Advanced School and Conference on Quantum Matter

*Trieste, Dec. 1st 2025*

# *Spin liquid phases in SU(N) antiferromagnets*

**Sylvain Capponi, Toulouse university**

UNIVERSITÉ DE TOULOUSE

cnrs

# Collaborators

LPT, Toulouse:

D. Poilblanc, M. Mambrini, F. Alet

PhD/postdocs: O. Gauthé, Ji-Yao Chen, P. Patil

LPTM, Cergy-Pontoise:

**Philippe Lecheminant**, D. Papoular

PhD/postdocs: V. Bois, P. Fromholz, M. Moliner, H. Nonne

YITP, Kyoto: **Keisuke Totsuka**

Also: G. Roux (Saclay), A. Läuchli (PSI/EPFL), A. Tsvelik (Brookhaven), A. Weichselbaum (Brookhaven), L. Herviou, P. Nataf (Grenoble), J.-W. Li, H.-H. Tu, J. von Delft (Munich), L. Devos (Ghent/New York), L. Vanderstraeten (Bruxelles)…

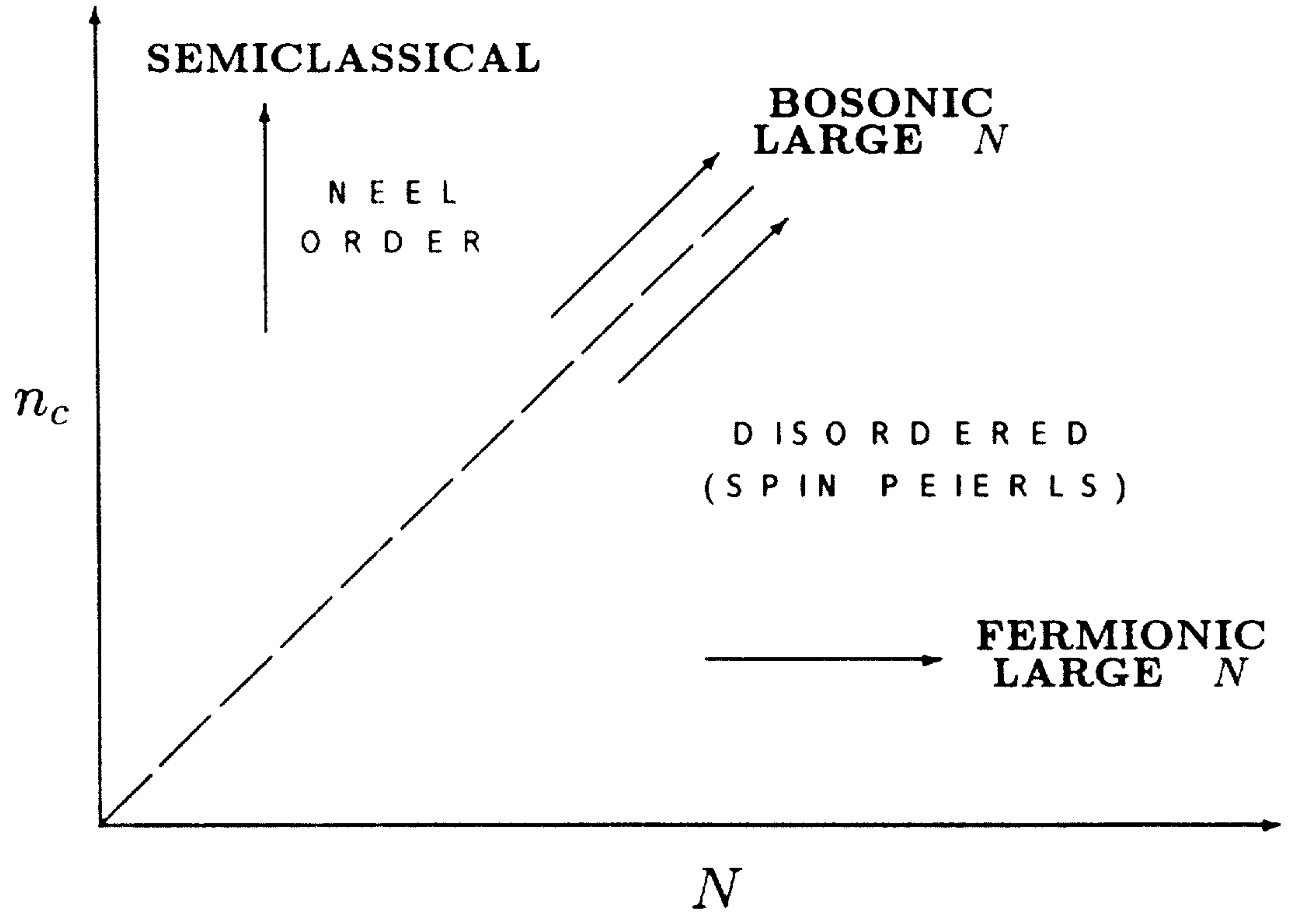
# SU(N): platform for spin liquids

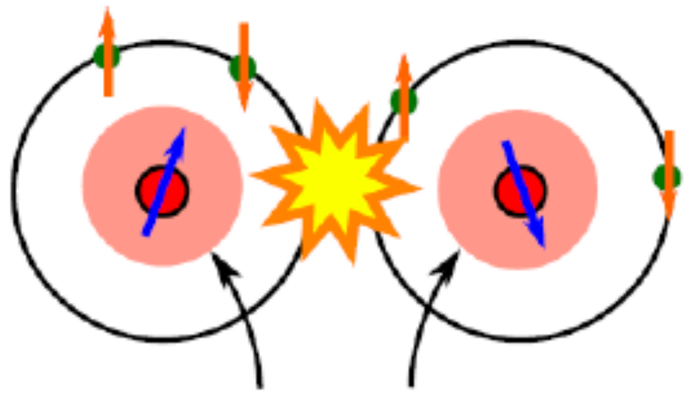

*A. Auerbach*

*Read, Sachdev '89 …*

Enlarging SU(2) into SU(N) allows to destabilize (semi)classical order (1d, 2d)

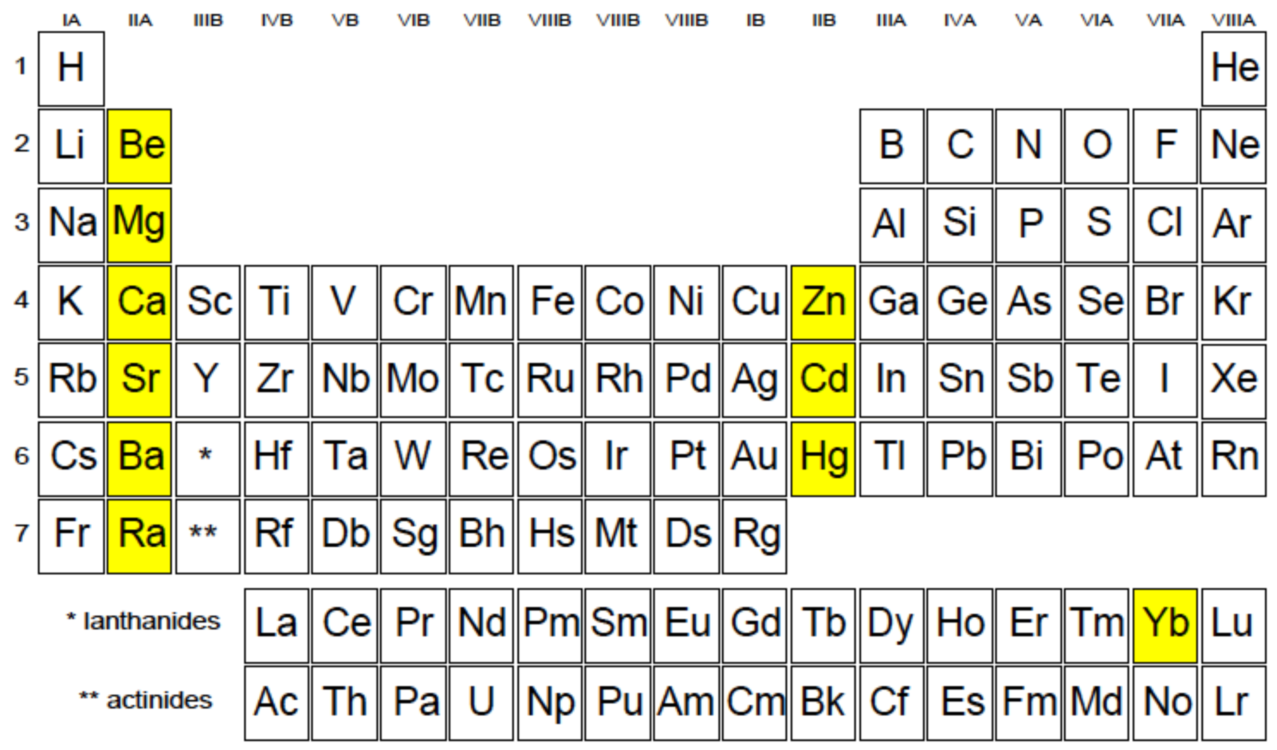

Illustration by Richard Codor





# alkaline-earth cold atoms

## ... an introduction




what's so nice ??

✓ decoupling of "nuclear spin *I*" from electronic states
➡ $I_z$-independent scattering length

✓ SU(*N*) (*N*=2*I*+1) symmetry (almost perfect...) *Gorshkov et al '10*

✓ $^{171}$Yb (I=1/2, SU(2)), $^{173}$Yb (I=5/2, SU(6)), $^{87}$Sr (I=9/2, SU(10)) ,.... *Takahashi group '10-'12*
*DeSalvo et al. '10*

✓ tunability: interactions, lattice, ....

✓ Pomeranchuk cooling *Zi Cai et al '13*

**Fermionic isotopes**

**IOP** Publishing — Reports on Progress in Physics

Rep. Prog. Phys. **77** (2014) 124401 (20pp) — doi:10.1088/0034-4885/77/12/124401

**Report on Progress**

**Ultracold Fermi gases with emergent SU(*N*) symmetry**

**Miguel A Cazalilla[1,2] and Ana Maria Rey[3]**

PRL 105, 030402 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

# Degenerate Fermi Gas of $^{87}$Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

Selected for a Viewpoint in *Physics*
PRL 105, 190401 (2010) PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2010

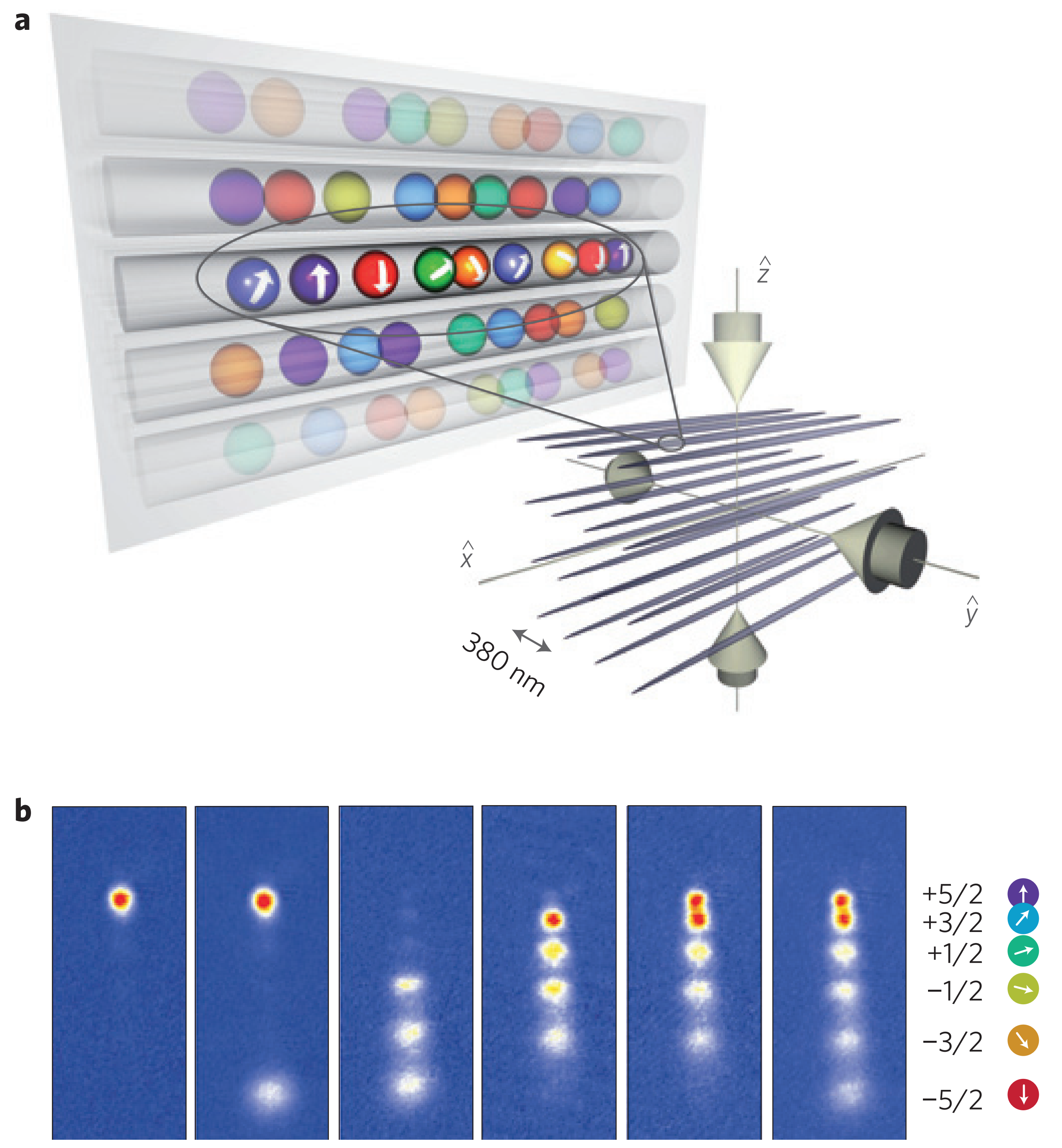
# Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

LETTERS
PUBLISHED ONLINE: 2 FEBRUARY 2014 | DOI: 10.1038/NPHYS2878
nature physics

# A one-dimensional liquid of fermions with tunable spin

Guido Pagano[1,2], Marco Mancini[1,3], Giacomo Cappellini[1], Pietro Lombardi[1,3], Florian Schäfer[1], Hui Hu[4], Xia-Ji Liu[4], Jacopo Catani[1,5], Carlo Sias[1,5], Massimo Inguscio[1,3,5] and Leonardo Fallani[1,3,5★]



# Exotic phases: topological order ?

*See e.g. X.-G. Wen's book*

✓ **Long-range entanglement**

- genuine topological order allowed in (2+1)D, (3+1)D
- robust against any perturbation
- Ex: fractional quantum Hall effect, toric code…

✓ **Short-range entanglement**

- without symmetry, all states are connected to a trivial product state
- with protecting symmetries, various topological phases

**Symmetry-protected topological (SPT) order**

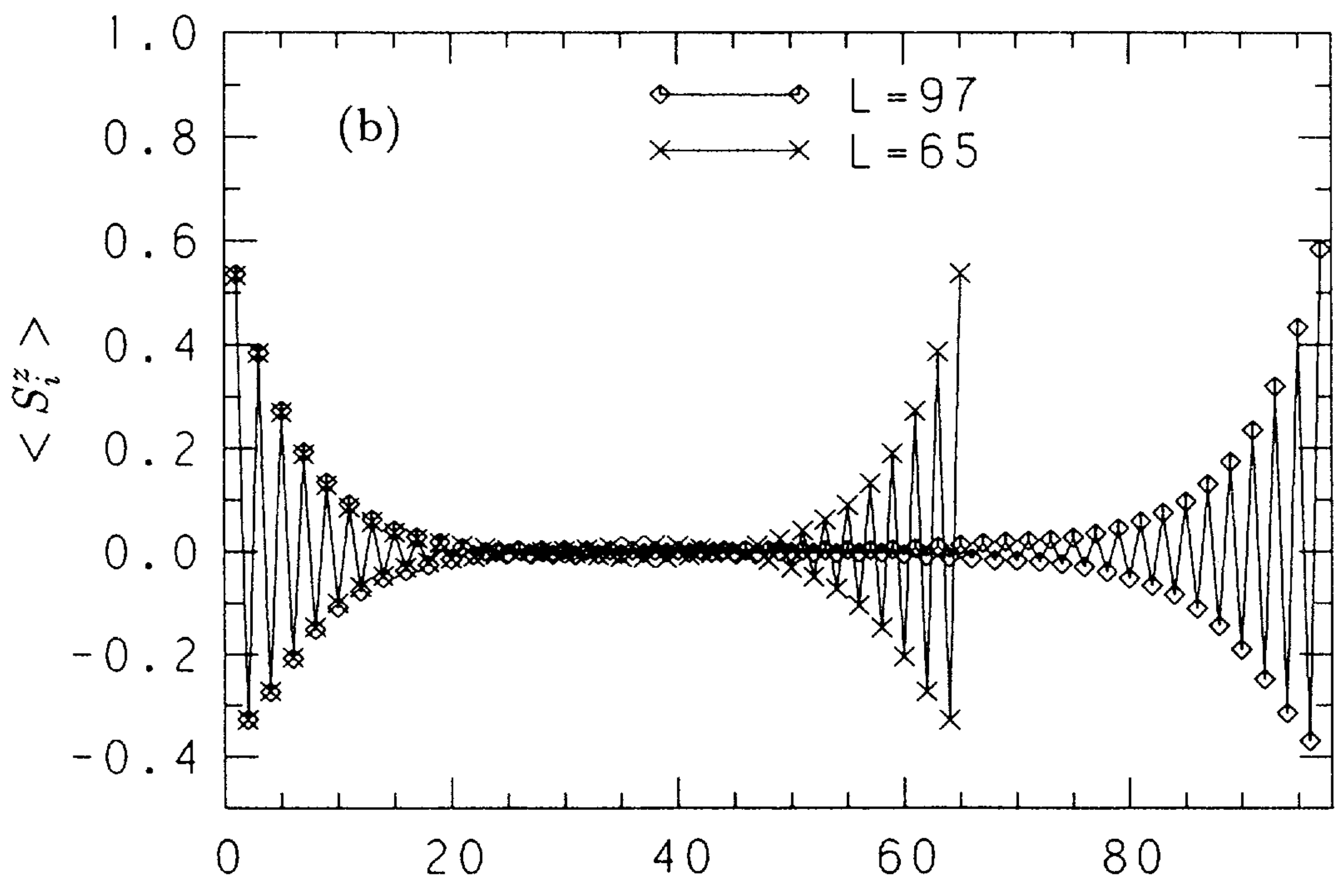
# Topological phases in 1d

✓ **Only SPT is possible** *Verstraete et al '05; Chen, Gu & Wen '10*

- protecting symmetry is mandatory
- only short-range entanglement and no topological degeneracy
- Different SPT phases cannot be connected adiabatically
- SPT phases are featureless in the bulk but exhibit various edge states

→ **Classification** *Chen et al. '11 Duivenvoorden and Quella '13*

- Example: S=1 Heisenberg chain and its famous Haldane phase




*Pollmann, Berg, Turner, Oshikawa '12*

***Emergent S=1/2 edge states can be measured !***

*QMC: Miyashita & Yamamoto '93*

# 1D SPT phase: the Haldane phase

Spin-1 AF Heisenberg spin chain: Haldane phase $\Delta \simeq 0.4104327J$

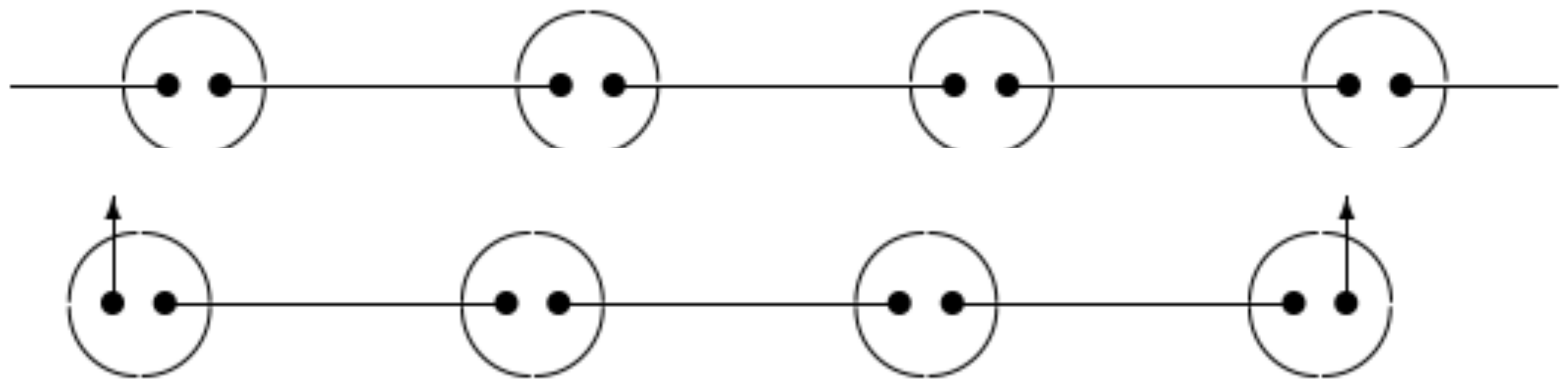
AKLT wave function exact ground state:

$$\mathcal{H}_{AKLT} = J \sum_i \left[ \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} \left( \vec{S}_i \cdot \vec{S}_{i+1} \right)^2 \right]$$

*Affleck, Kennedy, Lieb, Tasaki '88*

Edge states (spin-1/2):
Non degenerate GS with PBC

4-fold degenerate GS with OBC



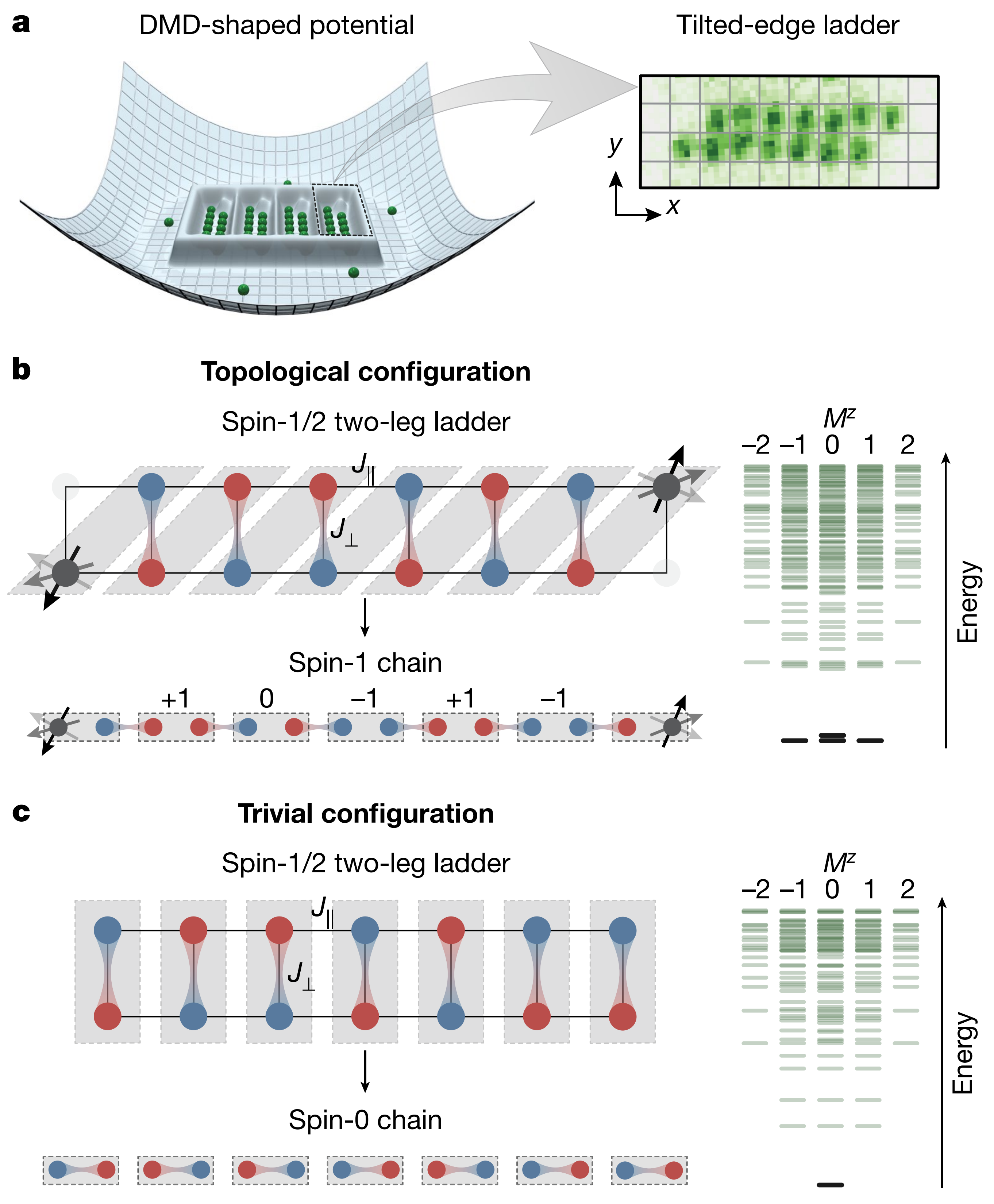
**SPT phase protected by T, parity, pi-rotations**

*Pollmann et al. '12*

Haldane phase S odd: SPT phase
Haldane phase S even: **not** a SPT phase

# Quantum simulator: Fermi-Hubbard SU(2) ladder

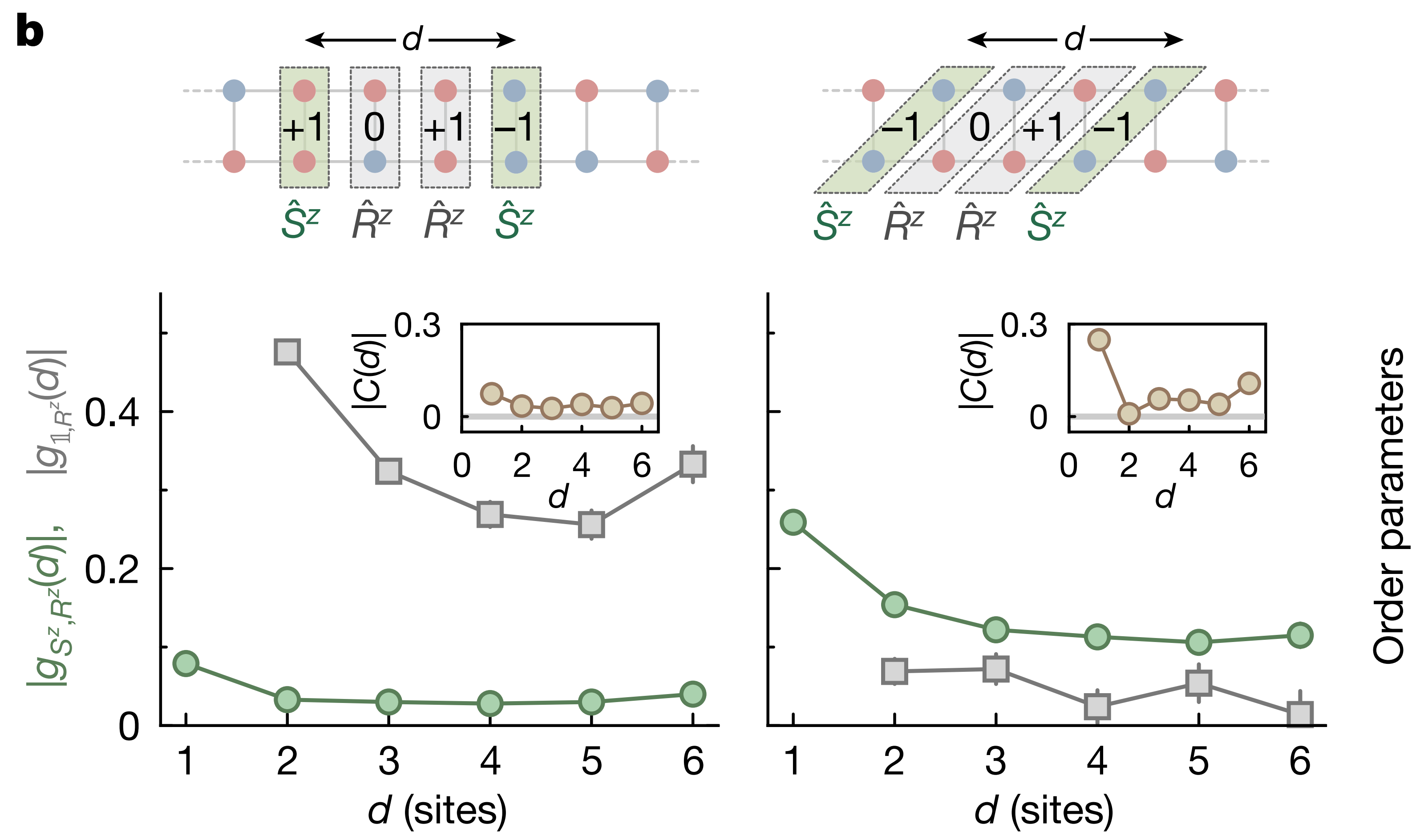
*Group of I. Bloch, Nature (2022)*




Measurement of « hidden » order

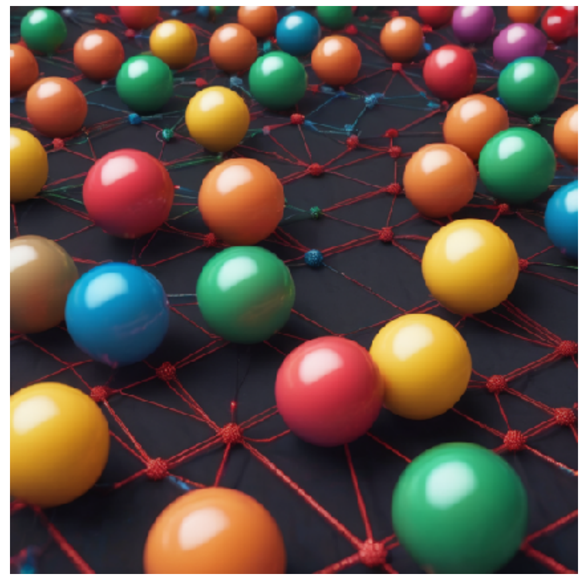
String order parameter

$$\langle S_i^z \left( \prod_{k=i+1}^{j-1} S_k^z \right) S_j^z \rangle$$




Note that the edge state is a physical spin

# Outline



1) How can we realize **SU(N) SPT phases** in 1d ?

2) How can we realize **chiral SU(N) spin liquids** in 2d ?

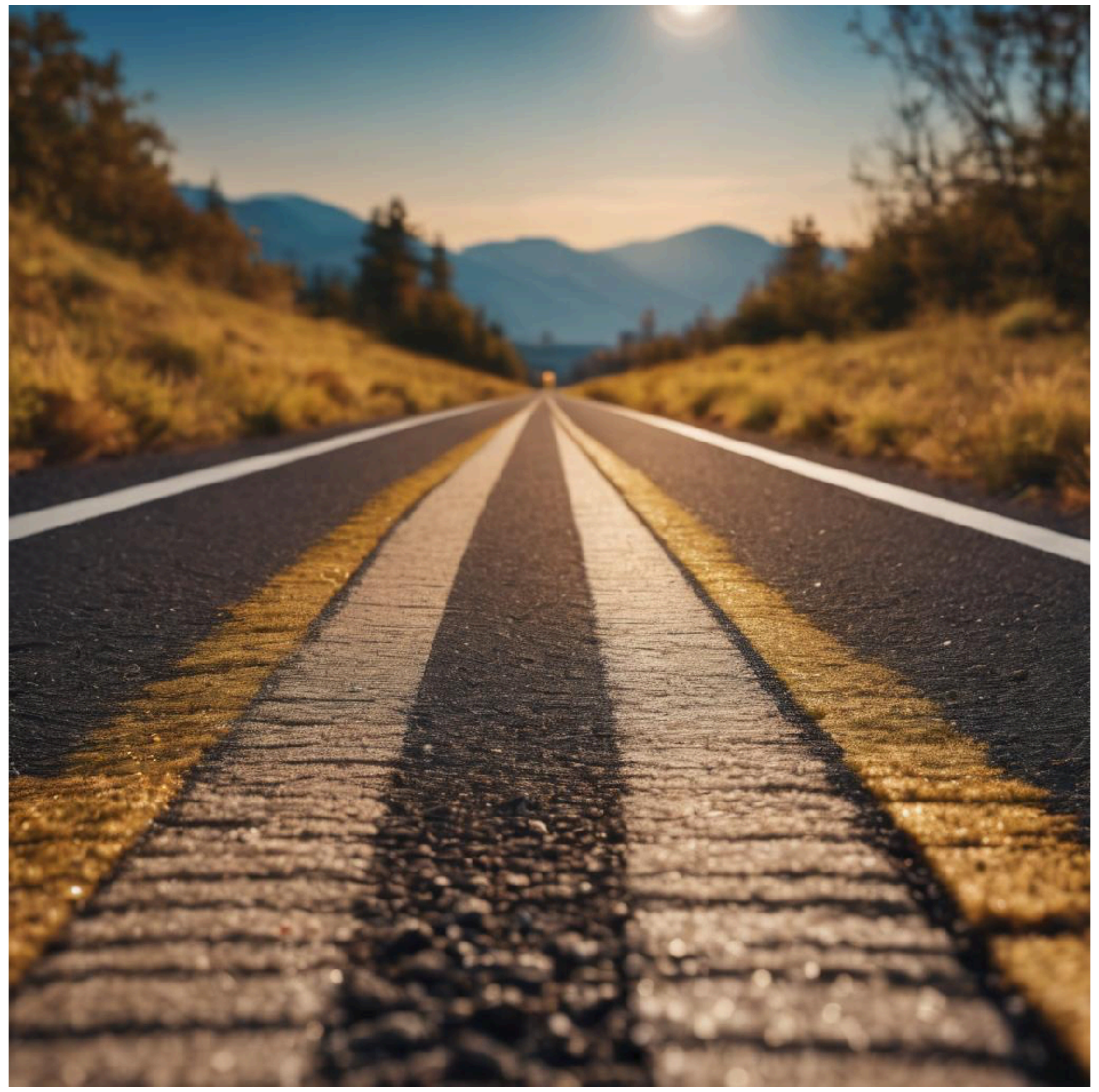
**Disclaimer**:

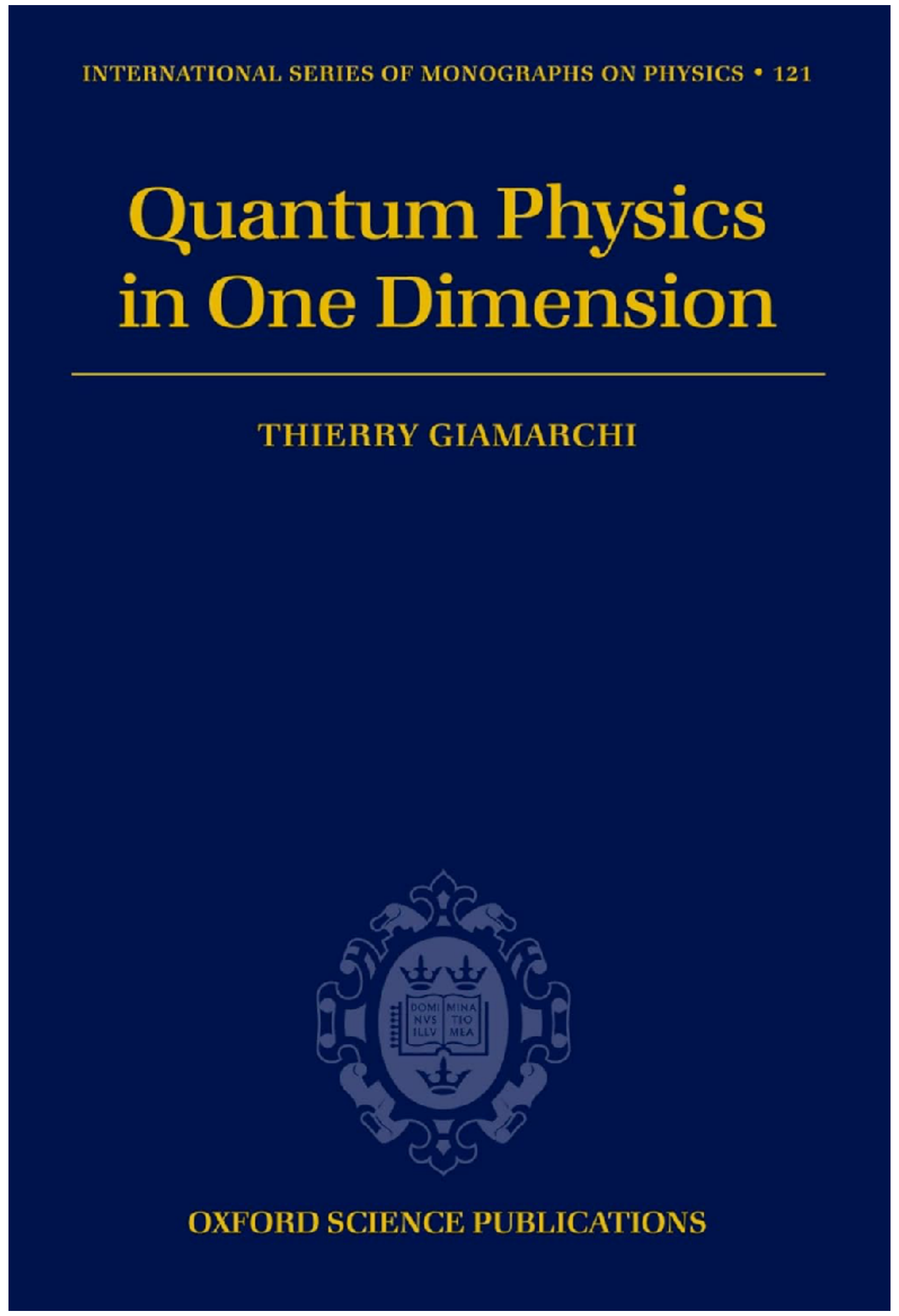
This talk will be about ***exact*** SU(N) microscopic models with equal population per color

But it could be interesting to investigate imbalanced populations, SU(N) symmetry breaking (e.g. in Lithium) etc.

# “Simple” SU(N) chains

One-dimensional world






Field theory, bosonization, integrable models,

density-matrix renormalization group (DMRG) ….

# SU(N) Hubbard model: Mott phase

SU(N) degree of freedom = « spin » = « color »

Filling 1 particle per site: fundamental representation of SU(N) □

$$\mathcal{H} = J \sum_i P_{i,i+1} \qquad P_{i,i+1} = \frac{1}{N} + \sum_{A=1}^{N^2-1} \mathcal{S}_i^A \mathcal{S}_{i+1}^A$$

$$J = 2t^2/U$$

✓ Bethe-ansatz solution *Sutherland '75*

✓ $SU(N)_1$ Wess-Zumino-Witten CFT *Affleck '86 '88*

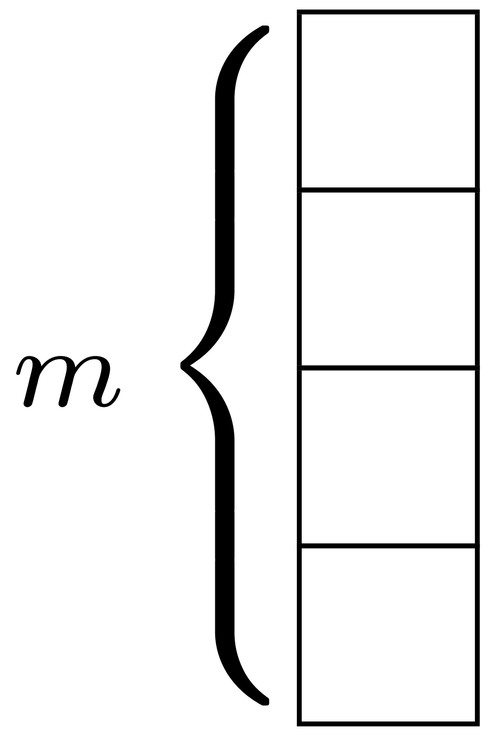
✓ stable fixed point of generic SU(N) 1d gapless systems

✓ In the Hubbard case, no exact solution but it seems that there is a finite *U* BKT Mott transition *Assaraf et al. '99*

# SU(N) Hubbard model: Mott phase

Filling m particles per site: in the large-U regime, Mott phase

$$\mathcal{H} = J \sum_i \sum_{A=1}^{N^2-1} \mathcal{S}_i^A \mathcal{S}_{i+1}^A$$



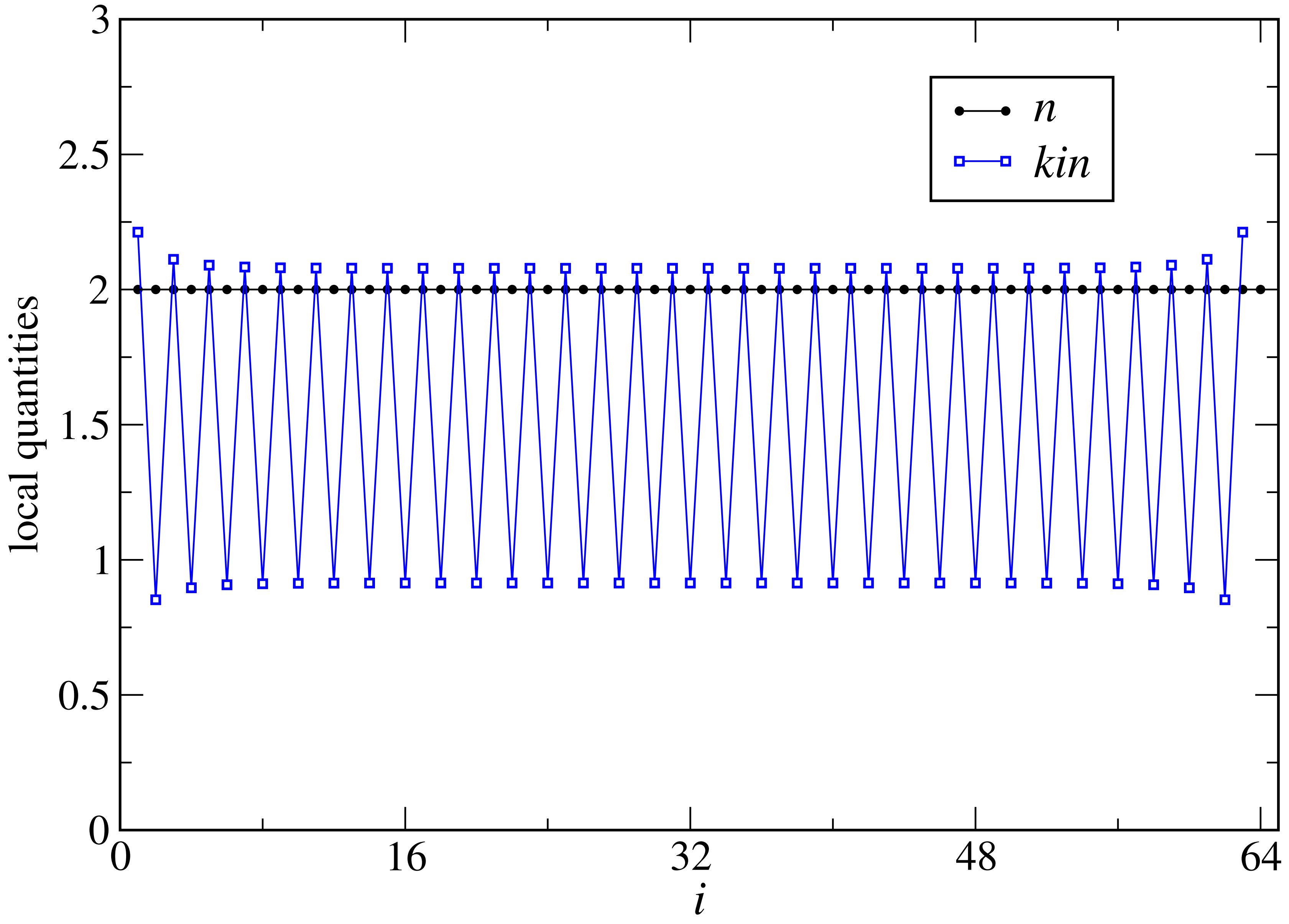

antisymmetric
rep. at each site

- ✓ CFT analysis *Affleck '88* *Lecheminant & Tsvelik '15*
- ✓ $SU(N)_1$ Wess-Zumino-Witten CFT if gcd(m,N)=1
- ✓ if N=pm, $SU(N)_1$ WZW or translation symmetry breaking
- ✓ Numerics (variational Monte-Carlo) confirms this

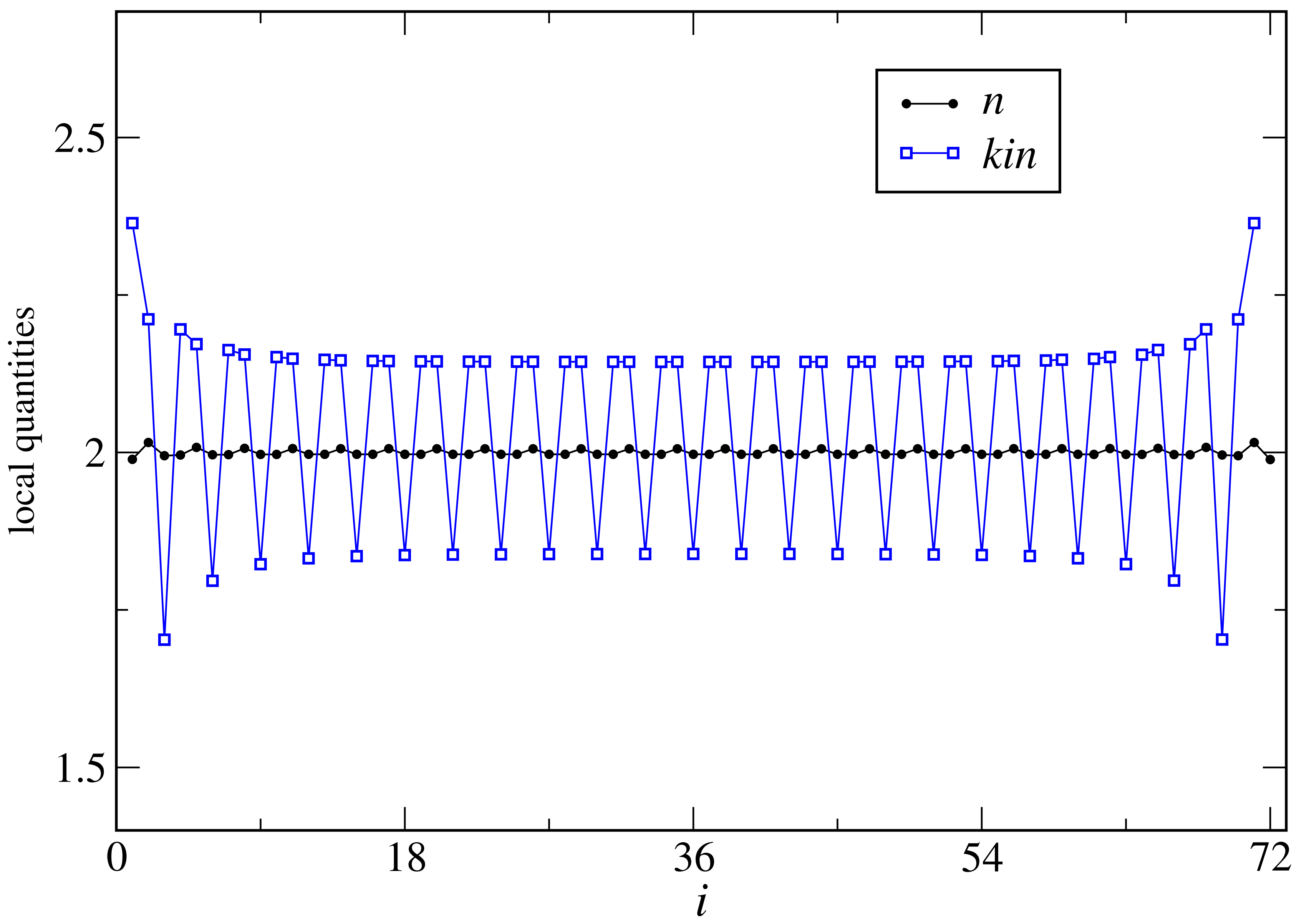
*Dufour, Nataf, Mila '15*

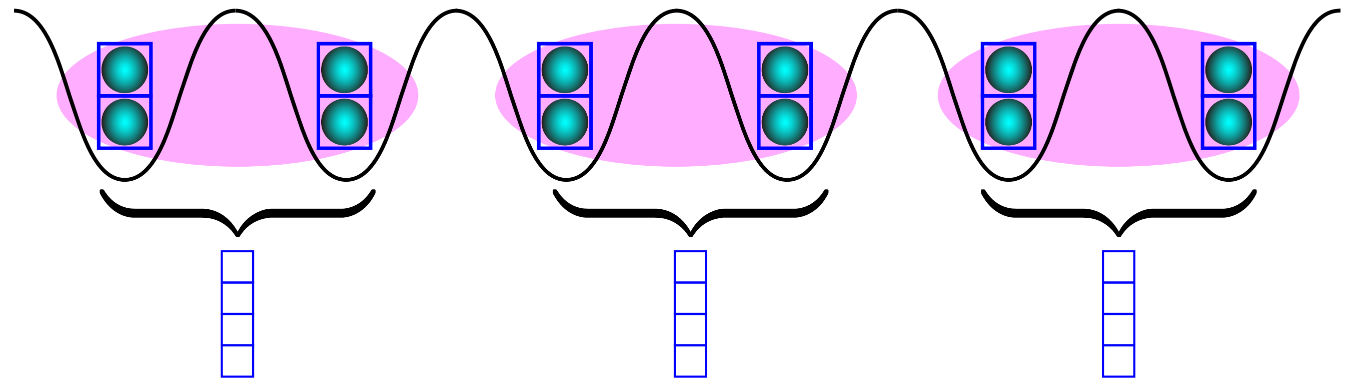
# Translation symmetry breaking

SU(4)  m=2




SU(6)  m=2



dimerization

trimerization

*Capponi, Lecheminant & Totsuka '16*

# SU(N) Hubbard: incommensurate filling

No Umklapp processes: gapless charge mode, metallic behavior

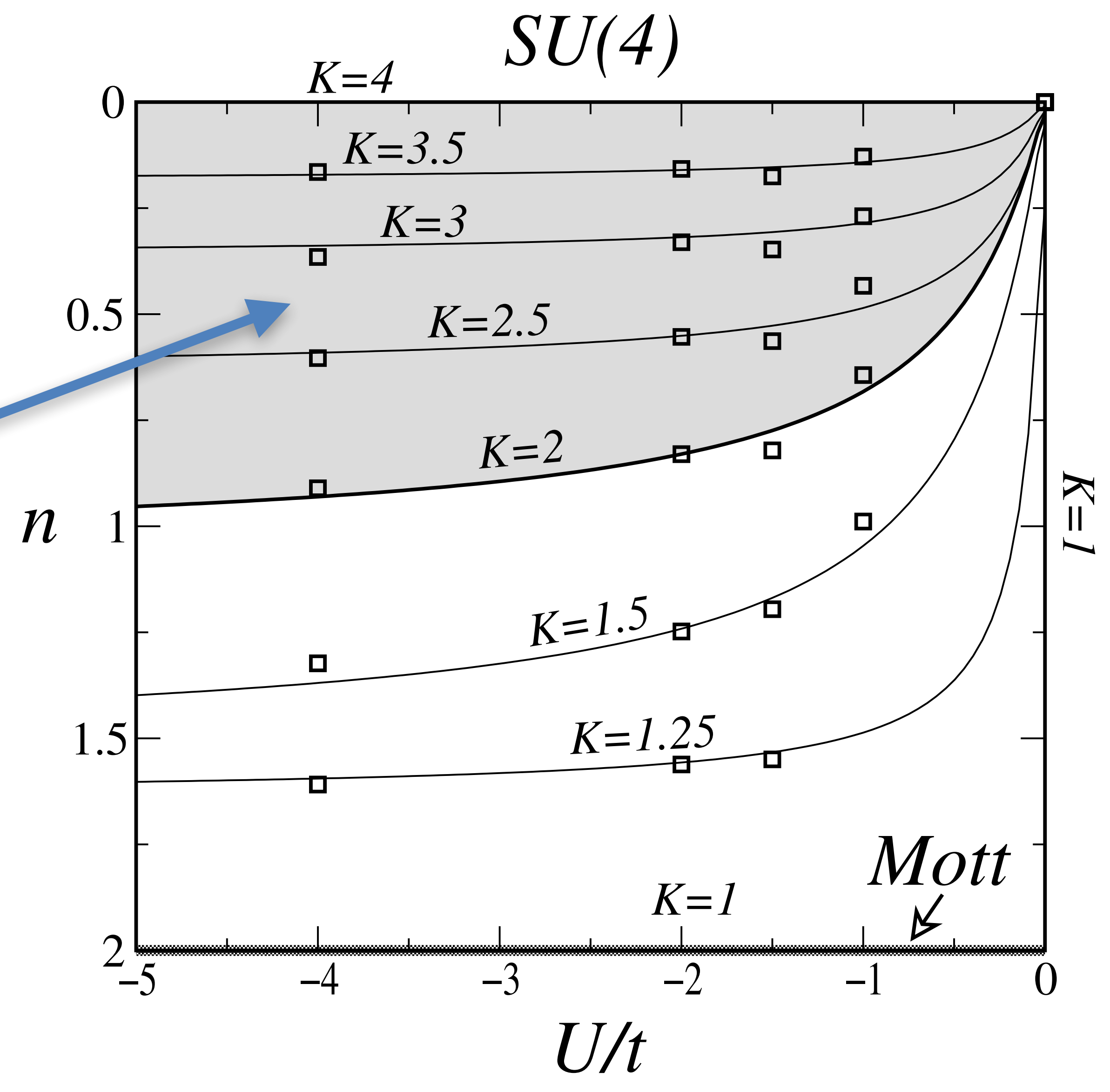
U>0 — all modes are gapless: N-component **Luttinger liquid**

U<0 — spin gap for the SU(N) degrees of freedom; gapless charge = **Luther-Emery** phase

N>2: no singlet pairing is possible !

SU(N) singlet formed by N fermions: **molecular superfluid**

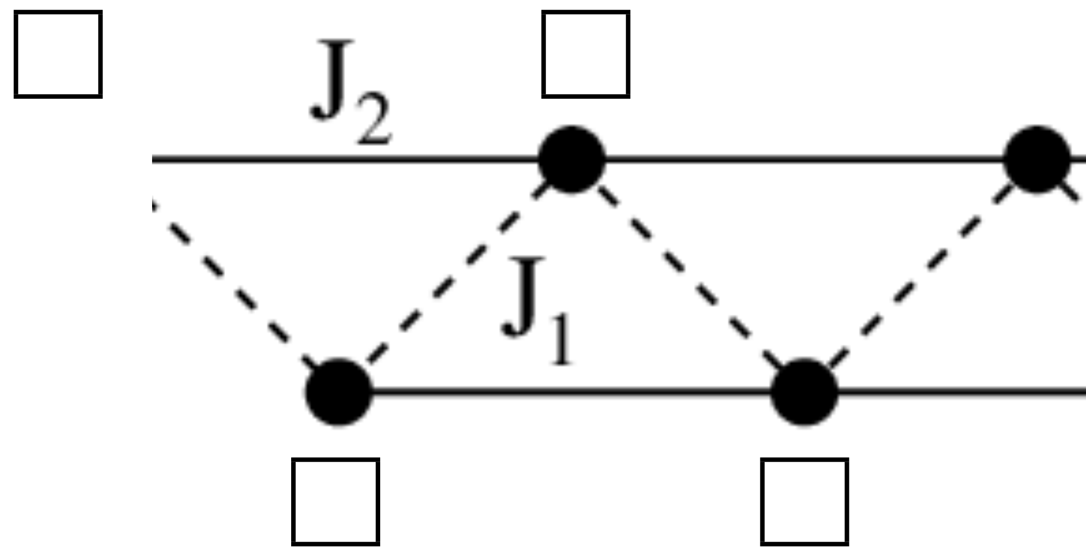
dominant molecular superfluid



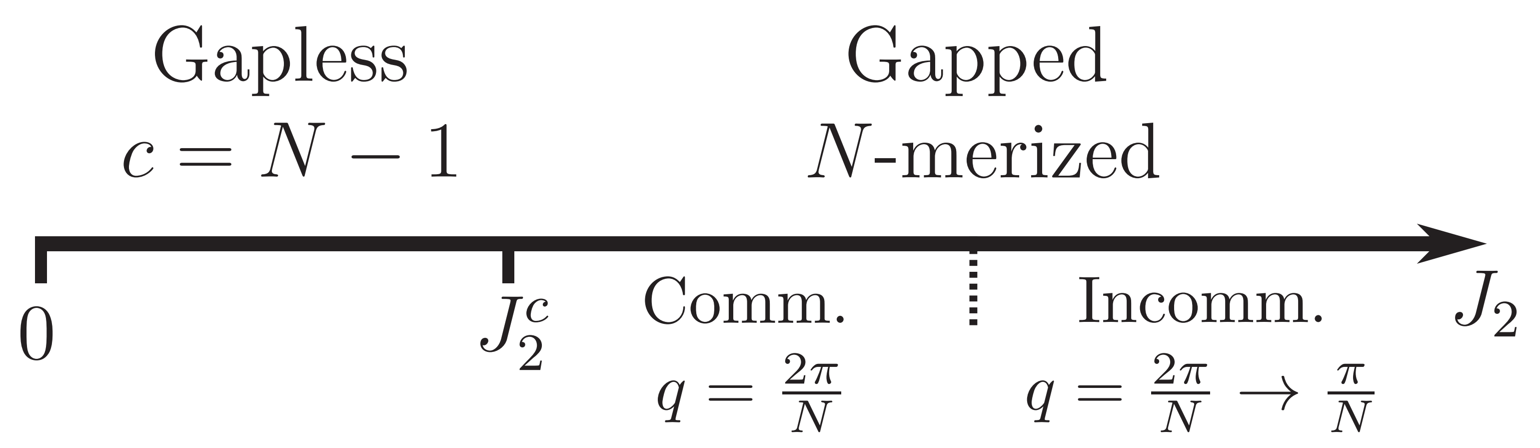

*Capponi et al. '08*

# 1d zigzag SU(N) chain

*Herviou, Capponi, Lecheminant '23*

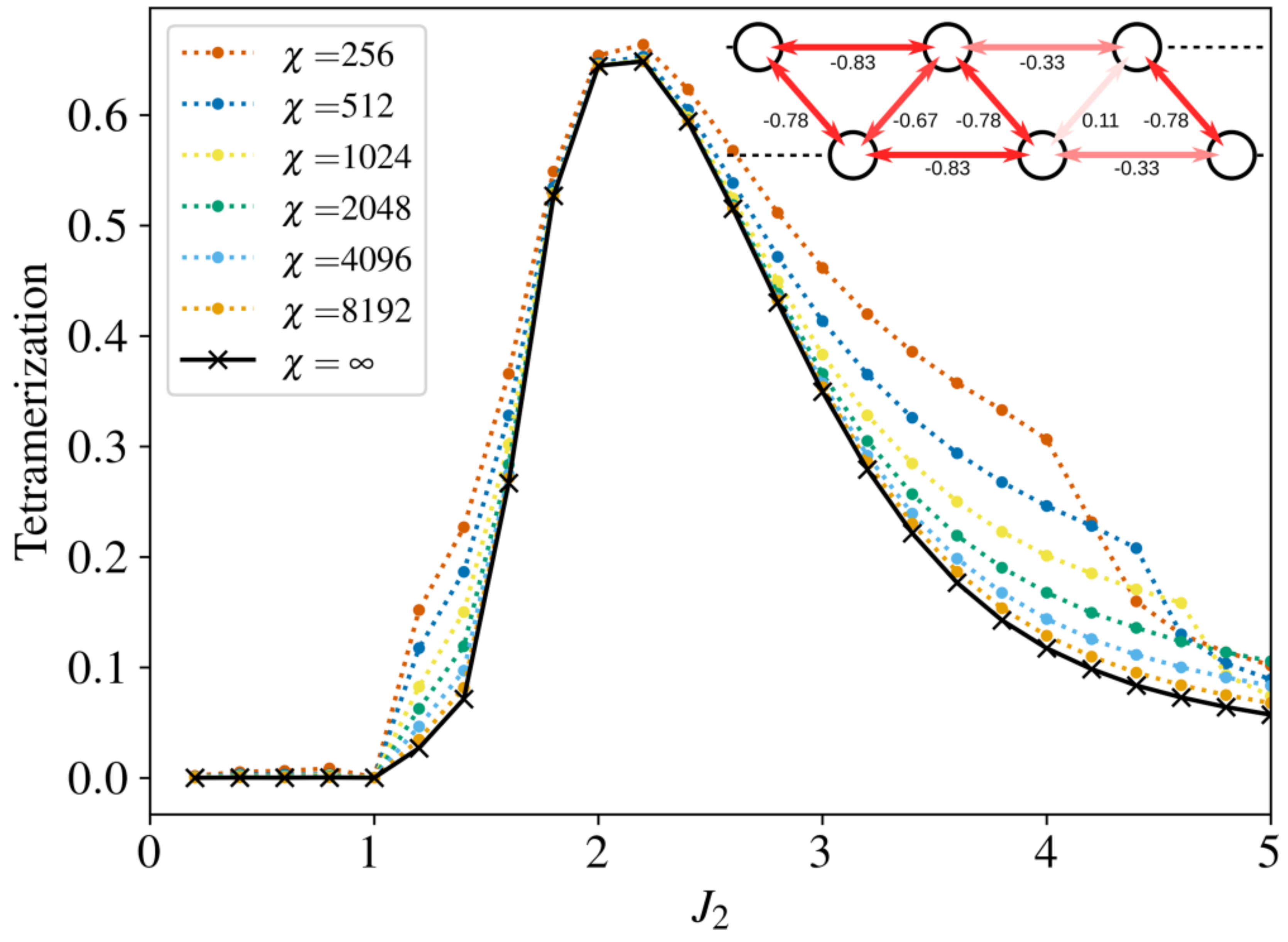



## Even N

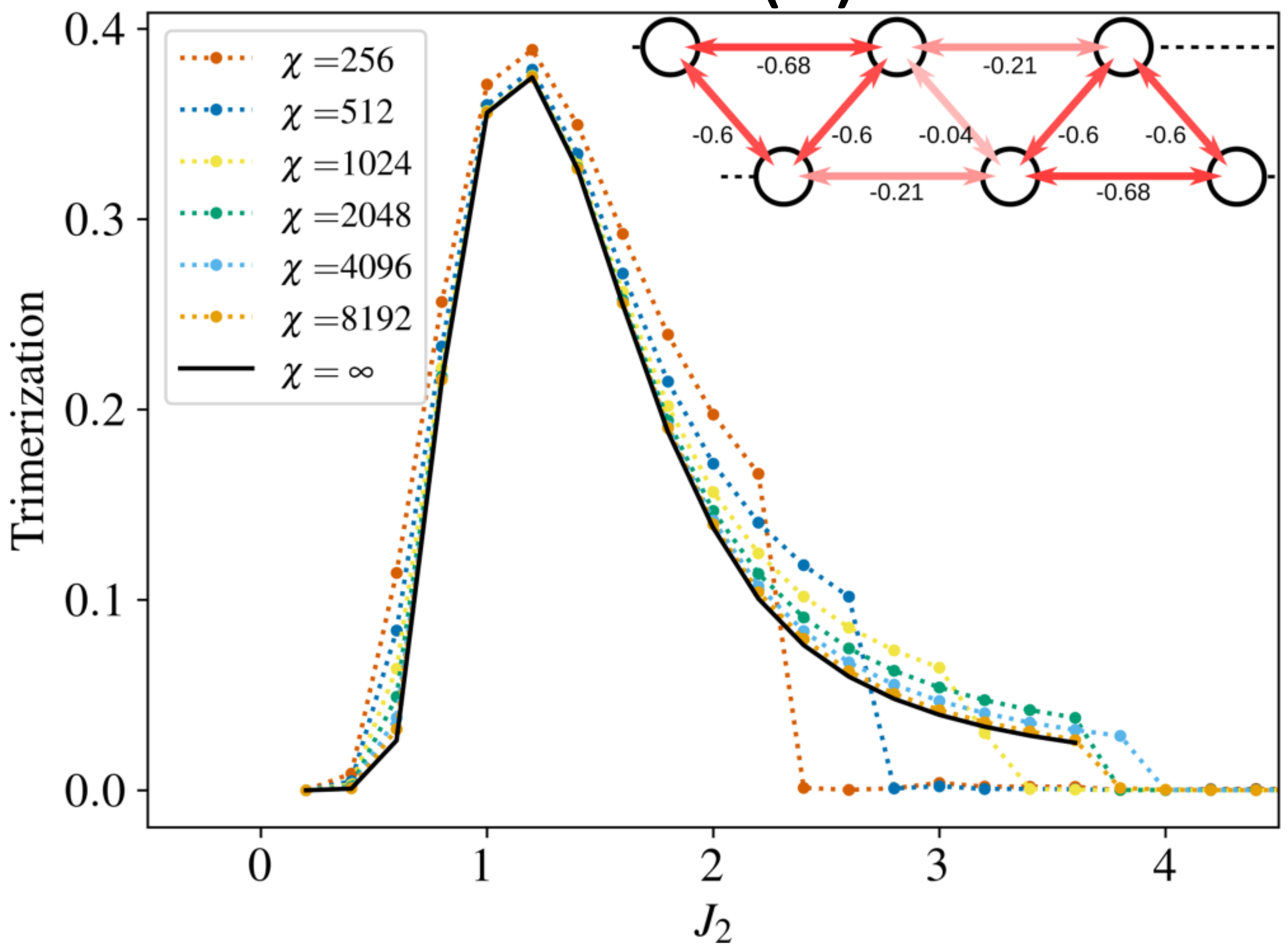



## Odd N


## SU(4)




## SU(3)

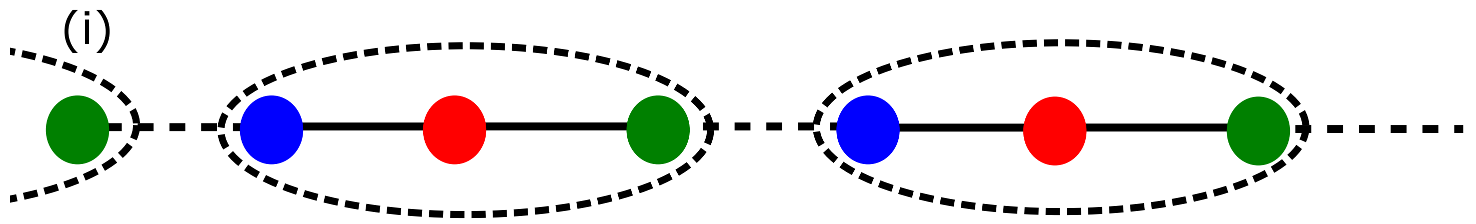



Take-home message: only trivial phases (no SPT)

# 1d modulated SU(N) chain

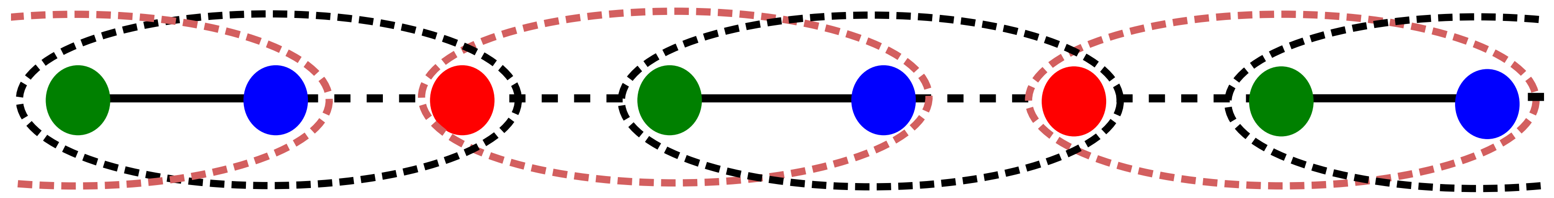
Heisenberg couplings are $1 + \delta \cos\left(\frac{2\pi r}{N}\right)$ e.g. for N=3: $1+\delta,\ 1-\delta/2,\ 1-\delta/2$

$\delta < 0$




Trivial phase

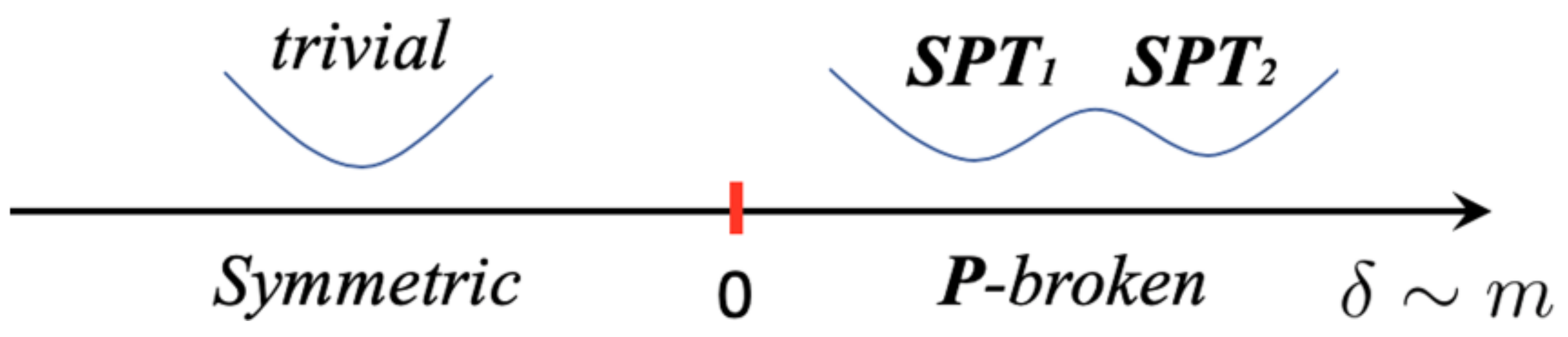
$\delta > 0$



**SPT phase**, breaks inversion sym

What is the nature of the transition at $\delta = 0$ ?

What is the nature of the excitations ?




*Bi, Lake, Senthil (2020)*


PHYSICAL REVIEW B **111**, L020404 (2025)

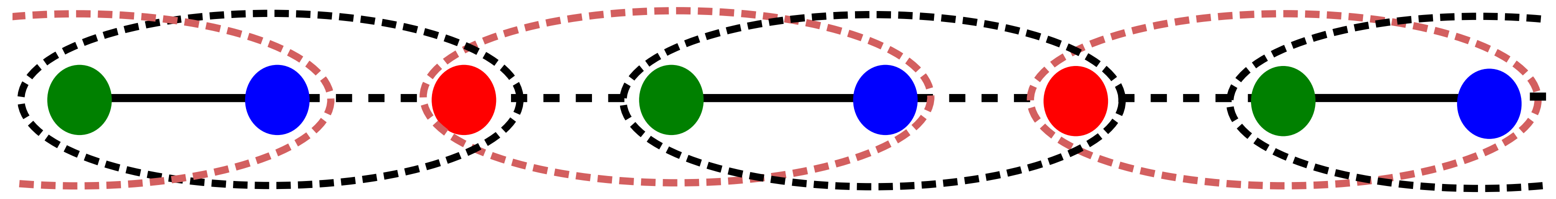
Letter


**Non-Landau quantum phase transition in modulated SU(N) Heisenberg spin chains**


Sylvain Capponi,[1] Lukas Devos,[2] Philippe Lecheminant,[3] Keisuke Totsuka,[4] and Laurens Vanderstraeten[5]


TensorKit.jl

# SU(3) chain with modulation

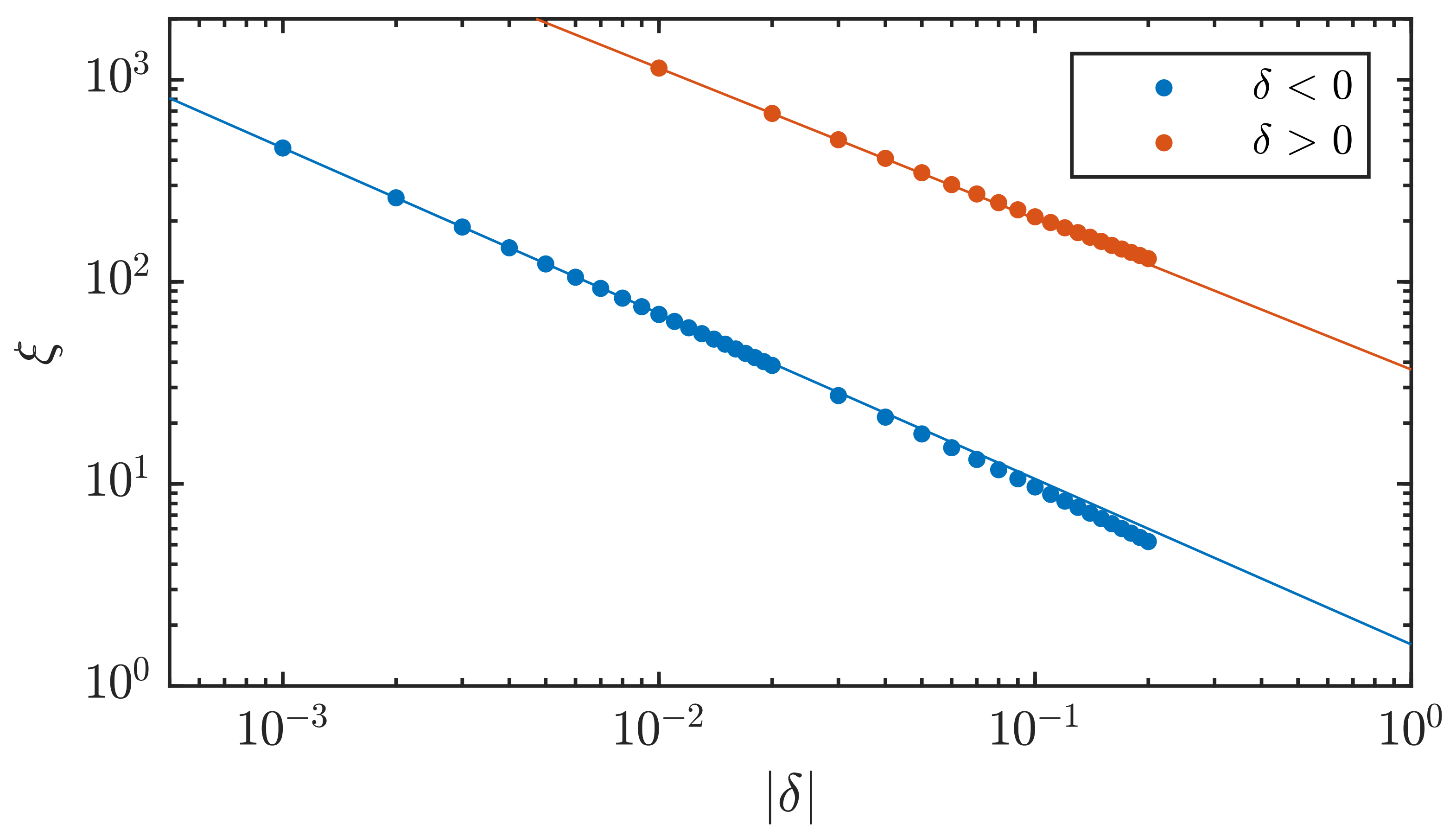


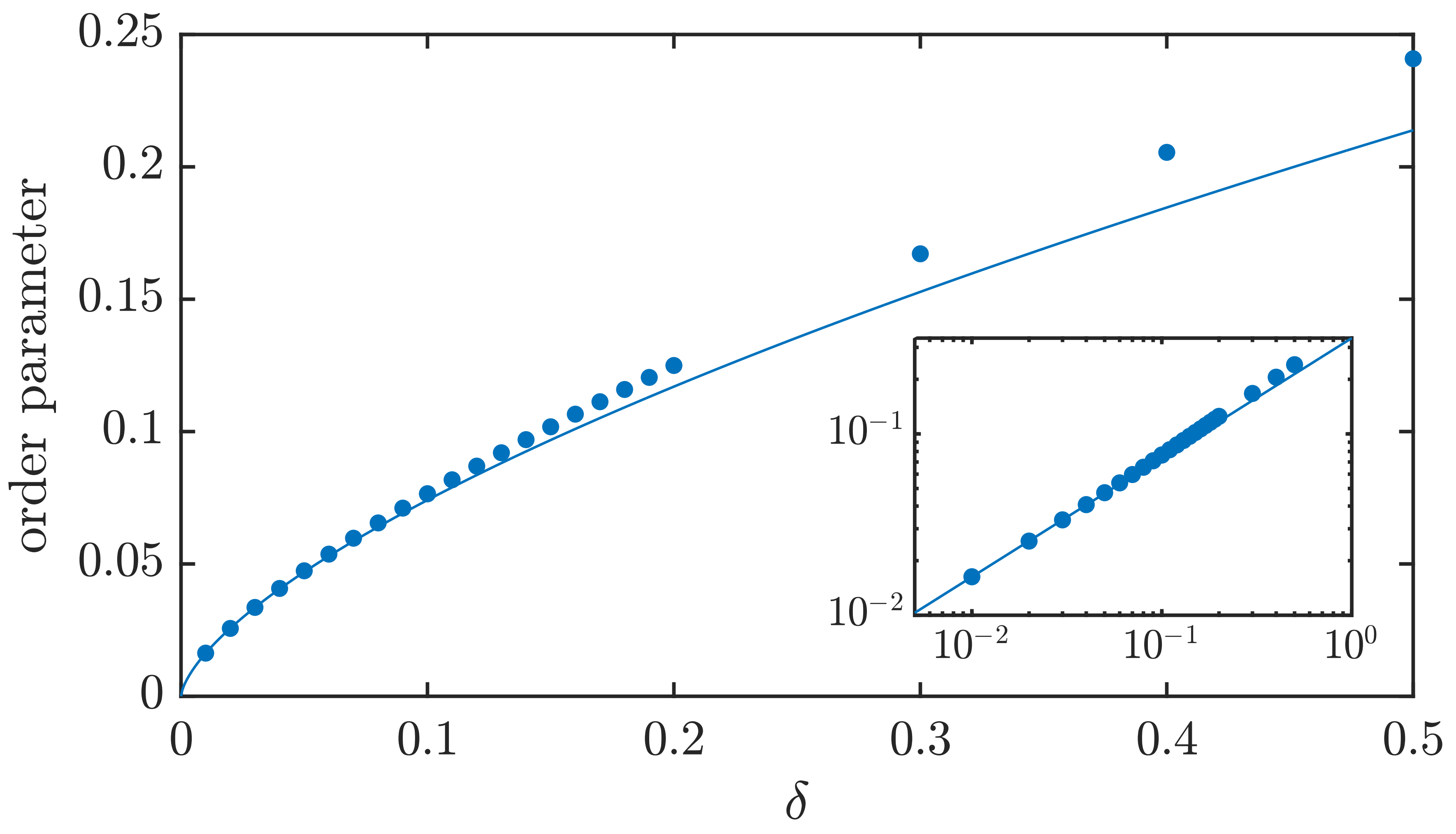
Strong-coupling: it maps onto a $\mathbf{3} - \bar{\mathbf{3}}$ chain, known to be dimerized *Afflect '90*

Field-theory predicts opening of a spectral gap $\Delta \sim |\delta|^{N/(N+1)}$

Despite the Z2 inversion symmetry breaking, the phase transition is not Ising-like (c=1/2) but belongs to the $SU(N)_1$ universality class with central charge c=N-1

iDMRG results using SU(3)



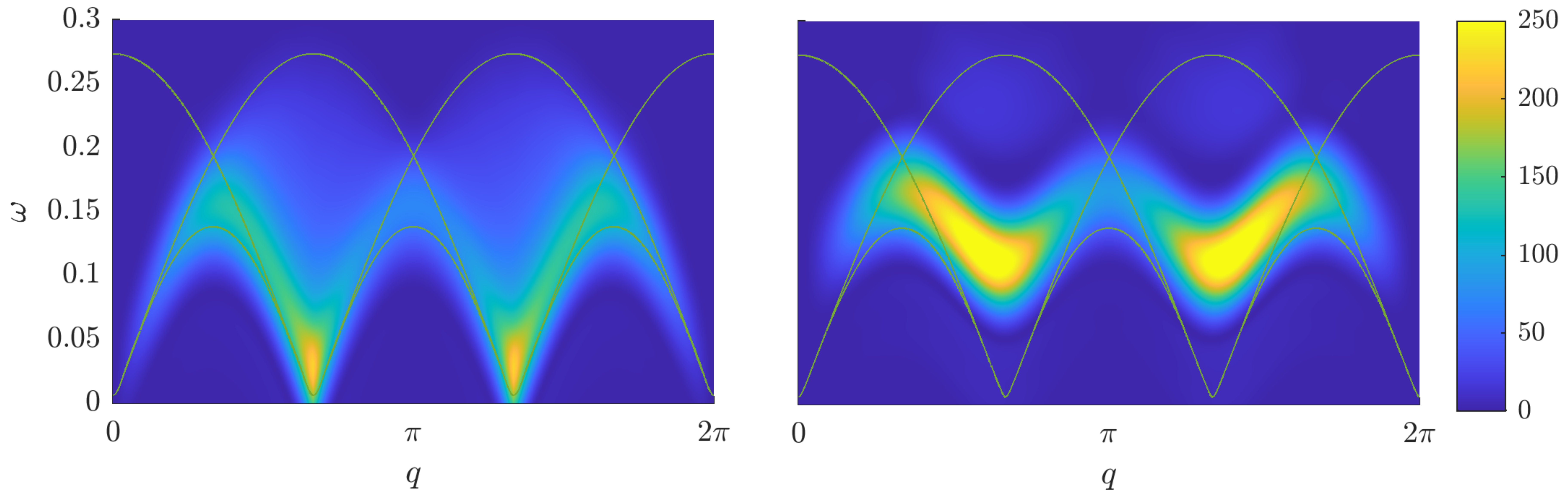



# SU(3) chain: spectral functions

Using MPS excitation ansatz, one can compute the spectral functions

$$S(q,\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle\Psi_0| e^{-iHt} S^A_{-q} e^{iHt} S^A_q |\Psi_0\rangle$$

*Vanderstraeten, Verstraete et al.*




Compatible with a 2-spinon continuum

$\delta > 0$

(Small) Explicit symmetry breaking

Spinons are confined into bound states

# Conclusion about 1d

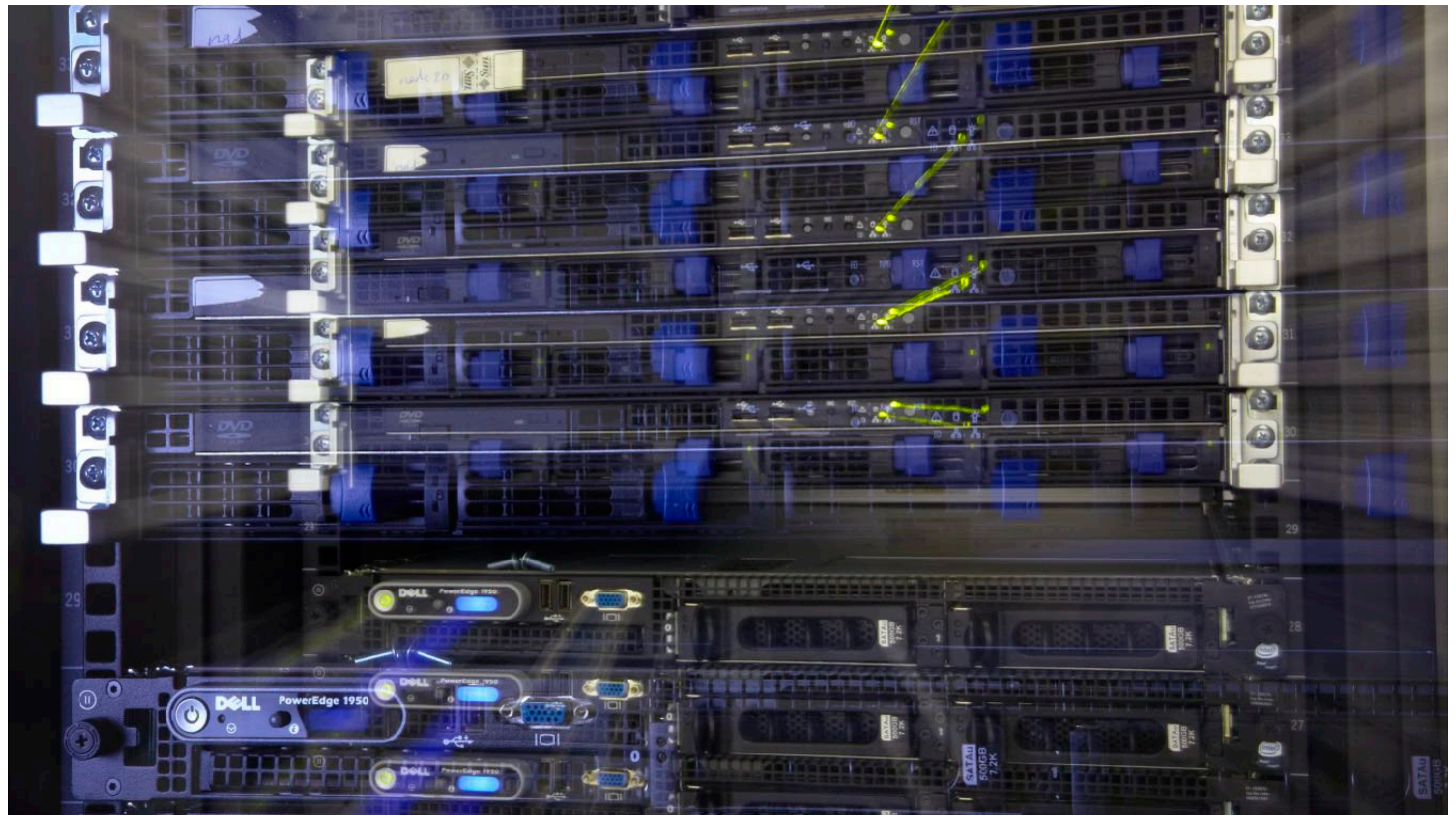
- Often, SU(N) models have nontopological groundstates
- Some realistic *microscopic* SU(N) models can stabilize all SPT phases
- Some SPT phases also break inversion symmetry breaking (chiral)
- A simple modulated SU(N) chain exhibits a non-Landau quantum phase transition
- **Perspective**: quantum phase transition between a trivial phase and an SPT is conjectured to have central charge $c \geq \log_2 d$

*Verresen, Moessner, Pollmann (2017)*

# Going to two dimensions

Quite fun !


But challenging too…



# Topological phases of matter

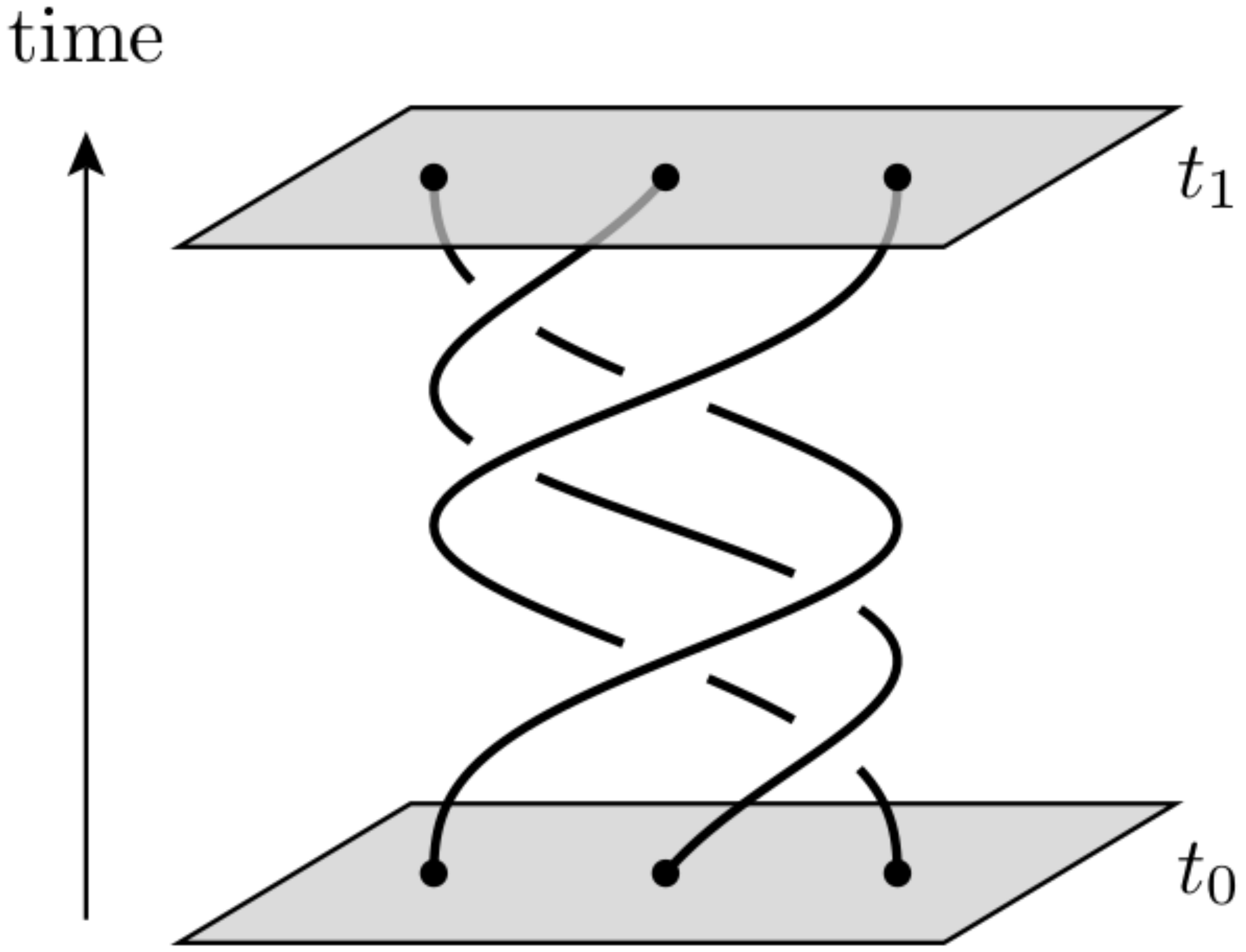
- Robustness of topological states

→ **Topological quantum computation** (no error correction needed !)

- Quasiparticles are anyons (fractional statistics) i.e. not necessarily bosons or fermions (spin statistics theorem breaks down in 2+1D)

- Excitations can be abelian or not

topological quantum field theory
(e.g. Chern-Simons),
braid group, fusion rules…





$\neq$

# Chiral topological spin liquids

Chiral topological phase is found in the fractional quantum Hall (FQH) effect

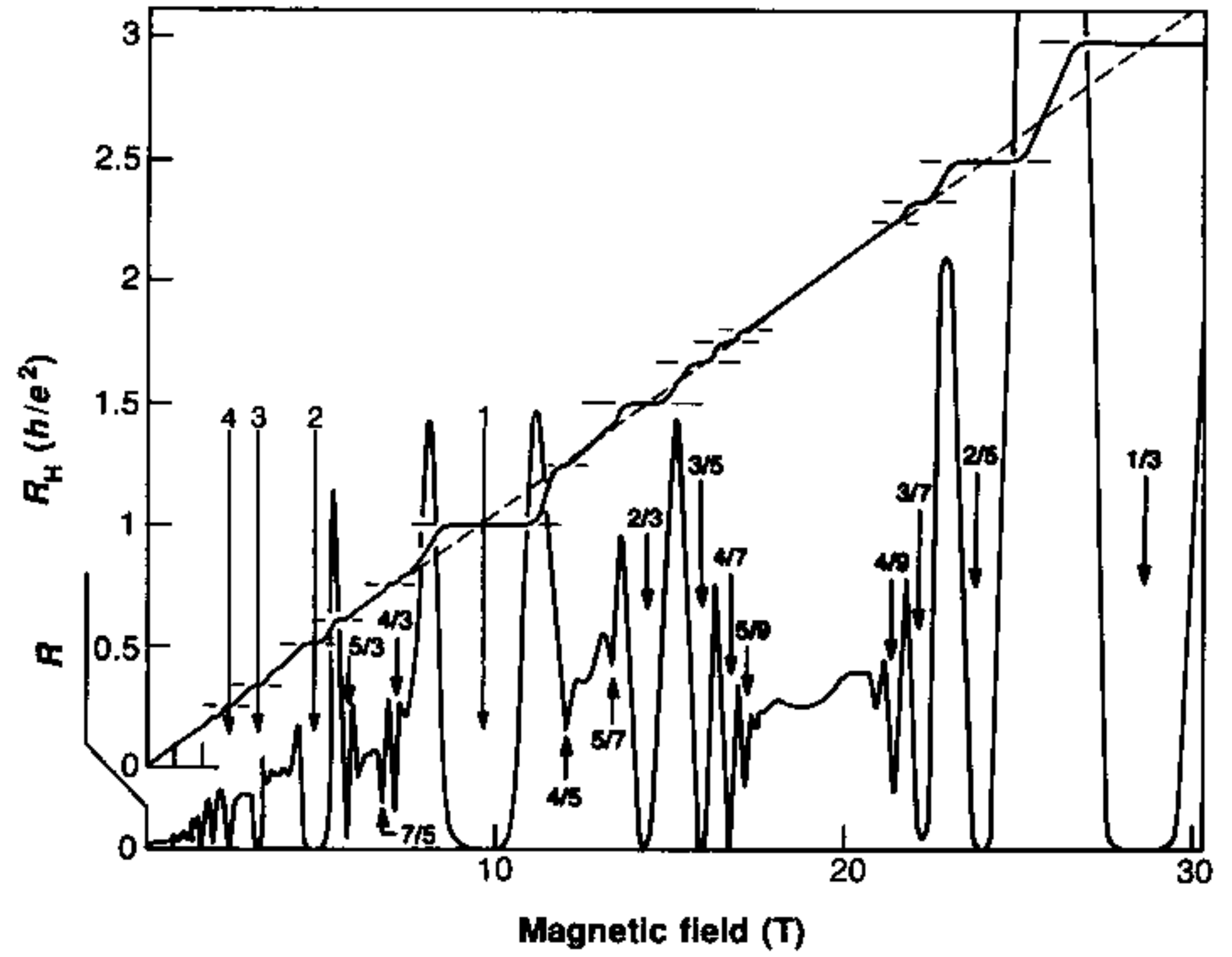



**Fig. 2.** Composite view showing the Hall resistance $R_H$ and longitudinal

**topological** phases, exotic excitations
(**abelian or not**)
unconventional superconductor when doped

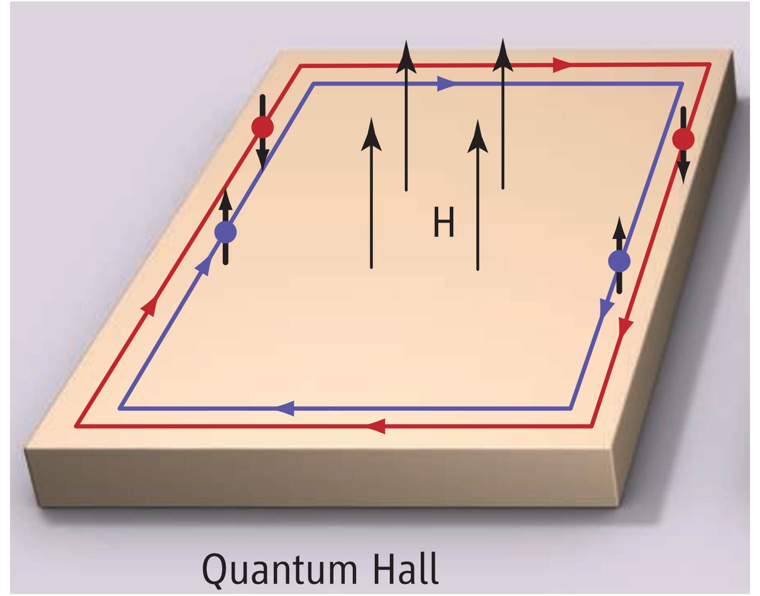
Is it possible to reach the same physics without Landau levels, on a lattice ?

Mimic an effective magnetic field, flat bands etc.

**Fractional Chern insulators**

Look for lattice models with similar wavefunctions

# Chiral spin liquids (CSL)




# = lattice analogue of FQH states

Low-energy physics described by 2+1 Chern-Simons theory

| $\nu = \frac{1}{2}$ FQH state | | lattice spin S=1/2 model |
|---|---|---|
| incompressible (gapped) in the bulk | | same |
| charged e/2 fractional excitation |  | neutral s=1/2 fractional excitation |
| robust gapless chiral edge states | | same |
| $SU(2)_1$ CFT | | |

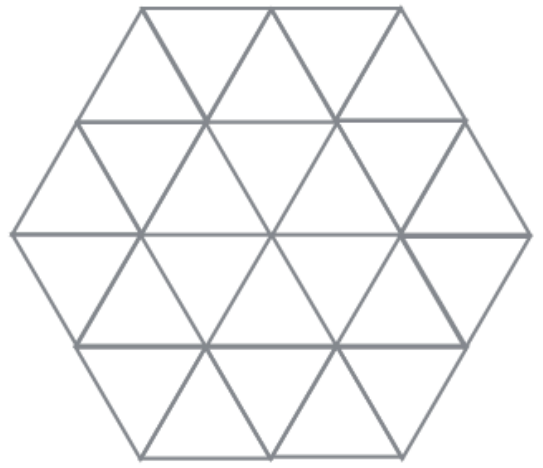
triangular lattice: *Kalmeyer-Laughlin, 1987*

# Abelian CSL in spin-1/2 SU(2) models on frustrated lattices

breaks T

## S=1/2 on triangular lattice

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{\triangle/\nabla} (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k$$



PHYSICAL REVIEW B **96**, 075116 (2017)

**Global phase diagram and quantum spin liquids in a spin-$\frac{1}{2}$ triangular antiferromagnet**

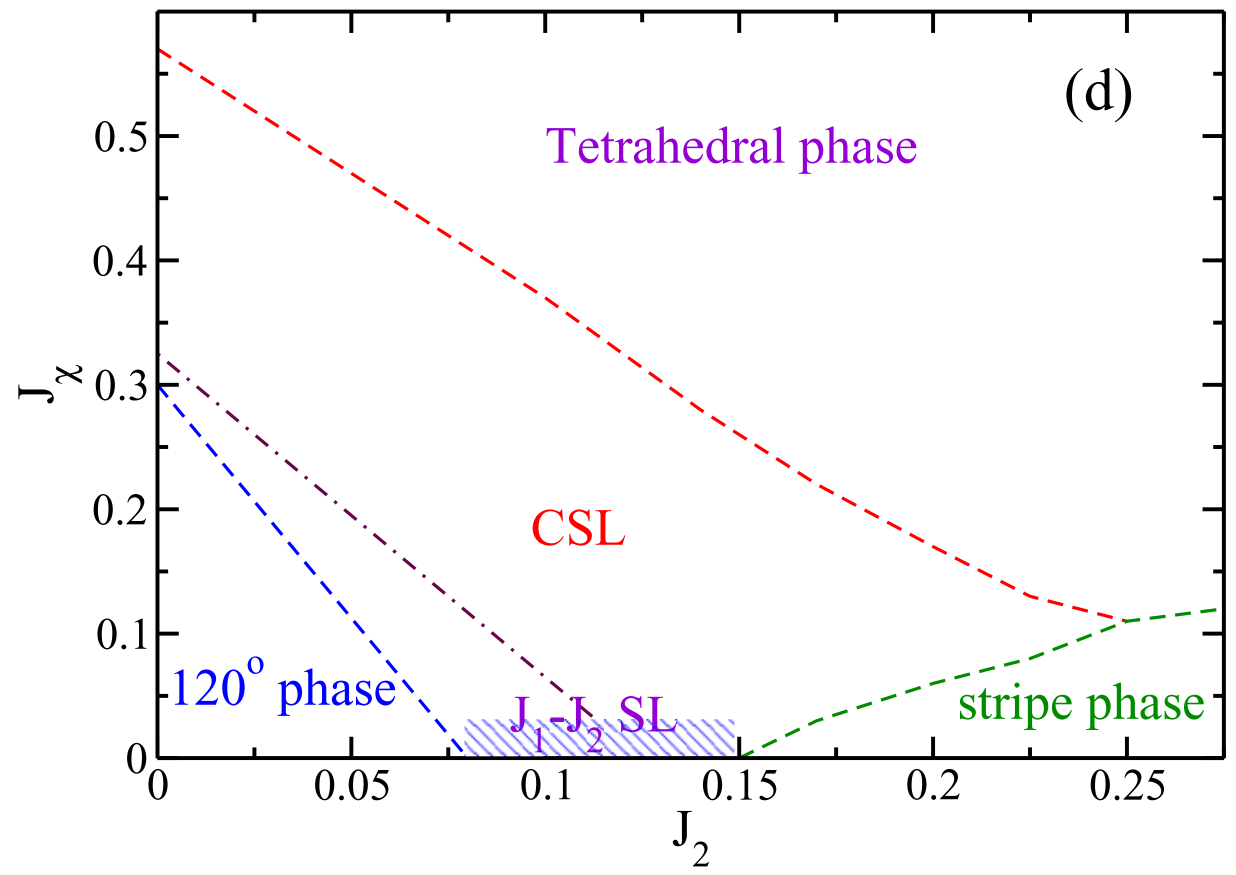
Shou-Shu Gong,[1] W. Zhu,[2] J.-X. Zhu,[2,3] D. N. Sheng,[4] and Kun Yang[5]

PHYSICAL REVIEW B **95**, 035141 (2017)

**Chiral spin liquid and quantum criticality in extended $S = \frac{1}{2}$ Heisenberg models on the triangular lattice**

Alexander Wietek* and Andreas M. Läuchli

DMRG

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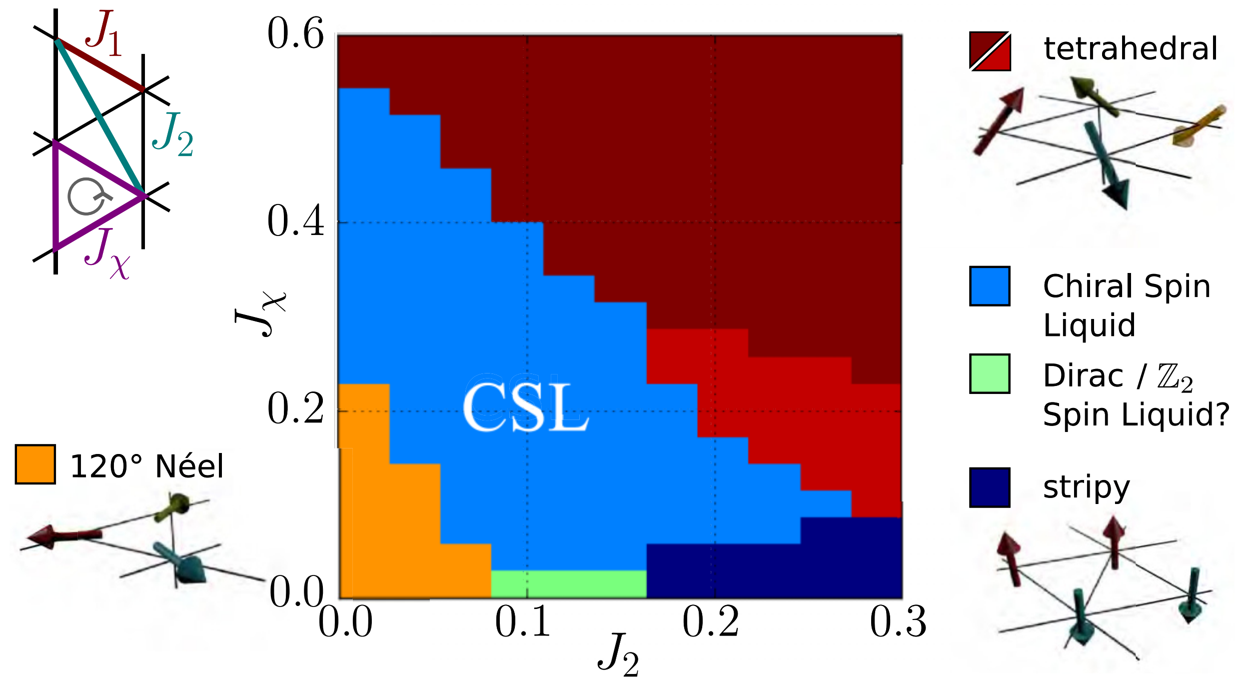
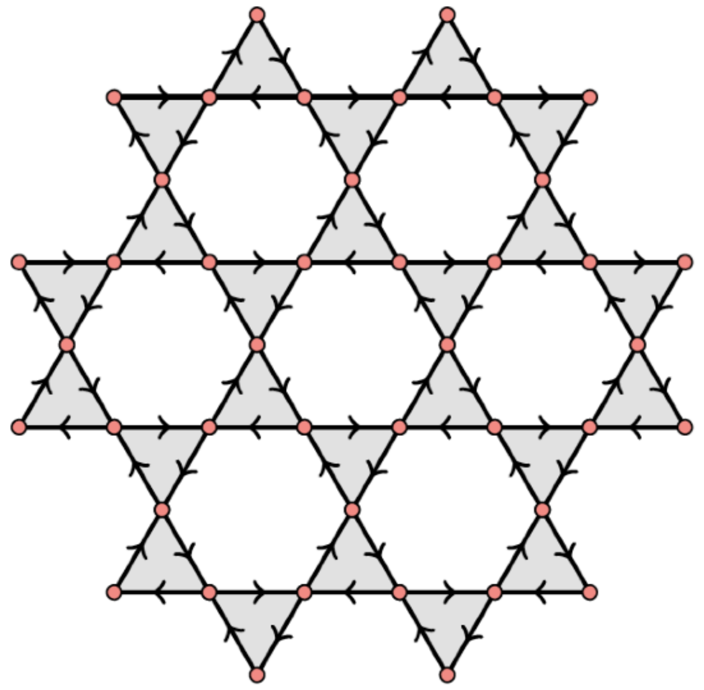



## S=1/2 on kagome lattice

DMRG

Chiral spin liquid and emergent anyons in a Kagome lattice Mott insulator

B. Bauer[1], L. Cincio[2], B.P. Keller[3], M. Dolfi[4], G. Vidal[2], S. Trebst[5] & A.W.W. Ludwig[3]



## S=1/2 on frustrated square lattice

PHYSICAL REVIEW B **96**, 121118(R) (2017)

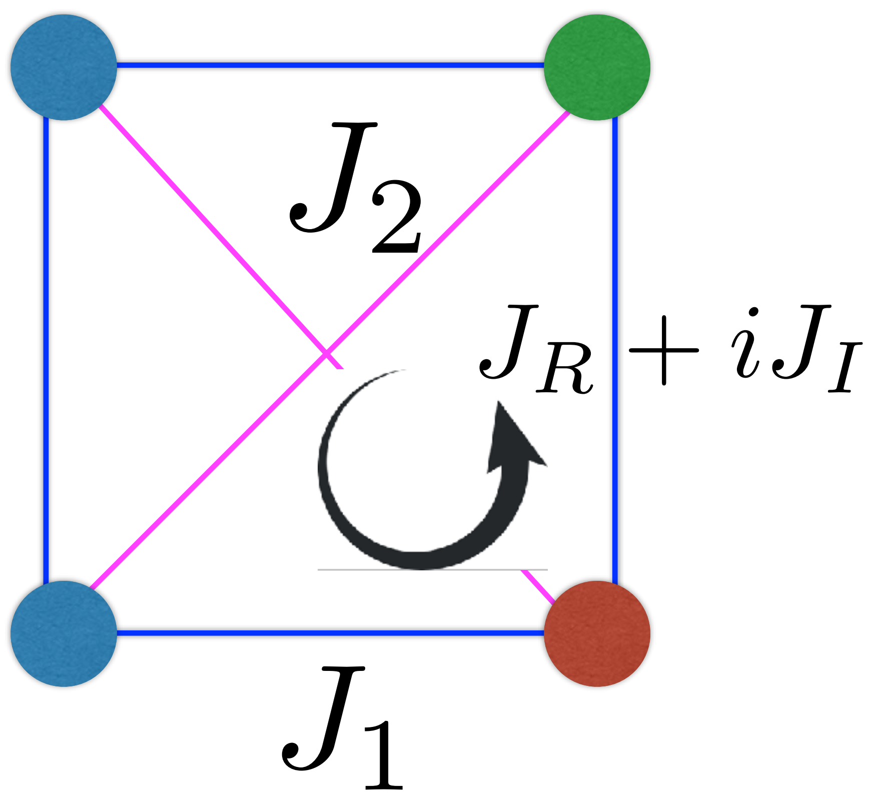
PEPS

**Investigation of the chiral antiferromagnetic Heisenberg model using projected entangled pair states**

Didier Poilblanc

# SU(N) extension

$$H = J_1 \sum_{\langle i,j \rangle} P_{ij} + J_2 \sum_{\langle\langle k,l \rangle\rangle} P_{kl} + J_{\mathrm{R}} \sum_{\triangle ijk} (P_{ijk} + P_{ijk}^{-1}) + iJ_{\mathrm{I}} \sum_{\triangle ijk} (P_{ijk} - P_{ijk}^{-1})$$




imaginary 3-site permutation explicitly breaks T and P

Square lattice, C4 symmetry

PHYSICAL REVIEW B **104**, 235104 (2021)

**Abelian SU($N$)$_1$ chiral spin liquids on the square lattice**

Ji-Yao Chen,[1,2] Jheng-Wei Li,[3] Pierre Nataf,[4] Sylvain Capponi,[5] Matthieu Mambrini,[5] Keisuke Totsuka,[6] Hong-Hao Tu,[7] Andreas Weichselbaum,[8] Jan von Delft,[3] and Didier Poilblanc[5]

See also on the triangular lattice:

PRL **117**, 167202 (2016) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2016

**Chiral Spin Liquids in Triangular-Lattice SU($N$) Fermionic Mott Insulators with Artificial Gauge Fields**

Pierre Nataf,[1] Miklós Lajkó,[2] Alexander Wietek,[3] Karlo Penc,[4,5] Frédéric Mila,[1] and Andreas M. Läuchli[3]

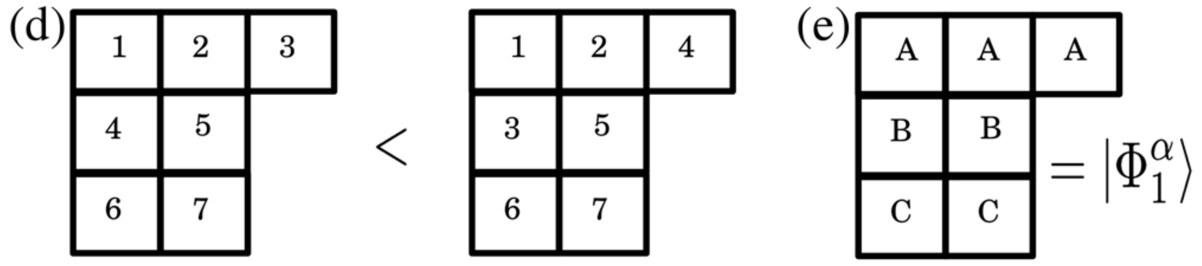
# Numerical methods for SU(N)

## ... an introduction

Analytics: large-N, mean-field, parton wavefunctions

- **Exact diagonalization** (U(1)+lattice symmetries or SU(N) symmetry)

using standard Young tableaux




PRL 113, 127204 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

**Exact Diagonalization of Heisenberg SU($N$) Models**

Pierre Nataf and Frédéric Mila

| $SU(N)$ | $n$ | $f^{[k,\dots,k]}$ | $\frac{(n-1)!}{k!^N}$ | $\mathcal{E}_{GS}$ |
|---|---|---|---|---|
| $SU(5)$ | 25 (tilted) | 701149020 | $2.5 \times 10^{13}$ | $-1.154324$ |
| $SU(5)$ | 25 $(5 \times 5)$ | 701149020 | $2.5 \times 10^{13}$ | $-1.164712$ |
| $SU(5)$ | 20 | 1662804 | $1.5 \times 10^{10}$ | $-1.215377$ |
| $SU(8)$ | 16 | 1430 | $5.1 \times 10^{9}$ | $-1.572223$ |
| $SU(10)$ | 20 | 16796 | $1.2 \times 10^{14}$ | $-1.589218$ |

using SU(N) U(1) (N-1 Cartan)

# Numerical methods for SU(N)

## ... an introduction

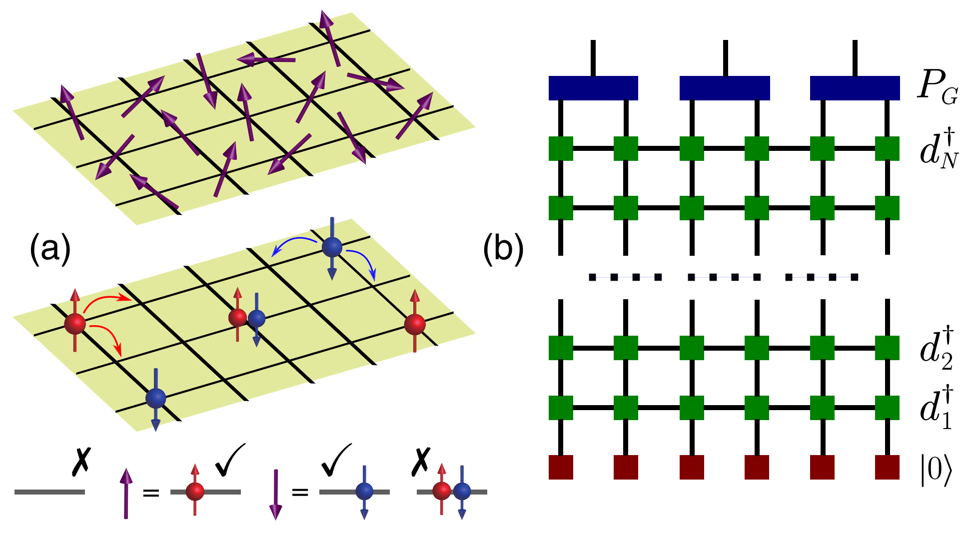
Analytics: large-N, mean-field, parton wavefunctions

- Exact diagonalization (U(1)+lattice symmetries or SU(N) symmetry)
- **DMRG** (U(1) or SU(N) symmetry) + parton wavefunction

Projected Fermi sea

has a tensor network representation
MPO: D=2

using MPO-MPS compression -> MPS




PHYSICAL REVIEW LETTERS **124,** 246401 (2020)

**Tensor Network Representations of Parton Wave Functions**

Ying-Hai Wu,[1] Lei Wang,[2,3] and Hong-Hao Tu[4,*]

# Numerical methods for SU(N)

## ... an introduction

Analytics: large-N, mean-field, parton wavefunctions

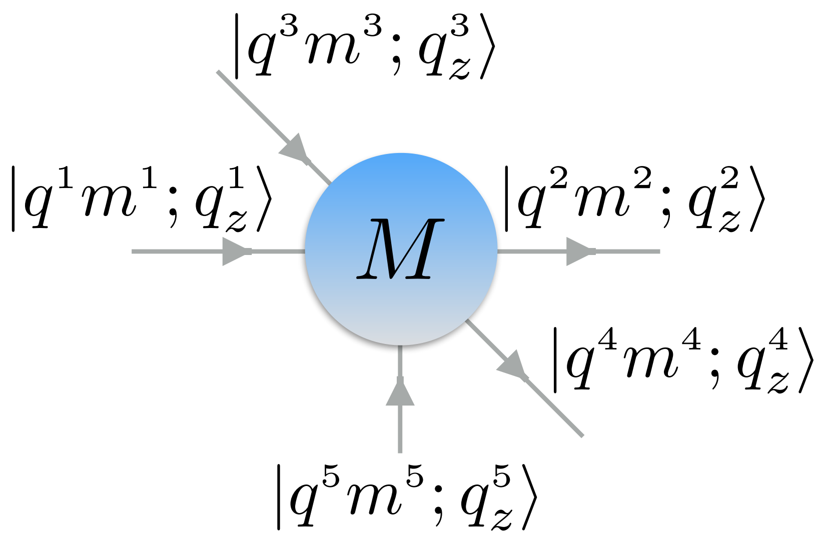
- Exact diagonalization (U(1)+lattice symmetries or SU(N) symmetry)
- DMRG (U(1) or SU(N) symmetry) + parton wavefunction
- **PEPS** using SU(N) symmetric tensors + lattice point-group symmetry

PHYSICAL REVIEW B **94**, 205124 (2016)

**Systematic construction of spin liquids on the square lattice from tensor networks with SU(2) symmetry**

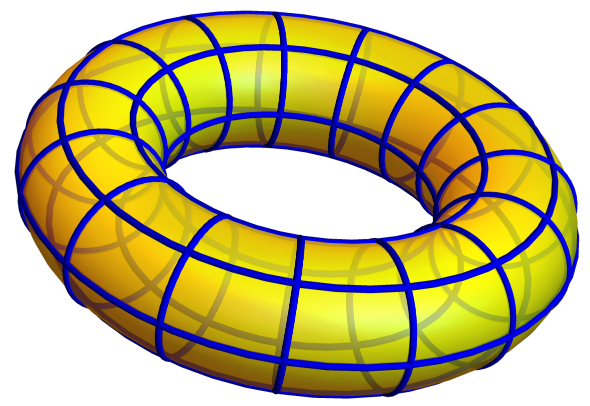
Matthieu Mambrini,[1] Román Orús,[2] and Didier Poilblanc[1]

Both DMRG and PEPS can use SU(N) symmetry, e.g. ***QSpace*** or ***TensorKit*** libraries



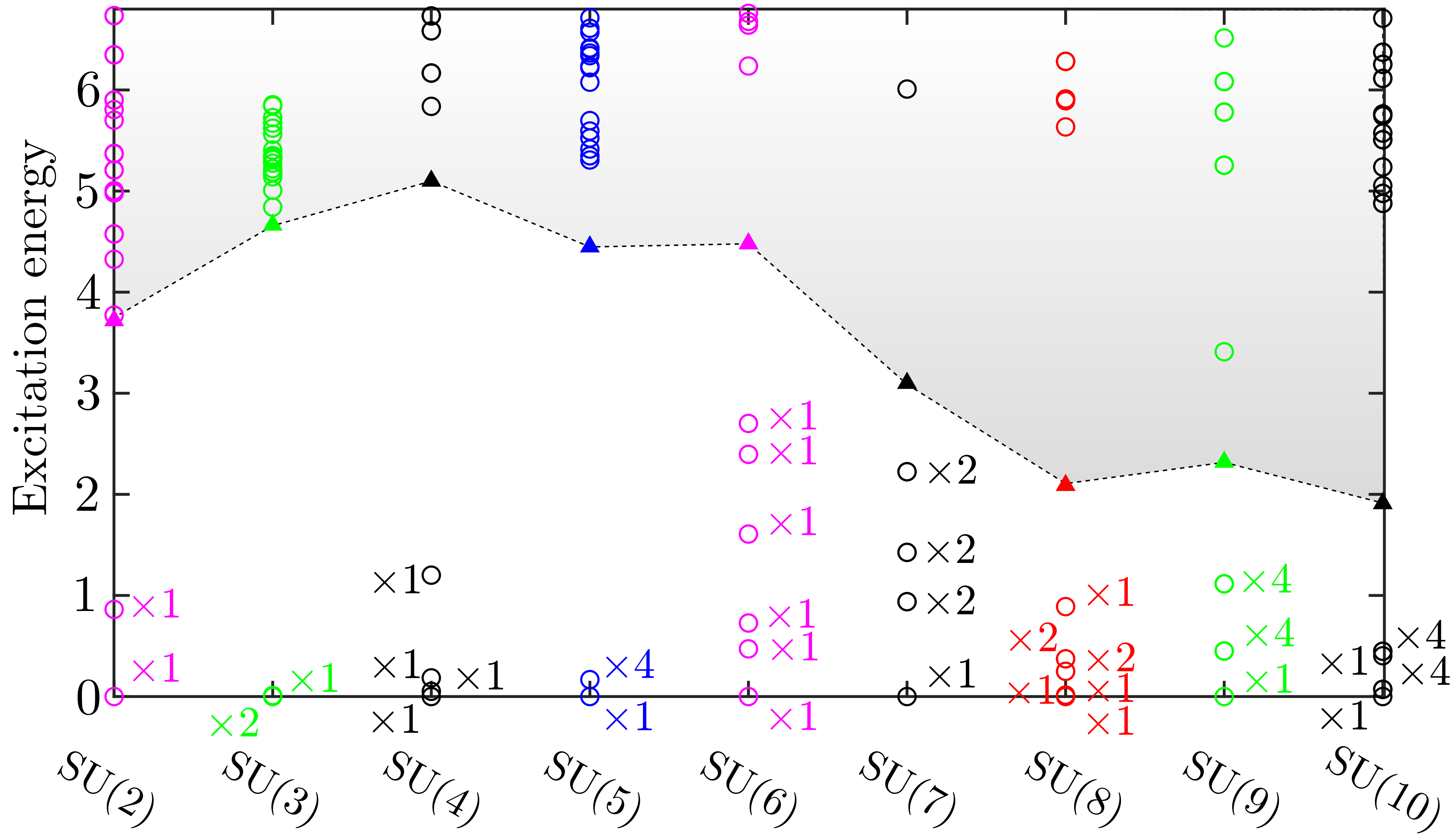

*Andreas Weichselbaum et al. / Lukas Devos & Jutho Haegeman*

# Exact Diagonalization on torus

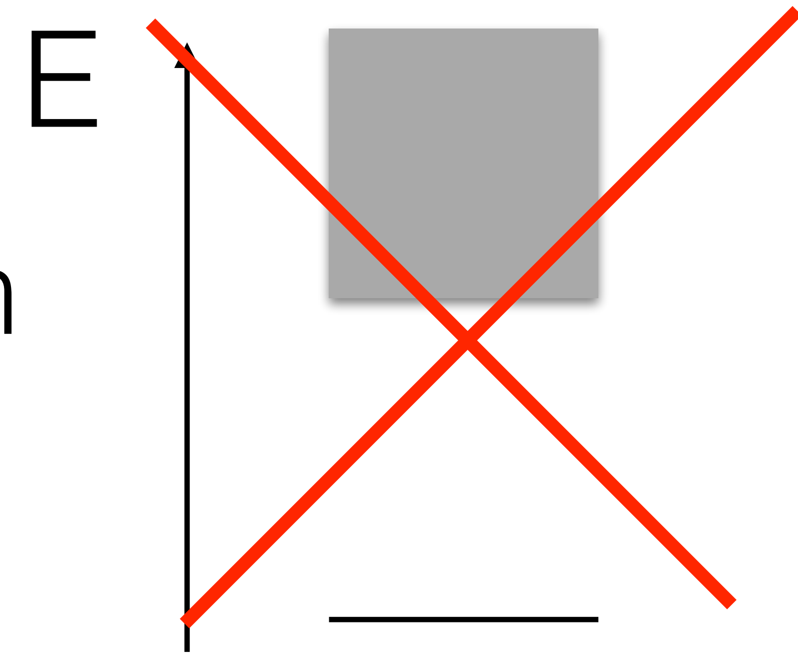


**Predictions:**

- If $\#\text{sites} = kN$ : singlet ground-state, degeneracy on a torus = N




gap above the N quasi-deg ground-states




In 2d: generalization of Hastings-Oshikawa-Lieb-Schultz-Mattis theorem forbids a non-degenerate gapped state

gapless **or** discrete symmetry breaking **or** topological

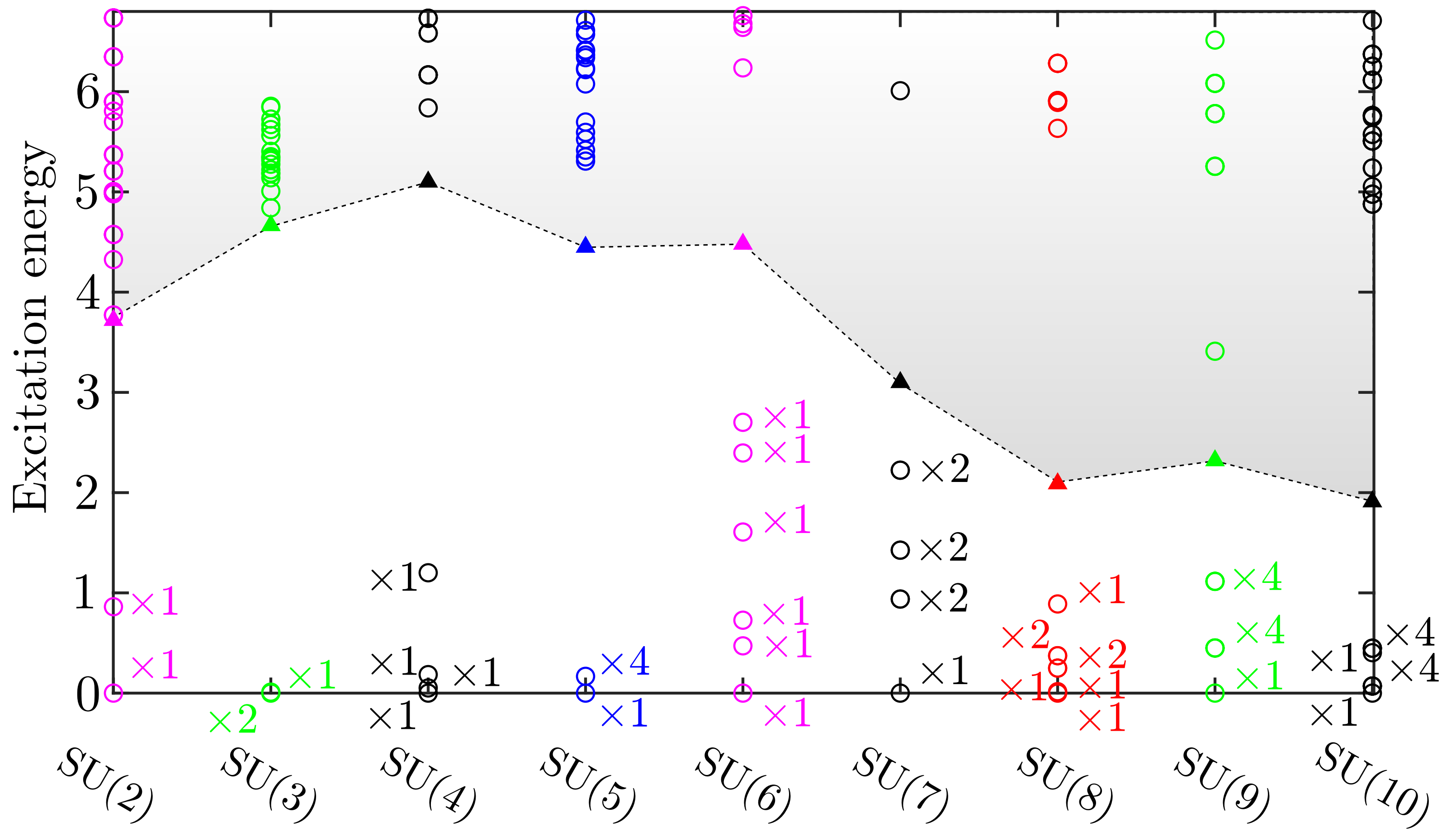
# Exact Diagonalization on torus

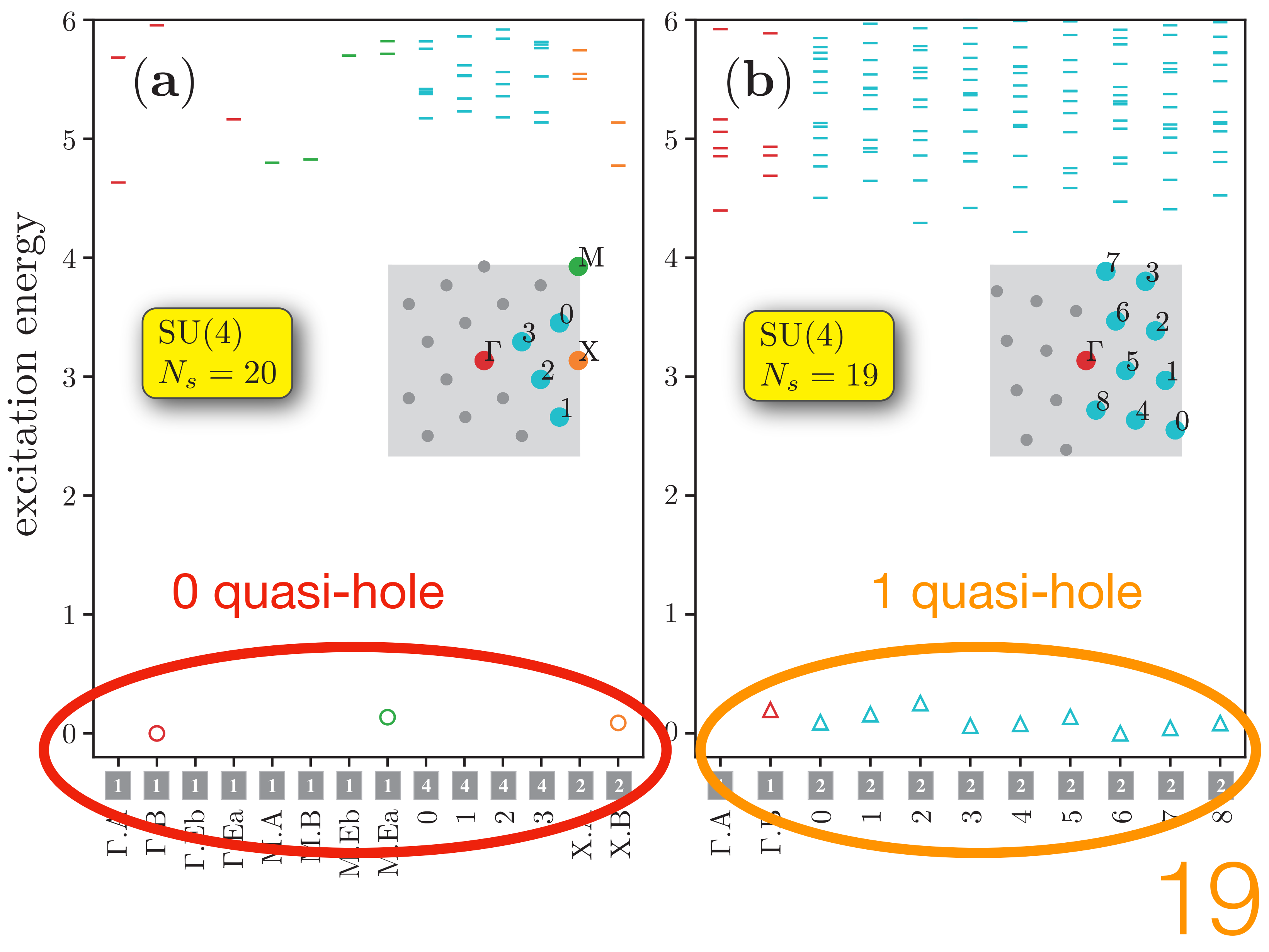
**Predictions:**

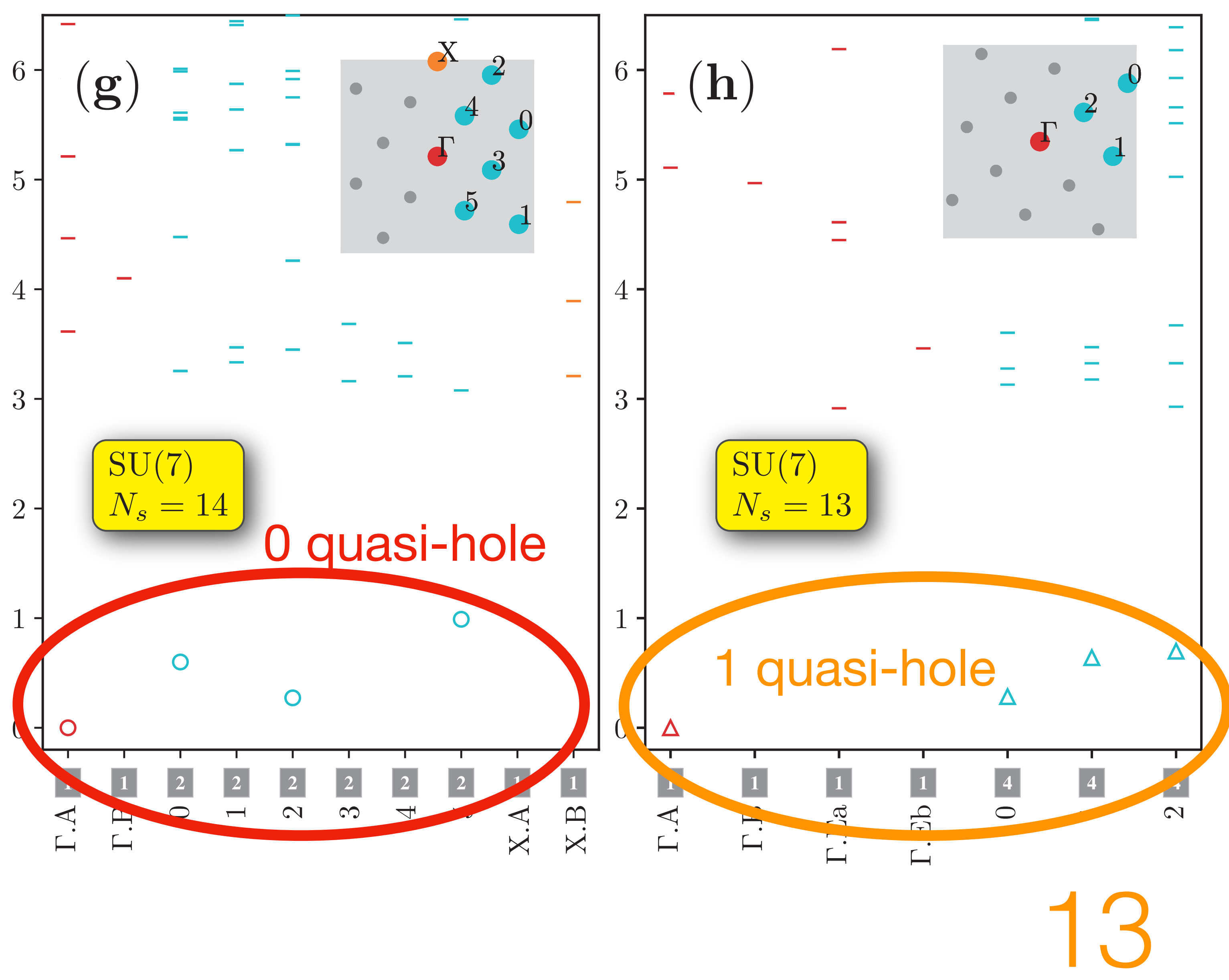
- If $\#\text{sites} = kN$ : singlet ground-state degeneracy on a torus = N
- Lattice momenta can be obtained from a generalized Pauli principle

*Haldane, Bernevig, Regnault,….*

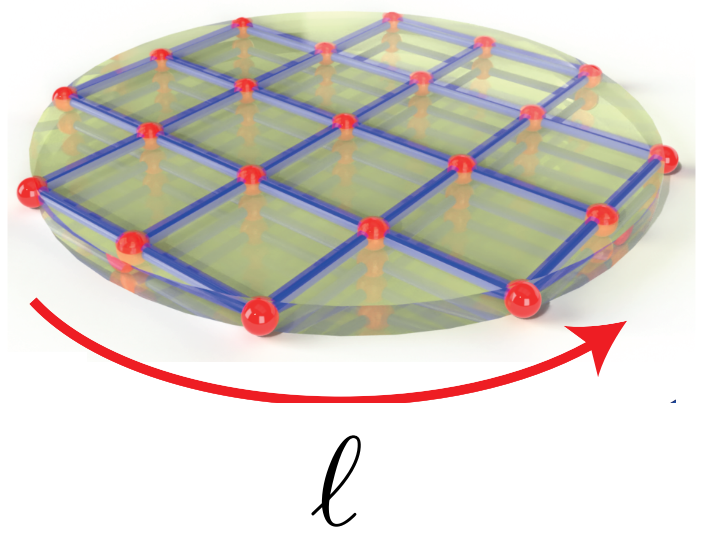
- Quasi-hole counting: deg= $\#\text{sites}$ , 1 per momentum sector



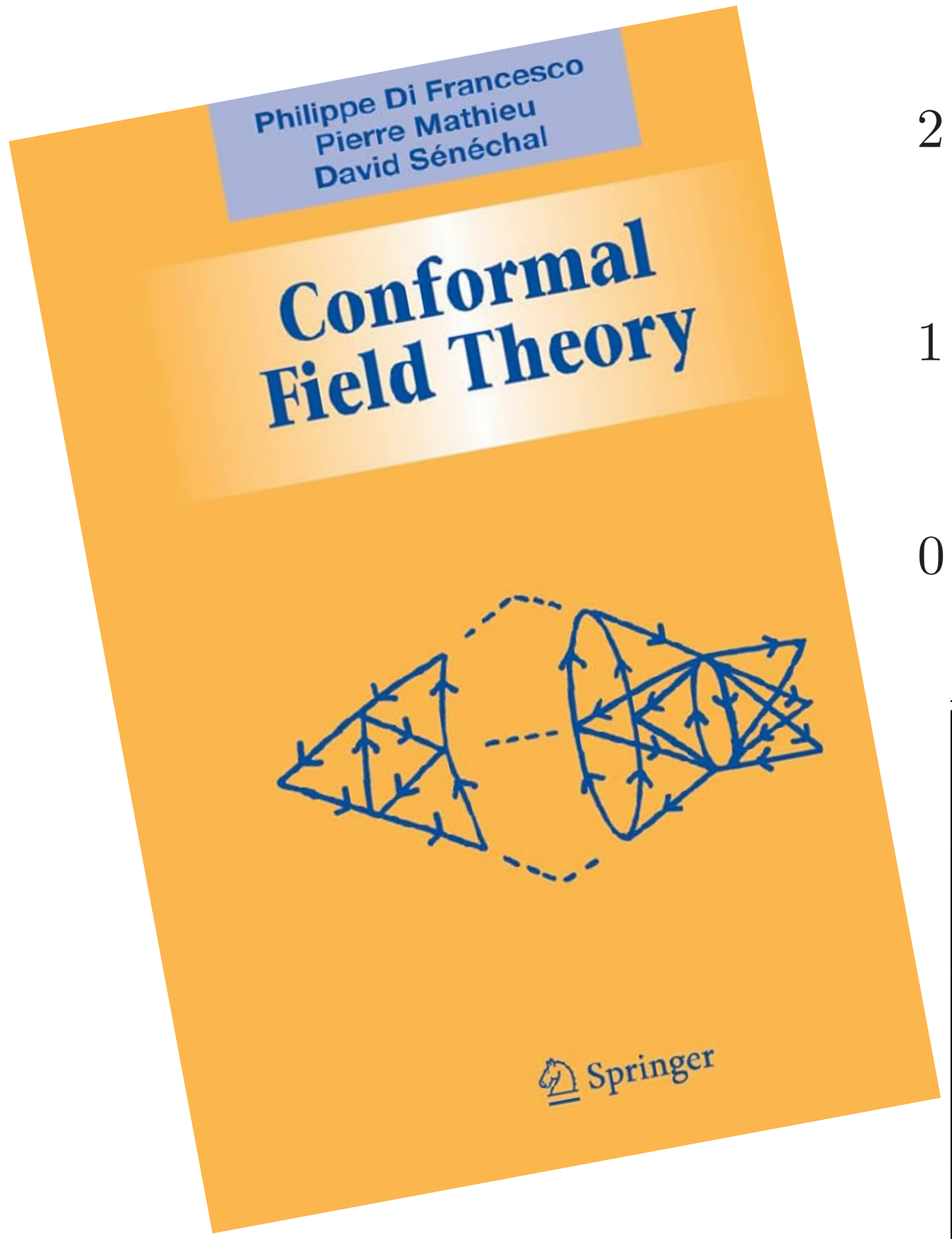



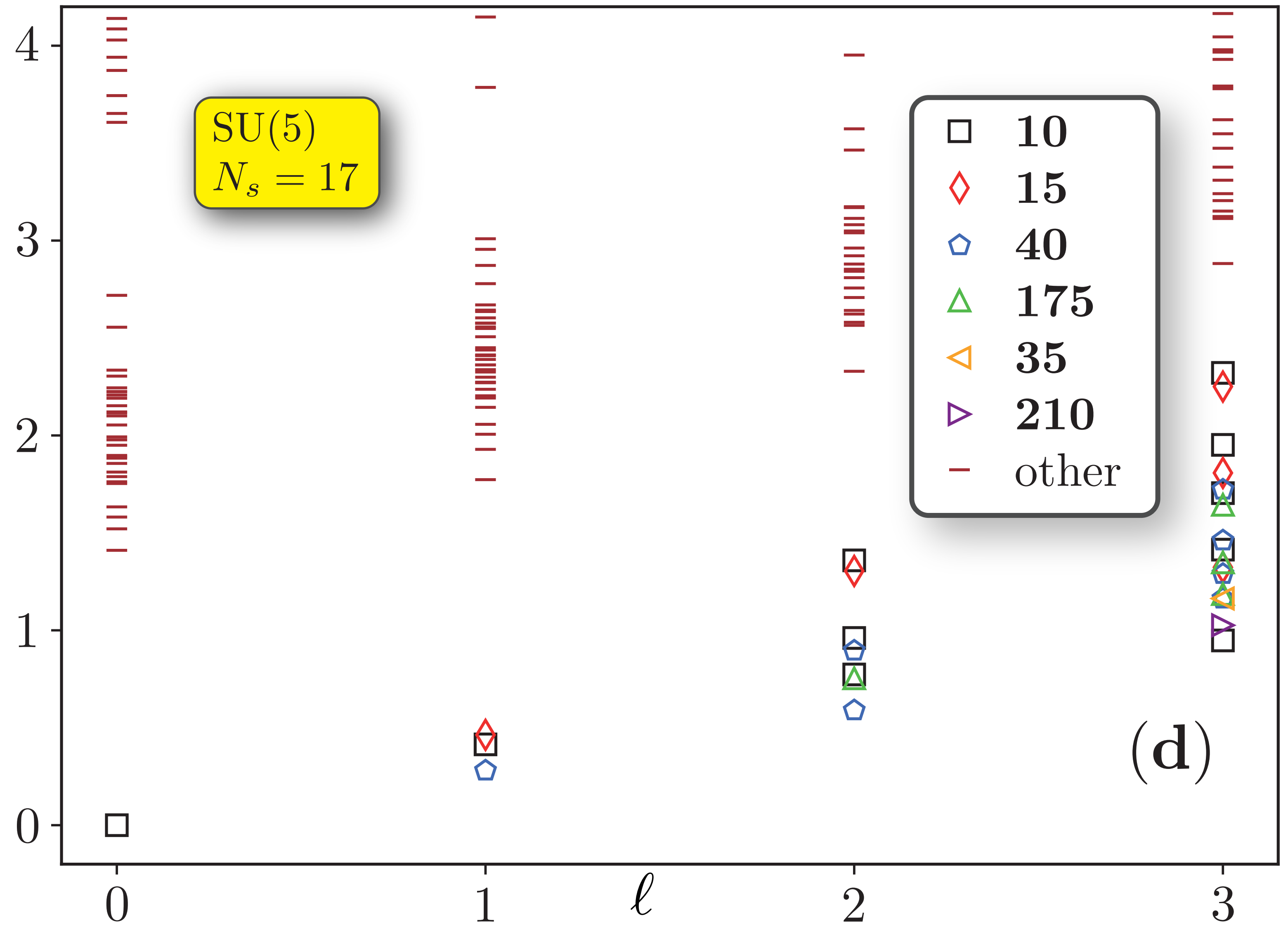



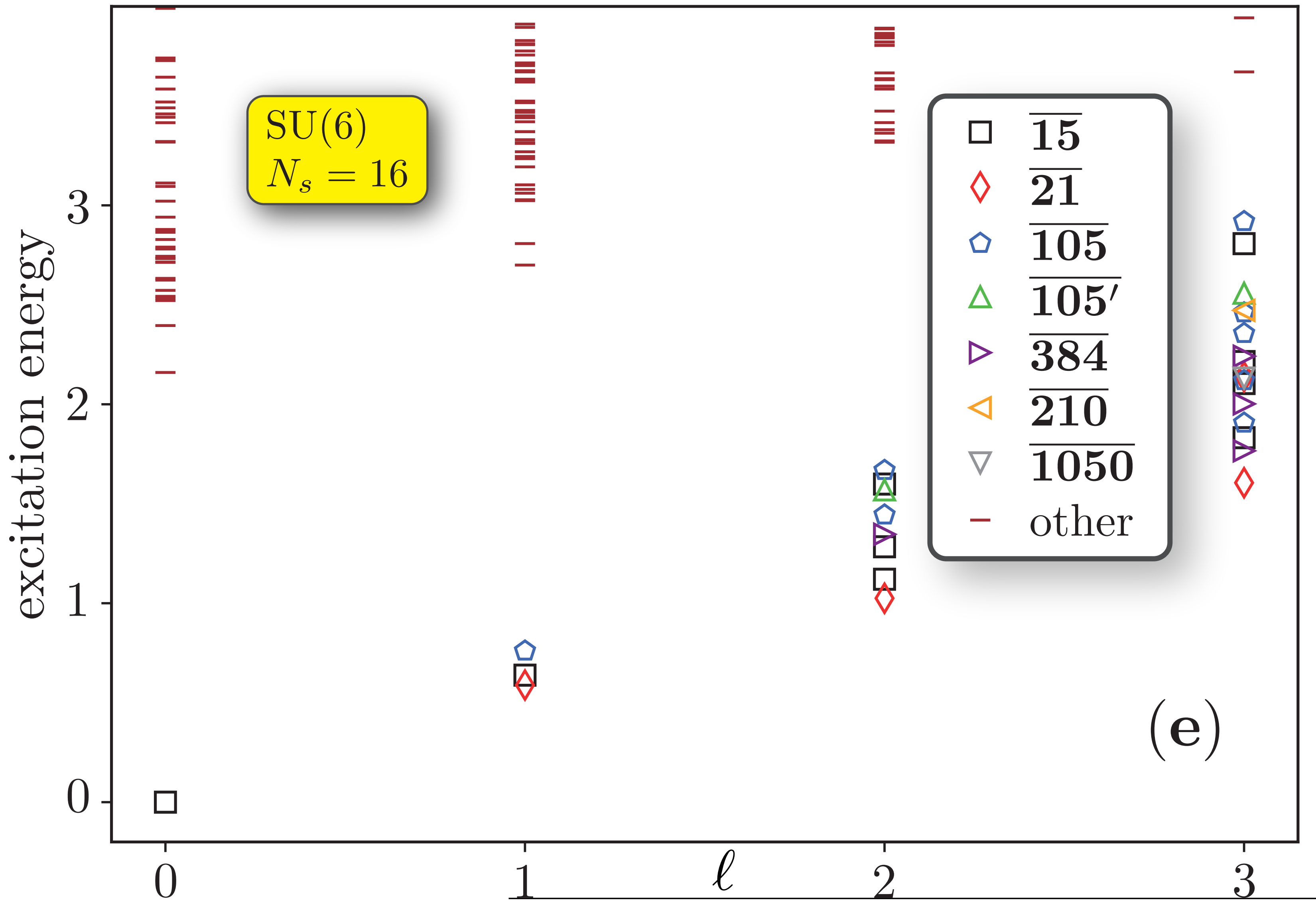
# Exact Diagonalization on **open** cluster




- SU(N)$_1$ chiral CFT counting (number of sites = Ns)




| $l - l_0$ | Order | Irreps / Multiplicities |
|---|---|---|
| 0 | $q^{3/5}$ | 1 [10] |
| 1 | $q^{8/5}$ | 1 [10] ⊕ 1 [15] ⊕ 1 [40] |
| 2 | $q^{13/5}$ | 3 [10] ⊕ 1 [15] ⊕ 2 [40] ⊕ 1 [175] |
| 3 | $q^{18/5}$ | 5 [10] ⊕ 3 [15] ⊕ 1 [35] ⊕ 4 [40] ⊕ 3 [175] ⊕ 1 [210] |

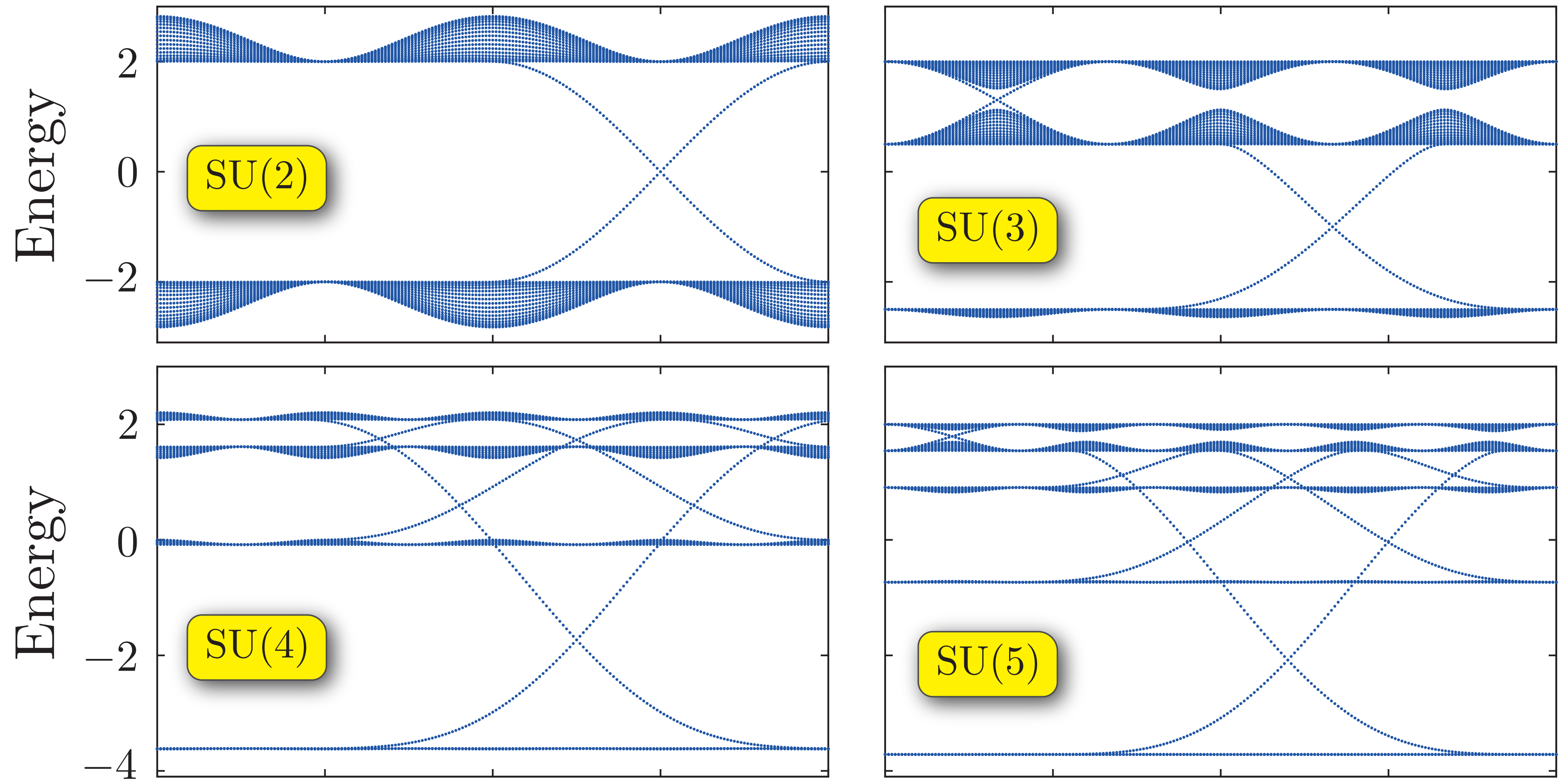
**perfect agreement !**

| $l - l_0$ | Order | Irreps / Multiplicities |
|---|---|---|
| 0 | $q^{2/3}$ | 1 [15] |
| 1 | $q^{5/3}$ | 1 [15] ⊕ 1 [21] ⊕ 1 [105] |
| 2 | $q^{8/3}$ | 3 [15] ⊕ 1 [21] ⊕ 1 [105′] ⊕ 2 [105] ⊕ 1 [384] |
| 3 | $q^{11/3}$ | 5 [15] ⊕ 3 [21] ⊕ 1 [105′] ⊕ 5 [105] ⊕ 1 [210′] ⊕ 3 [384] ⊕ 1 [1050] |

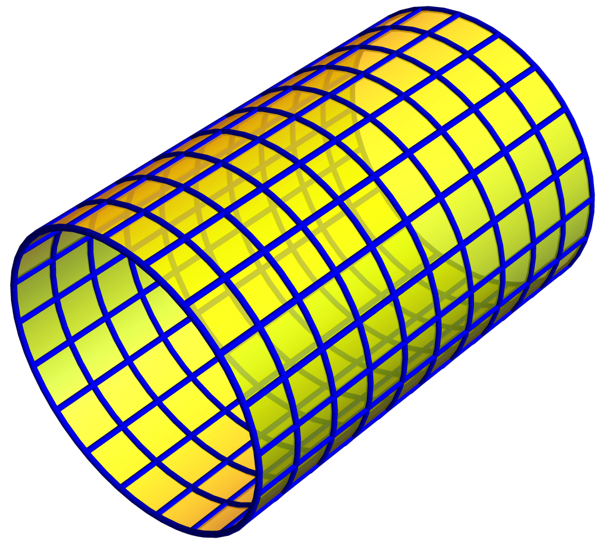
# DMRG

*Wu, Wang, Tu, PRL 124, 246401 (2020)*

- Parton construction is useful to boost DMRG convergence

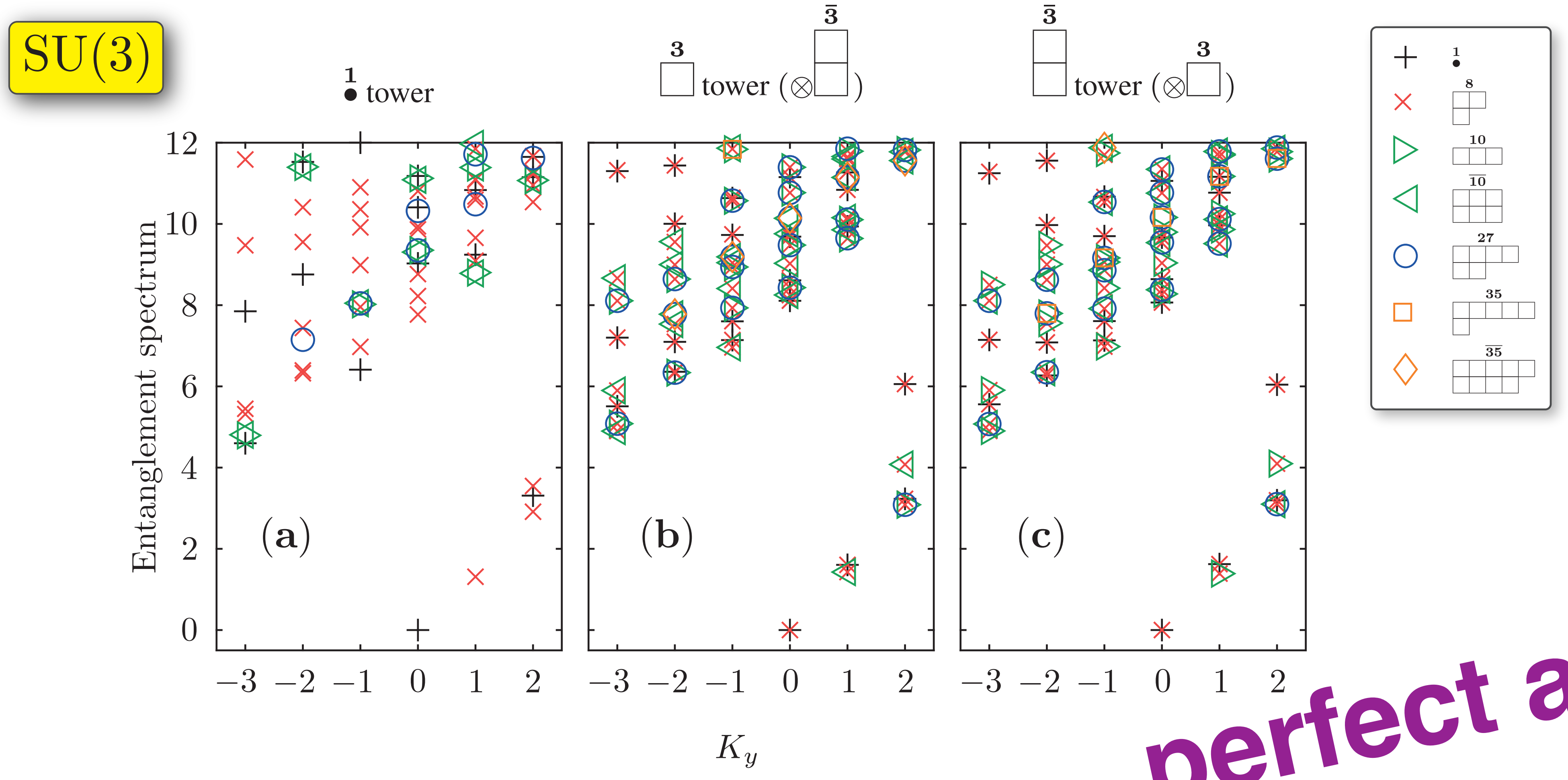


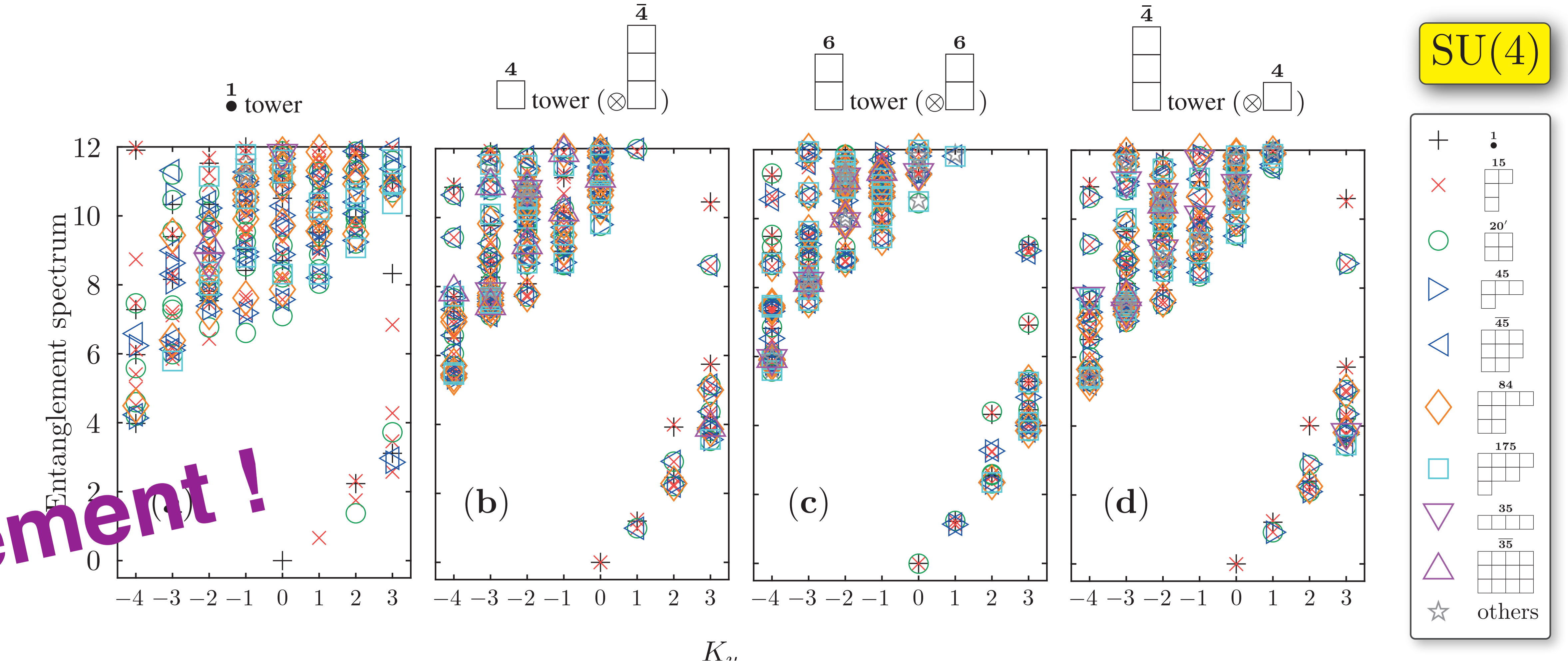

Spectrum on cylinder vs $K_y$
Exact zero-mode edge states



Construct N different minimally entangled states to target different excitations

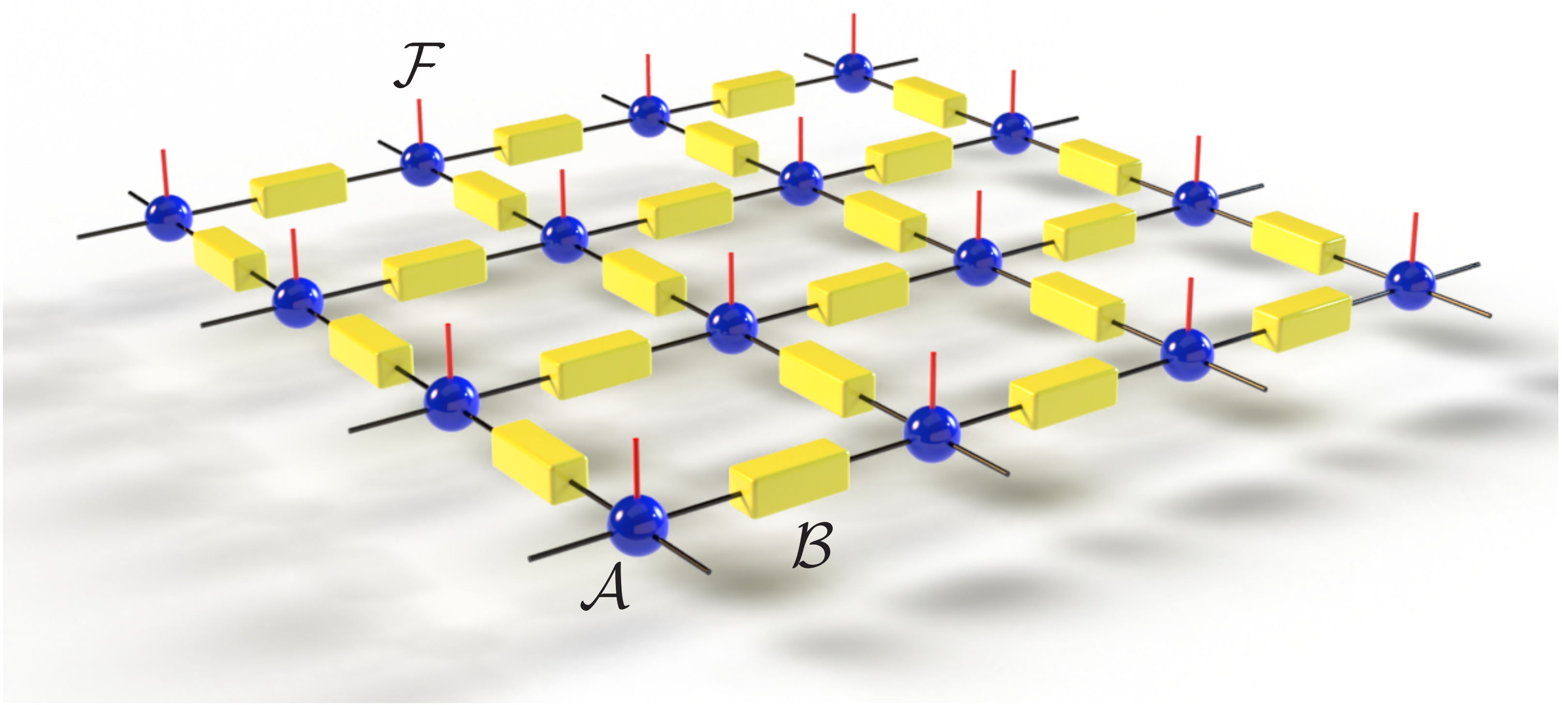
- Probe entanglement spectrum as fingerprint of topological order




perfect agreement !

# PEPS




- Symmetric PEPS construction

virtual space: $\mathcal{V}_N = \bullet \oplus \square \oplus \cdots \oplus \left.\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline\end{array}\right\} N-1$

$\mathcal{A} : (\mathcal{V}_N)^{\otimes z} \to \mathcal{F}$ $\qquad$ $\mathcal{B} : (\mathcal{V}_N)^{\otimes 2} \to \bullet$

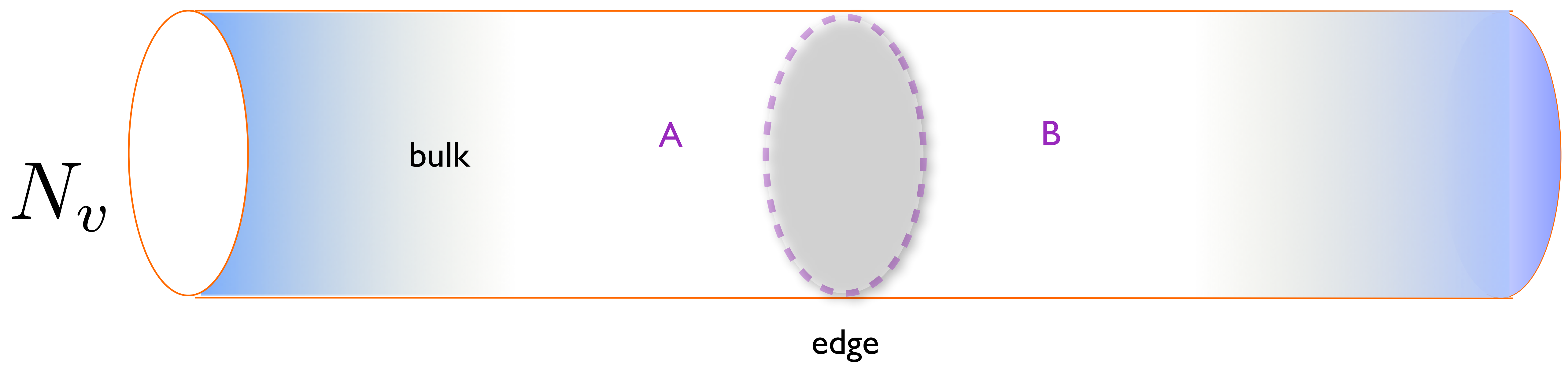
CSL breaks P and T but not PT

$$\mathcal{A} = \mathcal{A}_R + i\mathcal{A}_I = \sum_{a=1}^{N_R} \lambda_a^R \mathcal{A}_R^a + i \sum_{b=1}^{N_I} \lambda_b^I \mathcal{A}_I^b$$

Tensor is a linear combination of point-group SU(N) symmetric ones


- Optimization is performed using CTMRG

# PEPS: entanglement spectrum

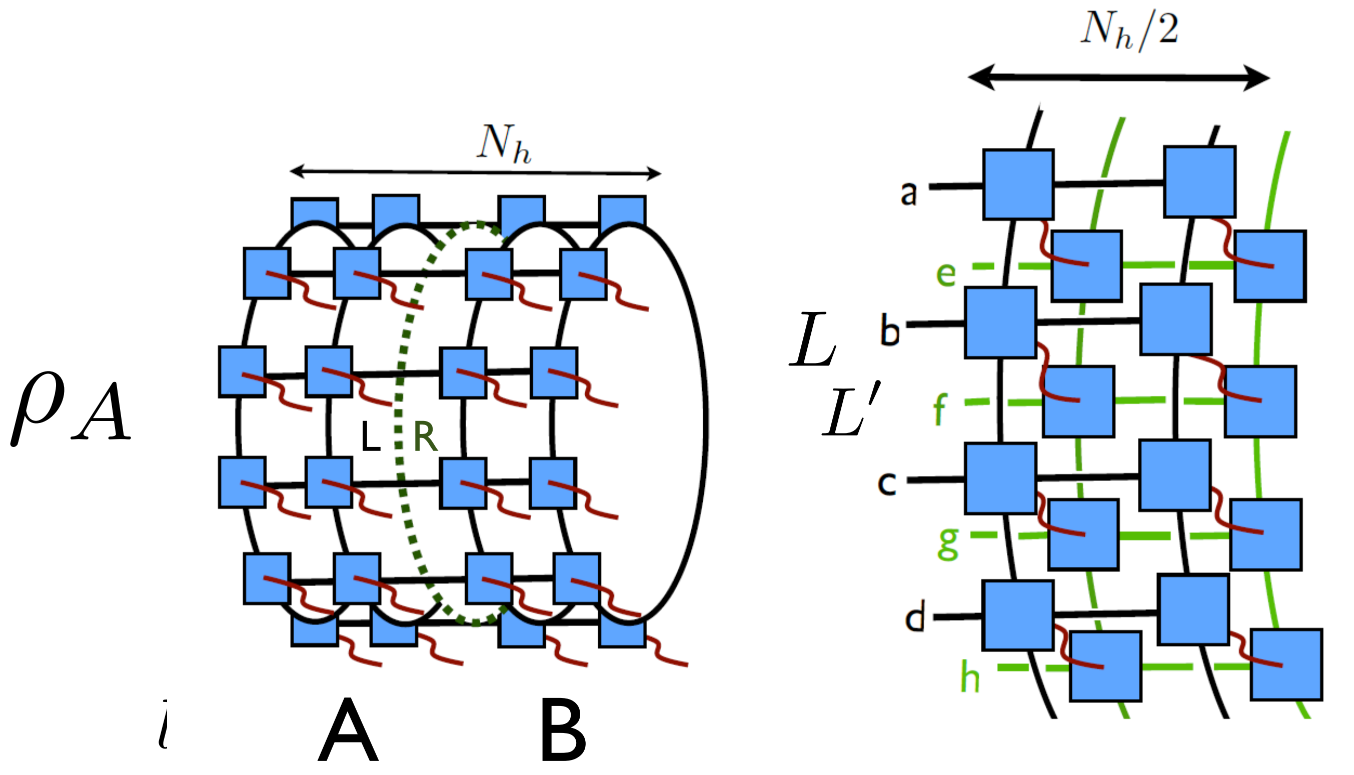



infinite PEPS cylinder

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$ reduced density matrix

Li & Haldane '08

Entanglement spectrum is identical to a CFT boundary spectrum
a.k.a. holographic bulk-edge correspondence




$\sigma_b^2$

lives" on the boundary

Basic formula: $\rho_A = U \sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

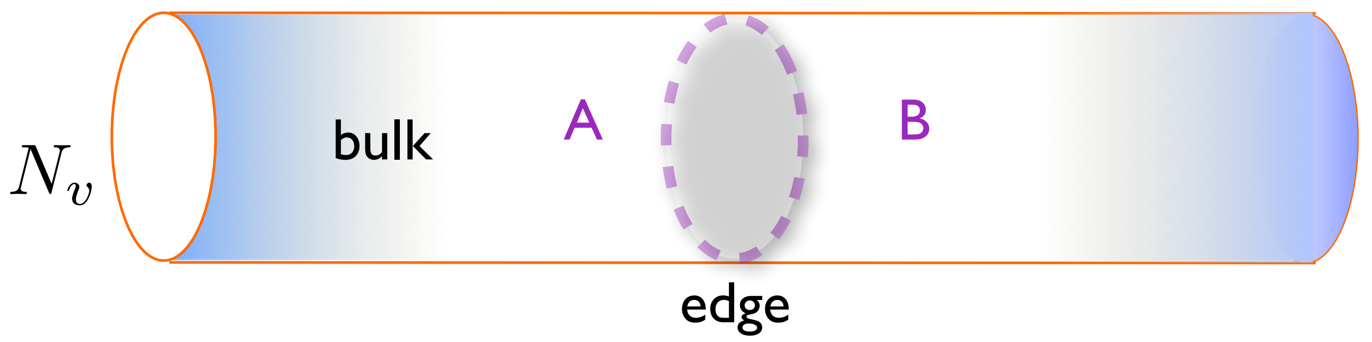
PHYSICAL REVIEW B **83**, 245134 (2011)

**Entanglement spectrum and boundary theories with projected entangled-pair states**

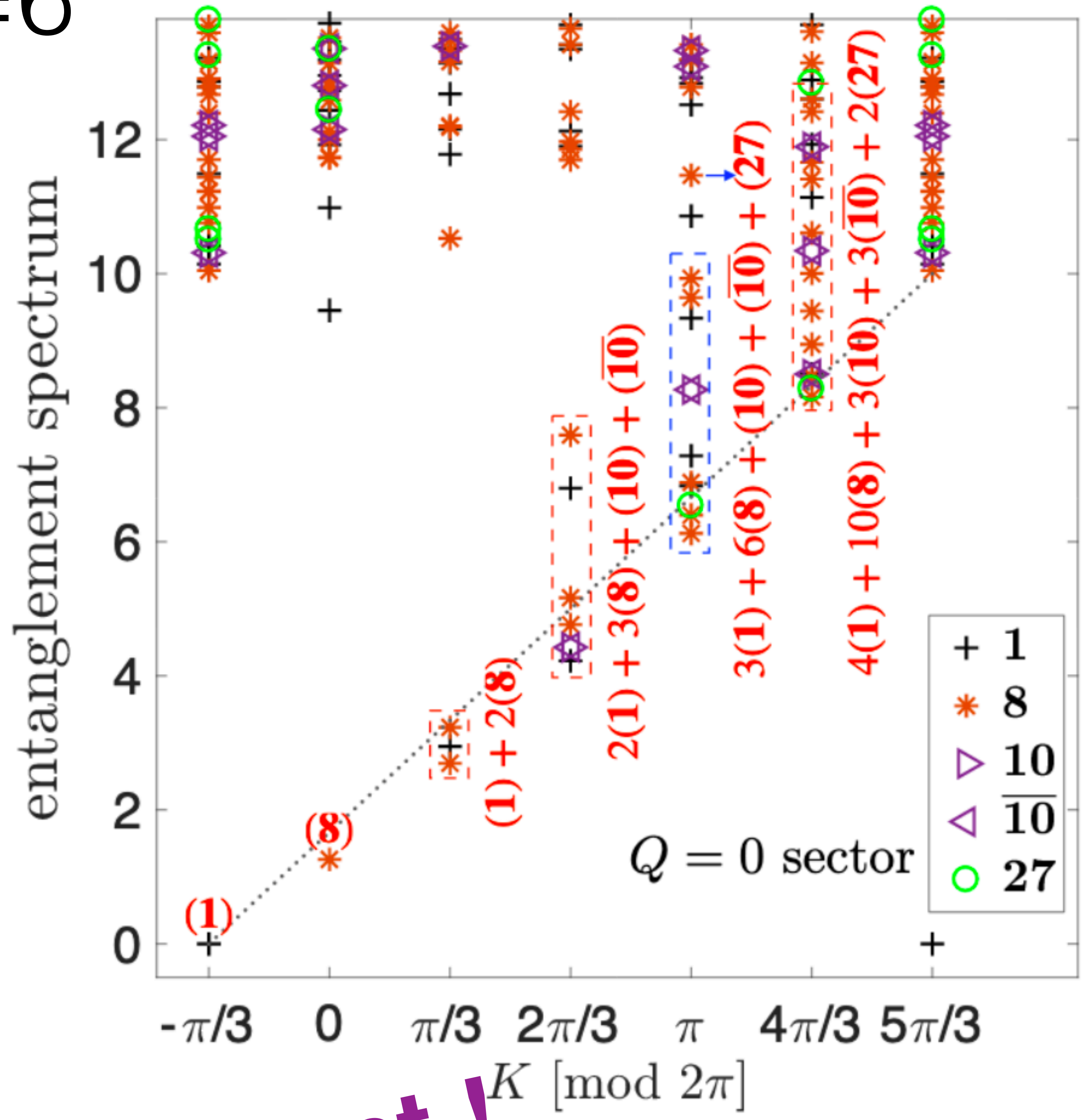
J. Ignacio Cirac,[1] Didier Poilblanc,[2] Norbert Schuch,[3] and Frank Verstraete[4]

# PEPS: entanglement spectrum

infinite PEPS cylinder



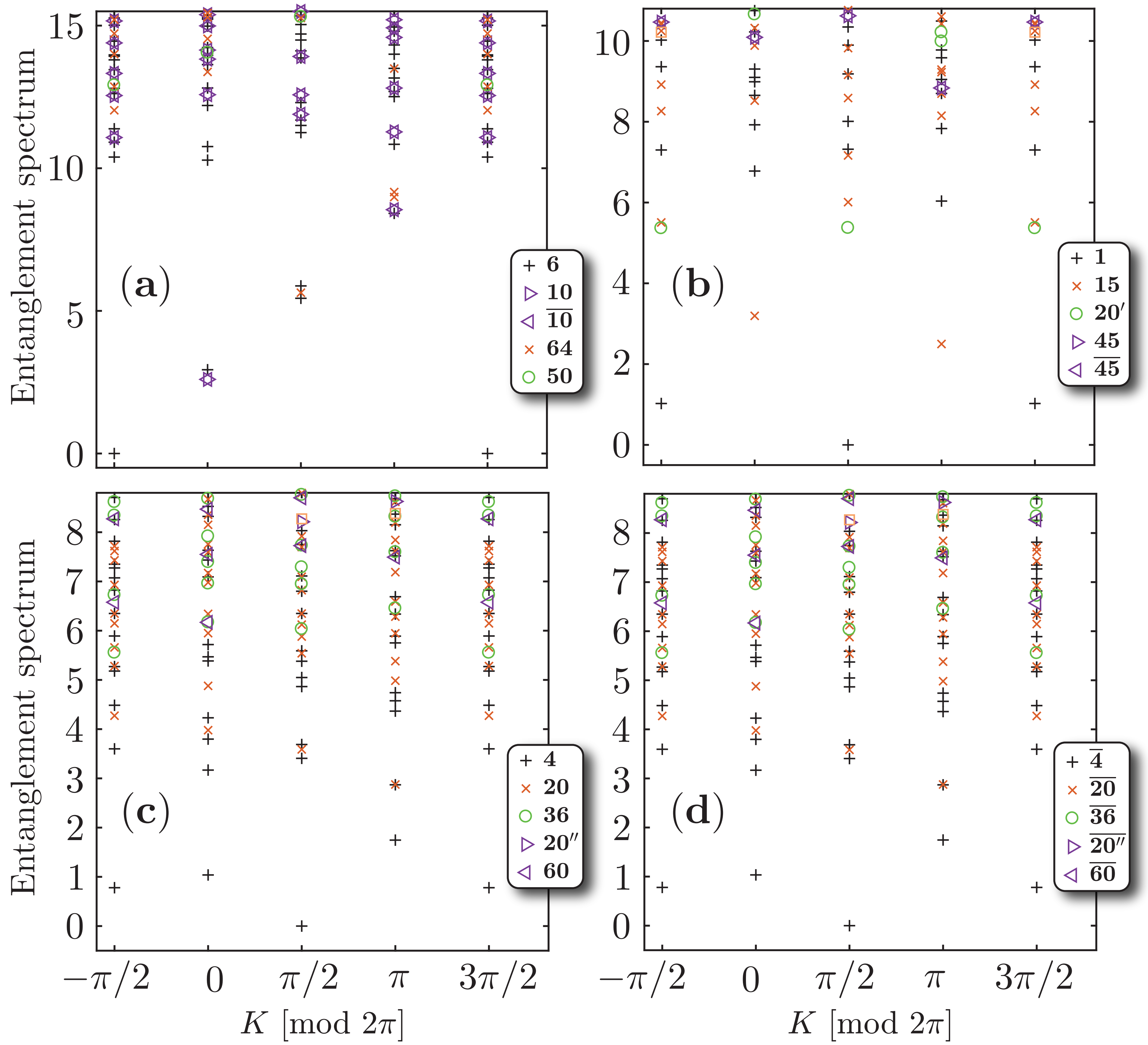

SU(3) $N_v$=6




SU(4), $N_v$=4, full SU(N) symmetry

D=15

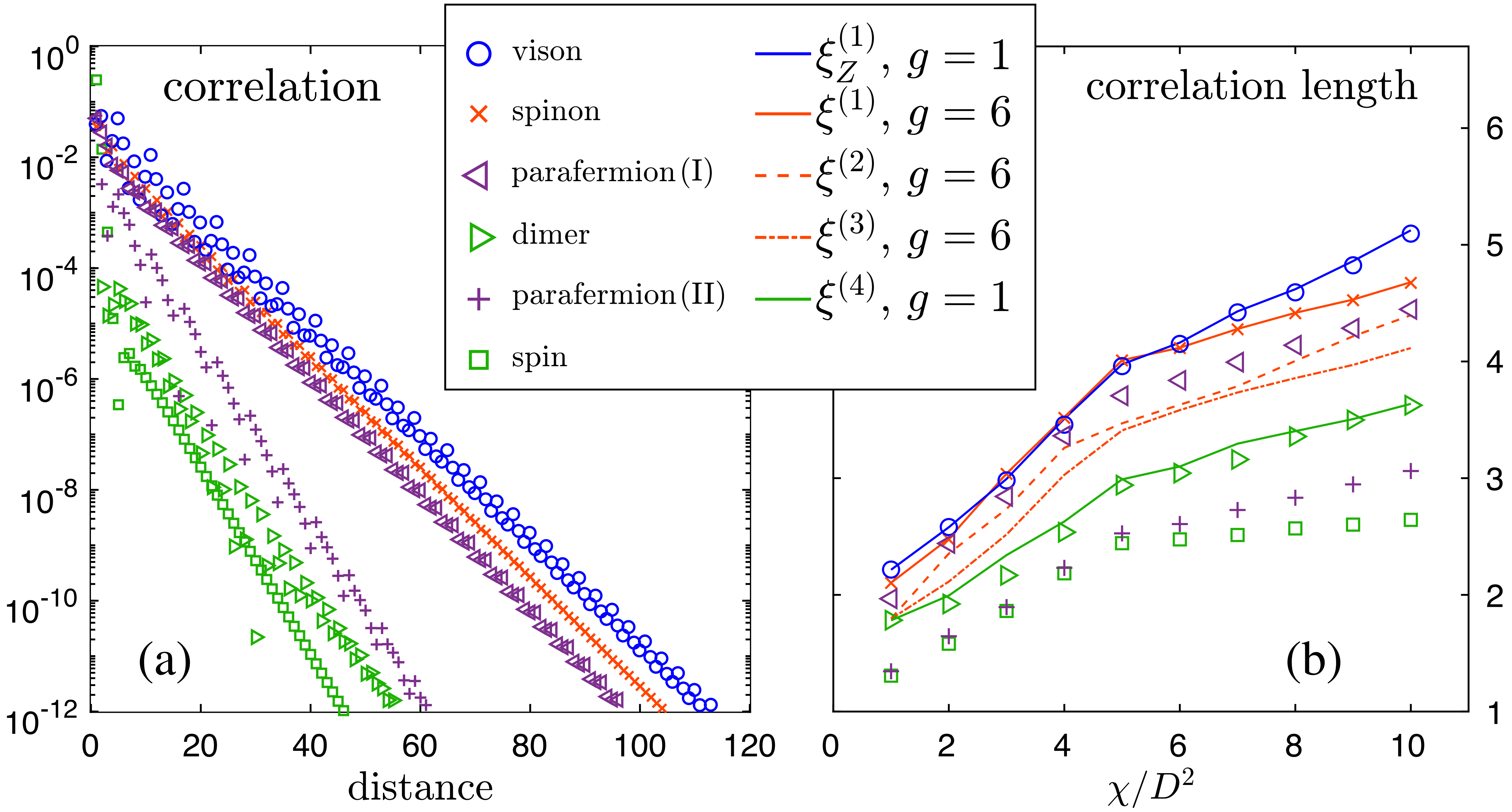
$\chi = 1350$



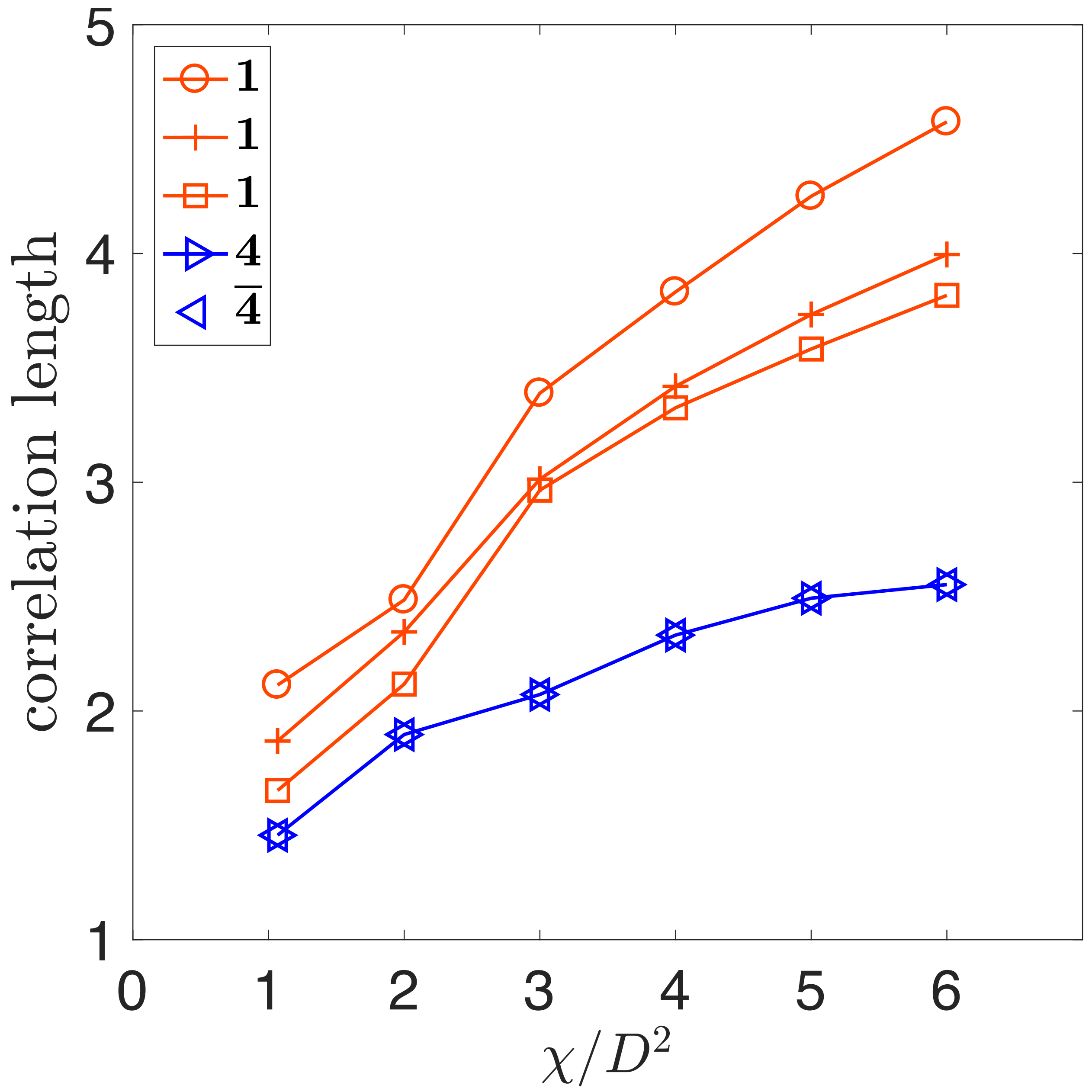

perfect agreement !

# PEPS: correlation lengths

## SU(3)




## SU(4)




Correlations in the bulk

Correlation length directly from transfer matrix

No saturation so presumably **gapless state** !?

No-go theorem for a free-fermion PEPS to have a bulk gap Dubail & Read '15

Also true in the interacting case Li, Lin, McGreevy, Shi '25

# Abelian CSL: spontaneous T-breaking

Topological CSL can also be found in the **absence** of explicit T-breaking

SU(2) Heisenberg

PRL **112,** 137202 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

**Chiral Spin Liquid in a Frustrated Anisotropic Kagome Heisenberg Model**

Yin-Chen He,[1] D. N. Sheng,[2] and Yan Chen[1,3]

PHYSICAL REVIEW B **92**, 125122 (2015)

**Nature of chiral spin liquids on the kagome lattice**

Alexander Wietek,[*] Antoine Sterdyniak, and Andreas M. Läuchli

SU(2) Hubbard

**Quantum Spin Liquid with Emergent Chiral Order in the Triangular-lattice Hubbard Model**

Bin-Bin Chen,[1,2] Ziyu Chen,[1] Shou-Shu Gong,[1,*] D. N. Sheng,[3] Wei Li,[1,4,†] and Andreas Weichselbaum[5,2,‡]

PHYSICAL REVIEW X **10,** 021042 (2020)

**Chiral Spin Liquid Phase of the Triangular Lattice Hubbard Model: A Density Matrix Renormalization Group Study**

Aaron Szasz,[1,2,3,*] Johannes Motruk,[1,2] Michael P. Zaletel,[1,2,4] and Joel E. Moore[1,2]

SU(4)

**An $SU(4)$ chiral spin liquid and quantized dipole Hall effect in moiré bilayers**

Ya-Hui Zhang[1], D. N. Sheng[2], and Ashvin Vishwanath[1]

# Non-abelian case: SU(2)

non abelian FQHS

Moore-Read
Read-Rezayi

incompressible (gapped) in the bulk

**non abelian** fractional excitation

gapless chiral edge states

$SU(2)_k$ CFT

↔

Spin analogue ?

Parent Hamiltonian approach

Coupled wire construction

***These topological phases can host $SU(2)_k$ non-abelian anyons !***

# Moore-Read state corresponds to **spin-1** lattice model

**New Journal of Physics**
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Deutsche Physikalische Gesellschaft DPG
IOP Institute of Physics

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FAST TRACK COMMUNICATION

Exact parent Hamiltonians of bosonic and fermionic Moore–Read states on lattices and local models

Ivan Glasser[1], J Ignacio Cirac[1], Germán Sierra[2,3] and Anne E B Nielsen[1]

Proposed parent Hamiltonian is rather complicated, long-range

$$
\begin{aligned}
H = {} & J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l \\
& + K_1 \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + K_2 \sum_{\langle\langle k,l \rangle\rangle} (\mathbf{S}_k \cdot \mathbf{S}_l)^2 \\
& + K_c \sum_{\square} [\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) + \mathbf{S}_j \cdot (\mathbf{S}_k \times \mathbf{S}_m) \\
& + \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_m) + \mathbf{S}_i \cdot (\mathbf{S}_k \times \mathbf{S}_m)],
\end{aligned}
$$

→ truncated approximate spin-1 model on the square lattice

→ ***numerics needed***

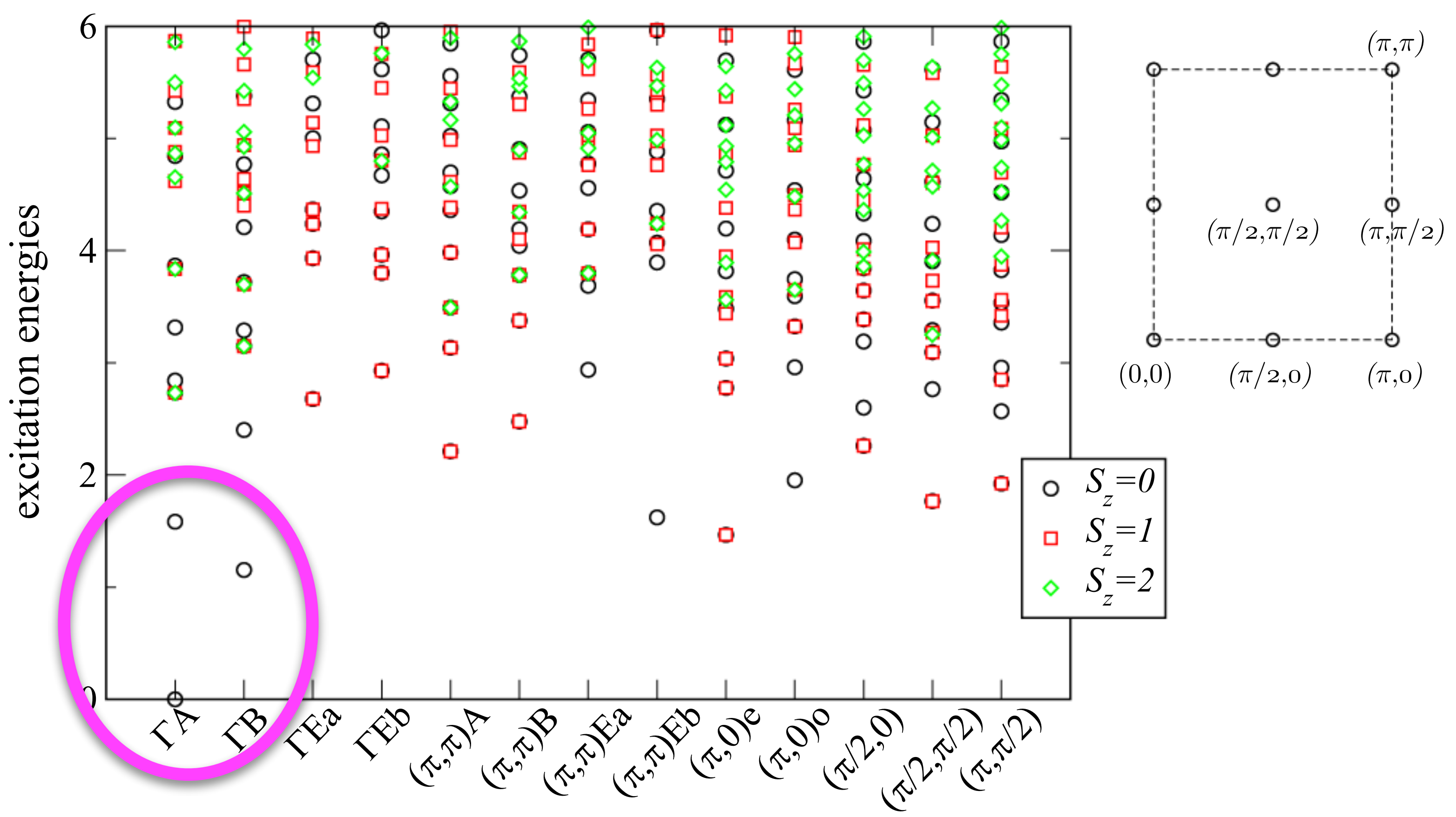
# Combined ED/DMRG/PEPS study

PHYSICAL REVIEW B **98**, 184409 (2018)

**Non-Abelian chiral spin liquid in a quantum antiferromagnet revealed by an iPEPS study**

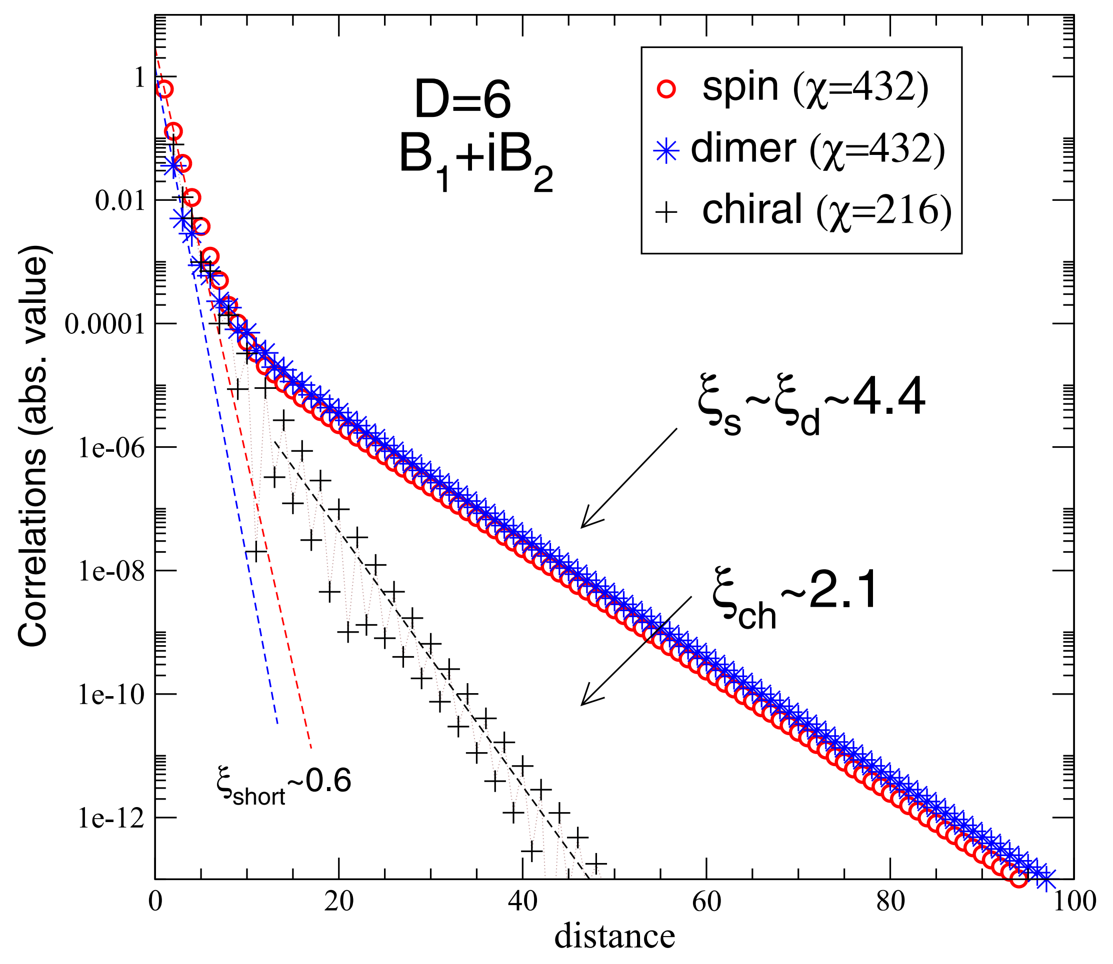
Ji-Yao Chen,[1] Laurens Vanderstraeten,[2] Sylvain Capponi,[1] and Didier Poilblanc[1,3]

## ED on a torus



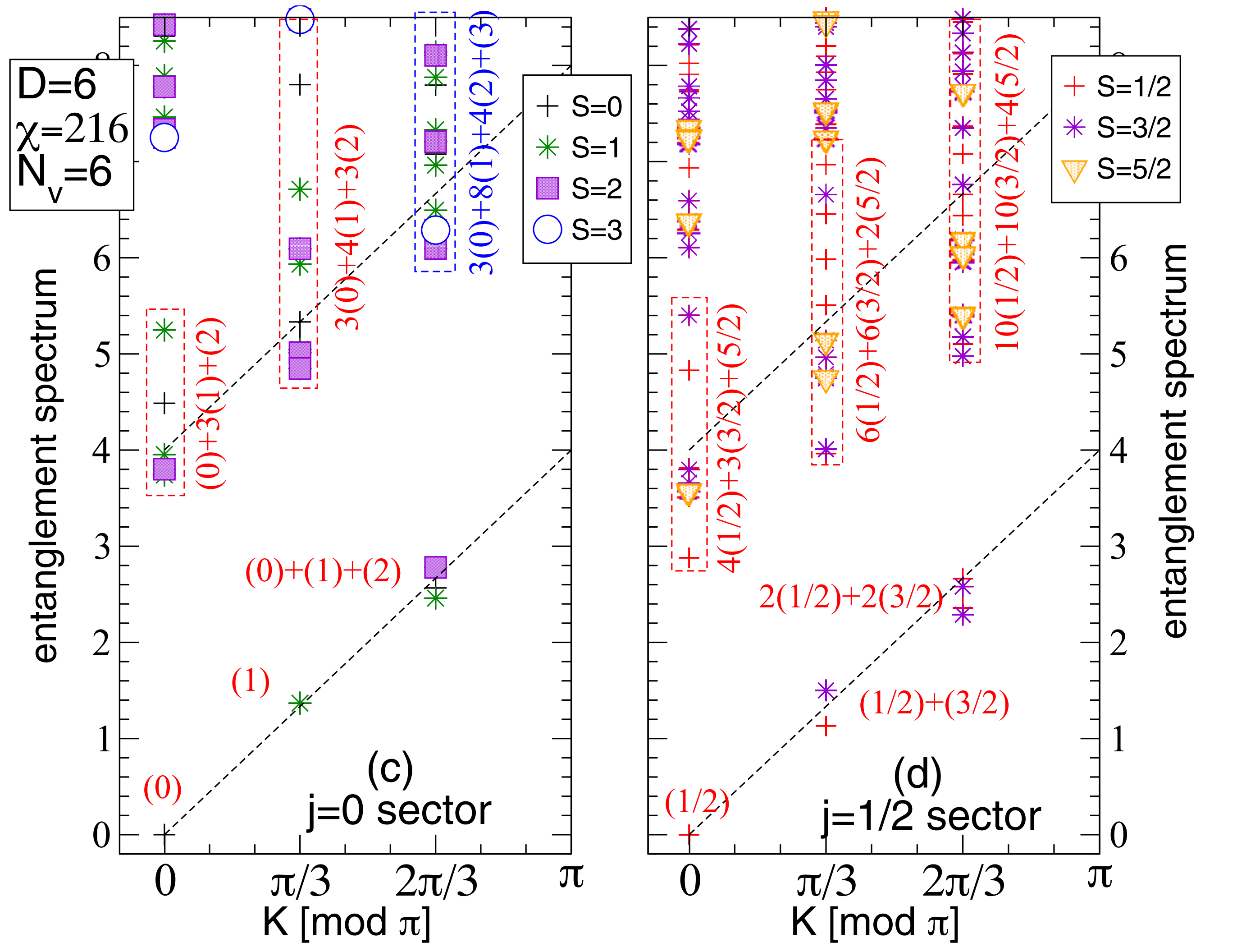

3 states

## PEPS bulk correlation



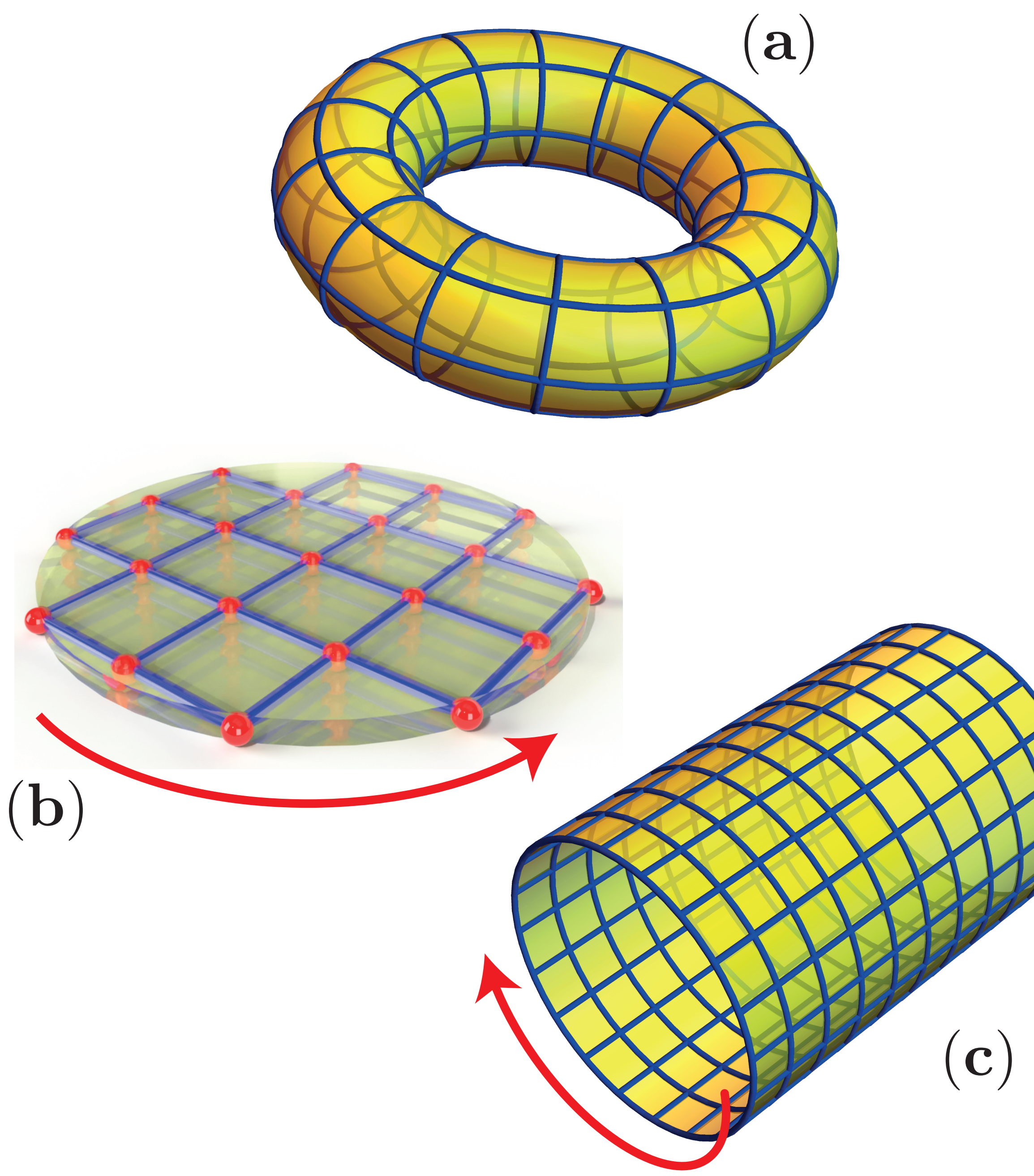

gossamer critical tail

## PEPS edge entanglement



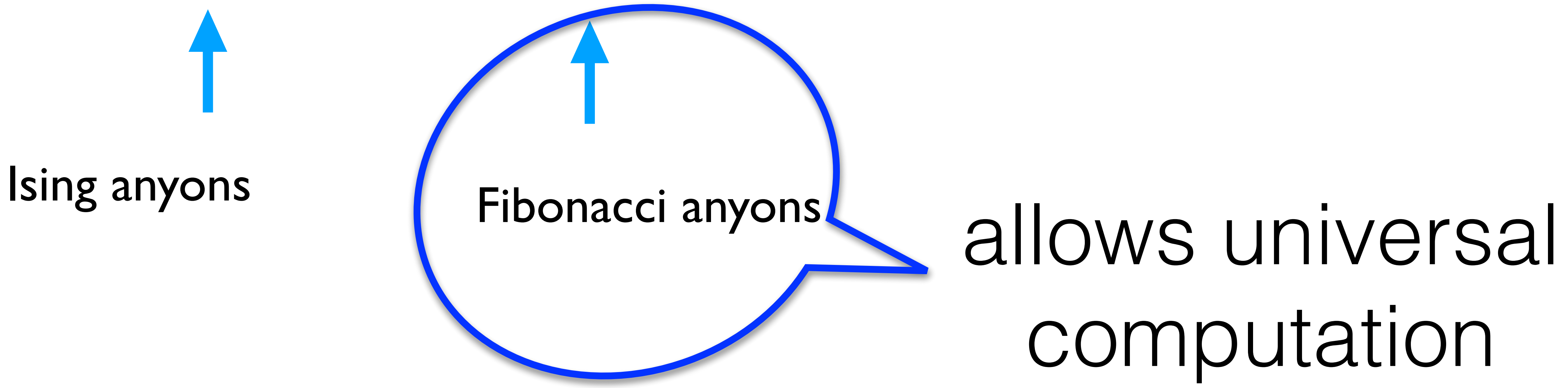

agrees with **SU(2)$_2$** WZW

c=3/2




# Conclusion and outlook

- Simple SU(N) spin models hosting **topological chiral spin liquids**
- Important to combine different numerical techniques to validate all properties
- Characterization of edge states and entanglement properties
- Also non-Abelian CSL with $SU(2)_2$, $SU(2)_3$, $SU(3)_2$, etc… edges physics




*Luo, Huang, Sheng, Zhu '23*