Pulse Shaping

Joint ICTP-IAEA School on Detector Signal Processing and Machine Learning for Scientific Instrumentation and Reconfigurable Computing

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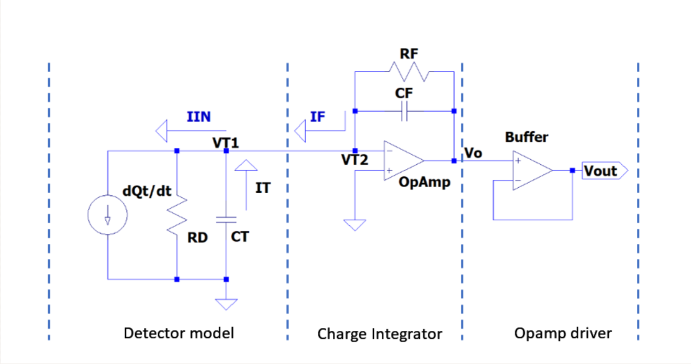
IAEA
International Atomic Energy Agency

Shape of the Detector Signal



$$I_{in}(t) = I_0 e^{-\frac{t}{\tau_2}}$$

$$I_{in}(s) = \frac{I_0}{s + \frac{1}{\tau_2}} \qquad Q = I_0\,\tau_2$$

$$H(s) = \frac{V_o(s)}{I_{in}(s)} = \frac{Z_F}{1 + \frac{1}{A\beta}}$$

$\beta = Z_T/(Z_T + Z_F)$ is feedback factor,
A is the open loop gain, $A\beta$>>1

$$H(s) = \frac{V_o(s)}{I_{in}(s)} = \frac{Z_F}{1 + \frac{1}{A\beta}} \cong Z_F = \frac{R_F}{1 + s\tau_1}$$

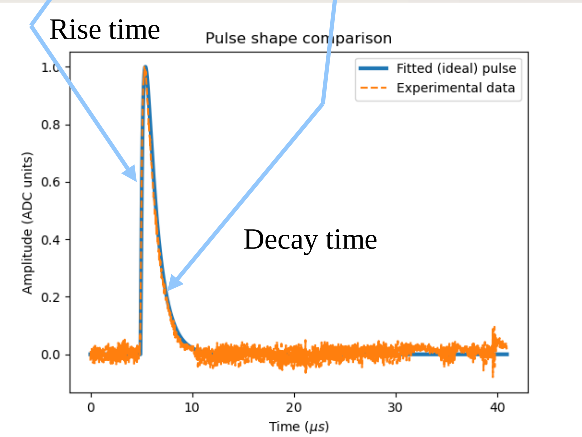
$$\boxed{\tau_2 = scintillator\ decay\ time}$$

$$\boxed{\tau_1 = \frac{1}{2\pi f_{c1}} = R_F C_F}$$

$$V(s) = H(s)\,I_{in}(s) = \frac{R_F}{s+\frac{1}{\tau_1}}\,\frac{I_0}{s+\frac{1}{\tau_2}}$$

$$L\left\{\frac{b_1 b_2}{b_2 - b_1}\left(e^{-b_1 t} - e^{-b_2 t}\right)\right\} = \frac{b_1 b_2}{(b_1 + s)(b_2 + s)}$$

$$\boxed{V_o(t) = \frac{Q}{C}\,\frac{\tau_1}{\tau_1 - \tau_2}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right)}$$



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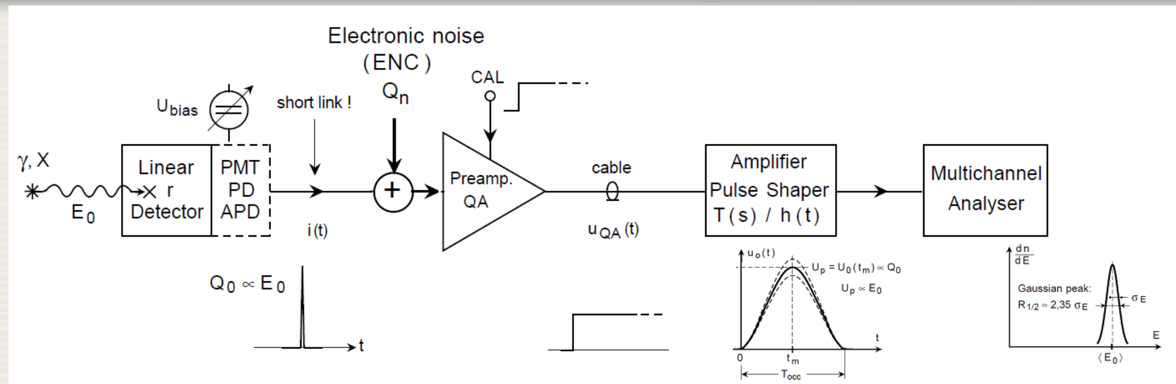
Equivalent Noise Charge (ENC)



- The ENC is the number of electrons Qn to be injected as a delta pulse into the electronic processor input in order to get at its output a pulse whose peak amplitude equals the root-mean-square noise there present.

- The ENC depends on the
  1) electronic noise sources present in the detector, bias and reset circuitry, amplifier itself,
  2) pulse shaping, which acts as a filter on the different noise sources.

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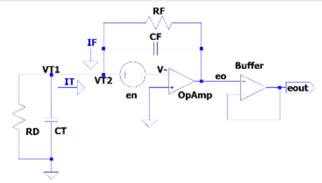
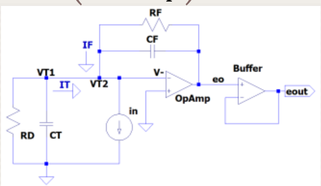
Equivalent Noise Charge (ENC)

Noise at output of CSA:

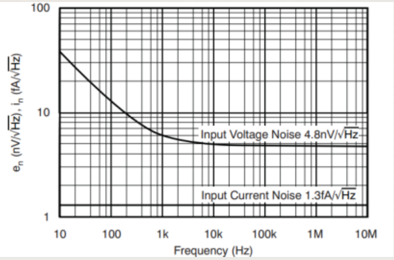
$$e_o(s) = G_{in}(s)i_n(s) = \frac{1}{C_F s} i_n(s)$$

$$e_o(s) = G_{en}(s)e_n(s) = \left(1 + \frac{C_T}{C_F}\right) e_n(s)$$





$$V_{rms-PA}^2 = \int_0^\infty (|e_n G_{en}|^2 + |i_n G_{in}|^2)df$$



SOURCE	PARALLEL NOISE A^2/Hz	SERIES NOISE V^2/Hz
	white	white
Detector	$q M^2 I_{db}(1+v_M)$	
Bias resistor	$2kT/R_b$	
Feedback resistor	$2kT/R_f$	
J-FET:		
gate current	$q I_G$	
channel thermal noise		$\frac{2kT}{g_m} \cdot \frac{2}{3}$
induced gate noise		$\frac{2kT}{g_m} \cdot \frac{1}{4}\left(C_{GS}/C_t\right)^2$
SUB-TOTAL	b	a
	violet	pink
J-FET		$c/\lvert\omega\rvert$
Dielectrics	$d \cdot \lvert\omega\rvert$	
TOTAL	$i_n^2 = b + d\lvert\omega\rvert = b$	$e_{ns}^2 = a + \frac{c}{\lvert\omega\rvert}$

Signal and Noise at output of Shaper:

$$S_{max} = G\frac{Q}{C_F}$$

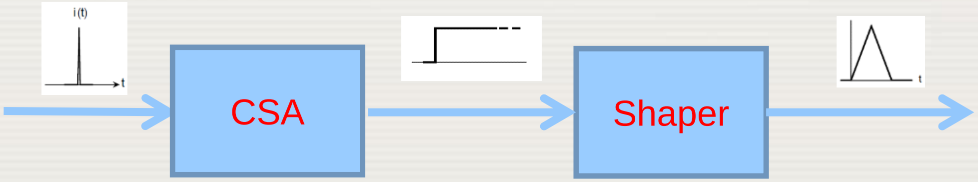
$$V_{rms-SA}^2 = \int_0^\infty (|e_n G_{en} T|^2 + |i_n G_{in} T|^2)df$$

$$V_{rms-SA}^2 = \frac{G^2}{2\pi}\int_0^\infty \left(|e_n|^2\left(1 + \frac{C_T}{C_F}\right)^2 + |i_n|^2 \frac{1}{|i\omega C_F|^2}\right)|T(i\omega)|^2 d\omega$$

$$\frac{S}{N} = \frac{S_{max}}{\sqrt{V_{rms-SA}^2}} = \frac{Q}{\sqrt{\frac{1}{\pi}\int_0^\infty \left(|e_n|^2(C_F + C_T)^2 + |i_n|^2 \frac{1}{\omega^2}\right)|T(i\omega)|^2 d\omega}}$$

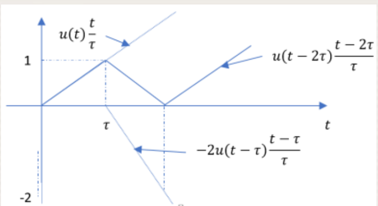
$$ENC = Q_{ENC} = \sqrt{\frac{1}{\pi}\int_0^\infty \left(|e_n|^2(C_F + C_T)^2 + |i_n|^2 \frac{1}{\omega^2}\right)|T(i\omega)|^2 d\omega}$$

$$\boxed{\boldsymbol{ENC^2 = Q_{ENC}^2 = \frac{1}{\pi}\int_0^\infty \left[\left(b + \frac{c}{\omega}\right)(C_F + C_T)^2 + a\frac{1}{\omega^2}\right]|T(i\omega)|^2 d\omega}}$$



Equivalent Noise Charge (ENC) for triangular shaper

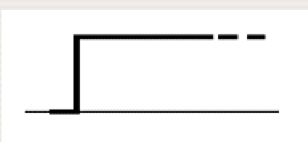
Output signal from Shaper:



$$L\big(f(t)\big) == \frac{(1-e^{-\tau s})^2}{\tau s^2}$$

$$f(t) = u(t)\frac{t}{\tau} - 2u(t-\tau)\frac{t-\tau}{\tau} - u(t-2\tau)\frac{t-2\tau}{\tau}$$

Input signal into CSA:



$$g(t) = u(t) \qquad L\big(g(t)\big) = \frac{1}{s}$$

Calculation of Transfer function:

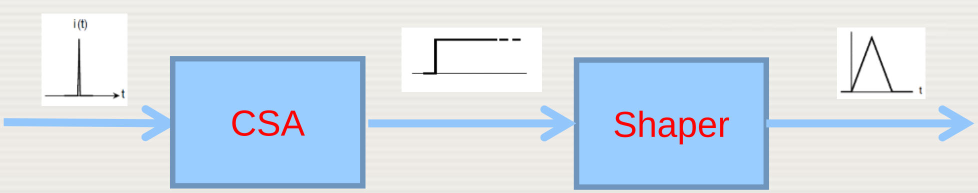
$$T(s) = \frac{L\big(f(t)\big)}{L\big((g(t)\big)} = \frac{(1-e^{-\tau s})^2}{\tau s}$$

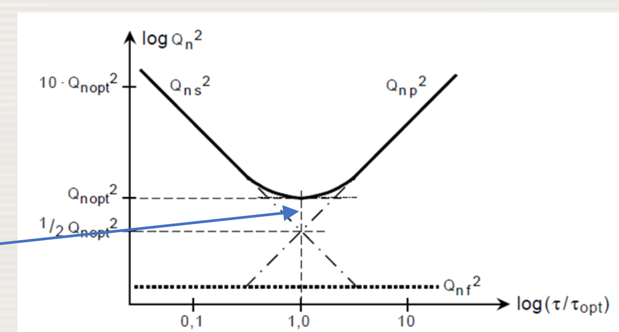
$$|T(i\omega)|^2 = 16\frac{\sin^4\left(\frac{\tau\omega}{2}\right)}{(\tau\omega)^2}$$

Calculation of ENC Integral

$$ENC^2 = \frac{1}{\pi}\int_0^\infty \left[\left(b+\frac{c}{\omega}\right)(C_F+C_T)^2 + a\frac{1}{\omega^2}\right]|T(i\omega)|^2 d\omega$$

$$\boxed{ENC^2 = \frac{2b(C_F+C_T)^2}{\tau} + \frac{2a}{3}\tau + \frac{4ln2}{\pi}(C_F+C_T)^2 c}$$



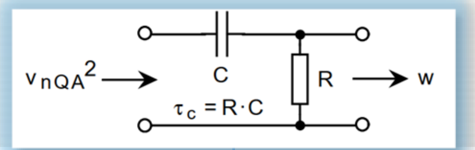


$$for\ c = 0 \Rightarrow ENC_{min} = \left(\sqrt[4]{4/3}\right) 2C_t\sqrt{ab}$$

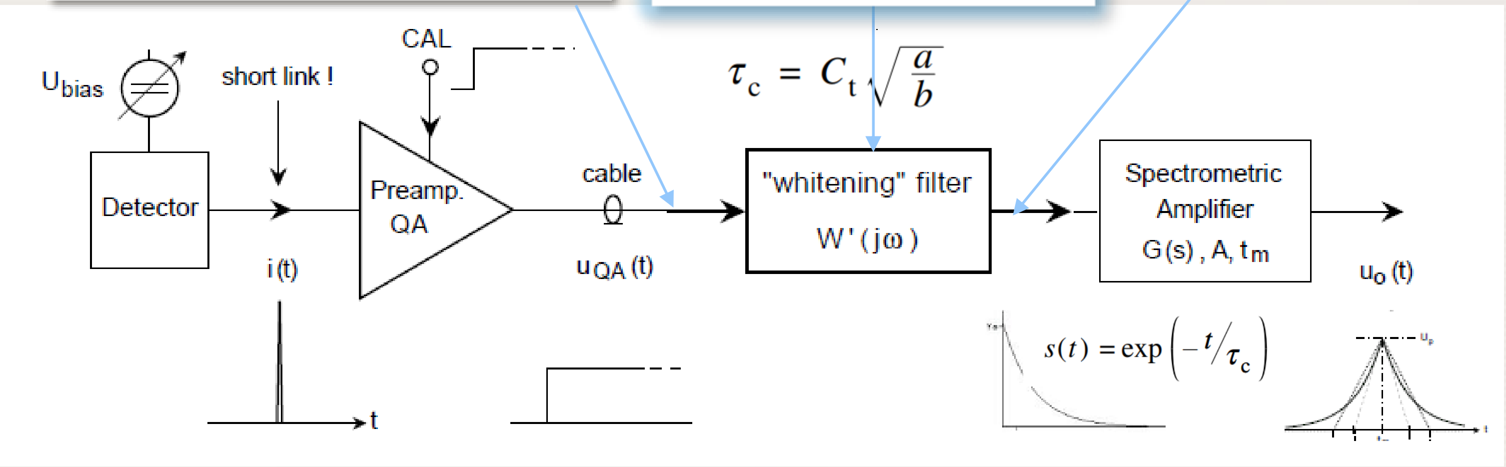
Optimal Filtering (cusp filter)

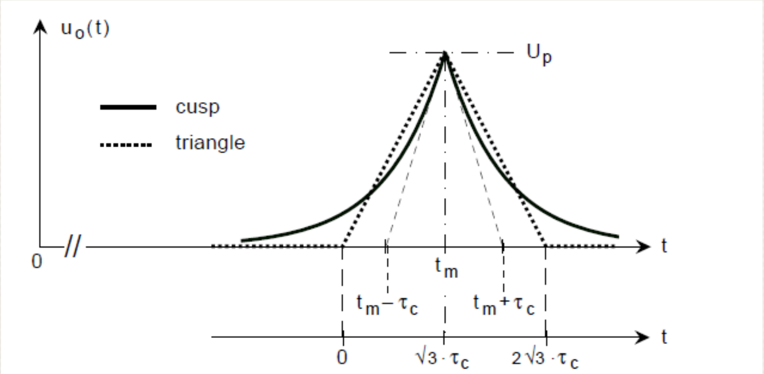
$v = input\ noise$

$$v_{nQA}^2(\omega) = \frac{1}{C_t^2}\left(a\,C_t^2 + \frac{b}{\omega^2} + \cancel{\frac{c}{|\omega|}C_t^2}\right)$$



$w = output\ white\ noise$





$$u_0(t) = \int_{-\infty}^{+\infty} s(\tau)\cdot s(t+\tau)\,d\tau = \exp\left(-|t|/\tau_c\right)$$

$$ENC_{cusp} = 2C_t\sqrt{ab} \qquad for\ \tau_c = C_t\sqrt{\frac{a}{b}}$$

$$ENC_{triangle} = \left(\sqrt[4]{4/3}\right)2C_t\sqrt{ab} \qquad for\ \tau_c = \sqrt{3}\,\tau_c$$

$$\frac{ENC_{triangle}}{ENC_{cusp}} = \sqrt[4]{4/3} = 1.07$$

Cusp shaper is 7% better then triangle

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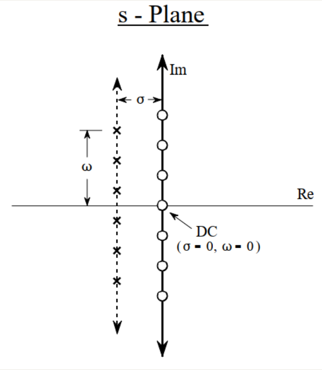
Comparison between shapers

	Definition of τ	τ_{opt}	F	Reference	RMS flicker noise[19] (arbitrary scale)
Infinite cusp (Subsection 4.3.2)	—	—	1·00	(9, 14)	—
Triangular	Total duration $= \tau$	$2\sqrt{3}\,\tau_c$	1·075	(15, 17, 19)	1.665
DL–RC	Total delay time $= 1{\cdot}036\,\tau$ RC time constant $= \tau$	$1{\cdot}29\,\tau_c$	1·098	(15, 17, 19)	1·626
Gaussian			1·120	(16, 18, 19)	1·773
Semi-Gaussian CR–(RC)^4	CR time constant $= \tau$ RC rime constant $= \tau$	$0{\cdot}378\,\tau_c$	1·165	(16, 19, 20)	1·810
CR–(RC)^2	CR time constant $= 1{\cdot}06\,\tau$ RC time constant $= \tau$	$0{\cdot}57\,\tau_c$	1·215	(15, 16, 19)	1·847
CR–RC (Subsection 4.4.1)	CR time constant $= \tau$ RC time constant $= \tau$	τ_c	1·359	(9, 14)	1·992
(CR)^2–(RC)^4			1·380	(20)	
(CR)^2–RC	CR time constant $= 1{\cdot}38\,\tau$ RC time constant $= \tau$	$1.40\,\tau_c$	1·410	(15, 20)	—

Filtering Math

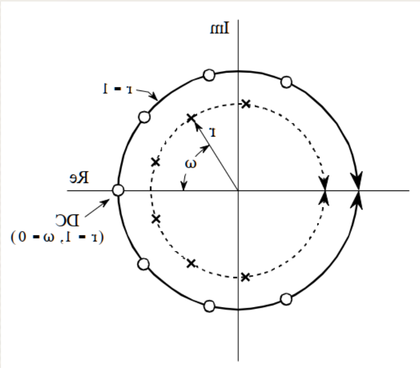
s-transform

$$X(s) = \int_{t=-\infty}^{\infty} x(t)\, e^{-st}\, dt$$



z-transform

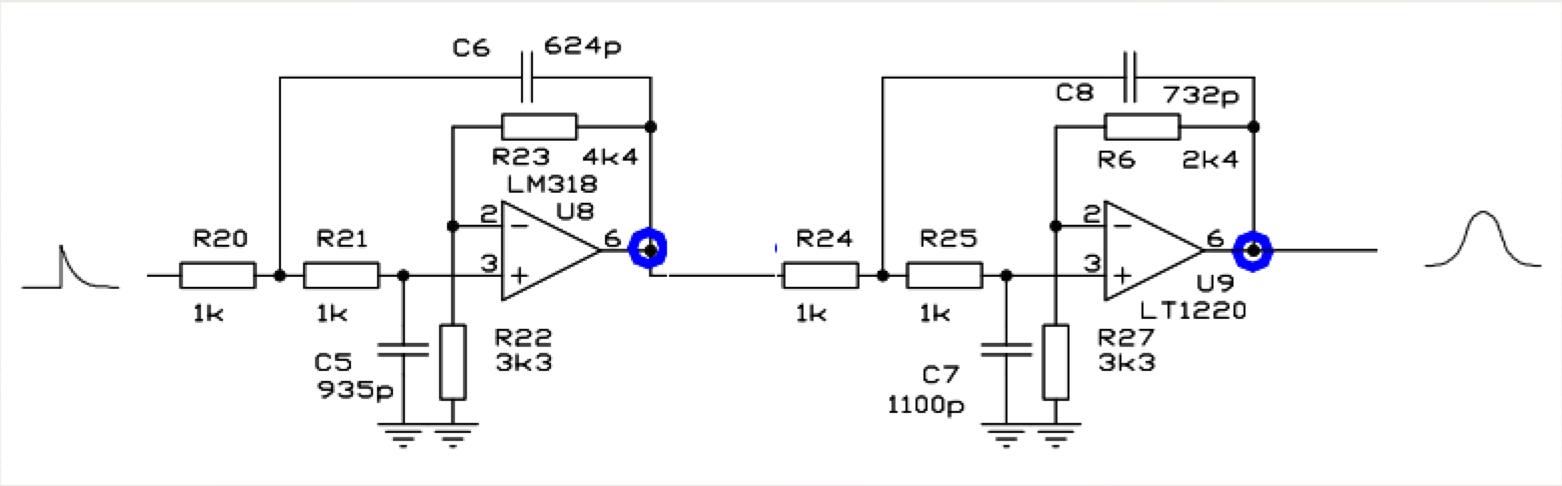
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

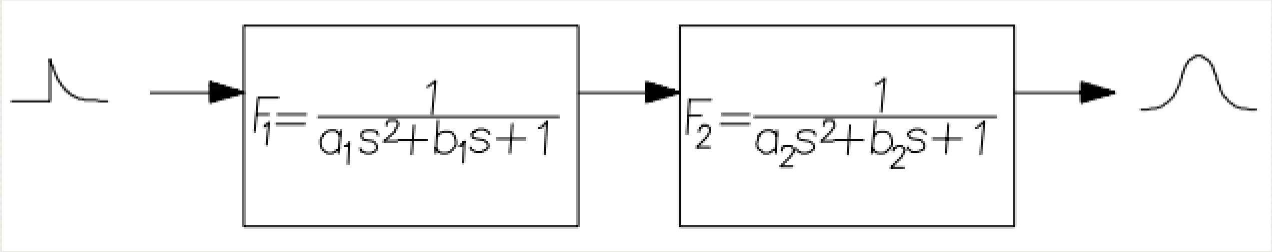


$F(s)$ is the Laplace transform of $f(t)$, and $F(z)$ is the z-transform of $f(kT)$. Note: $f(t)=0$ for $t<0$.

Number	$F(s)$	$f(kT)$	$F(z)$
1	1	$\delta(t)$, unit impulse	1
2	e^{-k_0Ts}	$\delta(t-k_0T)$	z^{-k_0}
3	$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
4	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
5	$\frac{1}{s^3}$	$\frac{1}{2!}(kT)^2$	$\frac{T^2}{2}\left[\frac{z(z+1)}{(z-1)^3}\right]$
6	$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z-e^{-aT}}$
7	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
8	$\frac{a}{s(s+a)}$	$1-e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
9	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(akT-1+e^{-akT})$	$\frac{z[(aT-1+e^{-aT})z+(1-e^{-aT}-aTe^{-aT})]}{a(z-1)^2(z-e^{-aT})}$
10	$\frac{b-a}{(s+a)(s+b)}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$
11	$\frac{s}{(s+a)^2}$	$(1-akT)e^{-akT}$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
12	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bkT}-ae^{-akT}$	$\frac{z[z(b-a)-(be^{aT}-ae^{-bT})]}{(z-e^{-aT})(z-e^{-bT})}$

Analog domain
Sallen – Key Complex Pole Filtering





$$F = \frac{1}{a\,s^2 + b\,s + 1} \quad \text{or} \quad F = \frac{\lambda^2 + \omega^2}{[s + (\lambda + i\omega) \cdot (s - (\lambda + i\omega)]}$$

Z transform – IIR Filter

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + \cdots + b_1 y[n-1] + b_2 y[n-2] + b_3 y[n-3] + \cdots$$

- Transfer function H(z) = Y[z]/X[z]

$$H[z] = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3} - \cdots}$$

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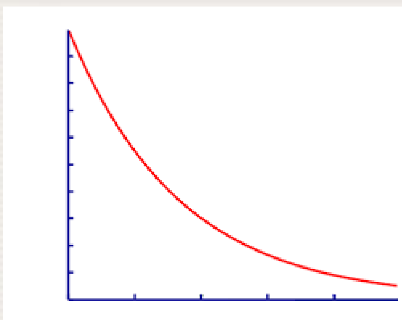
From Exp to Triangle,Trapezoid

Exponential

$$y_n = e^{-\frac{nT_{clk}}{\tau}}$$

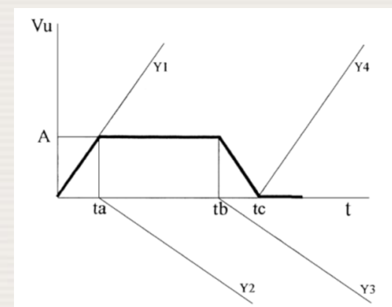


$$X(z) = \frac{A_0}{1 - e^{-T/\tau} z^{-1}}$$

$$H(z)$$

Traingle/Trapezoid

$$y_n = \frac{A_0}{n_a} h_n - \frac{A_0}{n_a}(n - n_a) h_{n-n_a}$$
$$-\frac{A_0}{n_a}(n - n_b) h_{n-n_b} + \frac{A_0}{n_a}(n - n_b - n_a) h_{n-n_b-n_a}$$



$$Y(z) = \frac{A_0}{n_a} z^{-1} \frac{1 - z^{-n_a}}{1 - z^{-1}} \frac{1 - z^{-n_b}}{1 - z^{-1}}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{1}{n_a} z^{-1} \left(1 - e^{-T/\tau} z^{-1}\right) \left(\frac{1 - z^{-n_a}}{1 - z^{-1}}\right) \left(\frac{1 - z^{-n_b}}{1 - z^{-1}}\right)}$$

C. Imperiale, A. Imperiale / Measurement 30 (2001) 49 –73

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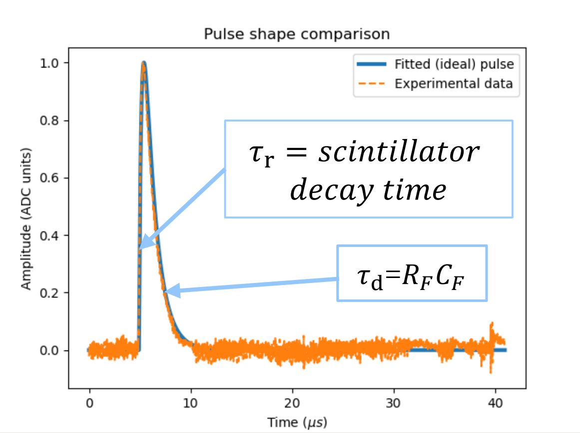
Second pole correction

Input signal

$$V(t) = \frac{Q}{C}\frac{\tau_d}{\tau_d - \tau_r}\left(e^{-\frac{t}{\tau_d}} - e^{-\frac{t}{\tau_r}}\right)$$

$$V(s) = \frac{Q}{C}\frac{1}{\left(s+\frac{1}{\tau_d}\right)\left(s+\frac{1}{\tau_r}\right)}$$



Trapezoidal Filter Transfer function

$$H(z) = \frac{1}{n_a}\frac{e^{-\frac{T_{clk}}{\tau_d}m} - e^{-\frac{T_{clk}}{\tau_r}m}}{e^{-\frac{T_{clk}}{\tau_d}} - e^{-\frac{T_{clk}}{\tau_r}}}\left(1 - e^{-\frac{T_{clk}}{\tau_d}}z^{-1}\right)\left(1 - e^{-\frac{T_{clk}}{\tau_r}}z^{-1}\right)\frac{1-z^{-n_a}}{1-z^{-1}}\frac{1-z^{-n_b}}{1-z^{-1}}$$

$$m = \begin{cases} m_1 = ceil\left(\frac{1}{T_{clk}}\frac{\tau_d\tau_r}{\tau_d-\tau_r}\ln\frac{\tau_d}{\tau_r}\right) & f(m_1T_{clk}) > f(m_2T_{clk}) \\ m_2 = floor\left(\frac{1}{T_{clk}}\frac{\tau_d\tau_r}{\tau_d-\tau_r}\ln\frac{\tau_d}{\tau_r}\right) & f(m_1T_{clk}) < f(m_2T_{clk}) \end{cases}$$

$$n_a = \frac{t_{trapez\_peaking}}{T_{clk}}$$

$$n_b = \frac{t_{trapez\_flat\_top}}{T_{clk}}$$

$$\tau_d = input\ signal\ decay\ time$$

$$\tau_r = input\ signal\ rise\ time$$

$$T_{clk} = clock\ period$$

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From Exp to Triangle,Trapezoid

[1]*V*. Jordanov at al *NIM A*345 (1994)337

$$H(z) = \frac{z^{-1}}{n_a}\boxed{(1 - z^{-n_a})(1 - z^{-n_b})}\boxed{\left(\frac{1 - e^{-\frac{T_{clk}}{\tau_d}} z^{-1}}{1 - z^{-1}}\right)}\boxed{\left(\frac{1}{1 - z^{-1}}\right)}$$

Original work notations[1]

$$d^{k,l}(n) = v(n) - v(n-k) - v(n-l) + v(n-k-l), \quad (1)$$

$$p(n) = p(n-1) + d^{k,l}(n), \quad n \geq 0, \quad (2)$$

$$r(n) = p(n) + Md^{k,l}(n), \quad (3)$$

$$s(n) = s(n-1) + r(n), \quad n \geq 0, \quad (4)$$

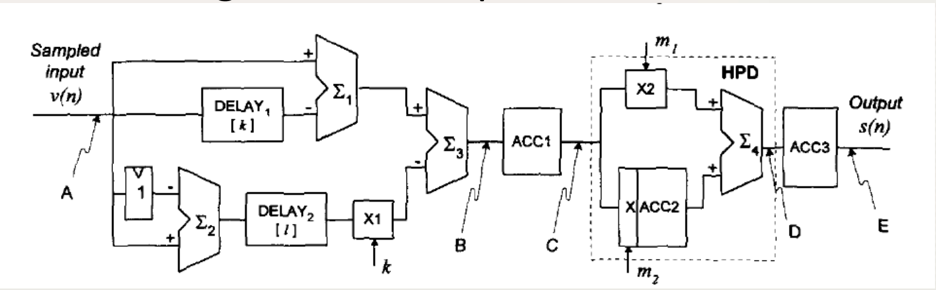
This work notations

$$d_n = x_n - x_{n-n_a} - x_{n-n_b} + x_{n-n_a-n_b}$$

$$p_n = p_{n-1} + d_n$$
$$r_n = p_n + Md_n \qquad M = 1/\left(e^{T_{clk}/\tau_d} - 1\right)$$

$$s_n = s_{n-1} + r_n$$

Original work implementation



From Exp to Triangle,Trapezoid

[2]A. Georgiev and W. Gast   IEEE Trans Nucl Sci 40-4(1993)770

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{n_a} \boxed{\frac{(1-e^{-T/\tau} z^{-1})(1-z^{-n_a})}{1-z^{-1}}} \boxed{\left(\frac{1-z^{-n_b}}{1-z^{-1}}\right)}$$

Original work notations[2]

$$G(n) = \sum_{j=n-M}^{n} g(j) = U(n) - U(n-M) + (1-k) \sum_{j=n-M}^{n-1} U(j) \quad \text{for any } n > z+M \qquad (9)$$

$$G_e = \frac{1}{L} \sum_{n=z+R}^{z+M} G(n) = \frac{1}{L} \sum_{n=z+R}^{z+M} \sum_{j=n-M}^{n} g(j); \qquad (11)$$

This work notations

$$MWD_k = x_k - x_{k-n_a} + (1 - e^{-\frac{T_{clk}}{\tau_d}}) \sum_{i=k-n_a}^{k-1} x_i$$

$$G_k = \frac{1}{n_a} \sum_{i=1}^{n_b} MWD_{k-i}$$

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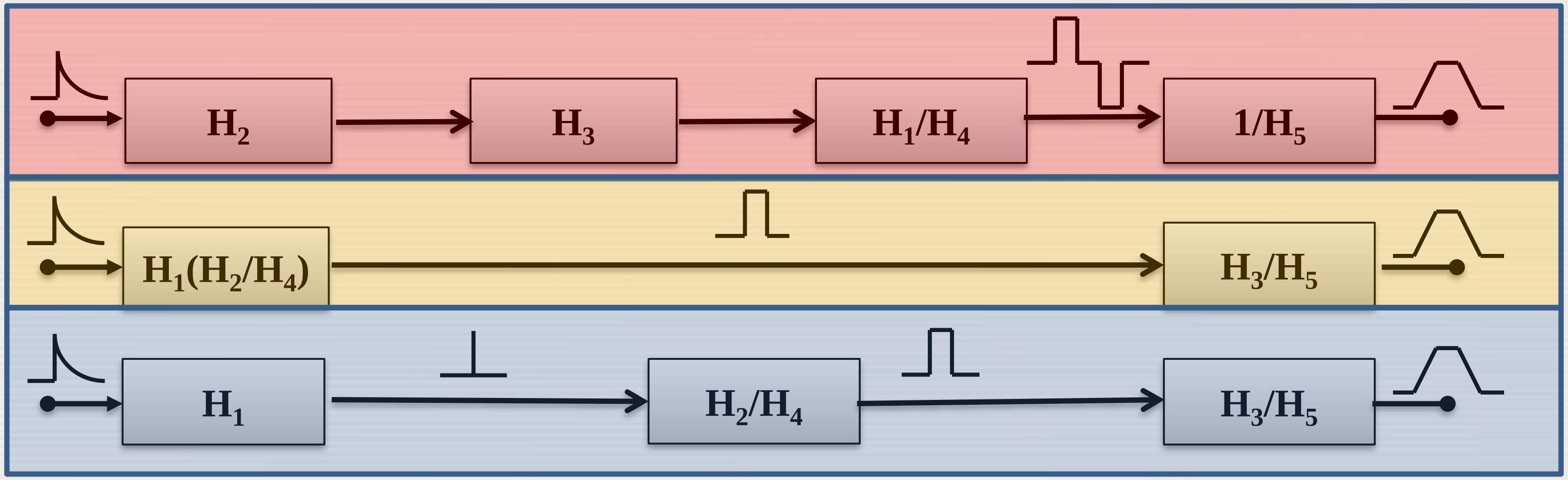
From Exp to Triangle,Trapezoid

$$H(z) = \frac{H_1(z)H_2(z)H_3(z)}{H_4(z)H_5(z)} = \frac{\left(1 - e^{-T/\tau}z^{-1}\right)(1 - z^{-n_a})(1 - z^{-n_b})}{(1 - z^{-1})(1 - z^{-1})}$$



[1]*V*. Jordanov at al *NIM A*345 (1994)337

[2]*A*. Georgiev and W. Gast IEEE Trans Nucl Sci 40-4(1993)770

Lab excercise (Impulse unfolding)

$$H(z) = \left(1 - e^{-T/\tau}z^{-1}\right)\frac{1 - z^{-n_a}}{1 - z^{-1}}\frac{1 - z^{-n_b}}{1 - z^{-1}}$$

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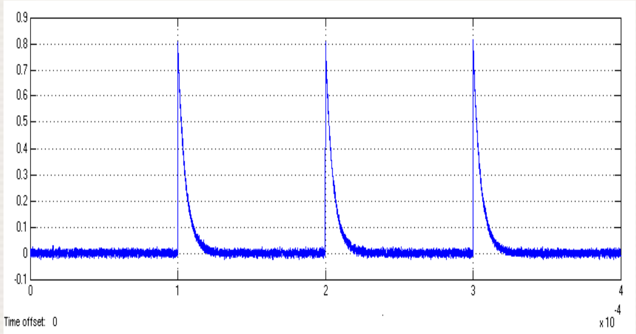
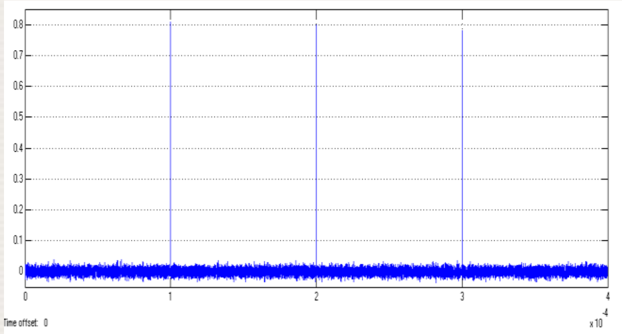
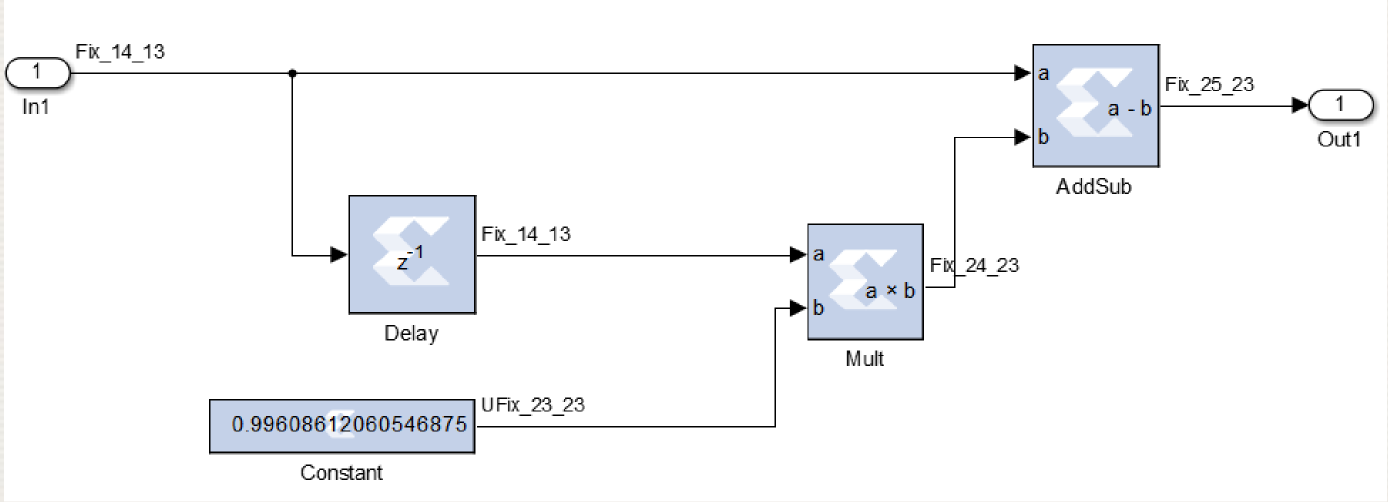
Design of the filter – Stage I



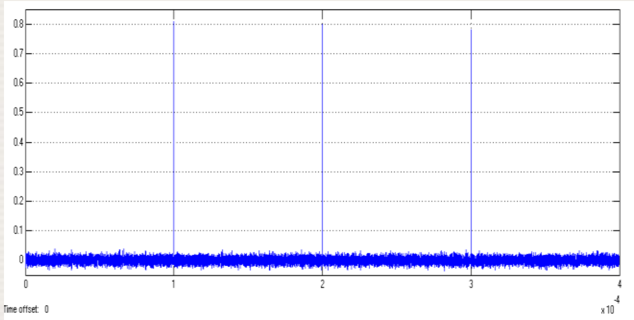
$$H(z) = \left(1 - e^{-\frac{T_{clk}}{\tau_d}} z^{-1}\right)$$

$$y_n = x_n - e^{-\frac{T_{clk}}{\tau_d}} x_{n-1}$$





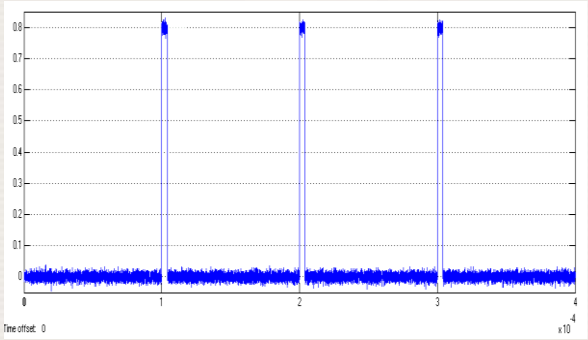
Design of the filter – Stage II

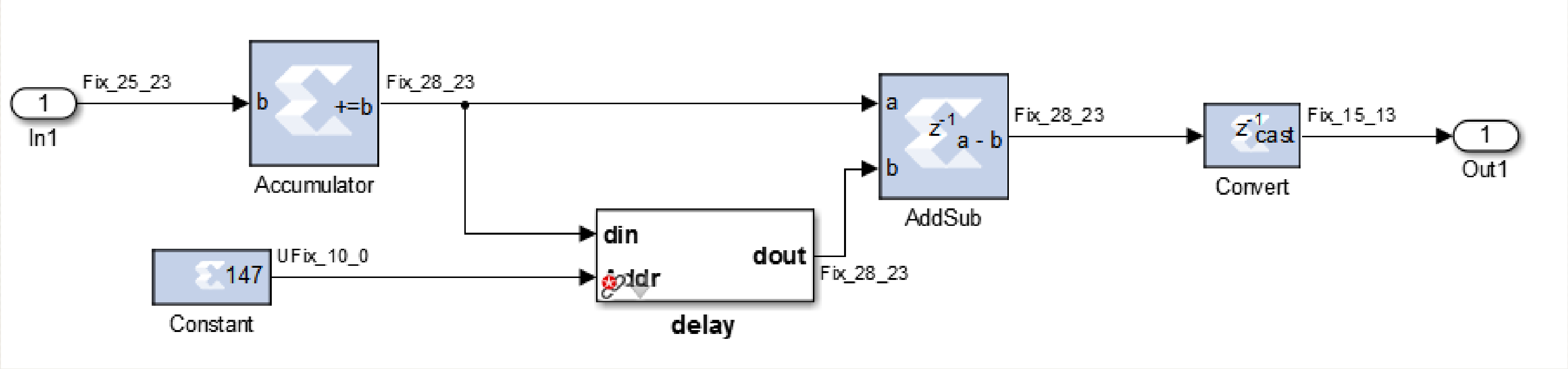


$$H_2(z) = \frac{1}{1-z^{-1}}(1 - z^{-n_a})$$

$$y_n = y_{n-1} + x_n$$

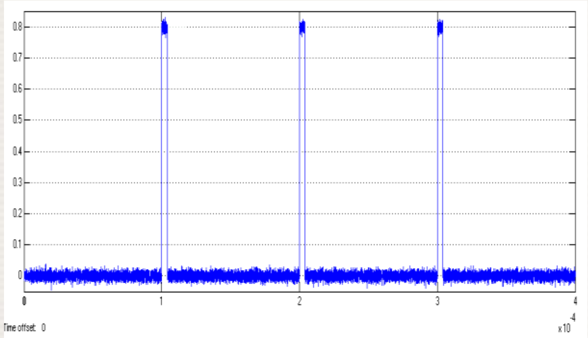
$$z_n = y_n - y_{n-n_a}$$





IAEA

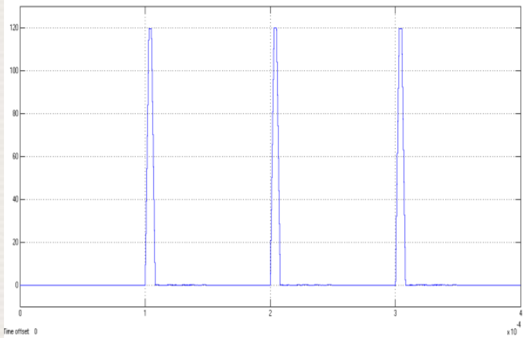
Design of the filter – Stage III

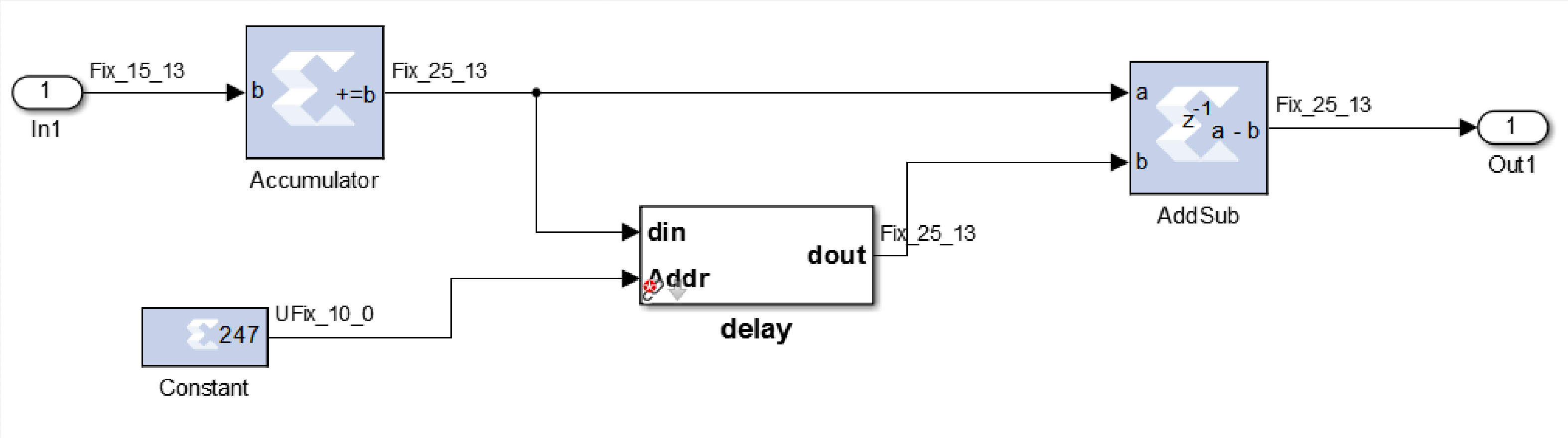


$$H_2(z) = \frac{1}{1-z^{-1}}\left(1 - z^{-n_b}\right)$$

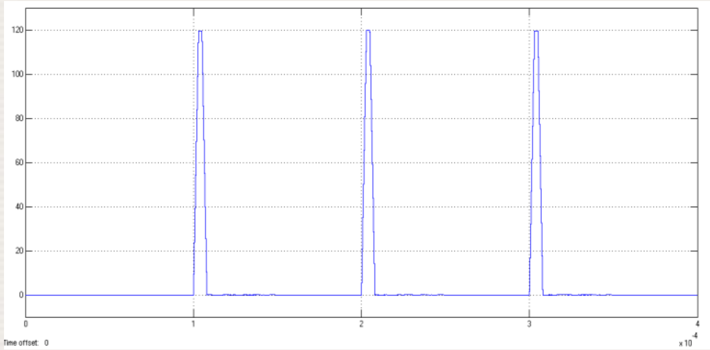
$$y_n = y_{n-1} + x_n$$

$$z_n = y_n - y_{n-n_b}$$



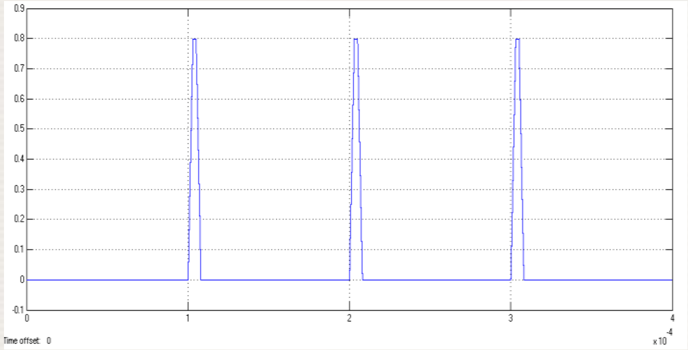


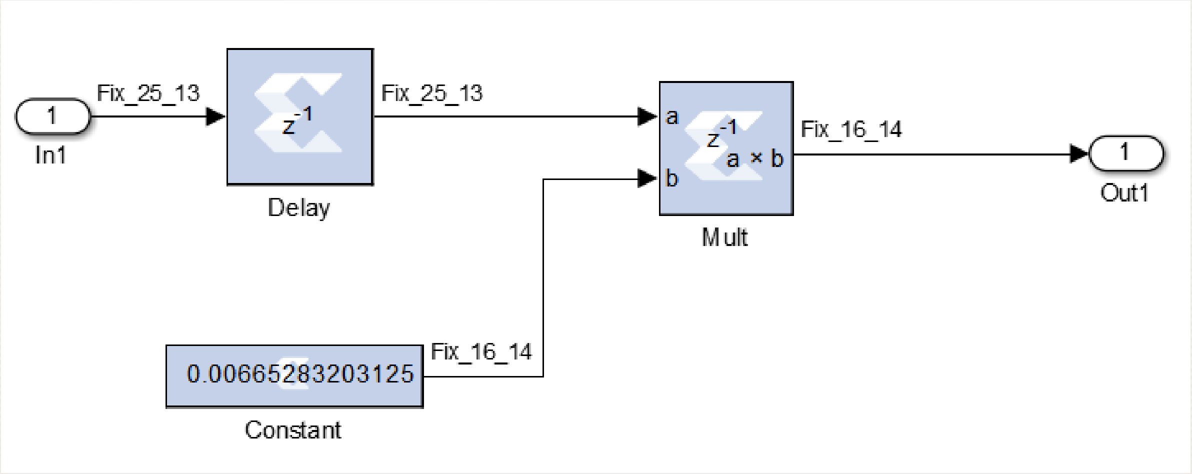
Design of the filter – Stage IV



$$H_3(z) = \frac{z^{-1}}{n_a}$$

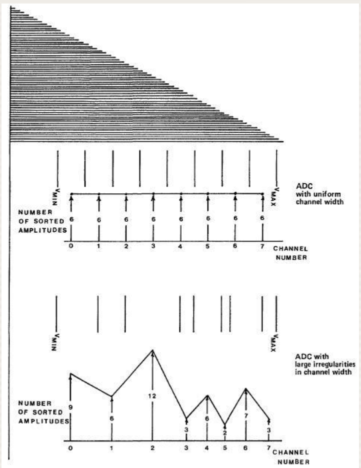
$$y_n = \frac{1}{n_a} x_{n-1}$$

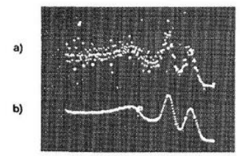




Design of the filter – Precision

Analog system with MCA/ADC without Sliding scale





Scintillation detector spectra obtained with a 1024 channel ADC.
a) without sliding-scale correction
b) with sliding-scale correction

Digital MCA filter

Stage	Gain at DC	Element	Sign	Width	Precision	Overflow	Digital noise
I	$1-b_{10} = 1-e^{-\frac{T_{clk}}{\tau_d}} \approx \frac{T_{clk}}{\tau_d}$						
		Coefficient b_{10}	unsigned	$B_m = B + B_{n_a}$	$B_m = B + B_{n_a}$		
		Multiplier	signed	$B + B_{n_a} + 1$	$B + B_{n_a}$		
		Subtractor	signed	$B + B_{n_a} + 2$	$B + B_{n_a}$		
II	$n_a = \frac{\tau_p}{\tau_d}$	Accumulator		$B + B_{n_a} + 2 + B_{st}$		wrap	$\approx 2\frac{2^{-2B}}{12}$
		Subtractor	signed	$B + B_{n_a} + 2 + B_{st}$	$B + B_{n_a}$	wrap	
		Converter	signed	$B + 2$	B		
III	$n_b = \frac{\tau_p + \tau_{ft}}{T_{clk}}$	Accumulator				wrap	$\approx 2n_b \frac{2^{-2B}}{12}$
		Subtractor	signed	$B + B_{n_b} + 2$	B	wrap	
IV	$\frac{1}{n_a} = \frac{T_{clk}}{\tau_p}$	Multiplier	signed	$B + 2$	$B + 1$		$\approx \frac{2n_b}{n_a{}^2}\frac{2^{-2B}}{12}$
		Mult n_{a_inv}	unsigned	18	18		

- Error sources
  - rounding/truncation of the filter coefficient b_{10}
  - ADC quantization $2\frac{2^{-2B}}{12}$
  - finite precision multiplication $n_a \frac{2^{-2Bm}}{12}$
- Variance at the output of stage IV $2n_b \frac{2^{-2B}}{12}$
- Example:
  - ADC 14 bit, $\tau_d = 5\mu sec, \tau_p = 3\mu sec, \tau_{ftop} = 2\,T_{clk} = 20\,nsec$:

$$\sigma_4^2 = \frac{2^{-2\times 16.5}}{12}$$