\nabla} \Pi$$

Relaxation for order parameter $\mathbf{P} \propto \mathbf{v}$

Shankar et al <u>Topological active matter</u>
Nature Reviews Physics (2022)

Toner-Tu equations

Polar active particle with self-propulsion speed v_0.



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Relaxation for order parameter $\mathbf{P} \propto \mathbf{v}$

No derivatives: $\mathbf{P} = \text{const} = \sqrt{a_2/a_4}$

Spontaneous symmetry breaking far from equilibrium

Shankar et al Topological active matter *Nature Reviews Physics* (2022)

Derivations of the Toner-Tu equations

(1) Write down based on symmetries

[Toner & Tu. *Phys Rev Lett* 1995]

(2) Derive from taking the overdamped limit

[e.g., AS, van Zuiden, Bartolo, Vitelli. *Nature Physics* 2017]

(3) Derive from a microscopic model

[Bertin, Droz, Gregoire. *Phys Rev E* 2006/*J Phys A* 2009]

(3) Derive from a microscopic model

$$\frac{\partial \mathbf{w}}{\partial t} + \gamma(\mathbf{w}\cdot\nabla)\mathbf{w} = -\frac{v_0^2}{2}\nabla\rho + \frac{\kappa}{2}\nabla\mathbf{w}^2 + (\mu - \xi\mathbf{w}^2)\mathbf{w} + \nu\nabla^2\mathbf{w}$$
$$- \kappa(\nabla\cdot\mathbf{w})\mathbf{w} + 2\nu'\nabla\rho\cdot\mathbf{M} - \nu'(\nabla\cdot\mathbf{w})\nabla\rho, \qquad (27)$$

with $\nu' = \partial\nu/\partial\rho$ and where $\mathbf{M} = \frac{1}{2}(\nabla\mathbf{w} + \nabla\mathbf{w}^{\mathbf{T}})$ is the symmetric part of the momentum gradient tensor. The different coefficients appearing in this equation are given by

$$\nu = \frac{v_0^2}{4}\left[\lambda\left(1 - e^{-2\sigma_0^2}\right) + \frac{16}{3\pi}d_0 v_0 \rho\left(\frac{7}{5} + e^{-2\sigma^2}\right)\right]^{-1}, \qquad (28)$$

$$\gamma = \frac{16\nu d_0}{\pi v_0}\left(\frac{16}{15} + 2\,e^{-2\sigma^2} - e^{-\sigma^2/2}\right), \qquad (29)$$

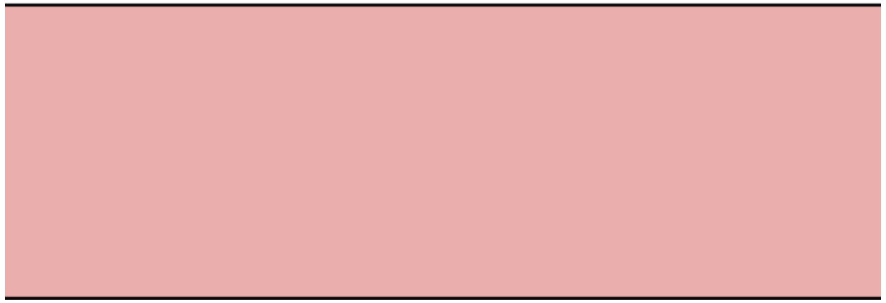
$$\kappa = \frac{16\nu d_0}{\pi v_0}\left(\frac{4}{15} + 2\,e^{-2\sigma^2} + e^{-\sigma^2/2}\right), \qquad (30)$$

$$\mu = \frac{8}{\pi}d_0 v_0 \rho\left(e^{-\sigma^2/2} - \frac{2}{3}\right) - \lambda\left(1 - e^{-\sigma_0^2/2}\right), \qquad (31)$$

$$\xi = \frac{256\nu d_0^2}{\pi^2 v_0^2}\left(e^{-\sigma^2/2} - \frac{2}{5}\right)\left(\frac{1}{3} + e^{-2\sigma^2}\right). \qquad (32)$$

Bertin, Droz, Gregoire. *Phys Rev E* 2006
J Phys A 2009

Hydrodynamics of a liquid



$$\partial_t \varrho + \nabla_i(\varrho v_i) = 0$$

Mass conservation
(Continuity equation)

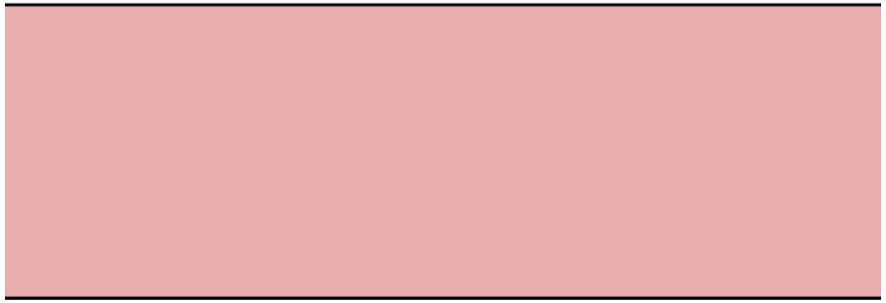
$$\partial_t(\varrho v_j) + \nabla_i\left(\varrho v_i v_j\right) = \nabla_i \sigma_{ij}$$

Momentum conservation
(Navier-Stokes equation)

Simple fluid hydrodynamics results from conservation laws

AS, B. C. v. Zuiden, D. Bartolo, V. Vitelli. *Nat. Phys.* (2017)

Hydrodynamics of a polar liquid



$$\partial_t \varrho + \nabla_i(\varrho v_i) = 0$$ Mass conservation

$$\partial_t(\varrho v_j) + \nabla_i\left(\varrho v_i v_j\right) = \nabla_i \sigma_{ij}$$ Momentum conservation

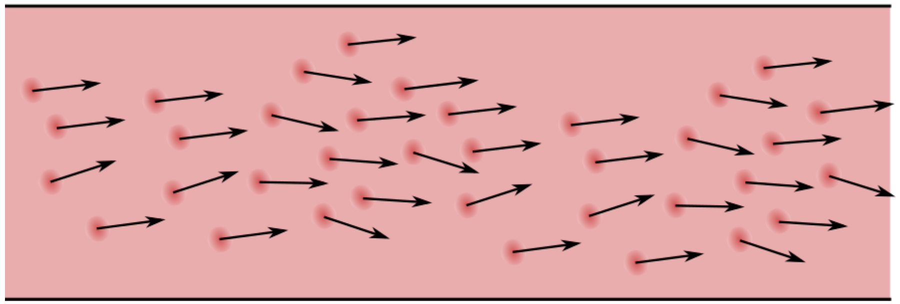
$$\partial_t p_j + v_i \nabla_i p_j + \omega_{ji} p_i = \nu_2 v_{ji} p_i - \Gamma^p \frac{\delta \mathcal{H}}{\delta p_j}$$ Polarization relaxation

$$\omega_{ji} \equiv \tfrac{1}{2}\left(\partial_j v_i - \partial_i v_j\right) \qquad v_{ji} \equiv \tfrac{1}{2}\left(\partial_j v_i + \partial_i v_j\right)$$

Complex fluid has slow variables due to broken symmetries

AS, B. C. v. Zuiden, D. Bartolo, V. Vitelli. *Nat. Phys.* (2017)

Hydrodynamics of a polar active liquid



$$\partial_t \varrho + \nabla_i(\varrho v_i) = 0$$

Mass conservation

$$\partial_t(\varrho v_j) + \nabla_i\left(\varrho v_i v_j\right) = \nabla_i \sigma_{ij} - \Gamma^v(v_j - v_0 p_j)$$

Velocity-polarization coupling

$$\partial_t p_j + v_i \nabla_i p_j + \omega_{ji} p_i = \nu_1 v_j + \nu_2 v_{ji} p_i - \Gamma^p \frac{\delta \mathcal{H}}{\delta p_j}$$

Polarization relaxation

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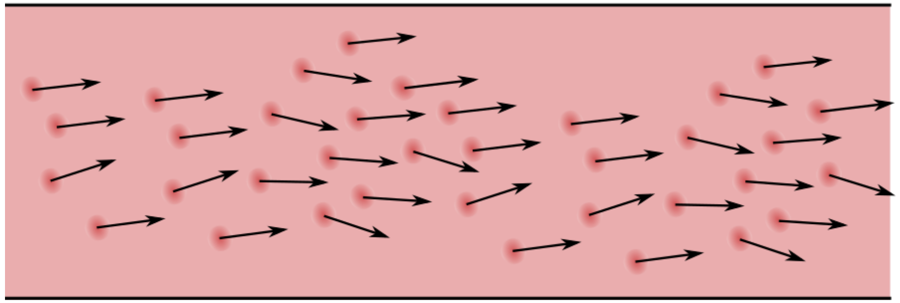
Polar active liquid has velocity coupled to polarization

Toner and Tu. *Phys. Rev. Lett.* (1995)

Marchetti et al. *Rev. Mod Phys.* (2013)

AS, B. C. v. Zuiden, D. Bartolo, V. Vitelli. *Nat. Phys.* (2017)

(overdamped) Hydrodynamics of a polar active liquid



$$\partial_t \varrho + \nabla_i(\varrho v_i) = 0$$ Mass conservation

$$\cancel{\partial_t(\varrho v_j) + \nabla_i(\varrho v_i v_j) =} \quad \boxed{\mathbf{v} = v_0 \mathbf{P}}$$ Velocity-polarization coupling

$$\partial_t p_j + v_i \nabla_i p_j + \omega_{ji} p_i = \nu_1 v_j + \nu_2 v_{ji} p_i - \Gamma^p \frac{\delta \mathcal{H}}{\delta p_j}$$ Polarization relaxation

$$\omega_{ji} \equiv \frac{1}{2}(\partial_j v_i - \partial_i v_j) \qquad v_{ji} \equiv \frac{1}{2}(\partial_j v_i + \partial_i v_j)$$

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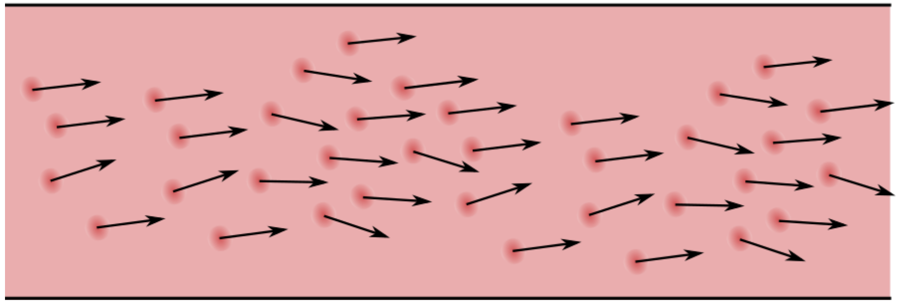
AS, B. C. v. Zuiden, D. Bartolo, V. Vitelli. *Nat. Phys.* (2017)

(overdamped) Hydrodynamics of a polar active liquid



$$\partial_t \rho + v_0 \boldsymbol{\nabla} \cdot (\rho \mathbf{P}) = 0$$

Mass conservation

$$\partial_t \mathbf{P} + \lambda \mathbf{P} \cdot \boldsymbol{\nabla} \mathbf{P} = [a_2 - a_4 |\mathbf{P}|^2] \mathbf{P} + K \nabla^2 \mathbf{P} - \boldsymbol{\nabla} \Pi$$

Polarization relaxation

$$\mathbf{v} = v_0 \mathbf{P}$$

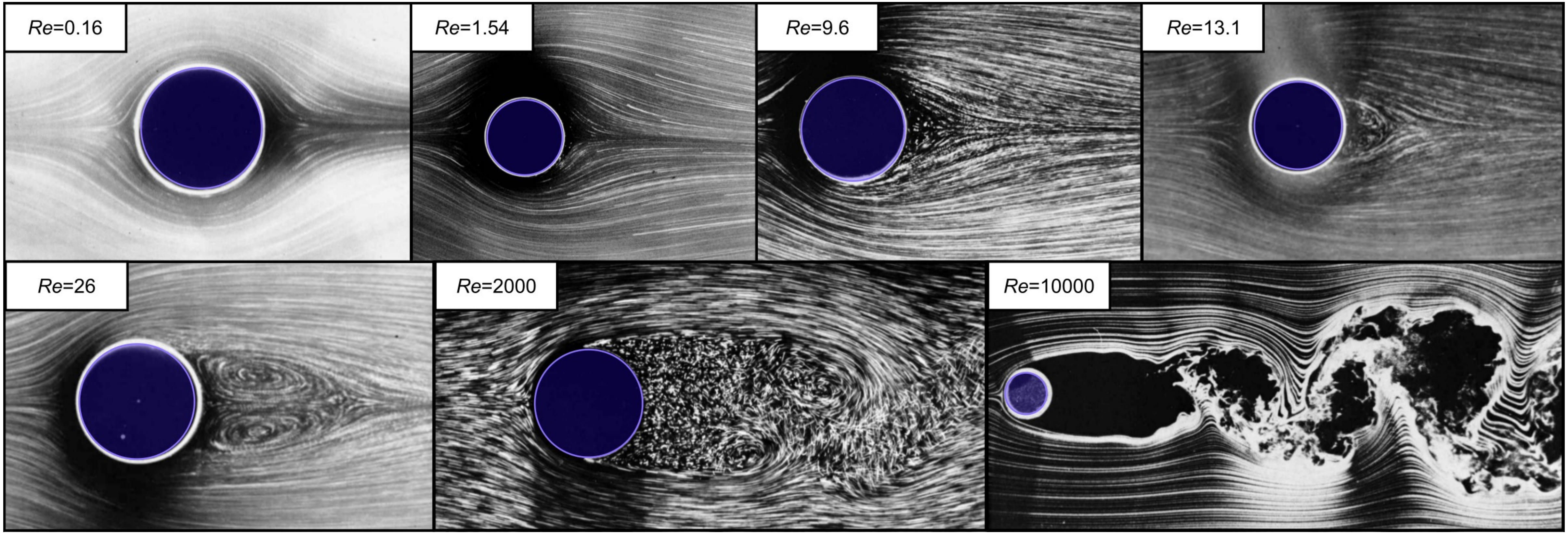
Overdamped dynamics reminiscent of inertial dynamics!

Toner and Tu. *Phys. Rev. Lett.* (1995)

Marchetti et al. *Rev. Mod Phys.* (2013)

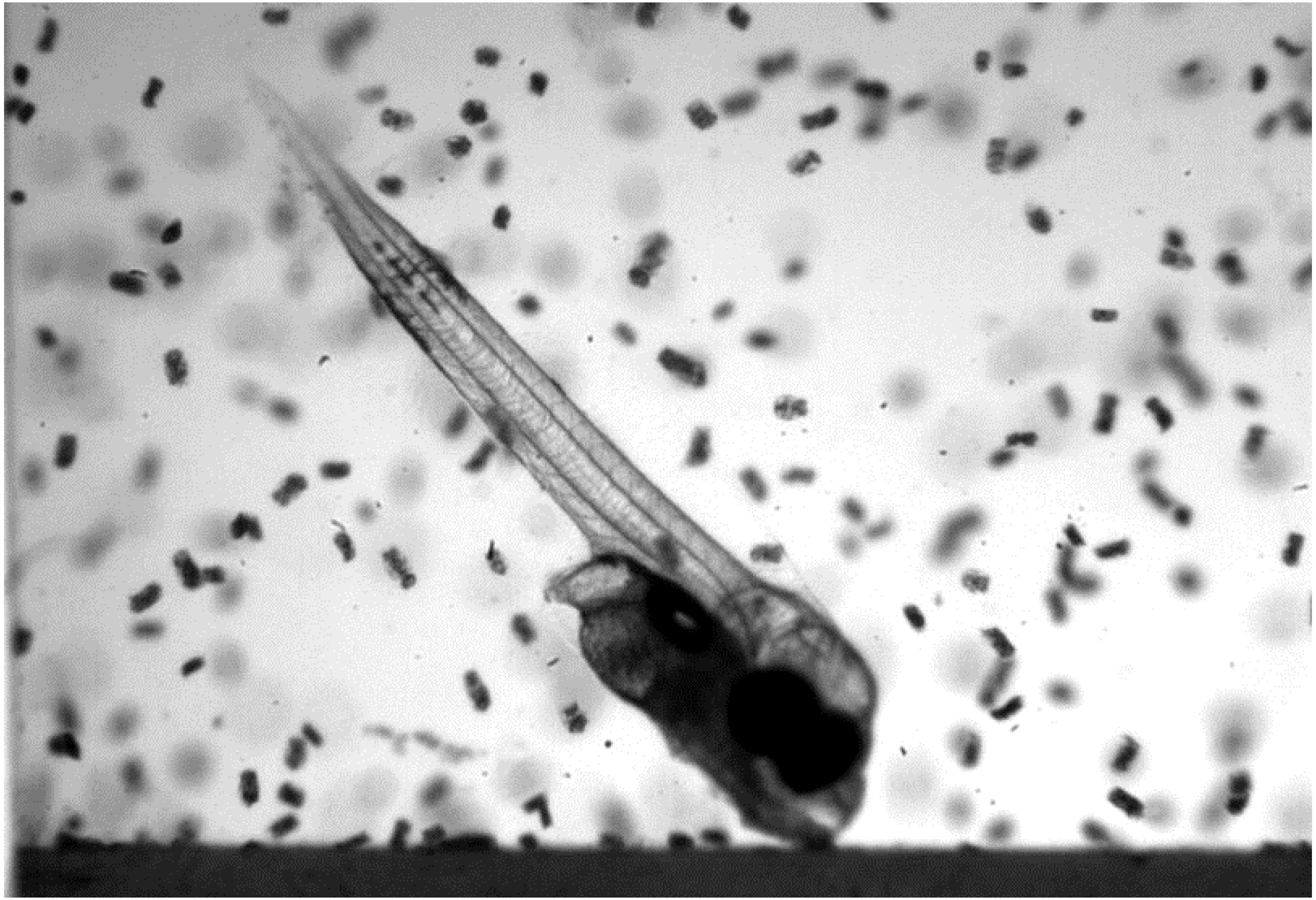
AS, B. C. v. Zuiden, D. Bartolo, V. Vitelli. *Nat. Phys.* (2017)

How to classify active fluids: Role of inertia



$$Re = \frac{\rho L u}{\eta} = \frac{\text{inertia}}{\text{viscosity}}$$

How to classify active fluids: Role of inertia

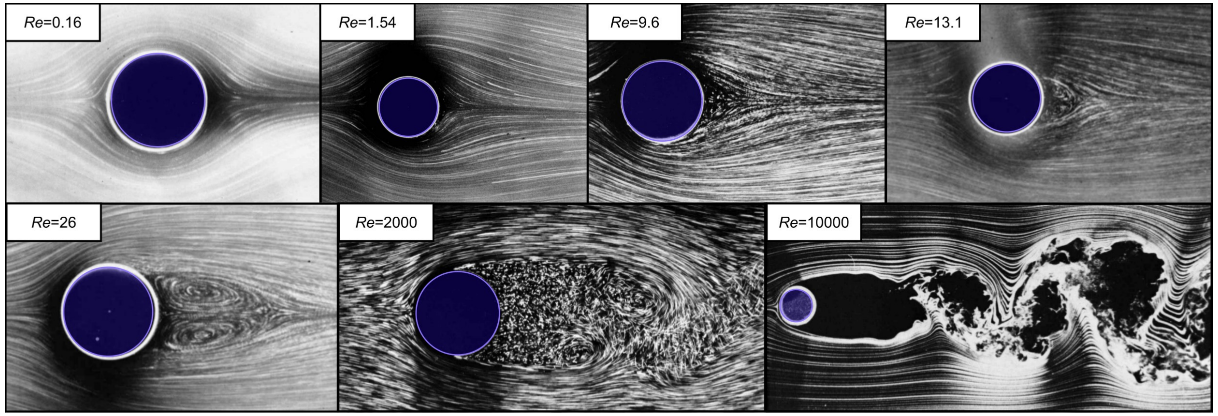


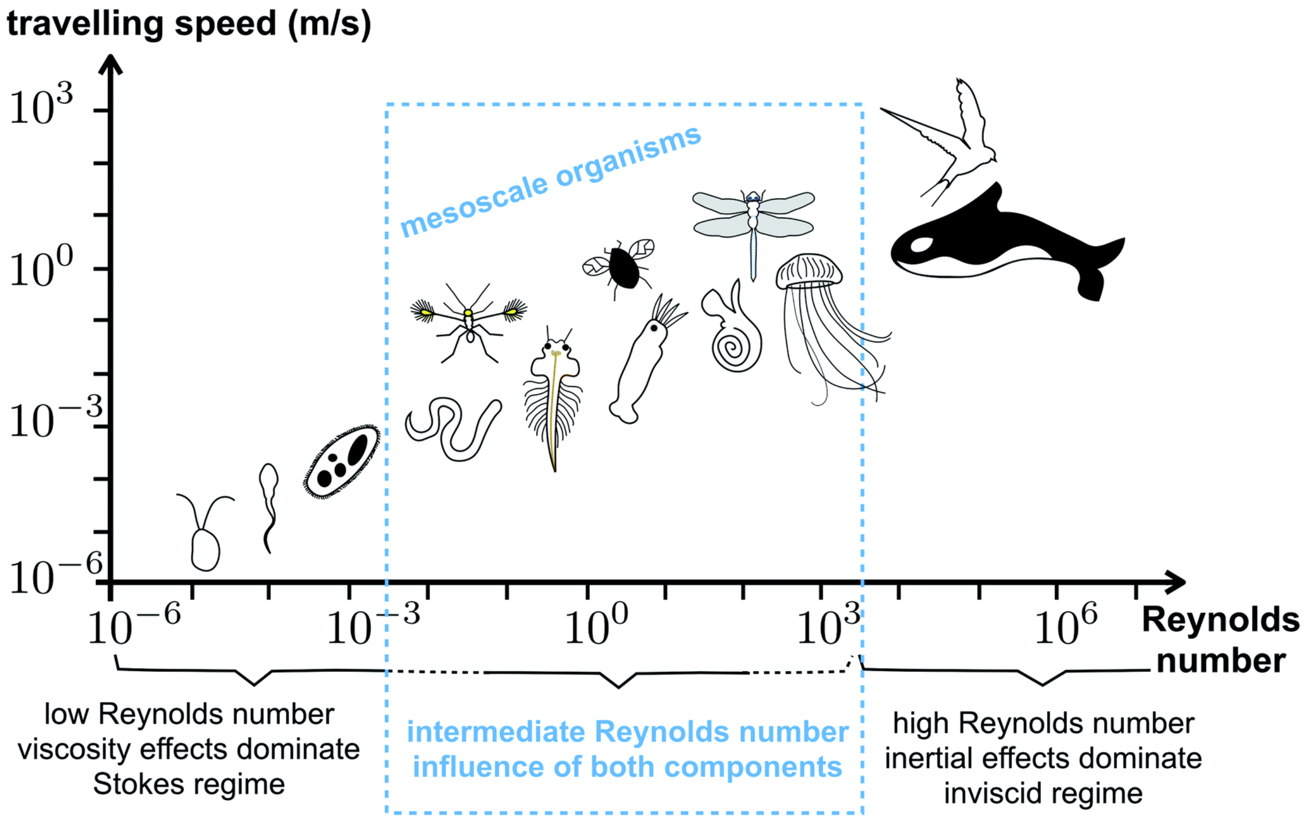
$$Re = \frac{\text{inertia}}{\text{viscosity}}$$

Low Re: time-reversal symmetry

China, V. *Proc Natl Acad Sci* (2014)

How to classify active fluids: Role of inertia





$$Re = \frac{\rho L u}{\eta} = \frac{\text{inertia}}{\text{viscosity}}$$

Active matter outside the low Re regime is not well understood

Klotsa, D. *Soft Matter* (2019)

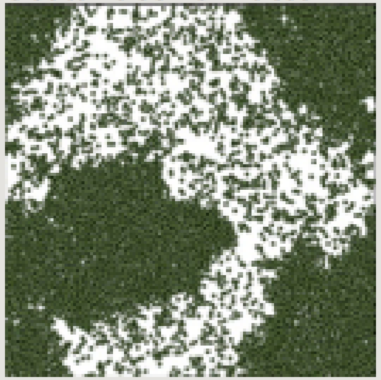
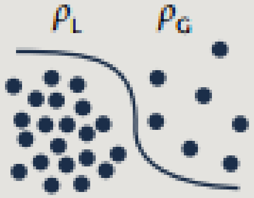
How to classify active fluids: Momentum conservation

	Nematic	Polar
Dry		Migrating animal herds (Parrish and Hamner, 1997)
	Melanocytes (Kemkemer *et al.*, 2000)	Migrating cell layers (Serra-Picamal *et al.*, 2012)
	Vibrated granular rods (Narayan, Ramaswamy, and Menon, 2007)	Vibrated asymmetric granular particles (Kudrolli *et al.*, 2008)
		Films of cytoskeletal extracts (Surrey *et al.*, 2001)
Wet		Cell cytoskeleton and cytoskeletal extracts in bulk suspensions (Bendix *et al.*, 2008)
	Suspensions of catalytic colloidal rods (Paxton *et al.*, 2004)	Swimming bacteria in bulk (Dombrowski *et al.*, 2004)
		Pt catalytic colloids (Palacci *et al.*, 2010)

Wet active systems conserve momentum, *dry* active systems do not

Marchetti et al Hydrodynamics of soft active matter
Reviews of Modern Physics 85, 1143 (2013)

Scalar active matter

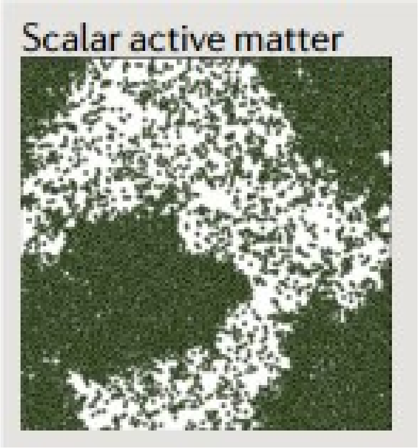
System	Particles	Order parameter	Example model	Model notes
Scalar active matter 	Scalar active particle with no alignment 	Phase separation: scalar order parameter, the density difference between liquid (ρ_L) and gas (ρ_G) phases $\rho=\langle\Sigma_i\delta(\mathbf{r}-\mathbf{r}_i)\rangle$, $\phi=\frac{(2\rho-\rho_L-\rho_G)}{(\rho_L-\rho_G)}$ 	Motility-induced phase separation described by Cahn–Hilliard dynamics involving the density (ρ)[253]: $\partial_t\rho=\nabla\cdot[D(\rho)\nabla\mu]$, $\mu=\ln[\rho v(\rho)]+\kappa(\rho)\nabla^2\rho$	The effective chemical potential μ includes the density suppression of motility $v(\rho)$ and nonintegrable gradient terms ($\kappa'(\rho)\neq 0$); density also suppresses the diffusion constant ($D\propto[v(\rho)]^2$)

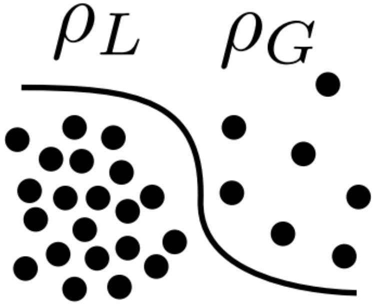
$$\partial_t\rho = \boldsymbol{\nabla}\cdot[D(\rho)\boldsymbol{\nabla}\mu]$$ Continuity equation

$$\mu = \ln[\rho v(\rho)] + \kappa(\rho)\nabla^2\rho$$

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Scalar active matter





$\rho = \langle \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \rangle$

$\phi = (2\rho - \rho_L - \rho_G)/(\rho_L - \rho_G)$

Cahn-Hilliard equation for the density:

$$\partial_t \rho = \boldsymbol{\nabla} \cdot [D(\rho) \boldsymbol{\nabla} \mu]$$

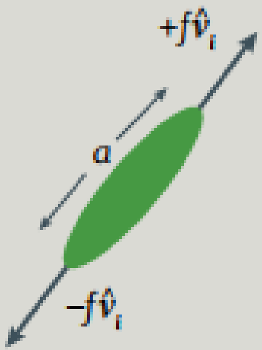
Continuity equation

$$\mu = \ln[\rho v(\rho)] + \kappa(\rho) \nabla^2 \rho$$

Motility-induced phase separation (MIPS)
in purely repulsive active particles

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active nematic fluids

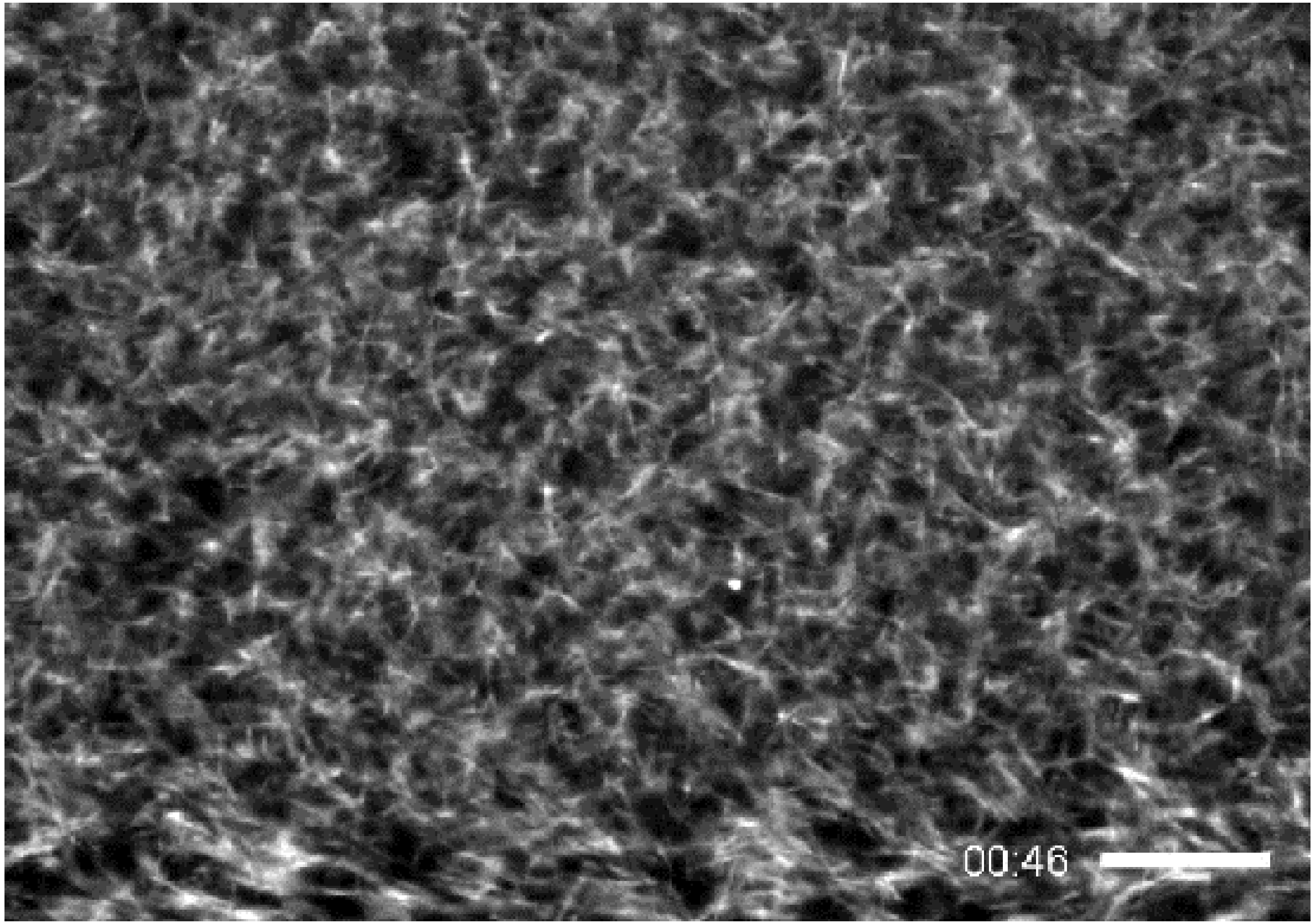
System	Particles	Order parameter	Example model	Model notes
Active nematic fluids, such as a microtubule–kinesin film 	Apolar: exerts force dipole $\alpha \sim f\ell$ 	Nematic order: tensor order parameter (in d dimensions) $\mathbf{Q} = \langle \sum_i (\hat{\nu}_i\hat{\nu}_i - \frac{1}{d})\delta(\mathbf{r}-\mathbf{r}_i)\rangle$ 	Incompressible hydrodynamics of nematic order ($\mathbf{Q}$) coupled with flow ($\mathbf{u}$) driven by an active stress ($\boldsymbol{\sigma}_a = \alpha\mathbf{Q}$)[39]: $\partial_t\mathbf{Q} + \mathbf{u}\cdot\nabla\mathbf{Q} + [\boldsymbol{\omega},\mathbf{Q}] = \lambda\mathbf{E} + [a_2 - a_4S^2]\mathbf{Q} + K\nabla^2\mathbf{Q}$, $\eta\nabla^2\mathbf{u} - \Gamma\mathbf{u} + \nabla\cdot\boldsymbol{\sigma}_a - \nabla\Pi = \mathbf{0}$, $\nabla\cdot\mathbf{u} = 0$	Force balance involves friction (Γ), viscosity (η) and pressure (Π); $\mathbf{E}$ and $\boldsymbol{\omega}$ are the symmetric and antisymmetric parts, respectively, of the strain rate tensor ($\nabla\mathbf{u}$); λ is the flow alignment parameter; nematic ordering [$S^2 = \mathrm{tr}(\mathbf{Q}^2)d/(d-1)$] is controlled by a_2 and a_4; and K is the elastic stiffness

$$\partial_t\mathbf{Q} + \mathbf{u}\cdot\boldsymbol{\nabla}\mathbf{Q} + [\boldsymbol{\omega},\mathbf{Q}] = \lambda\mathbf{E} + [a_2 - a_4S^2]\mathbf{Q} + K\nabla^2\mathbf{Q}$$

$$\eta\nabla^2\mathbf{u} - \Gamma\mathbf{u} + \boldsymbol{\nabla}\cdot\boldsymbol{\sigma}_a - \boldsymbol{\nabla}\Pi = \mathbf{0} \qquad \boldsymbol{\nabla}\cdot\mathbf{u} = 0$$

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active nematic fluids



Isolated biological motors

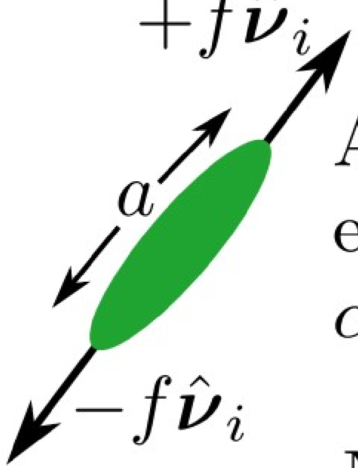
Dogic lab. *Nature* (2012)

$$\partial_t \mathbf{Q} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{Q} + [\boldsymbol{\omega}, \mathbf{Q}] = \lambda \mathbf{E} + [a_2 - a_4 S^2] \mathbf{Q} + K \nabla^2 \mathbf{Q}$$

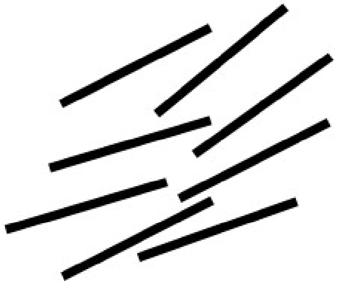
$$\eta \nabla^2 \mathbf{u} - \Gamma \mathbf{u} + \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}_a - \boldsymbol{\nabla} \Pi = \mathbf{0} \qquad \boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active Nemato-hydrodynamics



Apolar active particle exerting a force dipole $\alpha \sim fa$.



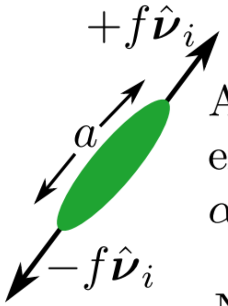
Nematic order is captured by a tensor order parameter (in d dimensions).

$$\mathbf{Q} = \langle \textstyle\sum_i \left(\hat{\boldsymbol{\nu}}_i \hat{\boldsymbol{\nu}}_i - \mathbf{1}/d\right) \delta(\mathbf{r} - \mathbf{r}_i) \rangle$$

Dogic lab. *Nature* (2012)

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active Nemato-hydrodynamics



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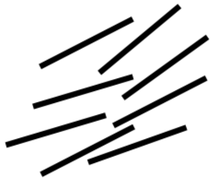

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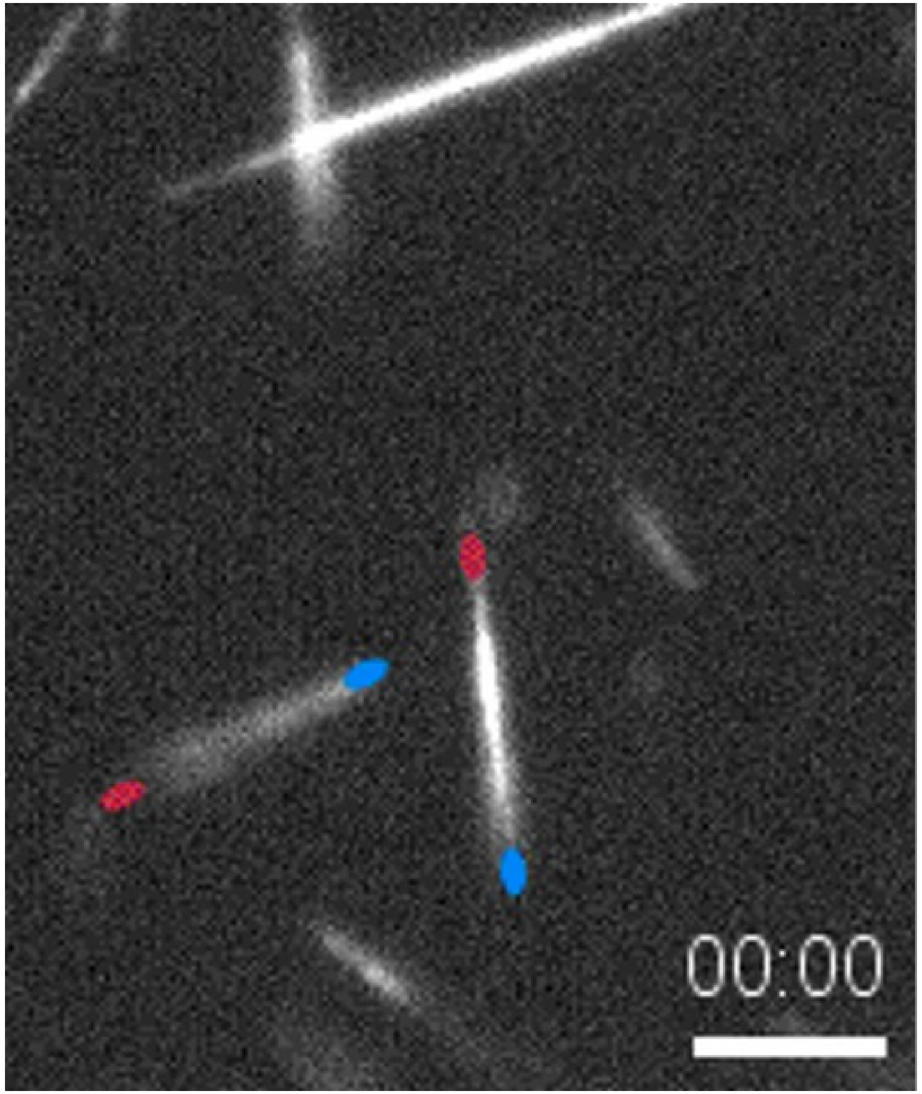
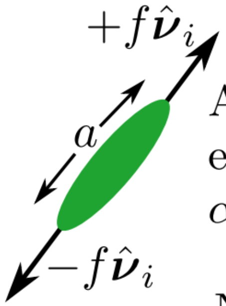


Dogic lab. *Nature* (2012)

Shankar et al Topological active matter *Nature Reviews Physics* (2022)

Active Nemato-hydrodynamics



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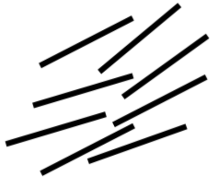

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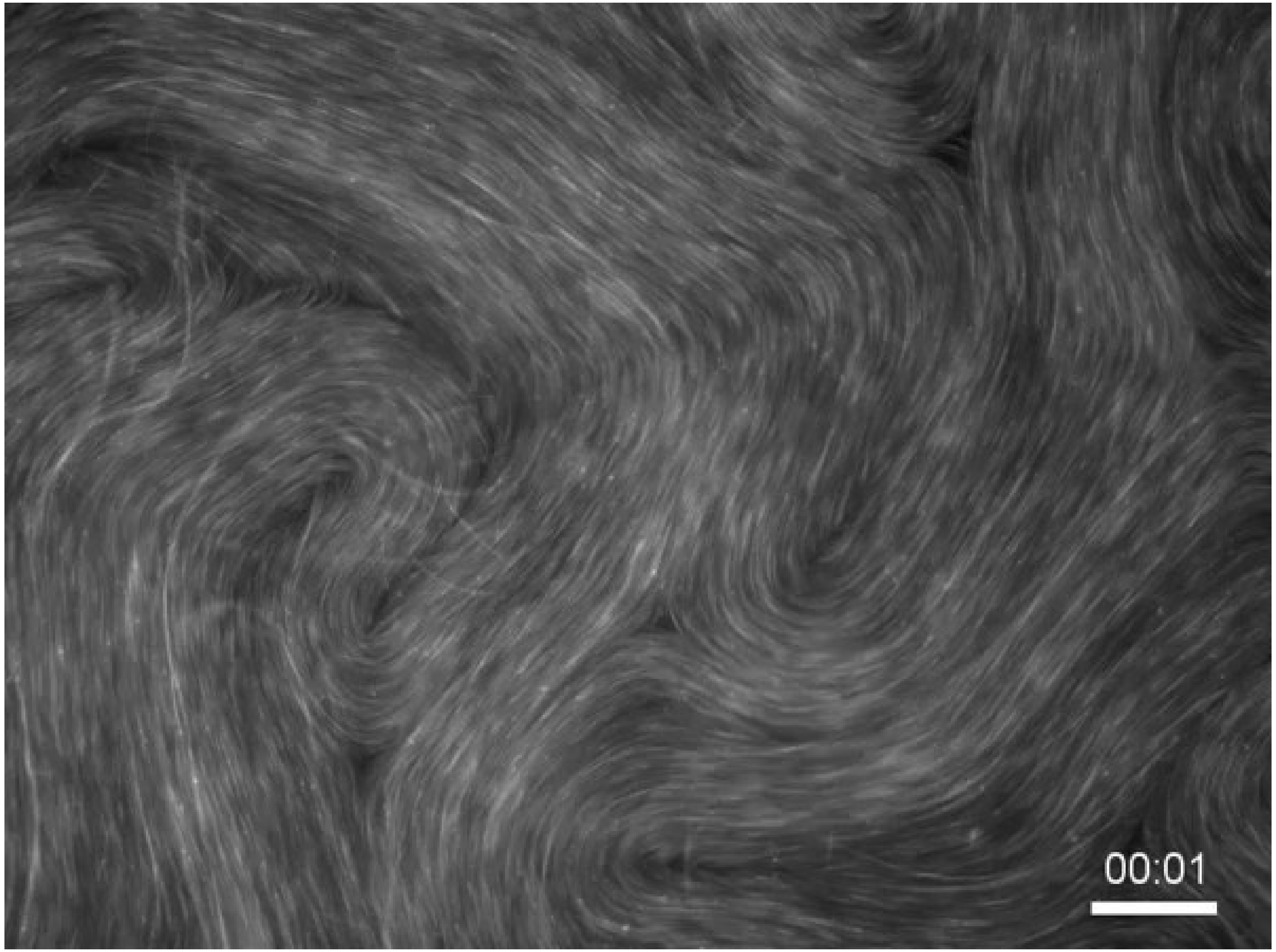
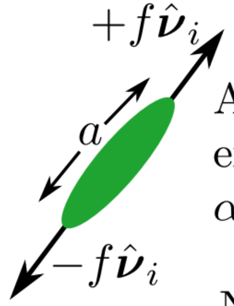


Dogic lab. *Nature* (2012)

Shankar et al Topological active matter
Nature Reviews Physics (2022)

Active nematic stress



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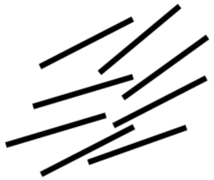

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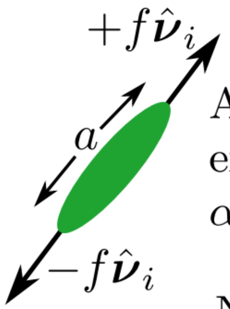
$$\mathbf{Q} = \langle \textstyle\sum_i \left(\hat{\boldsymbol{\nu}}_i \hat{\boldsymbol{\nu}}_i - \mathbf{1}/d\right) \delta(\mathbf{r} - \mathbf{r}_i) \rangle$$

$$\boldsymbol{\sigma}_a = \alpha\, \boldsymbol{Q}$$

Effect of activity is captured in one extra term relating the order parameter to fluid stress

Shankar et al <u>Topological active matter</u>
Nature Reviews Physics (2022)

Active nematic stress



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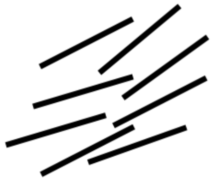

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$$\boldsymbol{\sigma}_a = \alpha\, \boldsymbol{Q}$$

Effect of activity is captured in one extra term relating the order parameter to fluid stress

Velocity $\mathbf{u}$
Vorticity $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{u}$
Pressure Π
$S \propto \mathrm{Tr}\, \mathbf{Q}^2$

$$\partial_t \mathbf{Q} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{Q} + [\boldsymbol{\omega}, \mathbf{Q}] = \lambda \mathbf{E} + [a_2 - a_4 S^2] \mathbf{Q} + K \nabla^2 \mathbf{Q}$$

$$\eta \nabla^2 \mathbf{u} - \Gamma \mathbf{u} + \boxed{\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}_a} - \boldsymbol{\nabla} \Pi = \mathbf{0} \qquad \boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

Shankar et al <u>Topological active matter</u>
Nature Reviews Physics (2022)

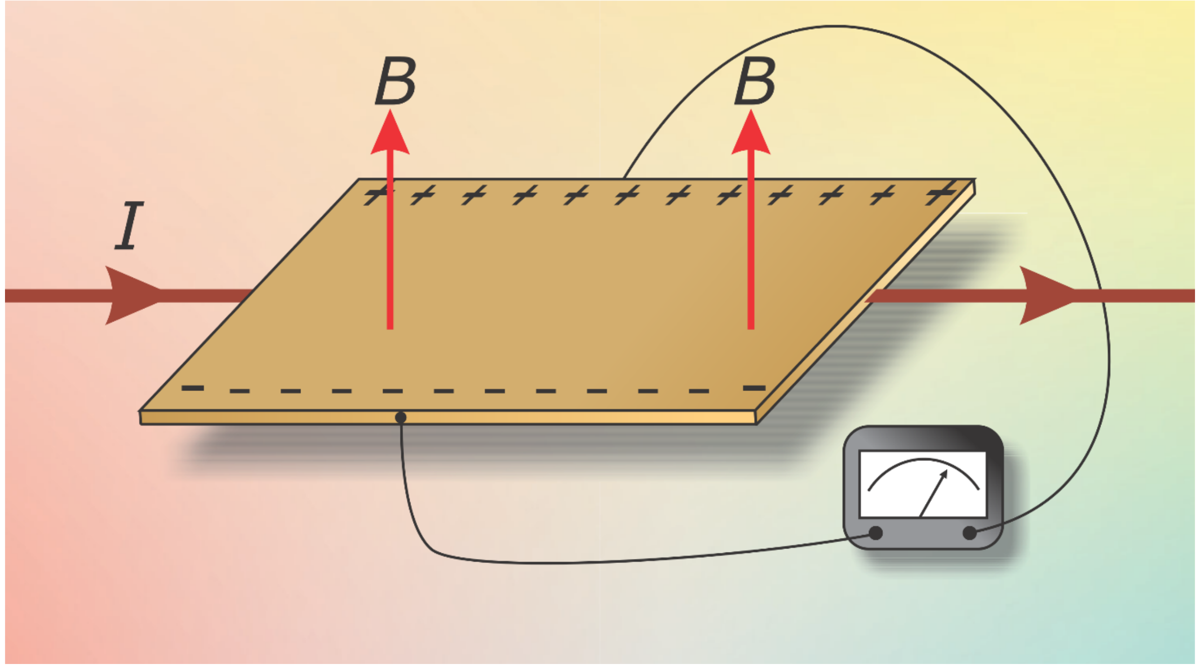
Chiral active fluids

Inspiration: 2D electron fluids

2D Hall probe using,

e.g., Graphene.

Transverse magnetic field



Avron, Osadchy, Seiler. *Physics Today* (2003)

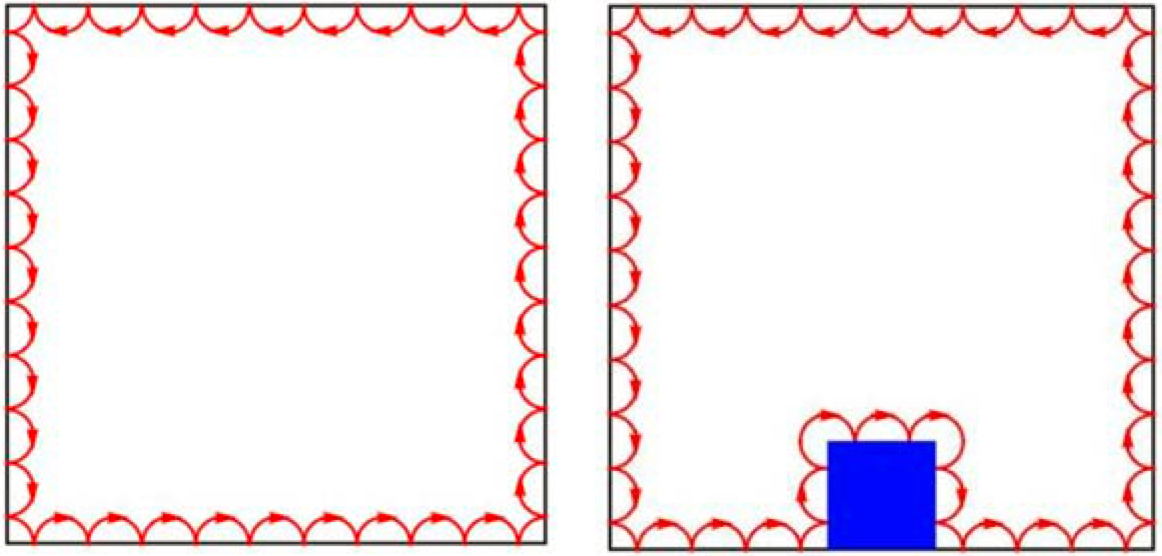
v. Klitzing et al. *PRL* (1980)
Hasan and Kane, *Rev. Mod. Phys.* (2010)

Quantum Hall effect in 2D electron fluids

2D Hall probe using,

e.g., Graphene.

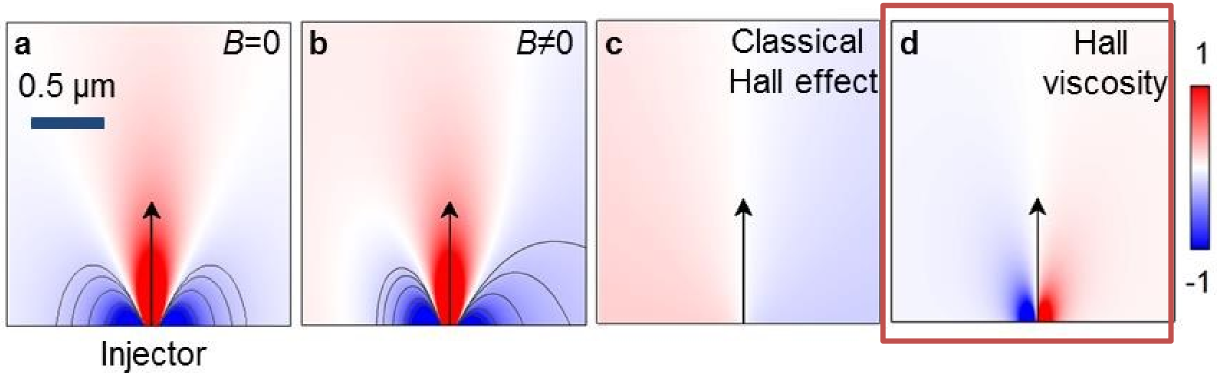
Transverse magnetic field



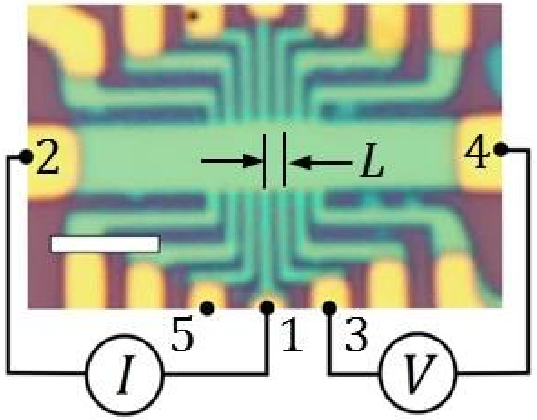
Topological protection of edge states

v. Klitzing et al. *PRL* (1980)
Hasan and Kane, *Rev. Mod. Phys.* (2010)

Hall viscosity in 2D electron fluids



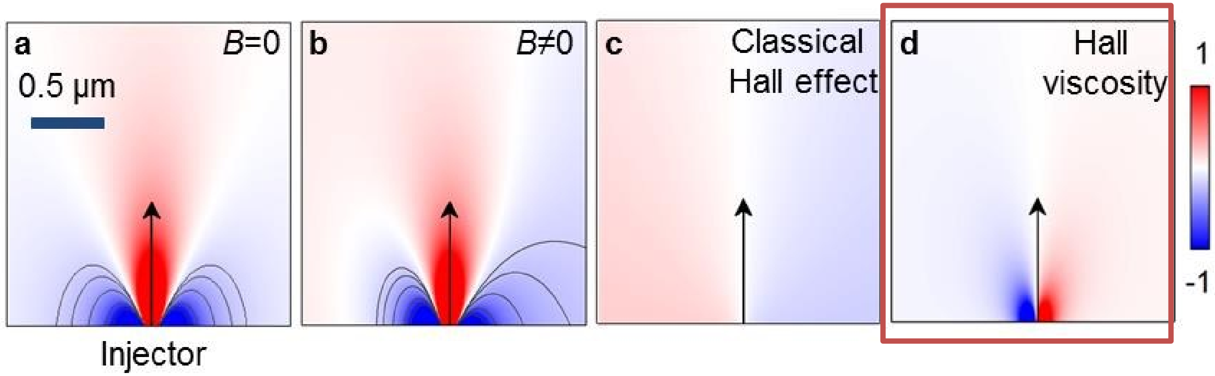
Electric potential distribution due to various effects, used to measure Hall viscosity



Theory:
"Odd viscosity" Avron. *J. Stat. Phys.* (1998)
Avron et al *PRL* (1995)

Expt: Science (2019)
"Measuring Hall Viscosity of Graphene's Electron Fluid"
Berdyugin, … Geim, Bandurin

Hall viscosity in 2D electron fluids



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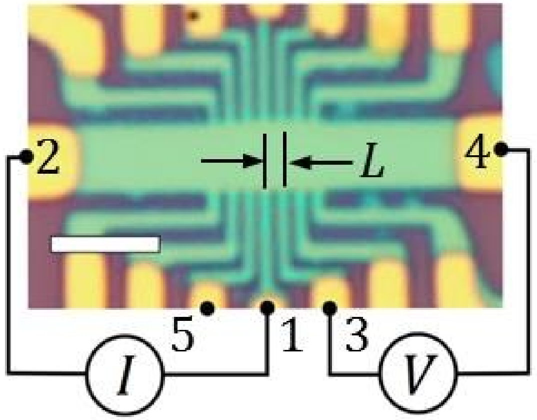

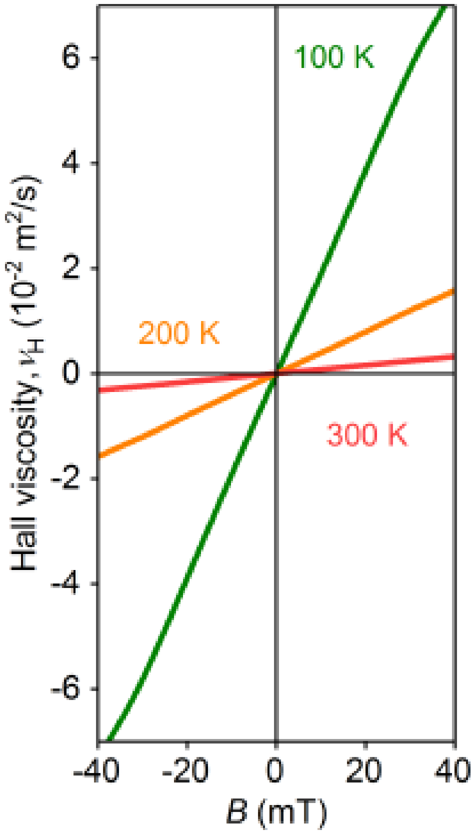

Hall viscosity is time-reversal (and B-field) **odd**

Theory:
"Odd viscosity" Avron. *J. Stat. Phys.* (1998)
Avron et al *PRL* (1995)

Expt: Science (2019)
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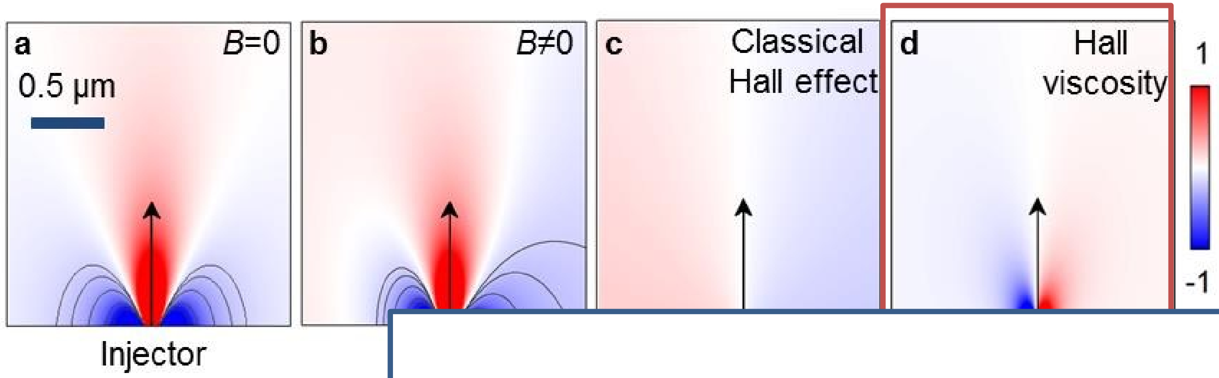
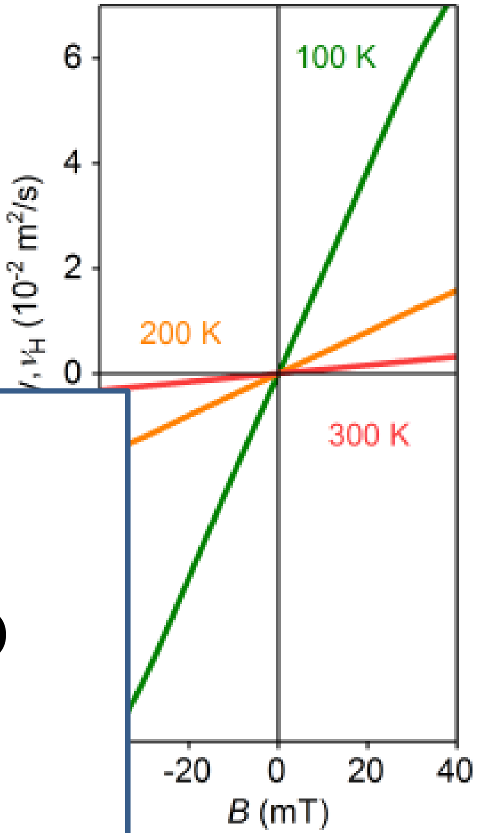
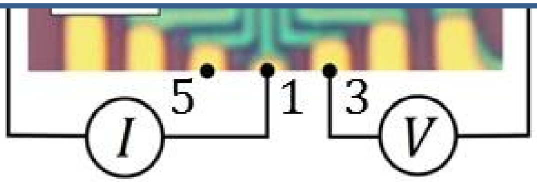
Hall viscosity in 2D electron fluids





Electric potential d
used to measure H

Mechanical analogs?



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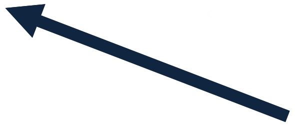
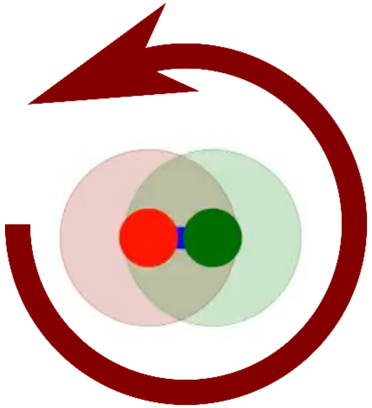
Avron et al *PRL* (1995)

Expt: Science (2019)
"Measuring Hall Viscosity of Graphene's Electron Fluid"
Berdyugin, … Geim, Bandurin

Self-propelled vs *self-rotating* particles



Force



Torque

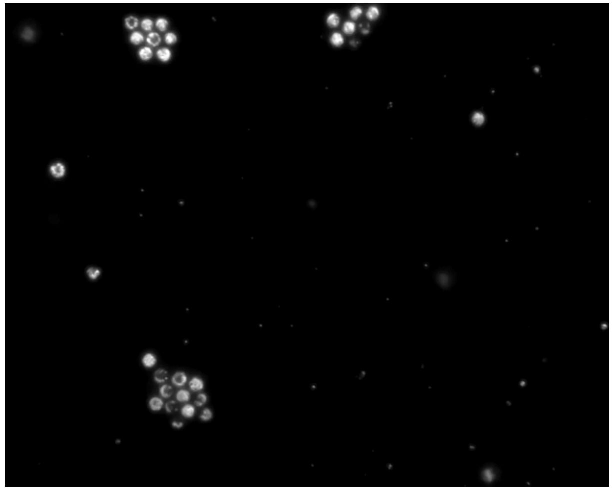
Self-propelled particle

Active rotor

Experimental realizations of active rotors

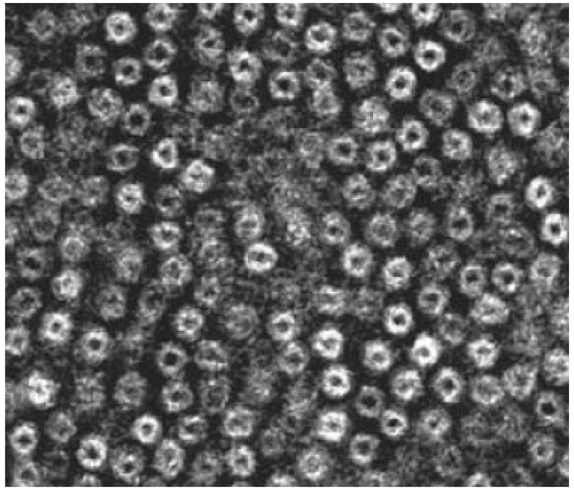
Biological:

T. Majus bacteria



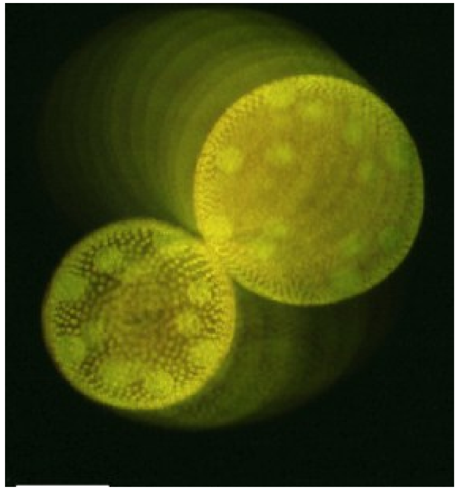
Petroff et al. *PRL* (2015)

Sperm cells



Riedel et al. *Science* (2005)

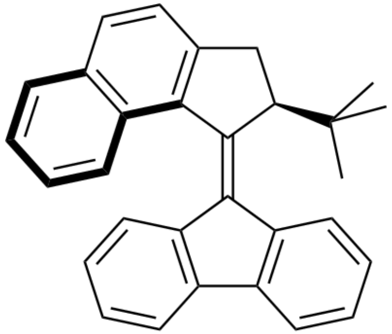
Volvox algae



Drescher et al. *PRL* (2009)

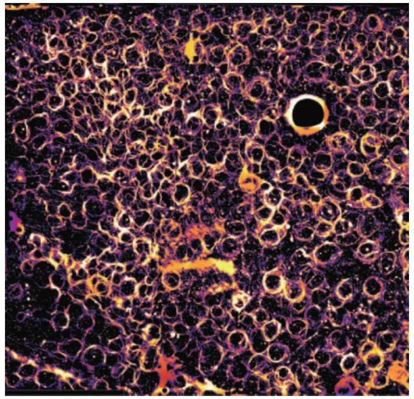
Synthetic:

Molecular motors



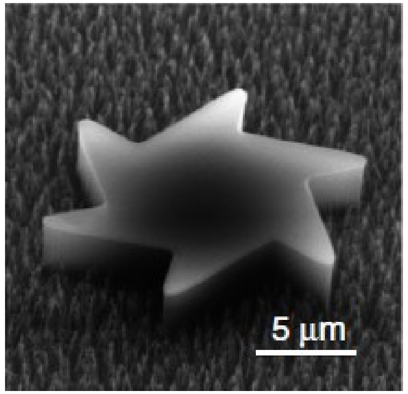
Feringa et al.
Nature (1999)

Microtubules



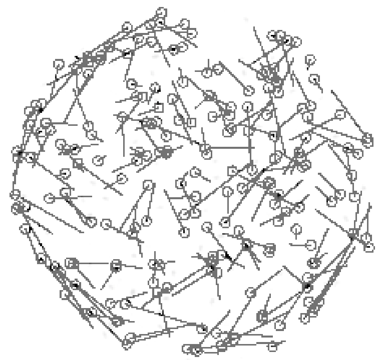
Sumino et al.
Nature (2012)

Colloids



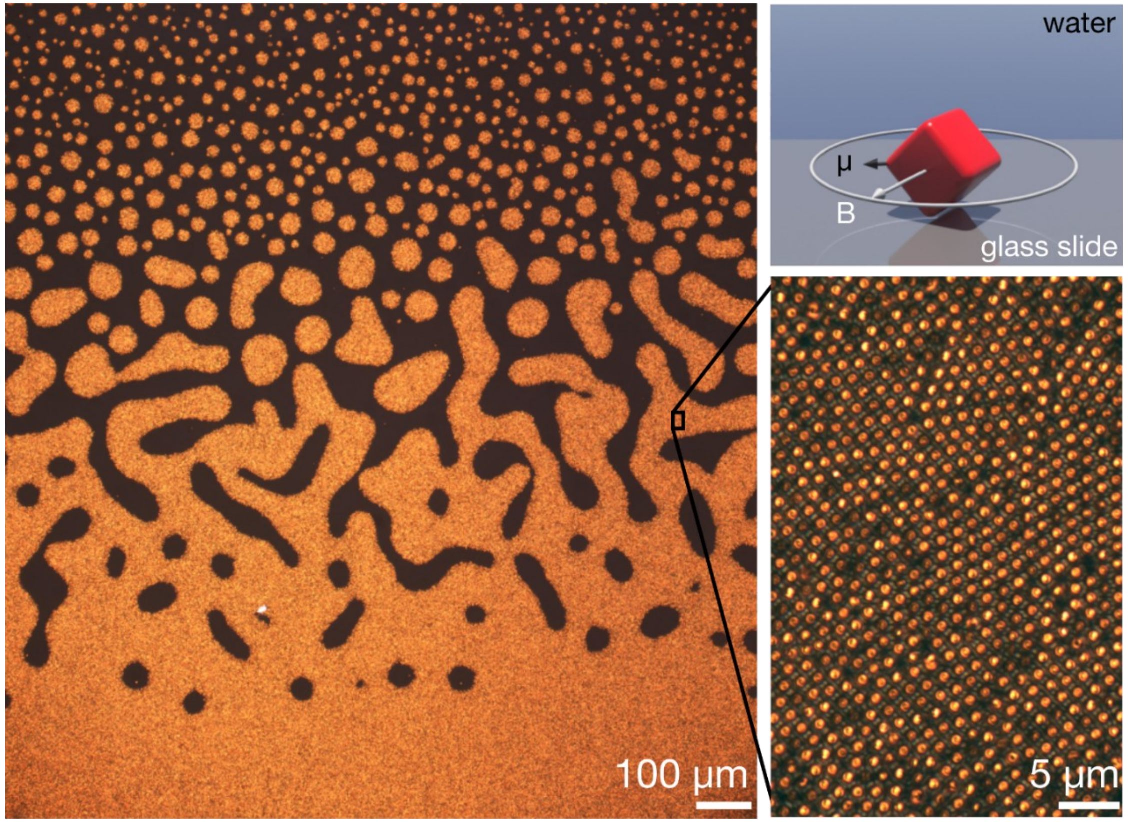
Maggi et al.
Nat. Comm. (2015)

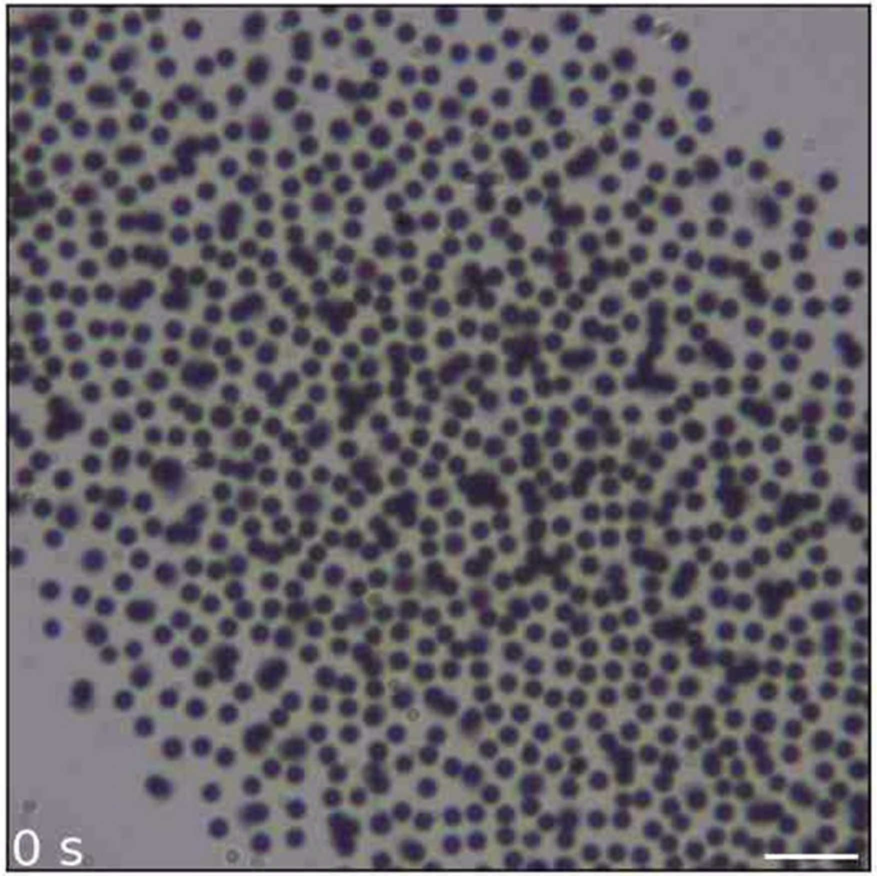
Granular gas



Tsai et al.
PRL (2005)

Colloidal realization of odd viscosity

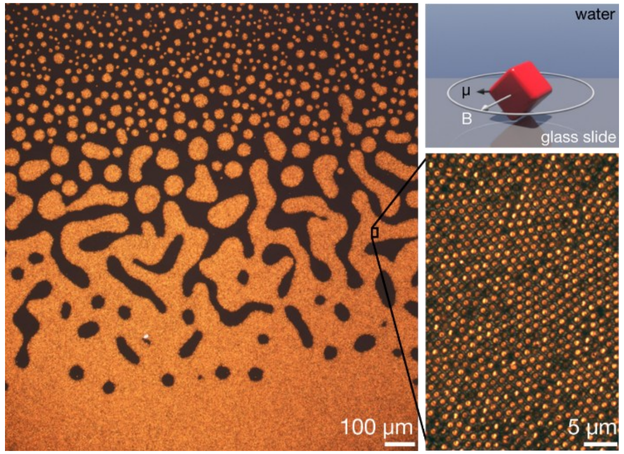


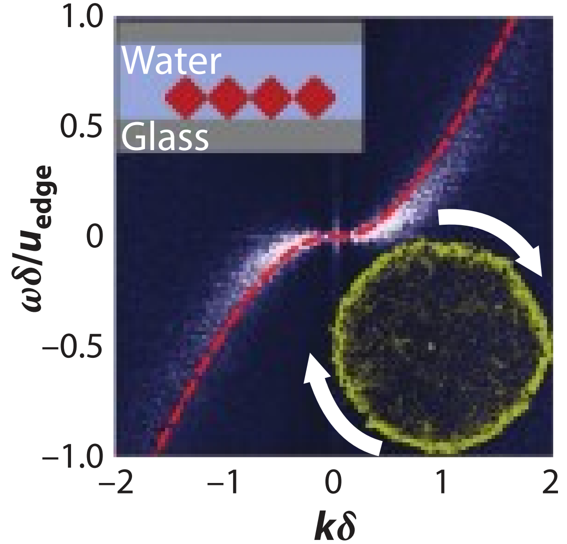


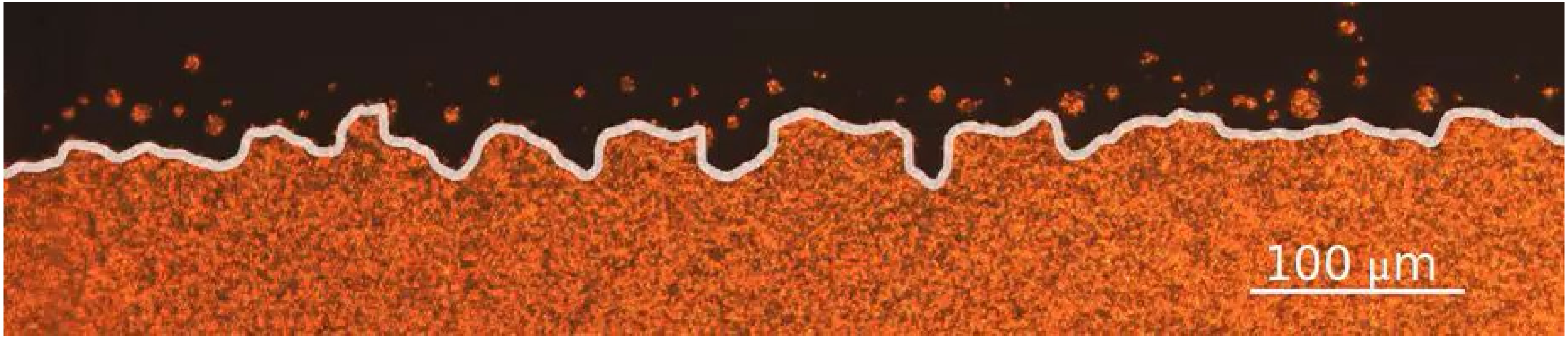
By tracking the dispersion relation of surface waves,
odd viscosity has been recently measured in chiral active fluids

Soni*, Bililign*, Magkiriadou*,..., Irvine. Nature Physics (2019)

Colloidal realization of odd viscosity



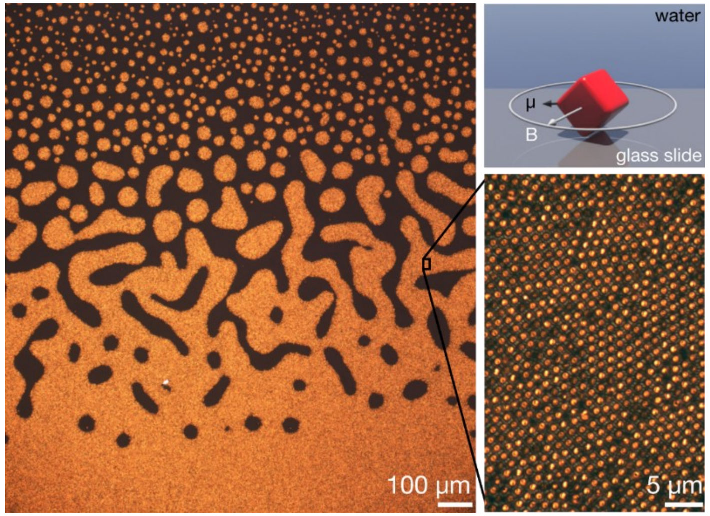


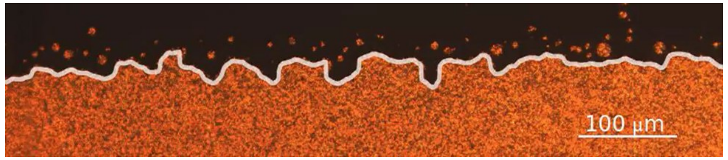


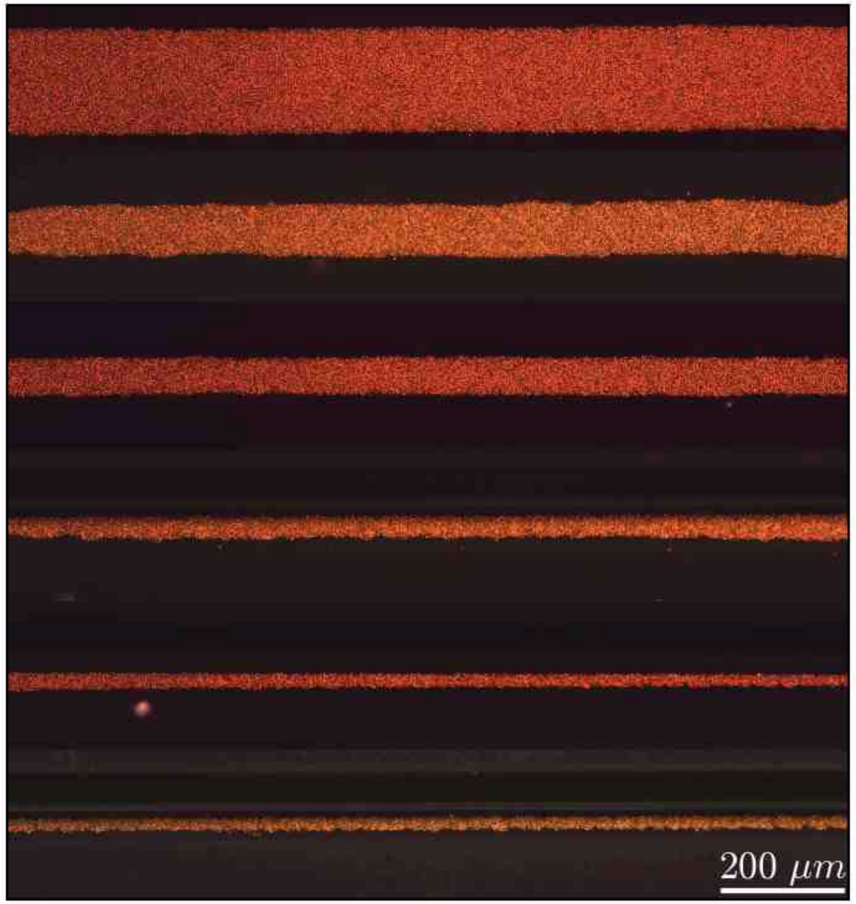
By tracking the dispersion relation of surface waves,
odd viscosity has been recently measured in chiral active fluids

Soni*, Bililign*, Magkiriadou*,…, Irvine. Nature Physics (2019)

Colloidal realization of odd viscosity



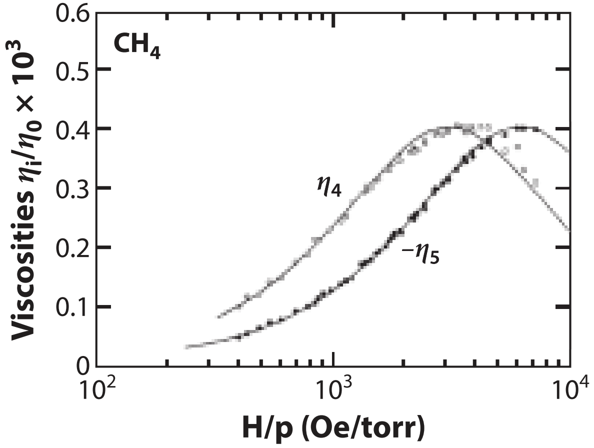


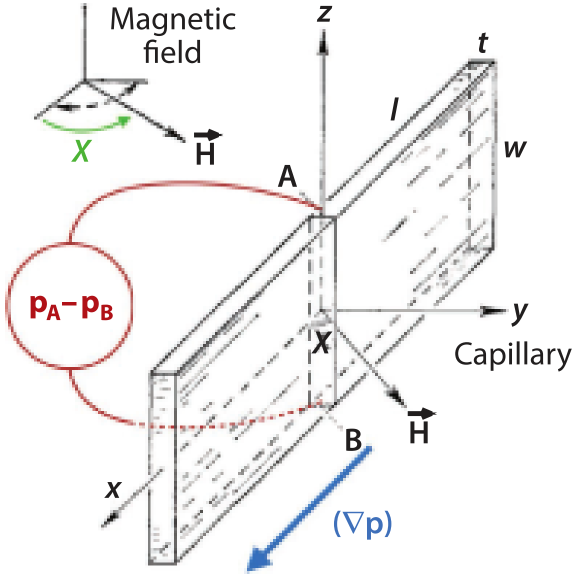


Soni*, Bililign*, Magkiriadou*,…, Irvine. Nature Physics (2019)

Experimental realizations of odd viscosity

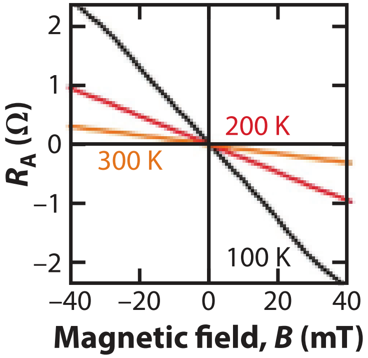
Polyatomic gases

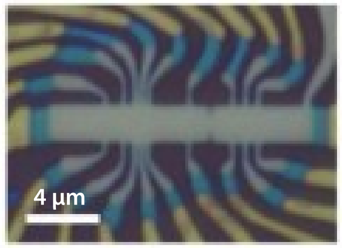




Hulsman et al, Physica 50, 53 (1970)

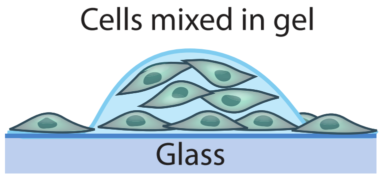
Graphene

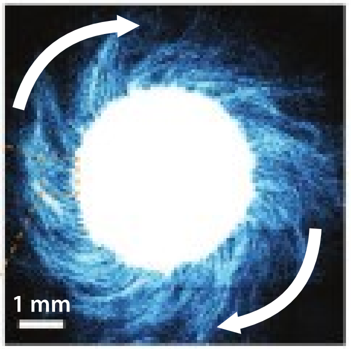




Berdyugin et al, Science (2019)

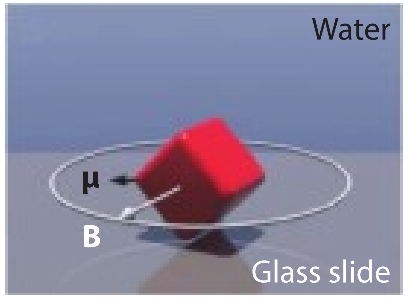
Neural progenitor cells

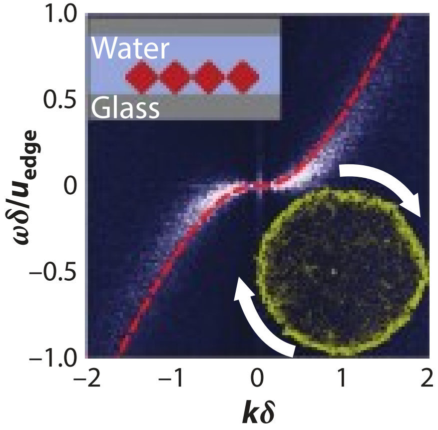




Yamauchi L, et al (2020)

Spinning colloids





Soni et al, Nat Phys (2019)

M. Fruchart , C. Scheibner, V. Vitelli. "Odd viscosity and odd elasticity." Annual Review of Condensed Matter Physics 14, 471 (2023)

Viscosity: measurement

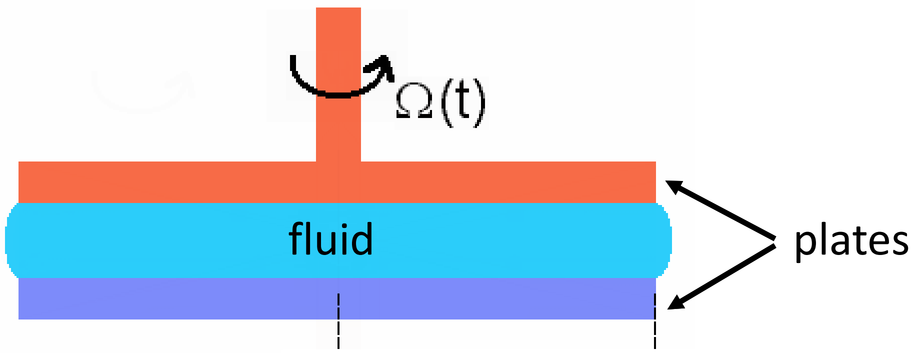


Plate-plate rheometer

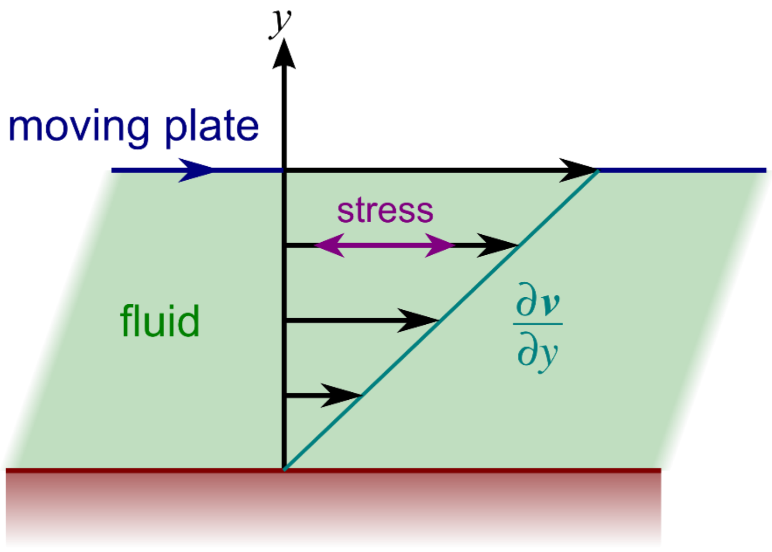
Measure stress due to shear rate

wikipedia.org

Isotropic viscous stress tensor

$$D_t v_j = \nabla_i \sigma_{ij} \qquad v_{kl} = \nabla_k v_l$$

$$\sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$



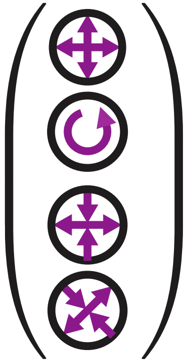
$$\underset{\sigma_\alpha}{\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}} = \underset{\eta_{\alpha\beta}}{\begin{pmatrix} \zeta & \eta^{B} & 0 & 0 \\ \eta^{A} & \eta^{R} & 0 & 0 \\ 0 & 0 & \eta & \eta^{o} \\ 0 & 0 & -\eta^{o} & \eta \end{pmatrix}} \underset{\dot e_\beta}{\begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}}$$

Deformation	Deformation rate	Stress	Geometric meaning
$e_0 = \; = \partial_x u_x + \partial_y u_y$	$\dot e_0 = \; = \partial_x v_x + \partial_y v_y$	$\sigma_0 = \; = [\sigma_{xx} + \sigma_{yy}]/2$	isotropic area change
$e_1 = \; = \partial_x u_y - \partial_y u_x$	$\dot e_1 = \; = \partial_x v_y - \partial_y v_x$	$\sigma_1 = \; = [\sigma_{yx} - \sigma_{xy}]/2$	rotation
$e_2 = \; = \partial_x u_x - \partial_y u_y$	$\dot e_2 = \; = \partial_x v_x - \partial_y v_y$	$\sigma_2 = \; = [\sigma_{xx} - \sigma_{yy}]/2$	pure shear 1
$e_3 = \; = \partial_x u_y + \partial_y u_x$	$\dot e_3 = \; = \partial_x v_y + \partial_y v_x$	$\sigma_3 = \; = [\sigma_{xy} + \sigma_{yx}]/2$	pure shear 2

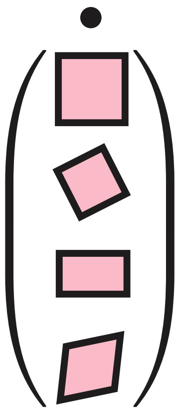
Isotropic viscous stress tensor

$$D_t v_j = \nabla_i \sigma_{ij} \qquad v_{kl} = \nabla_k v_l$$

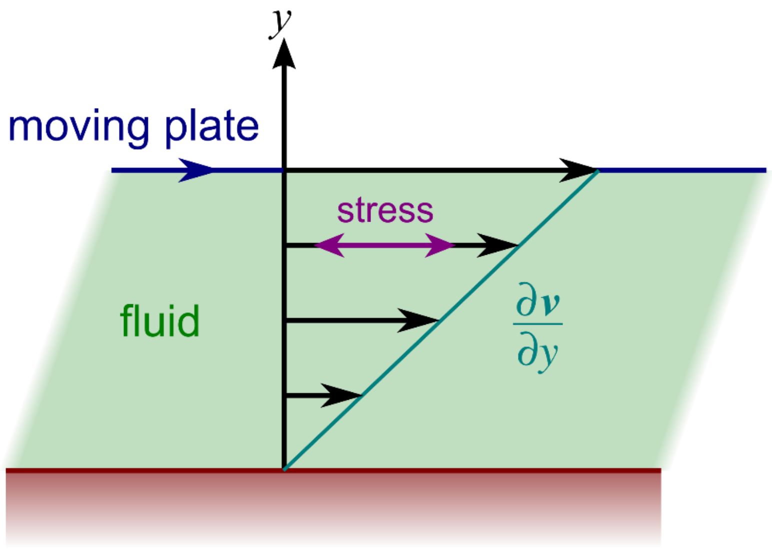
$$\sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$



$$= \begin{pmatrix} \zeta & \eta^{\mathrm{B}} & 0 & 0 \\ \eta^{\mathrm{A}} & \eta^{\mathrm{R}} & 0 & 0 \\ 0 & 0 & \eta & \eta^{\mathrm{o}} \\ 0 & 0 & -\eta^{\mathrm{o}} & \eta \end{pmatrix}$$



$$\sigma_\alpha \qquad \eta_{\alpha\beta} \qquad \dot{e}_\beta$$

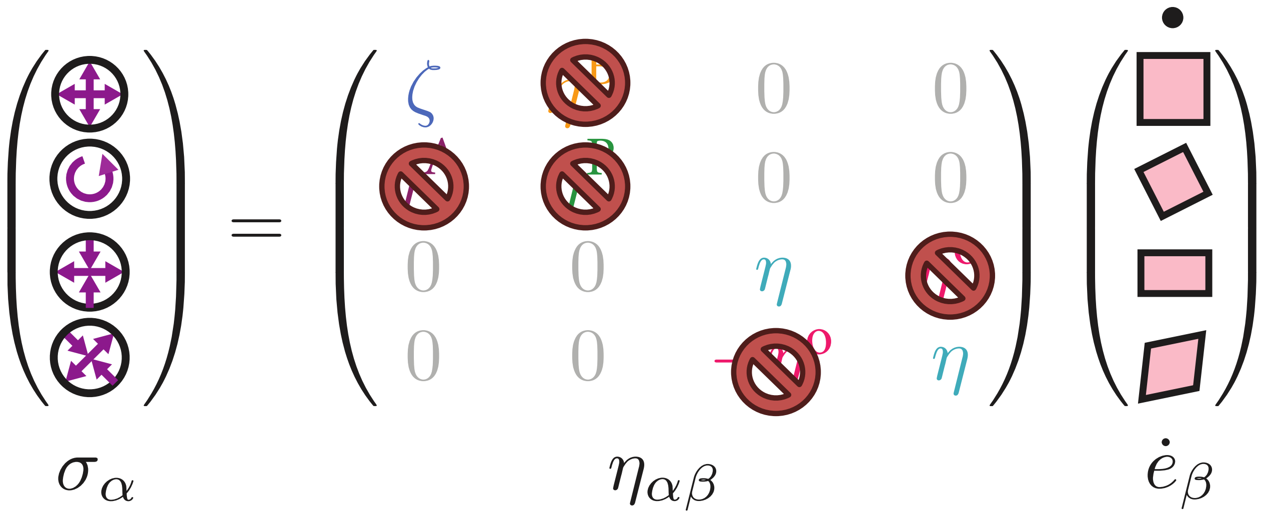


Deformation	Deformation rate	Stress	Geometric meaning
	$\dot{e}_0 = = \partial_x v_x + \partial_y v_y$	$\sigma_0 = = [\sigma_{xx} + \sigma_{yy}]/2$	isotropic area change
	$\dot{e}_1 = = \partial_x v_y - \partial_y v_x$	$\sigma_1 = = [\sigma_{yx} - \sigma_{xy}]/2$	rotation
	$\dot{e}_2 = = \partial_x v_x - \partial_y v_y$	$\sigma_2 = = [\sigma_{xx} - \sigma_{yy}]/2$	pure shear 1
	$\dot{e}_3 = = \partial_x v_y + \partial_y v_x$	$\sigma_3 = = [\sigma_{xy} + \sigma_{yx}]/2$	pure shear 2

Isotropic viscous stress tensor in equilibrium

$$D_t v_j = \nabla_i \sigma_{ij} \qquad v_{kl} = \nabla_k v_l$$

$$\sigma_{ij} = \sigma_{ji} \qquad \sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$



$$\eta_{ijkl} = \boxed{\eta}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \left(\boxed{\zeta} - \tfrac{2}{3}\eta\right)\delta_{ij}\delta_{kl}$$

Shear viscosity

Bulk viscosity

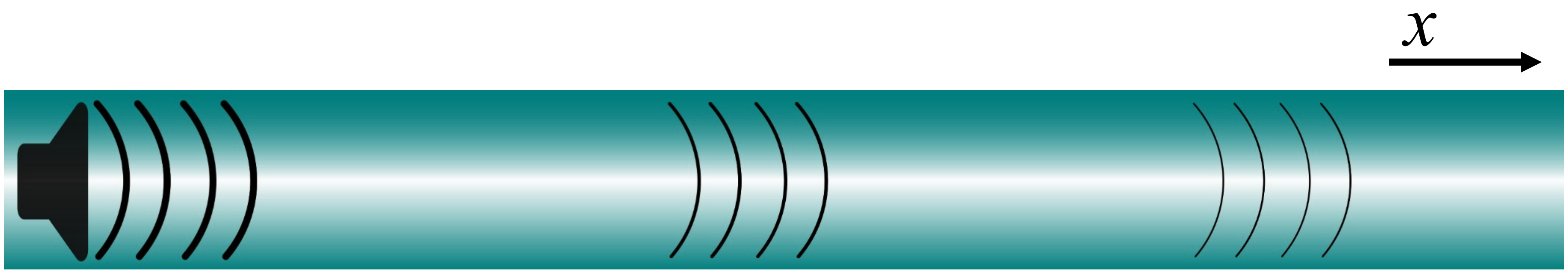
In equilibrium isotropic fluids, only shear and bulk viscosities

Viscosity and Stokes' law of sound attenuation

$$D_t v_j = \nabla_i \sigma_{ij} \qquad v_{kl} = \nabla_k v_l$$

$$\sigma_{ij} = \sigma_{ji} \qquad \sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$

$$A \sim e^{-\frac{2\omega^2}{3\rho c^3}\left(\boxed{\eta + \frac{3}{2}\zeta}\right)x}$$



Both shear and bulk viscosities enter sound attenuation in fluids

G.G. Stokes (1845)

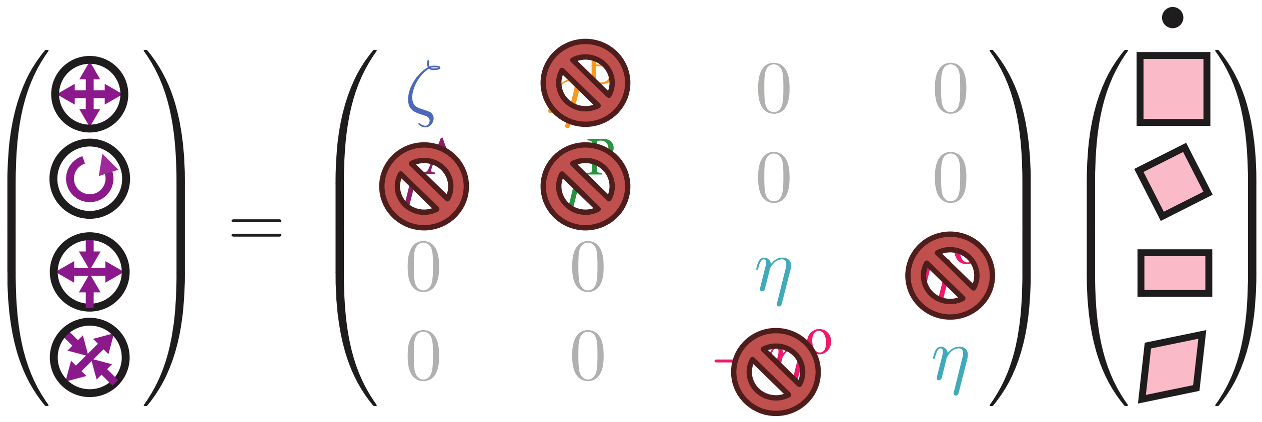
Constitutive relation for equilibrium fluids

$$D_t v_j = \nabla_i \sigma_{ij} \qquad v_{kl} = \nabla_k v_l$$

$$\sigma_{ij} = \sigma_{ji} \qquad \sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$

$$\eta_{ijkl} = \eta^s_{ijkl}$$

$$\eta^s_{ijkl} = \eta^s_{klij}$$

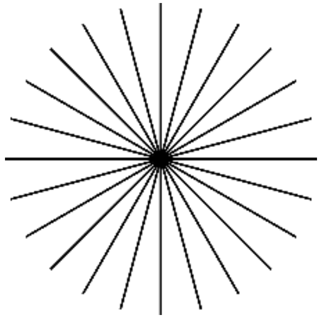


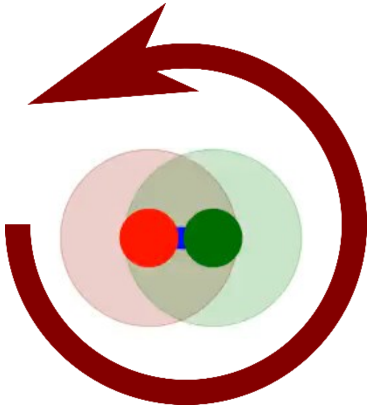
$\sigma_\alpha \qquad \eta_{\alpha\beta} \qquad \dot{e}_\beta$

Symmetry of viscosity tensor
guaranteed by Onsager reciprocal relation

Constitutive relation for chiral active fluids

$$D_t v_j = \nabla_i \sigma_{ij} \qquad\qquad \sigma_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$





Torque

2D isotropic fluid that break time-reversal and chiral symmetries

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Odd viscosity for chiral active fluids

$$D_t v_j = \nabla_i \sigma_{ij}$$

$$\sigma_{ij} = \sigma^s_{ij} + \sigma^a \epsilon_{ij}$$

$$\sigma^s_{ij} = \sigma^s_{ji} \qquad \epsilon_{ij} = -\epsilon_{ji}$$

$$\sigma^s_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$

$$\eta_{ijkl} = \eta^s_{ijkl} + \eta^o_{ijkl}$$

$$\eta^s_{ijkl} = \eta^s_{klij}$$

$$\eta^o_{ijkl} = -\eta^o_{klij}$$

Onsager relation does not apply without TRS

Active rotors create antisymmetric stress and modify symmetric stress

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Viscosity tensor and symmetry

$$D_t v_j = \nabla_i \sigma_{ij}$$

$$\sigma_{ij} = \sigma^s_{ij} + \sigma^a \epsilon_{ij}$$

$$\sigma^s_{ij} = \sigma^s_{ji} \qquad \epsilon_{ij} = -\epsilon_{ji}$$

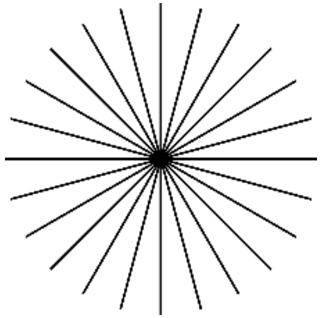
$$\sigma^s_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$

$$\eta_{ijkl} = \eta^s_{ijkl} + \eta^o_{ijkl}$$

$$\eta^s_{ijkl} = \eta^s_{klij}$$

$$\eta^o_{ijkl} = -\eta^o_{klij}$$

2D fluid with broken TRS
and spatial isotropy



$$\eta^o_{ijkl} = \frac{1}{2}\eta^o \left(\epsilon_{ik}\delta_{jl} + \epsilon_{il}\delta_{jk} + \epsilon_{jk}\delta_{il} + \epsilon_{jl}\delta_{ik}\right)$$

Odd viscosity is a *generic* property based on symmetries!

Avron, Seiler, Zograf, *PRL* (1995); Avron. *J. Stat. Phys.* (1998); D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Dissipationless viscosity

$$D_t v_j = \nabla_i \sigma_{ij}$$

$$\sigma_{ij} = \sigma^s_{ij} + \sigma^a \epsilon_{ij}$$

$$\sigma^s_{ij} = \sigma^s_{ji} \qquad \epsilon_{ij} = -\epsilon_{ji}$$

$$\sigma^s_{ij} \equiv -p\delta_{ij} + \eta_{ijkl} v_{kl}$$

$$\eta_{ijkl} = \eta^s_{ijkl} + \eta^o_{ijkl}$$

$$\eta^s_{ijkl} = \eta^s_{klij}$$

$$\eta^o_{ijkl} = -\eta^o_{klij}$$

$$\eta^o_{ijkl} = \frac{1}{2}\eta^o \left(\epsilon_{ik}\delta_{jl} + \epsilon_{il}\delta_{jk} + \epsilon_{jk}\delta_{il} + \epsilon_{jl}\delta_{ik}\right)$$

$$\partial_t v_y \sim \eta^o \nabla^2 v_x$$ Transverse response

This transverse response does not lead to dissipation

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

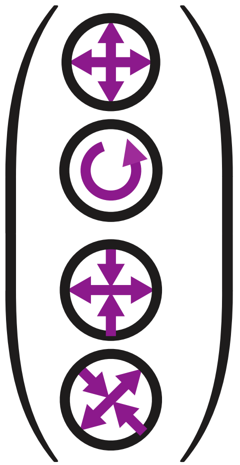
Dissipationless viscosity

$$T\eta^o = -\eta^o$$

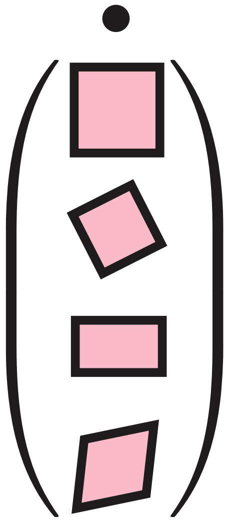
Dissipationless because it changes sign under time reversal

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Full tensor



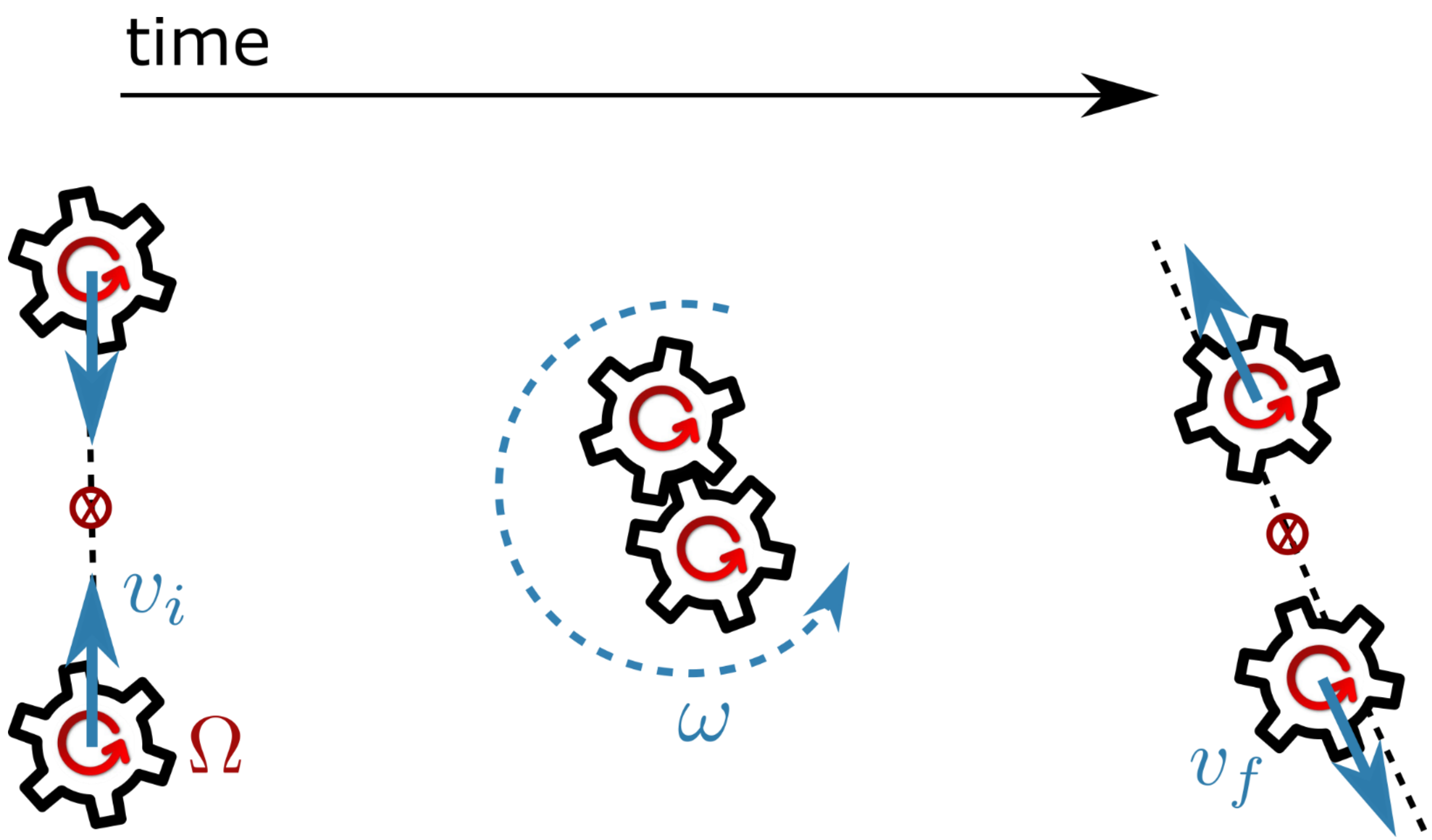
$$= \begin{pmatrix} \zeta & \eta^{B} & 0 & 0 \\ \eta^{A} & \eta^{R} & 0 & 0 \\ 0 & 0 & \eta & \eta^{o} \\ 0 & 0 & -\eta^{o} & \eta \end{pmatrix}$$



σ_α $\eta_{\alpha\beta}$ $\dot{e}_\beta$

M. Fruchart , C. Scheibner, V. Vitelli. "Odd viscosity and odd elasticity." Annual Review of Condensed Matter Physics 14, 471 (2023)

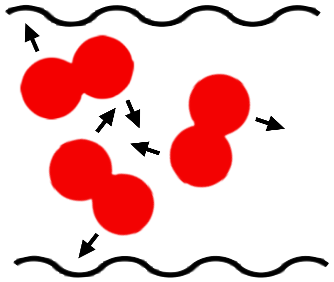
Collisions between particles III



Intrinsic spinning relaxes, vorticity is generated

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Regime of constant Ω



Chiral active gas limit

$$\frac{\Gamma}{I} \ll \frac{v_0}{r_0}$$
$$\frac{\tau}{\Gamma} \gg \frac{v_0}{r_0}$$
$$\tau I \gg \Gamma^2$$

Density

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

Intrinsic rotation Ω

$$\partial_t(\rho\Omega) + \nabla \cdot (\rho\Omega\mathbf{v}) = D^{\Omega}\nabla^2\Omega - \Gamma^{\Omega}\Omega + \tau - \Gamma(\Omega - \omega)$$

Flow velocity

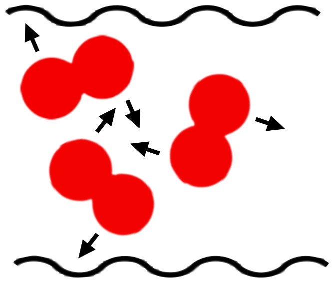
$$\partial_t(\rho\mathbf{v}) + \nabla \cdot (\rho\mathbf{v}\mathbf{v}) = \eta\nabla^2\mathbf{v} - \nabla p + \frac{I}{2}\nabla \cdot (\Omega(\partial_i v_j^* + \partial_i{}^* v_j)) + \frac{\Gamma}{2}\nabla^*(\Omega - \omega)$$

Nonlinear coupling (Anti-symmetric stress)

$$v_i^* = \epsilon_{ij} v_j$$

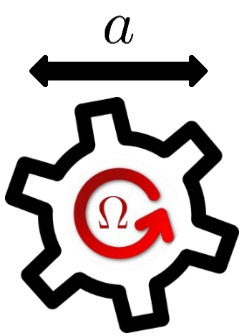
D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Moment of inertia density



Chiral active gas limit

$$\frac{\Gamma}{I} \ll \frac{v_0}{r_0}$$
$$\frac{\tau}{\Gamma} \gg \frac{v_0}{r_0}$$
$$\tau I \gg \Gamma^2$$



$$I \sim \rho a^2$$

Density

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

Constant intrinsic rotation Ω

$$\Omega = \frac{\tau}{\Gamma^\Omega}$$

Flow velocity

$$\partial_t (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = \eta \nabla^2 \mathbf{v} - \nabla p + \eta^o \nabla^2 \mathbf{v}^*$$

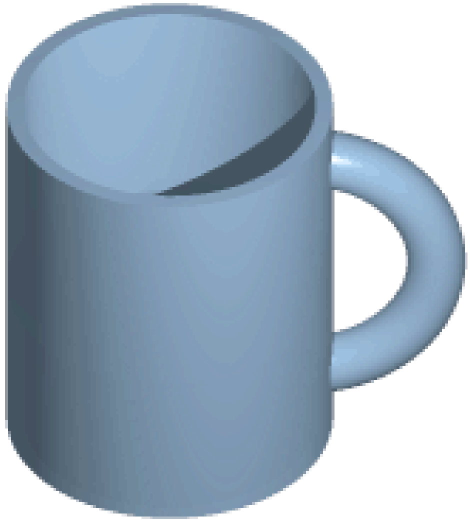
$$v_i^* = \epsilon_{ij} v_j$$

$$\boxed{\eta^o = \frac{\ell}{2} = \frac{I\Omega}{2}}$$

Navier-Stokes equations, but with extra odd viscosity term!

D. Banerjee, AS, A. Abanov, and V. Vitelli. *Nat. Commun.* (2017)

Part 1.3: Topology

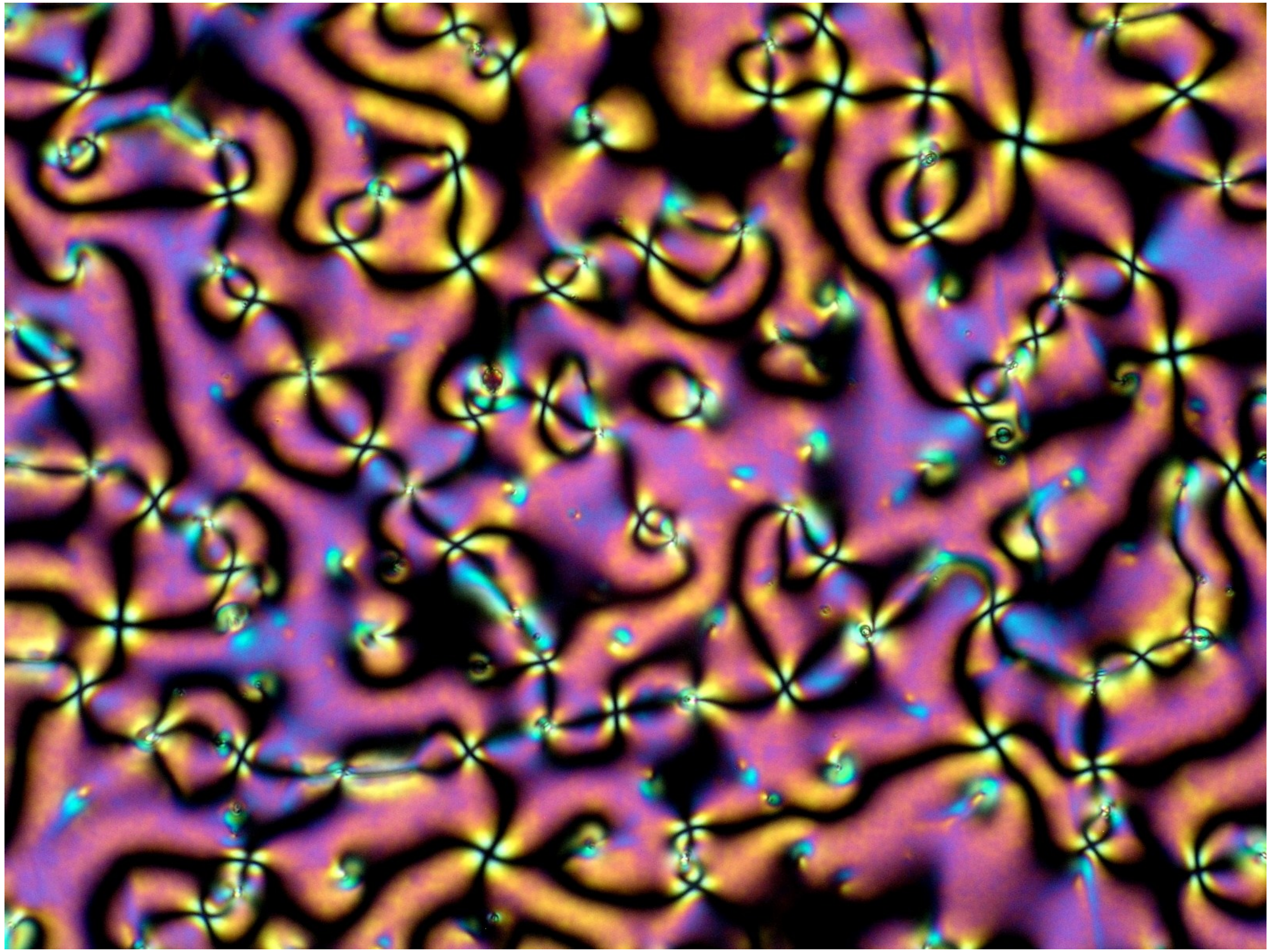


Torus: Genus (“number of holes”) = 1

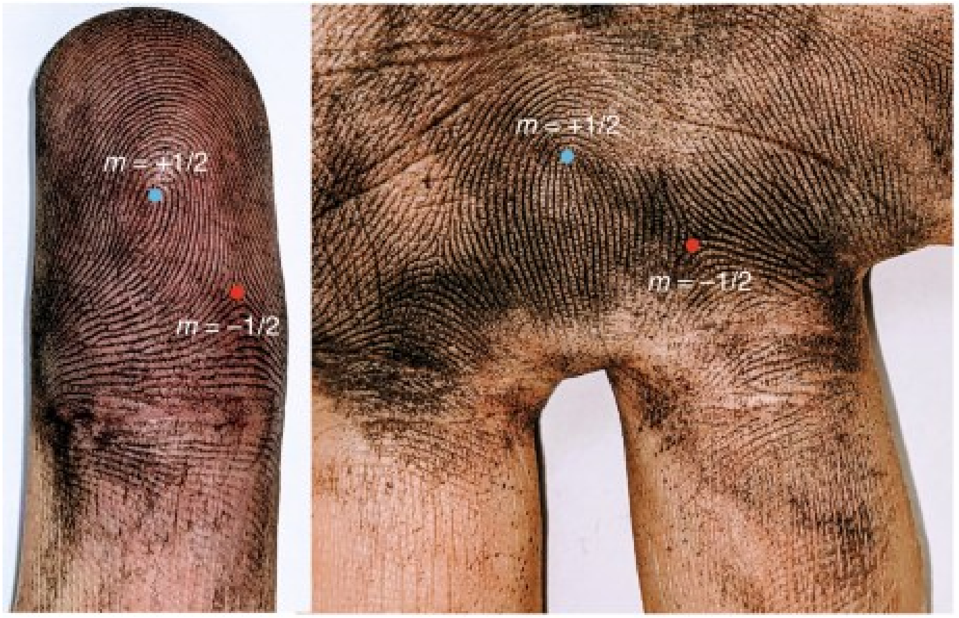
Sphere: Genus = 0

Topology is the mathematical study of properties that remain invariant under continuous change

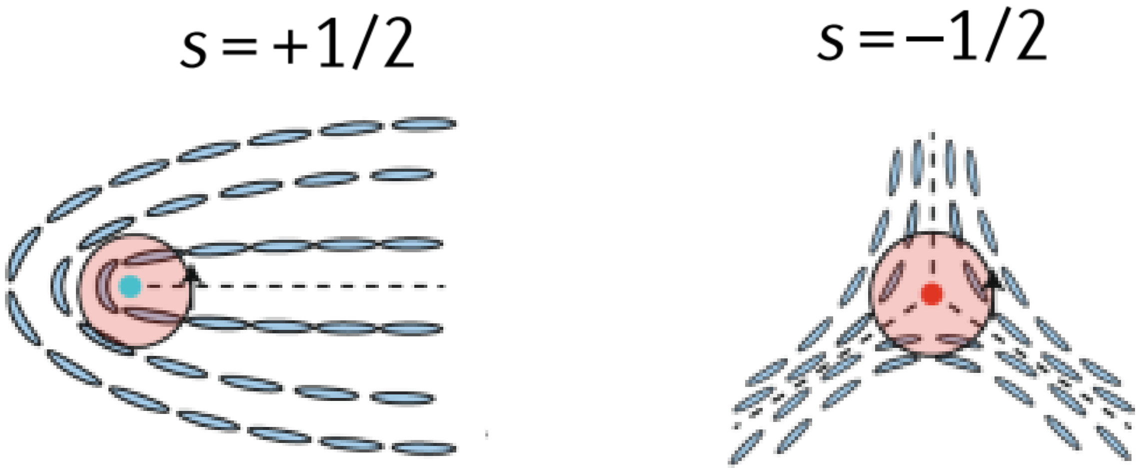
How is topology useful?



Topological Defects: Disclinations



Fardin, Ladoux, Nat. Phys. (2020)

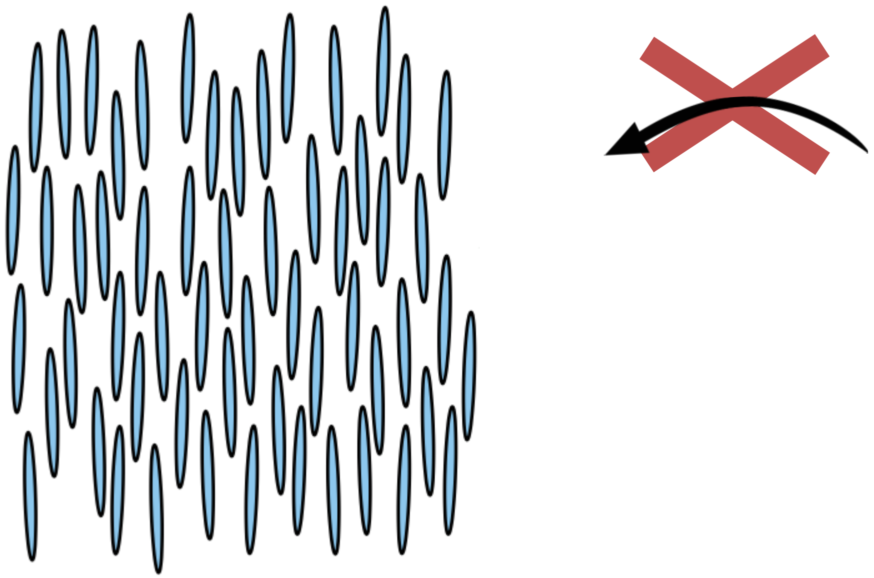


Places where continuous order breaks down

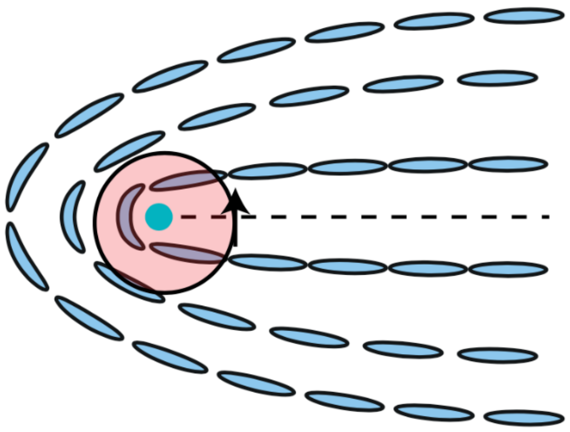
Point defects

2D line field

Topological Defects

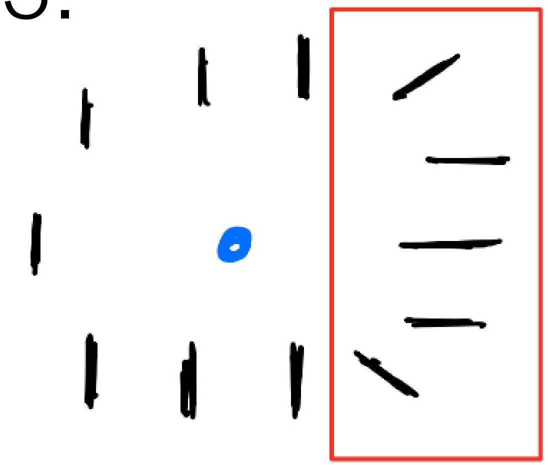


$s = +1/2$

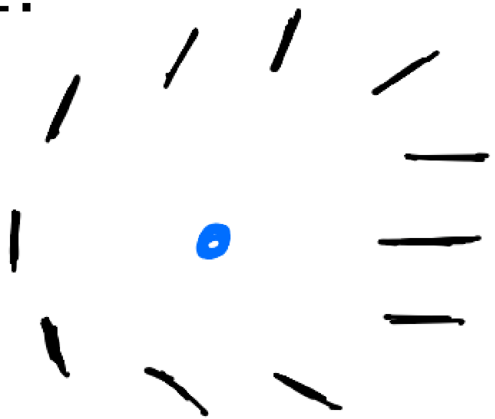


+1/2 turn

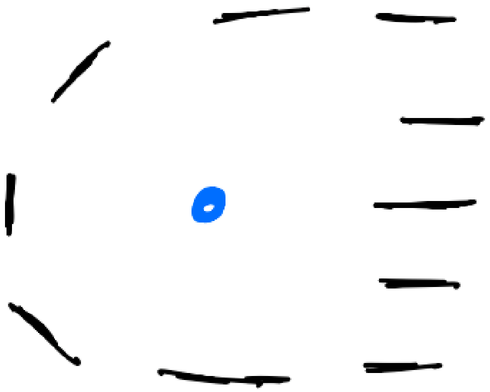
3.



2.



1.



Where can you find them?

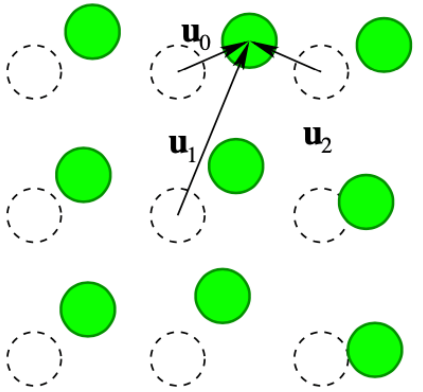
Any physical system described by continuous order (a field):

- <u>Periodic structures</u>:
  displacement vector field $\mathbf{u}(\mathbf{x})$

- <u>Magnets</u>: magnetization vector field $\mathbf{m}(\mathbf{x})$

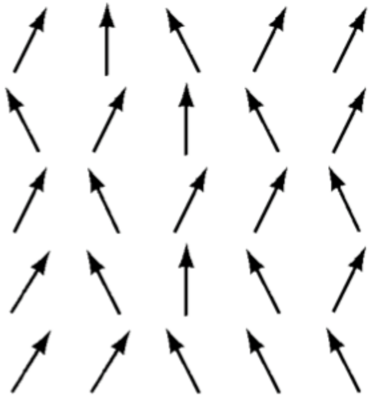
- <u>Liquid crystals</u>: “Director” field

$$\mathbf{n}(\mathbf{x}) \sim -\mathbf{n}(\mathbf{x})$$

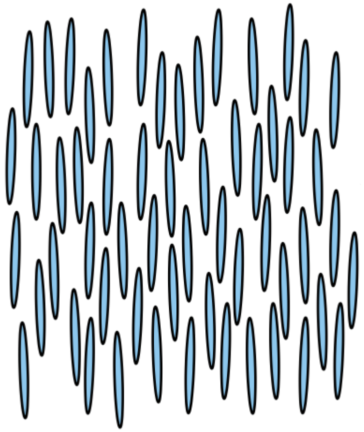


$\mathbf{u}(\mathbf{x})$

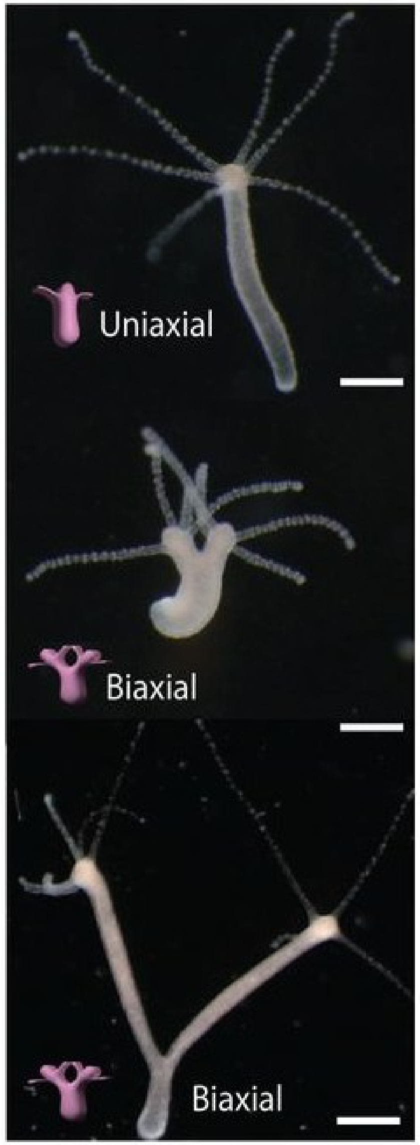
$\mathbf{m}(\mathbf{x})$

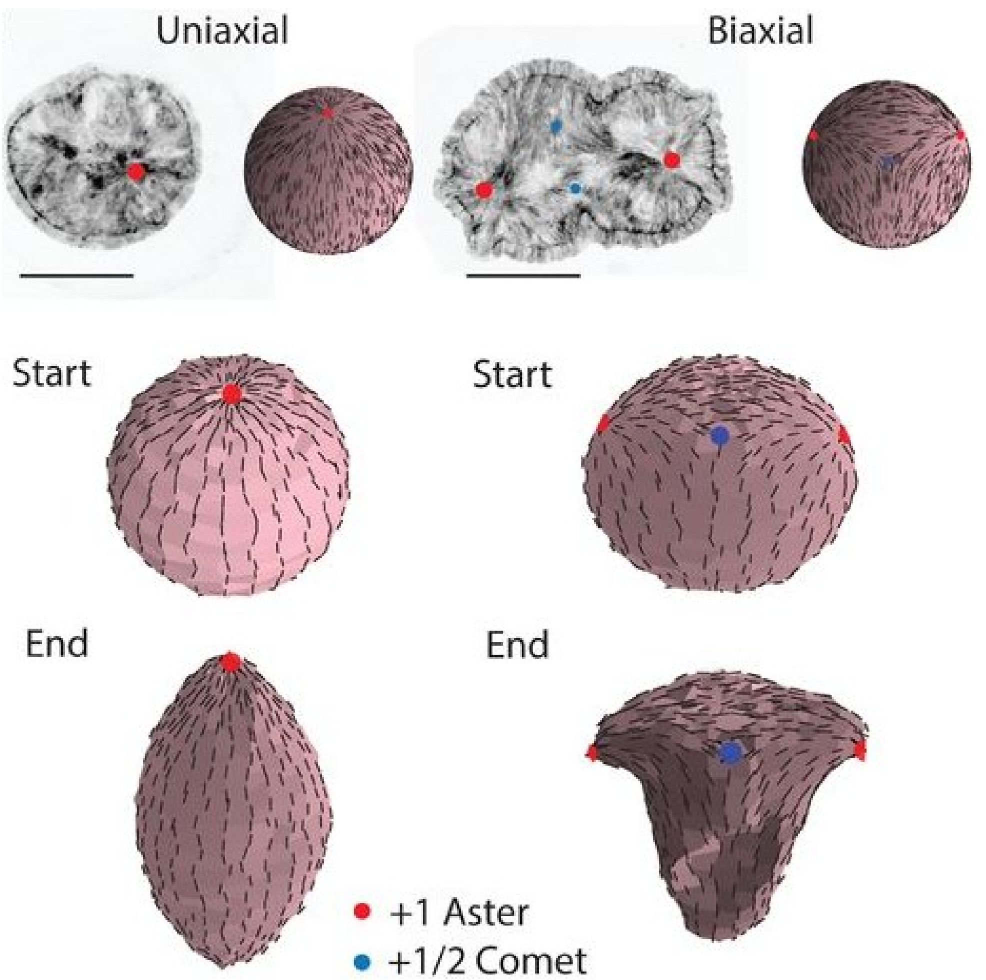


$\mathbf{n}(\mathbf{x}) \sim -\mathbf{n}(\mathbf{x})$



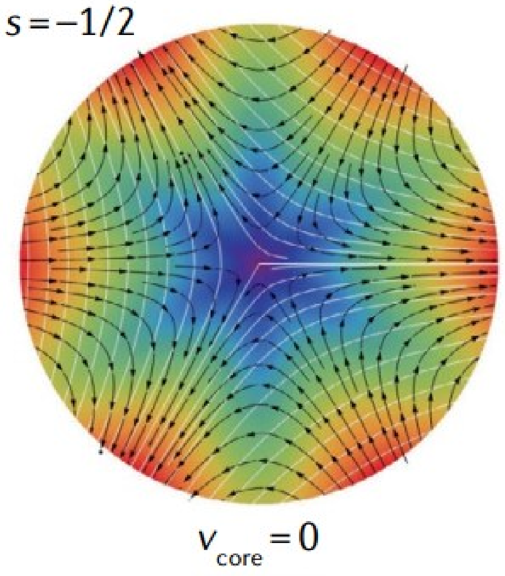
Biological systems: *Hydra* development



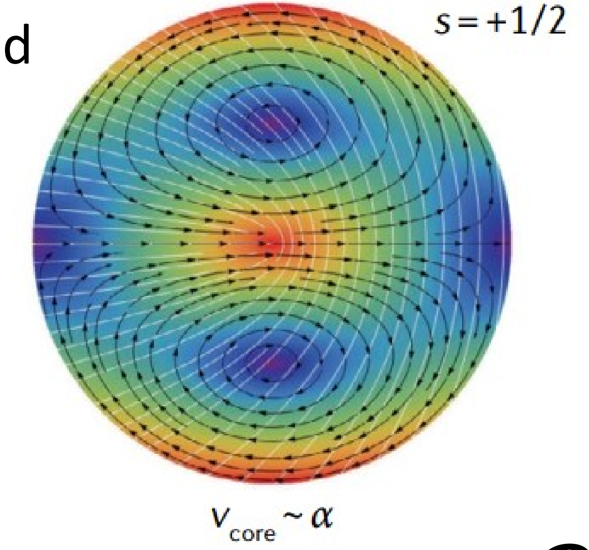


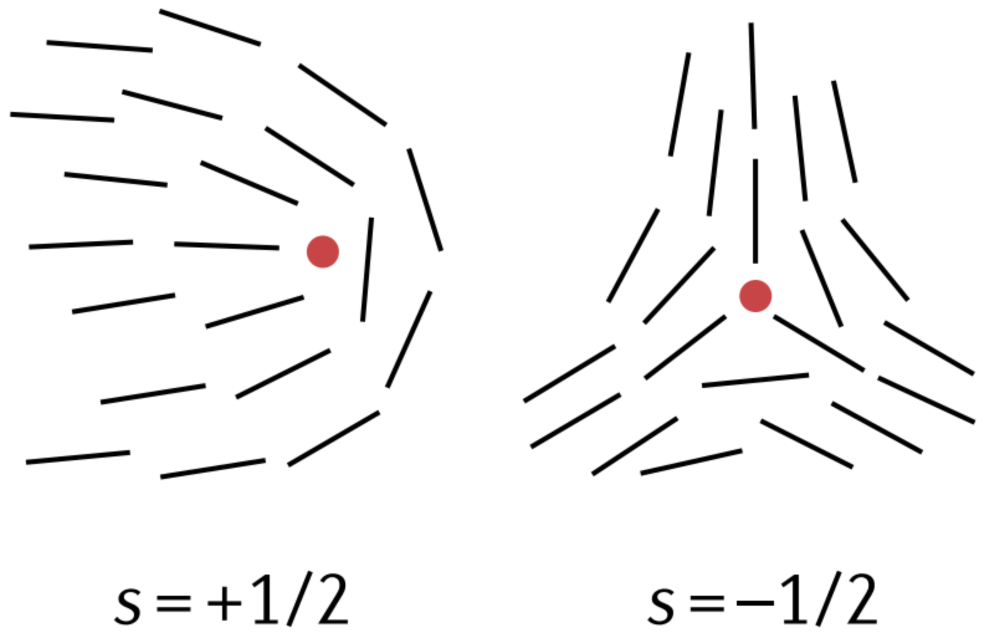
Topological defects in active nematics

Static



Self-propelled





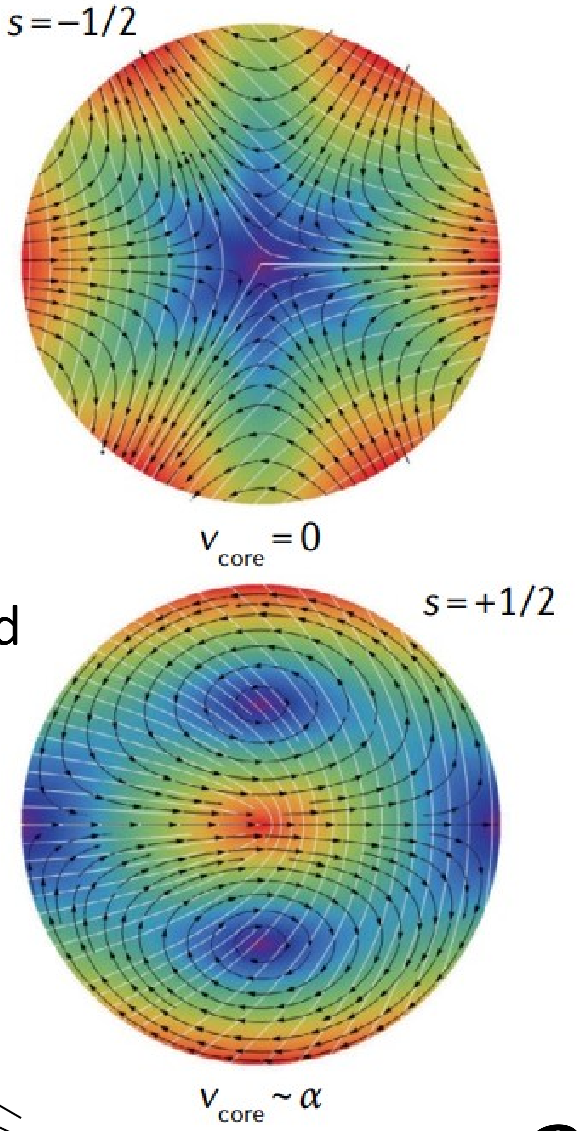
$$\mathbf{Q} = \langle \textstyle\sum_i \left(\hat{\boldsymbol{\nu}}_i \hat{\boldsymbol{\nu}}_i - \mathbf{1}/d\right) \delta(\mathbf{r}-\mathbf{r}_i) \rangle$$

$$\boldsymbol{\sigma}_a = \alpha\, \boldsymbol{Q}$$

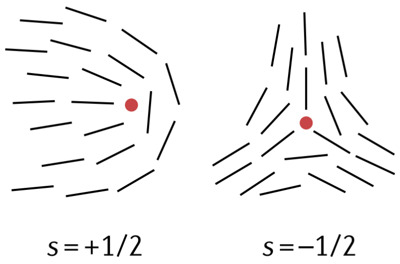
Propulsion of +1/2 defects leads to a defect unbinding transition and active chaos

Topological defects in active nematics

Static



Self-propelled

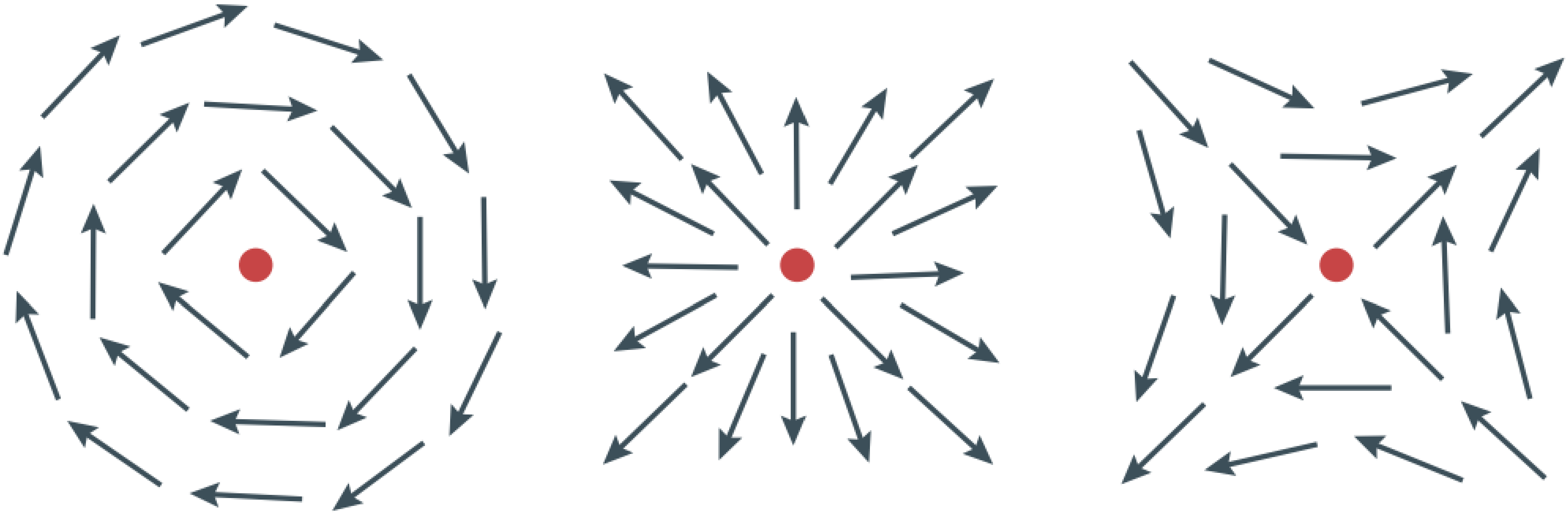


$$\boldsymbol{\sigma}_a = \alpha\, \boldsymbol{Q}$$

Propulsion of +1/2 defects leads to a defect unbinding transition and active chaos

Topological defects in polar active matter

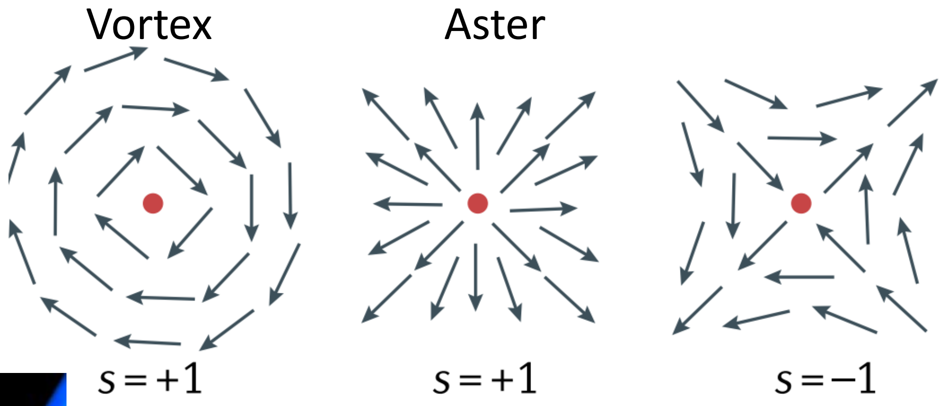
Vortex Aster

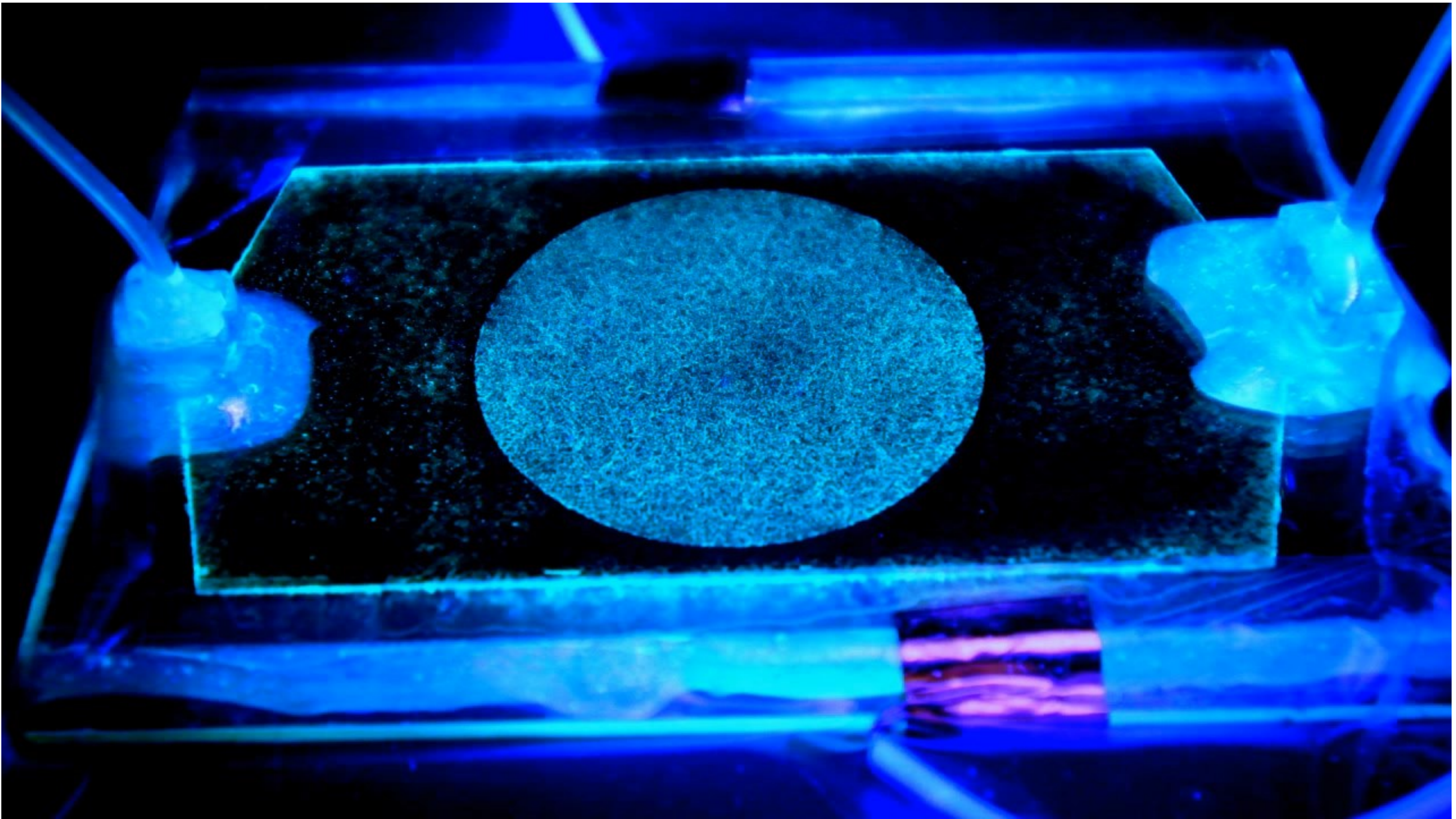


$s = +1$ $s = +1$ $s = -1$

Bartolo lab

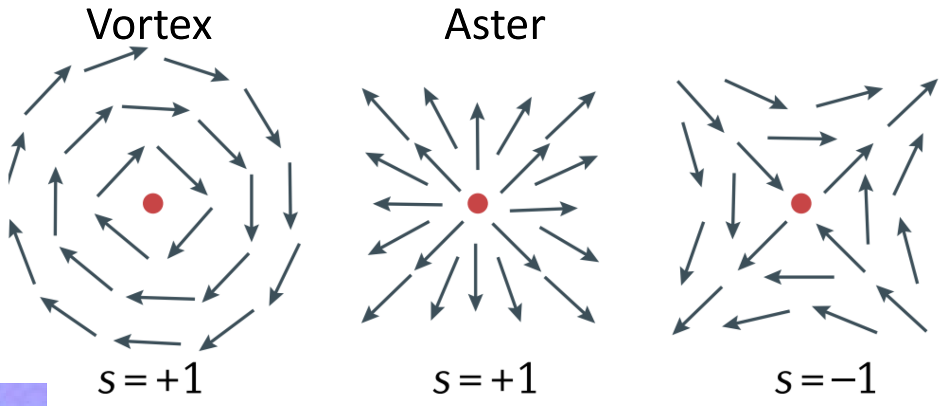
Topological defects in polar active matter

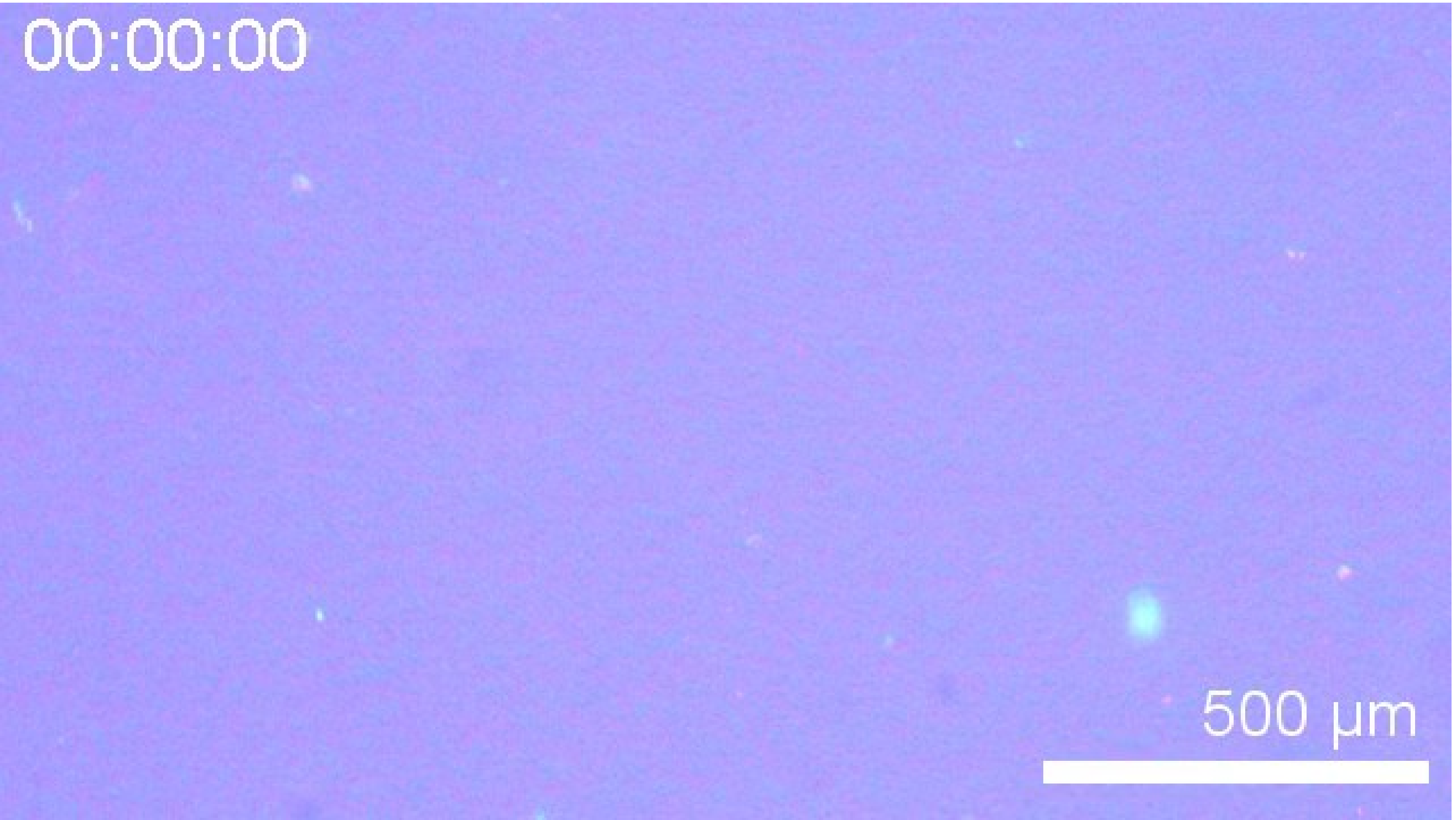




Bartolo lab

Topological defects in polar active matter



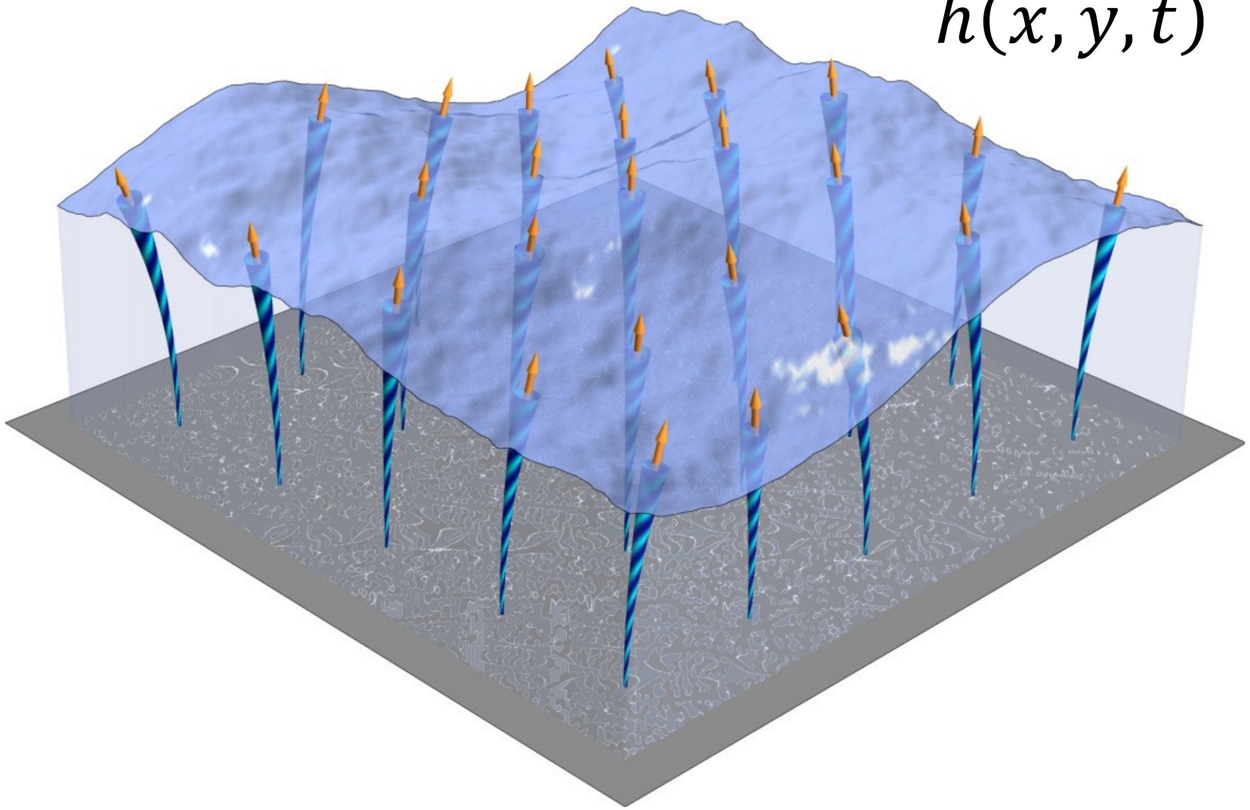


Dogic lab

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

Topological waves in fluids with odd viscosity

$h(x, y, t)$



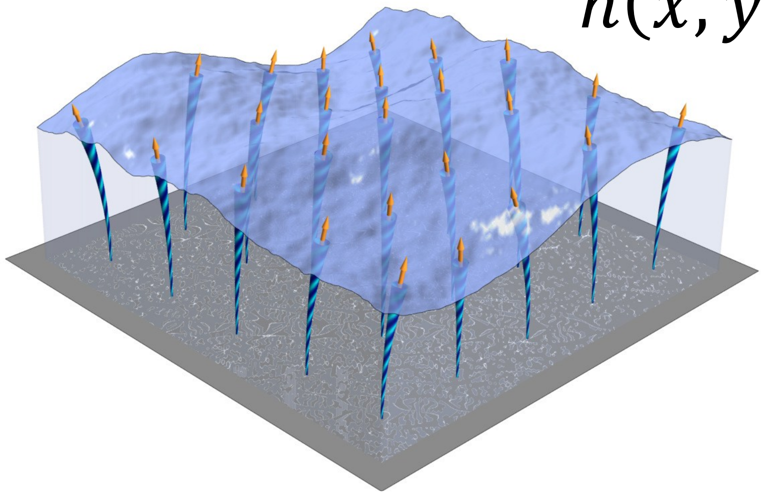
Fluid of vortices has odd viscosity due to breaking time-reversal and parity symmetries

Wiegmann and Abanov. *PRL* (2014)

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)
Shankar, AS, Bowick, Marchetti, Vitelli. Nat. Rev. Phys. (2022)

Linear water waves

$h(x, y, t)$



Height

$$\partial_t h(\mathbf{x}, t) = -h_0 \nabla \cdot \boldsymbol{u}(\mathbf{x}, t)$$

$$\boldsymbol{u} = (u_x, u_y)$$

Velocity

$$\partial_t \boldsymbol{u} = -c^2 \nabla h / h_0 + \omega_B \boldsymbol{u}^* + \nu^o \nabla^2 \boldsymbol{u}^*$$

$$\epsilon_{ij} = -\epsilon_{ji}$$

$$u_i^* = \epsilon_{ij} u_j$$

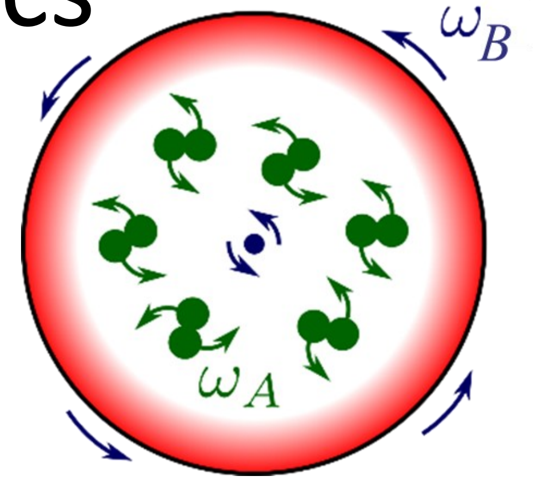
Coriolis force

Odd (Hall) viscosity

Derive terms or write down based on symmetry

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)
Shankar, AS, Bowick, Marchetti, Vitelli. Nat. Rev. Phys. (2022)

Correspondence with acoustics



$$h(x,y,t) \rightarrow \rho(x,y,t)$$

Density

$$\partial_t \rho(\mathbf{x},t) = -\rho_0 \nabla \cdot \boldsymbol{u}(\mathbf{x},t)$$

$$\boldsymbol{u} = (u_x, u_y)$$

Velocity

$$\partial_t \boldsymbol{u} = -c^2 \nabla \rho / \rho_0 + \omega_B \boldsymbol{u}^* + \nu^o \nabla^2 \boldsymbol{u}^*$$

$$\epsilon_{ij} = -\epsilon_{ji}$$

$$u_i^* = \epsilon_{ij} u_j$$

Coriolis force

Odd (Hall) viscosity

At linear order, acoustics same as water waves

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

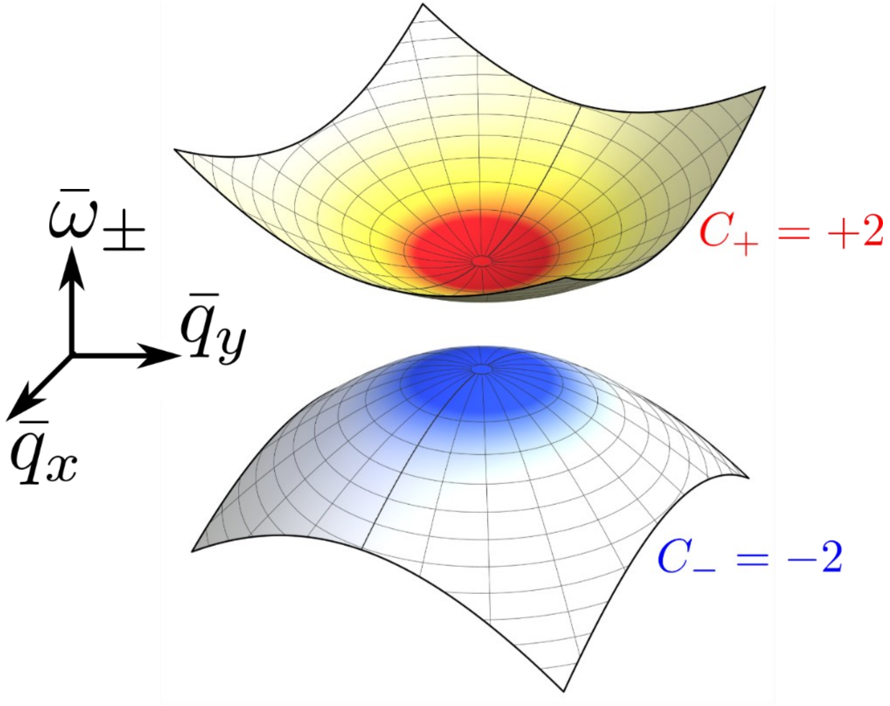
Eigensystem for rotating fluids with odd viscosity

$$\omega \begin{bmatrix} \rho \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} 0 & \rho_0 q_x & \rho_0 q_y \\ c^2 q_x/\rho_0 & 0 & -i(\omega_B - \nu^o q^2) \\ c^2 q_y/\rho_0 & i(\boxed{\omega_B} - \boxed{\nu^o q^2}) & 0 \end{bmatrix} \begin{bmatrix} \rho \\ u_x \\ u_y \end{bmatrix}$$

Coriolis force **Odd viscosity**

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

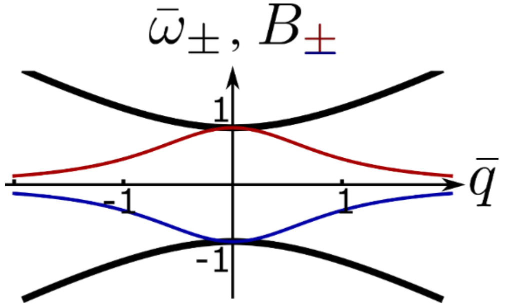
Odd viscosity compactifies reciprocal space, Chern number can be defined



Berry curvature

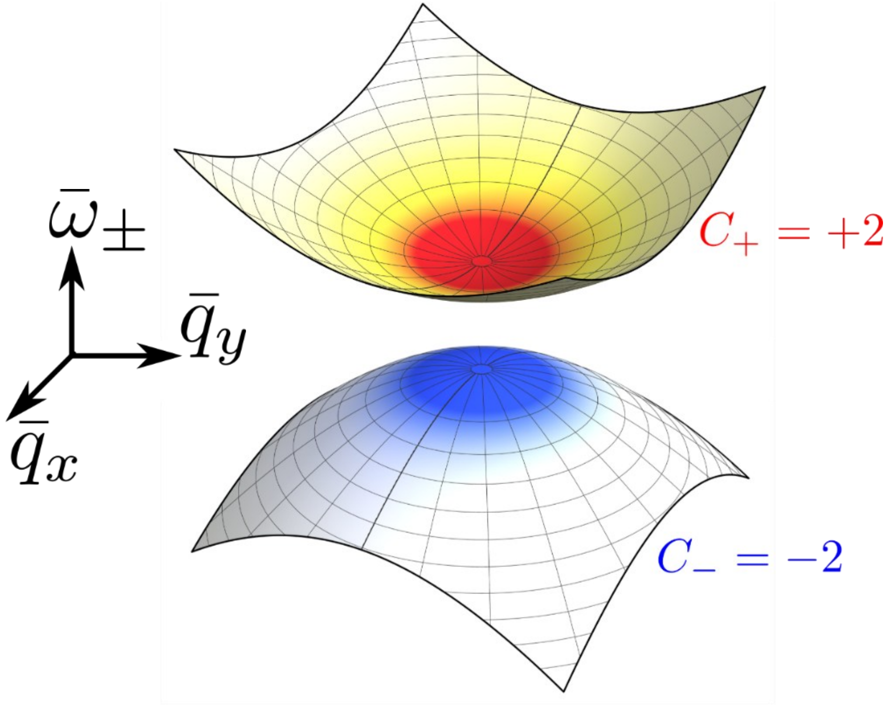
$$B_\pm(\mathbf{q}) \equiv i\nabla_\mathbf{q} \times \left[(\mathbf{u}_\mathbf{q}^\pm)^\dagger \cdot (\nabla_\mathbf{q}\mathbf{u}_\mathbf{q}^\pm)\right]$$

$$C_\pm \equiv \frac{1}{2\pi}\int B_\pm(\mathbf{q})d\mathbf{q}$$ Chern number

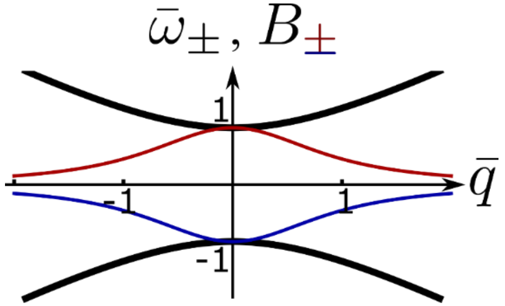


Only integer if reciprocal space is a closed surface!

Odd viscosity compactifies reciprocal space, Chern number can be defined



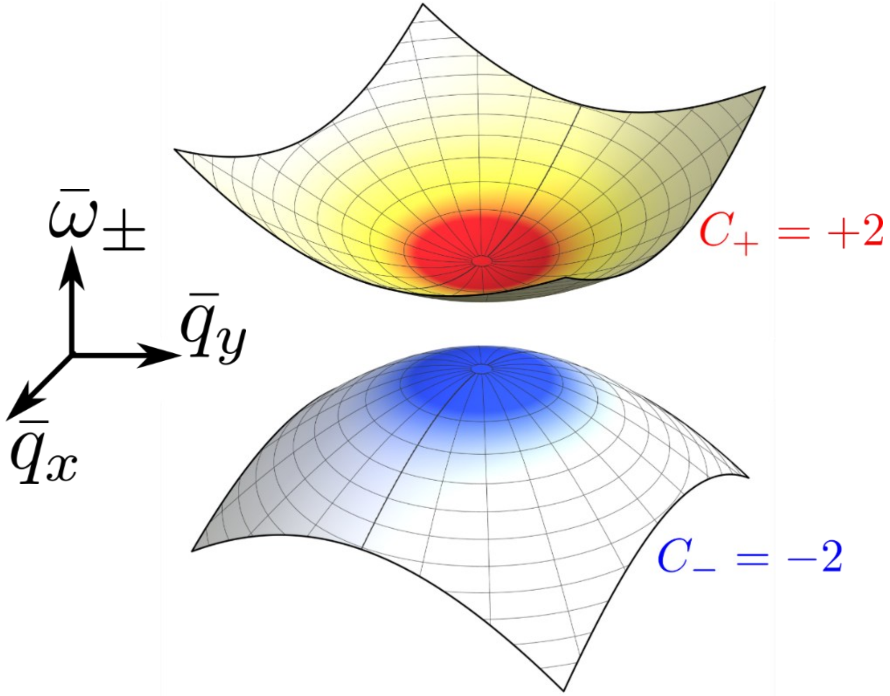
$$\mathcal{C}_- = \mathrm{sign}(\nu^{\mathrm{o}}) + \mathrm{sign}(\omega_B)$$



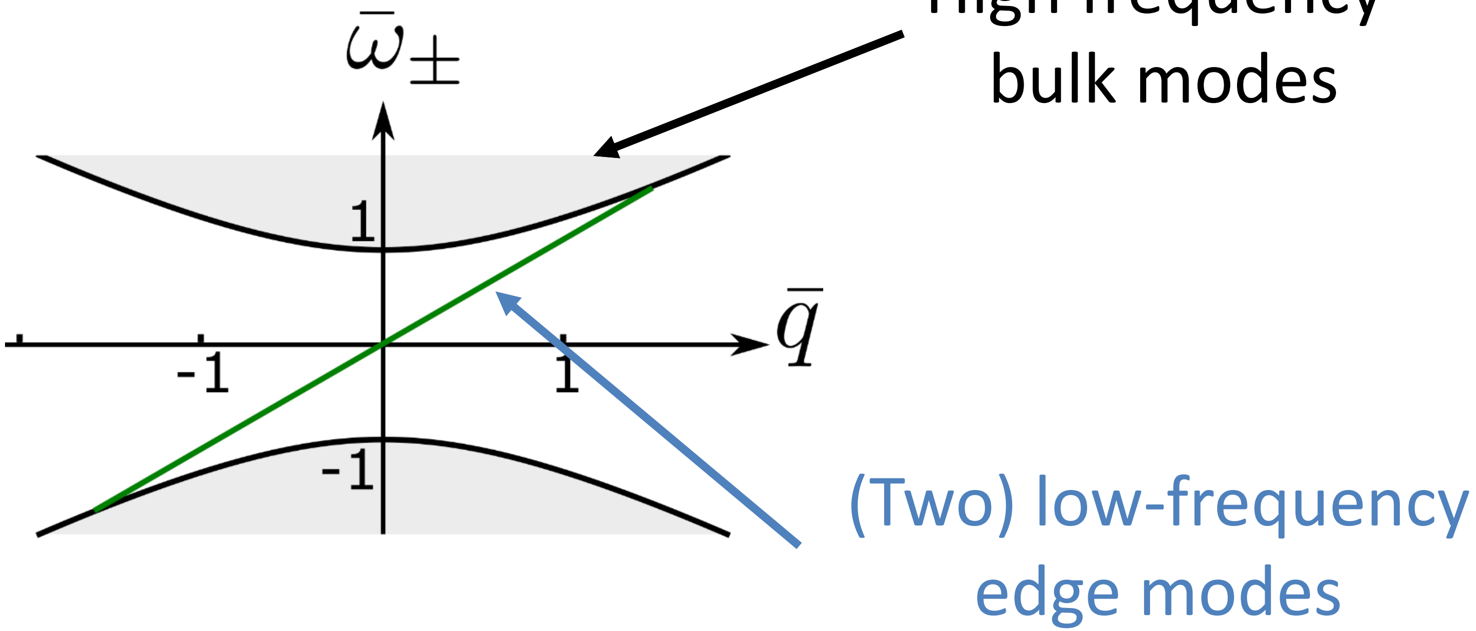
Only integer if reciprocal space is a closed surface!

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

Chern number counts number of chiral edge states



(bulk)

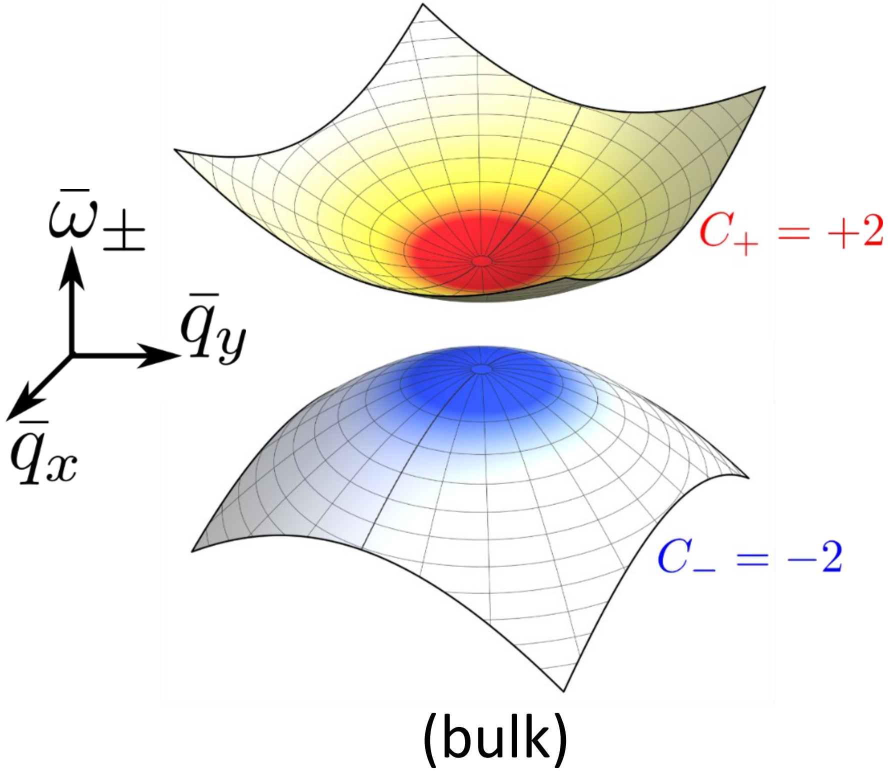


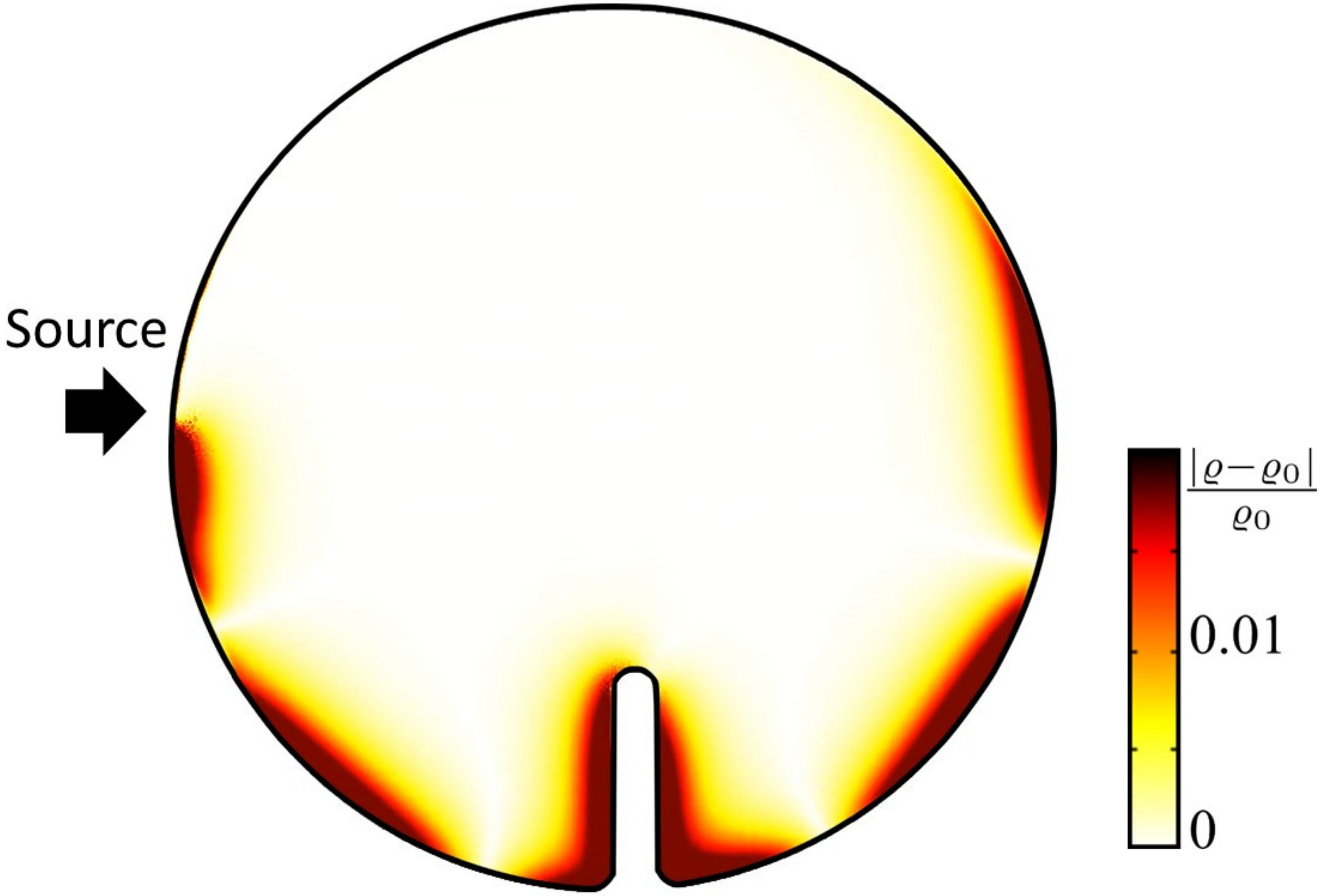
(boundary)

Bulk-boundary correspondence can translate to active fluids

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. Phys. Rev. Lett. (2019)

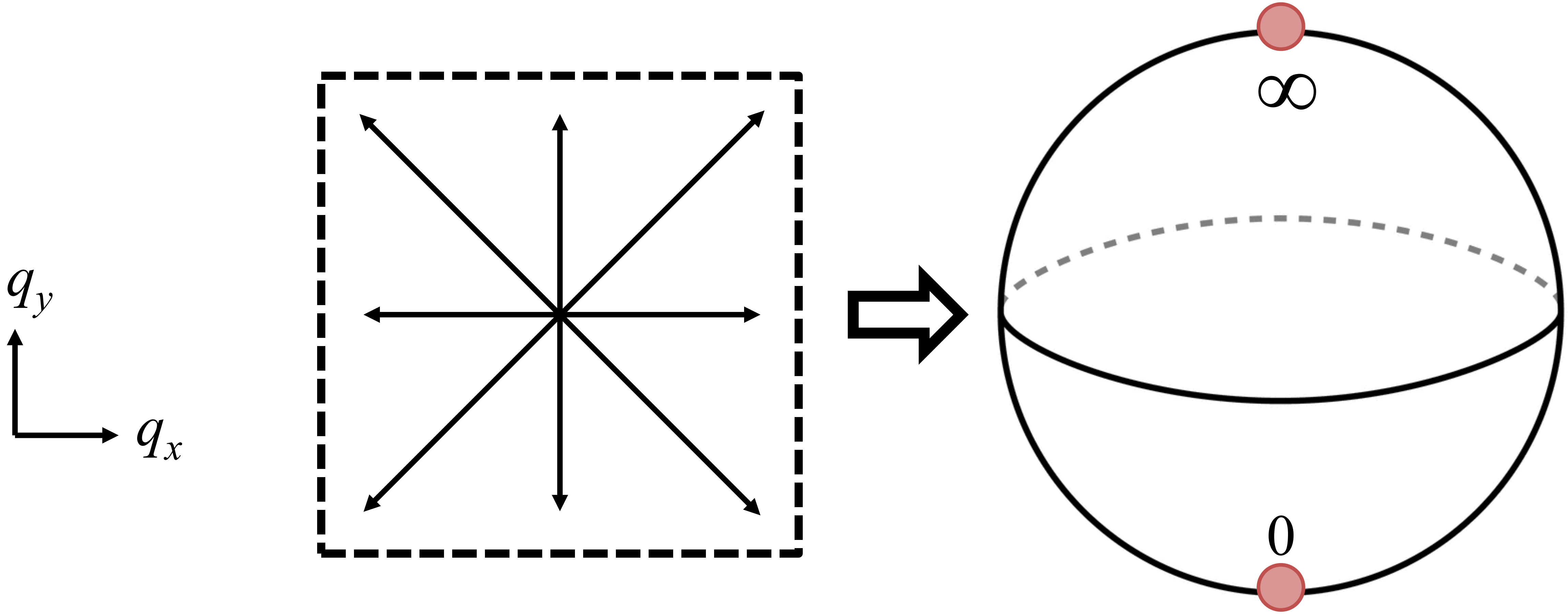
Chern number counts number of chiral edge states





Bulk-boundary correspondence can translate to active fluids

Reciprocal space: fluids



If large-wavevector dynamics are independent of wavevector direction, then fluid reciprocal space can be compactified

AS, Dasbiswas, Fruchart, Vaikuntanathan, Vitelli. arxiv:1802.09649 *Phys. Rev. Lett.* (2019)

About these lectures

Lecture 1. Topological active matter
Part 1.1:
Overview; Definition of active matter
Part 1.2:
Classification of active fluids
Part 1.3:
Topological active matter

Lecture 2. Non-reciprocal active solids
Part 2.1:
Introduction to active solids
Part 2.2:
Odd elasticity
Part 2.3:
Current topics: active percolation, pattern formation

Review articles on active matter:
Shankar et al Topological active matter
Nature Reviews Physics (2022)

Fruchart, Scheibner, Vitelli. Odd viscosity and odd elasticity.
Annual Review of Condensed Matter Physics 14, 471 (2023)

Marchetti et al Hydrodynamics of soft active matter
Reviews of Modern Physics 85, 1143 (2013)

Background textbook:
P. M. Chaikin and T. C. Lubensky (1995) Ch. 6-10
Principles of Condensed Matter Physics

Topology:
David Mermin *Rev Mod Phys* (1979)
The topological theory of defects in ordered media

This lecture:

Broad introduction to active matter and connections to topology