Vacancy-induced local moment instability of RVB spin liquids

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@
Advanced School and Conference on Quantum Matter (ICTP Dec 2025)

unpublished:

Bhola, KD, arXiv:2311.05634v2 (2025); Ansari, Kundu, KD (in preparation)

recent:

Ansari, KD, PRL 132 226504 (2024)

Bhola, Biswas, Islam, KD, PRX 12, 021058 (2022)

KD, PRB 105 235118 (2022)

background:

Sanyal, KD, Chalker, Moessner PRL 127 127201 (2021)

Sanyal, KD, Motrunich, PRL 117 116806 (2016)






Antiferromagnetism and Frustration

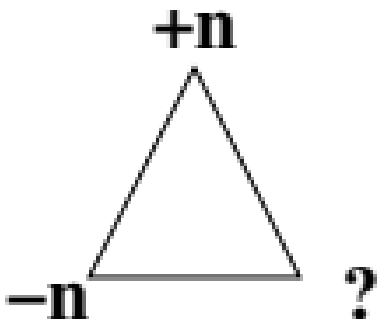
Bipartite lattices: A-sublattice spins point “up”, B-sublattice spins point “down”

up and down about what axis? Spontaneous symmetry breaking

Geometric frustration

Triangles in the nearest neigbhor connectivity

Collinear antiferromagnet frustrated

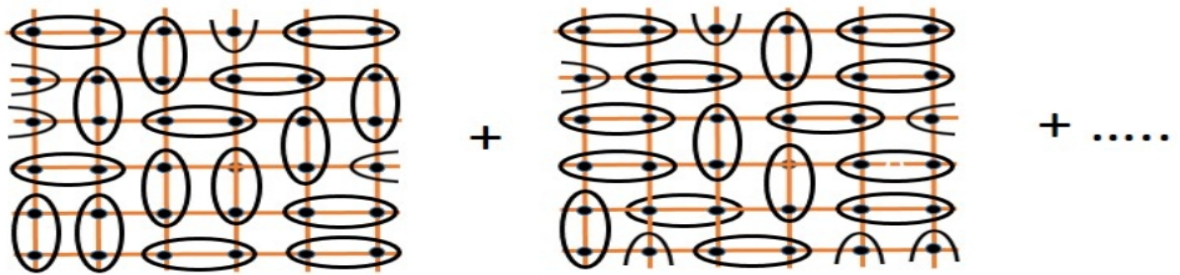


Higher orders in strong coupling expansion:

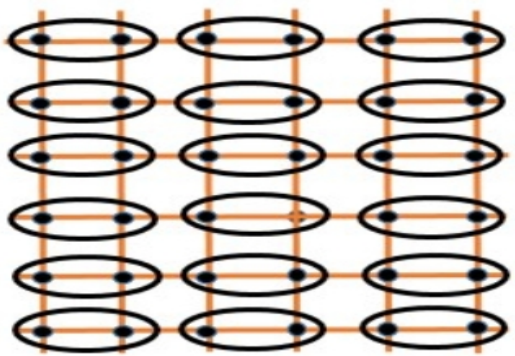
Four-spin couplings: Ring exchange around plaquettes

Simple antiferromagnetic state again frustrated.

Frustration induced quantum disordered states



Short-range resonating valence bond (sRVB) spin liquid



Valence bond solid (VBS)

(with spontaneous lattice symmetry breaking)

Our focus: Effect of quenched disorder aka dirt

Substitutional impurities, interstitial adatoms, structural defects...

Quenched (on electronic timescales).

Not to be confused with doping with mobile holes...

Our focus: Non-magnetic substitutional impurities--- e.g Zn for Cu, Ga for Cr

e.g. in Herbertsmithite, SCGO...

Variety of effects

Weak disorder: Can be irrelevant for low energy properties (not always).

Strong disorder: new phases of matter (e.g. spin glasses)

Can probe correlations of underlying state (e.g. spin textures in frustrated magnets)

Quantum dimer model framework for RVB/VBS states

Rokhsar and Kivelson: Effective Hamiltonian living in subspace of singlets spanned by nn VB

$$H_{QDM} = -t(\,|=\rangle\langle\,\shortparallel\,| + |\,\shortparallel\,\rangle\langle=|\,) + \ldots$$

More generally: Ring-exchange kinetic terms on “flippable” plaquettes, and local interactions

Additional terms incorporate the effect of matrix elements to further-neighbor singlet states

Z2 spin liquid example: Triangular QDM

Triangular lattice: Moessner-Sondhi (within QDM framework):

Triangular lattice QDM has truly quantum disordered phase

Short-range spin correlations, valence bond correlations, genuine Z2 spin liquid

(also for kagome lattice)

Language primer: Fully-packed dimers (perfect matchings)

Fully-packed hard-core dimer models in stat-mech: Match **each** site to an adjacent site monogamously

In graph theory/computer science: The perfect matching problem

Easy to see (for regular lattices like square, triangular, honeycomb, kagome...):

Extensive entropy of fully-packed dimer covers (perfect matchings)

(exact computation of entropy on planar graphs: Classic papers by Kasteleyn & Fisher)

(also exact results on special non-planar graphs: Chandra & Dhar)

QDM framework: Maximum matchings of disordered lattices

Basic question arises: Can a diluted lattice with even number of vertices be perfectly matched?

If bipartite, need |A| = |B|

But: generally not possible (even with |A|=|B|)

Then have *maximum matching* but not *perfect matching*

Maximum matchings have unmatched sites that host *monomers*

Generally, nonzero vacancy density implies nonzero density of monomers (multi-vacancy effect)

Bhola, Biswas, Islam, KD, PRX 12, 021058 (2022)

Sanyal, KD, Motrunich, PRL 117 116806 (2016)

Non-bipartite case more subtle

Same question: Can a lattice with even number of vertices be perfectly matched?

Two classes of disordered lattices

Generic disordered case (e.g. site-diluted triangular lattice):

nonzero vacancy density implies nonzero density w of monomers (multi-vacancy effect)

Non-generic ‘’claw-free’’ case (e.g. site-diluted kagome lattice):

Vanishing bulk density of monomers for arbitrary vacancy concentraions/correlations

Ansari, KD, PRL 132 226504 (2024)
Bhola, KD, in preparation

Monomers correspond to “emergent” local moments in spin system

Each monomer corresponds to a disorder-induced “emergent” local moment
(purely kinematic effect, independent of VBS vs RVB nature of ground state)

Signature: Large intermediate temperature range with Curie tail in susceptibility

Quenched below scale set by residual interactions

$$\chi_{\rm imp} \sim \frac{\mathcal{C}}{T} \text{ for } J_{\rm eff} \ll T \ll J$$
$$\mathcal{C} \propto n_{\rm monomer}$$

But wait: This conclusion seems to rely too much on having only nearest-neighbor singlets?
Does it hold for more generic short-range RVB liquid?

To answer: large-N route to quantum dimer model

$$
\begin{aligned}
H &= J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \dots \\
&= -J \sum_{\langle rr' \rangle} \left(\mathcal{P}_{rr'} - \frac{1}{4} \right) + \dots
\end{aligned}
$$

Enlarge symmetry group:

$$
H = -\frac{J_m}{N} \sum_{\langle r_1 r_2 \rangle} \sum_{\alpha,\beta=1}^{N} |\alpha\rangle_{r_1} |\alpha\rangle_{r_2} \langle\beta|_{r_1} \langle\beta|_{r_2} + \cdots ,
$$

Affleck, Read, Sachdev, Auerbach, Penc, Mila, Coleman, Sandvik, Alet, Kawashima, Beach, Kaul...(1988 - now)

What's the enlarged symmetry?

$$
\begin{aligned}
\mathcal{A}_{\alpha\beta}(r) &= -i(|\alpha\rangle_r\langle\beta|_r - |\beta\rangle_r\langle\alpha|_r) \;\; \forall \;\; \text{pairs} \;\; \alpha < \beta \\
\mathcal{S}_{\alpha\beta}(r) &= (|\alpha\rangle_r\langle\beta|_r + |\beta\rangle_r\langle\alpha|_r) \;\; \forall \;\; \text{pairs} \;\; \alpha < \beta \\
\mathcal{Q}_{\alpha\alpha}(r) &= (|\alpha\rangle_r\langle\alpha|_r - 1/N) \;\; \forall \;\; \alpha = 1 \ldots N-1
\end{aligned}
$$

$$\mathcal{A}^{\text{tot}}_{\alpha\beta} = \sum_r \mathcal{A}_{\alpha\beta}(r)$$

SO(N) symmetry on any arbitrary lattice

$$\mathcal{S}^{\text{tot}}_{\alpha\beta} = \sum_r (-1)^r \mathcal{S}_{\alpha\beta}(r)$$

$$\mathcal{Q}^{\text{tot}}_{\alpha\beta} = \sum_r (-1)^r \mathcal{Q}_{\alpha\beta}(r)$$

Bipartite case: Enhanced "staggered" SU(N) symmetry

Large N limit in pure case

Any perfect (fully packed) dimer cover is a ground state (each dimer interpreted as singlet state)

Leading 1/N corrections: Captured precisely by QDM Hamiltonian with ring-exchange

Higher orders in 1/N: Additional local terms in QDM Hamiltonian

(Affleck, Read, Sachdev, Kaul...)

Recover the same QDM framework---without nearest-neighbor singlet assumption.

Disordered case: Large N limit

Any maximum matching now gives a large-N ground state.

Monomers correspond to free moments (additional degeneracy)

Leading 1/N corrections: QDM Hamiltonian with ring-exchange + monomer kinetic energy terms

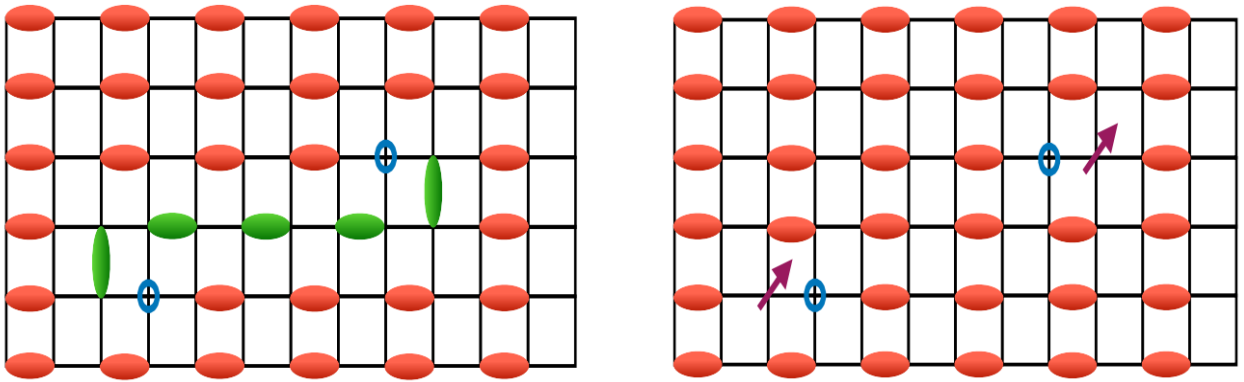
Higher orders in 1/N: Additional local terms in QDM Hamiltonian

Correspond to residual interactions between local moments...(?)

These control fate of system at lowest energies

So: Large N also gives maximally-packed QDM description of disorder effects in short-range RVB liquid

Contrast with VBS state



Each vacancy, even if isolated from other vacancies, seeds a local moment in a VBS state

(even when perfect matchings are possible, i.e even when there are no monomers)

In contrast, for sRVB case: Monomers of maximum matchings are sole mechanism

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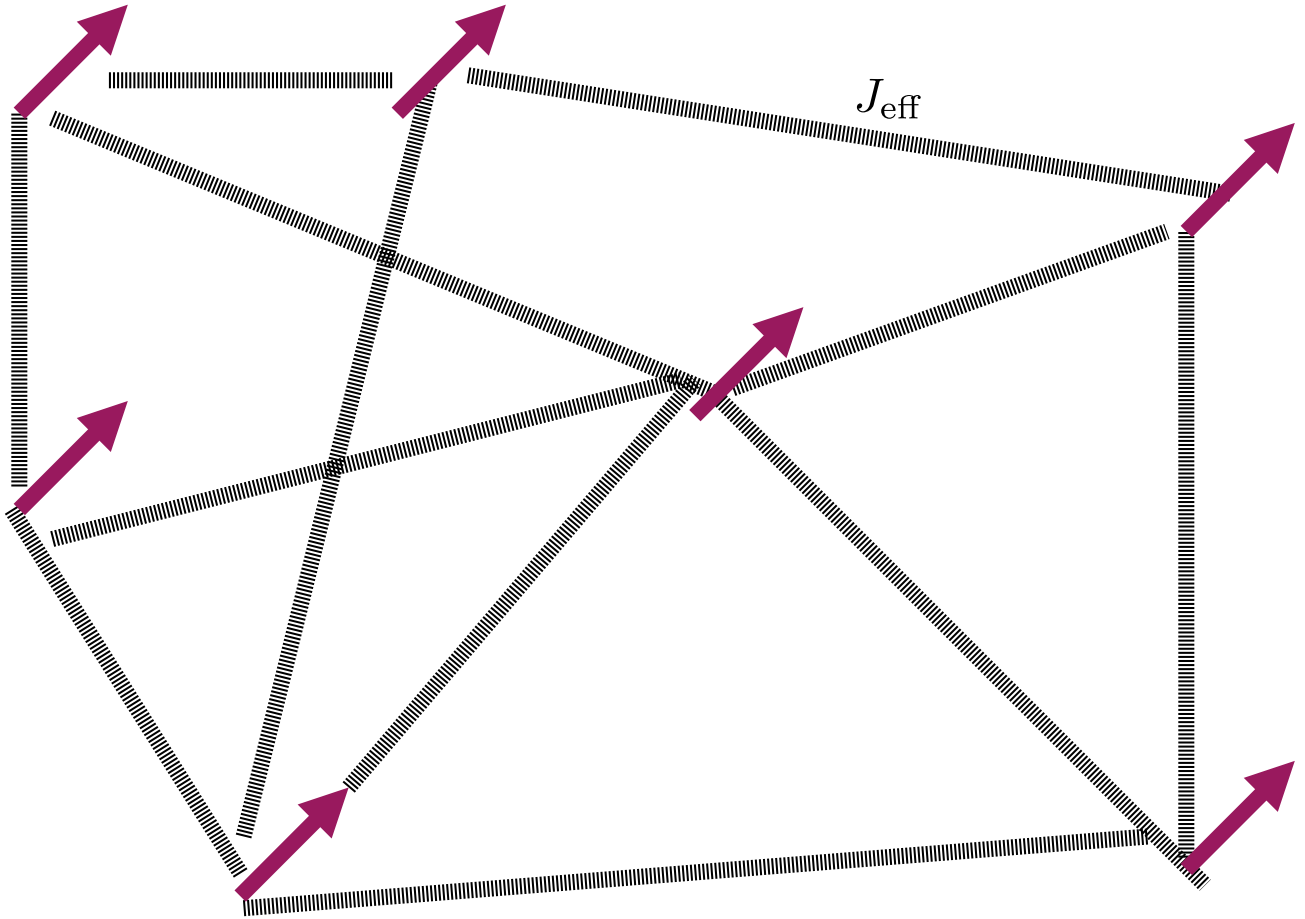
Summary: Distinct vacancy-induced local moment instabilities of RVB and VBS states



In RVB case, only if

$$w \neq 0$$

In VBS case, even when

$$w = 0 \;\; \text{but} \;\; n_v \neq 0$$

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Striking implication: Stability of the kagome RVB liquid

w=0 in the thermodynamic limit of the diluted kagome lattice with nonzero vacancy density

Short-range RVB state stable to vacancy disorder on kagome lattice (!)

Generally true on all claw-free lattices (pyrochlore lattice, star lattice etc)

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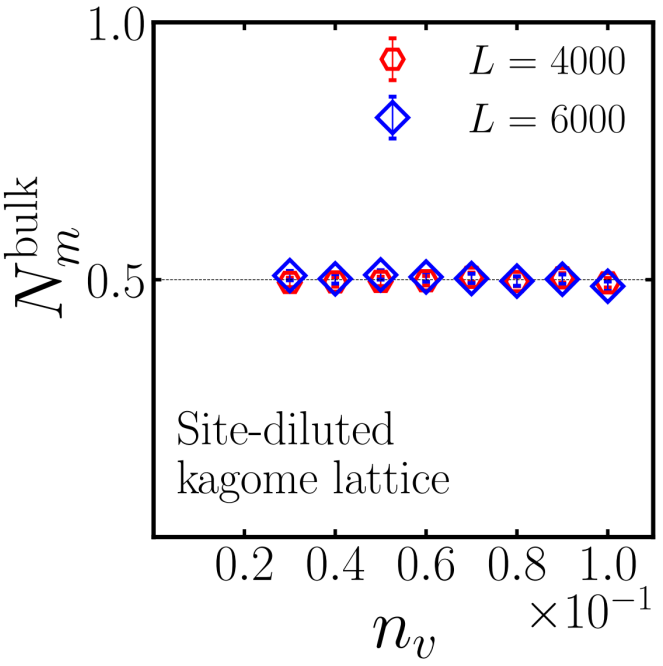
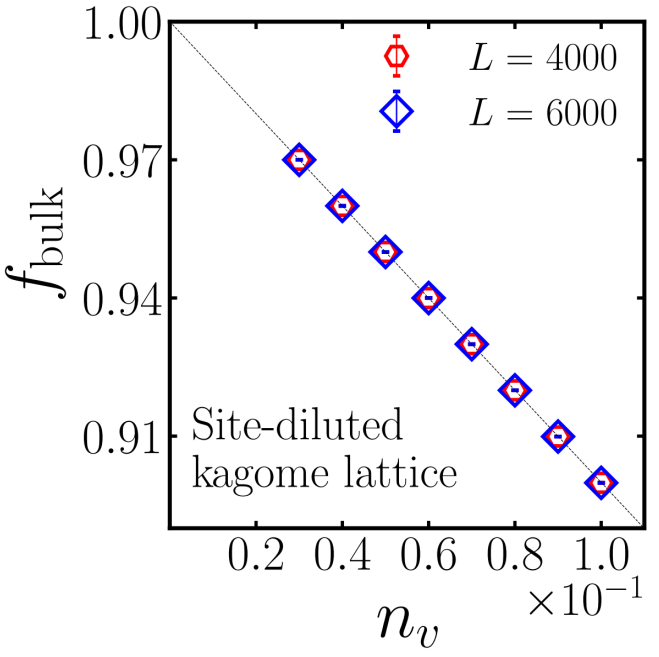
Explicit check for site-diluted kagome:





Any maximum matching has at most 1 monomer in each connected component of lattice(!)

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Story so far:

VBS states always have vacancy-induced local moment instability (single-vacancy effect)

sRVB states have such an instability if maximum matchings have nonzero bulk monomer density.
(multi-vacancy effect)

Key implication: Kagome sRVB liquid is stable

When there's an instability:
Nature of the actual many-body ground state controlled by random geometry of monomer-carrying regions

Motivates study of this random geometry

But first: key claims need computational test

Isolated vacancies do not seed local moments in sRVB states, but do so in VBS states.

Monomer-carrying regions of lattice correspond to local moments in both kinds of states

Primer: Computational tests

O(N) models on non-bipartite lattices, SU(N) models on bipartite lattices

Ideal unified test: $\chi^{\mathcal{A}}$ (runs into computational difficulties)

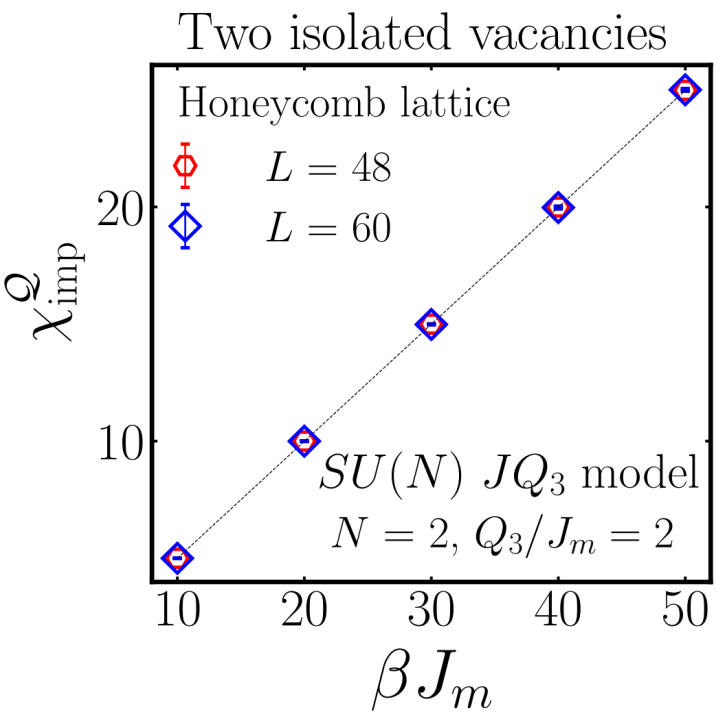
For SU(N) systems, equivalent to checking: $\chi^{\mathcal{Q}}$

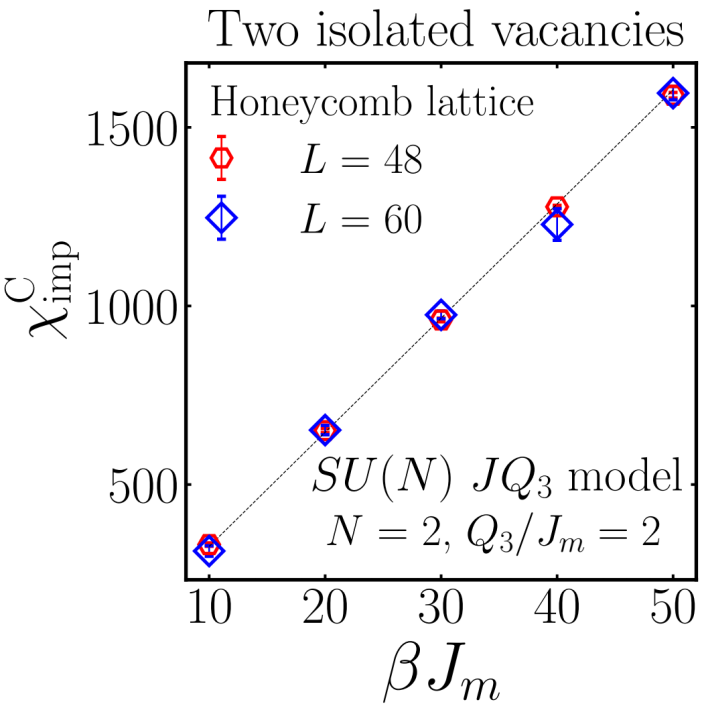
This is not defined for nonbipartite O(N) models

For O(N) systems, can instead check: χ^{C} $\qquad C^{\rm tot}_{\alpha\alpha} = \sum_r \mathcal{Q}_{\alpha\alpha}(r)$

expected to be equivalent for $J_m \gg T \gg J_{\rm eff}$

Isolated vacancies: VBS state (bipartite)





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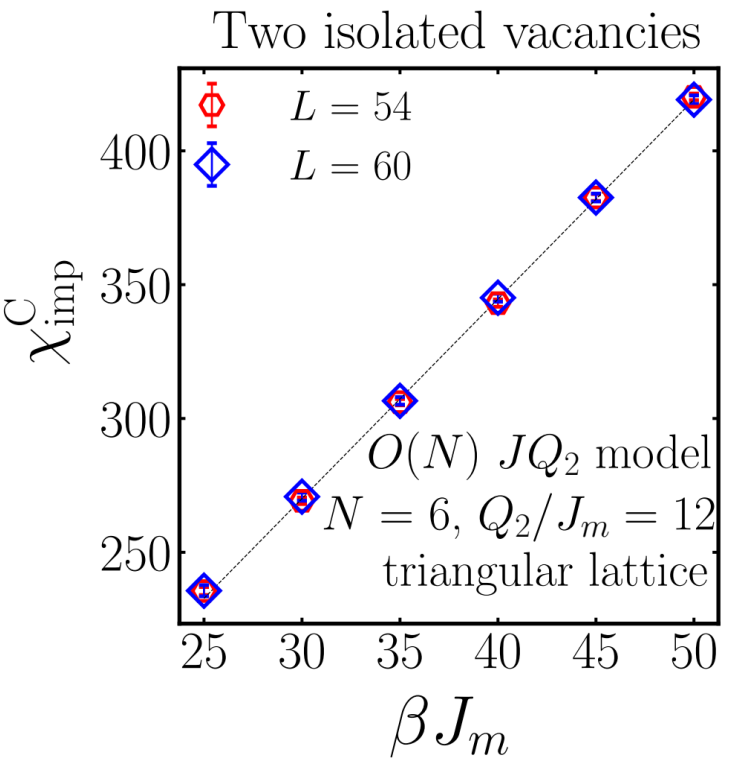
Isolated vacancies: VBS state (nonbipartite)



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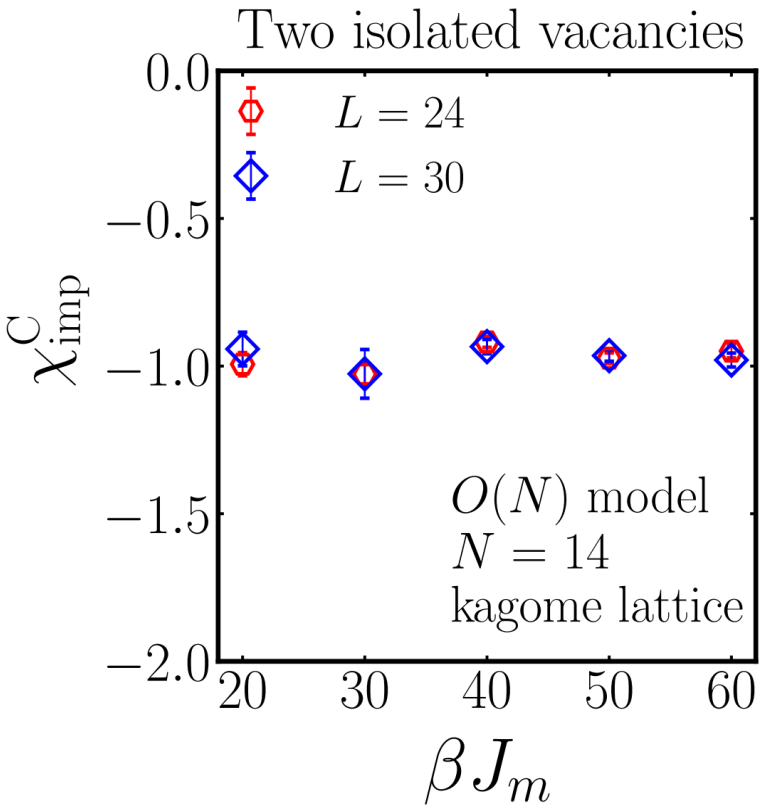
Isolated vacancies: kagome RVB state (non-bipartite)

RVB state established in Block,D'Emidio, Kaul 2020



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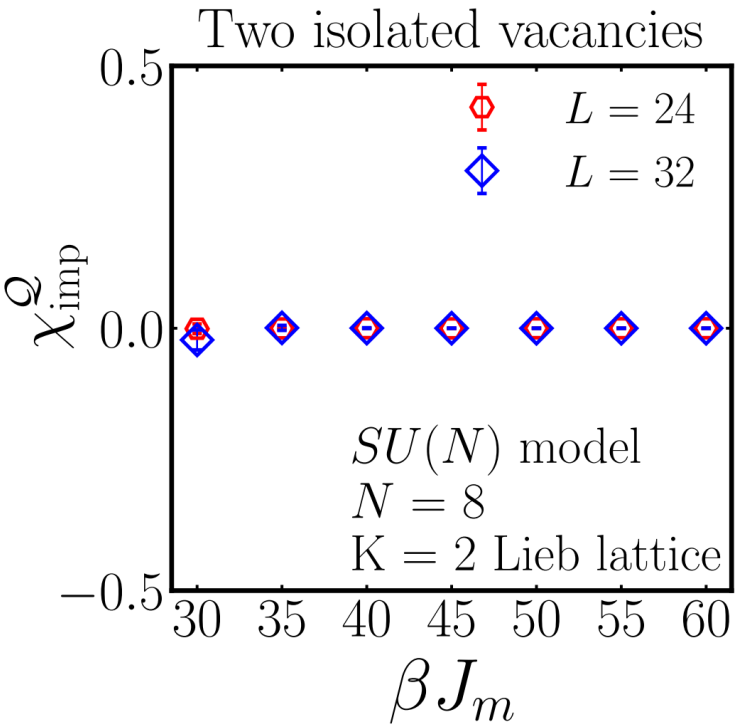
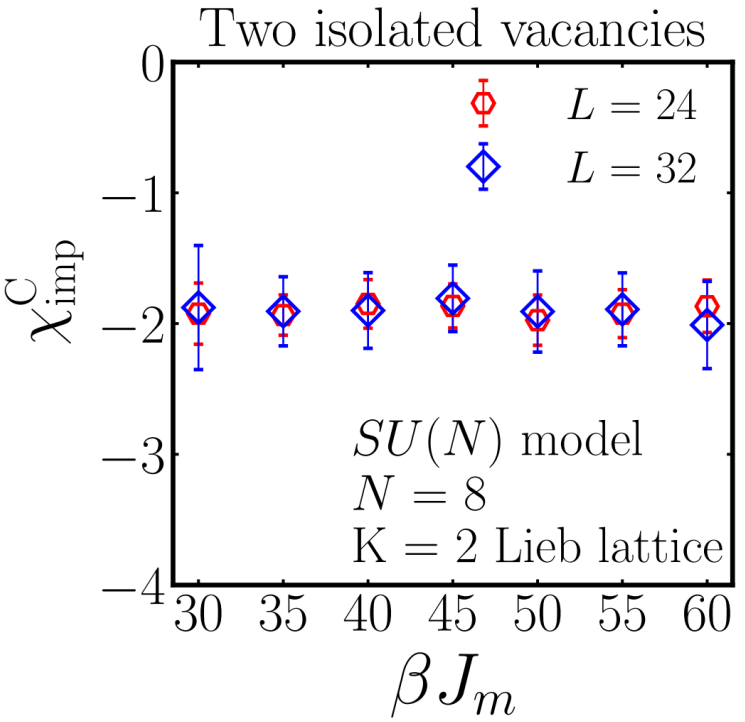
Isolated vacancies: sRVB regime (bipartite)





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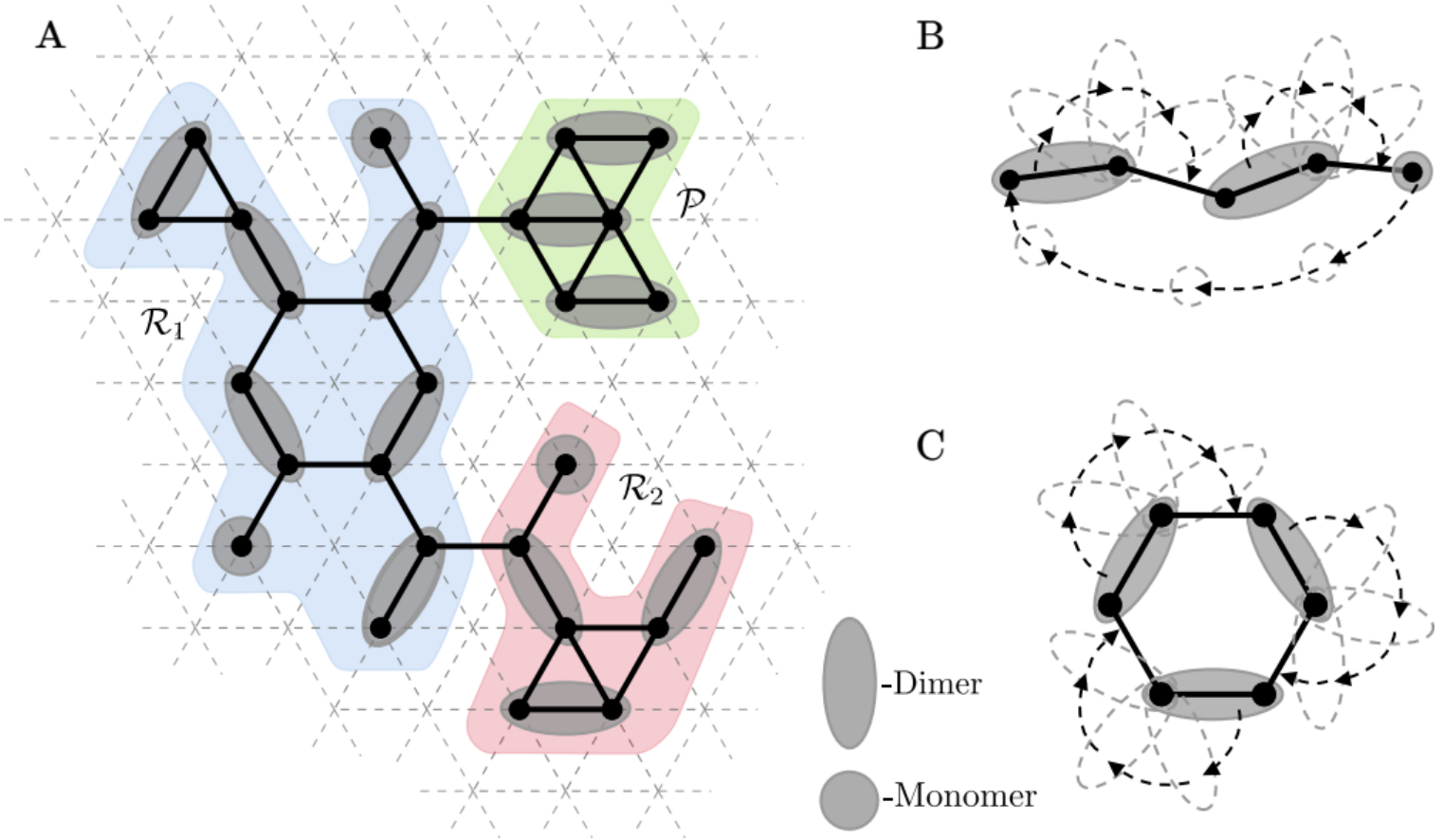
Where do the monomers live?

(Theory for geometry of monomer-carrying regions)

The setting: Maximum-density dimer packings of diluted lattices



Some conclusions (from pictures):

Pure case:

Most regular lattices have nonzero entropy density of fully-packed dimer coverings

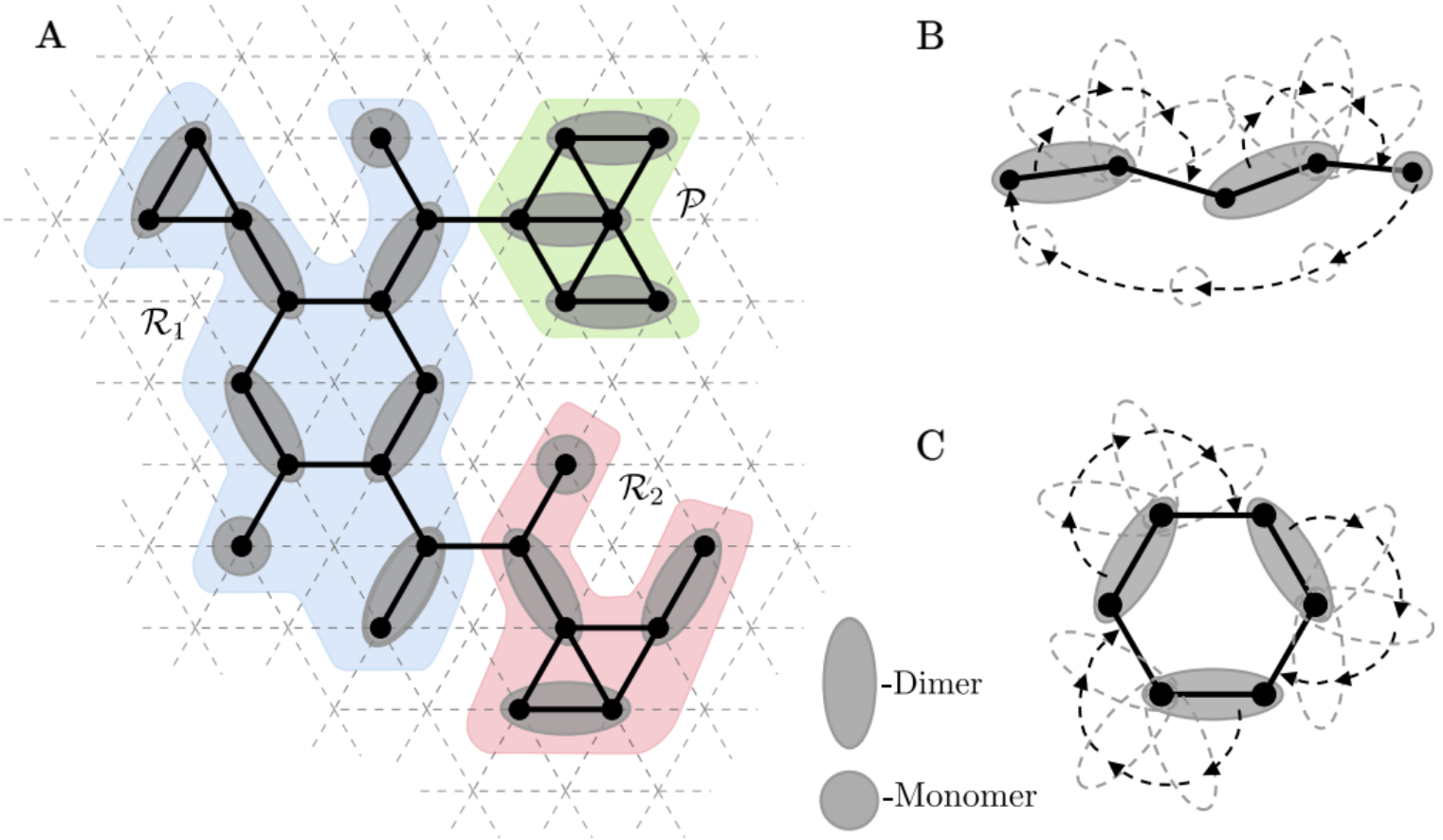
(if bipartite, require |A|=|B| of course)

Weak vacancy disorder or bond dilution:

Typically have nonzero density of monomers in any maximum-density dimer packing

(and nonzero entropy density of such packings)

Constraints on maximum-density dimer packings



More conclusions (from pictures):

Consequences of hard-core and maximum-density constraints:

Constrained kinematics: ring-exchange or monomer-hopping

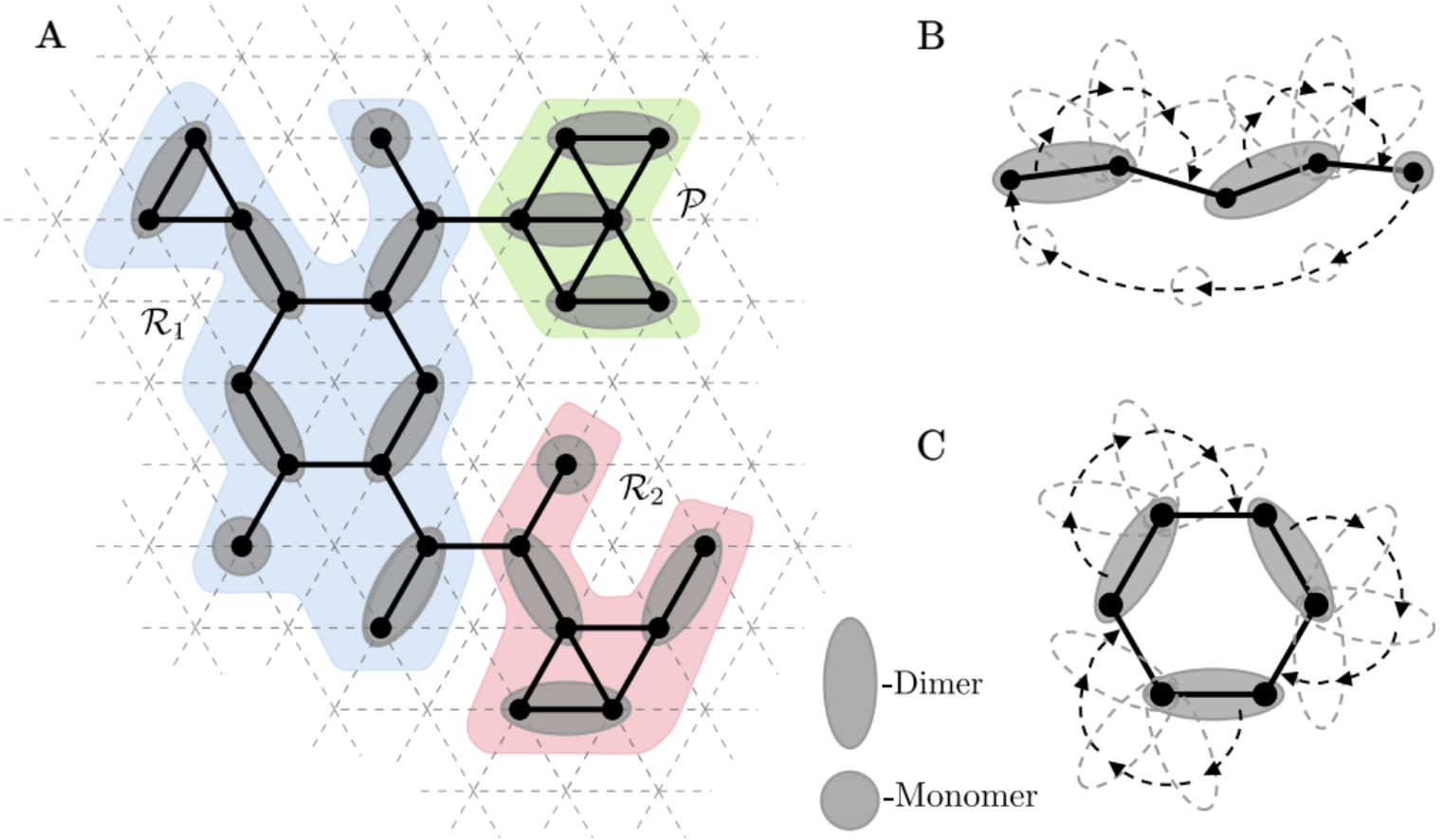
Constraint on links of ring-exchange and monomer-hopping process paths:

Each such link must be occupied by a dimer in at least one such dimer packing

Constraint on monomer and dimer motion:

Monomers confined to well-defined regions of disordered lattice. Other regions fully-packed.

Geometry of monomer-carrying and fully-packed regions



Another conclusion (from pictures):

Boundaries of monomer-carrying $\mathcal{R}$-type, fully-packed $\mathcal{P}$-type regions:

Some "forbidden" links of disordered lattice can never be occupied by a dimer in any such packing

Boundaries of these regions demarcated by the "forbidden" links

These regions are properties of disordered lattice, not any one maximum-density packing

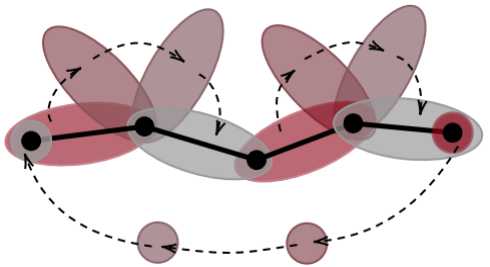
T. Gallai 1963,'64

Formal justification

J. Edmonds, 1965

PATHS, TREES, AND FLOWERS

JACK EDMONDS



Prescription:

Pick favorite maximum-density dimer packing

Explore forest of alternating paths starting from all monomers

Label vertices e (even) if they can be reached along an even-length path of this forest

Label vertices u (unreachable) if they cannot be reached along any paths of this forest

Label vertices o (odd) otherwise (i.e. can be reached by odd-length path but not even-length path)

Gallai-Edmonds Theory

T. Gallai 1963,'64

J. Edmonds, 1965

PATHS, TREES, AND FLOWERS

JACK EDMONDS

Labeling independent of choice of favorite maximum-density dimer packing

Property of underlying disordered lattice

Labeling comes with structural guarantees about disordered lattice

No e – u links possible

Deleting e – o organizes all e vertices into odd-cardinality connected components: "Blossoms"

Labeling also comes with guarantees about ensemble of maximum-density dimer packings

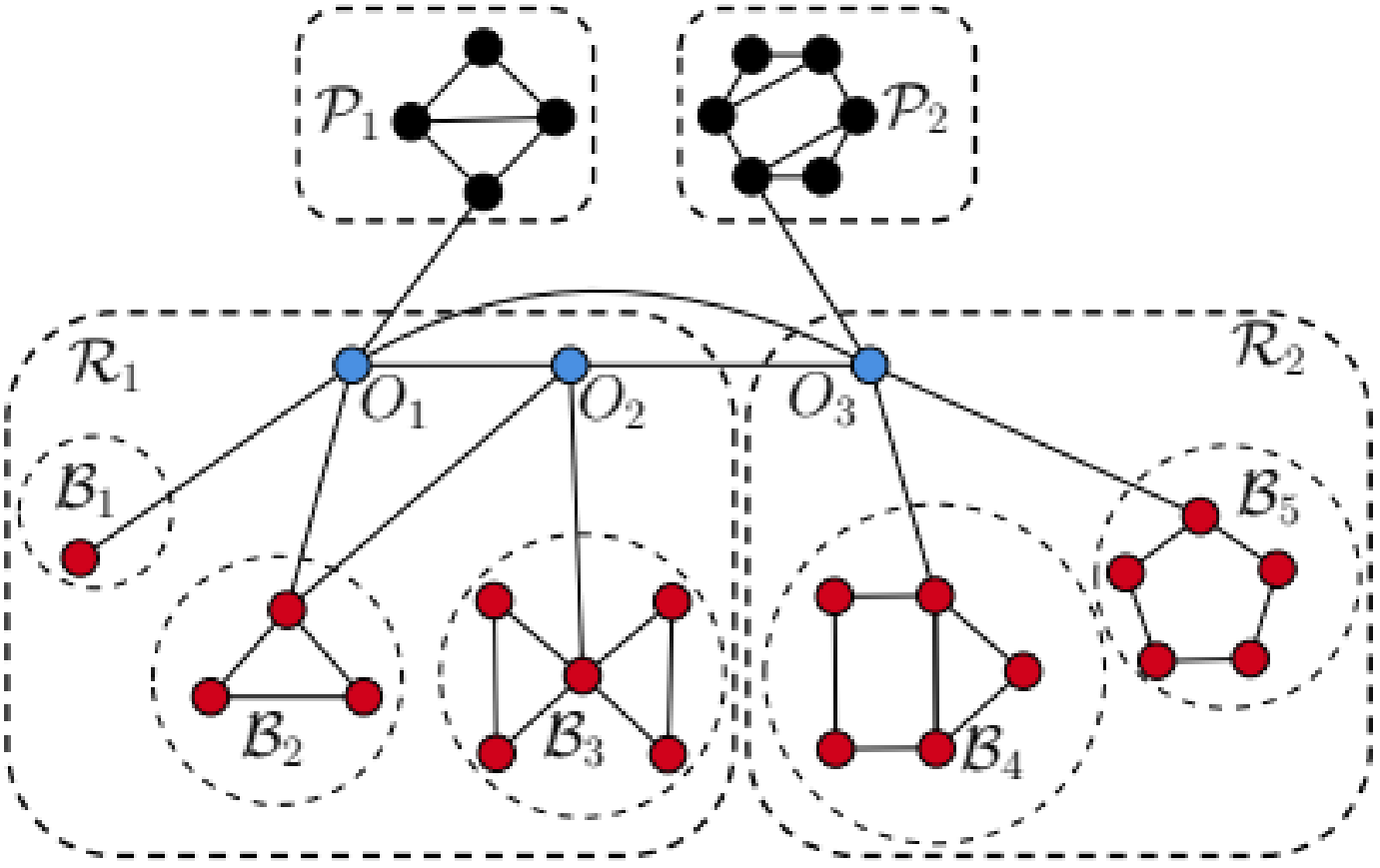
All u vertices connected to another u vertex by a dimer

All o vertices connected to some e vertex by a dimer

All monomers live on blossoms, no blossom has more than one monomer on it.

Construction of $\mathcal{R}$-type and $\mathcal{P}$-type regions

Key observation: o – o and o – u links are the "forbidden" links. Delete!



KD, PRB 105 235118 (2022)

Aside: significance of $\mathcal{R}$-type and $\mathcal{P}$-type regions

Quantum monomer-dimer models

Monomer-hopping and ring-exchange processes cannot cross boundaries

All eigenstates of quantum/classical monomer-dimer models factorize

(for any dimer-interactions along flippable loops, but short-range monomer interactions)

Implies: If all regions small, area law entanglement in the middle of the many-body spectrum

To summarize: Vacany-induced local moments in sRVB liquids associated with monomers

Emergent local moments are a multi-vacancy effect, and confined to R-type regions of lattice

Dominant short-range interactions between these local moments also confined within R-type region

Geometry of $\mathcal{R}$-type regions expected to determine low-energy state and magnetic response

Very different from vacancy effects in VBS states:

Each vacancy individually nucleates a local moment bound to it

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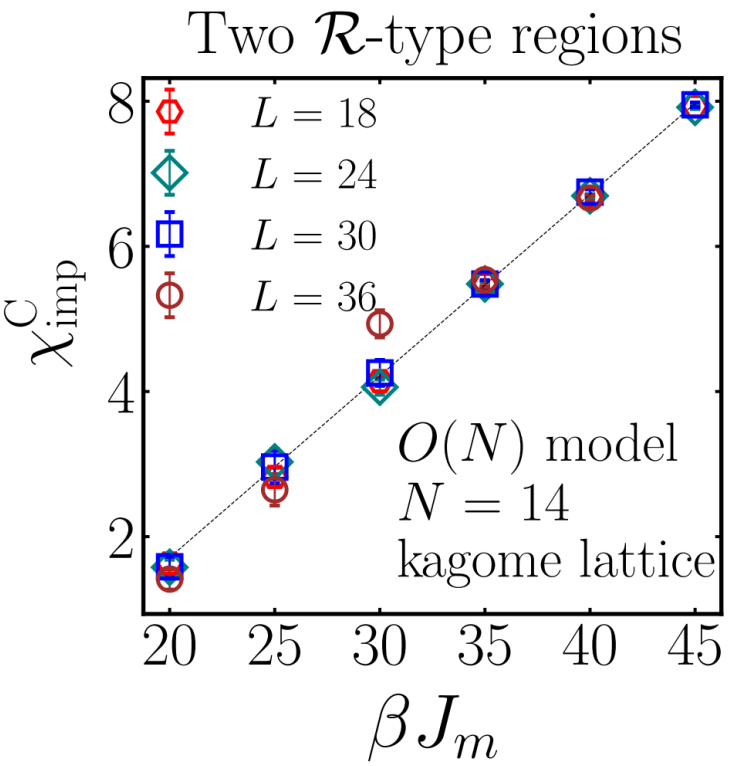
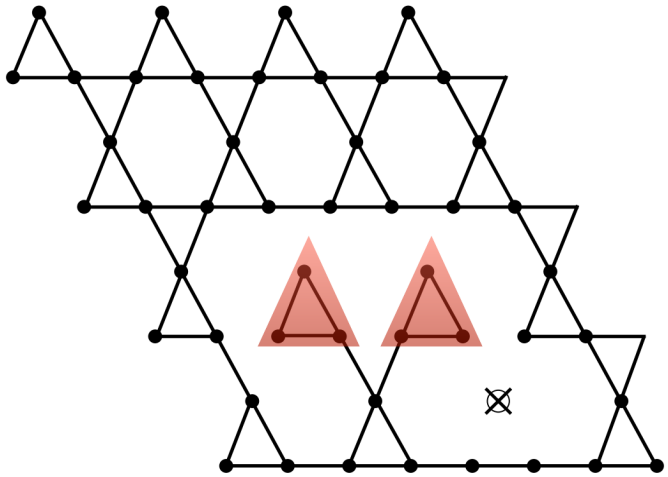
Back to tests: two R-type regions in RVB state (non-bipartite)





Note: deleted bonds, not sites

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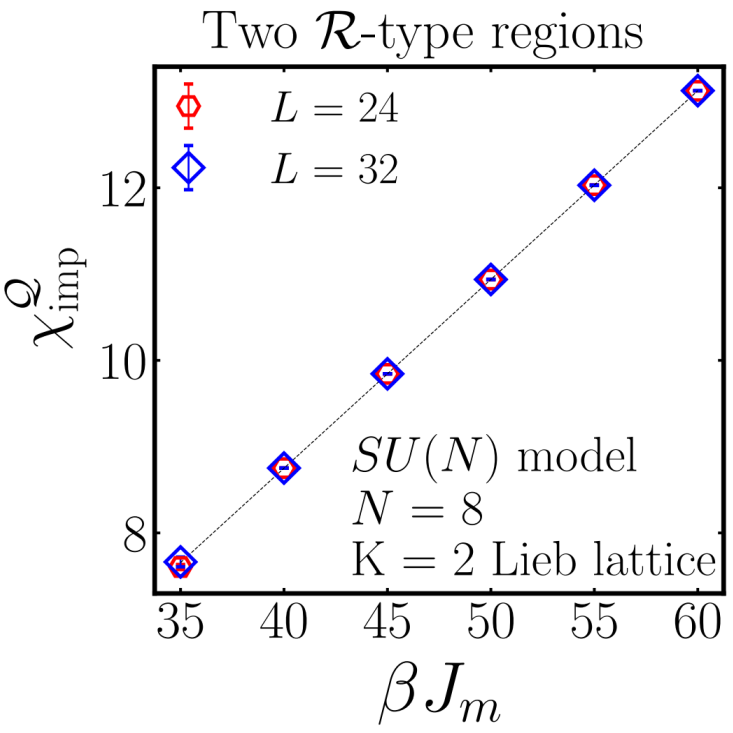
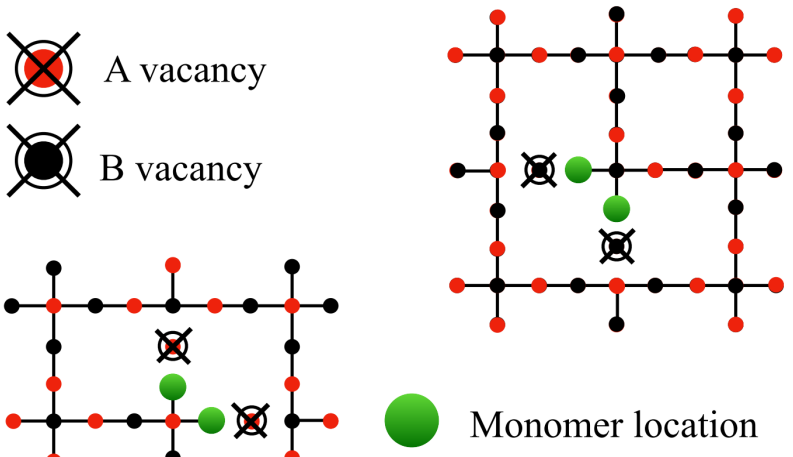
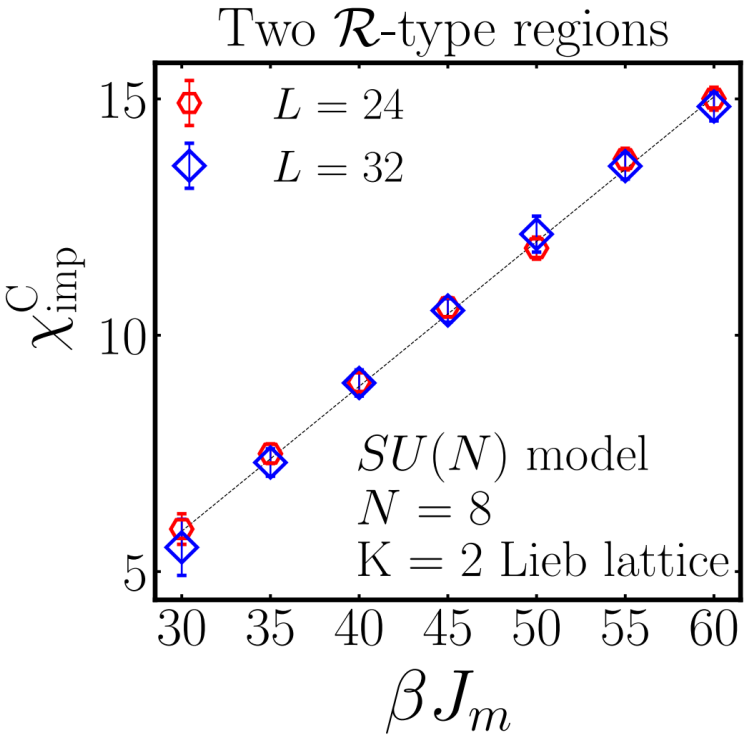
Back to tests: two R-type regions Q-response: RVB state (bipartite)

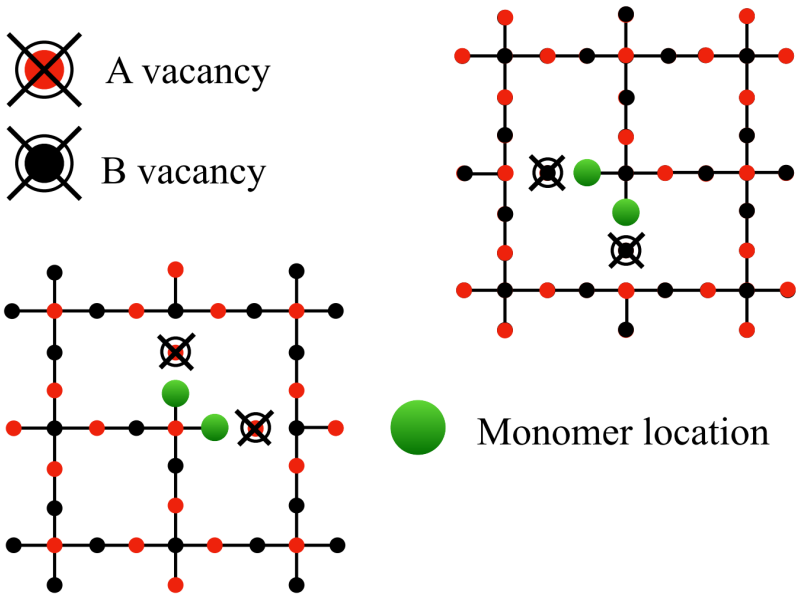




Ansari, Kundu, KD, in preparation

Back to tests: Two R-type regions C-response: RVB state (bipartite)





Ansari, Kundu, KD, in preparation

Large-scale geometry of monomer-carrying R-type, fully-packed P-type regions

Computationally tractable (but challenging) using Edmonds' polynomial time matching algorithm

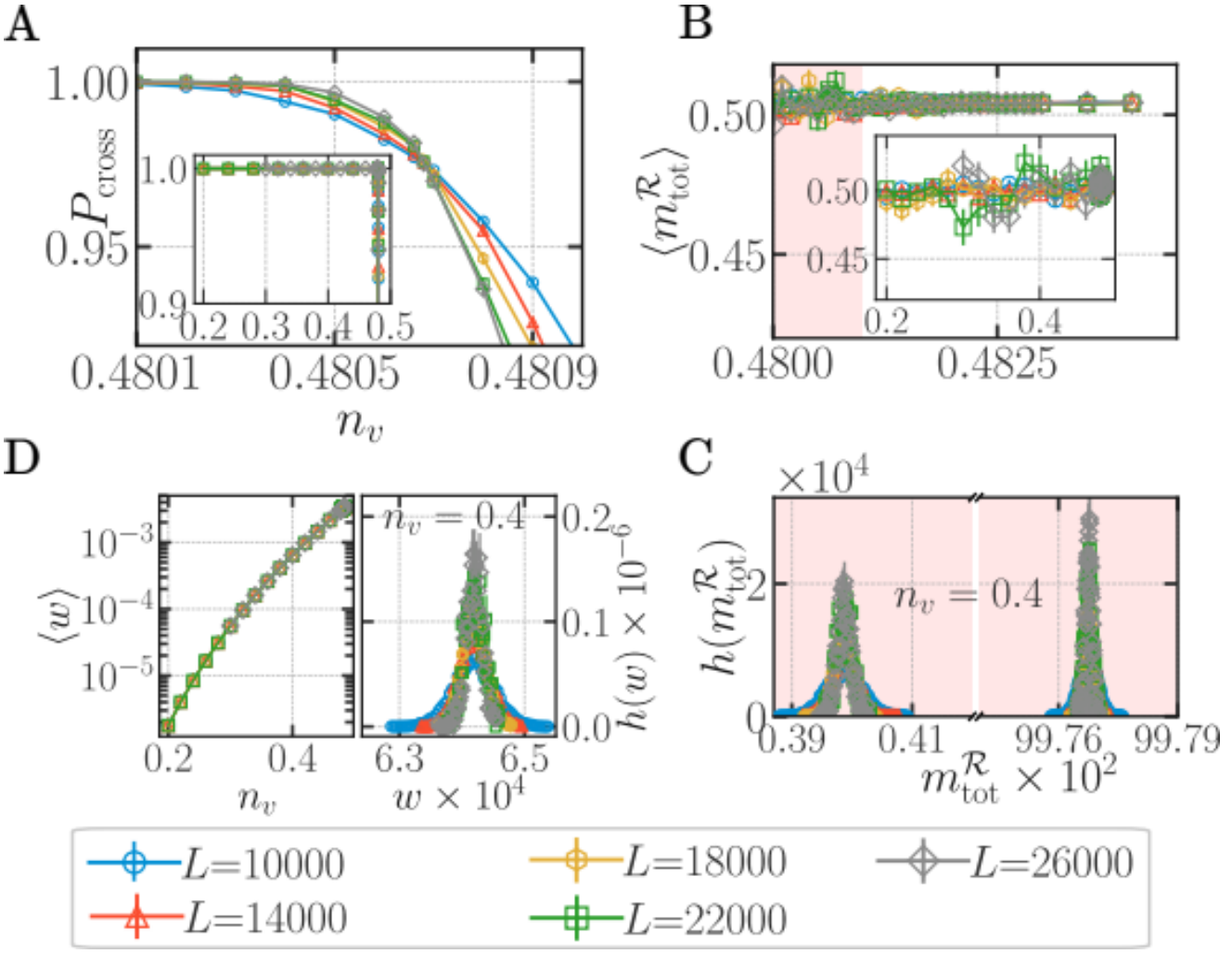
For site-diluted triangular lattice: typical regions are large at low dilution

Think in terms of percolation

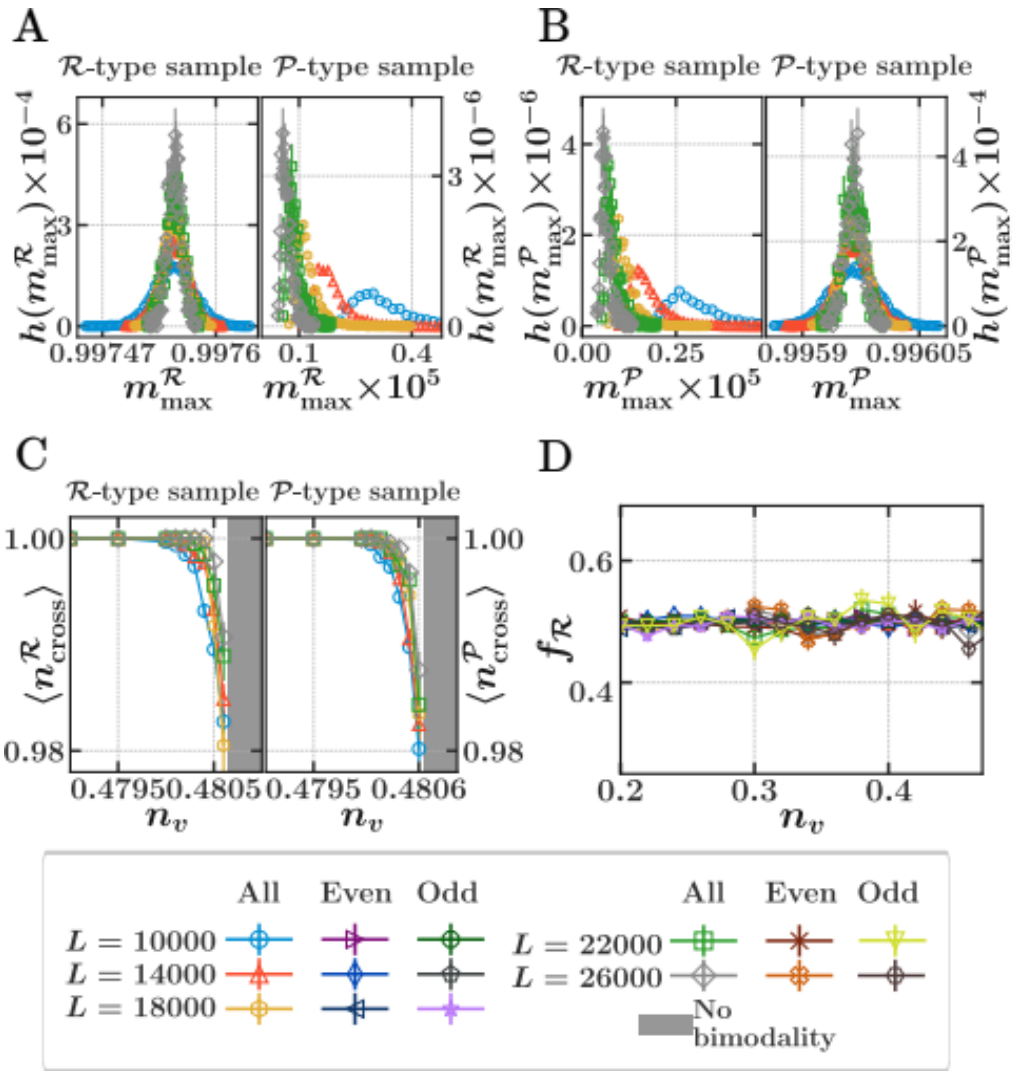
(Sharp threshold as function of some parameter in end-to-end connectivity of a medium)

The "right" yes/no question to ask: Can one walk from one end of a sample, staying within a single region?

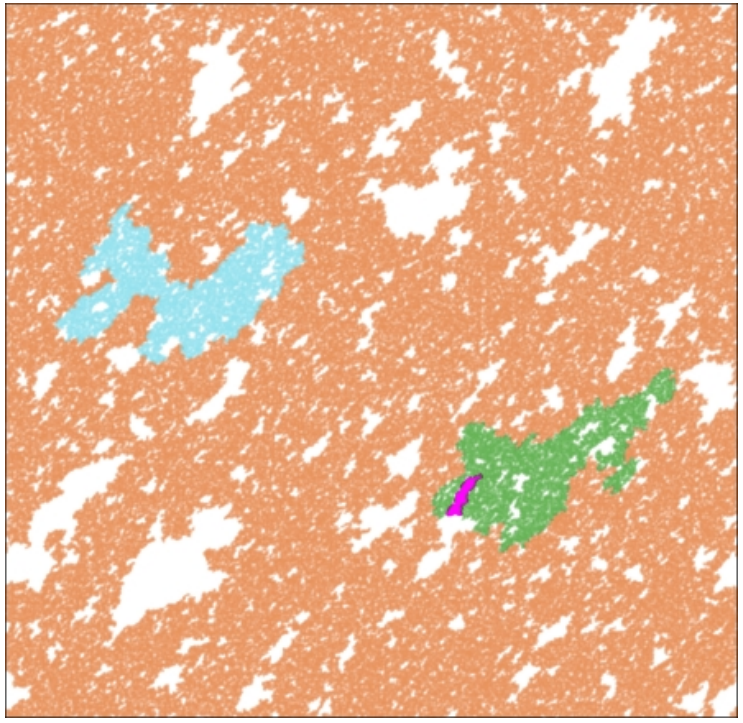
On the diluted triangular lattice



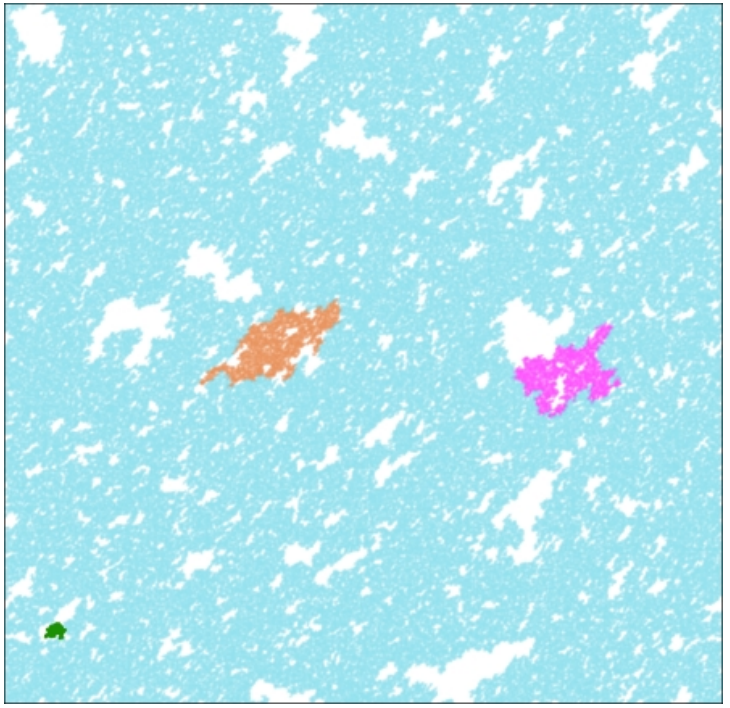
On the diluted triangular lattice



Pictorially on the diluted triangular lattice



R-type sample



P-type sample

On the diluted triangular lattice

$$f_{\mathcal{R}} = 1/2$$

Why???

Does this suggest some emergent symmetry between monomer-carrying and fully-packed regions

Again: Parity of largest geometric cluster plays no role!

“Central dogma” of thermodynamic self-averaging

In large-size limit -

Strong version:

Self-averaging of properties: Sample-to-sample fluctuations small (average = typical)

Violations exist – e.g. Disordered quantum spin chains (infinite-disorder fixed points)

Weak version:

At a minimum, two samples prepared using some protocol must be in same phase.

Violations? May exist in infinite-range spin glass models (?)

On the diluted triangular lattice

Violation of even the weak form of "central dogma" at low vacancy concentration:

Monomers delocalized in half the samples, localized to O(1) regions in the other half!

All samples identically prepared, randomly diluted, with the exact same density of vacancies

Our basic message

Violations of “central dogma”

Weak disorder can lead to:

Violations of not just strong but also weak form of the “central dogma”

Root cause: Kinematic constraints induce long-range correlations

(caveat emptor: merely post-facto rationalization, no detailed understanding)

Some predictions for observable effects

Consequences:

Weak vacancy disorder leads to similar effects in short-range RVB spin liquids on the triangular lattice

At a minimum: Strong violations of thermodynamic self-averaging in low-temperature suscepbtibility

Likely: "R-type samples" have spin-glass order but not "P-type" samples

Not discussed in this talk:

Weak vacancy disorder in pinned vortex lattice state of p+ip superconductors will also lead to similar effects

At a minimum: Strong violations of thermodynamic self-averaging in the thermal conductivity

Likewise for weak vacancy disorder in triangular lattice Majorana spin liquids

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vacancies in sRVB, VBS, & AFM states: S. Bhattacharjee, L. Balents, S. Sachdev, A. Sandvik

Percolation: Deepak Dhar, Subhajit Goswami

Other collaborations (2010-20) on disorder effects: Fabien Alet, Argha Banerjee, Sylvain Capponi, Pranay Patil, Arnab Sen

Computing cluster related: K. Ghadiali and A. Salve (DTP SysAds)

Chaotic response to vacancy motion

Suggests extreme sensitivity of large-scale geometry to micro-scale details of disorder configuration

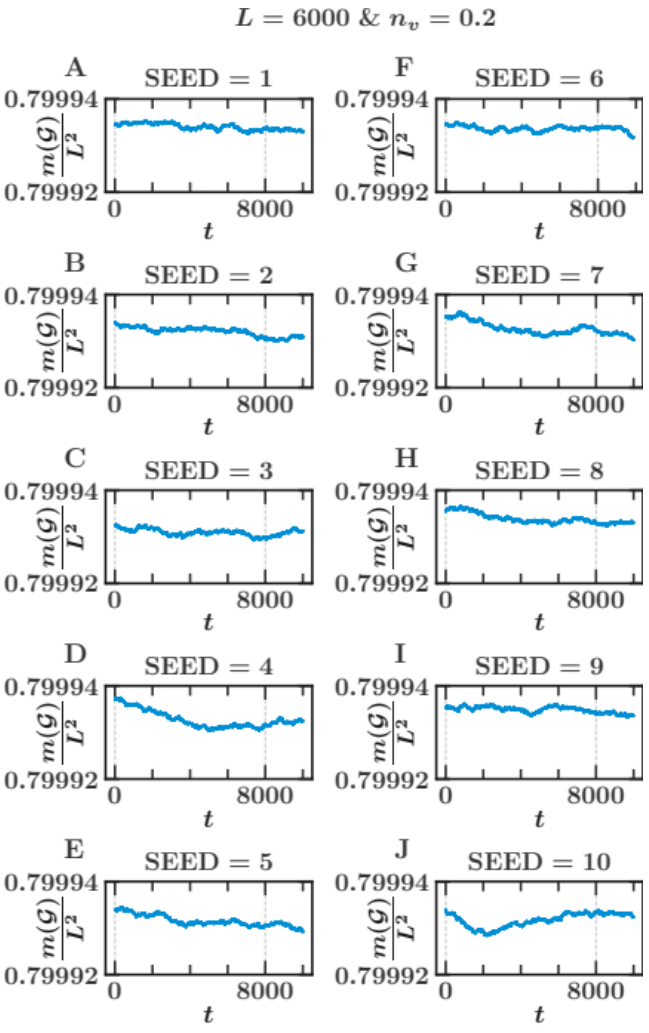
Can we quantify this?

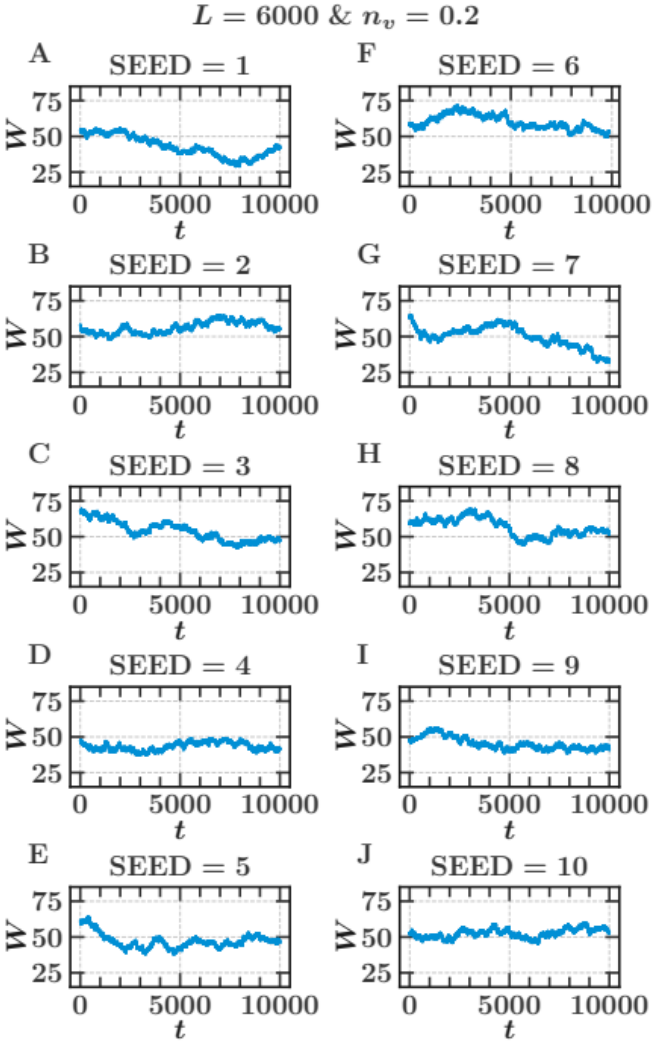
Model dynamics: Set vacancies in motion and watch what happens!

Small fraction of vacancies exchange position with neighboring surviving site at each time step

How does the large-scale geometry of these regions react?

Dynamics doesn't disturb underlying lattice much





Yet: Large-scale geometry of monomer-carrying/fully-packed regions responds chaotically