

Data Analysis and Filter Optimization for Online Pulse-Amplitude Measurement

A Case Study on High-Resolution X-ray Spectroscopy

Andres Cicuttin

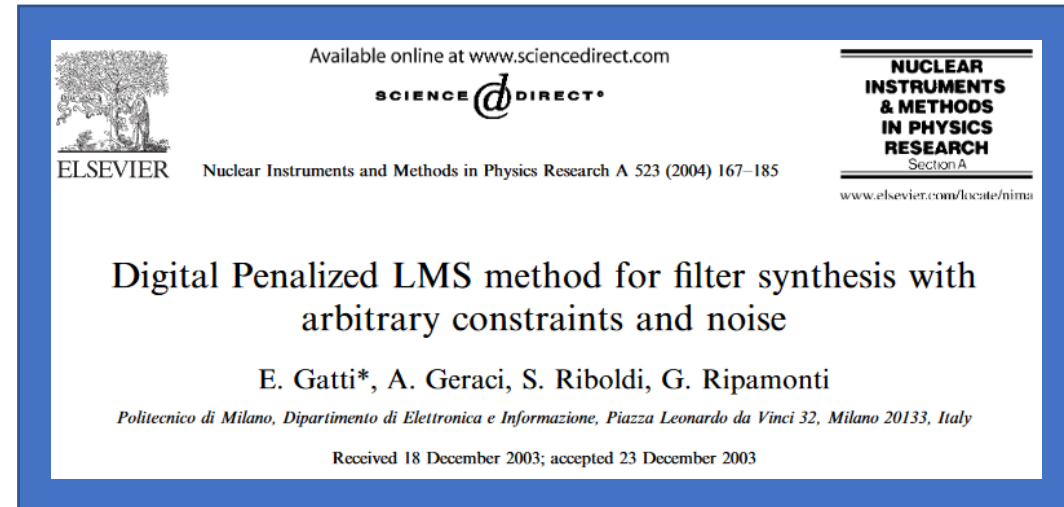
ICTP - Multidisciplinary Laboratory

Trieste, Italy

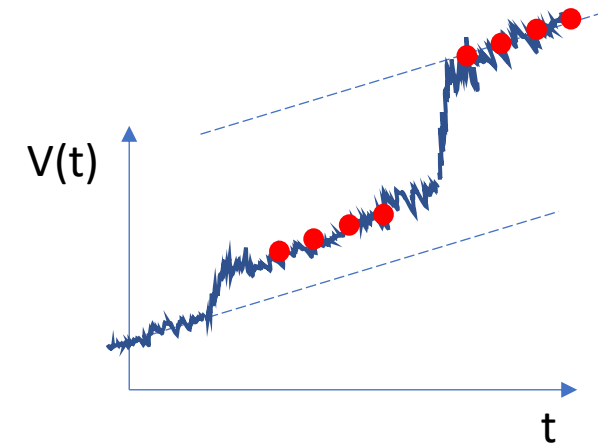
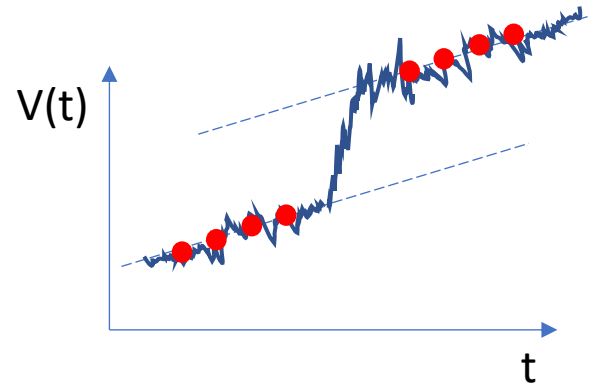
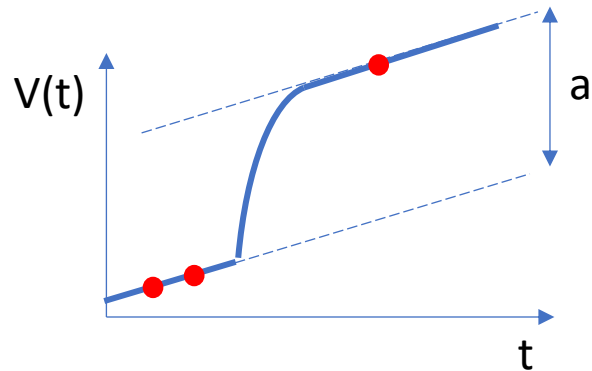
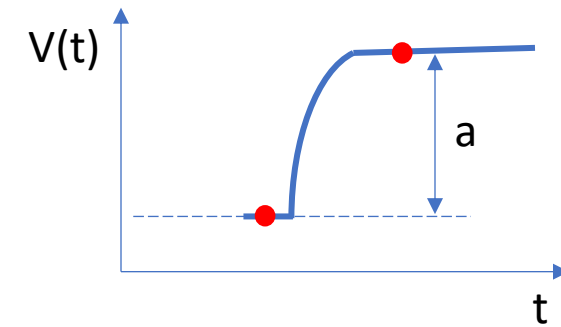
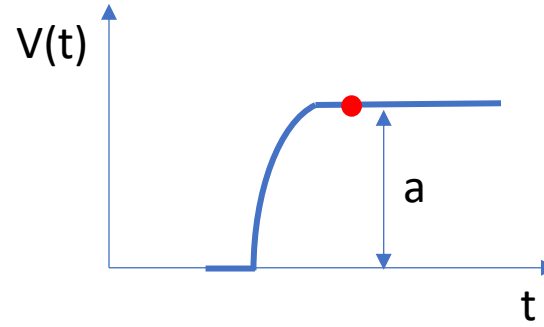
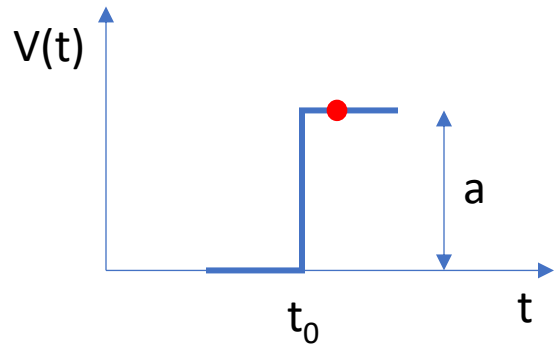
Outline

- Introduction
- Pulsed signals: Description levels
- Processing chain: Detector, Preamplification, Pulse shaping, Data acquisition, . . .
- Digital Pulse Processor (DPP): Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .
- DPP Optimization
 - Data analysis
 - Pulse modeling
 - Digital Penalized Least Mean Squares (DPLMS) method for filtering optimization
- Discussion and Conclusions

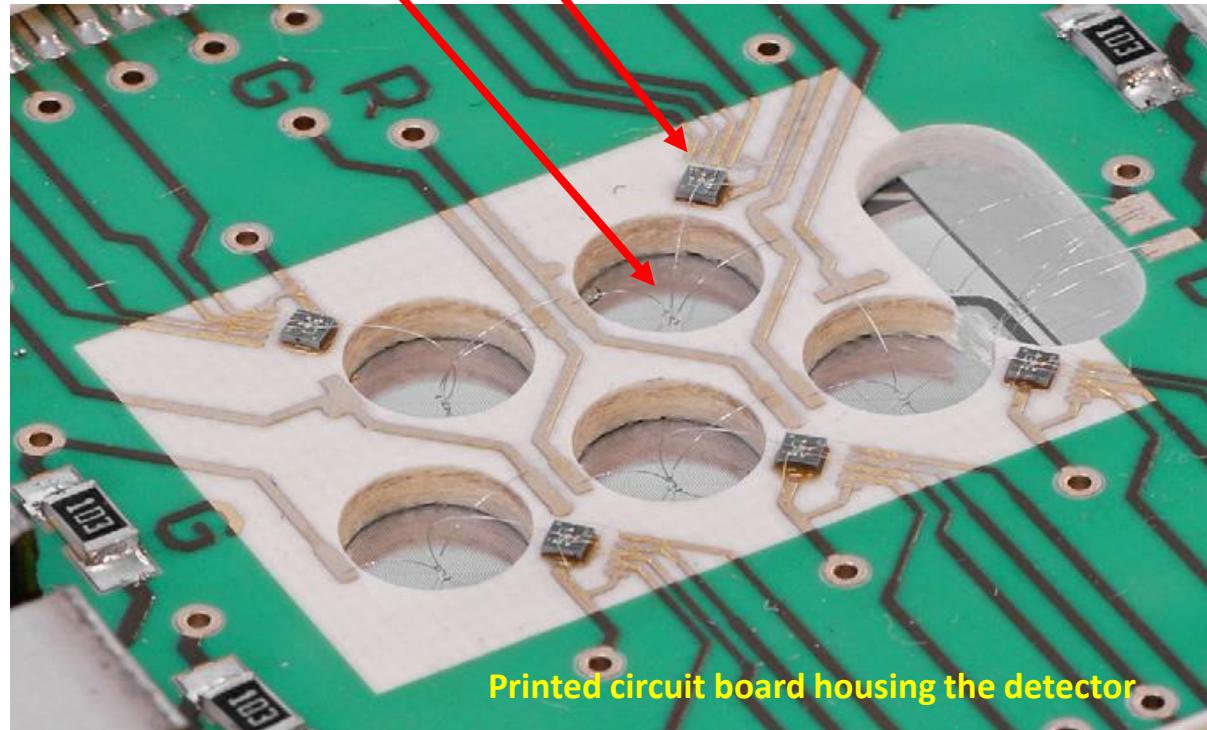
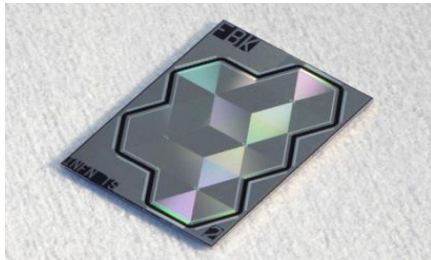
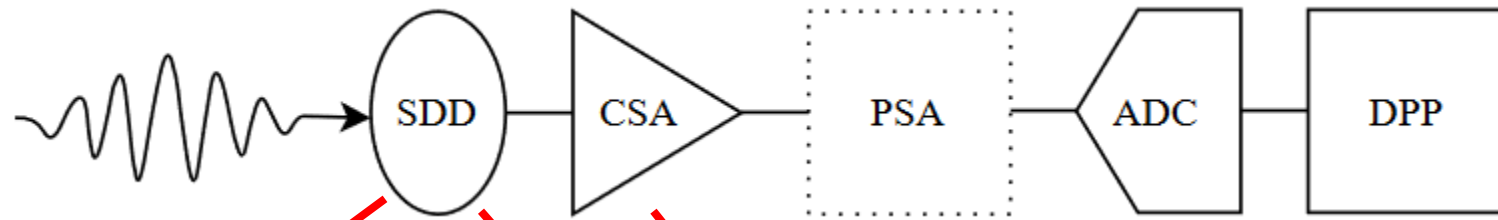
More technical details in



Pulsed signals: Description levels

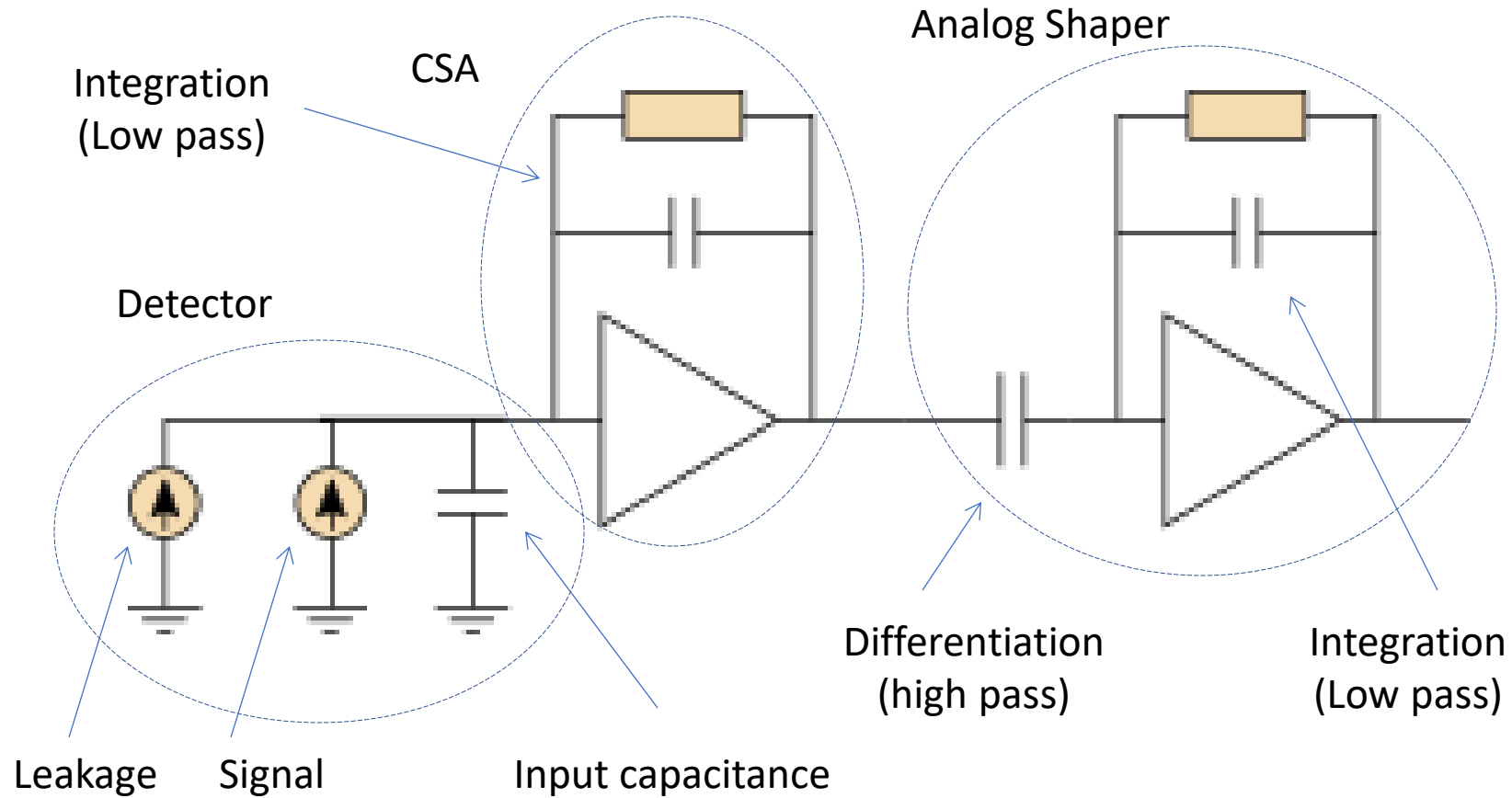
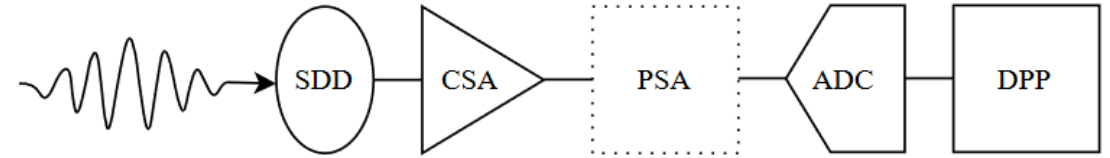


Processing chain

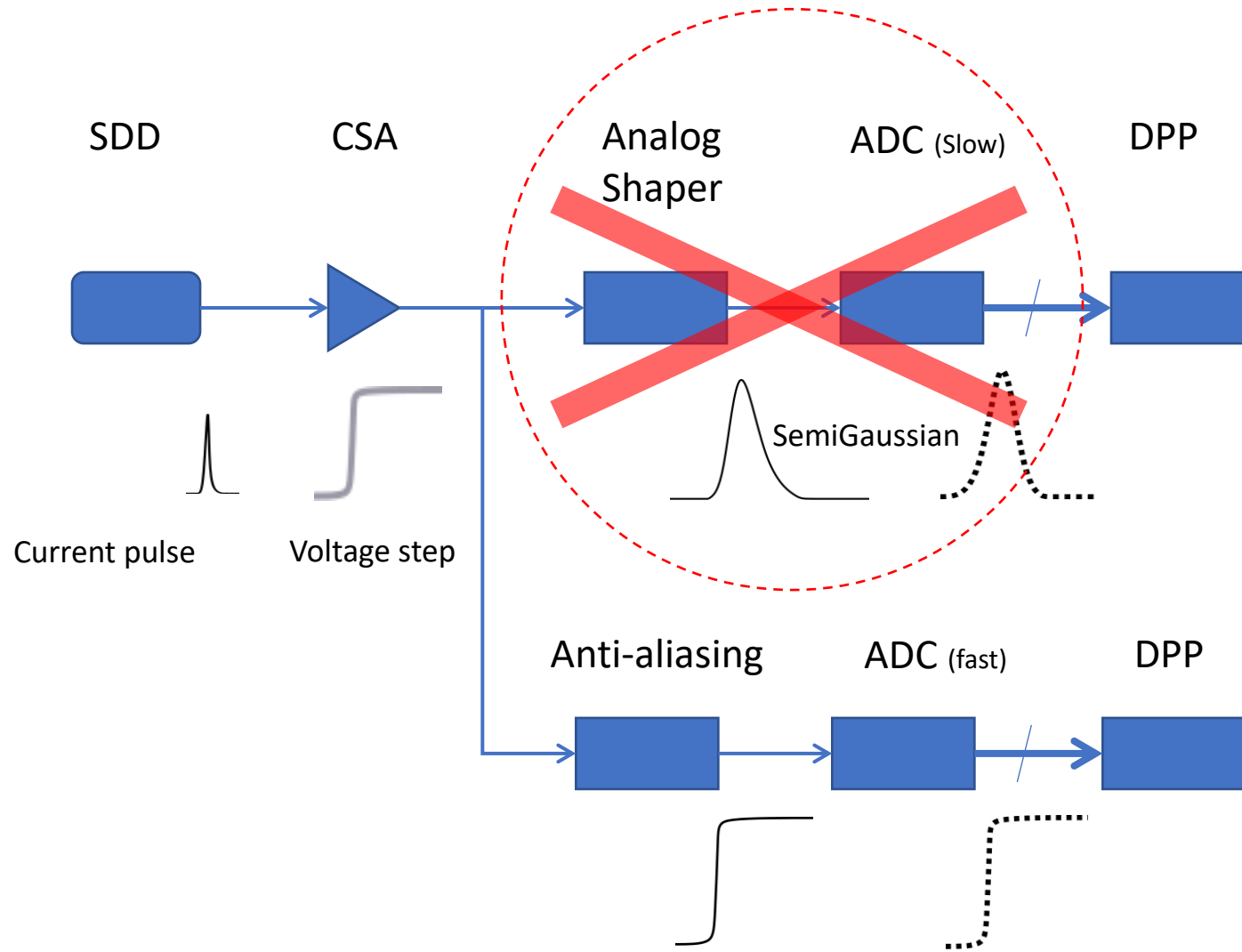


Printed circuit board housing the detector

Detector, CSA, Pulse Shaper

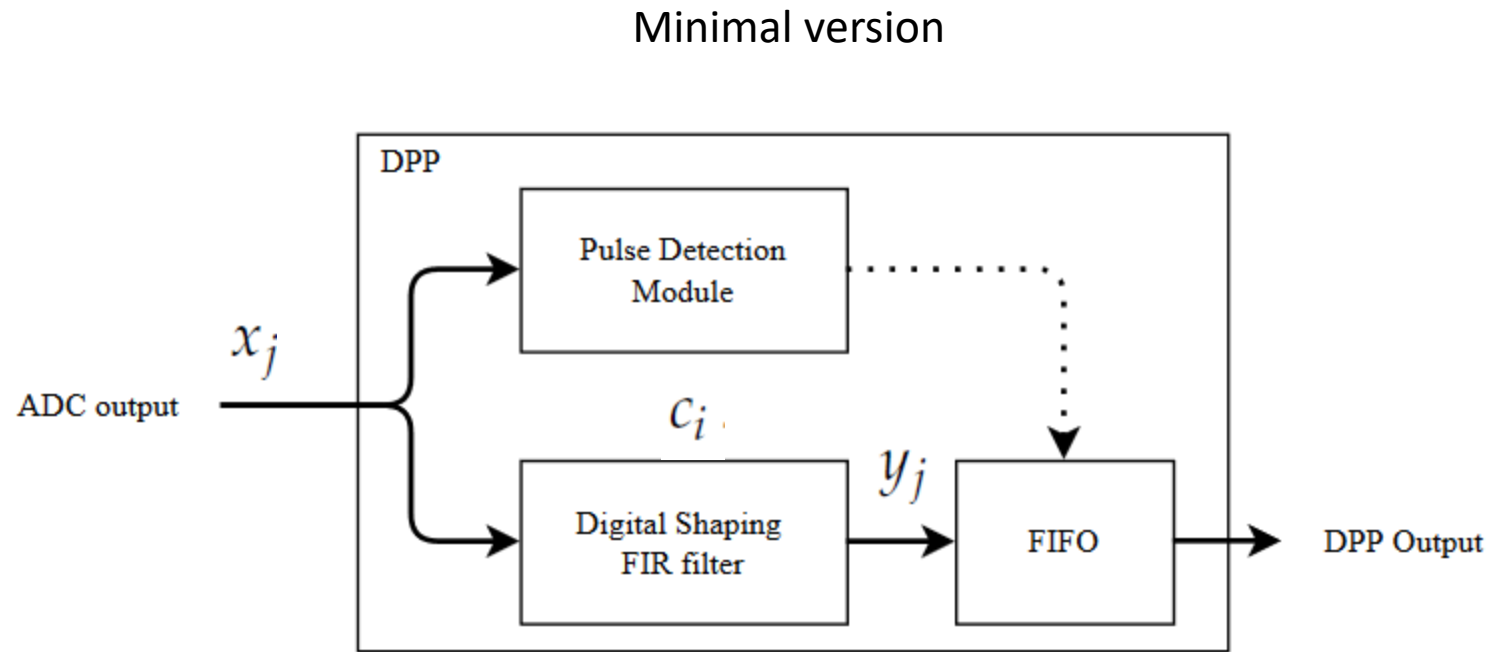


Pulse Processing Chain



Digital Pulse Processor (DPP)

Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .

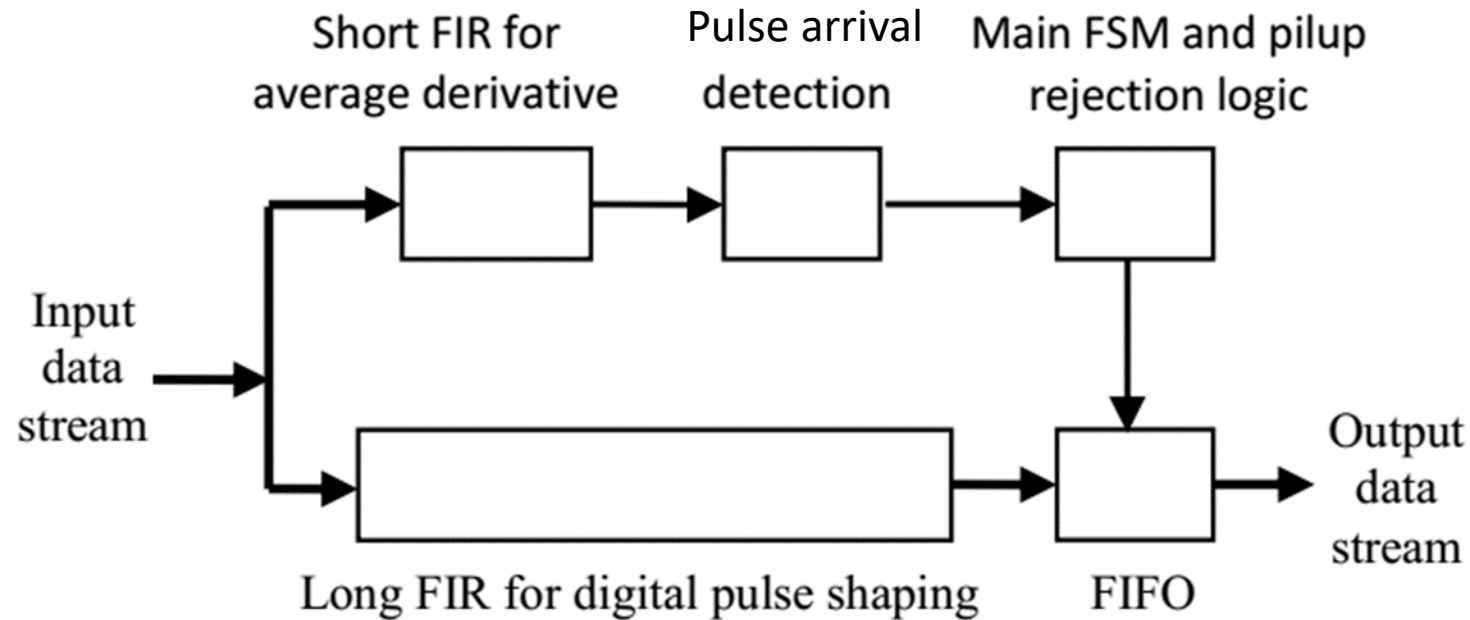


convolution

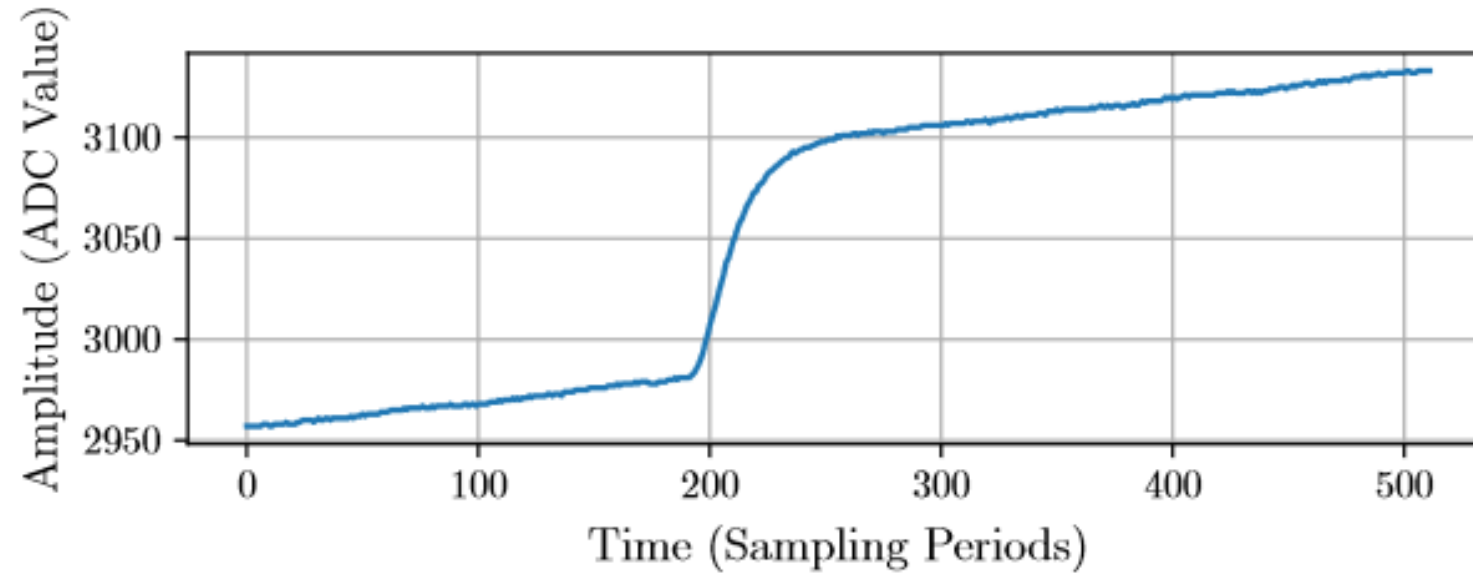
$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i}$$

Digital Pulse Processing Strategy for High-Resolution and High-Performance Amplitude Measurement

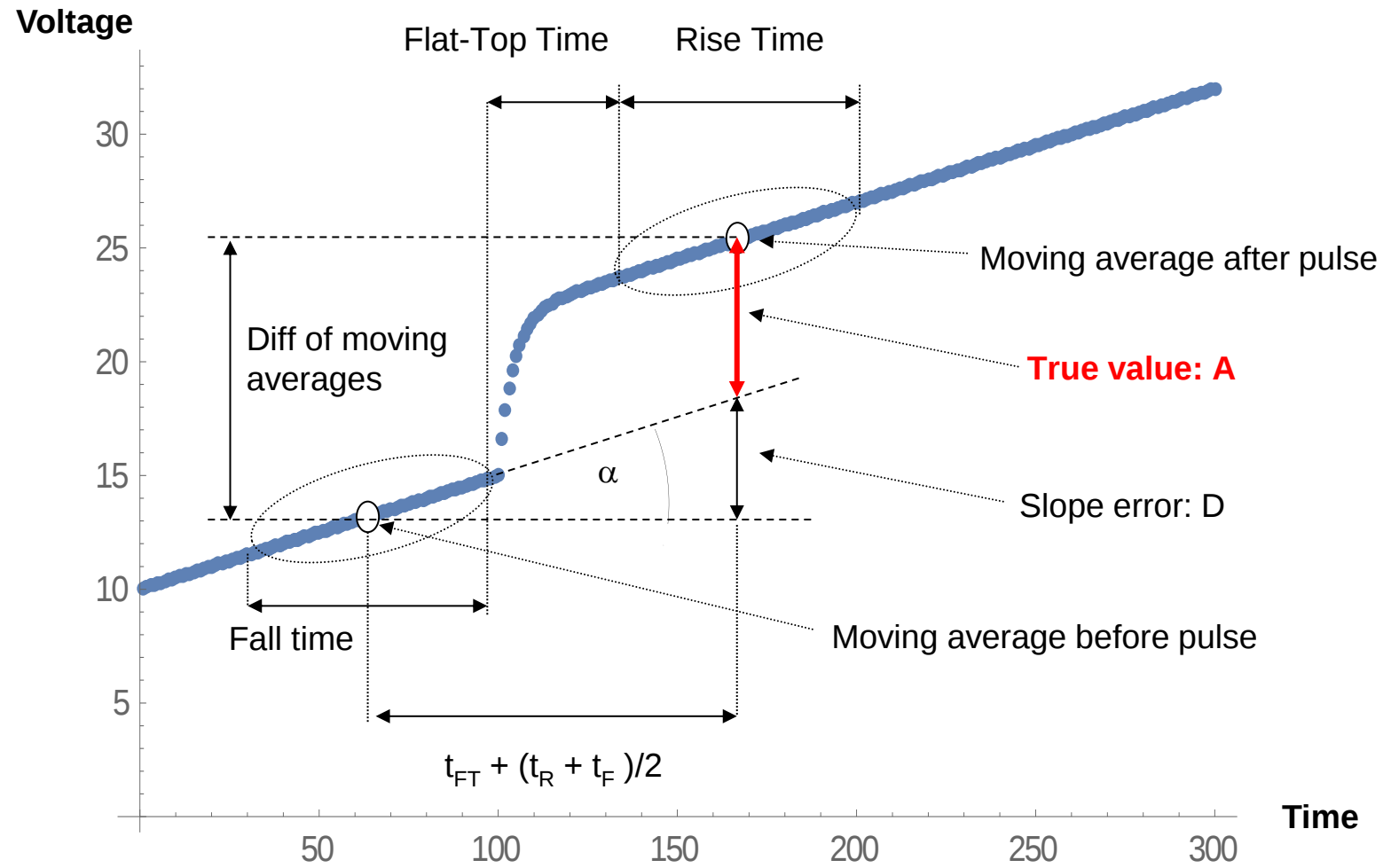
$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i}$$



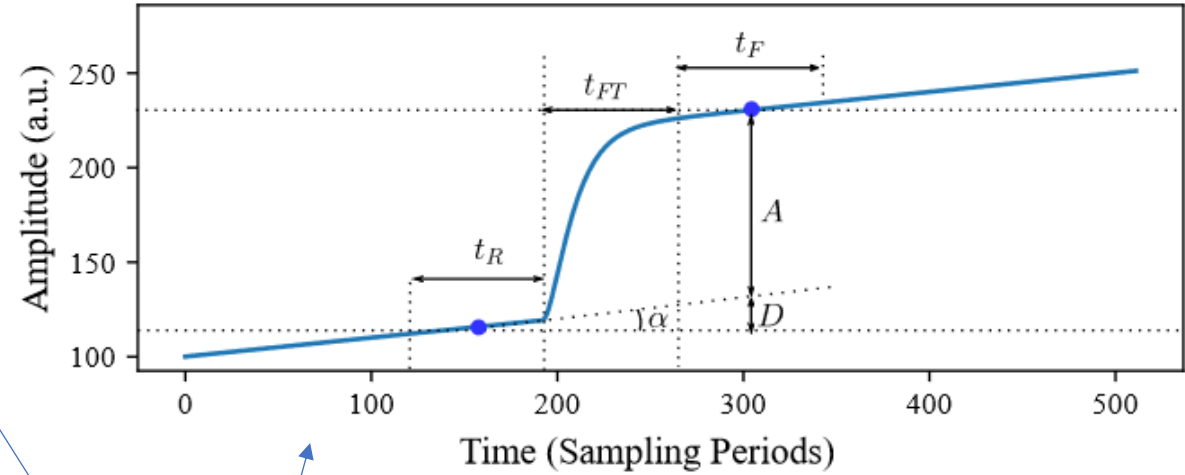
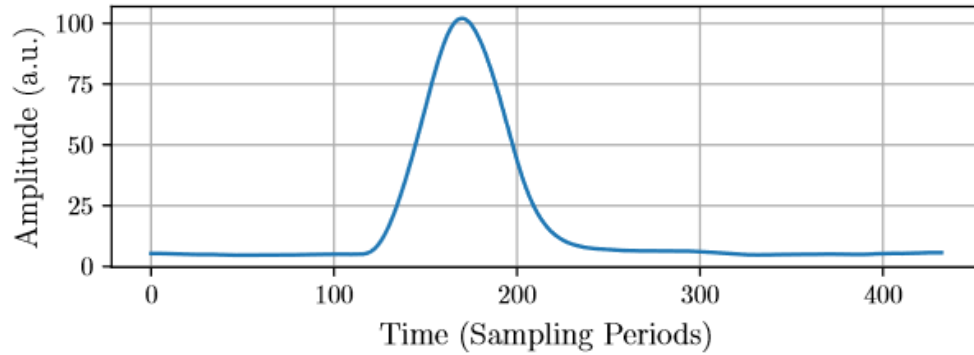
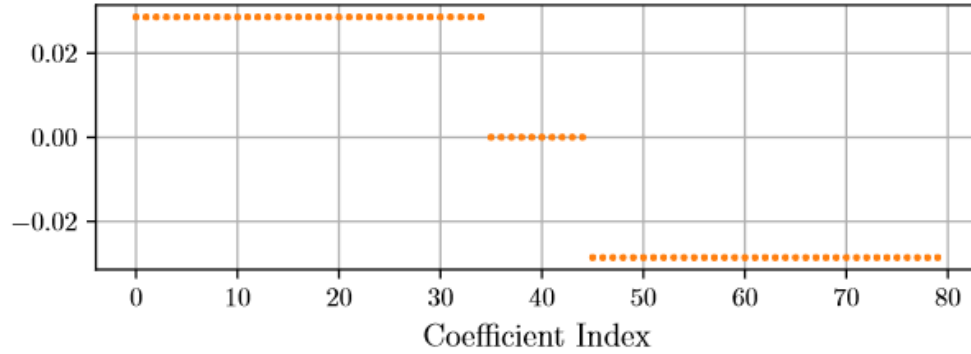
A typical experimental pulse



Pulse amplitude measurement



A simple trapezoidal shaper



$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i}$$

convolution

$$\tan \alpha = \frac{D}{\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F}$$

$$\tan \alpha \approx \sum_{i=0}^{t_R-1} -6 \left(\frac{1 + t_R - 2i}{t_R^3 - t_R} \right) x_i$$

After some algebra . . .

$$D = \sum_{i=0}^{t_R-1} -6 \left(\frac{1+t_R-2i}{t_R^3-t_R} \right) \left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F \right) x_i$$

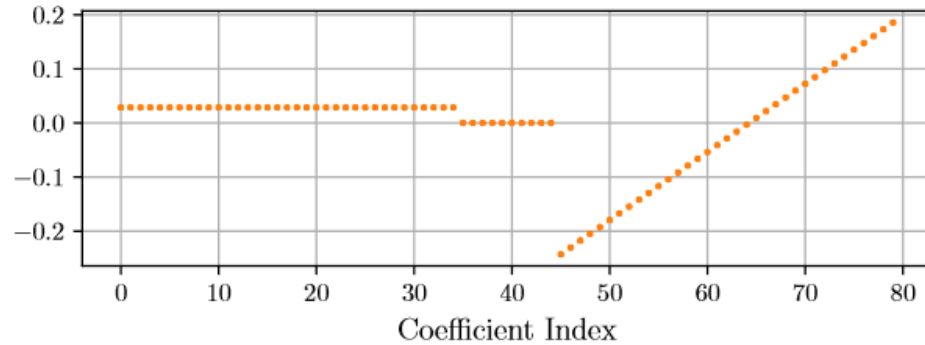
$$A = \frac{1}{t_F} \sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} x_i - \frac{1}{t_R} \sum_{i=0}^{t_R-1} x_i - \sum_{i=0}^{t_R-1} -6 \left(\frac{1+t_R-2i}{t_R^3-t_R} \right) \left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F \right) x_i$$

$$A = \sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} \frac{1}{t_F} x_i + \sum_{i=0}^{t_R-1} \left[-\frac{1}{t_R} + 6 \left(\frac{1+t_R-2i}{t_R^3-t_R} \right) \left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F \right) \right] x_i$$

← Linear combination

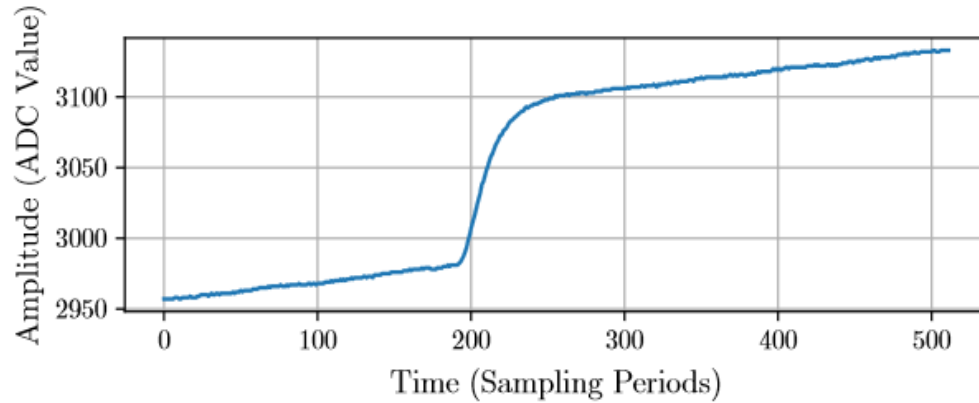
$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leq i < t_F; \\ 0, & t_F \leq i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6 \left(\frac{1+t_R-2i}{t_R^3-t_R} \right) \left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F \right), & t_F + t_{FT} \leq i < t_F + t_{FT} + t_R; \end{cases}$$

Geometrically Derived FIR Filter

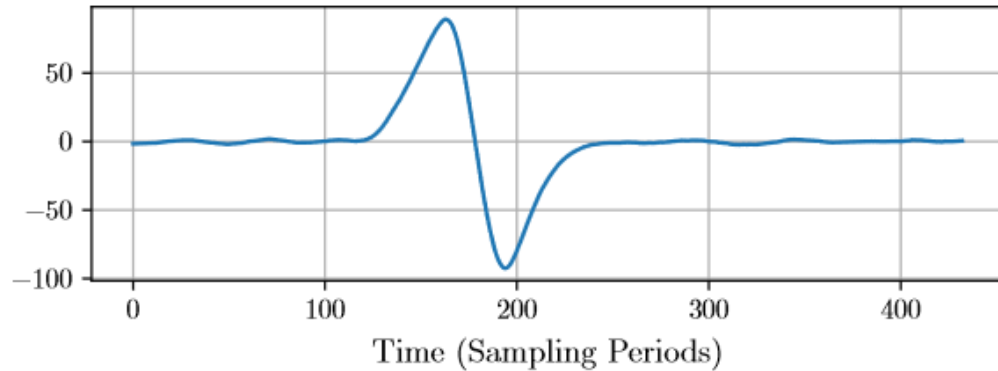


$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leq i < t_F; \\ 0, & t_F \leq i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6 \left(\frac{1+t_R-2i}{t_R^3-t_R} \right) \left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F \right), & t_F + t_{FT} \leq i < t_F + t_{FT} + t_R; \end{cases}$$

Input pulse



Output pulse



DPP Optimization

Pulse modeling

$$V(t) = \begin{cases} 0, & t \leq t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}), & t > t_0; \end{cases}$$

$$V(t) = \begin{cases} B_0 + B_1 t + n(t), & t \leq t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}) + B_0 + B_1 t + n(t), & t > t_0; \end{cases}$$

$$x_i = \begin{cases} B_0 + B_1 i + n_i, & i \leq t_0; \\ A(1 - e^{\frac{-(i-t_0)}{\tau}}) + B_0 + B_1 i + n_i, & i > t_0; \end{cases}$$

DPP Optimization

Pulse modeling

Deterministic component
(ideal pulse)

Increasing information

$$S_i = \begin{cases} B_0 + iB_1, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}) + B_0 + iB_1, & i > t_0 \end{cases}$$

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}), & i > t_0 \end{cases}$$

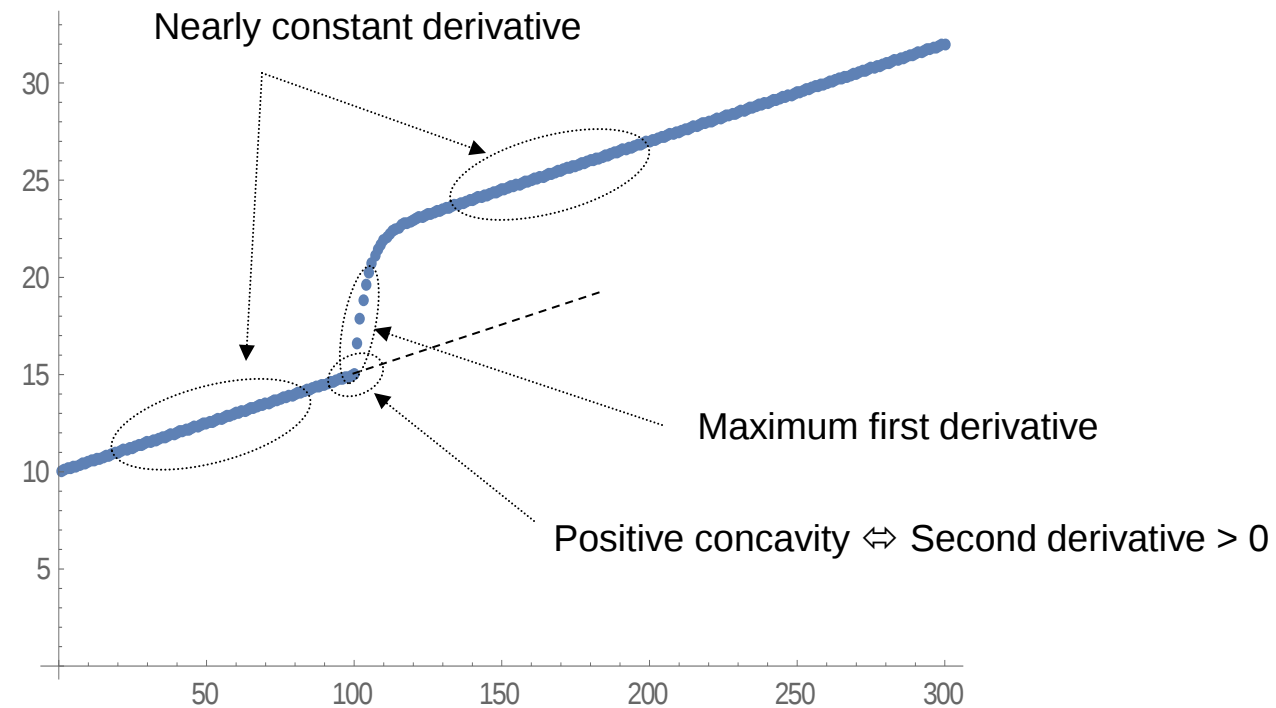
$$S_i = \begin{cases} 0, & i \leq t_0 \\ A, & i > t_0 \end{cases}$$

Deterministic component

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

Stochastic component

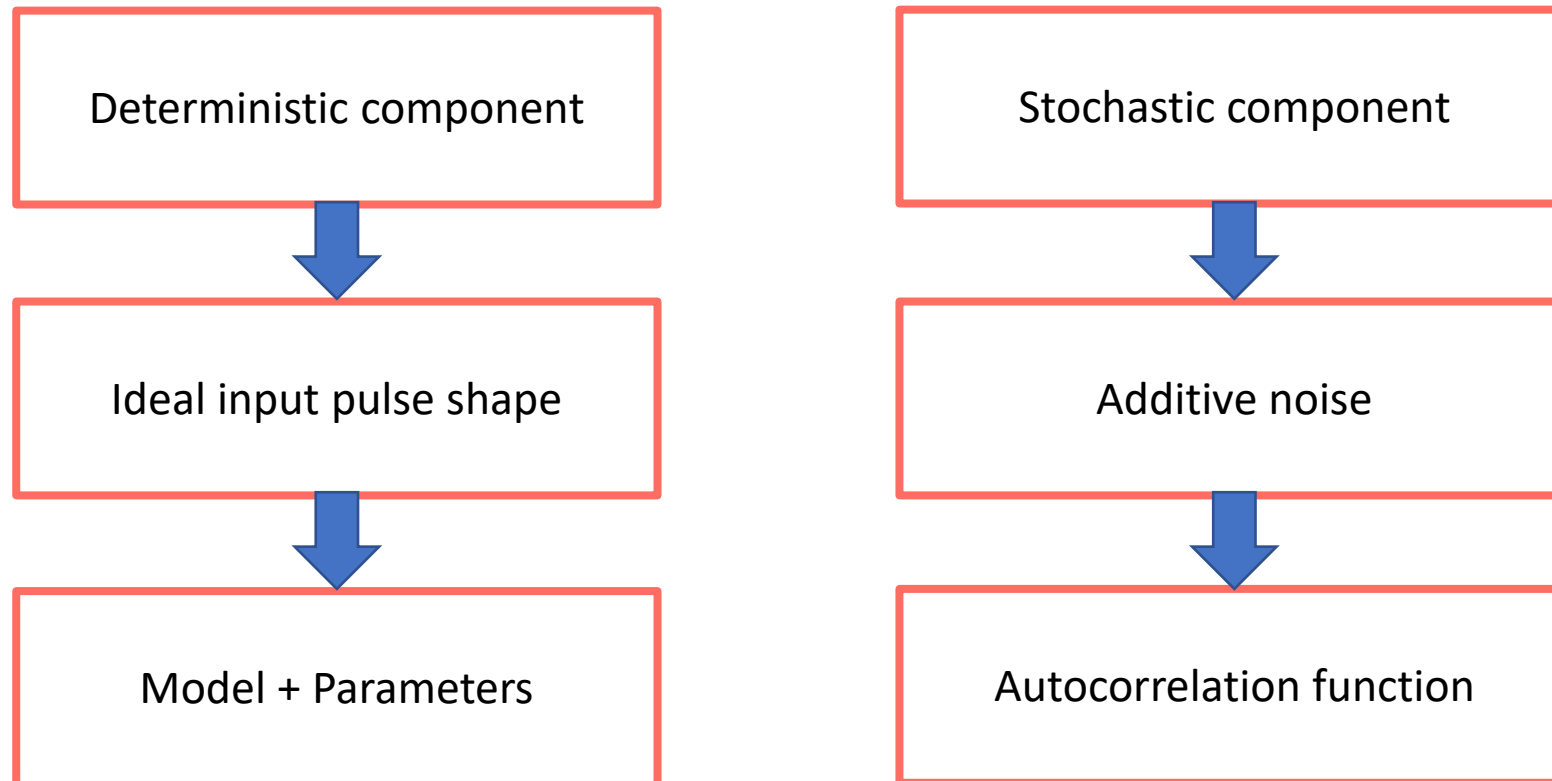
Digital Pulse Processing: Detecting Arrival Time



A short FIR can compute different discrete derivatives

FIR Design and Optimization

Input signal analysis



FIR Design and Optimization

Input pulse modeling I

The ideal case corresponding to a single photon detection is represented by the step function S_i

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A, & i > t_0 \end{cases}$$

The finite frequency response of the CSA determines a limited rise time that could be modeled (1st aprox) as an exponential growth

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}), & i > t_0 \end{cases}$$

A constant detector leakage current determines a baseline with a steady slope and a variable offset on top of which the signal segment must be processed

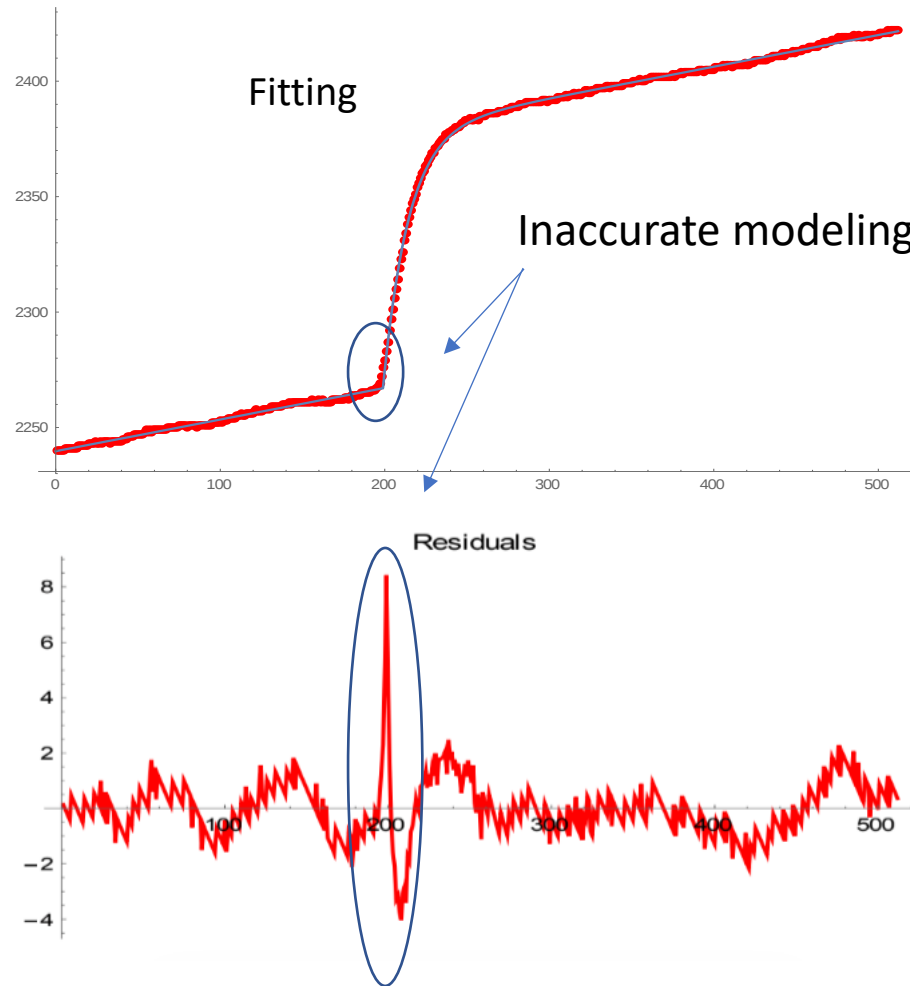
$$S_i = \begin{cases} B_0 + iB_1, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}) + B_0 + iB_1, & i > t_0 \end{cases}$$

Several sources of noise will contribute with an additive spurious signal n_i that degrades the voltage step measurement

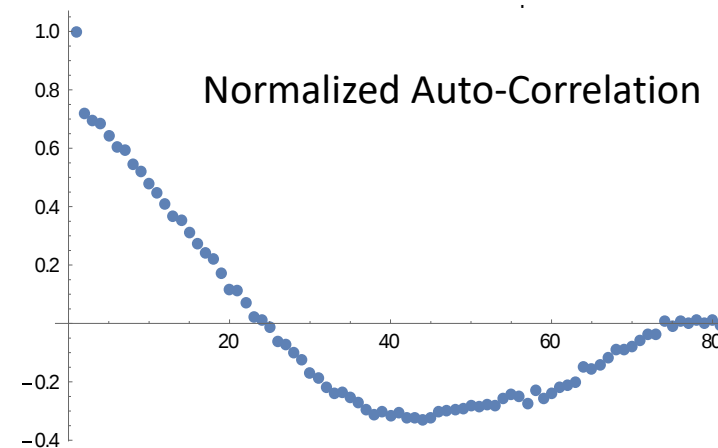
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A(1 - e^{-(i-t_0)/\tau}) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

FIR Design and Optimization

Input noise characterization



$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$



Some statistic results from the extracted parameters after fitting 1447 data segments with the special bi-exponential function.

The proposed signal model has five (quite independent) parameters

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0 \left(2 \left(1 - e^{-(i-t_0)/\tau} \right) - \left(1 - e^{-2(i-t_0)/\tau} \right) \right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

(1) Amplitude,

(2) Arrival Time,

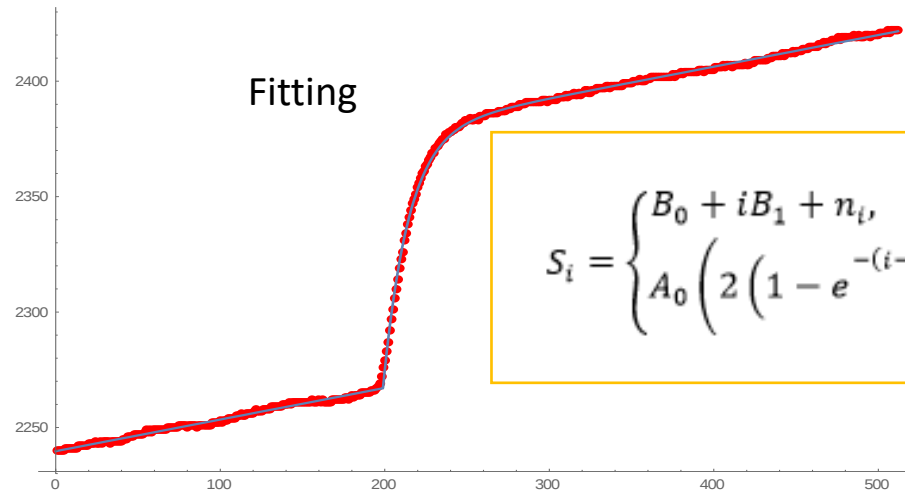
(3) Exponential Time,

(4) Offset,

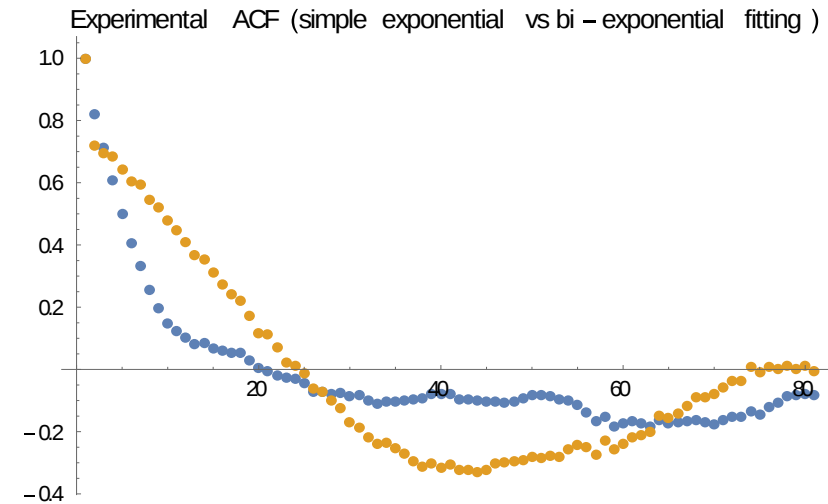
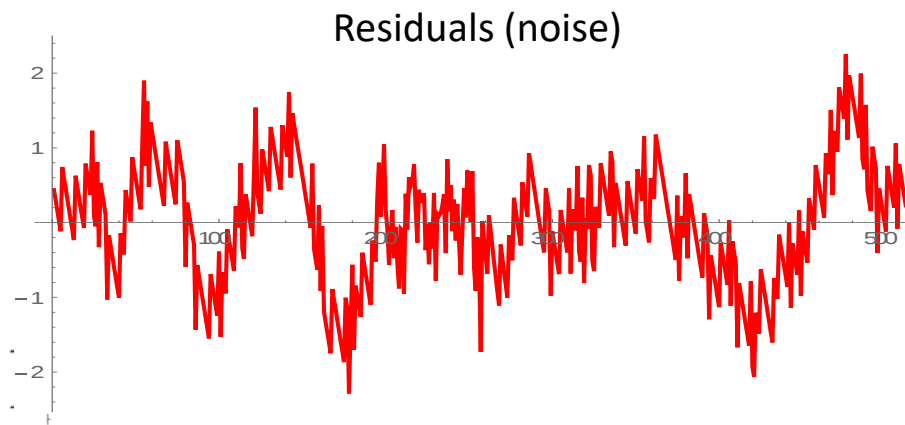
(5) Slope Coefficient

FIR Design and Optimization

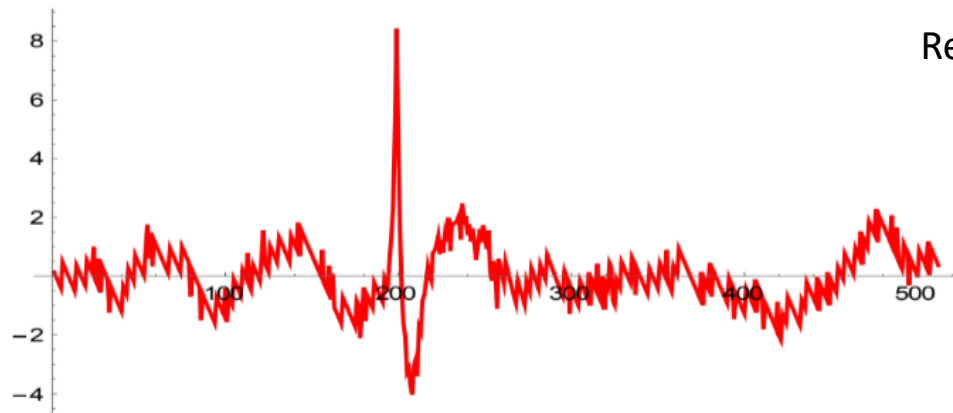
Input noise characterization



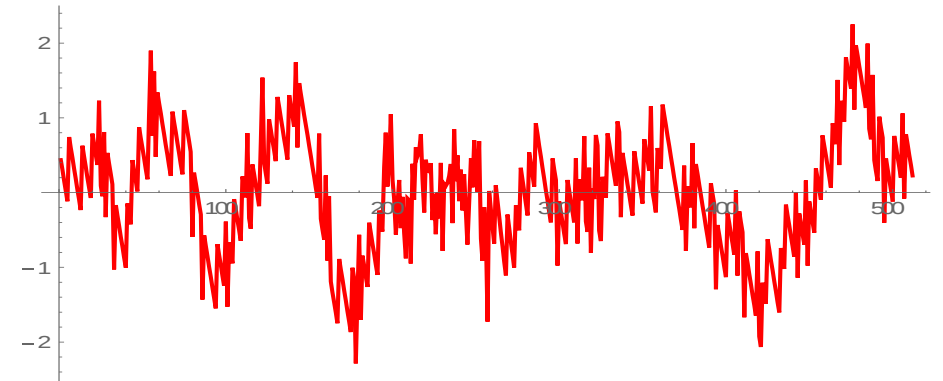
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0 \left(2 \left(1 - e^{-(i-t_0)/\tau} \right) - \left(1 - e^{-2(i-t_0)/\tau} \right) \right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$



Pulse models comparison

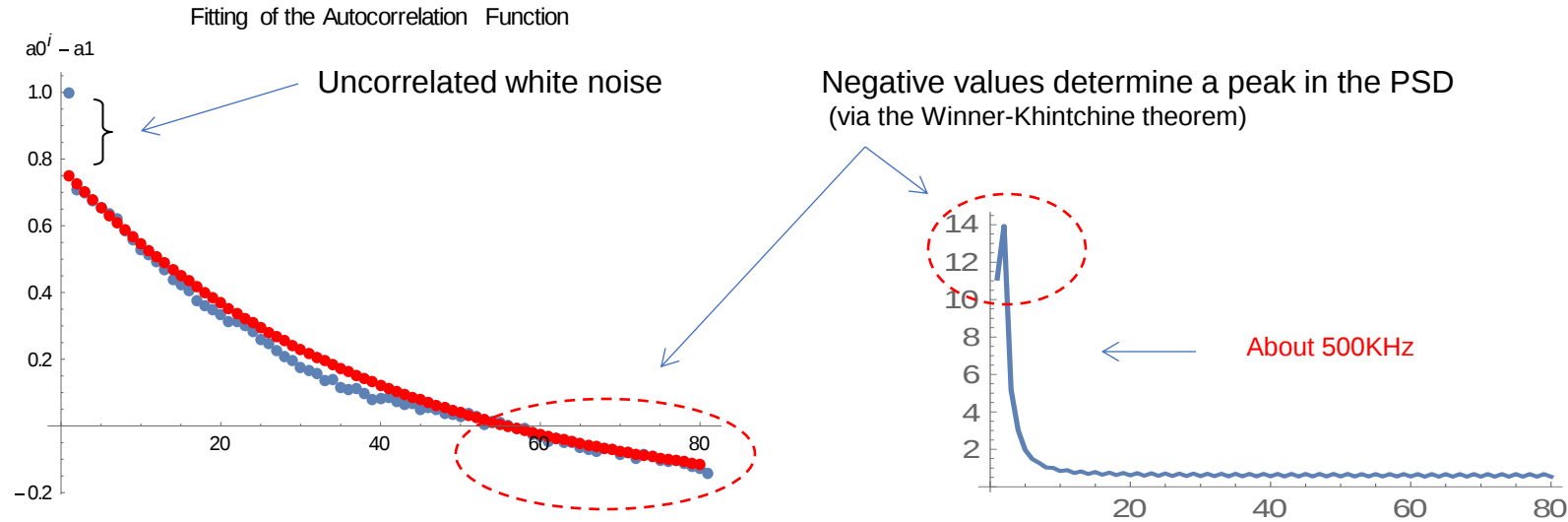


Residuals (noise)



	Exponential Model	Bi-Exponential Model
Mean quadratic residuals	6201	5914
Mean peak-to-peak residuals	13.7	6.6
Mean Akaike information criterion	1720	1397

Autocorrelation model



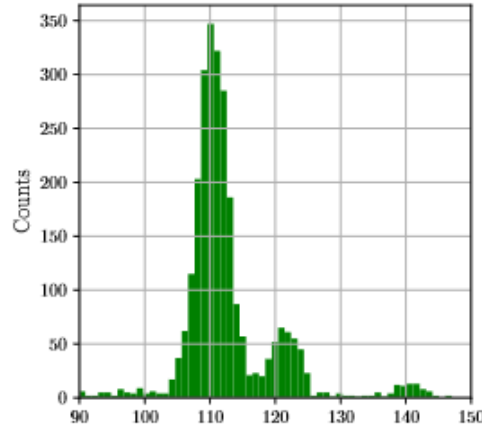
$$ACF(i, a_0, a_1) = \begin{cases} 1 & , \quad i = 0 \\ a_0^i + a_1 & , \quad i \geq 1 \end{cases}$$

In this case the average normalized ACF can be approximated with $a_0=0.965$ and $a_1=-0.2$

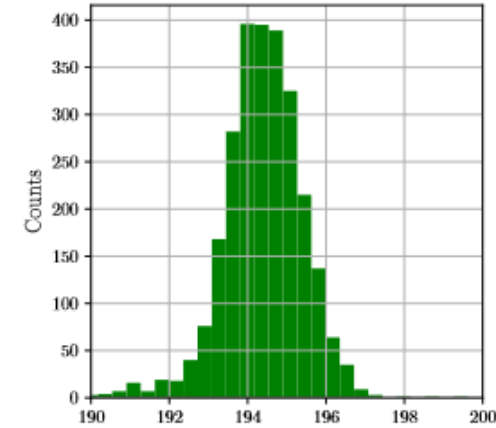
Histograms of fitting parameters corresponding to the bi-exponential model.

They look nice but . . .

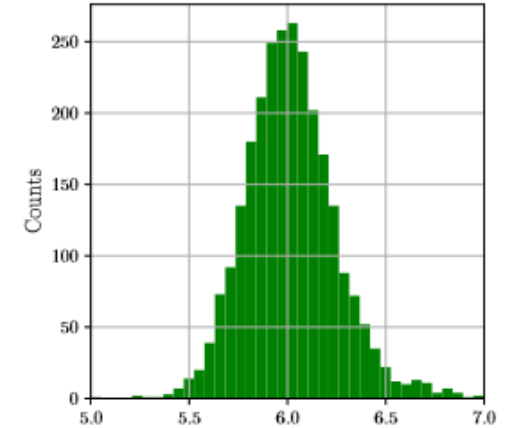
. . . scattered plots and correlation distances between pairs of parameter vectors may reveal non-idealities of the DAQ system.



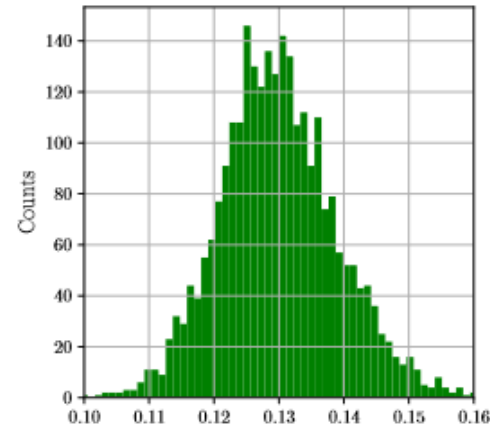
(a) Amplitude (A)



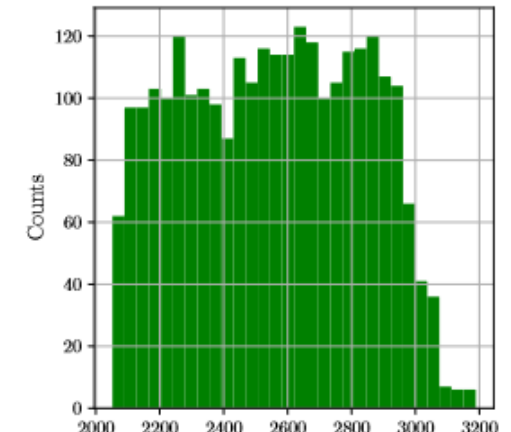
(b) Arrival time (t_0)



(c) Exponential time (τ)

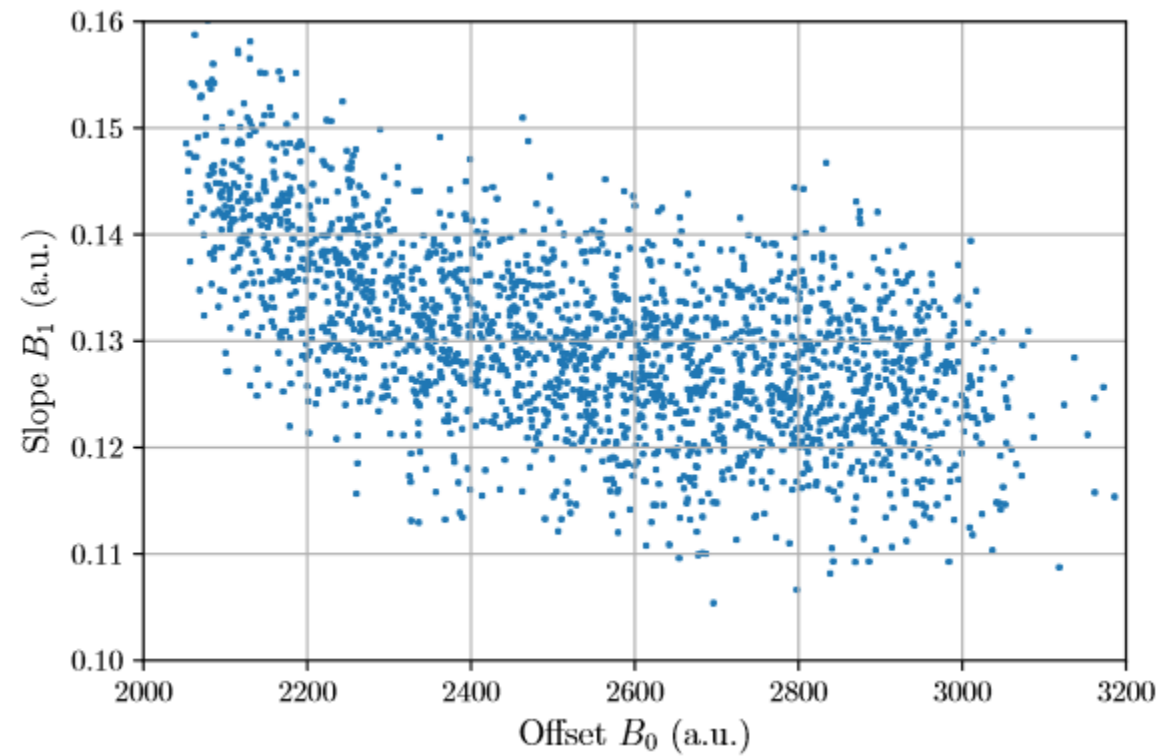


(d) Slope (B_1)

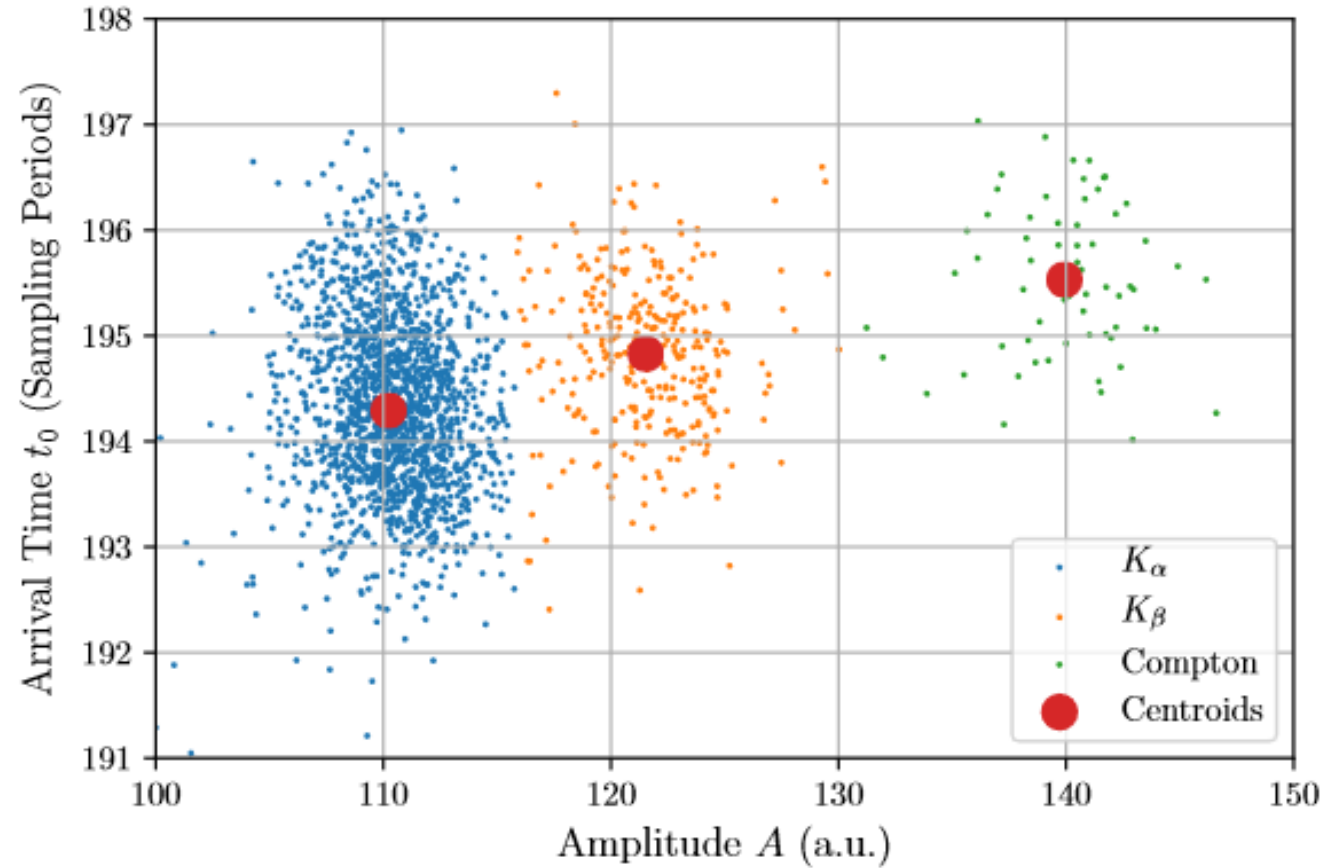


(e) Offset (B_0)

Nonlinearity: Amplification gain depends on offset (!)



Detection of arrival time depends on pulse amplitude (!)



DPP: Digital Penalized LMS Method for filtering optimization

$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i}$$

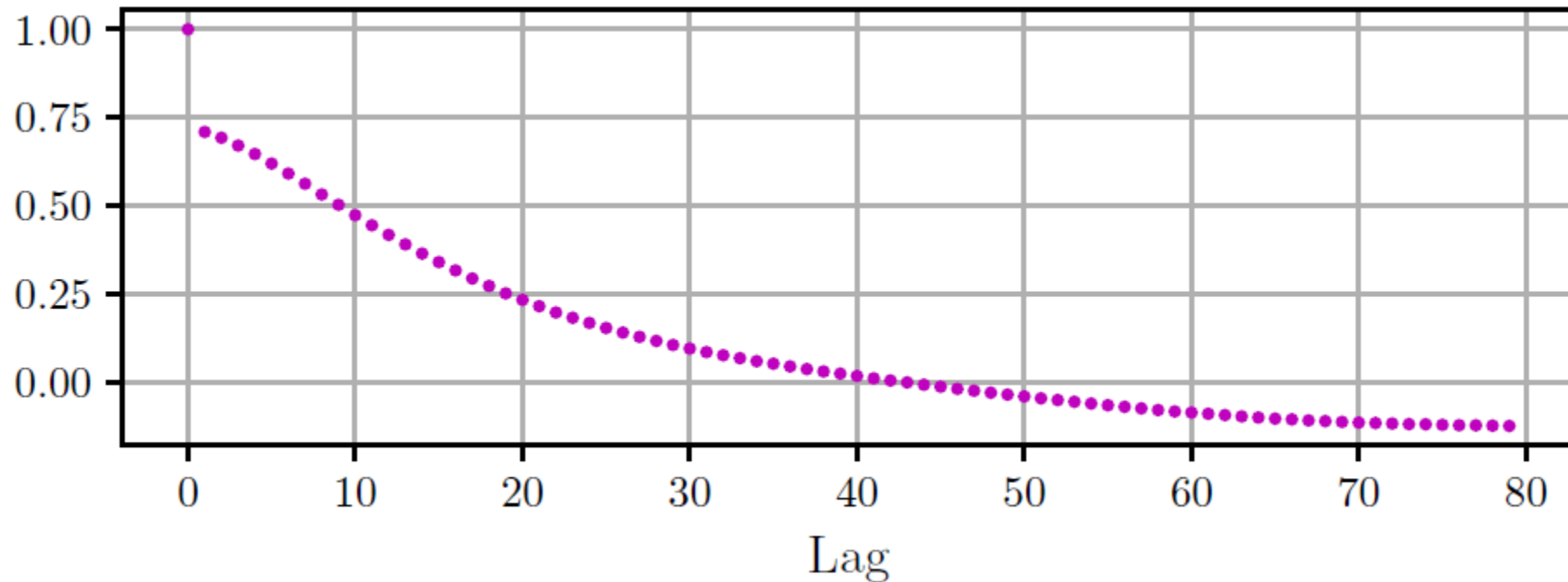
$$\sigma_y^2 = \left\langle (y - \langle y \rangle)^2 \right\rangle = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} c_i c_j \underbrace{\langle x_i - \langle x_i \rangle \rangle \langle x_j - \langle x_j \rangle \rangle}_{\text{Covariance Matrix } V_{i,j}}$$

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i c_j ACF(|i - j|)$$

$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$

DPP: Digital Penalized LMS Method for filtering optimization

Normalized average ACF



DPP: Digital Penalized LMS Method for filtering optimization

$$\sum_{i=0}^{k-1} c_i = 0$$

Offset rejection

$$\sum_{i=0}^{k-1} c_i i = 0$$

Slope rejection

Ideal
requirements

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i c_j ACF(|i - j|)$$

Output noise

$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i} = A, \quad j \in [t_R, t_R + t_{FT} - 1]$$

Output flat top

DPP: Digital Penalized LMS Method for filtering optimization

$$\Psi(c_0, c_1, \dots, c_{k-1}) = \alpha_1 \left(\sum_{i=0}^{k-1} c_i \right)^2 + \alpha_2 \left(\sum_{i=0}^{k-1} c_i i \right)^2 + \alpha_3 \sum_{j=t_R}^{t_R+t_{FT}-1} \left(\sum_{i=0}^{k-1} c_i x_{k+j-i} - A \right)^2 +$$

$$\alpha_4 \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i c_j ACF_{|i-j|}$$

$$y_j = \sum_{i=0}^{k-1} c_i x_{j-i} = A, \quad j \in [t_R, t_R + t_{FT} - 1]$$

$$\{c_0, c_1, \dots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \dots, c_{k-1}\}}{\operatorname{argmin}} \Psi(c_0, c_1, \dots, c_{k-1})$$

DPP: Digital Penalized LMS Method for filtering optimization

$$\{c_0, c_1, \dots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \dots, c_{k-1}\}}{\operatorname{argmin}} \Psi(c_0, c_1, \dots, c_{k-1})$$

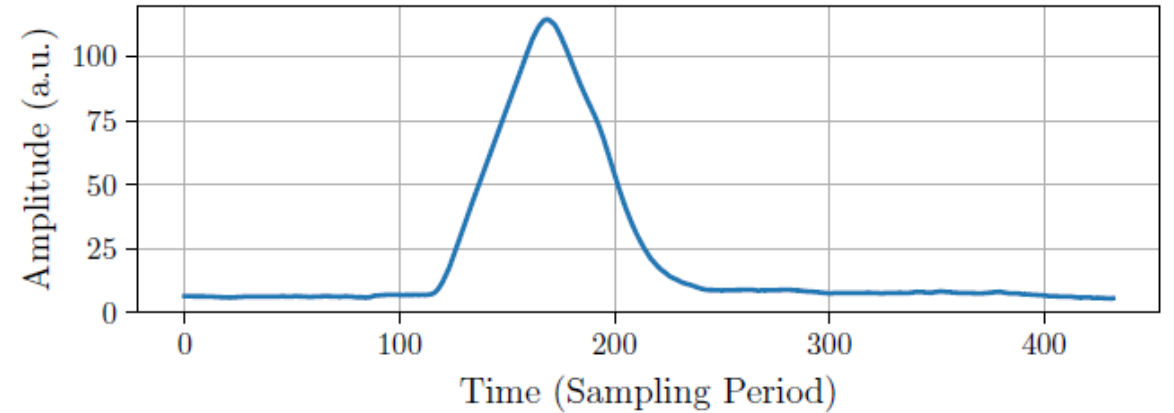
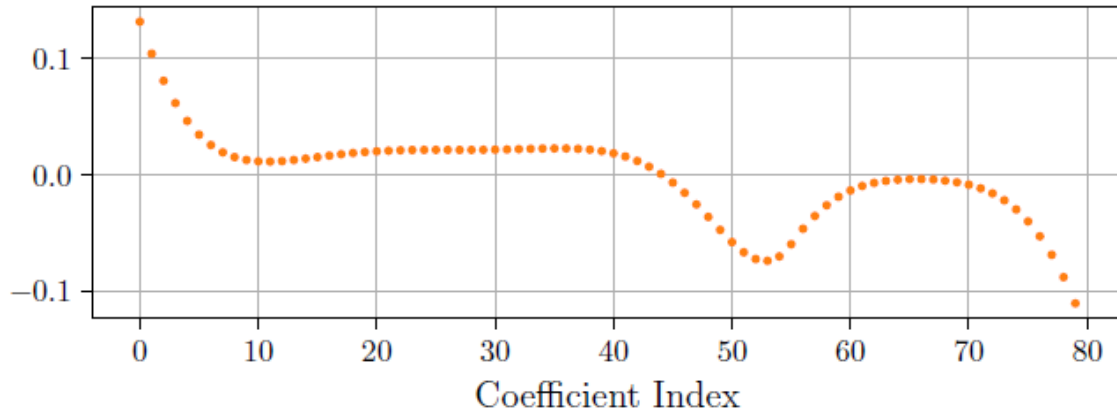


Table 3. Comparison of energy resolutions with different methods to estimate the energy spectrum.

Method	FWHM K_{α} [eV]	FWHM K_{β} [eV]	Slope-Error Correction
GD FIR	286 ± 4	316 ± 16	yes
Fitting [†]	267 ± 4	288 ± 17	yes
Trapezoidal FIR	207 ± 3	247 ± 17	no
DPLMS FIR	202 ± 2	233 ± 12	no

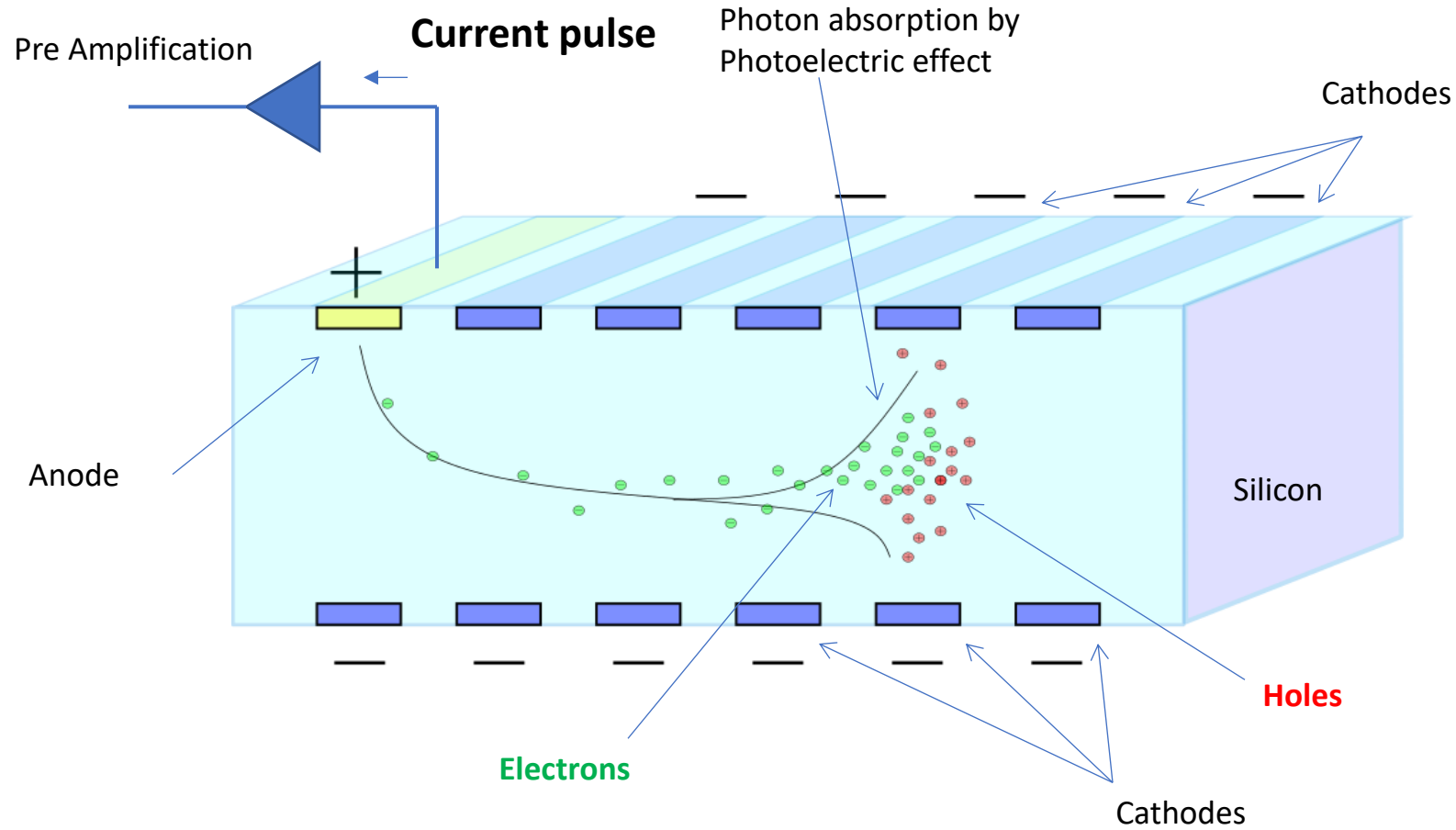
[†] These results correspond to the histogram of the amplitudes obtained by fitting all available photon traces.

Conclusions

- ✓ **High-resolution pulse amplitude measurement** can be achieved by accounting for actual experimental noise and using accurate pulse modeling.
- ✓ **Digital Pulse Processing (DPP)** can be optimized through the **DPLMS** method, providing a satisfactory trade-off among competing requirements that cannot all be simultaneously satisfied.
- ✓ Appropriate **data analysis** supplies the information needed to apply the DPLMS method and can also offer insights into the performance of the **frontend electronics** and **data acquisition system**.

Tank you !

Backup slides: X-Ray Photon detection with Silicon Drift Detectors (SDD)



Backup slides: Pile up (1)

Pile up: Being a Poissonian process, two or more photons may be absorbed in the SDD within an arbitrary small time-window. The superposition of two photons absorbed at times t_0 and t_1 , with respective amplitudes A_0 and A_1 , is then given by

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0 \left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & t_0 < i \leq t_1 \\ A_0 \left(1 - e^{-(i-t_0)/\tau}\right) + A_1 \left(1 - e^{-(i-t_1)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_1 \end{cases}$$

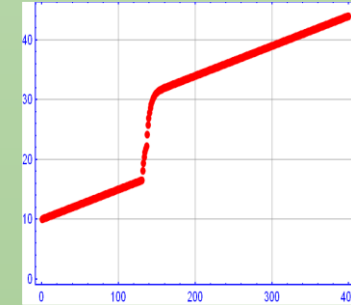
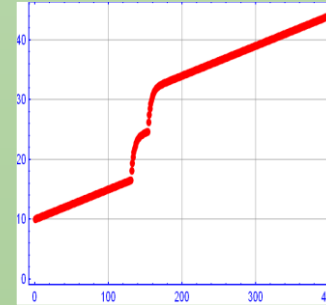
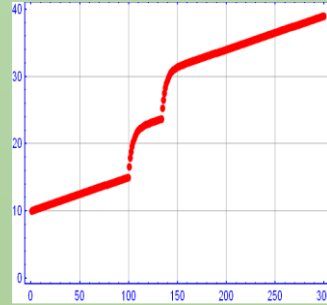
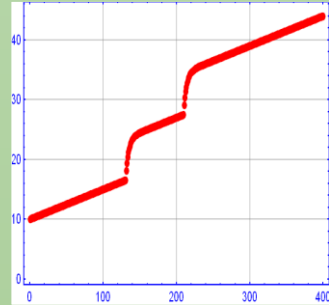
Backup slides: Pile up (2)

... and in general, for $m+1$ photons

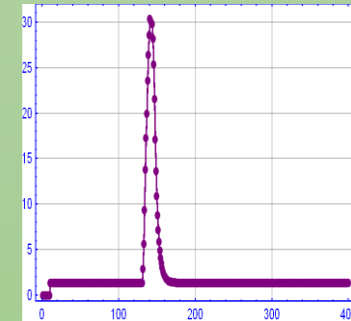
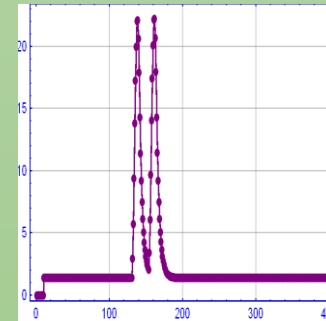
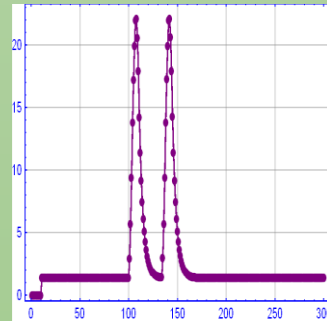
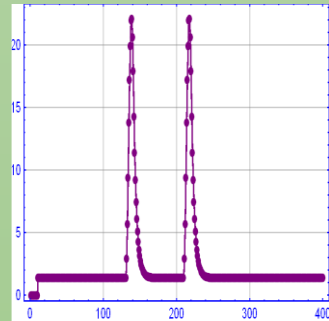
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0(1 - e^{-(i-t_0)/\tau}) + B_0 + iB_1 + n_i, & t_0 < i \leq t_1 \\ A_0(1 - e^{-(i-t_0)/\tau}) + A_1(1 - e^{-(i-t_1)/\tau}) + B_0 + iB_1 + n_i, & t_1 < i \leq t_2 \\ \vdots & \\ \sum_{j=0}^m A_j (1 - e^{-(i-t_j)/\tau}) + B_0 + iB_1 + n_i, & i > t_m \end{cases}$$

Backup slides: Pileup rejection

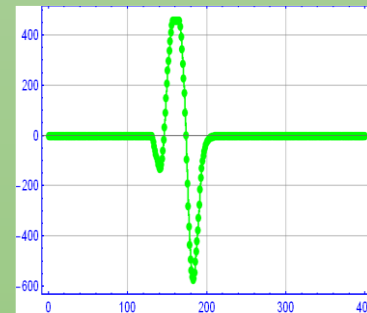
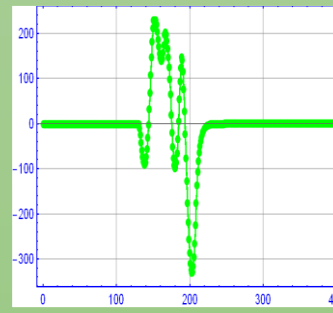
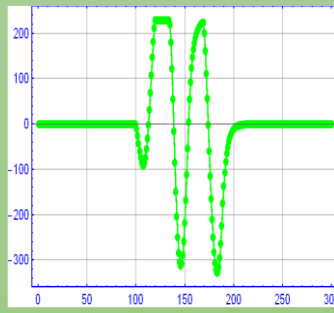
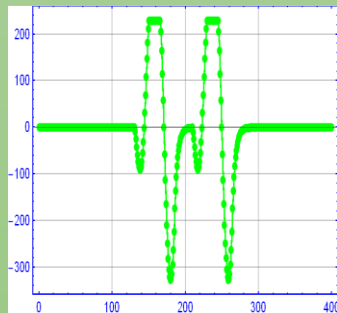
Input
Pulses



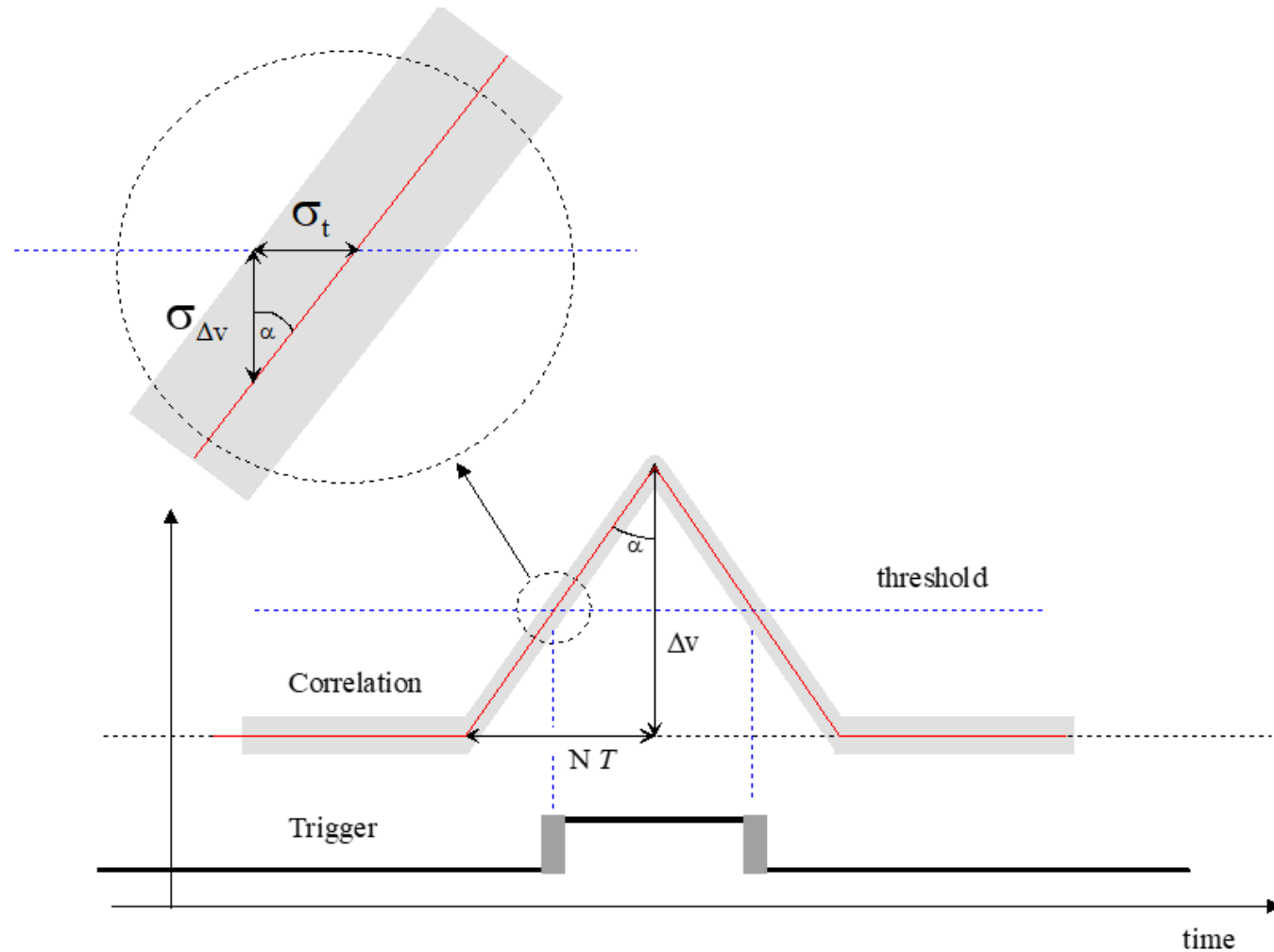
First
Derivs



Shaped
Pulses



Backup slides: Uncertainty relation between **Energy** and **Time**



$$\sigma_t \sigma_{\Delta V} = \frac{2T}{\Delta V} \sigma_x^2$$