Modelling the interaction of lava flows with topography and barriers

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Flows of volcanic lava, La Palma

Associated Press, 30 November 2021

Eruption Sept-Dec 2021

7000 people displaced.

Estimated damage € 850m

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Lava flow interactions with topography

- Lava flows slowly and as a relatively shallow layer.
- Very high viscosity (>> viscosity of water)
- Driven by gravity, guided by topography, interacts with infrastructure

Model as a viscously-dominated, shallow flowing layer.
Determine the flow speed,depth and inundated area.

Can lava flows be deflected or arrested?

- Infrastructure, lives and livelihoods are catastrophically impacted by lava inundation.
- Recent examples: La Palma, Canary Islands; Reykjanes Peninsula, Iceland; Kīlauea, Hawaii; Goma, D.R. Congo

Grindavík, Iceland 2024

Attempted strategies:

- Bombing (e.g. Hawaii, 1930s)
- Cooling (e.g. Heimaey, Iceland, 1973)
- Barriers

https://www.bbc.co.uk/news/magazine-29136747

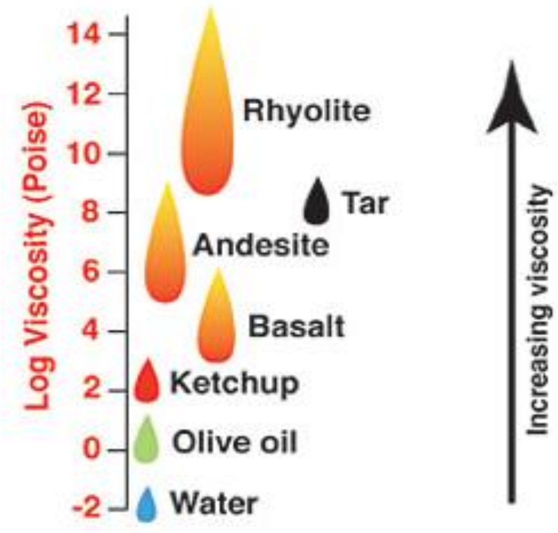
Barrier construction in Iceland (2023-)

- Programme of barrier construction to defend villages and infrastructure.
- Design choices: Size, location, materials

- Design based upon computational flow modelling
  (Hörn Hrafnsdóttir, Verkis)

Lava rheology

- Three-phase mixture of melt, crystals, and gas bubbles.
- The rheology – *the relationship between stresses and rates of deformation* - is a function of:
  chemical composition, volume fractions of crystals and bubbles and temperature.
- High viscosity
  (10^6-10^{12} times viscosity of water)



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- Vital role of cooling:
  - increases viscosity
  - changes rheology

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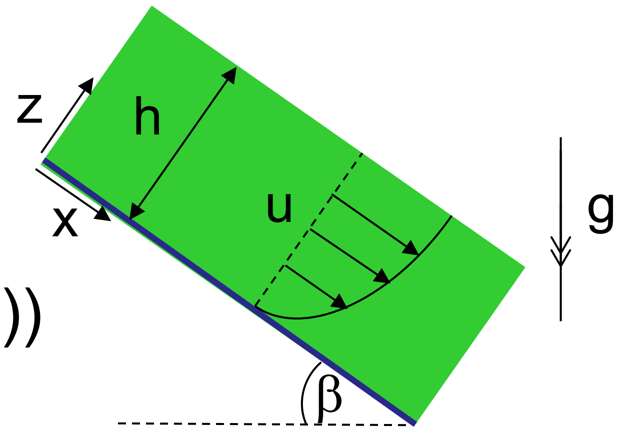
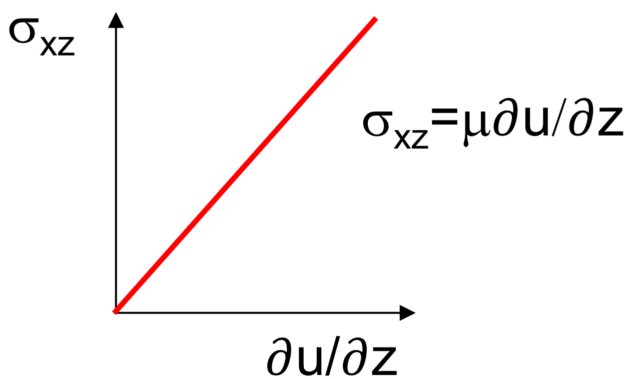
Free surface flows (i)



- Simple shear flow down a plane, driven by gravity. (Flow field **u**=(u(z),0,0))
- Balance of momentum (p(z)=pressure, σ_{xz}(z)=deviatoric shear stress):

$$0 = -\frac{\partial p}{\partial z} - \rho g \cos\beta \qquad\qquad 0 = \frac{\partial \sigma_{xz}}{\partial z} + \rho g \sin\beta$$

- Integrate: $p = p_a + \rho g \cos\beta\,(h-z)$ $\sigma_{xz} = \rho g \sin\beta\,(h-z)$
- Newtonian rheology



Velocity $u = \dfrac{\rho g \sin\beta}{2\mu}\,(2hz - z^2)$

Volume flux $q = \displaystyle\int_0^h u\,dz = \frac{\rho g \sin\beta\, h^3}{3\mu}$

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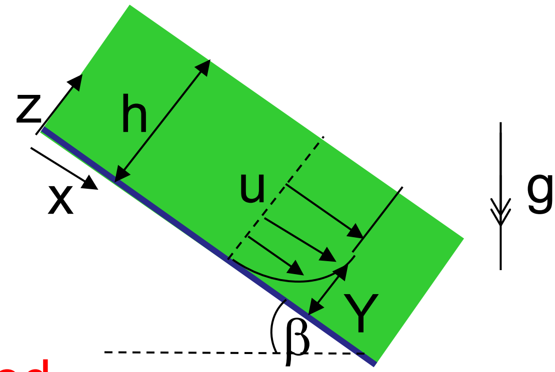
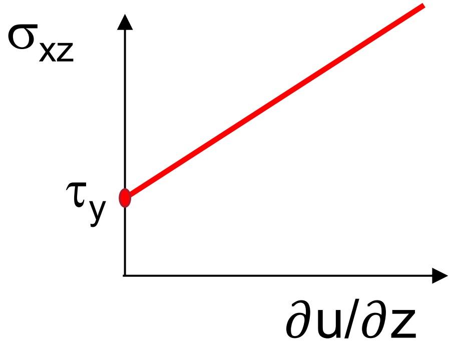
Free surface flow: yield stress



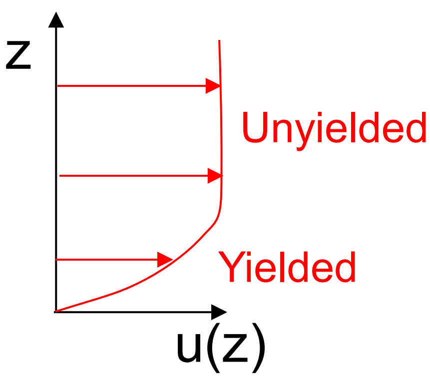
- Flows with a yield stress, τ_y



$\sigma_{xz}=\tau_y+\mu\partial u/\partial z$ if $\sigma_{xz}>\tau_y$ Yielded

$\partial u/\partial z=0$ otherwise Unyielded

- Flow is unyielded for $0<z<Y$ $\sigma_{xz}(z=Y)=\tau_y$ *Yield height*

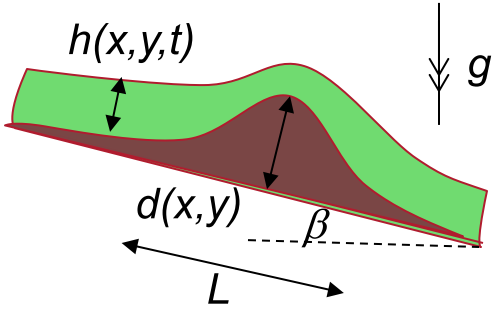


- Yielded region ($z<Y$) $u = \dfrac{\rho g \sin\beta}{2\mu} z(2Y-z)$
- Unyielded region ($z>Y$) $u = U_p = \dfrac{\rho g \sin\beta\, Y^2}{2\mu}$ *Plug velocity*
- Volume flux

$$q = \int_0^h u\, dz = \frac{\rho g \sin\beta\, Y^2(3h-Y)}{3\mu}$$

Only flows when Y>0: $\rho g \sin\beta\, h > \tau_y$

Mathematical model: *lubrication*



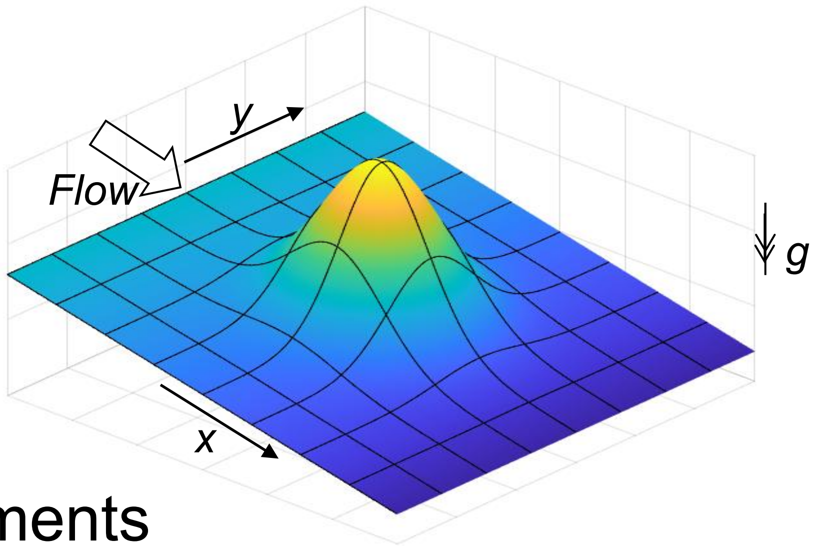
- Extend flow model to shallow flows ($h/L<<1$), flowing over topography with $z=d(x,y)$.
- Hydrostatic pressure $p = \rho g \cos\beta\,(h + d - z)$
- Deviatoric shear stresses σ_{xz} and σ_{yz}

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g \sin\beta \qquad\qquad 0 = -\frac{\partial p}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}$$

- The flow yields when $\sigma_{xz}^2 + \sigma_{yz}^2 = \tau_y^2$
  - Defines a yield surface $Y(x,y)$ and plug velocity $U_p(x,y)$
- Find velocity field parallel to boundary: $\mathbf{u}=(u,v,0)$ and volume flux $\mathbf{q}=(q_x,q_y)$ as function of $h(x,y,t)$
- Mass conservation $$\frac{\partial h}{\partial t} + \nabla . \boldsymbol{q} = 0$$

Dimensionless parameters

- Steady flow driven by a uniform upstream flux of fluid per unit width, Q. Newtonian fluid flows with depth $h_\infty = \left(\frac{3\mu Q}{\rho g \sin\beta}\right)^{1/3}$. Flow interacts with topography of characteristic width L and height d_m
- Dimensionless parameters:
  - Flow parameter, $F = \frac{h_\infty}{L \tan\beta}$
  - Topography parameter, $M = \frac{d_m}{L \tan\beta}$
  - Yield stress parameter, $B = \frac{\tau_y}{\rho g \sin\beta h_\infty}$
- Topographic effects illustrated using $d(x,y)=M \exp(-x^2-y^2)$
- Numerical solutions using finite elements



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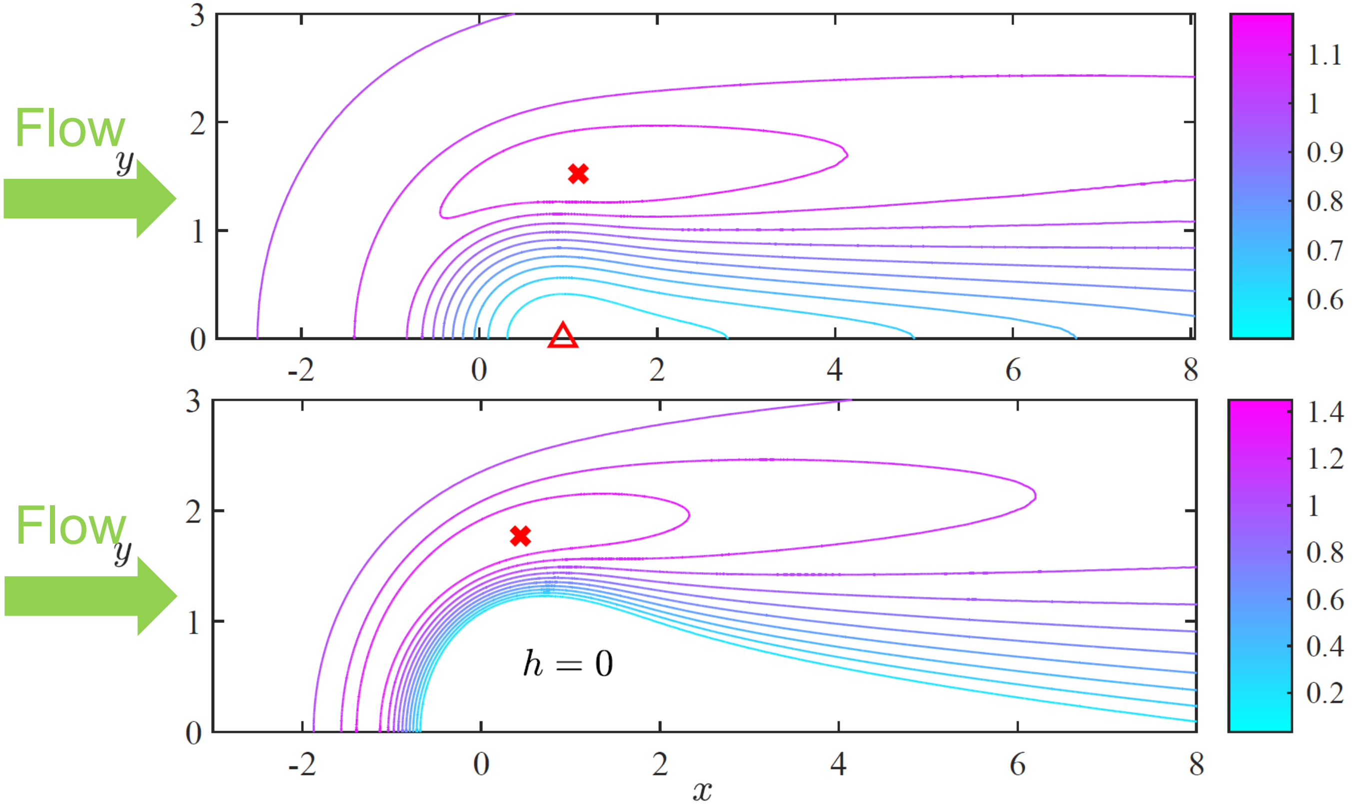
Flow over a mound (F=0.1, B=0)

- Contours of flow thickness: **deflection** and **overtopping**



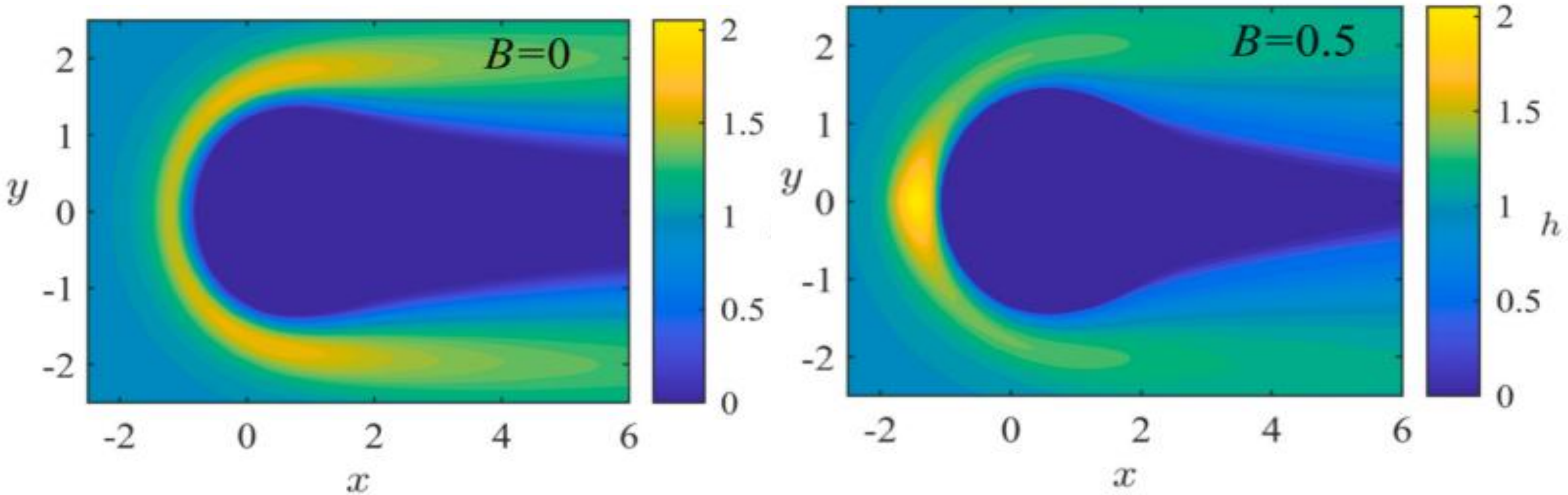
Small mound
M=0.5

Large mound
M=1.5

Creation of ‘dry’ zone into which no fluid flows

Dimensions of dry zone are function of M and F (and $d(x,y)$)

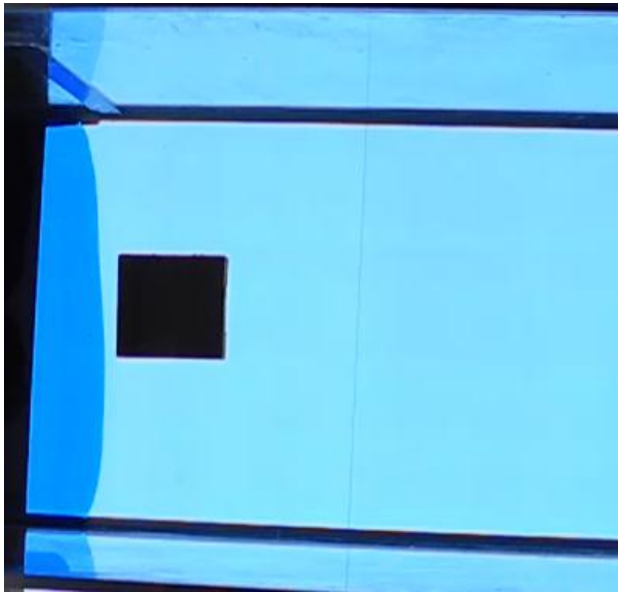
Flow over a mound: *effects of yield stress*

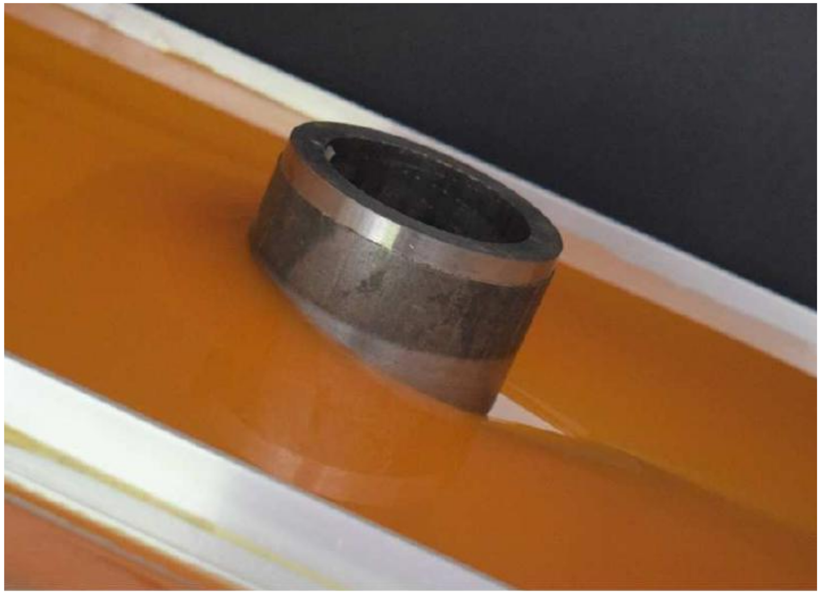


- Streamlines only weakly affected by yield stress
- Flow depths somewhat different:
  - Maximum depth on symmetry axis upstream of mound when B=0.5.

Flow around surface-piercing obstacle

- Obstacle can not be overtopped. Instead flow is deflected.
- Laboratory experiments with obstacles of different cross-section (circle, square, diamond)

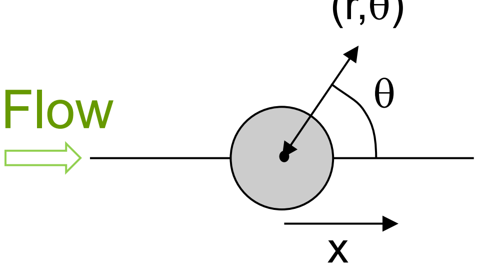




- Relatively wide obstacles $F = \frac{h_\infty}{L \tan \beta} \ll 1$:

  Upstream 'pond' of fluid. Downstream fluid-free region

Depth of fluid: *circular cylinder*

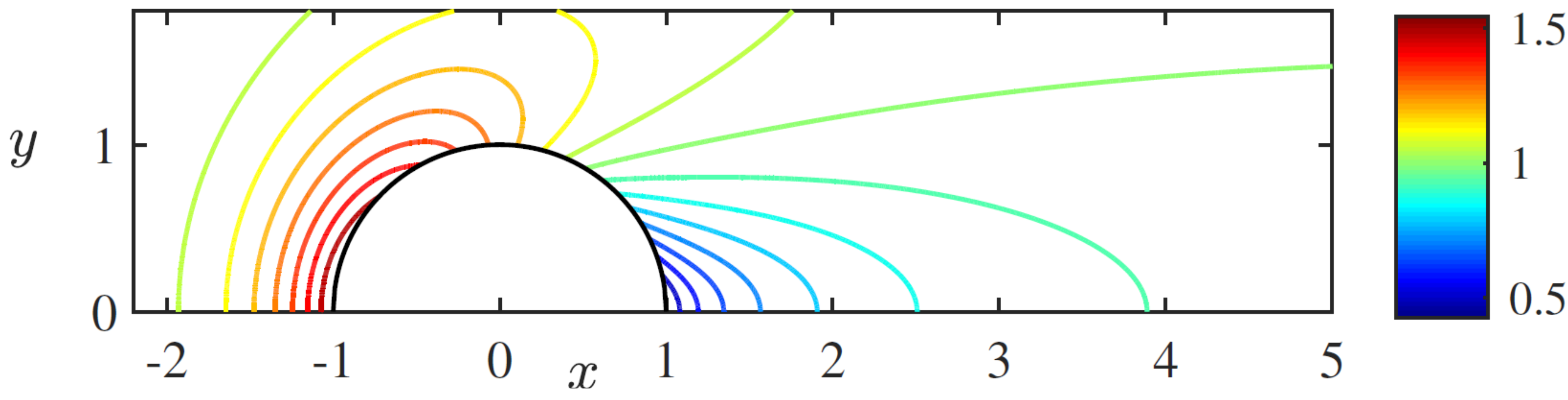


- Dimensionless governing equation

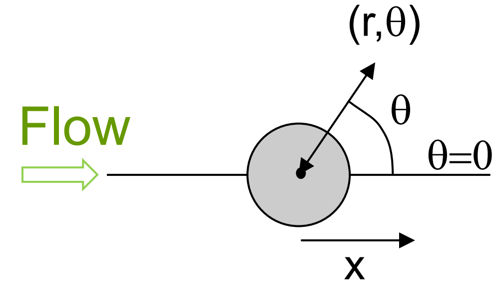
$$\frac{\partial h^3}{\partial x} = \nabla.\left(Fh^3\nabla h\right)$$

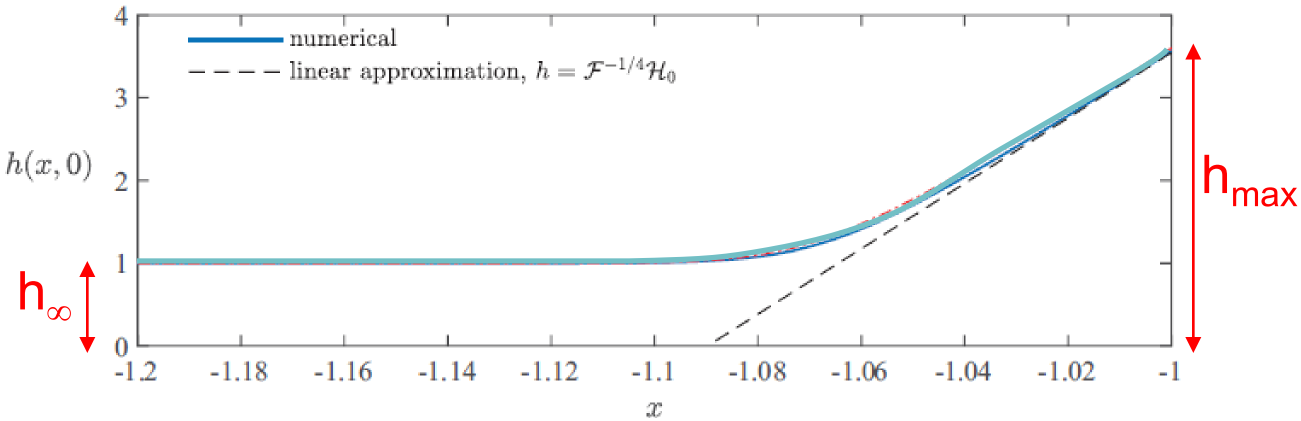
subject to $h \to 1$ as $r = |\boldsymbol{x}| \to \infty$ *[Far field uniform thickness]*
and $h^3\left(F\frac{\partial h}{\partial r} - \cos\theta\right) = 0$ on *r* = *1* *[Impermeable boundary]*

- Numerical solution (*F*=1)

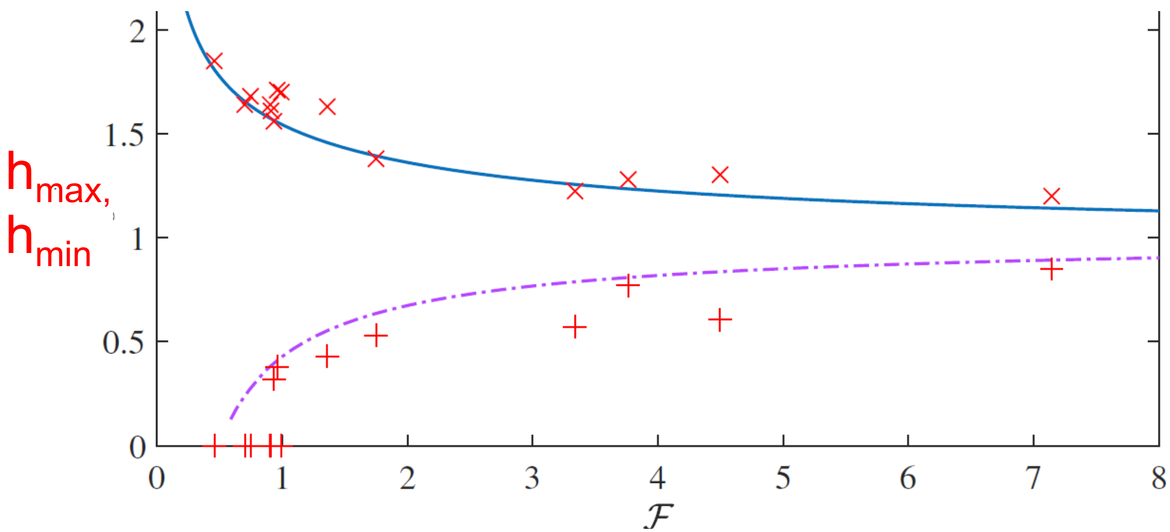


Circular cylinder results





Flow depth along symmetry axis (F=0.025)



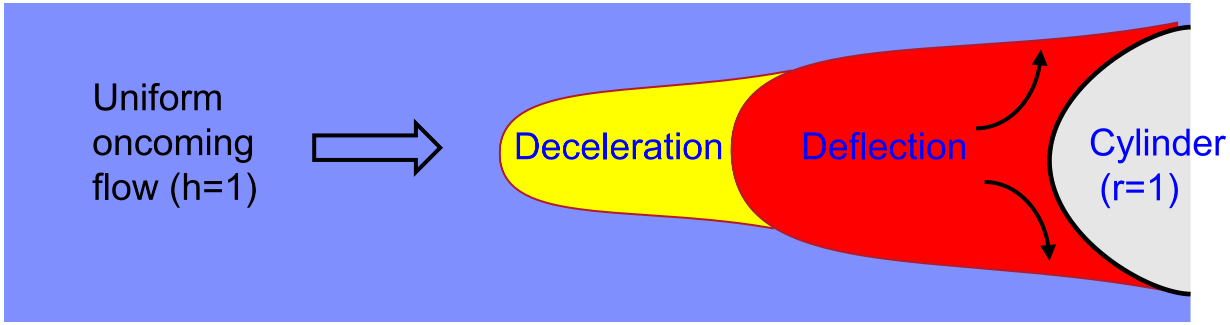
The maximum (θ=π) and minimum (θ=0) flow depths at cylinder.

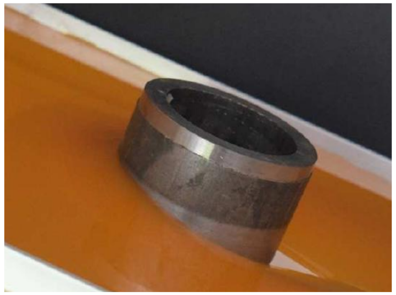
Close agreement with experiments.

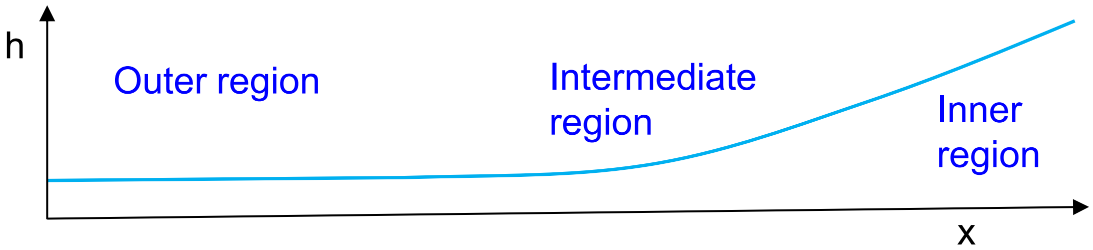
h_{max} is large when F<<1

Can this dependence be explained?

Maximum flow depth: *F<<1*

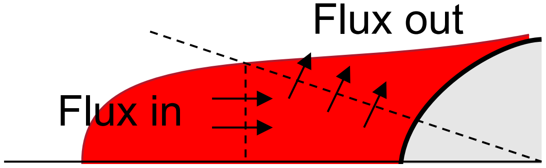




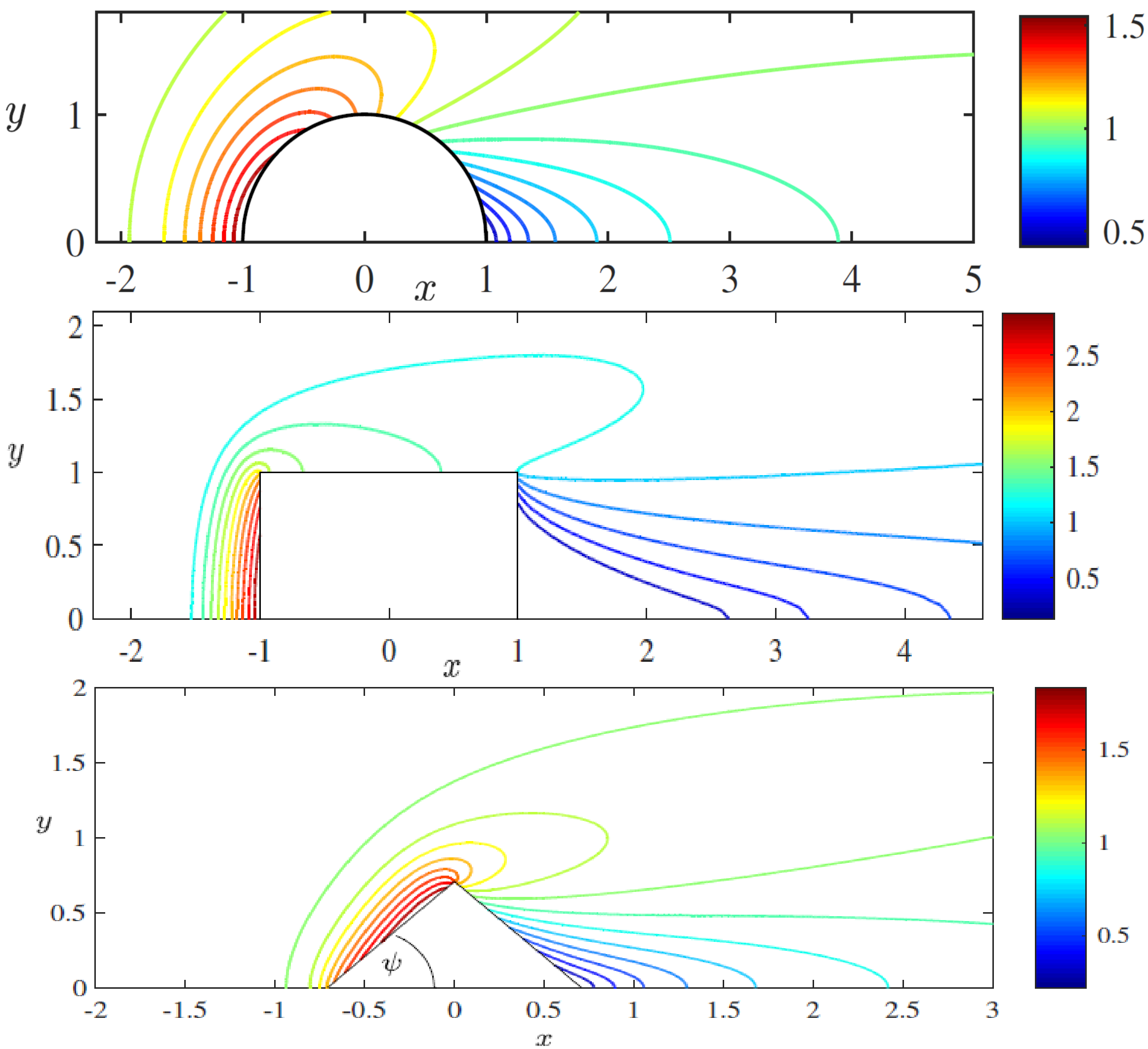


The full asymptotic description requires three regions and matched expansions between them.

- Leading order result more readily deduced:
  - Within deep-ponded inner region, no radial flux as cylinder impermeable
  - Balance flux into inner region from upstream with deflected flux
  - Deduce Maximum height: $h(1,\pi) = \left(\frac{4}{F}\right)^{1/4}$



Flow around other shapes



Asymptotic results (F<<1)

- Circles:

$$h_{max} = \left(\frac{4}{F}\right)^{1/4}$$

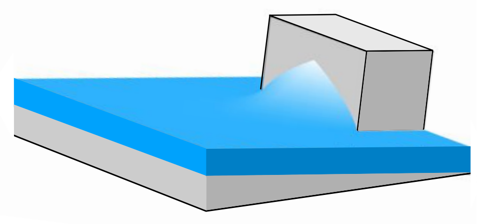
- Squares:

$$h_{max} = \left(\frac{10}{F^2}\right)^{1/5}$$

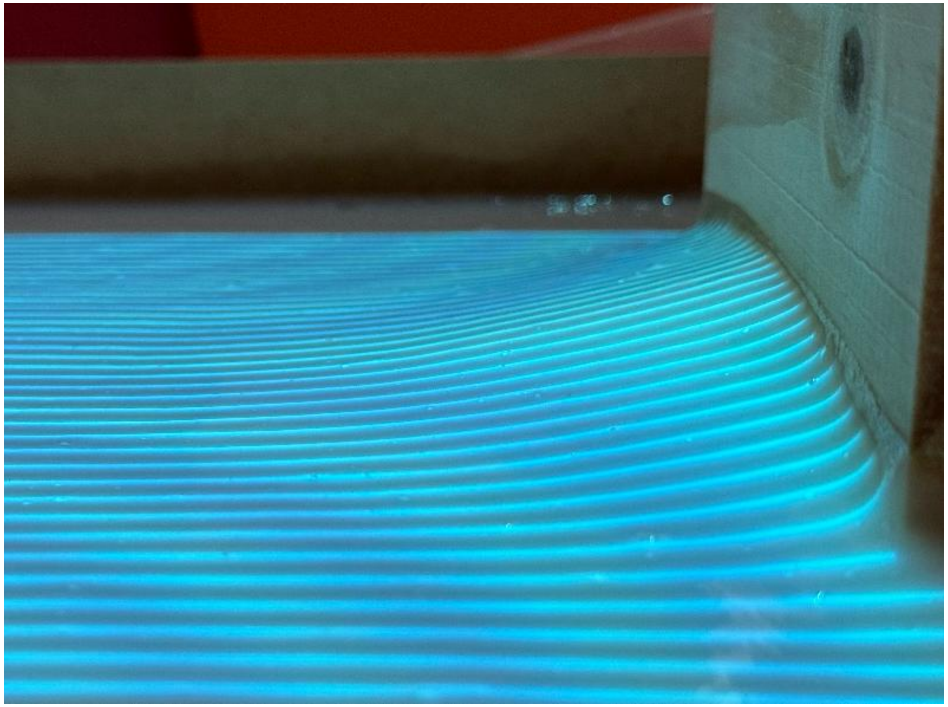
- Diamond:

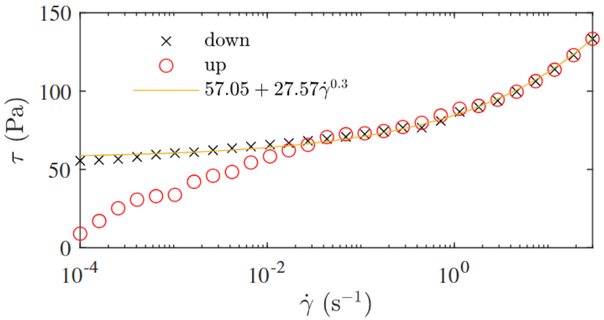
$$h_{max} = \left(\frac{4\sin^2\psi}{\cos\psi\, F}\right)^{1/4}$$

What happens when source is turned off?

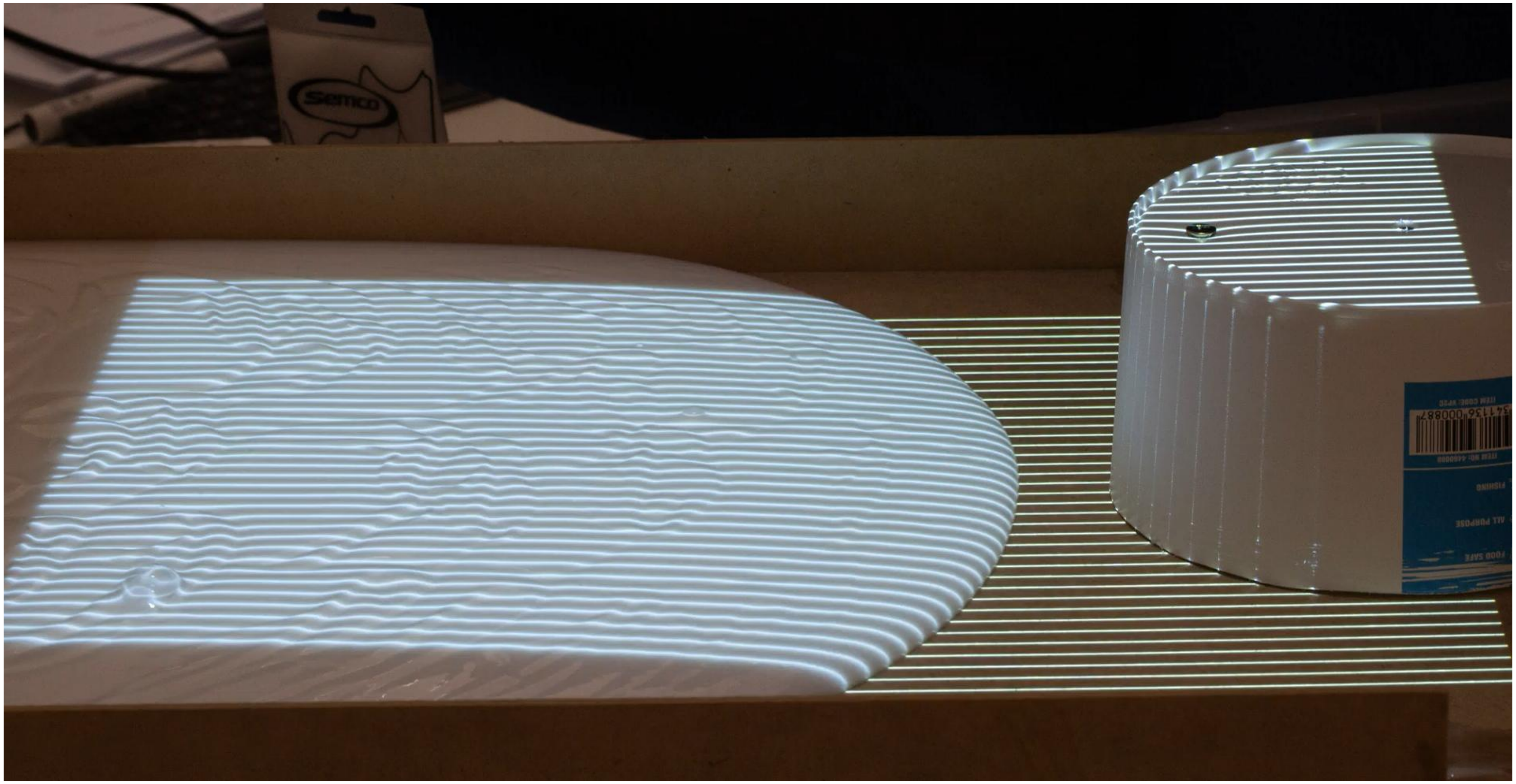


- For a purely viscous fluid, all fluid drains away.
- With a yield stress, fluid is retained upstream of the obstacle
  - Can yield stress be inferred from material left behind?



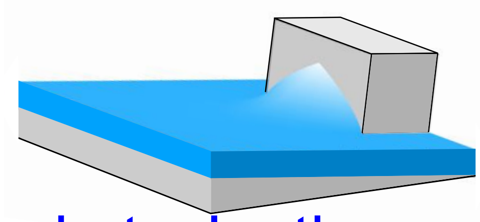


Laboratory Experiments

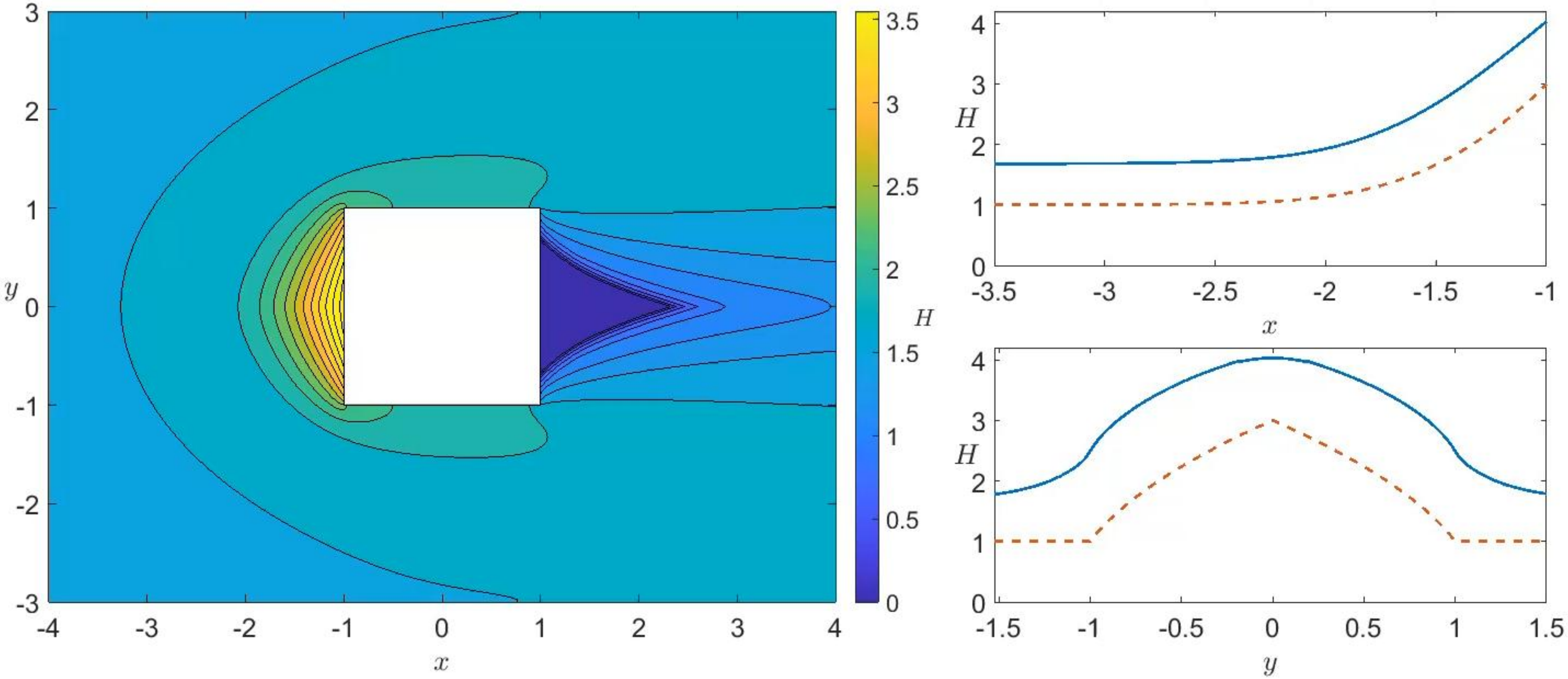


Experiments run for over 10 hours to adjust to final state

Simulations of yield stress fluid



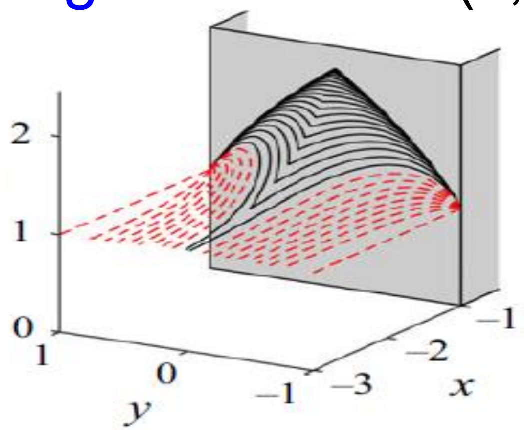
- Starting from the steady state of flow around obstacle, the source is turned off to compute h(x,y,t)

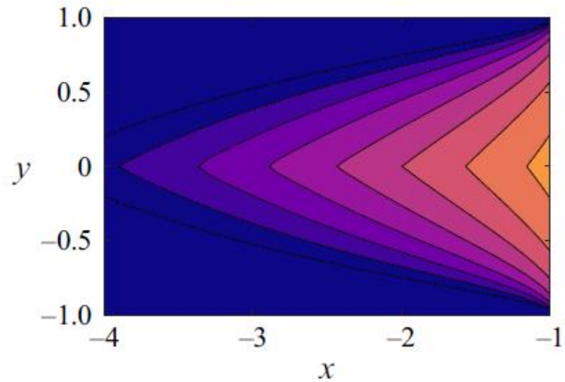


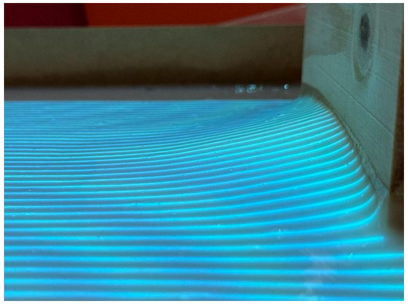
Slow evolution to a final state with fluid upstream of obstacle

Final arrested state

- Flow arrested when shear stresses no longer exceed yield stress. $\sigma_{xz}(z=0)^2 + \sigma_{yz}(z=0)^2 = \tau_y^2$
- Governing equation
$$h^2\left(\frac{1}{\tan\beta}\frac{\partial h}{\partial x} - 1\right)^2 + h^2\left(\frac{1}{\tan\beta}\frac{\partial h}{\partial y}\right)^2 = \left(\frac{\tau_y}{\rho g \sin\beta}\right)^2$$
- Integrate to find h(x,y) *[using Charpit's method]*







- Compute volume of material retained and force exerted on obstacle by retained flow.

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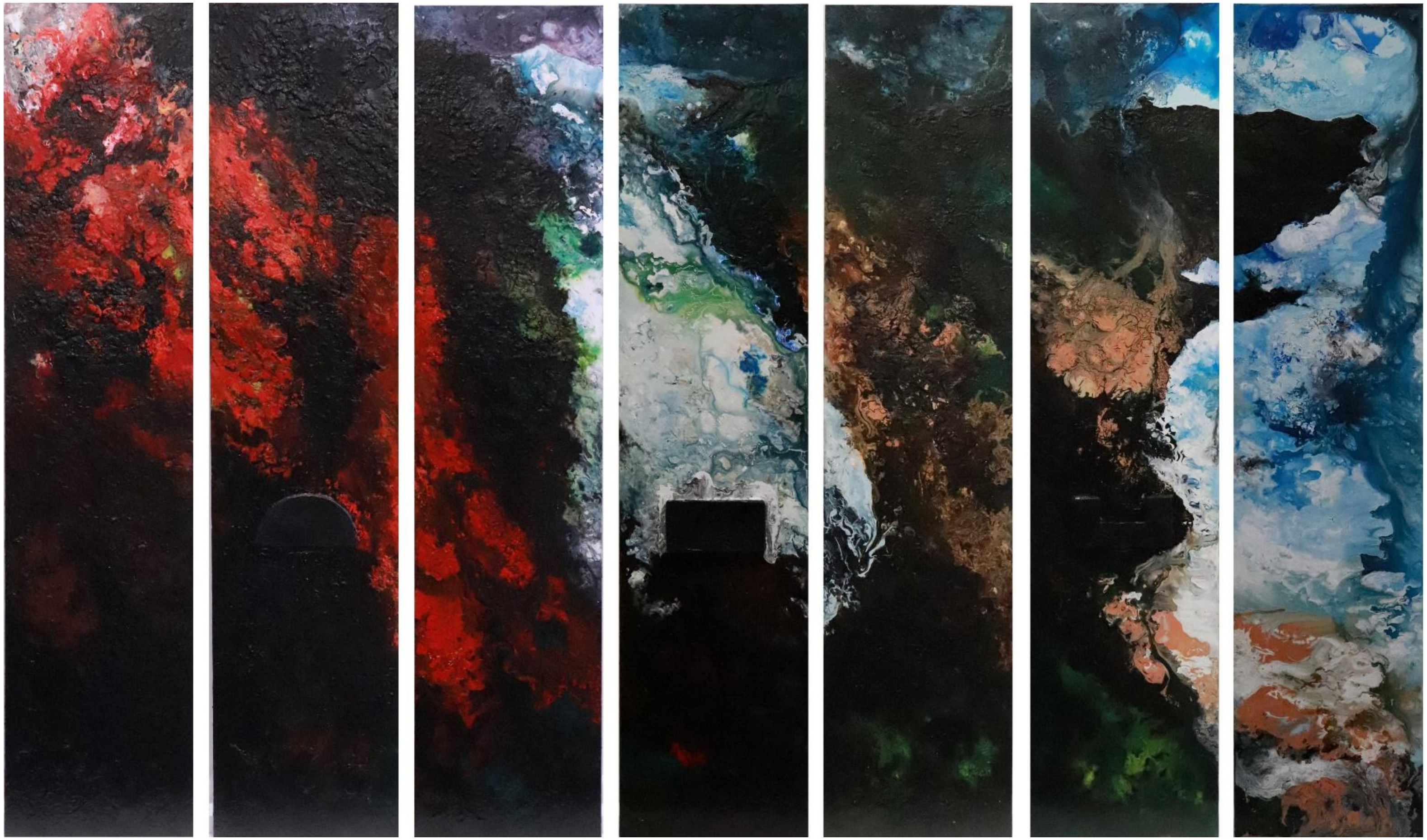
Future challenges

- Compare deflection patterns with natural observations
- Inform engineering design of barriers that defend infrastructure
- Include more realistic rheology, including temperature dependence

Creative reactions

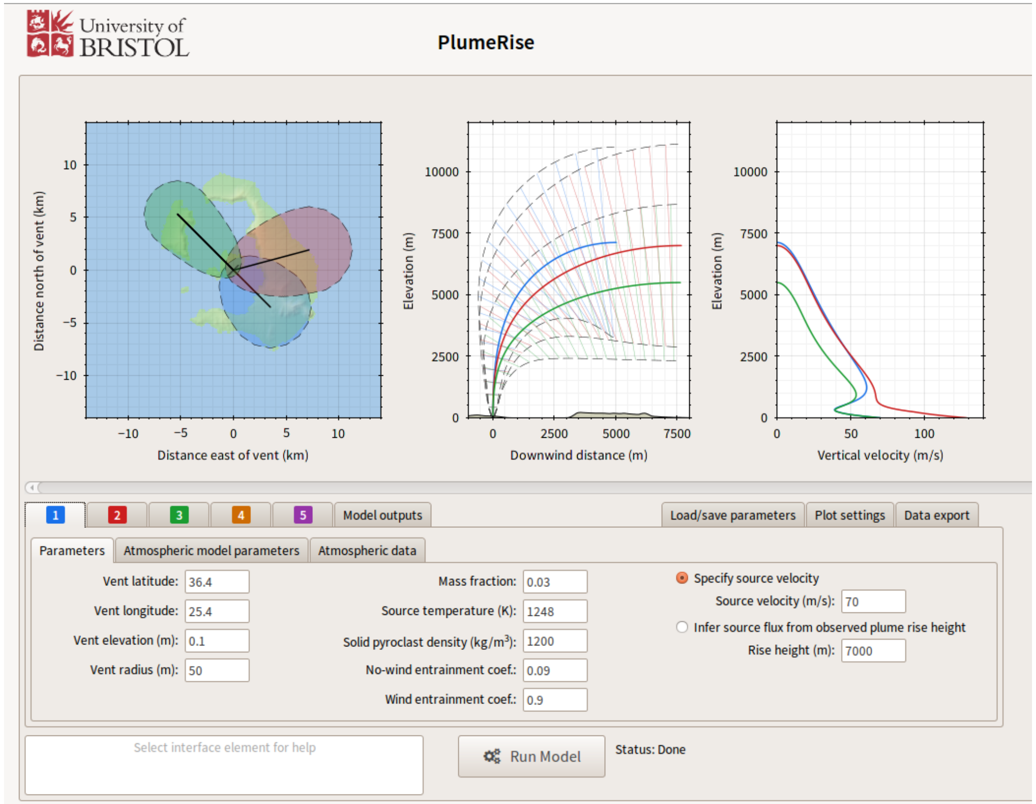


Viscosity, Huw Richard Evans, 2019

Operational models

Dr Mark Woodhouse, School of Earth Science, University of Bristol
Mark.Woodhouse@bristol.ac.uk

Ash Plumes modelling using PlumeRise



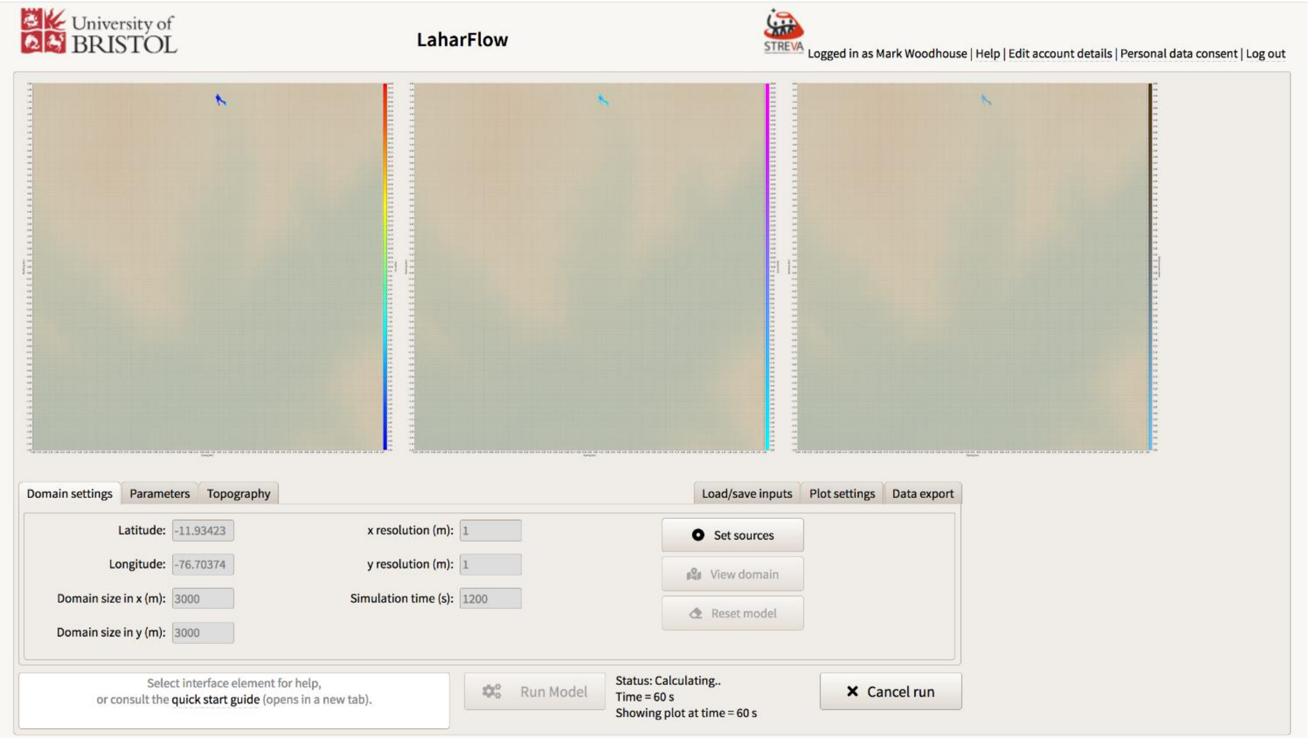
- Launched 2013
- Key users in ash hazard community

✶Met Office (London VAAC)
✶Darwin VAAC
✶Icelandic Met Office
✶Wellington VAAC
✶GNS Science New Zealand
✶BGS

www.plumerise.bris.ac.uk

Lahar modelling using LaharFlow.

LaharFlow solves equations that model the motion of a concentrated mixture of water and sediments, and includes erosion and deposition of solid materials.



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