Introduction to measurement uncertainty in dosimetry

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Overview

Introduction to the GUM

Some definitions

Type A and type B uncertainties

Expanded uncertainty

Measurement equation

Degrees of freedom

Example

Dose verification of a clinical IMRT plan, recalculated on a cylindrical phantom; planned dose 2 Gy

Zakithi measures 2.01 Gy, Karen measures 1.97 Gy

Which statement is correct?

A. These results disagree by 2%

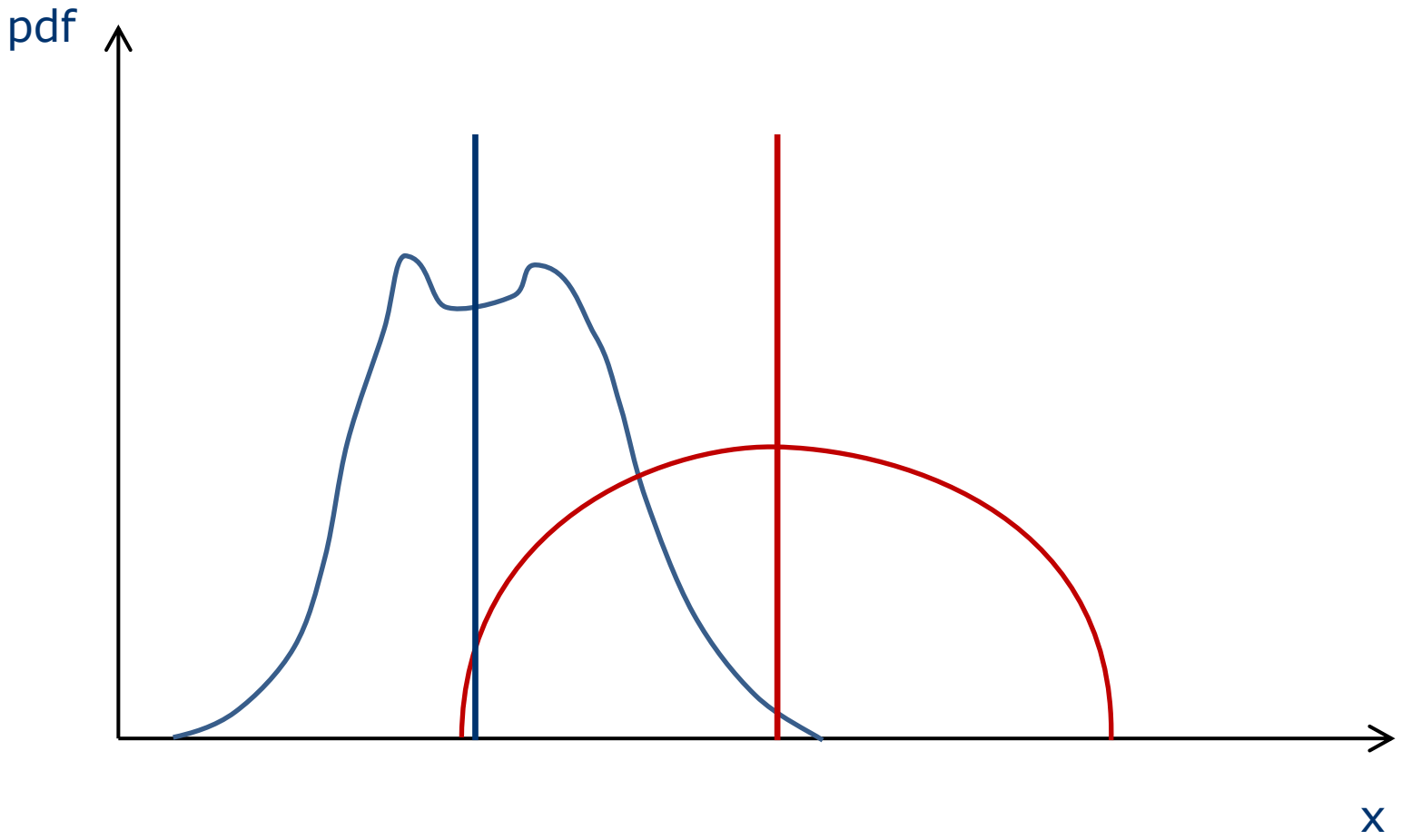
B. These results are in good agreement

C. Z and K cannot agree, invite another expert

D. None of the above

A MEASUREMENT WITHOUT UNCERTAINTY IS MEANINGLESS

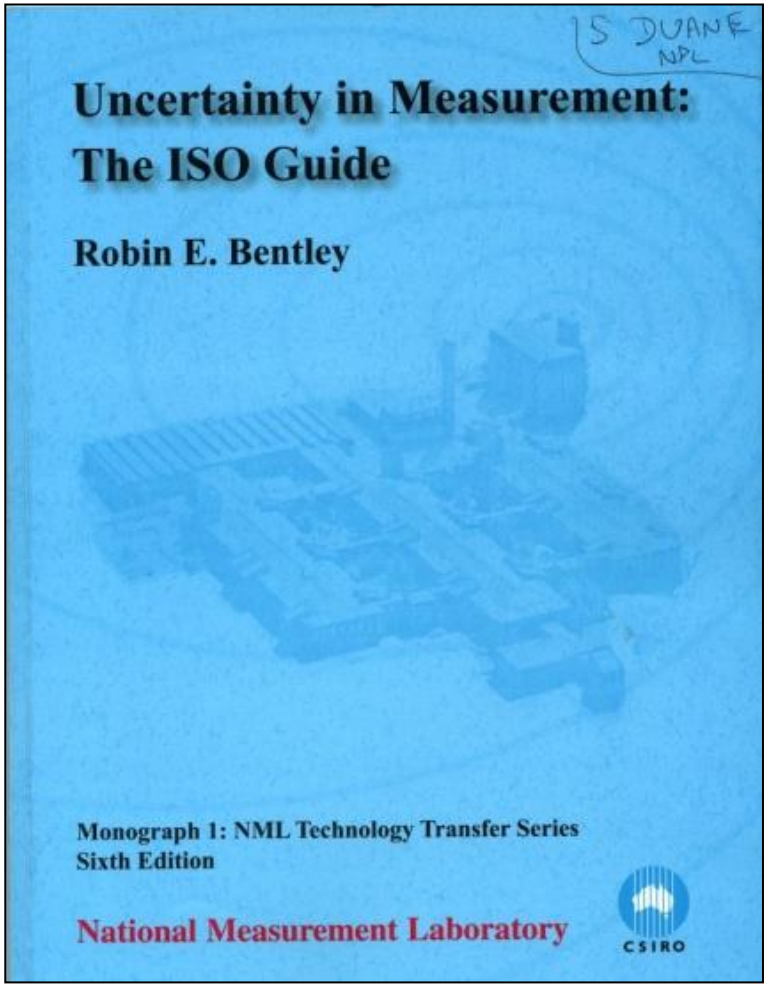
Testing if results agree

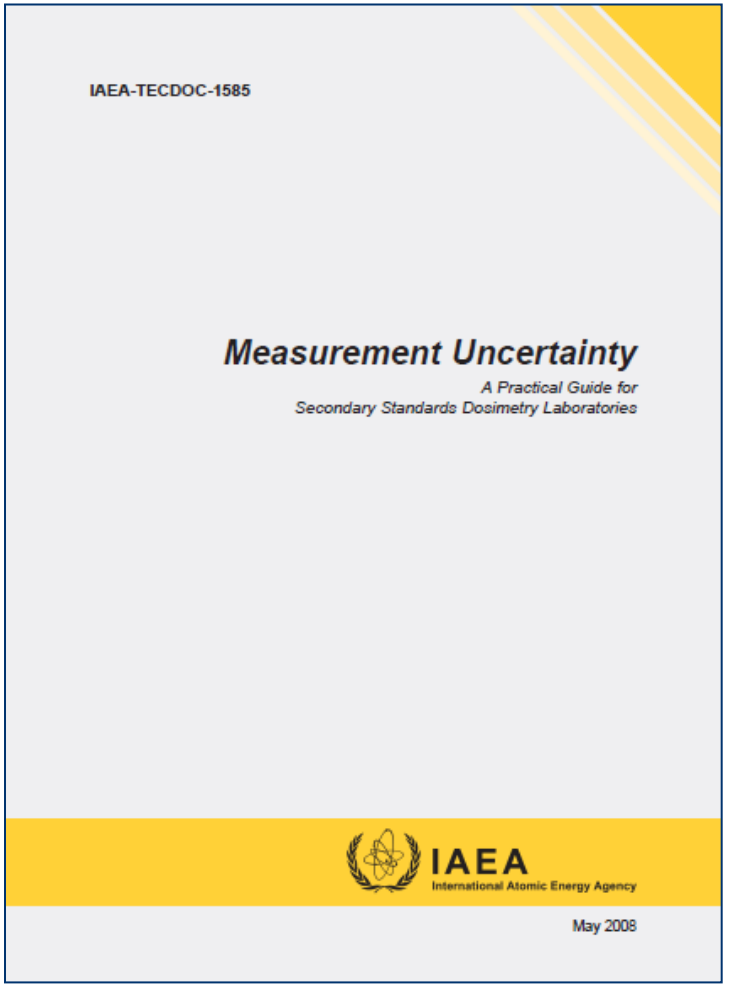


Guide to the Expression of Uncertainty in Measurement

Evaluation of measurement data – Guide to the expression of uncertainty in measurement
JCGM 100:2008
(GUM 1995 with minor corrections)
http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

More friendly introductions to the *ISO Guide*





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Review article

Uncertainty estimation for dosimetry in radiation oncology

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ARTICLE INFO

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Uncertainty budget

ABSTRACT

Accurate dosimetry in radiation oncology is one of the main pillars of a successful cancer treatment. An integral part of dose measurements and calculations is the thorough and adequate assessment of the involved uncertainties. Unfortunately, this step is often neglected or done in a way which makes it challenging for others to understand the quality of presented results. In this review we summarize the concepts on the expression of uncertainties and apply these to practical examples in the context of radiation oncology. Basic concepts such as Type A and Type B uncertainties, their assessment and potential correlations as well as more advanced concepts such as effective degrees of freedom are introduced. Finally, this review gives suggestions on how the results can be presented, how the resulting uncertainty budget can be used to identify the main contributors and how the combined uncertainty can be used to establish tolerance and action limits for quality assurance

Measurement

A measurement without uncertainty is meaningless

A measured value is an estimate of the "true" value of a measurand

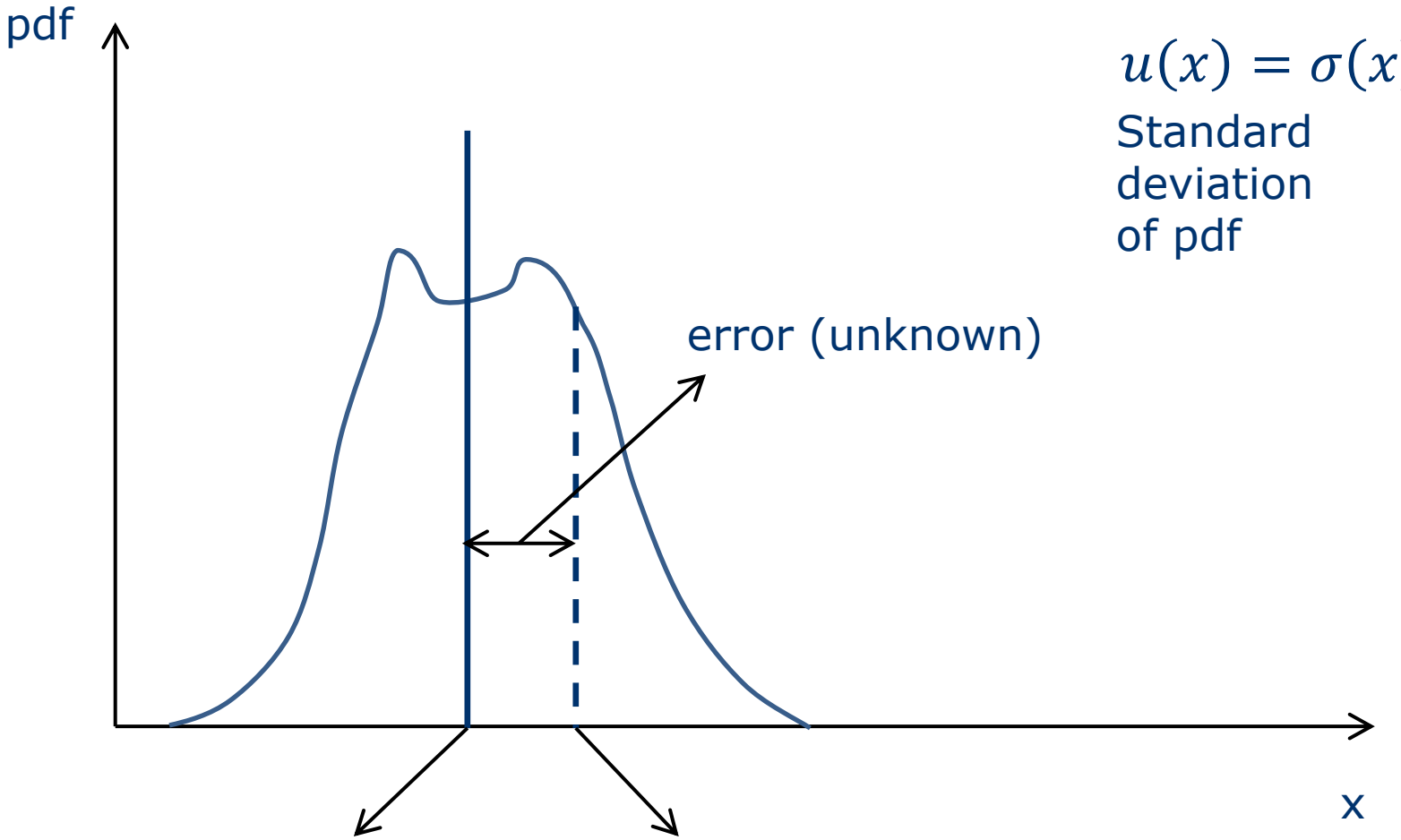
Error is the difference between the estimate and the "true" value

Uncertainty *is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand*

Coverage intervals should provide quantitative information on how good the estimate is…

In addition: we can also quantify the quality of the uncertainty assessment by # degrees of freedom

True value, measured value, error, uncertainty



$u(x) = \sigma(x)$

Standard deviation of pdf

Standard uncertainty - notation

$D_w = 2.01\ \text{Gy}, \quad u(D_w) = 0.03\ \text{Gy}$

$D_w = (\ 2.01\ \pm 0.03\ )\ \text{Gy}$

$u(D_w) = \pm\ 0.03\ \text{Gy}$

Type-A and type-B uncertainties

The uncertainty of a measurement consists of components which are grouped into two categories according to the way in which their numerical value is estimated:

A. those which are evaluated by the statistical analysis of a series of observations,

B. those which are evaluated by other means.

Type A

Sampled quantity x_i

Mean value $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

Standard deviation $s(x_i) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

Standard deviation of the mean value $s(\bar{x}) = \frac{s(x_i)}{\sqrt{n}} = u_A(\bar{x})$

If x_i is normally distributed the mean value is also normally distributed with confidence limits according to a t-distribution

degrees of freedom: $\nu = n - 1$

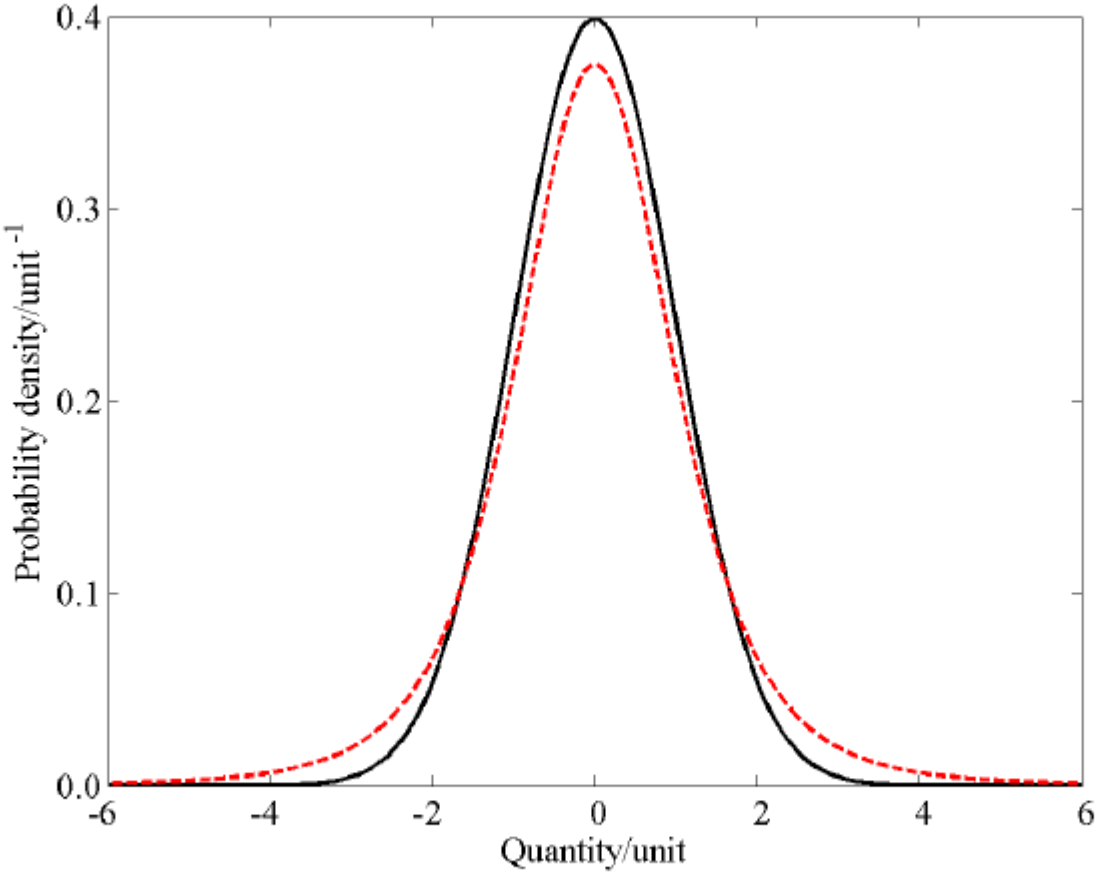


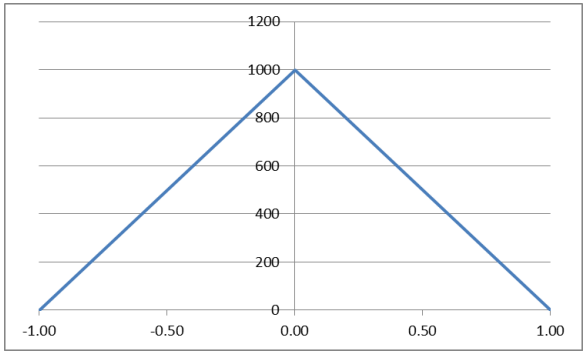
Figure 1 — A Gaussian distribution (continuous black curve) and a t-distribution with four degrees of freedom (broken red curve) ('unit' denotes any unit)

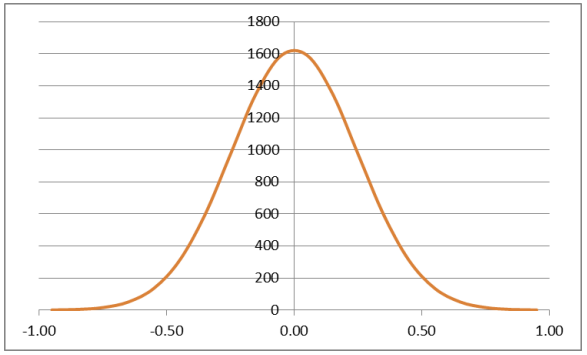
Type-A if underlying distribution not normal

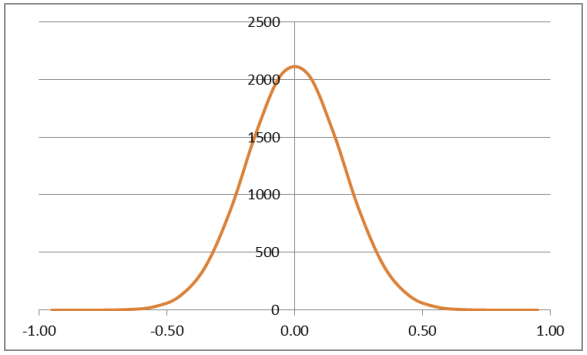
PDF

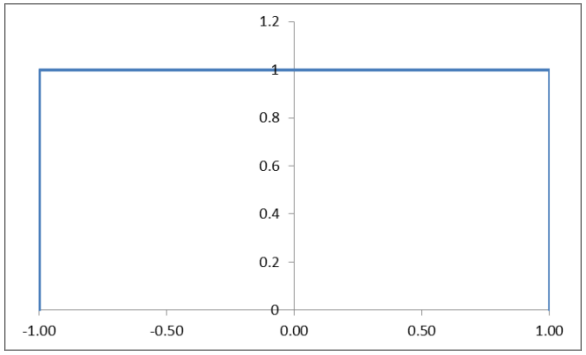
n = 3

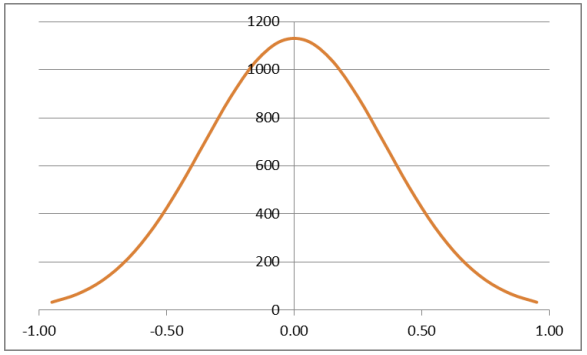
n = 5

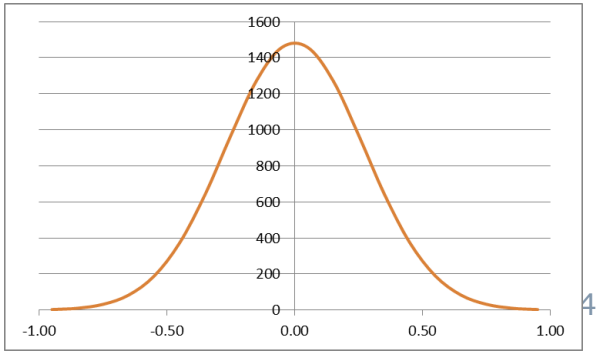






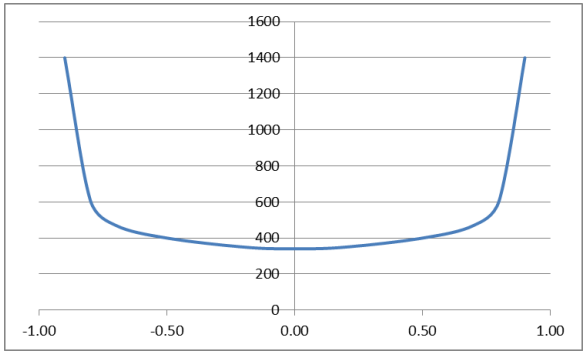




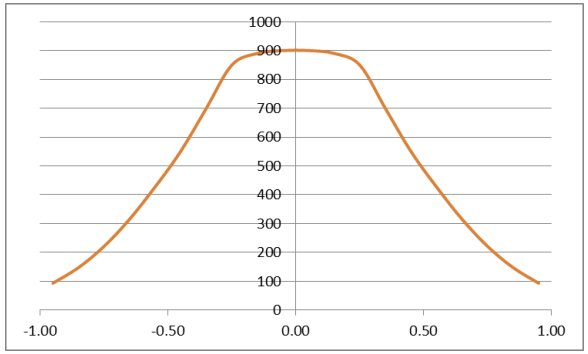


Type-A if underlying distribution not normal

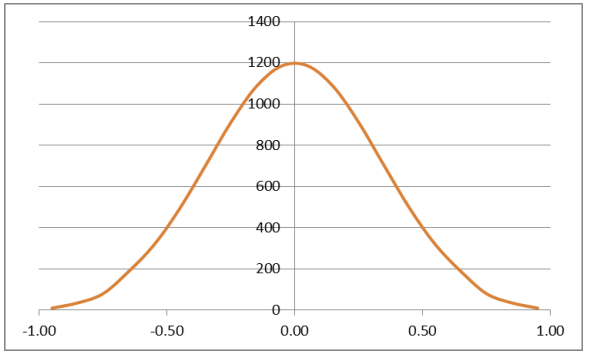
PDF

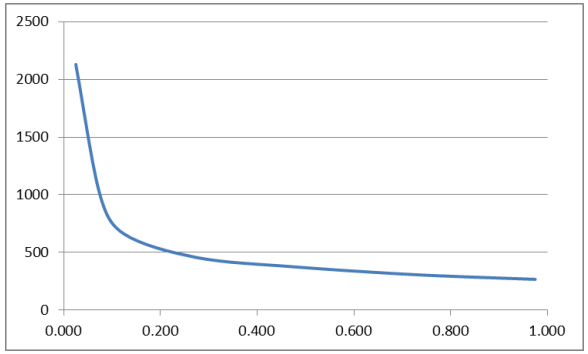


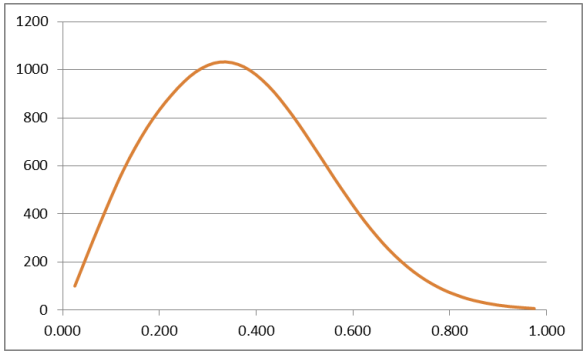
n = 3

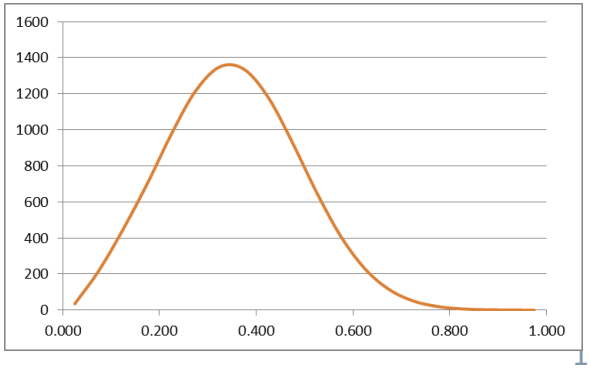


n = 5







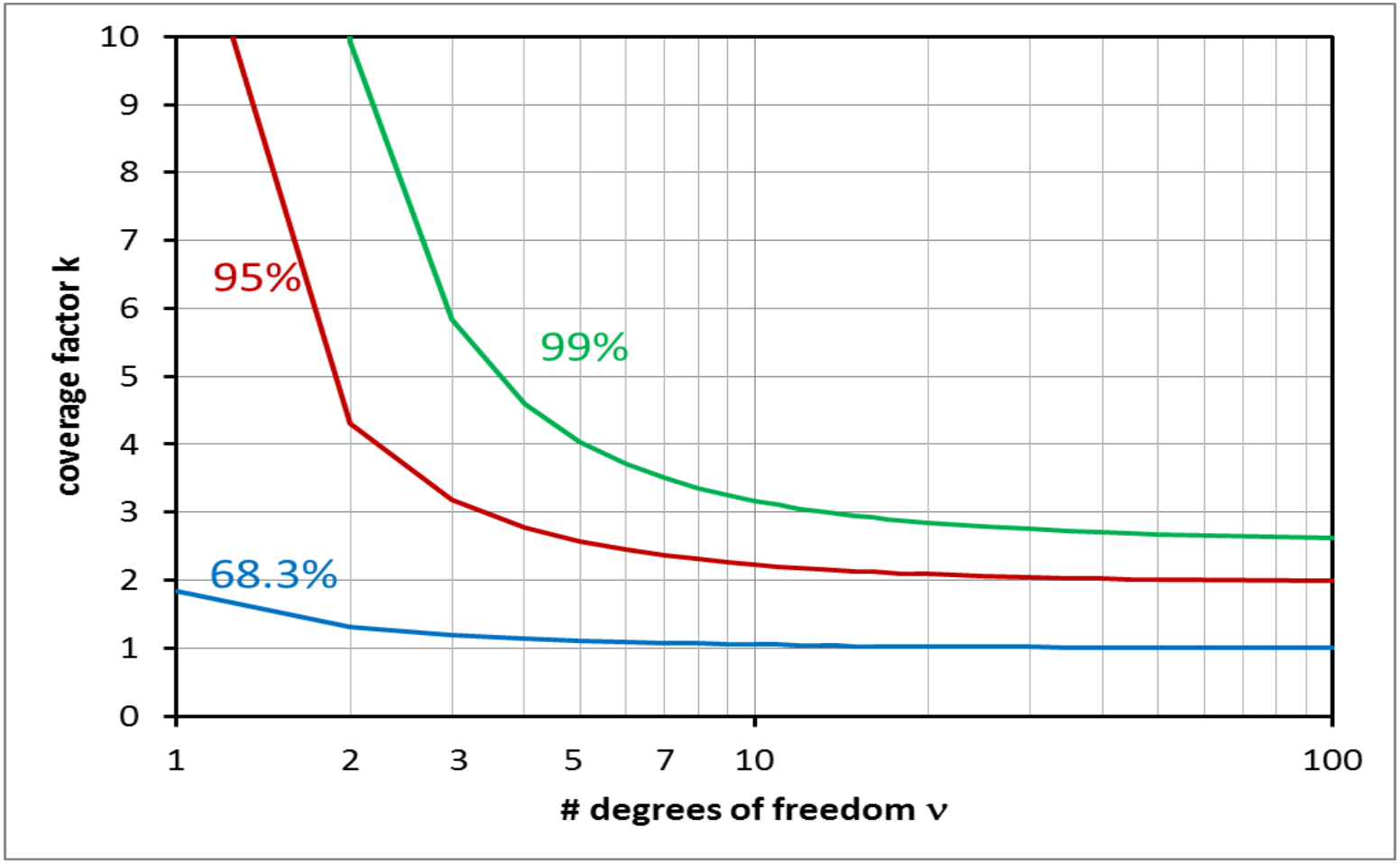


Expanded uncertainty

$$U(x) = k \cdot u(x)$$

with k the coverage factor corresponding with a given confidence level

Coverage factor k and confidence levels



Type-B uncertainty, u_B

Based on scientific judgement

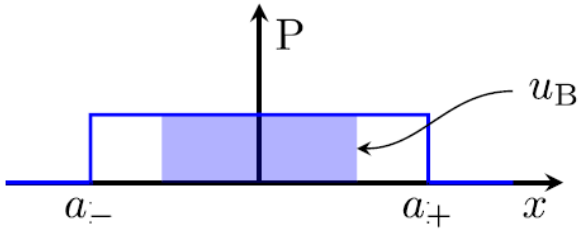
Investigator's knowledge and experience

Examples:

- Previous measurement data
- Instrument manufacturer specifications
- Calibration certificate
- Reference data from tables

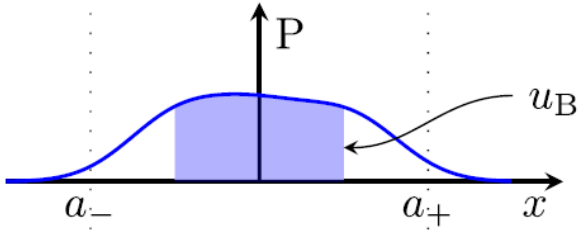
Type B estimates based on assumed distribution

a)



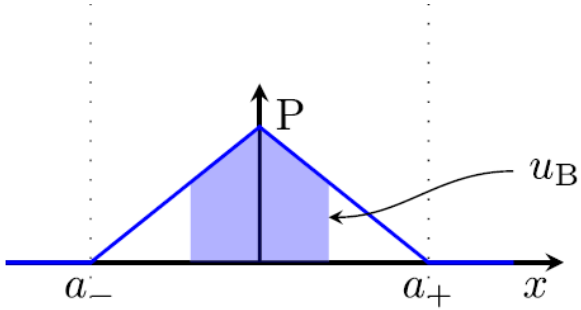
$$u_B = \frac{a_+ - a_-}{2\sqrt{3}}$$

b)



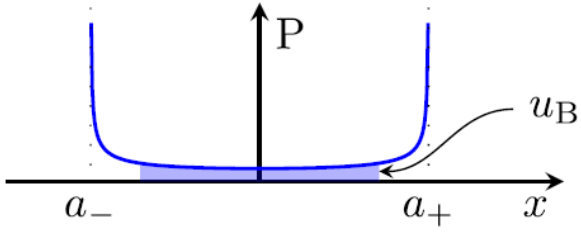
$$u_B \approx \frac{a_+ - a_-}{2 \cdot 2}$$

c)



$$u_B = \frac{a_+ - a_-}{2\sqrt{6}}$$

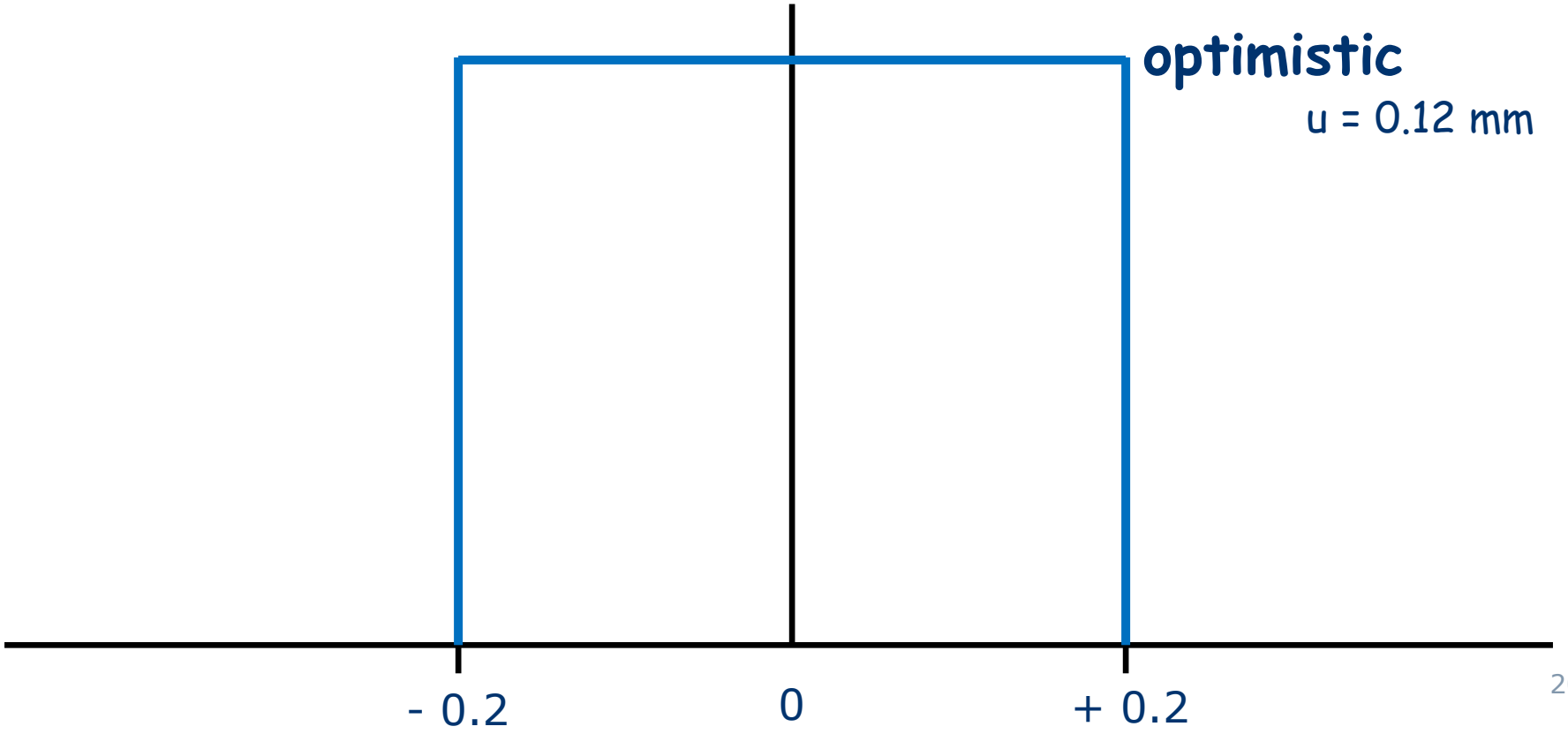
d)



$$u_B = \frac{a_+ - a_-}{2\sqrt{2}}$$

Example

Two independent methods for detector positioning: 0.4 mm difference



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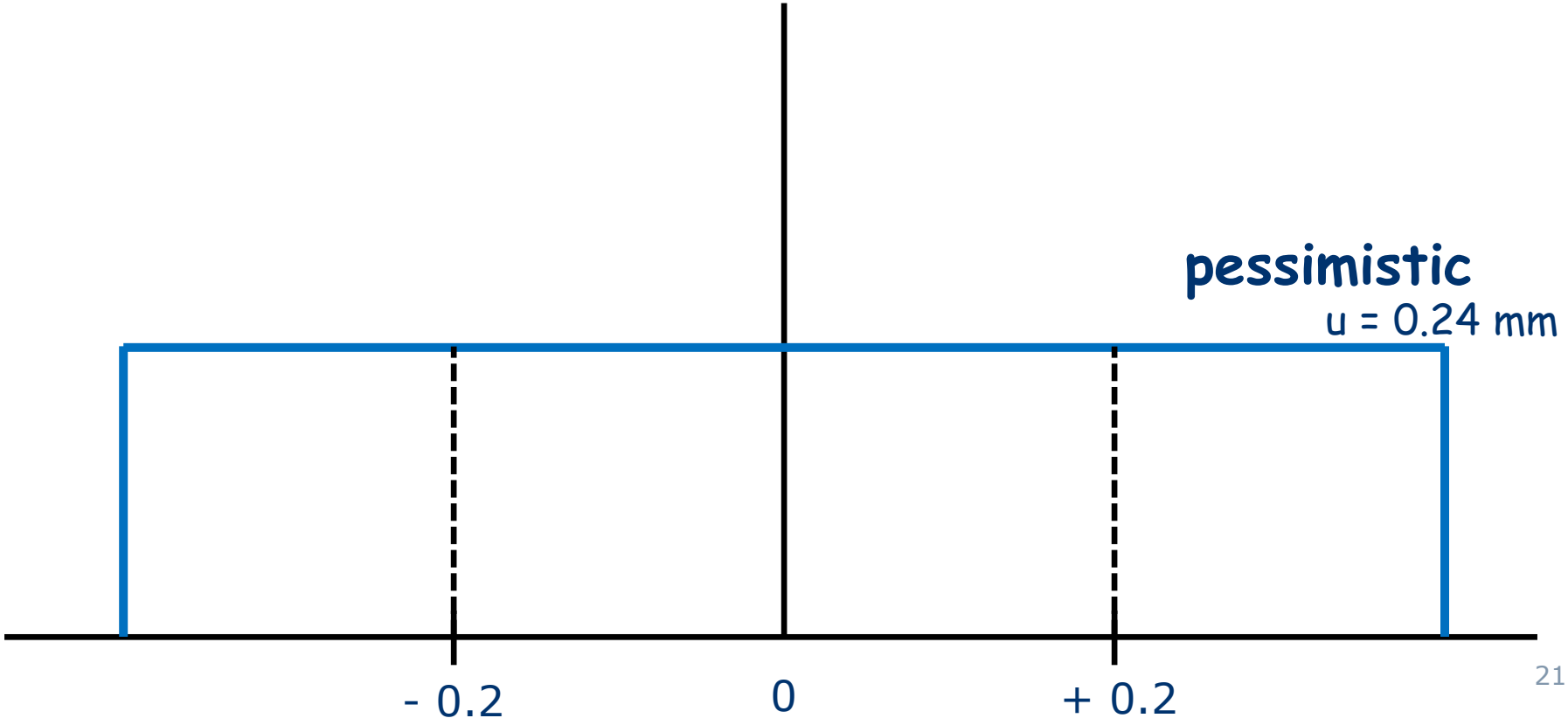
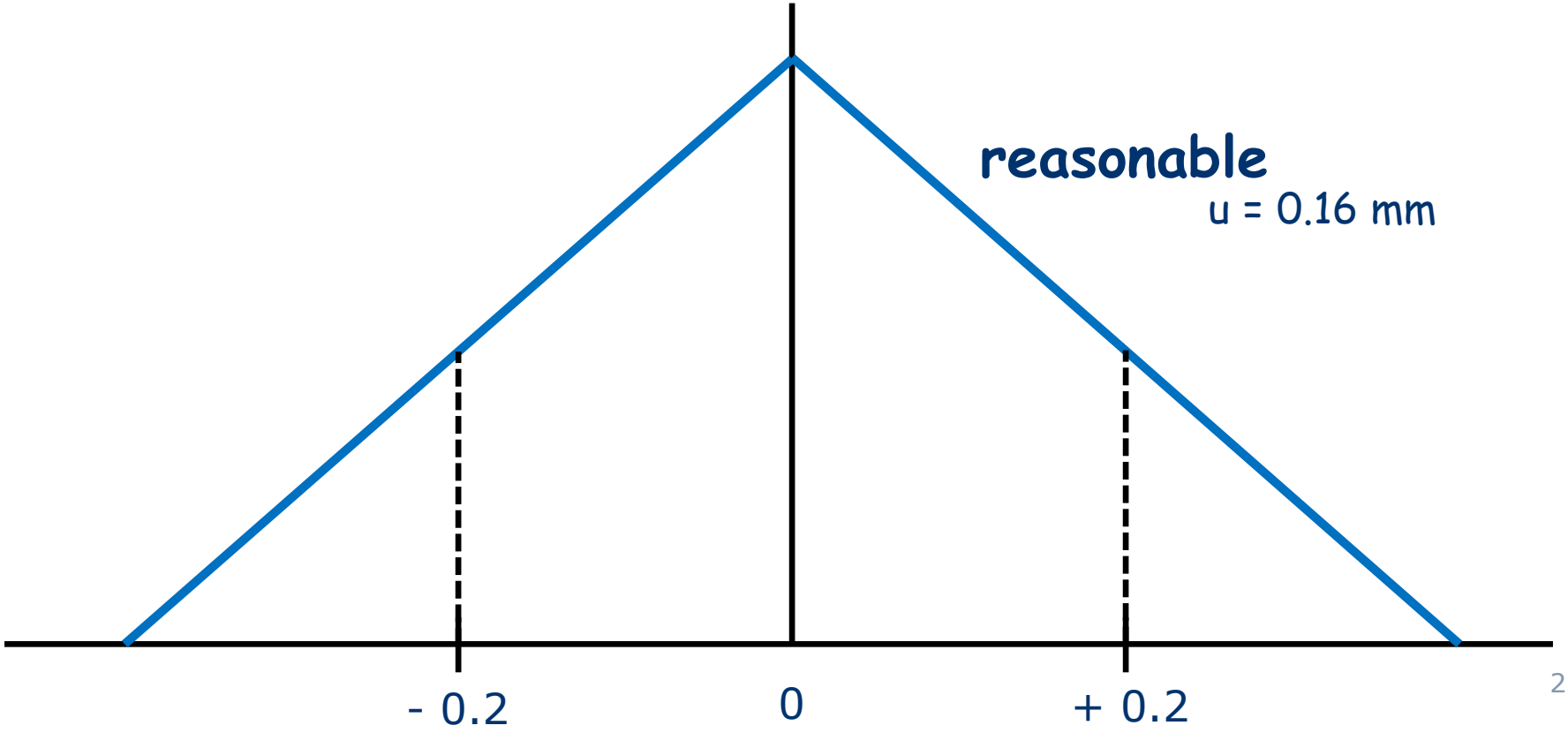
Two independent methods for detector positioning: 0.4 mm difference



Example

Two independent methods for detector positioning: 0.4 mm difference



Measurement equation

If uncorrelated contributions:

$$y = f(x_1, x_2, \dots, x_n) \rightarrow u(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u(x_i)^2}$$

Sensitivity coefficients: $c_i = \frac{\partial f}{\partial x_i}$

Simple product:

$$y = \prod_i x_i \rightarrow \quad \frac{u(y)}{y} = \sqrt{\sum_{i=1}^{n} \left(\frac{u(x_i)}{x_i}\right)^2}$$

Measurement equation

With correlated contributions:

$$y = f(x_1, x_2, \ldots, x_n) \rightarrow$$

$$u(y) = \sqrt{\sum_{i=1}^{n} {c_i}^2 \, u(x_i)^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_i \, c_j \, u\left(x_i, x_j\right)}$$

Covariances:

$$u\left(x_i, x_j\right) = u(x_i) \, u\left(x_j\right) r\left(x_i, x_j\right)$$

Measurement equation

$$D_{w,Q} = M_Q \cdot N_{D,w,Q_0} \cdot k_{Q,Q_0}$$

$$D_{w,Q} = M_{raw} k_{elec} k_{TP} k_h k_s k_{pol} N_{D,w,Q_0} \cdot k_{Q,Q_0}$$

All coefficients for relative uncertainty: 1 -> simple and no correlations

Except T in °C

$$k_{TP} = \frac{1013.25}{P}\frac{T + 273.15}{293.15} \qquad \frac{u(k_{TP})}{k_{TP}} = \sqrt{\frac{u(P)^2}{P^2} + \frac{T^2}{(T+273.15)^2}\frac{u(T)^2}{T^2}}$$

Better T in K

$$k_{TP} = \frac{1013.25}{P}\frac{T}{293.15} \qquad \frac{u(k_{TP})}{k_{TP}} = \sqrt{\frac{u(P)^2}{P^2} + \frac{u(T)^2}{T^2}}$$

Effective degrees of freedom

Combined standard uncertainty takes the form

$$u^2 = \sum_i (c_i u_i)^2$$

Now treat this as a model – what is the uncertainty?

Uncertainty on $(c_i u_i)^2$ is $(c_i u_i)^2 \Big/ \sqrt{\nu_{eff}}$

Combine uncertainties in quadrature:

$$u^4 / \nu_{eff} = \sum_i (c_i u_i)^4 / \nu_i$$

Or

$$\nu_{eff} = u^4 / \left(\sum_i (c_i u_i)^4 / \nu_i \right)$$

Welch-Satterthwaite.

Example uncertainty budget including degrees of freedom

Note	Quantity, source of uncertainty	Expanded uncertainty	Type of uncertainty	Confidence level	Coverage factor	Standard uncertainty	Uncertainty component	Effective degrees of freedom				
		U_i			k	u_i	$\lvert c_i u_i \rvert$	ν_{eff}	$\lvert c_i u_i \rvert^2$	$1/\nu_{eff}$	$\lvert c_i u_i \rvert^4$	$\lvert c_i u_i \rvert^4 / \nu_{eff}$
	Pressure (kPa)											
1	calibration	0.1	B	95%	2	0.05%	0.05%	30	2.4×10^{-7}	3.3×10^{-2}	5.7×10^{-14}	1.9×10^{-15}
2	repeatability	0.03	A	68%	1	0.03%	0.03%	9	8.6×10^{-8}	1.1×10^{-1}	7.4×10^{-15}	8.2×10^{-16}
3	resolution	0.005	B	100%	1.73	0.003%	0.003%	100	8×10^{-10}	1.0×10^{-2}	6.3×10^{-19}	6.3×10^{-21}
	Temperature (°C)											
5	calibration	0.5	B	95%	2	1.07%	0.08%	30	7.1×10^{-7}	3.3×10^{-2}	5.1×10^{-13}	1.7×10^{-14}
6	resolution	0.1	B	100%	1.73	0.25%	0.02%	100	3.8×10^{-8}	1.0×10^{-2}		
	Air density											
7	combined	0.0021	combined	95%	2	0.10%		59	1.07×10^{-6}	1.8×10^{-2}	1.1×10^{-12}	2.0×10^{-14}

Degrees of freedom of type-B uncertainties?

Rule of thumb:

Bad: n = 2 or 3
Reasonable: n = 10
Good: n = 30
Very good: n = 100

Example combining degrees of freedom

$$D_w = M_{raw}\, k_{TP}\, k_{pos}\, N_{D,w}\, k_Q$$

Quantity	rel u (%)	ν
M_{raw}	0.3	6
k_{TP}	0.2	3
k_{pos}	0.1	2
$N_{D,w}$	0.6	100
k_Q	1.0	30
total	1.2	63

Example combining degrees of freedom

$$D_w = M_{raw}\, k_{TP}\, k_{pos}\, N_{D,w}\, k_Q$$

Quantity	rel u (%)	ν
M_{raw}	0.3	6
k_{TP}	0.2	3
k_{pos}	3.0	2
$N_{D,w}$	0.6	100
k_Q	1.0	30
total	3.2	3

Example

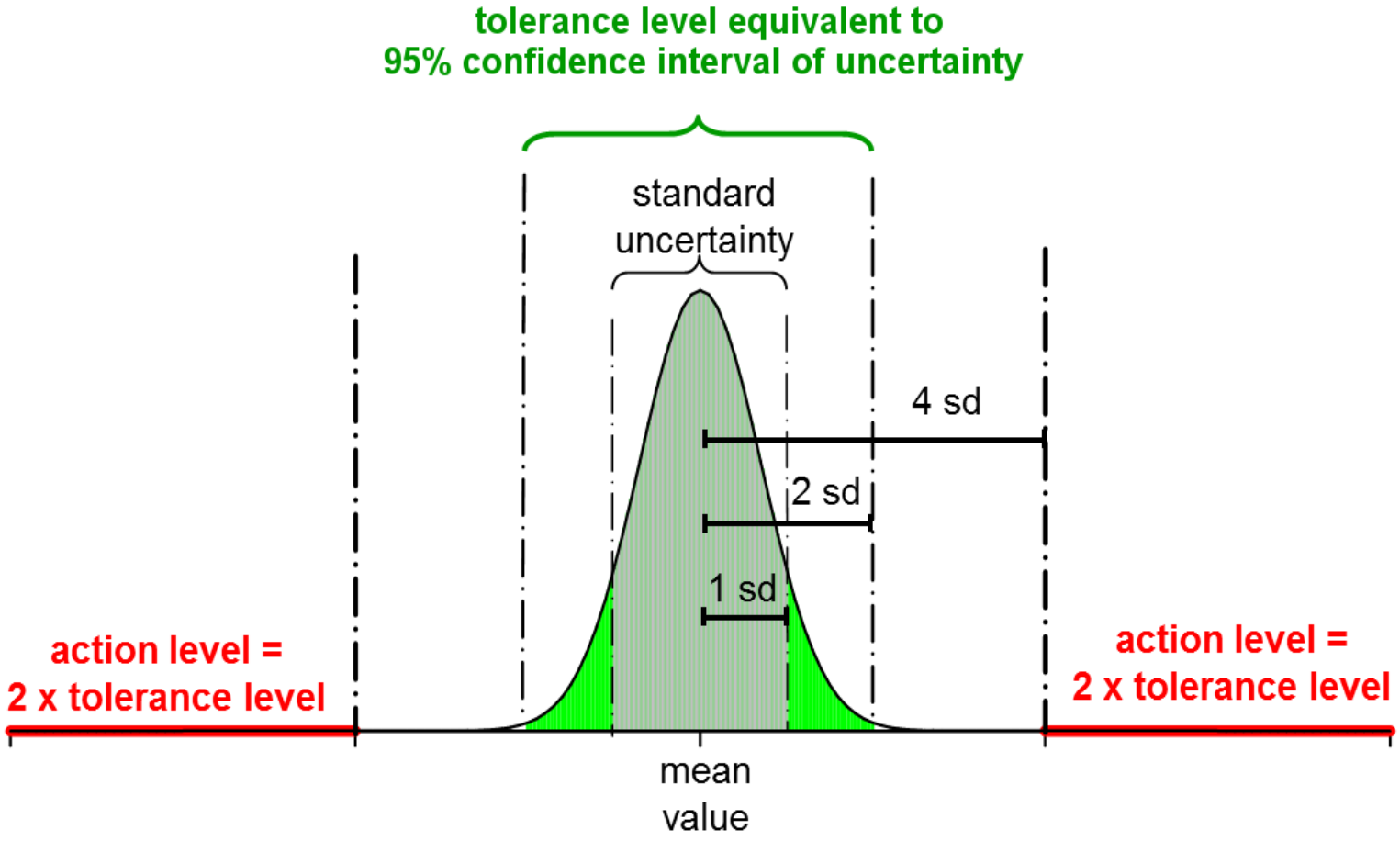
Dose verification of a clinical IMRT plan, recalculated on a cylindrical phantom; planned dose 2 Gy

Zakithi measures 2.01 ± 0.03 Gy

Karen independently measures 1.97 ± 0.03 Gy

Do these results agree or disagree?

Use of uncertainties in QA; tolerances, action levels



Documentation of uncertainties

Level 0: A list of sources of uncertainty without numerical values

Level 1: Basic uncertainty budget including additional information on individual sources of uncertainty for which a deeper explanation is required. Correlation of sources of uncertainty are neglected

Level 2: Level 1 including an investigation of correlation of sources of uncertainty

Level 3: Level 2 including effective degrees of freedom (advanced uncertainty budget)

Conclusions

Uncertainty evaluation is essential

Always make your own uncertainty budget and consider to assign degrees of freedom to type-B uncertainties

Beware of potential non-normal PDF, model uncertainties, overestimated uncertainties and dominant uncertainties with low ν

Type-B uncertainties of Monte Carlo simulations are an interesting issue

Examples from Lechner and Palmans 2025 (PhiRO 34:100773)

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Review article

Uncertainty estimation for dosimetry in radiation oncology

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[b] Medical Physics, MedAustron Ion Therapy Center, Wiener Neustadt, Austria
[c] Radiotherapy and Radiation Dosimetry, National Physical Laboratory, Teddington, United Kingdom

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Examples from Lechner and Palmans 2025 (PhiRO 34:100773)

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.phro.2025.100773.

[17] IAEA. Dosimetry of small static fields used in external beam radiotherapy. Technical report series No. 483., Vienna: International Atomic Energy Agency; 2017.

[18] Cook H, Lambert J, Thomas R, Palmans H, Hussein M, Clark C, et al. Development of a heterogeneous phantom to measure range in clinical proton therapy beams. Phys Med 2022;93:59–68. http://dx.doi.org/10.1016/j.ejmp.2021.11.006.

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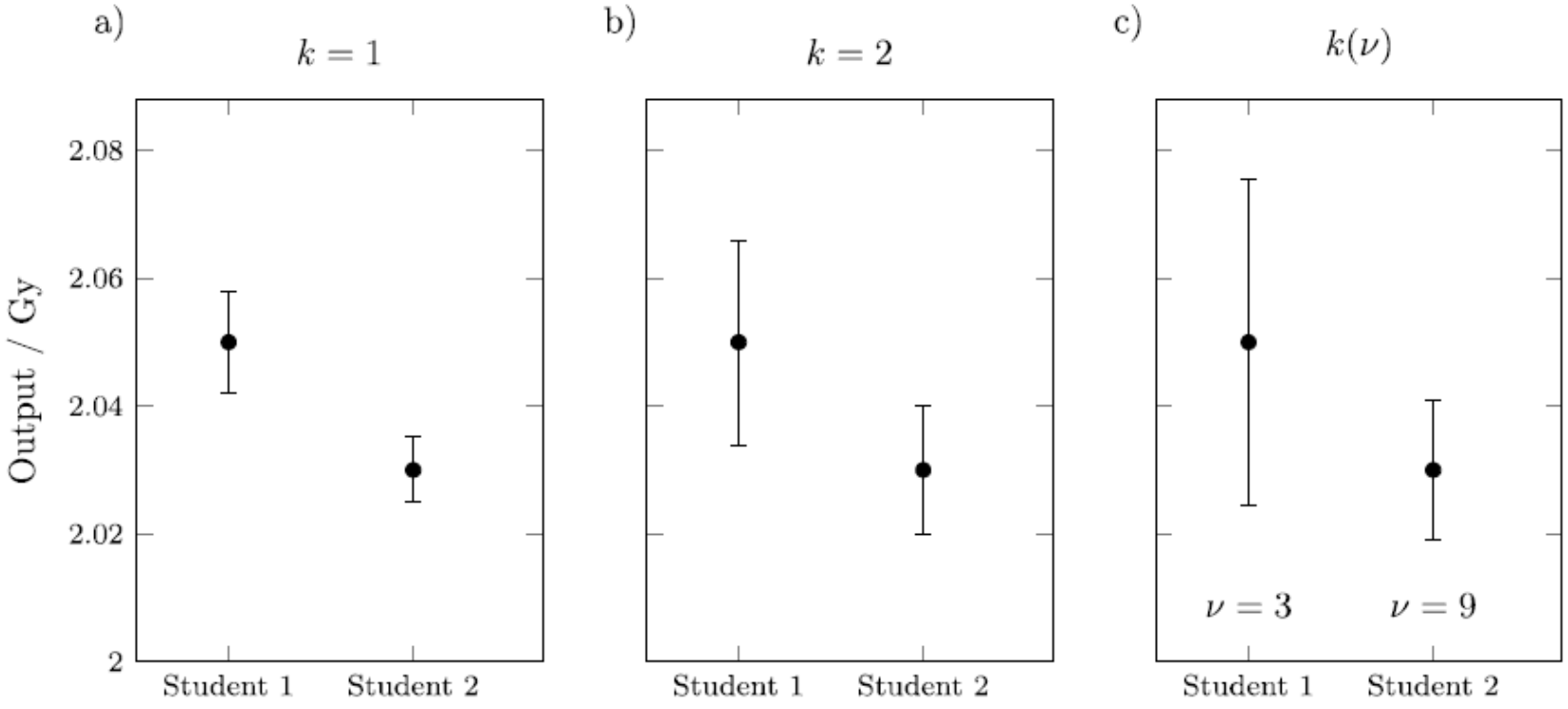
concepts such as effective degrees of freedom are introduced. Finally, this review gives suggestions on how the results can be presented, how the resulting uncertainty budget can be used to identify the main contributors and how the combined uncertainty can be used to establish tolerance and action limits for quality assurance

1. Type-A uncertainty

For this example, consider the following scenario: Two students are asked to determine the dose emitted by a radioactive source (corrected for the decay



2. Using cross-calibrated ionization chamber in electron beams

$$D_{\mathrm{w,Q}} = (M^{\mathrm{PP}}_{\mathrm{raw}})_{\mathrm{Q}} \frac{(M^{\mathrm{CYL}}_{\mathrm{raw}})_{\mathrm{Q_{cross}}}}{(M^{\mathrm{PP}}_{\mathrm{raw}})_{\mathrm{Q_{cross}}}} k^{\mathrm{overall}}_{\mathrm{infl}} N^{\mathrm{CYL}}_{\mathrm{D,w,Q_0}} k^{\mathrm{CYL}}_{\mathrm{Q_{cross},Q_0}} k^{\mathrm{PP}}_{\mathrm{Q,Q_{cross}}}, \qquad \text{(C.1)}$$

where

$$k^{\mathrm{overall}}_{\mathrm{infl}} = k^{\mathrm{PP}}_{\mathrm{TP,Q}} k^{\mathrm{PP}}_{\mathrm{pol,Q}} k^{\mathrm{PP}}_{\mathrm{s,Q}} k^{\mathrm{PP}}_{\mathrm{elec,Q}} \frac{k^{\mathrm{CYL}}_{\mathrm{TP,Q_{cross}}} k^{\mathrm{CYL}}_{\mathrm{pol,Q_{cross}}} k^{\mathrm{CYL}}_{\mathrm{s,Q_{cross}}} k^{\mathrm{CYL}}_{\mathrm{elec,Q_{cross}}}}{k^{\mathrm{PP}}_{\mathrm{TP,Q_{cross}}} k^{\mathrm{PP}}_{\mathrm{pol,Q_{cross}}} k^{\mathrm{PP}}_{\mathrm{s,Q_{cross}}} k^{\mathrm{PP}}_{\mathrm{elec,Q_{cross}}}}. \qquad \text{(C.2)}$$

k_{TP} (level 0)

No. i	Quantity or source of uncertainty	
1	T reading	
2	T calibration	
3	P reading	
4	P calibration	
	Uncertainty of k_{TP}	

Table C-1: An example uncertainty budget where all influence quantities are uncorrelated. All standard uncertainties are shown using a coverage factor of $k = 1$. The number of digits were intentionally increased in in the last column for the reader to be able to follow the calculations.

k_{TP} (level 1-a)

No. i	Quantity or source of uncertainty		rel. standar uncertainty $u_{i,r}$	
			%	
1	T reading		0.1	
2	T calibration		0.5	
3	P reading		0.2	
4	P calibration		0.6	
	Uncertainty of k_{TP}		0.6	

Table C-1: An example uncertainty budget where all influence quantities are uncorrelated. All standard uncertainties are shown using a coverage factor of $k = 1$. The number of digits were intentionally increased in in the last column for the reader to be able to follow the calculations.

k_{TP} (level 1-b)

No. i	Quantity or source of uncertainty	Uncertainty type	PDF	rel. sensitivity coefficient $c_{i,r}$	rel. standard uncertainty $u_{i,r}$	Calculation $(c_i u_i)^2$
					%	$\times 10^{-6}$
1	T reading	A	Normal	0.08	0.1	0.0064
2	T calibration	B	Normal	0.08	0.5	0.16
3	P reading	A	Normal	-1	0.2	4
4	P calibration	B	Normal	-1	0.6	36
	Uncertainty of k_{TP}	Combined	Normal		0.6	40.1664

Table C-1: An example uncertainty budget where all influence quantities are uncorrelated. All standard uncertainties are shown using a coverage factor of $k = 1$. The number of digits were intentionally increased in in the last column for the reader to be able to follow the calculations.

Correlation of two k_{TP}'s (level 2)

No. i	Quantity or source of uncertainty	Uncertainty type	PDF	rel. sensitivity coefficient $c_{i,r}$	rel. standard uncertainty $u_{i,r}$	Correlation with No. j	Correlation coefficient $r(x_i, x_j)$	Calculation $(c_i u_i)^2$	Calculation $c_i u_i c_j u_j$
					%			$\times 10^{-6}$	$\times 10^{-6}$
1	T_1 reading	A	Normal	0.08	0.1			0.0064	
2	T_1 calibration	B	Normal	0.08	0.5	6	1	0.16	−0.16
3	P_1 reading	A	Normal	-1	0.2			4	
4	P_1 calibration	B	Normal	-1	0.6	8	1	36	−36
5	T_2 reading	A	Normal	-0.08	0.2			0.0256	
6	T_2 calibration	B	Normal	-0.08	0.5	2	1	0.16	−0.16
7	P_2 reading	A	Normal	1	0.1			1	
8	P_2 calibration	B	Normal	1	0.6	4	1	36	−36
	Uncertainty of $k_{\mathrm{TP}}^{1,2}$	Combined	Normal		0.22			77.352	−72.32

Range measurement (level 3)

No. i	Quantity or source of uncertainty	Uncertainty type	PDF	Sensitivity coefficient c_i	Standard uncertainty u_i	Effective degrees of freedom ν_{eff}	Calculation $(c_i u_i)^2$	Calculation $(c_i u_i)^4/\nu_{\text{eff}}$
					mm		$\times 10^{-4}$	$\times 10^{-8}$
1	Repeatability	A	Normal	1	0.03	4	9.0	20.25
2	WET entrance window	B	Normal	1	0.09	44	81	149
3	WET chamber window	B	Normal	1	0.20	3	400	53 333
4	Scan resolution	B	Rectangular	1	0.03	100	9.0	0.81
5	Mechanical reference depth	B	Normal	1	0.20	10	400	16 000
6	Linearity depth scale	B	Normal	1	0.15	10	225	5063
	Combined uncertainty	Combined	Normal		0.34	17	1124	74 566

Relative range measurement (WET) (level 3)

No. i	Quantity or source of uncertainty	Uncertainty type	PDF	Sensitivity coefficient c_i	Standard uncertainty u_i	Correlation with No. j	Correlation coefficient $r(x_i, x_j)$	Effective degrees of freedom ν_{eff}	Calculation $(c_i u_i)^2$	Calculation $r(x_i, x_j) c_i u_i c_j u_j$	Calculation $(c_i u_i)^4/\nu_{\text{eff}}$
					mm				$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-8}$
1	Repeatability 1	A	Normal	1	0.03			4	9.0		20
2	Resolution 1	B	Rectangular	1	0.03			100	9.0		0.81
3	Linearity depth scale 1	B	Normal	1	0.15	6	0.9	10	225	−203	51
4	Repeatability 2	A	Normal	-1	0.03			4	9.0		20
5	Resolution 2	B	Rectangular	-1	0.03			100	9.0		0.81
6	Linearity depth scale 2	B	Normal	-1	0.15	3	0.9	10	225	−203	51
	Combined uncertainty	Combined	Normal		0.09			46	486	−405	143

Table E-2: Uncertainty budget for the water-equivalent thickness as a difference between two range measurements (Eq. (E.1)) showing only the uncorrelated or partially correlated contributions. Note that the contribution of the correlated uncertainties to the Welch-Satterthwaite equation is calculated as $((c_i u_i)^2 + r(x_i, x_j) c_i u_i c_j u_j)^2 / \nu_{eff}$. All standard uncertainties are shown using a coverage factor of $k = 1$.