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M. K. Atatürk

RECENT DEVELOPMENTS IN MODELLING AND IMAGING OF IONOSPHERIC PARAMETERS

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The International Reference Ionosphere and NeQuick
Improving the Representation of the Real-Time Ionosphere
ICTP, Trieste, Italy; 6 -10 October, 2025

OUTLINE

- **INTRODUCTION**
- **RANDOM FIELD MODEL**
- **APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL**
- **IMAGING OF IONOSPHERE IN 2-D**
- **APPLICATION 2- IONOLAB-SMAP**
- **IMPROVEMENT OF THE CLIMATIC MODELS**
- **CONCLUSIONS AND FUTURE DIRECTIONS**

INTRODUCTION (1)

- **Ionosphere is anisotropic, inhomogeneous, spatio-temporally varying and dispersive in both space and time.**
- **The electron density, Ne, is the determining parameter of magnetoplasma.**
- **Ne is a complicated function of**
  - **solar, interplanetary and cosmic forcings from above and**
  - **gravitational, geomagnetic, atmospheric and seismic forcings from below.**
- **Like all geophysical signals, it has climatic cycles depending on**
  - **Sun's rotational and magnetic movement;**
  - **Earth's inclination on its axis and motion with respect to Sun;**
- **and diurnal cycles depending on Earth's rotation around itself.**
- **The forcings from topside and bottomside generate disturbances that are difficult to be represented with deterministic structural physics!**
- **The nature of ionosphere is stochastic with a large dynamic range in both space and time.**

INTRODUCTION (2)

- **In-situ measurement of Ne for all space-time (in 4D-latitude, longitude, height, time) is not possible.**
- **Even with highly sparse rocket, sounding or probe measurements for a limited region and for a short time, it is only possible to define some derived parameter set that can be related to Ne (which brings limitations in empirical modelling relying on measurement data).**
- **Two major sets of data to image Ne are**
  1. **Ionosonde/Ionogram inversions/reconstructions to estimate NmF2 (maximum Ne of F2 layer), foF2 (critical frequency of F2 layer) and virtual hmF2 (virtual maximum ionization height of F2 layer);**
  2. **Tomographic Reconstruction using Total Electron Content (TEC) estimated from GPS-GNSS measurements from the ground and/or on Radio Occultation (RO) satellite receivers.**
- **Both of the above mentioned measurement sets or inversion/reconstruction methods rely on deterministic trend models IRI, IRI-Plas or NeQuick as structural background model or ground truth.**
- **With the measurement data, the background deterministic trend model is ‘modified’ or ‘adapted’ or ‘updated’ to represent near-real time Ne profiles.**

INTRODUCTION (3)

- The most acclaimed models of Ionosphere are International Reference Ionosphere (IRI), International Reference Ionosphere Extended to Plasmasphere (IRI-Plas) and NeQuick which are also listed as International Standard Ionosphere Models.
- All of these models are empirical, climatic, BUT deterministic.
- They all have good accordance with each other for bottomside ionosphere and excellent representation for hourly-monthly midlatitude deterministic trend.
- Yet, they are unable to represent anomalous trend structures such as winter anomaly, midlatitude summer nighttime anomaly.
- Although there are some models to represent secondary field geomagnetic disturbances, these efforts still need time to mature.
- Thus, their use in near-real time applications such as HF communication, mitigation of positioning, guidance and navigational errors for signals of LEO and MEO satellites; and imaging/mapping/remote sensing are highly limited or non-existent.
- The need for a reliable, robust, accurate and cost-efficient ionospheric model for near-real time representation of ionosphere is inevitable!

INTRODUCTION (4)

- **Variation of geophysical signals/data in space and time can be represented by a '<u>Random Field</u>' made up of**
  - **a PRIMARY FIELD: TREND (or EXPECTATION) representing the climatic/seasonal/monthly/diurnal cyclic variability and its anomalies;**
  - **a SECONDARY FIELD representing high-magnitude short duration disturbances/outliers.**
- **Based on the success of IRI, IRI-Plas and NeQuick, the empirical deterministic models should evolve into empirical random field/stochastic models that represent the TREND-PRIMARY FIELD.**
- **High fidelity near-real time imaging of ionospheric parameters will improve the theoretical models, especially for disturbed ionosphere using Artificial Intelligence/Machine Learning algorithms to represent/classify/predict the SECONDARY DISTURBANCE FIELD.**
- **In this talk:**
  - **Introduce Random Field model and show examples on how to obtain**
  - **The trend (primary field): High-fidelity modeling of regional midlatitude ionosphere using GIM-TEC.**
  - **The disturbance (secondary field): IONOLAB-SMAP (High resolution interpolation algorithm for Slant Projected TEC to capture realizations of random field) minus the trend.**

RANDOM FIELD MODEL (1)

- **In Electrical Engineering, a 'signal' is defined as a function of time representing a physical variable that carries information.**
- **It can represent analog or discrete data (any measurement from a sensor such as current, voltage, and temperature).**
- **Some signals are multivariate functions of both space and time. They are named as FIELDS (such as electric field and magnetic field).**
- **In geophysics, the fields can be vectors (such as wind velocity and electric current density in ionosphere) or scalars (such as temperature and electric charge density).**
- **No REAL SIGNAL or MEASUREMENT is static or deterministic! DYNAMIC/STOCHASTIC**
- **Probability Theory; *Outcomes* of Experiments; Random Variable; Probability Density Function (PDF)**
- **Stochastic/Random Process involves a collection of random variables indexed by some set.**
- **It can be considered to be a family of time functions where each *realization* of the process is a function of time. For fixed time, it reduces to a random variable.**

RANDOM FIELD MODEL (2)

- **A Random Field represents random processes in both space and time.**
- **Random Field Theory seeks to model complex patterns of variation and interdependence in cases where deterministic treatment is inefficient and conventional statistics are insufficient.**
- **An ideal random field model can capture the essential features of a complex random phenomenon in terms of a minimum number of physically meaningful and experimentally accessible parameters.**
- **A space-time process is characterized by active and inherent (or intrinsic) uncertainty similar to those in decision systems of radar signal processing or biomedical imaging:**
  - **Properties at different points in space change randomly with time.**
  - **Measurements may be taken at selected points in time and space, or continually in time at specific locations.**
  - **The measurements may be used for forecasting the future values of the process at given locations and times, or the values of system-wide averages or measures of performance.**
- **Distributed disordered system (dds)**

RANDOM FIELD MODEL (3)

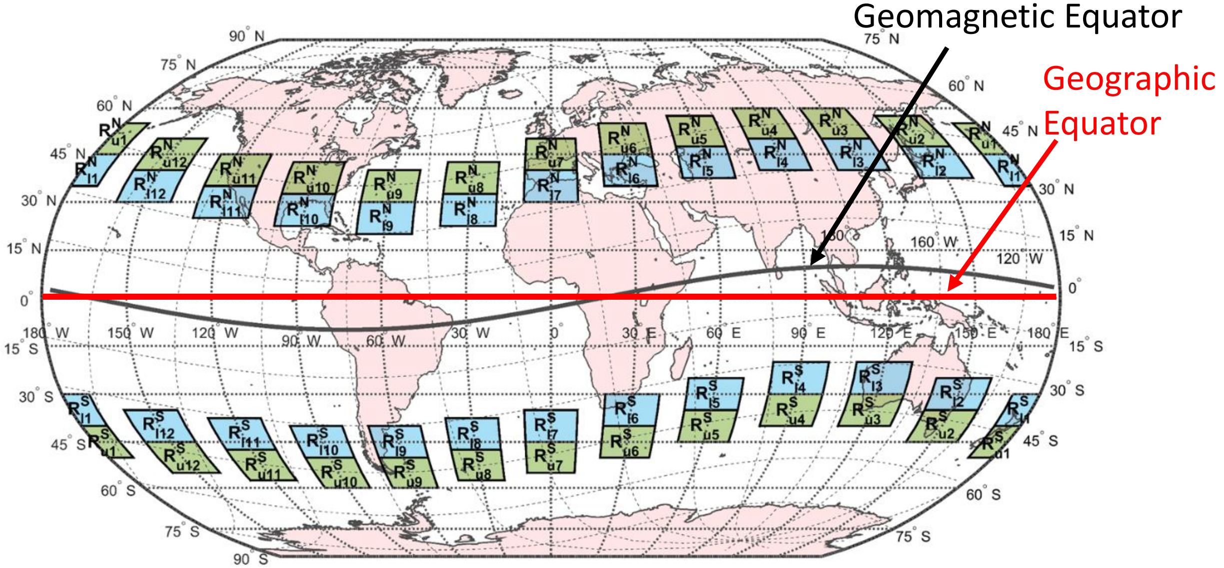
- **Proposed <u>additive</u> Random Field model for Ionospheric Parameters**
- $$Z(r,\theta,\phi;t) = \mu(r,\theta,\phi;t) + \nu(r,\theta,\phi;t)$$
- $Z(r,\theta,\phi;t)$ **denotes random field which is a function of height, *r*, latitude θ, longitude ϕ, and time *t*, such as Ne, foF2, hmF2, and TEC.**
- $\mu(r,\theta,\phi;t)$ **represents the <u>trend</u> part of the random field which includes the climatic and cyclic space-time variability plus trend anomalies.**
- $\nu(r,\theta,\phi;t)$ **represents the short duration/high amplitude <u>secondary</u> random field in space and time due to disturbances/forcings (outliers).**

APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL (1)

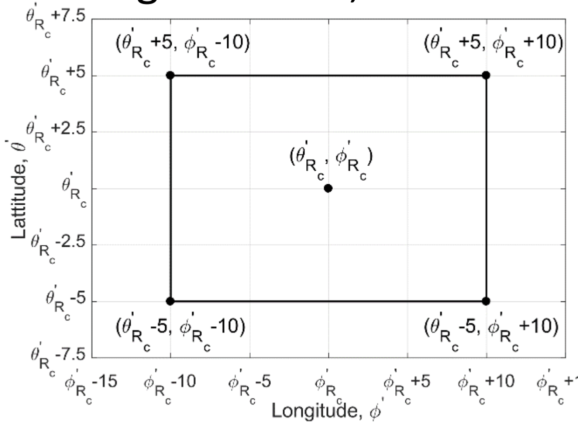
- **Midlatitude Ionosphere extends between 30^0 and 50^0 North and South of Geomagnetic Equator.**
- **48 Distinct Midlatitude Regions, each has an extend of 10^0 in latitude and 20^0 in longitude.**
- **Resolution is 2.5^0 in latitude and 5^0 in longitude.**
- **5 x 5 = 25 grid points, center longitude is placed at every 30^0 starting with 0^0 meridian which corresponds to center of R$_7$.**
- **In order to compare coefficients from one region to another, each region is normalized to set the center coordinate to (0,0). The normalized coordinates are denoted by primes.**
- **Proposed Midlatitude Planar Trend Model in normalized coordinates**
- $\boxed{\boldsymbol{\mu_R\,(\theta',\phi';t) = a_{\mu;R}(t) + a_{\theta;R}(t)\theta' + a_{\phi;R}(t)\phi'}}$

where $\boldsymbol{a_{\mu;R}(t)}$ is the base level; $\boldsymbol{a_{\theta;R}(t)}$ is the slope in latitude; $\boldsymbol{a_{\phi;R}(t)}$ is the slope in longitude.

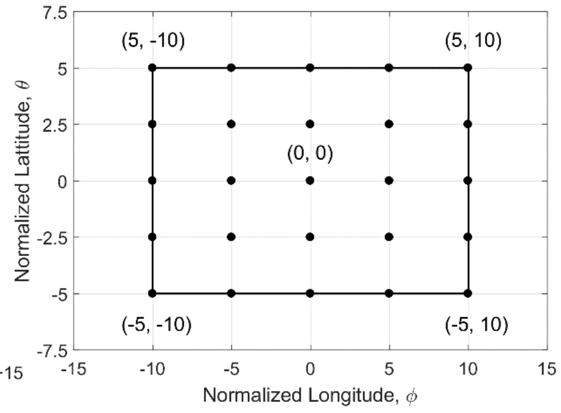
- **The coefficients have to be estimated in space (for each region) and in desired time resolutions.**
- **The model is analog which means for any coordinate and time, the ionospheric parameter can be obtained.**
- **One region, one year, one month, one day and one time interval is represented with only <u>three (3)</u> coefficients!**



Region N or S; l or u



Region in normalized coordinates



APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL (2)

- Total Electron Content (TEC): Line integral of electron density on a ray path

$$\mathrm{TEC} = \int \mathrm{N_e dl} \;\; (\mathrm{TECU}) \qquad 1\ \mathrm{TECU} = 10^{16}\ \mathrm{el/m^2}$$

- The regions are chosen to cover all midlatitude, use Global Ionospheric Maps (GIM) as source of TEC;
- Every GIM-TEC map denotes one realization of discrete random field: $g_R(\theta', \phi'; t)$.
- Time resolution of GIM is 2 h.
- The sampling or model matrix for the normalized coordinate map is
- $M_R = \begin{bmatrix} 1 & \theta'_1 & \phi'_1 \\ \vdots & \vdots & \vdots \\ 1 & \theta'_{25} & \phi'_{25} \end{bmatrix}$

$$\hat{a}_R(n_t) = [\hat{a}_{\mu;R}(n_t) \quad \hat{a}_{\theta;R}(n_t) \quad \hat{a}_{\phi;R}(n_t)]^T$$

vectorized estimated coefficient set

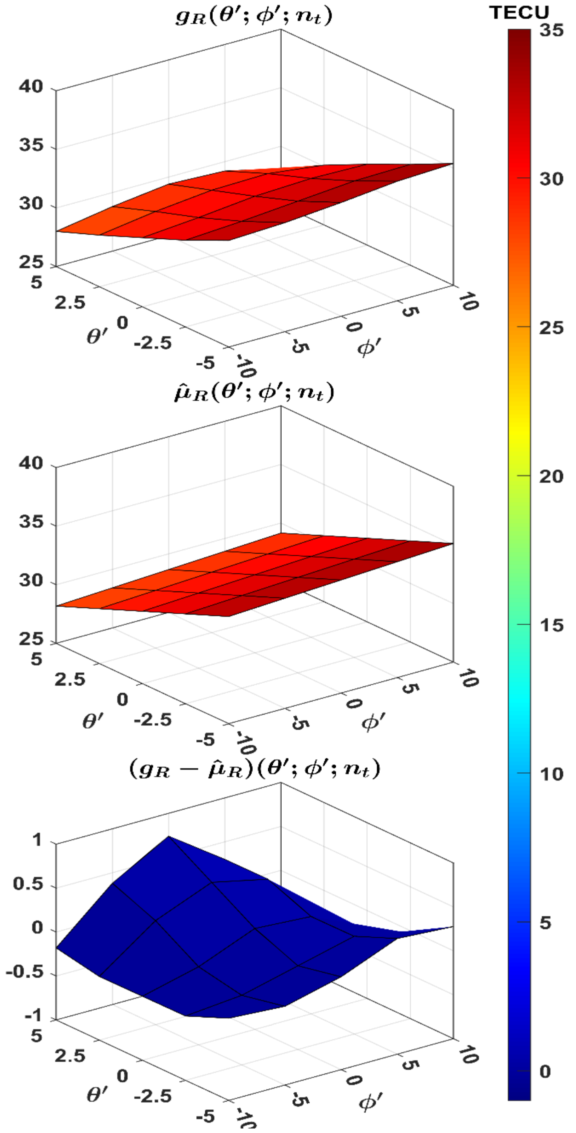
$$g_R(n_t) = [g_R(\theta'_1, \phi'_1; n_t) \quad \cdots \quad g(\theta'_{25}, \phi'_{25}; n_t)]^T$$

vectorized GIM values at each normalized coordinate

- In order to estimate the coefficients of the random field model, we used Least Squares Optimization
- The coefficients are estimated in CLOSED FORM in the LEAST SQUARES SENSE as

$$\boxed{\hat{a}_R(n_t) = (M_R^T M_R)^{-1} M_R^T\, g_R(n_t)}$$

APPLICATION 1-
MIDLATITUDE PLANAR TREND MODEL (3)



One realization of random field $Z(\theta',\phi';t)$
For one region and one time instant
from the GIM

$$g_R(\theta',\phi';n_t)$$

Estimated trend
For one region and one time instant in normalized coordinates

$$\hat{\mu}(\theta',\phi';n_t)$$

Residual or
estimate of secondary random field
For one region and one time instant

$$\hat{\nu}(\theta',\phi';n_t)=g_R(\theta',\phi';n_t)-\hat{\mu}(\theta',\phi';n_t)$$

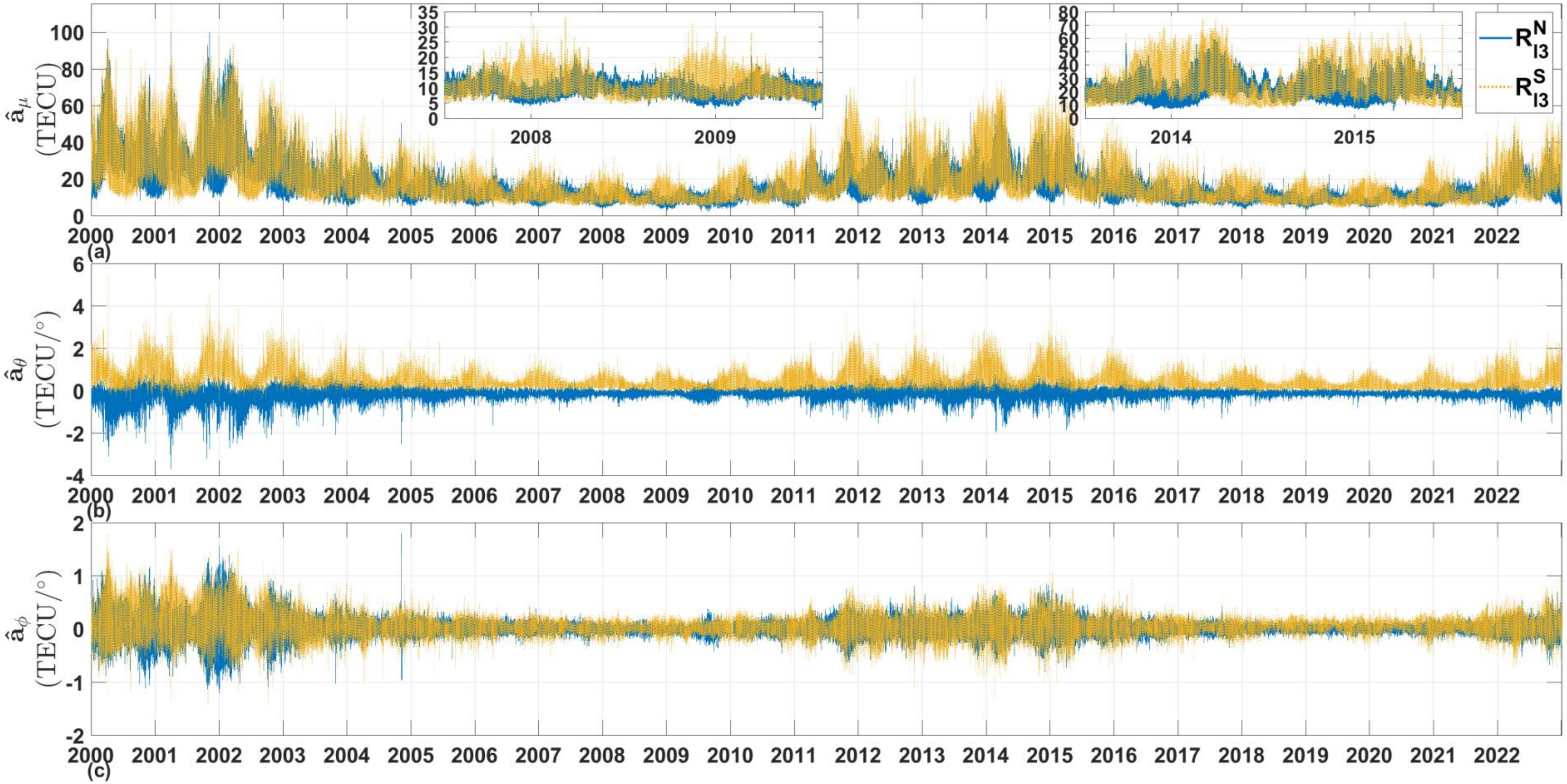
- $\hat{a}_{\mu;R}$, $\hat{a}_{\theta;R}$ and $\hat{a}_{\phi;R}$ are extracted in the LS sense
  - ✓ at every 2 hours
  - ✓ for 365 (or 366) days of
  - ✓ 25 years

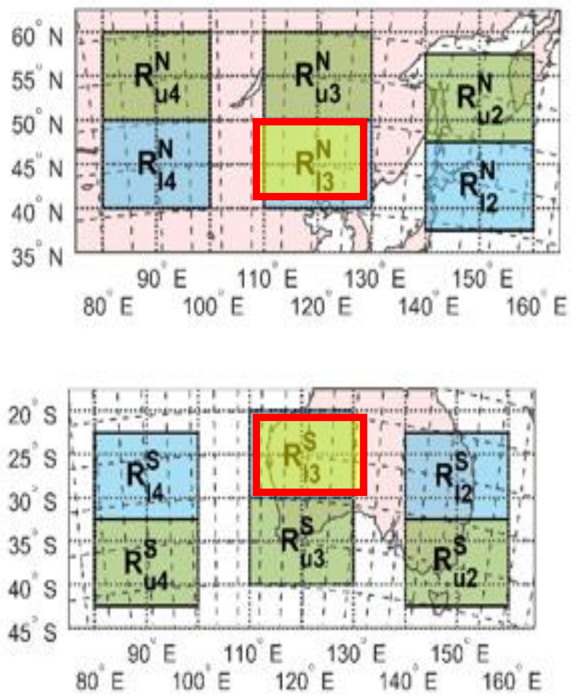
  (from Jan 1, 2000 at 00:00 UT to Dec 31, 2024 at 22:00 UT)
  - ✓ for ALL 48 midlatitude regions.
- The coefficients are open to public at
- https://www.ee.hacettepe.edu.tr/~solen/_PlanarTrendModel in ordered Excel files.

- SK Yildiz, O Arikan, F Arikan, Planar trend model for the midlatitude ionosphere, Advances in Space Research, 2025a.

APPLICATION 1-
MIDLATITUDE PLANAR TREND MODEL (4)

The estimated coefficients of R_{l3}^{N} (blue) and R_{l3}^{S} (dashed yellow) for 23 years at 2 h resolution. Center of the region: 120^0 E

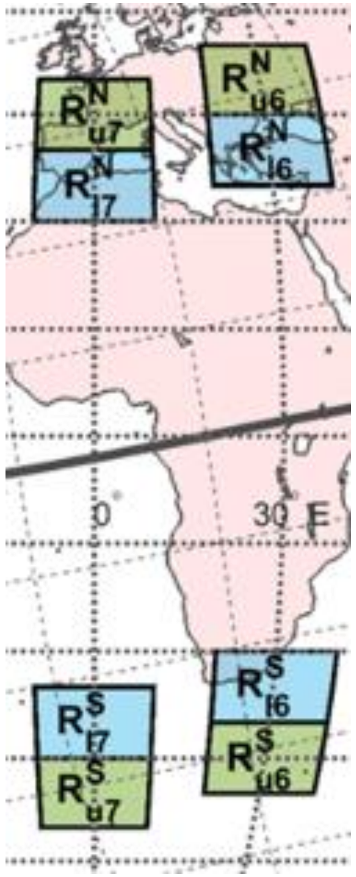




APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL (5)

- $\hat{a}_{\mu;R}(n_t)$ of $\mathbf{R^N_{l6}}$ and $\mathbf{R^S_{l6}}$; $\mathbf{R^N_{u6}}$ and $\mathbf{R^S_{u6}}$; $\mathbf{R^N_{l7}}$ and $\mathbf{R^S_{l7}}$; $\mathbf{R^N_{u7}}$ and $\mathbf{R^S_{u7}}$

for 25 years at 2 h resolution. Blue is north and dashed yellow is south.

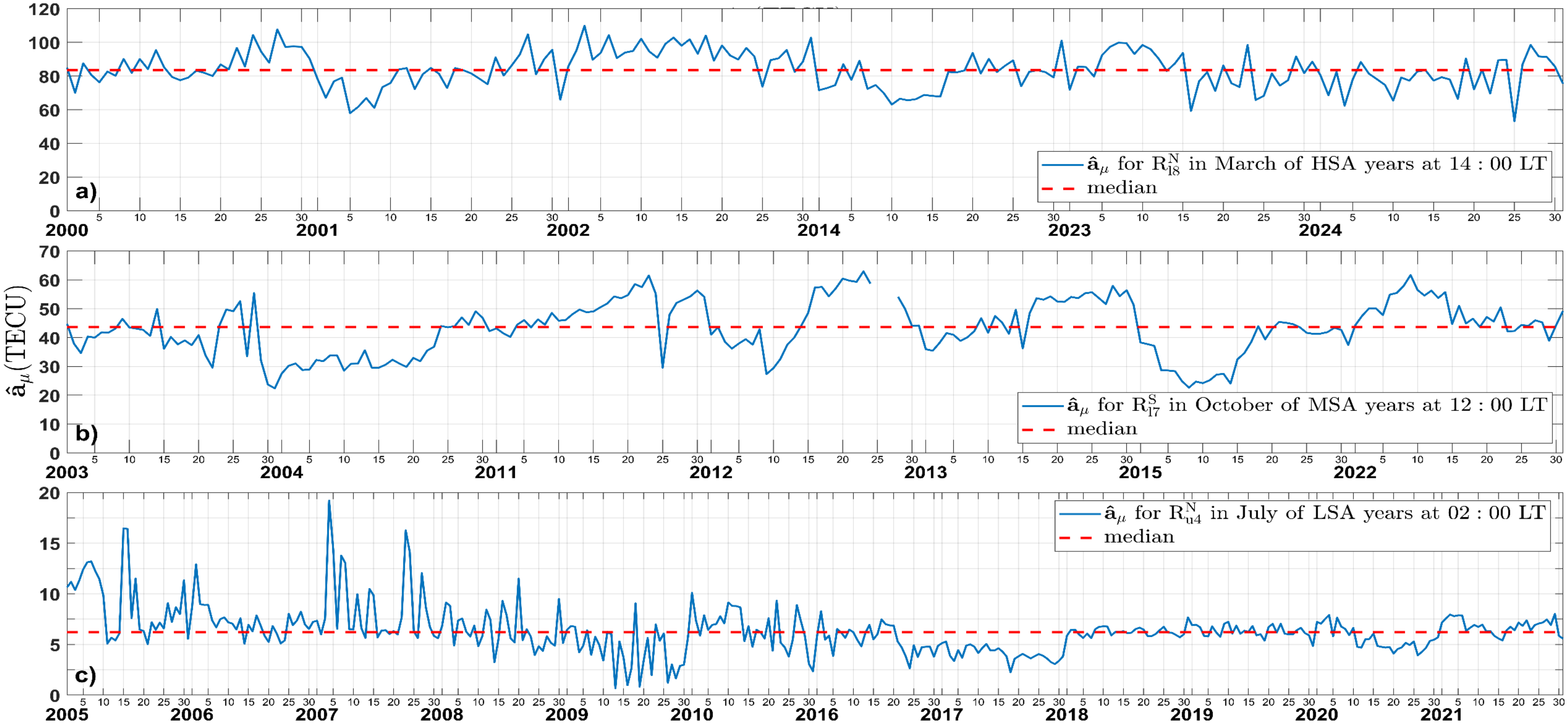


APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL (6)

- **The validity and accuracy of the model coefficients are tested using Square of the Normalized Metric Distance (SNMD) and Symmetric Kullback-Leibler Distance (SKLD).**
- **The coefficient of determination between the SNMD and SKLD reveals a fit of 99.995 % to the planar model (trend); deviations occur only on severe geomagnetic disturbance days and hours (outlier).**
- **Unlike previous region-specific models proposed in the literature with high computational complexity and problems in duplicability, the whole midlatitude region is covered in our model directly through region, date and time specific coefficients.**
- **In order to observe the trend with respect to solar activity, we obtained the medians of monthly-two hourly coefficients, for**
  - **each region**
  - **each Solar Activity (SA) group**
  - **each month**
  - **every two hours**
- **Open to public at https://www.ee.hacettepe.edu.tr/~solen/_AsymmetryAnalysis/MedianMaps**

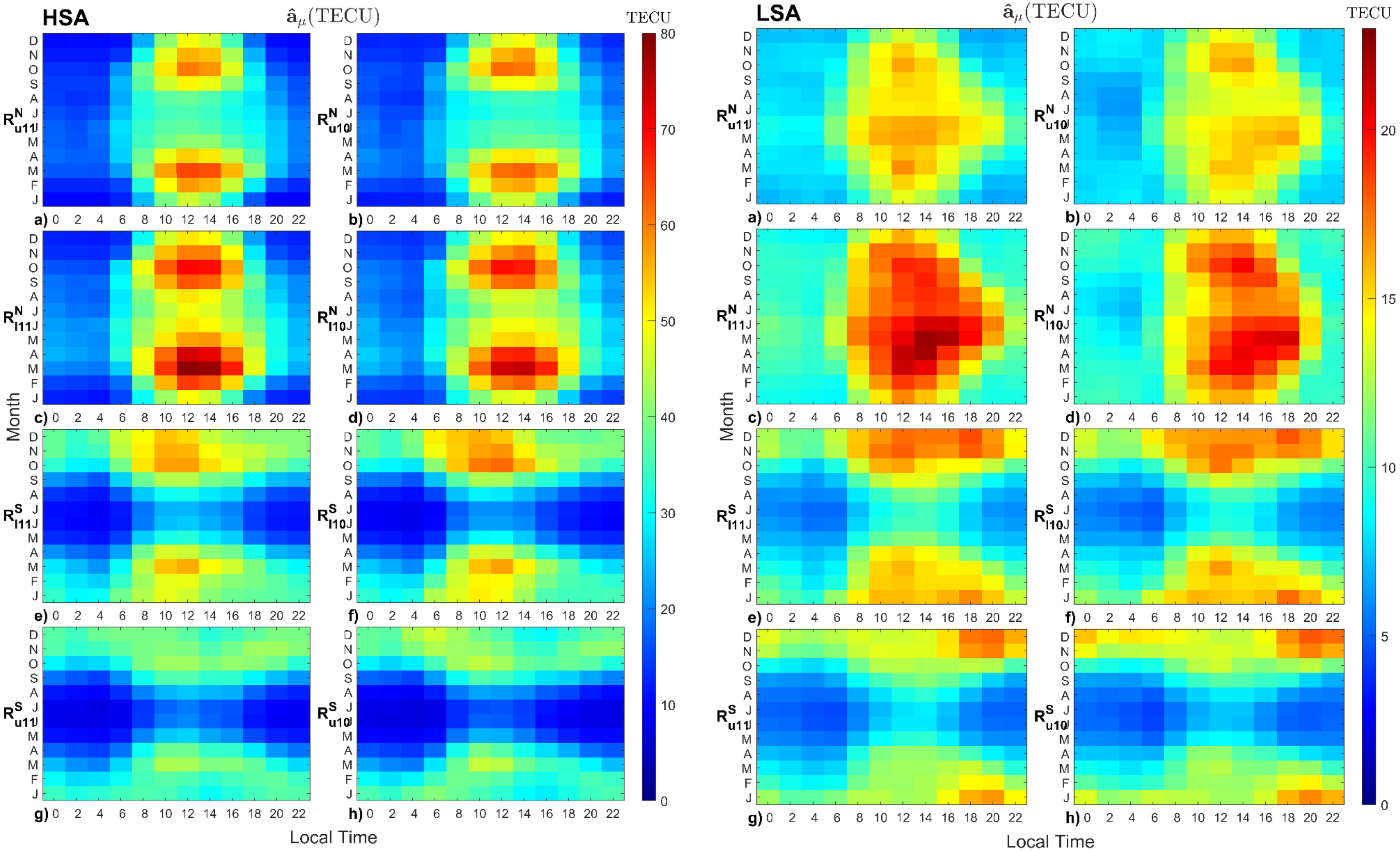
APPLICATION 1- MIDLATITUDE PLANAR TREND MODEL (7)

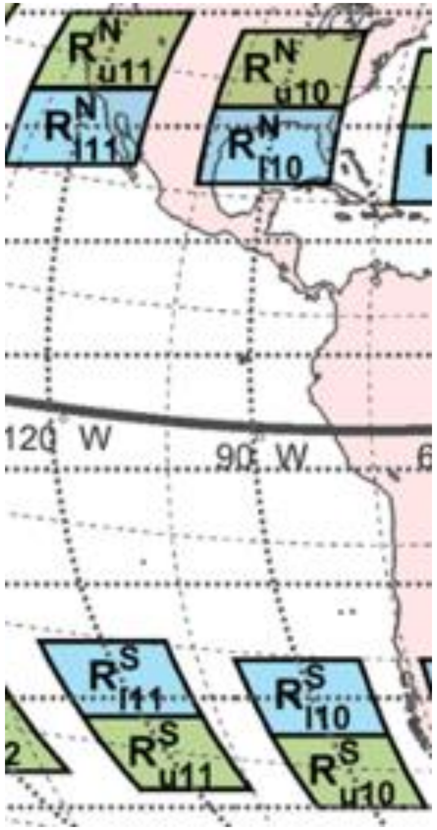
Median examples of various regions, solar activity, month and hours:



APPLICATION 1-
MIDLATITUDE PLANAR TREND MODEL (8)

- **Median heat maps for all regions and solar activity levels: https://www.ee.hacettepe.edu.tr/~solen/_AsymmetryAnalysis/MedianMaps**





APPLICATION 1-
MIDLATITUDE PLANAR TREND MODEL (9)

- **All datasets and visualizations generated in this study are publicly available.**
- **Estimated model coefficients of the locally planar TEC trend model from 2000 to 2024 for all 48 midlatitude regions, shared as Excel files: https://www.ee.hacettepe.edu.tr/~solen/_PlanarTrendModel**
- **2-hourly and monthly median TEC maps for each region, provided as high-resolution figures: https://www.ee.hacettepe.edu.tr/~solen/_AsymmetryAnalysis/MedianMaps**
- **Metric comparison figures (RMSD, DM, and SKLD) for all north–south conjugate and upper–lower band region pairs under low, moderate, and high solar activity conditions: https://www.ee.hacettepe.edu.tr/~solen/_AsymmetryAnalysis/North-SouthComparison**
- **Comprehensive east–west comparison figures across all regions and solar activity levels, made available as high-resolution figures for each region: https://www.ee.hacettepe.edu.tr/~solen/_AsymmetryAnalysis/East-WestComparison**
- **Kumbay Yildiz, S., Arikan, O., Arikan, F., Comprehensive Investigation of Hemispheric and Longitudinal Asymmetries in Midlatitude TEC, Accepted for publication in ASR, September 2025b.**

IMAGING OF IONOSPHERE IN 2-D

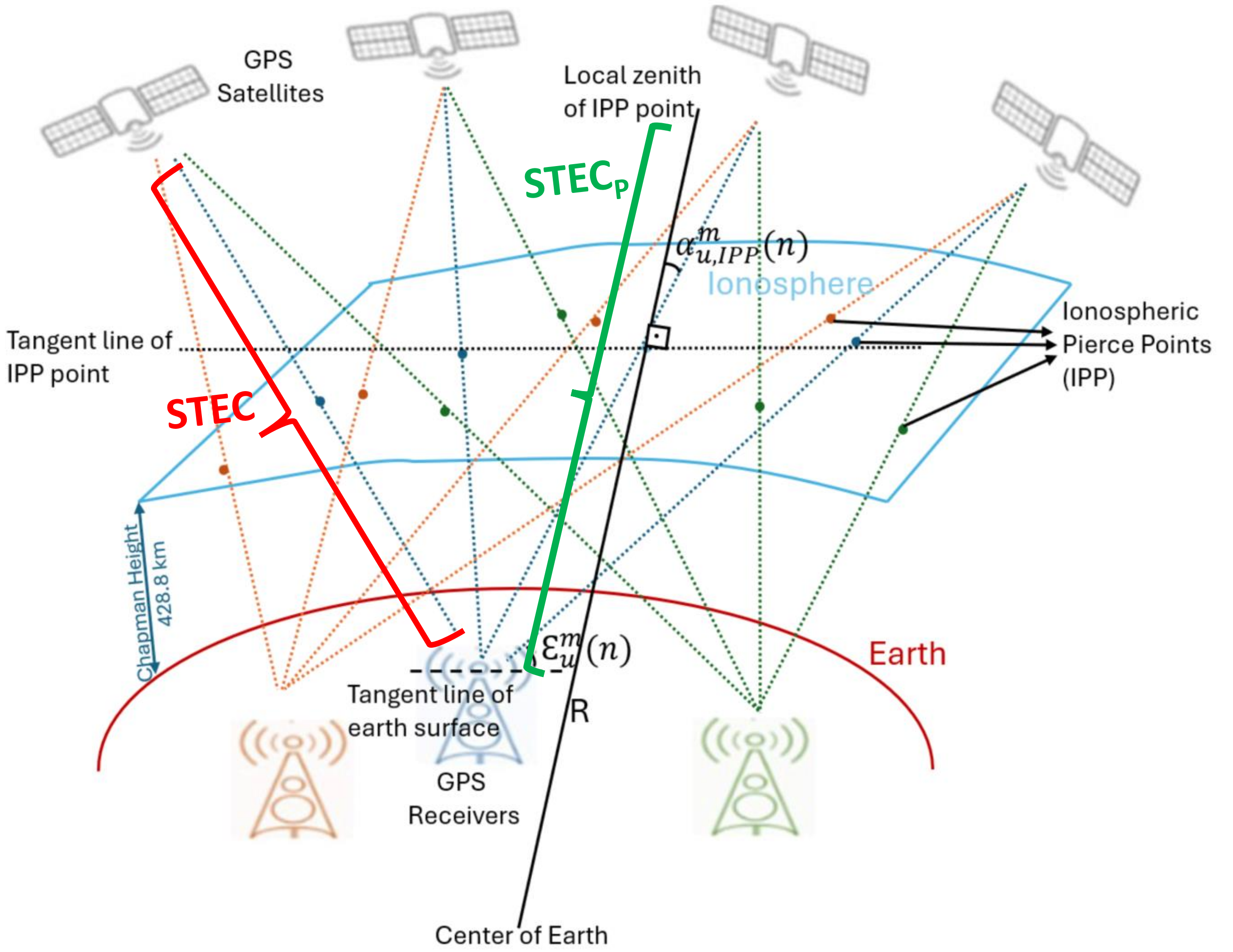
- Accurate Imaging of ionospheric parameters in
  - ✓time (1-D): IONOLAB-TEC; IONOLAB-STEC
  - ✓latitude and longitude (2-D/Mapping): IONOLAB-MAP; IONOLAB-SVD; IONOLAB-CoK
  - ✓3-D and 4-D as height, latitude, longitude and time: IONOLAB-CIT; IONOLAB-FUSION

using reliable, robust and cost-efficient algorithms is extremely important.

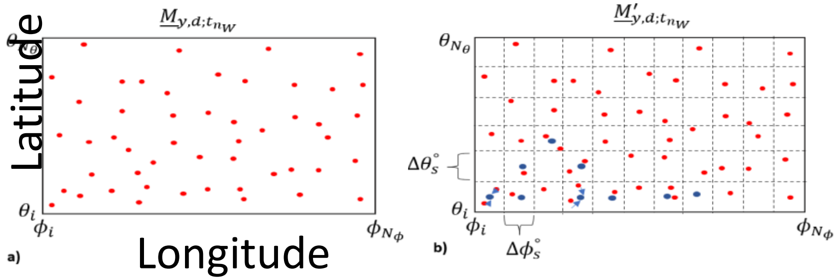
- The mapping algorithm in latitude, longitude and near-real time: IONOLAB-SMAP
- Developed to interpolate Slant Total Electron Content (STEC) which inherently includes anisotropicity, inhomogeneity, and spatio-temporal variability of ionosphere.
- IONOLAB-SMAP is a novel algorithm to map Projected STEC ($STEC_p$) automatically in the shortest wide-sense stationarity period of ionosphere (5 min).

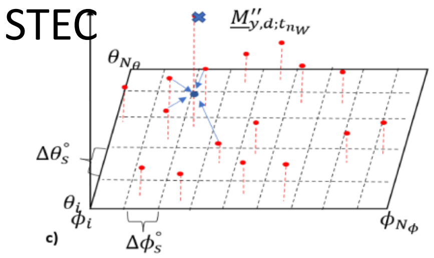
APPLICATION 2- IONOLAB-SMAP (1)

- **<u>Slant TEC (STEC)</u>: corresponds to the total number of electrons in a cylinder of 1 m^2 cross-section that joins the GPS receiver and the GPS satellite.**
- **STEC is estimated on each GPS satellite-receiver path (m,u) at each observation epoch n.**
- **Single Layer Ionospheric Model (SLIM) at Chapman Height of 428.8 km.**
- **STEC$_p$ is the projected STEC to the local zenith of Ionospheric Pierce Point (IPP).**



APPLICATION 2- IONOLAB-SMAP (2)





Red dots: projected STEC ($STEC_p$)
Blue dots: spatio-temporal filtered data
Blue cross: outlier data

- **Preprocess Stage: STEC estimation and spatio-temporal filtering in 5 min**

Daily RINEX files of each receiver

Daily IONEX and satellite ephemeris files

For each receiver and satellite in line of sight

1. Receiver Bias Estimation with IONOLAB-BIAS
2. STEC estimation with IONOLAB-STEC
3. Elimination of STEC values that have elevation angles lower than 40°
4. Projection of STEC values to the local zenith of related IPP locations ($STEC^m_{p;yd;u}$)
5. Conversion ECEF IPP coordinates to spherical coordinates
6. Temporal filtering to remove outlier data from projected STEC values for each station-satellite pair ($W_{y;d;u}$)
7. Spatial filtering to have uniform distributed projected STEC values ($\underline{M}_{y;d;t_{nw}}$)
8. Vectorization of median projected STEC values and corresponding IPP coordinates for each satellite and receiver pair

Projected STEC array of given region $\underline{M}''_{y;d;t_{nw}}$

- **Mapping Stage:**

IONOLAB-MAP

Projected STEC array of given region $\underline{M}''_{y;d;t_{nw}}$

For each map over the last 5 minutes

1. Interpolated map generation with first OK over STEC-IPP vector sets $\hat{Z}_{y;d;t_{nw}}$
2. New grid points specification to remove the outliers on the outer frame of interpolated maps
3. Sampling of the maps according to the determined new grid points $\underline{M}'''_{y;d;t_{nw}}$
4. Interpolation of the sampled values using second OK $\hat{Z}'_{y;d;t_{nw}}$

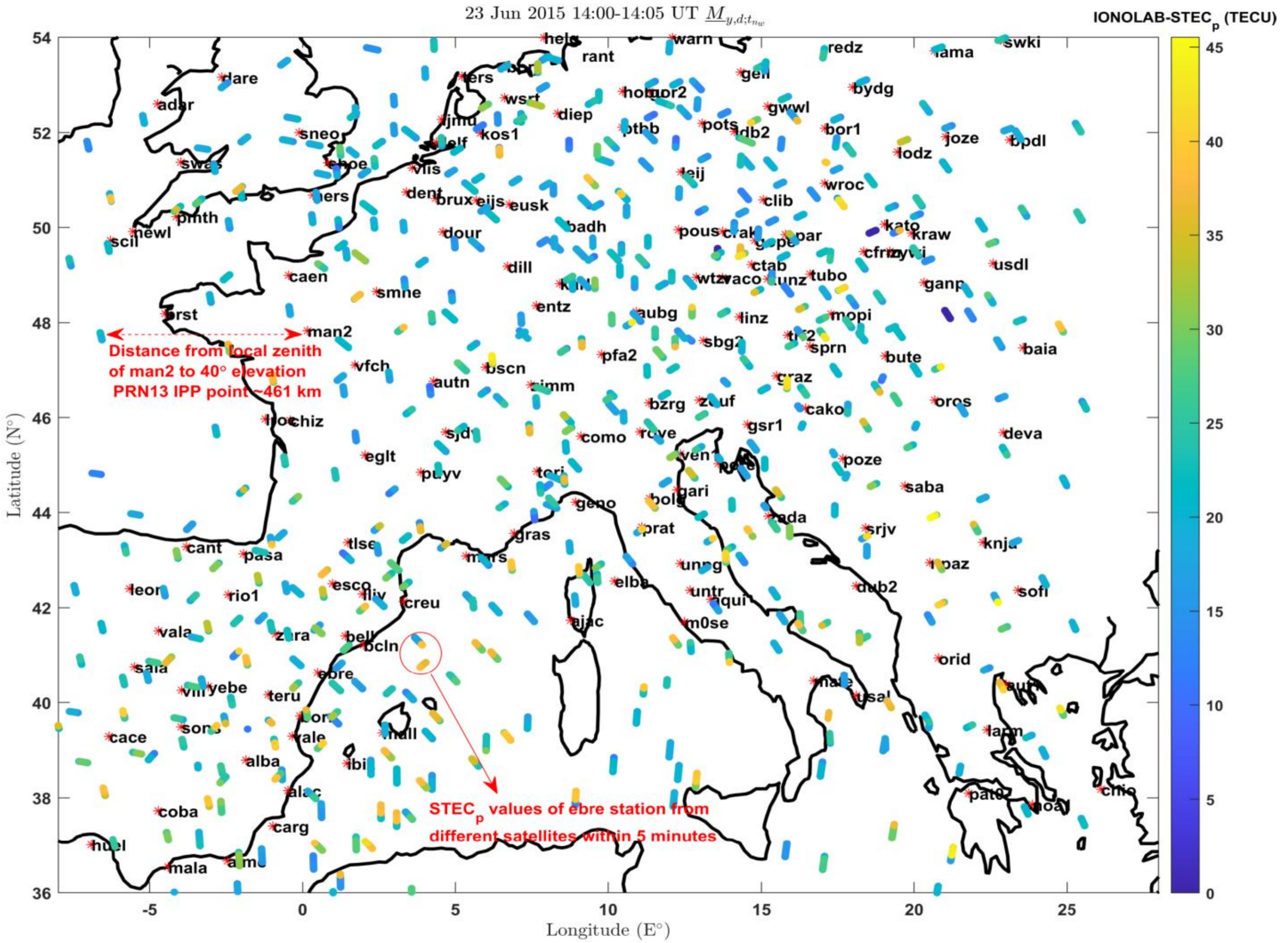
Calculation of the median map and its variance from interpolated maps at three different times $\hat{f}_{y;d;t_{nw3}}$

- **Repeat for every 5 min duration**
- **First Automatic Gridding (Rough)**
- **Apply first Ordinary Kriging (OK)**
- **Use Particle Swarm Optimization (PSO) over Matern Family Theoretical Semivariogram**
- **Second Automatic Gridding (Fine)**
- **Apply second OK**
- **Use PSO over Matern Family Semivariogram**

APPLICATION 2- IONOLAB-SMAP (3)

- STEC is computed using IONOLAB-TEC employing IONOLAB-BIAS with 30 s resolution (www.ionolab.org)
- STEC is projected to local zenith direction to obtain $STEC_p$.
- 163 IGS and EUREF receivers across Europe;
- On the average, there are four satellites visible in different directions for a receiver;
- Accumulate data for 5 min that corresponds to 10 data samples;
- 10 x 4 x 163 = 6,520 $STEC_p$ estimates.
- Apply 40° elevation mask to reduce multipath and phase lock problems.

A. Apply Preprocessing
B. Apply IONOLAB-MAP



- 23 June 2015: Geomagnetic Disturbance; 14:00 to 14:05 UT $STEC_p$ values before preprocessing

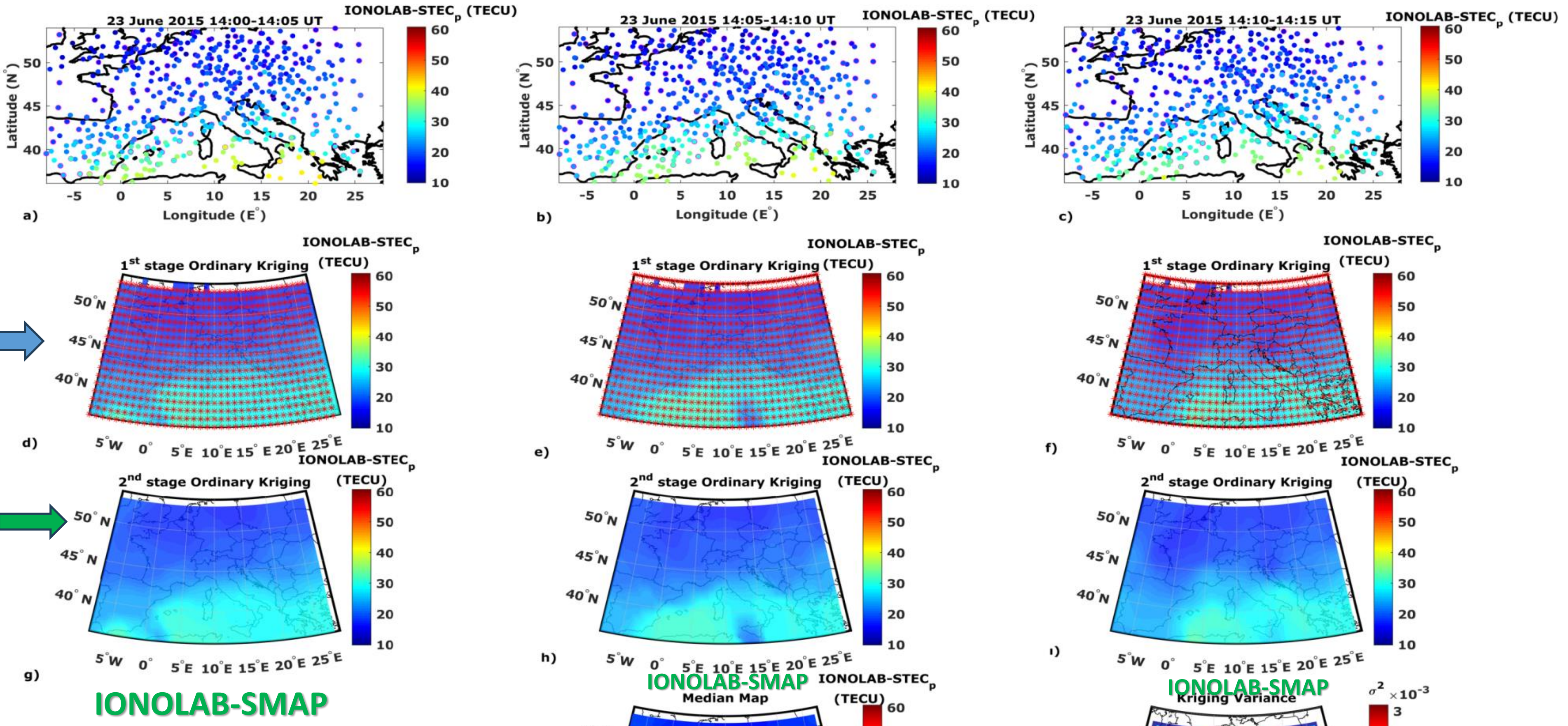
APPLICATION 2- IONOLAB-SMAP (4)

- **23 June 2015: Geomagnetic Disturbance**
- **14:00 to 14:05 UT** **14:05 to 14:10 UT** **14:10 to 14:15 UT**

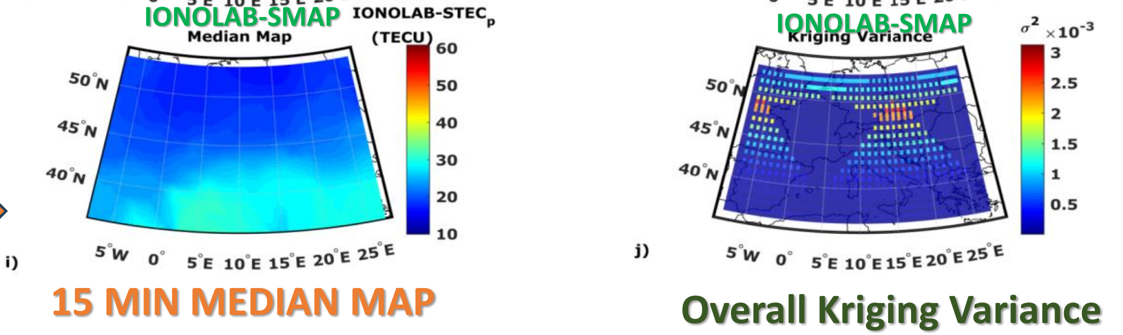
Preprocessed STEC$_p$ data
For all receivers and all satellites during 5 min interval

Preprocessed STEC$_p$ data distributed over Rough Grid
Application of first OK with Matern Semivariogram optimized using PSO (Resolution: 1^0 both in latitude and longitude)

Interpolated STEC$_p$ data on Fine Grid: IONOLAB-SMAP
Application of second OK with Matern Semivariogram optimized using PSO (Resolution: 0.48^0 both in latitude and longitude)



Median STEC$_p$ Map on Fine Grid for 14:00 – 14:15 UT
(Resolution: 0.48^0 both in latitude and longitude)



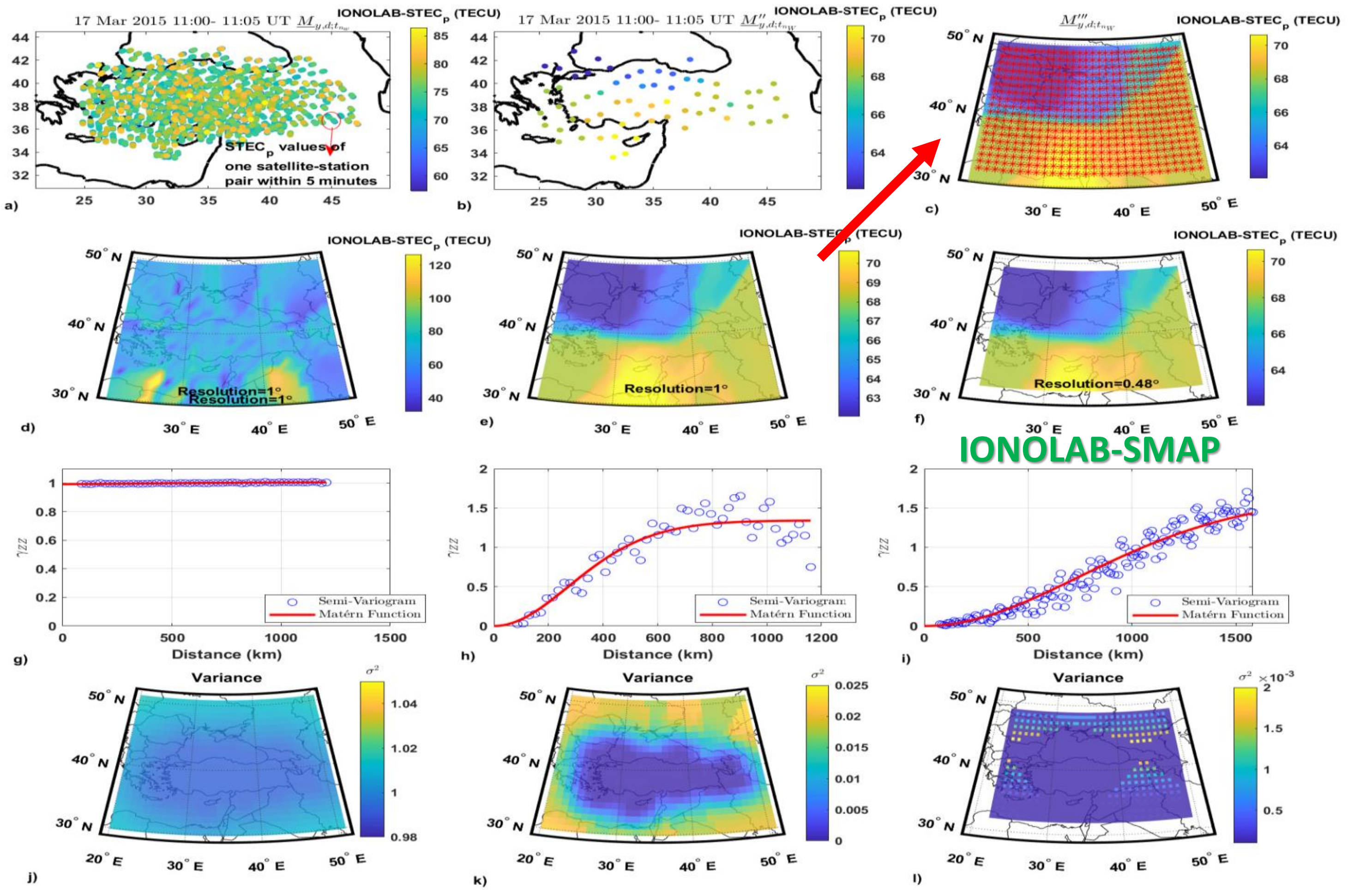
APPLICATION 2- IONOLAB-SMAP (5)

- Turkish National Permanent GPS Network (TNPGN-Active)
- 146 GPS stations
- Four visible satellites on the average
- 10 x 4 x 146 = 5,840 STEC estimates
- 17 March 2015: Geomagnetic Disturbance (St. Patrick's Day Storm) 11:00 to 11:05 UT
- Experimental Semivariogram: blue circles
- Theoretical Semivariogram: red curve
- Matern Function Family fit using Particle Swarm Optimization (PSO)
- Overall variance reduces 1000 fold
- Overall resolution is 0.48^0
- The interpolated map demonstrates both the local noon trend and the unusually high TECU values over the Black Sea due to the geomagnetic storm.

If all the data within 5 min are taken as they are and to be mapped using with one step OK

If preprocessed data within 5 min are taken to be mapped using with one step OK

If rough grid data within 5 min are taken to be mapped using second step OK



APPLICATION 2- IONOLAB-SMAP (6)

JPL-TEC and IONOLAB-SMAP Comparison

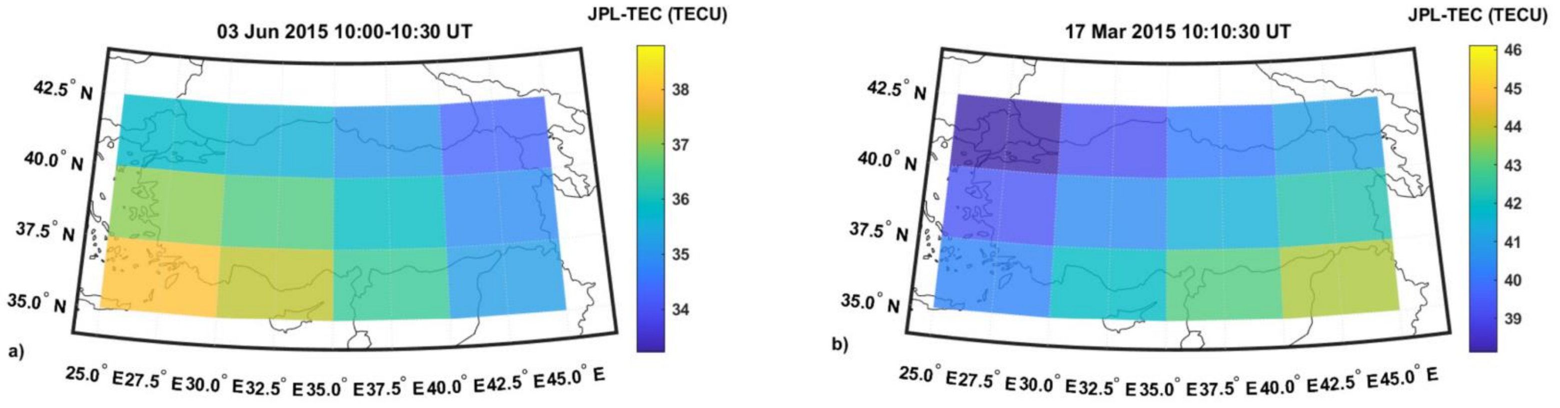
JPL-TEC

Spatial Resolution

2.5° x 5°

Update Rate

2 h



Quiet Day

SKLD:0.013 and NRMSE:0.12

Geomagnetic Storm Day

SKLD:0.01 and NRMSE:0.38

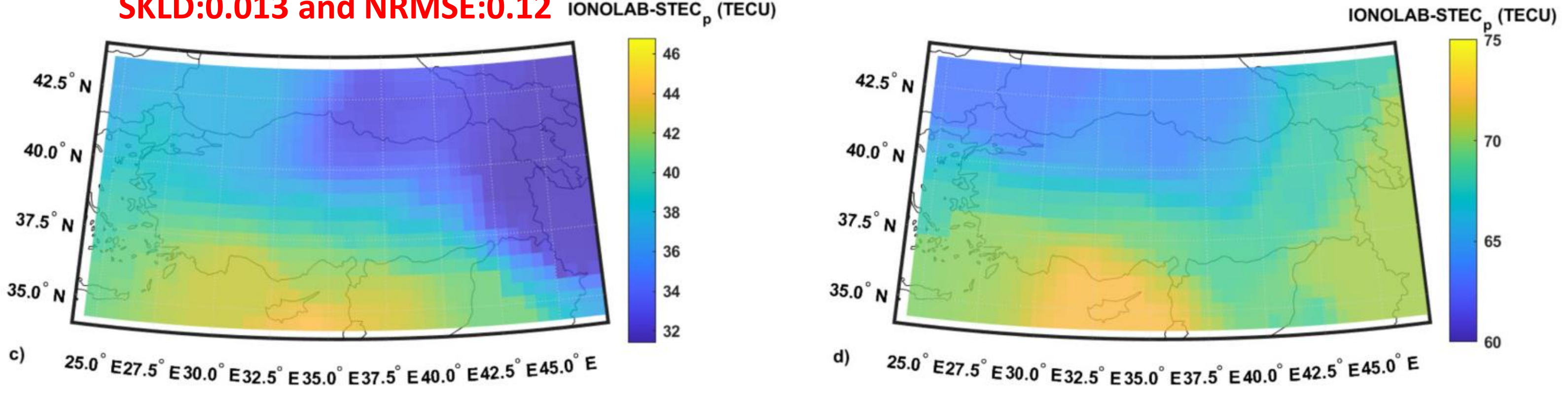
IONOLAB-SMAP

Spatial Resolution

0.48° x 0.48°

Update Rate

5 min



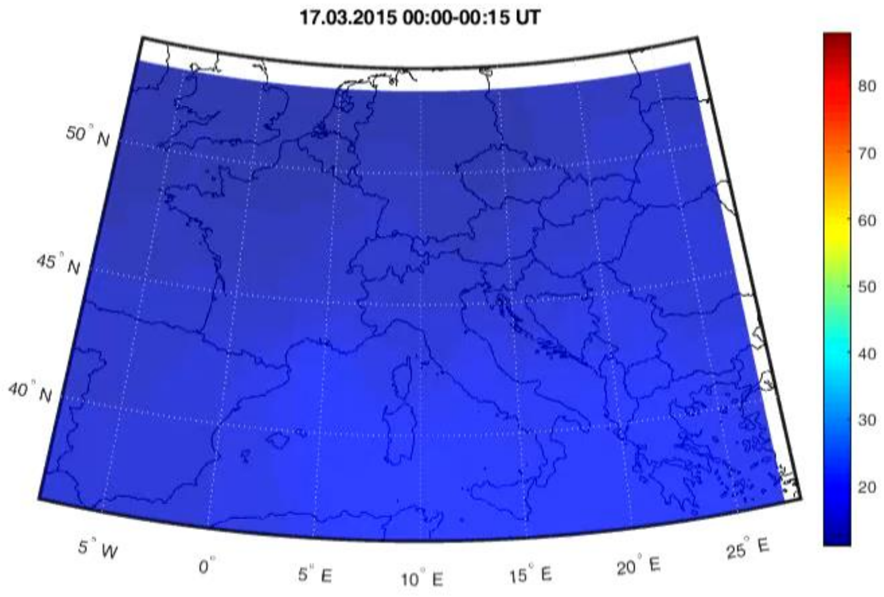
Jet Propulsion Laboratories (**JPL**), NASA (ftps://gdc.cddis.eosdis.nasa.gov/pub/gps/products/ionex/)

APPLICATION 2- IONOLAB-SMAP (7)

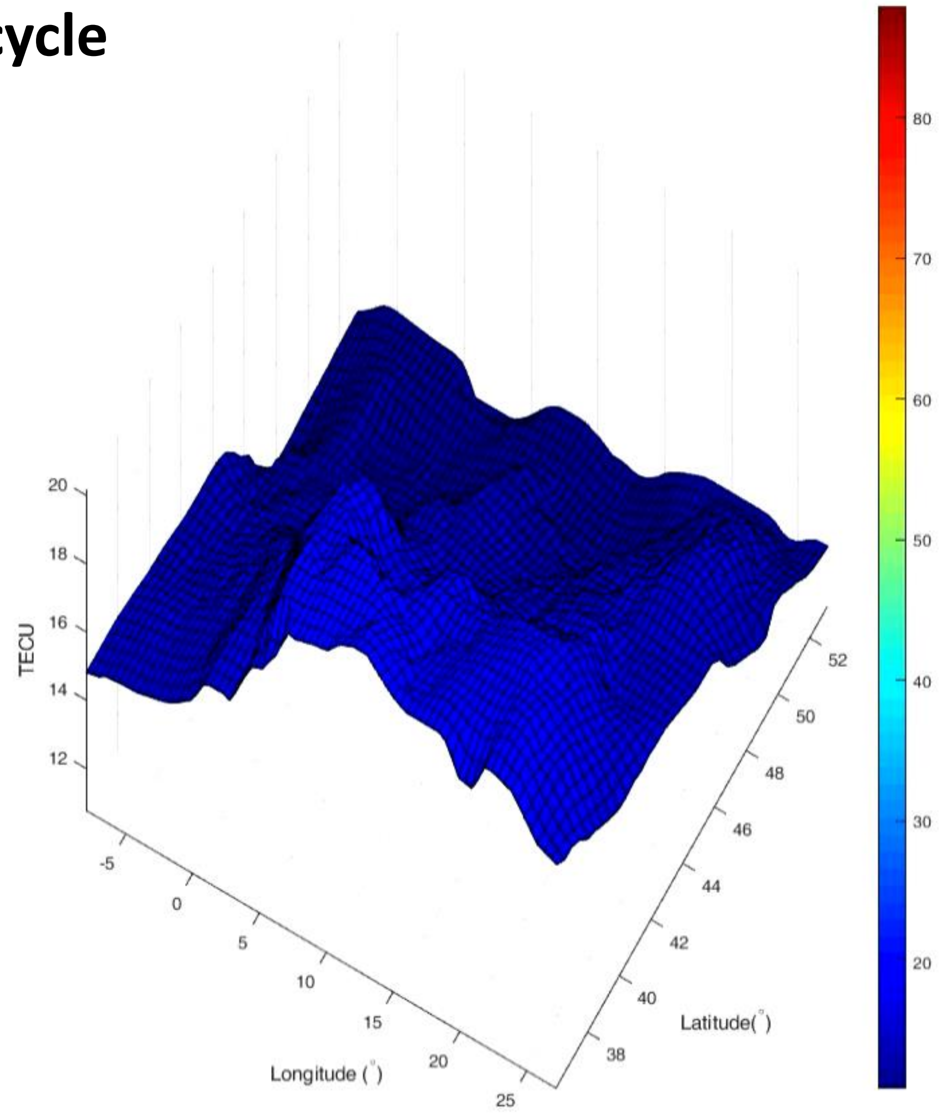
17 March 2015 – First superstorm of 24th solar cycle

St Patrick's Day Storm over Europe

TECU



TECU



APPLICATION 2- IONOLAB-SMAP (8)

- IONOLAB-SMAP is composed of a STECp data preprocess stage and IONOLAB-MAP stage which employes double Ordinary Kriging with Matern Family theoretical semivariogram whose parameters are optimized using PSO.
- IONOLAB-SMAP is a reliable, robust and accurate mapping method developed to overcome the limitations of imaging ionosphere during both quiet and disturbed states.
- It provides superior performance at a spatial resolution of 0.48° in both latitude and longitude; and a temporal resolution of 5 min.
- It can inherently capture detailed directional/anisotropic behavior globally or regionally.
- Excellent performance for sparse data networks: even with up to 70% reduction in the number of GPS stations, NRMSE remains below 10%.
- The preprocessing step reduces error margins in semivariogram estimation by up to 40 times.
- The algorithm can operate automatically over any duration RINEX files (even with the shortest reliable GPS RINEX package of 15 min) thanks to IONOLAB-TEC and IONOLAB-BIAS software which can be used online or downloaded from www.ionolab.org.
- IONOLAB-SMAP can revolutionize the understanding of physical processes of ionosphere both on quiet and disturbed days.

Koroglu, M., Koroglu, O., Arikan, O., Arikan, F. (August, 2025) Ionospheric Slant Total Electron Content Mapping Algorithm: IONOLAB-SMAP, available online in Advances in Space Research.

IMPROVEMENT OF THE CLIMATIC MODELS

- **The determining parameter of IRI, IRI-Plas and NeQuick is foF2.**
- **When foF2 is inputted from outside, the models scale all other parameters with respect to foF2.**
- **Using IONOLAB-CoKrig, sparse foF2 data can be mapped with a high resolution of 0.5^0 in both latitude and longitude using IONOLAB-SMAP as the background (denser modality) where available** $\Rightarrow$ $Z(\theta, \phi; t)$
- **The climatic trend part** $\mu(\theta, \phi; t)$ **can be obtained from IRI, IRI-Plas and NeQuick OR IONOLAB Midlatitude Planar Trend Model (in midlatitude for the time being)**
- **The residual (secondary high amplitude/short duration variability)** $\nu(\theta, \phi; t)$ **can be classified using machine learning algorithms such as Support Vector Machines.**
- **Empirical Probability Density Functions (EPDFs) can be obtained with respect to solar activity, region, month and hour (IONOLAB-PDF); provides stochastic characterization of both trend and secondary variability.**
- **Ionospheric Anomalies can be incorporated into stochastic trend models using IONOLAB-PDF.**

CONCLUSIONS AND FUTURE DIRECTIONS (1)

- **Random Field Model combines stochastic nature of ionosphere in both space and time.**
- **The trend part can be estimated using the medians of 2D Maps and 3D Tomographic Reconstructions with respect to solar activity level, month and hour.**
- **Midlatitude Planar Trend Model is analog (continuous in space) within the defined regions; for other times and coordinates, the coefficients can be interpolated. The model provides an excellent fit to GIM about 99% of the time.**
- **The empirical probability density functions (EPDF) of model parameters can be obtained.**
- **The EPDFs may serve as key parameters in tracking and estimation model coefficients in near-real time.**
- **IONOLAB-SMAP is a high-resolution imaging method which inherently includes the anisotropicity, inhomogeneity and space-time variability of ionosphere in latitude, longitude and time.**
- **It can be obtained from GPS networks for TEC.**
- **Sparse foF2 data (secondary modality) can be mapped using Co-Kriging in space and time using high resolution TEC as primary modality.**
- **The residuals obtained from realizations of Random Field minus the trend will yield secondary fields that can be classified or represented using machine learning algorithms.**
- **The climatic deterministic models IRI, IRI-Plas and NeQuick can input the realizations stochastic models or outcomes of probabilistic distributions (where available) to update the Ne profiles in a statistical sense.**

CONCLUSIONS AND FUTURE DIRECTIONS (2)

- **One realization of a random field** $G(\theta, \phi; t) = \hat{\mu}(\theta, \phi; t) + \nu_G(\theta, \phi; t)$

where $\hat{\mu}(\theta, \phi; t)$**: stochastic trend model from IRI, IRI-Plas and NeQuick**

$G(\theta, \phi; t)$ **: GIM or high resolution regional/global IONOLAB-SMAP**

$\nu_G(\theta, \phi; t) = G(\theta, \phi; t) - \hat{\mu}(\theta, \phi; t)$ **: one realization of secondary random field (geomagnetic disturbances/outliers due to atmospheric or seismic disturbances)**

- **The climatic deterministic models IRI, IRI-Plas and NeQuick can input the realizations stochastic models or outcomes of probabilistic distributions (where available) to update the Ne profiles in a statistical sense.**
- **The residuals obtained from realizations of Random Field minus the trend will yield secondary fields that can be classified or represented using machine learning algorithms.**

Thank you