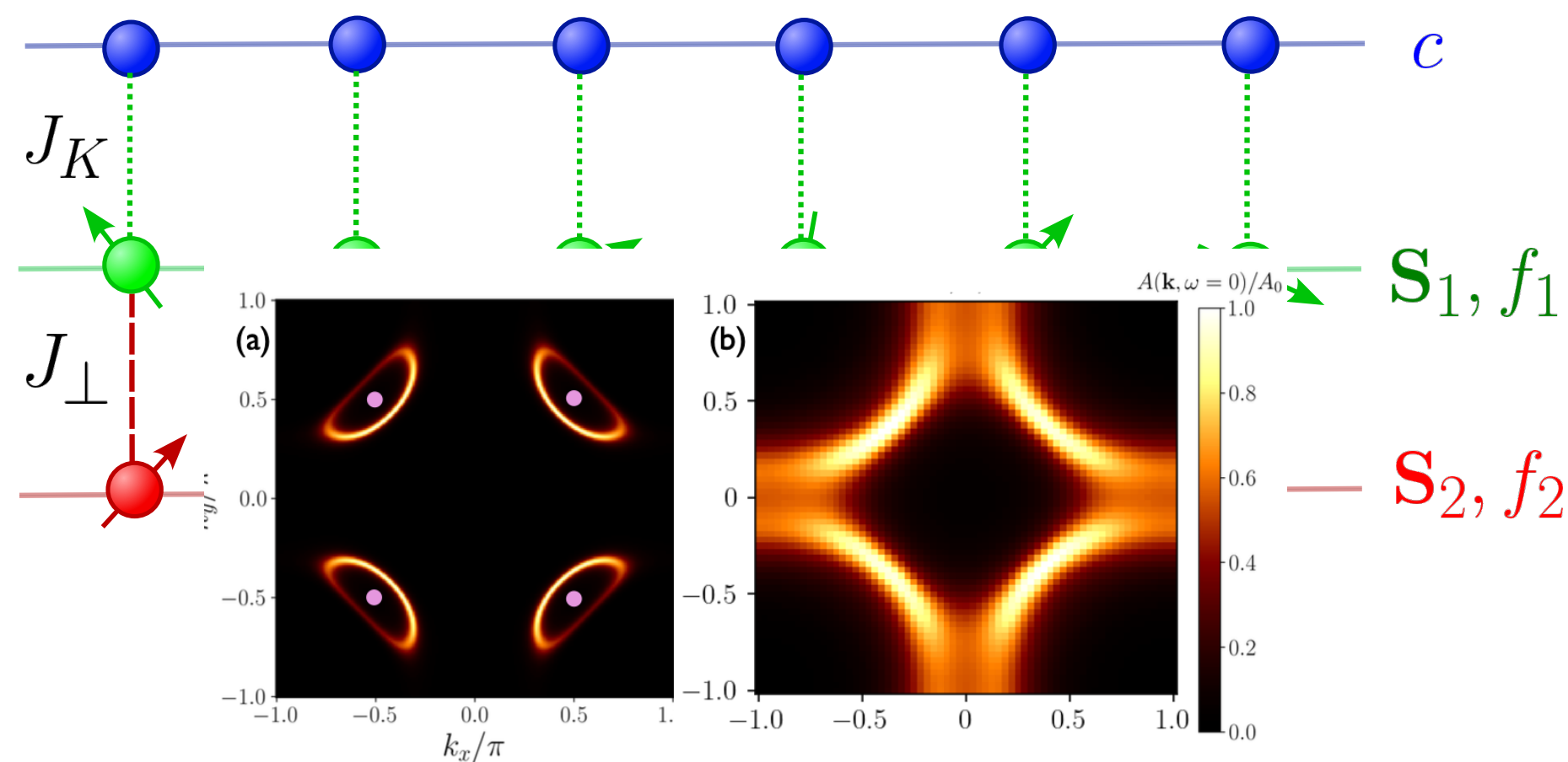
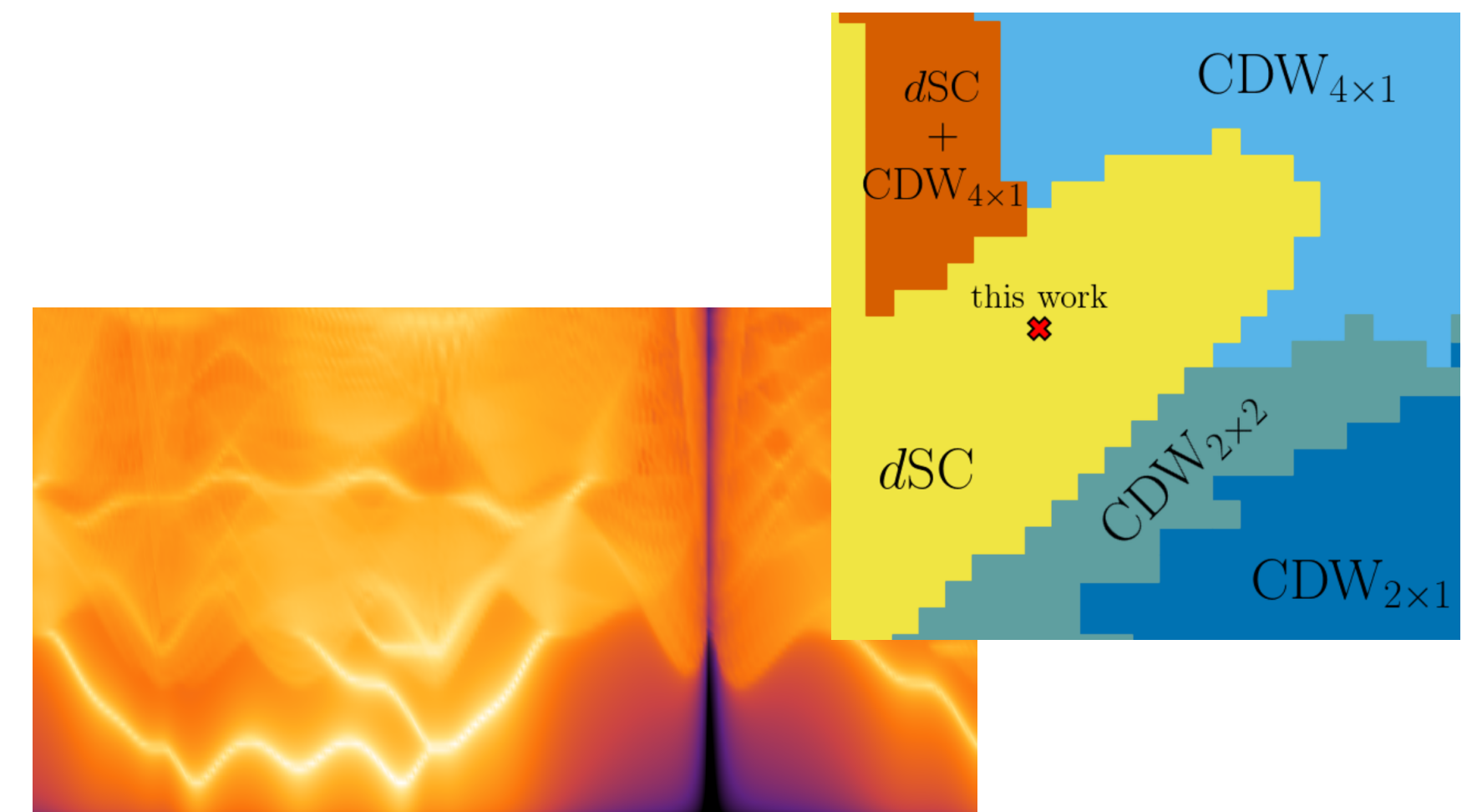
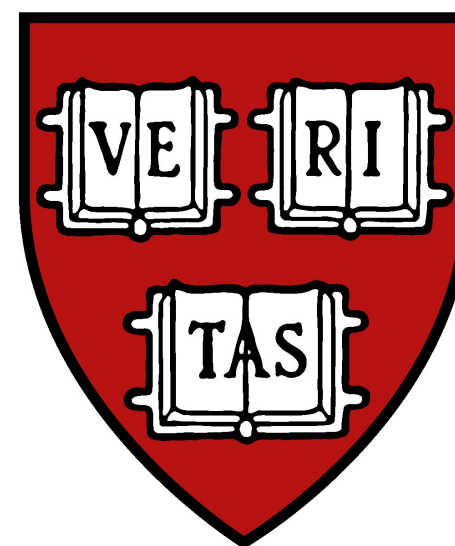


Gauge Fluctuations, Confinement, and the Pseudogap in Cuprates ^{45'}

Pietro M. Bonetti



Harvard University



School & Conference on Quantum Matter, ICTP (Trieste), December 11, 2025

Collaborators



Harshit Pandey
IMS Chennai



Ravi Shanker
IMS Chennai



Sayantan Sharma
IMS Chennai



Maine Christos
Harvard U → Caltech

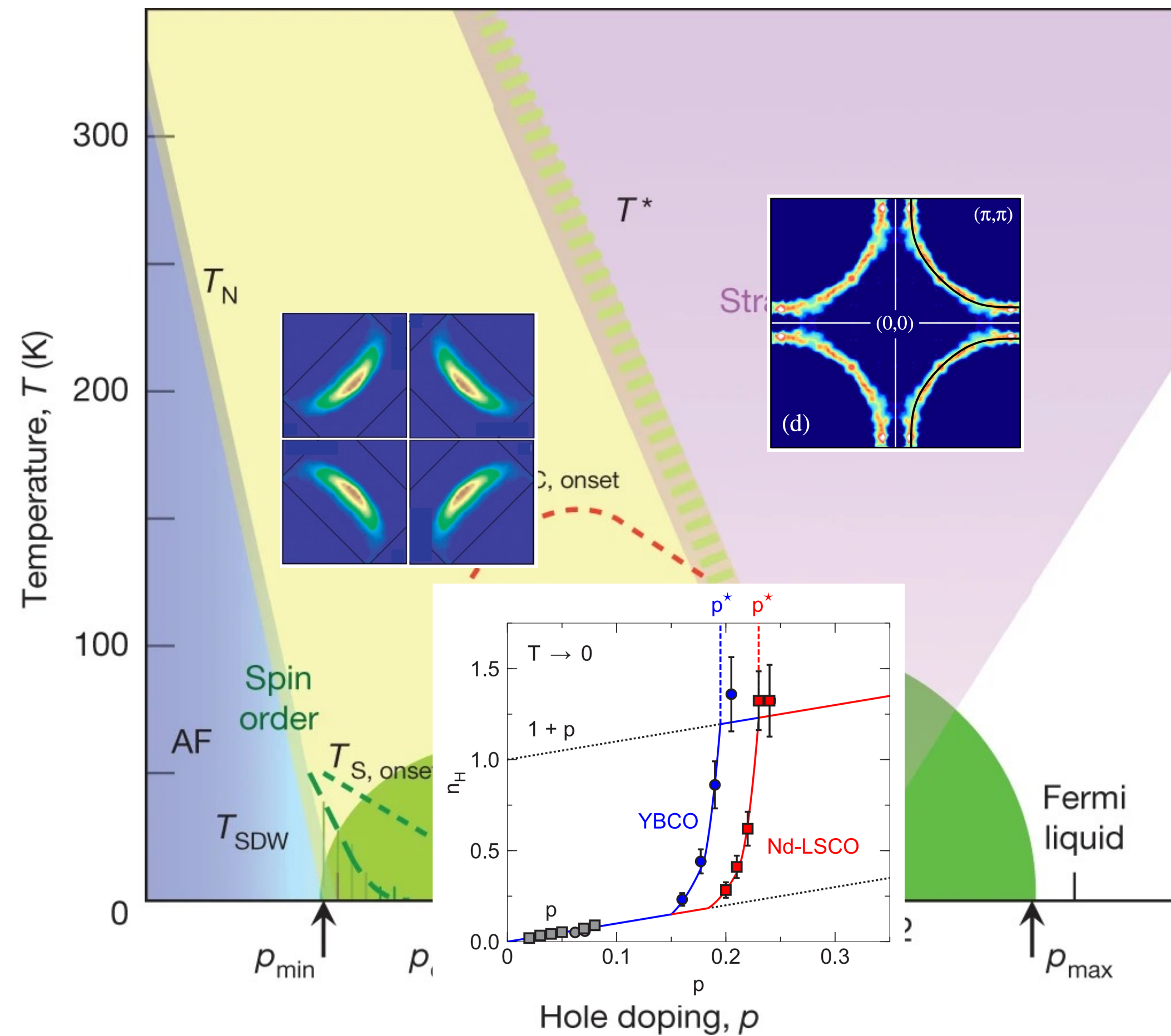


Alexander Nikolaenko
Harvard University

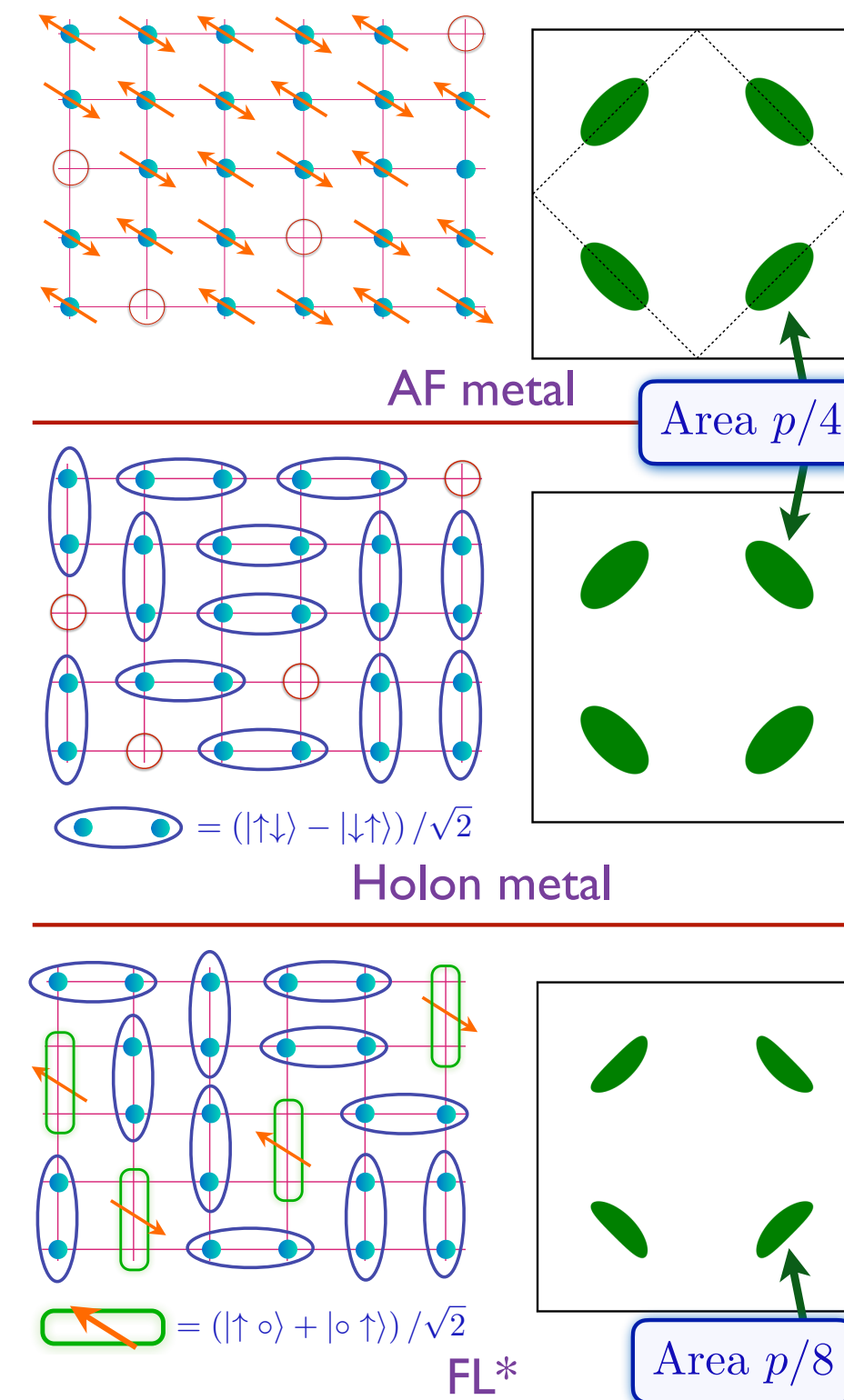


Subir Sachdev
Harvard University

Pseudogap Phase in Cuprates



Keimer *et al.*, Nature **518**, 179 (2015)



- Eberlein *et al.*, PRL **117**, 187001 (2016)
- Storey, EPL **113**, 27003 (2016)
- PMB*, Mitscherling*, *et al.*, PRB **101**, 165142 (2020)

- Scheurer *et al.*, PNAS **115** (16) E3665 (2018)
- PMB & Metzner, PRB **106**, 205152 (2022)

Zhang & Sachdev, PRR **2**, 023172 (2020)

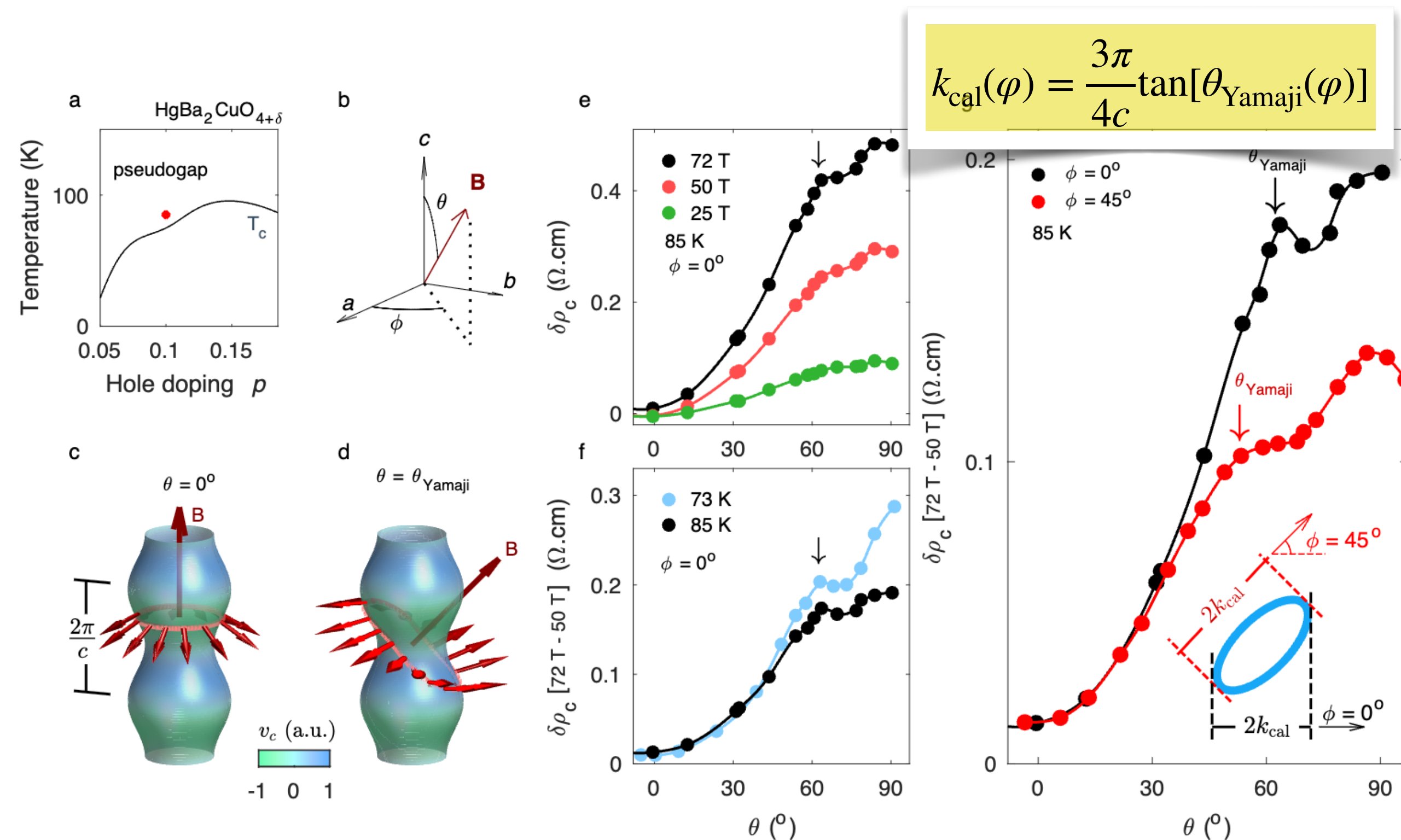
Observation of the Yamaji Effect

Nat. Phys. (2025)

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,^{1,*} Katherine A. Schreiber,¹ Oscar E. Ayala-Valenzuela,¹

Eric D. Bauer,² Arkady Shekhter,¹ and Neil Harrison¹



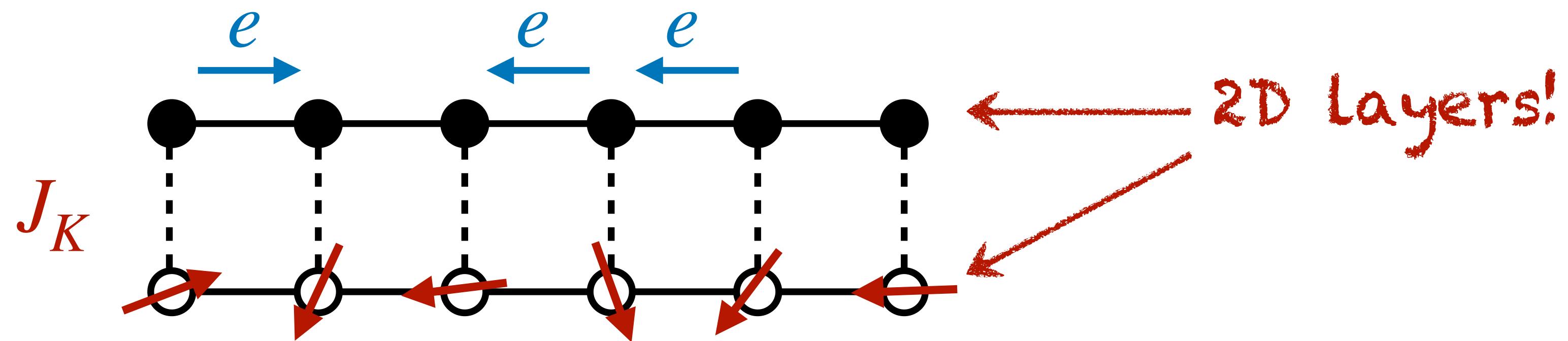
Hole pockets have sizes of 1.3% of the BZ

$$\frac{p}{8} = 1.25 \%$$

Compatible with FL* scenario!

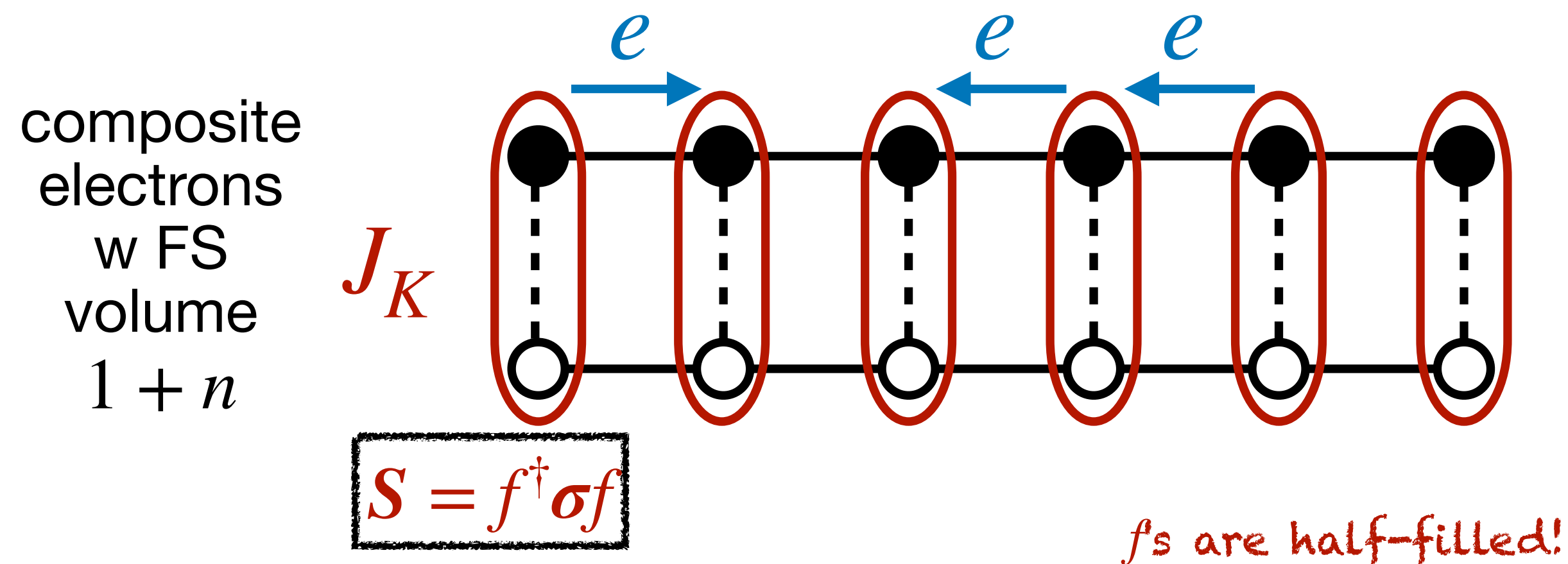
FL* Theory

Kondo Lattice



Kondo Lattice

Heavy Fermi Liquid



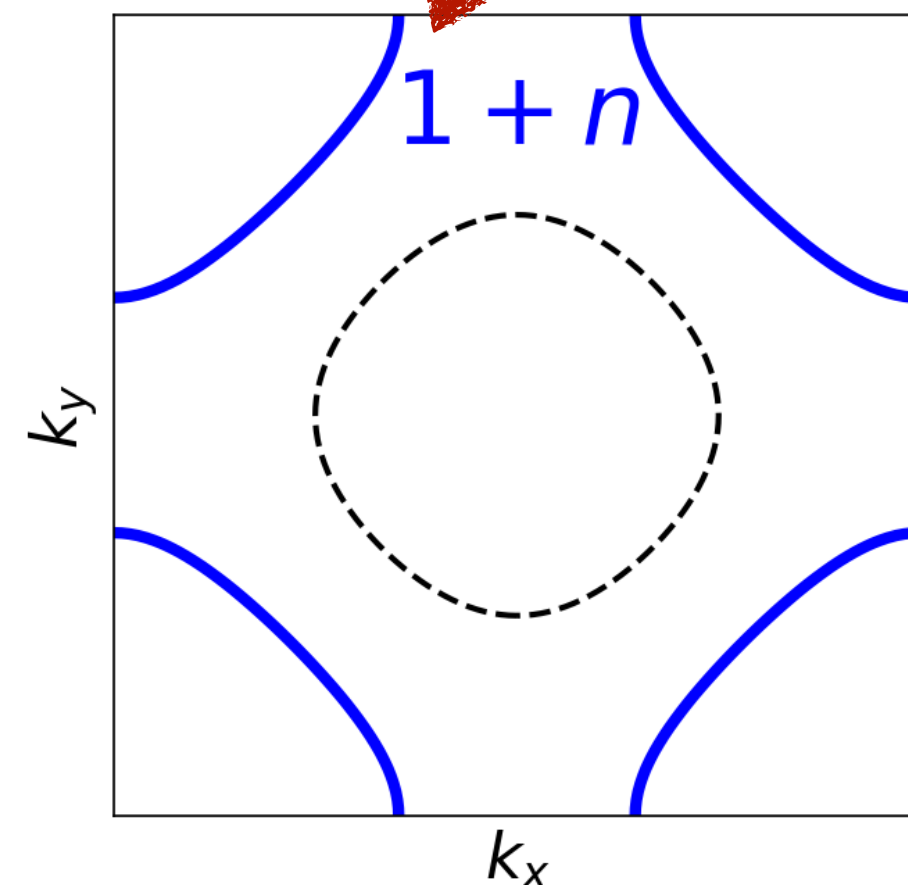
f's are half-filled!

$$\langle c^\dagger f \rangle \neq 0$$

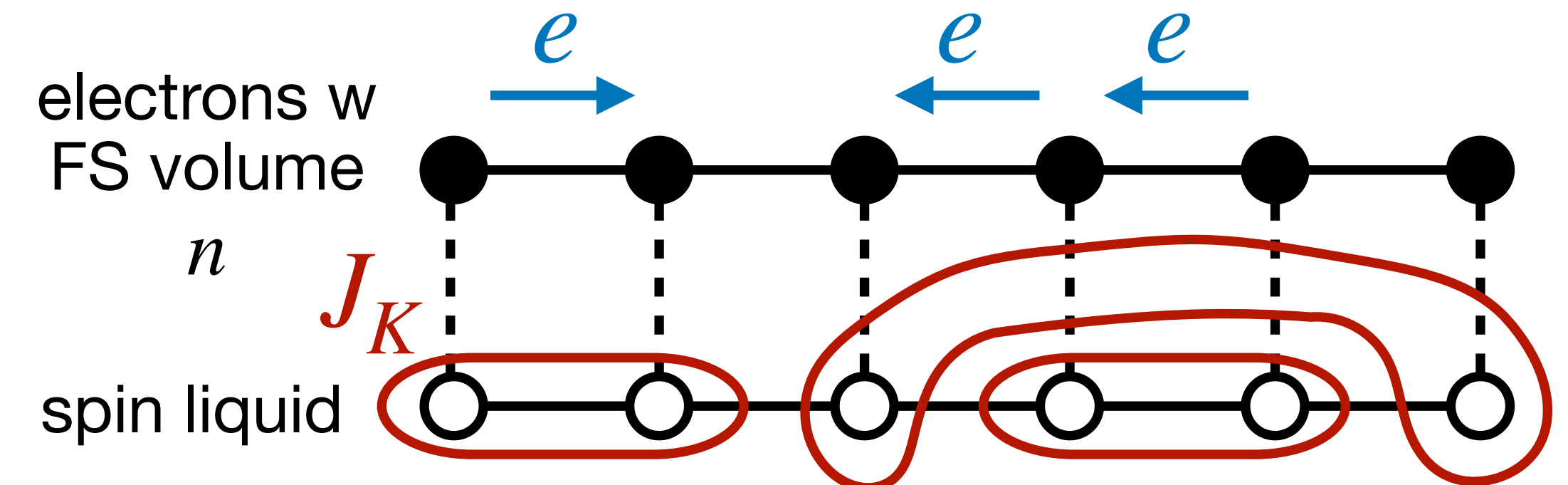
U(1) symmetry

$$c \rightarrow e^{i\alpha} c$$

$$f \rightarrow e^{i\alpha} f$$



Fermi Liquid *



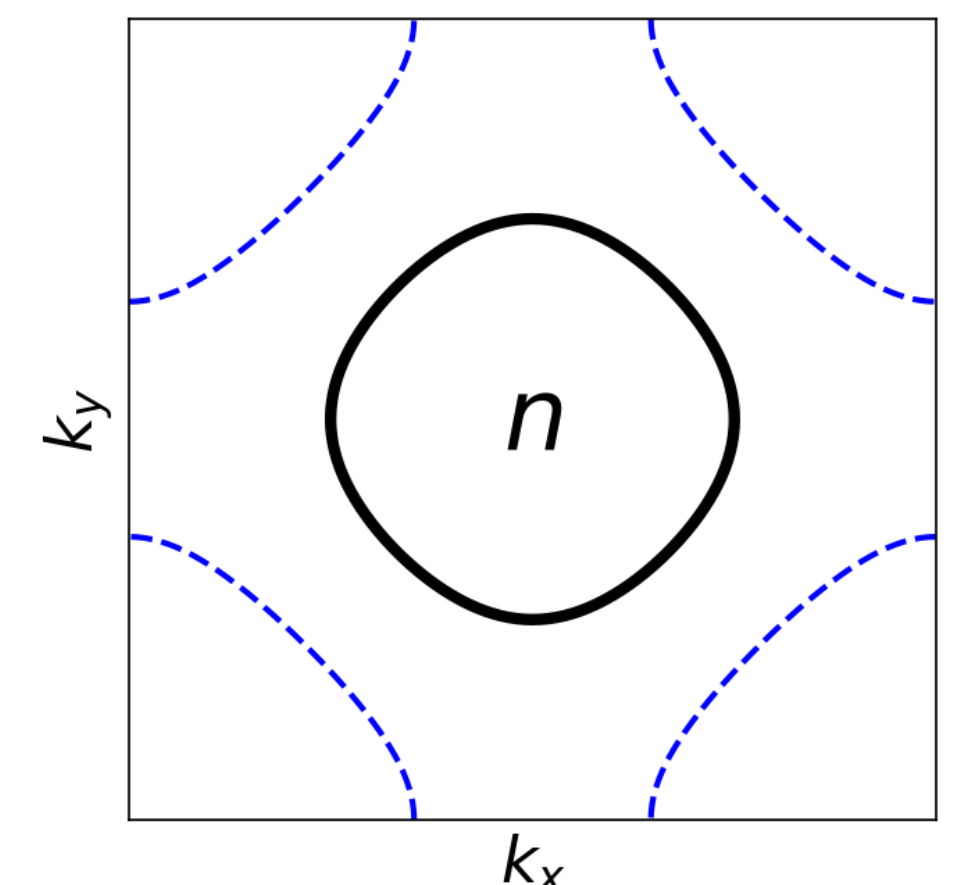
Senthil, Sachdev Vojta, PRL 2003
Senthil, Vojta, Sachdev, PRB 2004

$$\langle c^\dagger f \rangle = 0$$

U(1) symmetry

$$c \rightarrow e^{i\alpha} c$$

$$f \rightarrow f$$



The AKLT model: a recap

$$H = \sum_{\langle i,j \rangle} P_{ij}^{(2)}$$



$$\bullet \text{---} \bullet = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\bigcirc = |+\rangle\langle\uparrow\uparrow| + |0\rangle\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}} + |-\rangle\langle\downarrow\downarrow|$$

from Wikipedia.org

- **Enlarge Hilbert space:** $|+\rangle, |0\rangle, |-\rangle$
to $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle,$

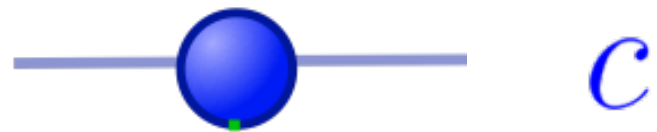
- **Project out local singlets**
 $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Ancilla Layer Model I

Zhang & Sachdev, PRR 2, 023172 (2020)

consider a model of interacting spin- $\frac{1}{2}$ electrons

4 local states $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$



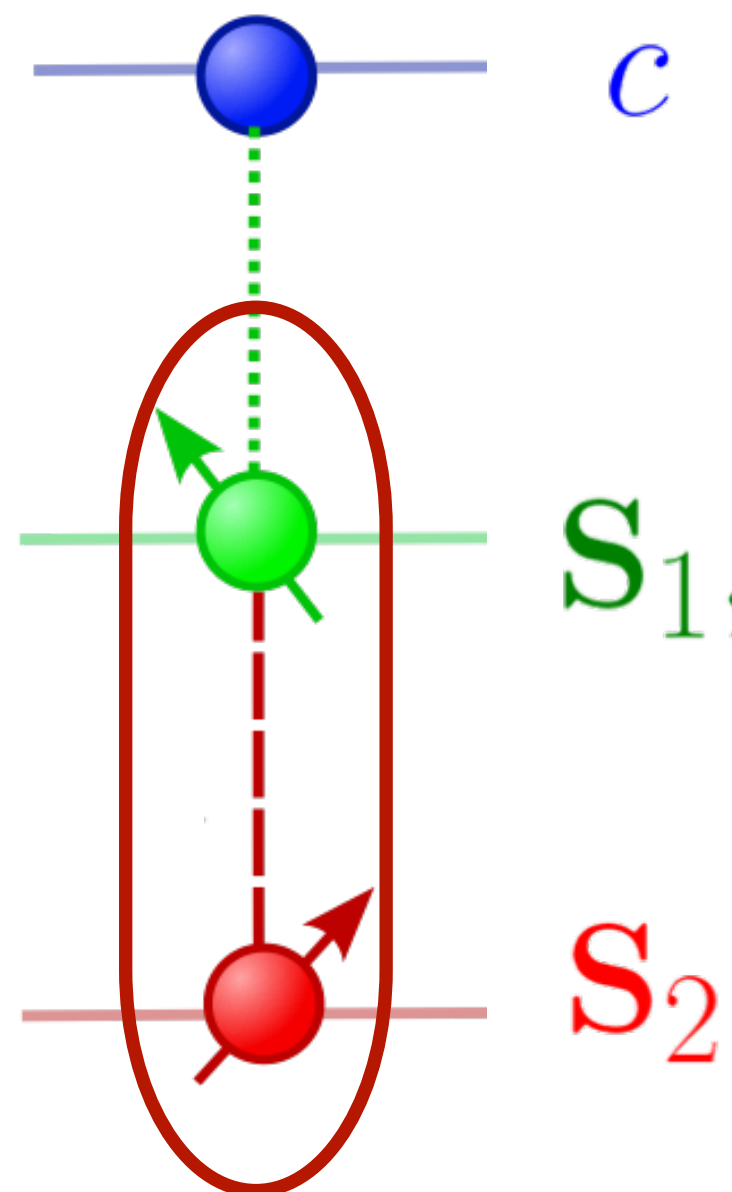
Ancilla Layer Model I

Zhang & Sachdev, PRR 2, 023172 (2020)

consider a model of interacting spin- $\frac{1}{2}$ electrons

4 local states $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

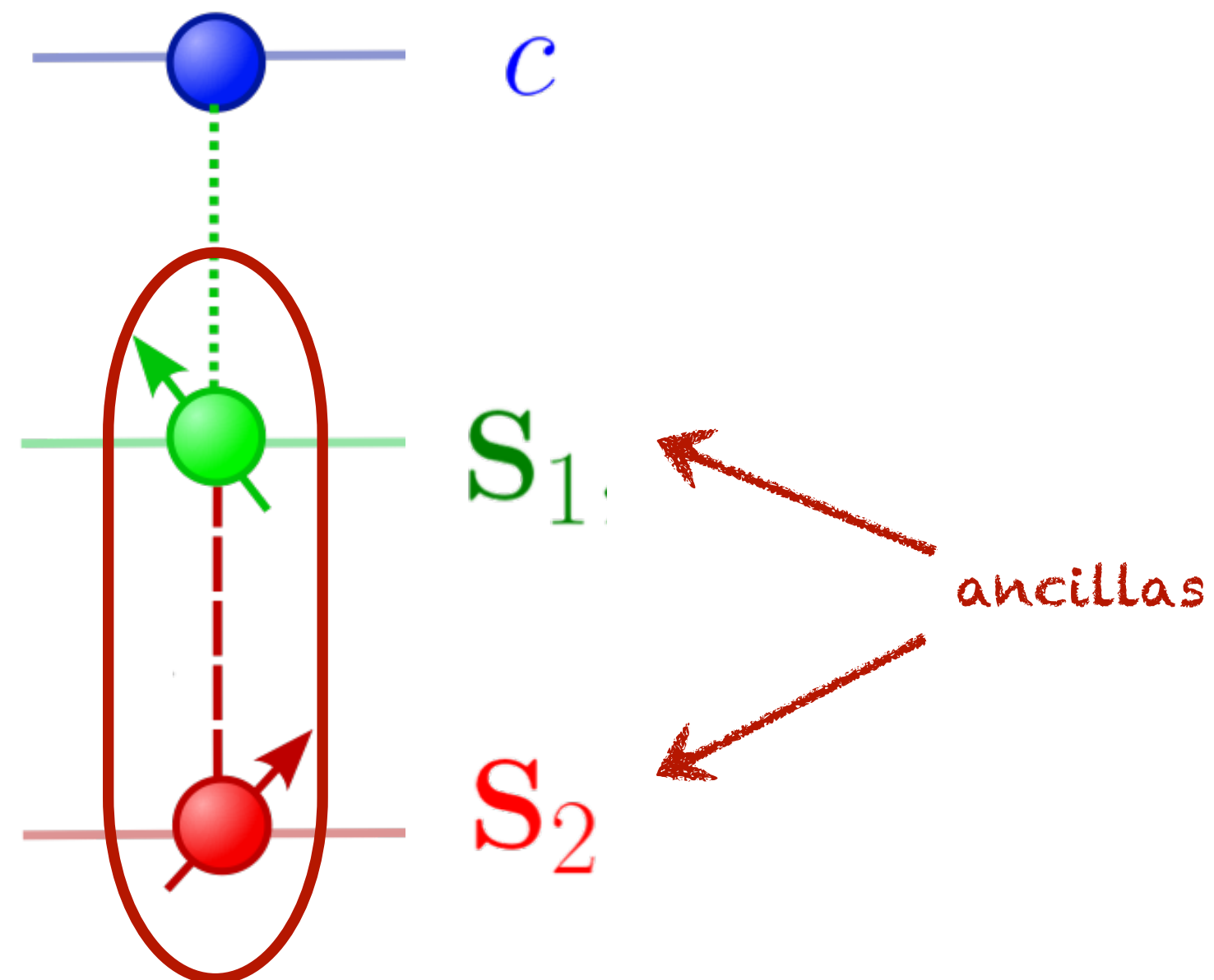
→ add a singlet $|0,s\rangle, |\uparrow,s\rangle, |\downarrow,s\rangle, |\uparrow\downarrow,s\rangle$



Ancilla Layer Model I

Zhang & Sachdev, PRR 2, 023172 (2020)

consider a model of interacting spin- $\frac{1}{2}$ electrons



4 local states $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

→ add a singlet $|0,s\rangle, |\uparrow,s\rangle, |\downarrow,s\rangle, |\uparrow\downarrow,s\rangle$

→ enlarge to

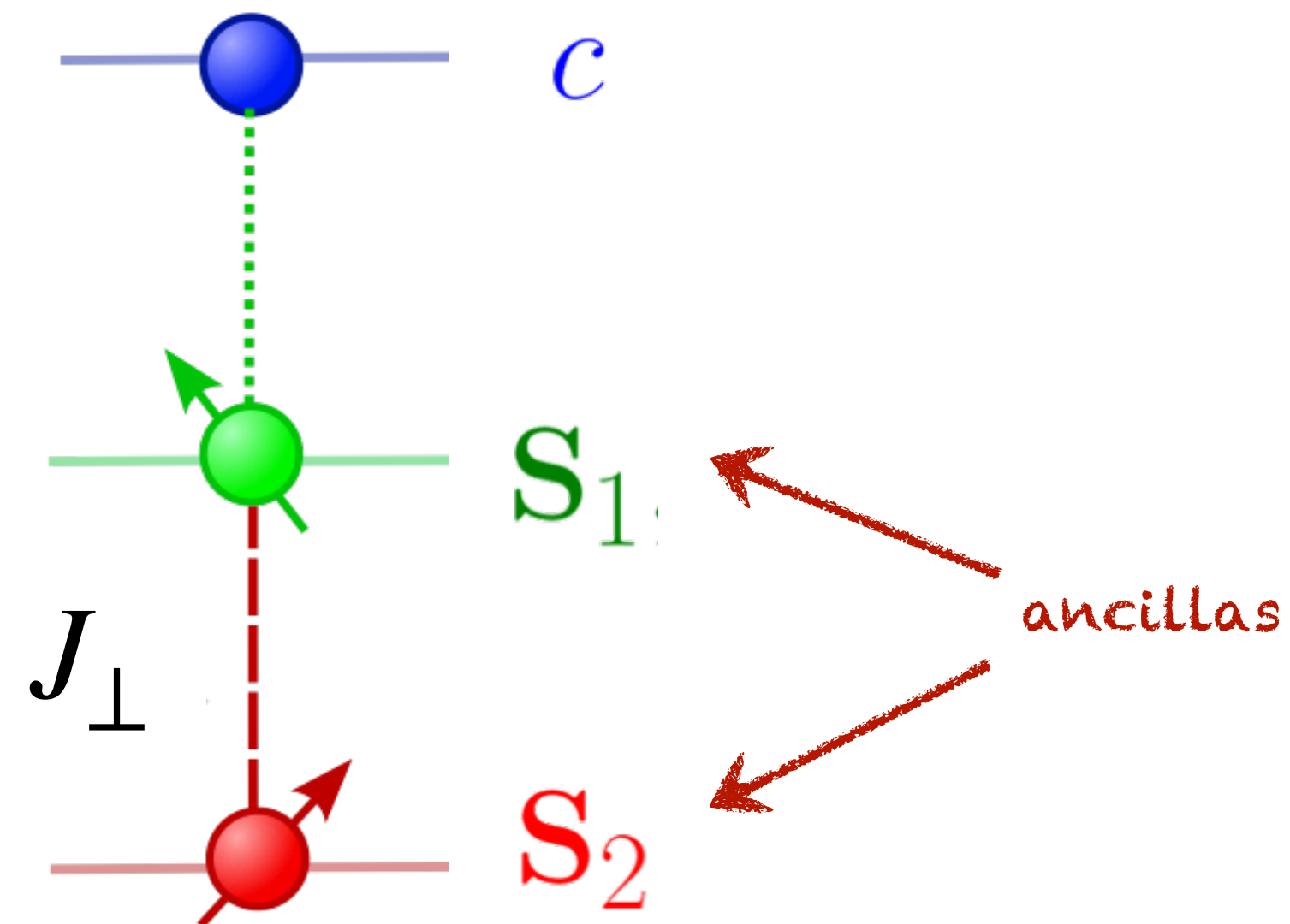
$$\begin{aligned} &|0, \uparrow\uparrow\rangle, |\uparrow, \uparrow\uparrow\rangle, |\downarrow, \uparrow\uparrow\rangle, |\uparrow\downarrow, \uparrow\uparrow\rangle \\ &|0, \uparrow\downarrow\rangle, |\uparrow, \uparrow\downarrow\rangle, |\downarrow, \uparrow\downarrow\rangle, |\uparrow\downarrow, \uparrow\downarrow\rangle \\ &|0, \downarrow\uparrow\rangle, |\uparrow, \downarrow\uparrow\rangle, |\downarrow, \downarrow\uparrow\rangle, |\uparrow\downarrow, \downarrow\uparrow\rangle \\ &|0, \downarrow\downarrow\rangle, |\uparrow, \downarrow\downarrow\rangle, |\downarrow, \downarrow\downarrow\rangle, |\uparrow\downarrow, \downarrow\downarrow\rangle \end{aligned}$$

→ project onto $\frac{1}{\sqrt{2}}(|\Omega, \uparrow\downarrow\rangle - |\Omega, \downarrow\uparrow\rangle)$

Ancilla Layer Model I

Zhang & Sachdev, PRR 2, 023172 (2020)

consider a model of interacting spin- $\frac{1}{2}$ electrons



4 local states $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

→ add a singlet $|0,s\rangle, |\uparrow,s\rangle, |\downarrow,s\rangle, |\uparrow\downarrow,s\rangle$

→ enlarge to

$$\begin{aligned} &|0, \uparrow\uparrow\rangle, |\uparrow, \uparrow\uparrow\rangle, |\downarrow, \uparrow\uparrow\rangle, |\uparrow\downarrow, \uparrow\uparrow\rangle \\ &|0, \uparrow\downarrow\rangle, |\uparrow, \uparrow\downarrow\rangle, |\downarrow, \uparrow\downarrow\rangle, |\uparrow\downarrow, \uparrow\downarrow\rangle \\ &|0, \downarrow\uparrow\rangle, |\uparrow, \downarrow\uparrow\rangle, |\downarrow, \downarrow\uparrow\rangle, |\uparrow\downarrow, \downarrow\uparrow\rangle \\ &|0, \downarrow\downarrow\rangle, |\uparrow, \downarrow\downarrow\rangle, |\downarrow, \downarrow\downarrow\rangle, |\uparrow\downarrow, \downarrow\downarrow\rangle \end{aligned}$$

→ project onto $\frac{1}{\sqrt{2}}(|\Omega, \uparrow\downarrow\rangle - |\Omega, \downarrow\uparrow\rangle)$

"soft projection" $-J_{\perp} S_1 \cdot S_2$, $J_{\perp} \gg E_{\text{model}}$

Ancilla Layer Model II

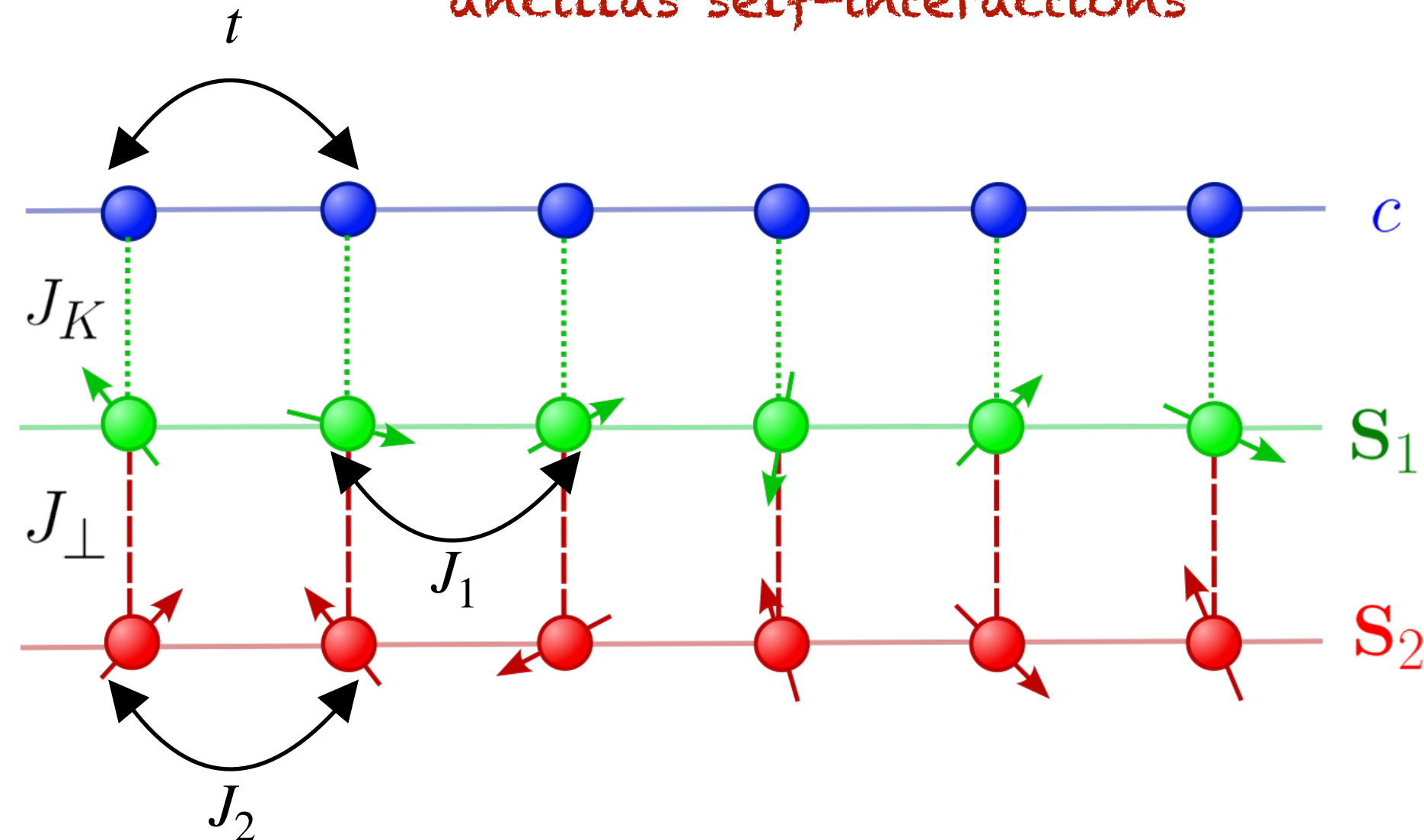
Zhang & Sachdev, PRR 2, 023172 (2020)

$$H = - \sum_{i,j} t_{ij} c_i^\dagger c_j + J_\perp \sum_i \mathbf{S}_{1,i} \cdot \mathbf{S}_{2,i} \quad \leftarrow \text{soft projector}$$

$$+ J_K \sum_i c_i^\dagger \boldsymbol{\sigma} c_i \cdot \mathbf{S}_{1,i} \quad \leftarrow \text{interaction}$$

$$+ \sum_{ij} J_{1,ij} \mathbf{S}_{1,i} \cdot \mathbf{S}_{1,j} + \sum_{ij} J_{2,ij} \mathbf{S}_{2,i} \cdot \mathbf{S}_{2,j}$$

ancillas self-interactions



project out triplet states perturbatively in $1/J_\perp$

$$\rightarrow H_{\text{el}} = - \sum_{ij} t_{ij} c_i^\dagger c_j + U \sum_i n_{i,\uparrow} n_{i,\downarrow} - \sum_{ij} J_{ij} (c_i^\dagger \boldsymbol{\sigma} c_i) \cdot (c_j^\dagger \boldsymbol{\sigma} c_j)$$

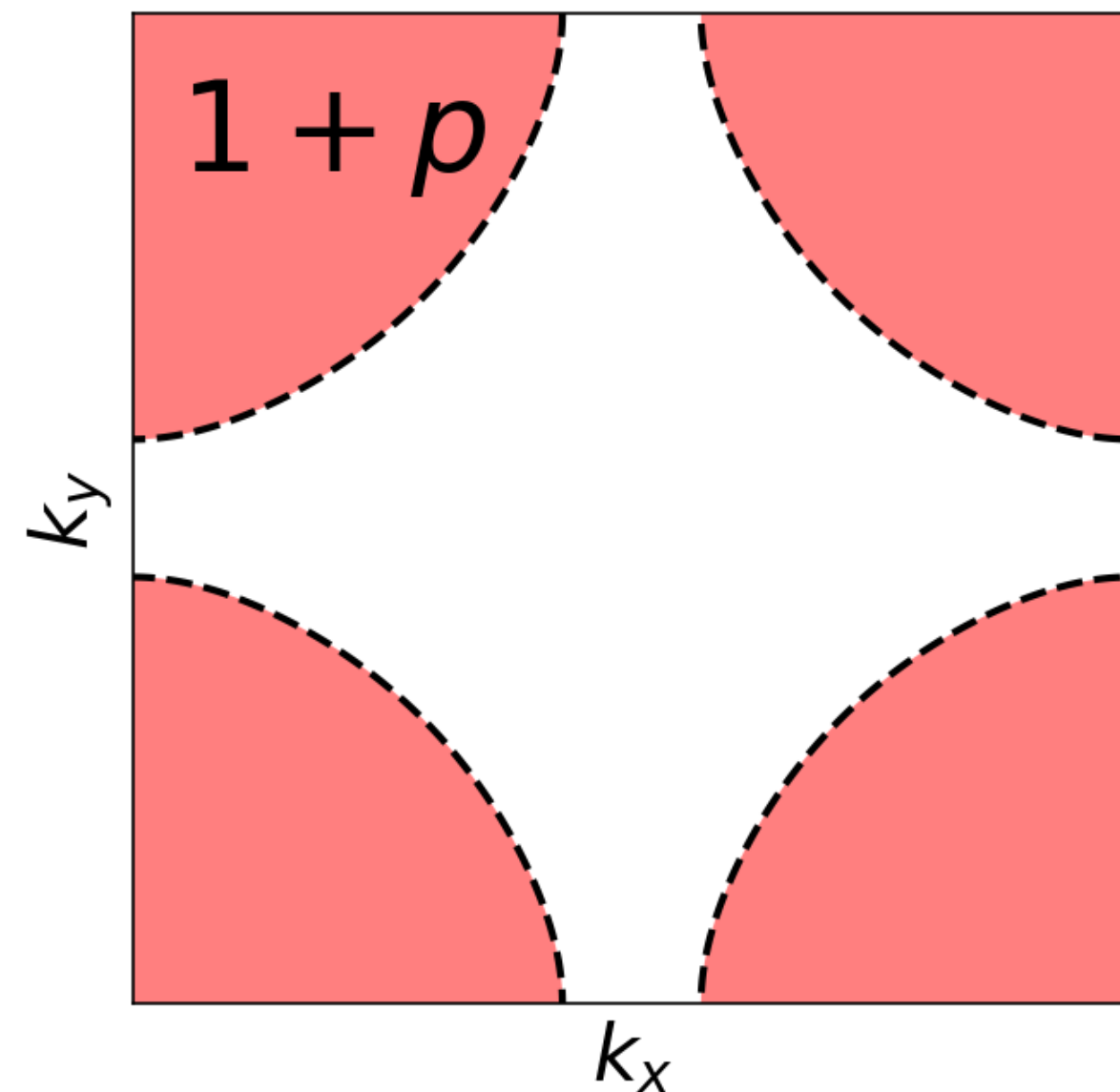
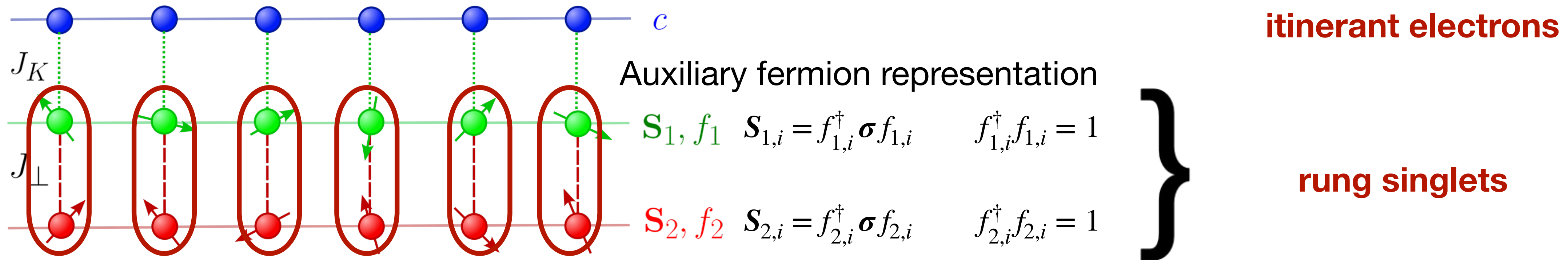
Nikolaenko et al., PRB 103, 235138 (2021)

$$U = \frac{3J_K^2}{8J_\perp} + \frac{3J_K^3}{16J_\perp^2} + \mathcal{O}(1/J_\perp^3)$$

$$J = \frac{J_K^2 (J_1 + J_2)}{4J_\perp^2} + \mathcal{O}(1/J_\perp^3)$$

FL phase

Zhang & Sachdev, PRR **2**, 023172 (2020)



U(1) symmetry

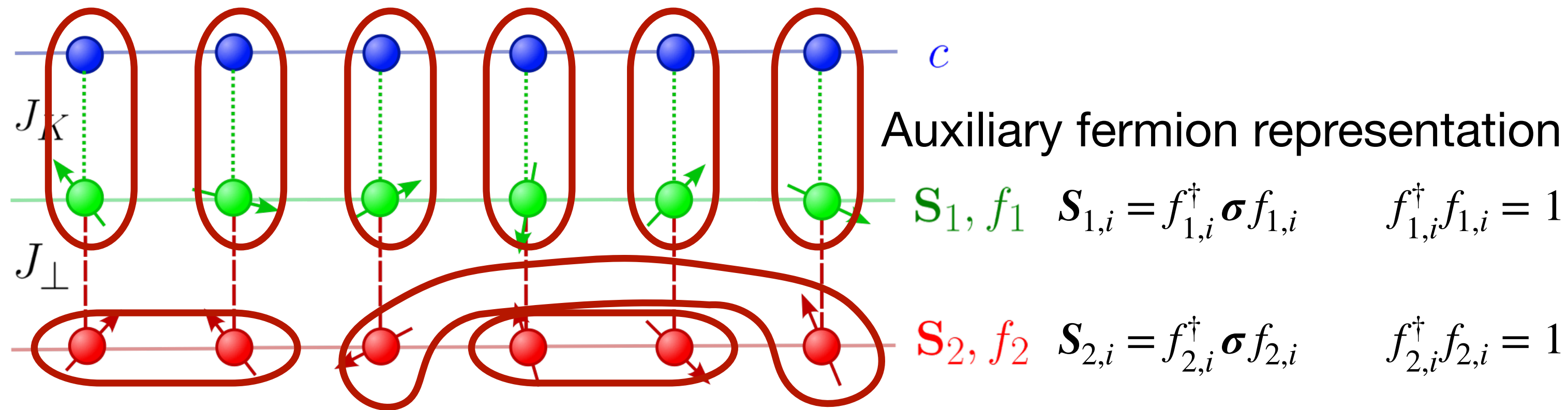
$$c \rightarrow e^{i\alpha} c$$

$$f_{1,2} \rightarrow f_{1,2}$$

$$V_{\text{FS}} = 1 + p$$

FL* phase

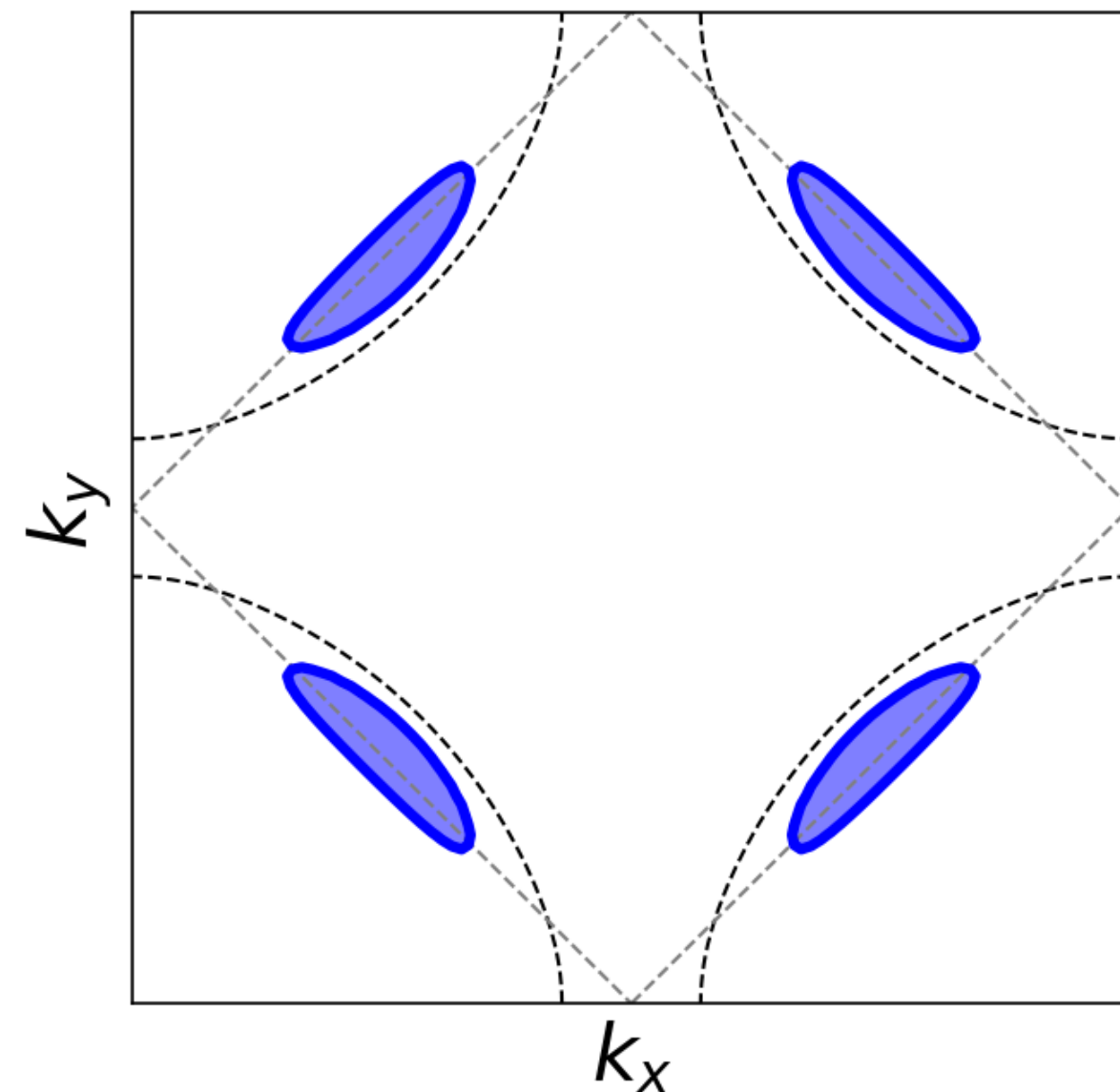
Zhang & Sachdev, PRR 2, 023172 (2020)



}

Hybridized $\phi \sim c_i^\dagger f_i$

spin liquid



U(1) symmetry

$$\langle c^\dagger f_1 \rangle \neq 0$$

$$c \rightarrow e^{i\alpha} c$$

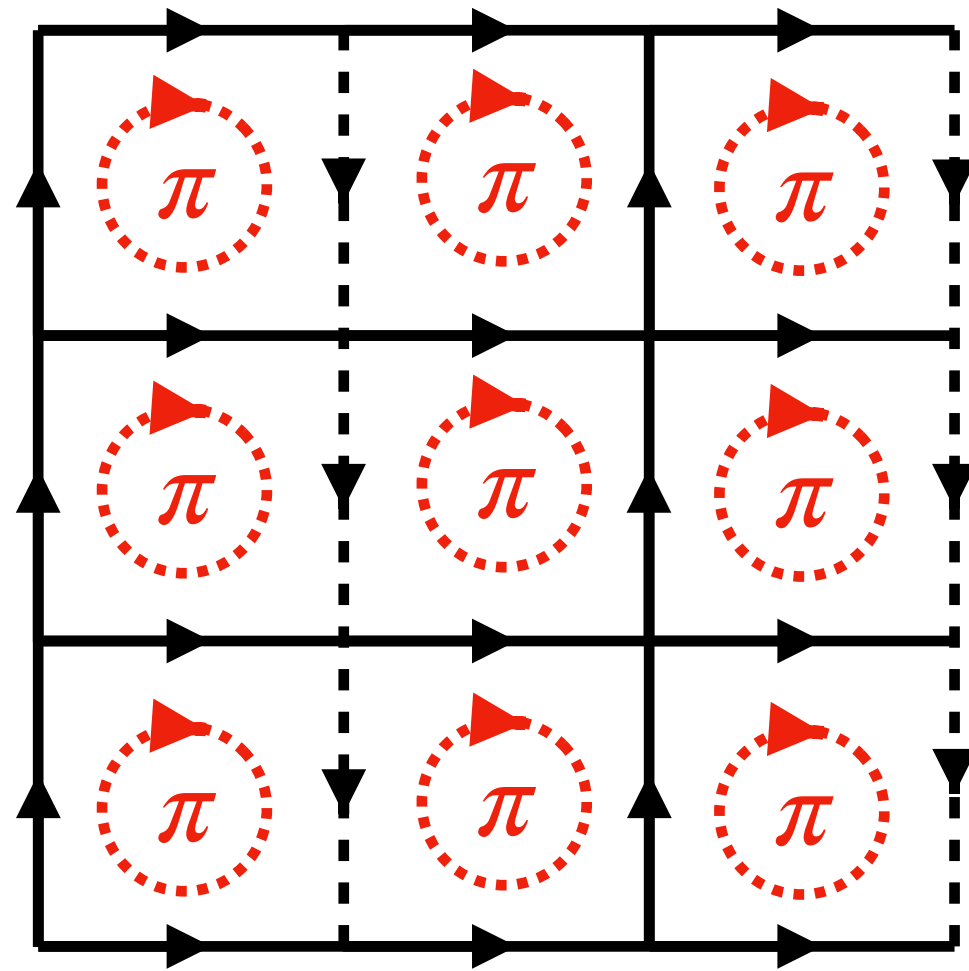
$$f_1 \rightarrow e^{i\alpha} f_1$$

$$f_2 \rightarrow f_2$$

$$V_{\text{FS}} = (1 + p + 1) \bmod 2 = p$$

π -Flux Spin Liquid

Affleck & Marston, PRB **37**, 3774(R) (1988)



$$\psi_i = (f_{2,i,\uparrow}, f_{2,i,\downarrow})^T$$

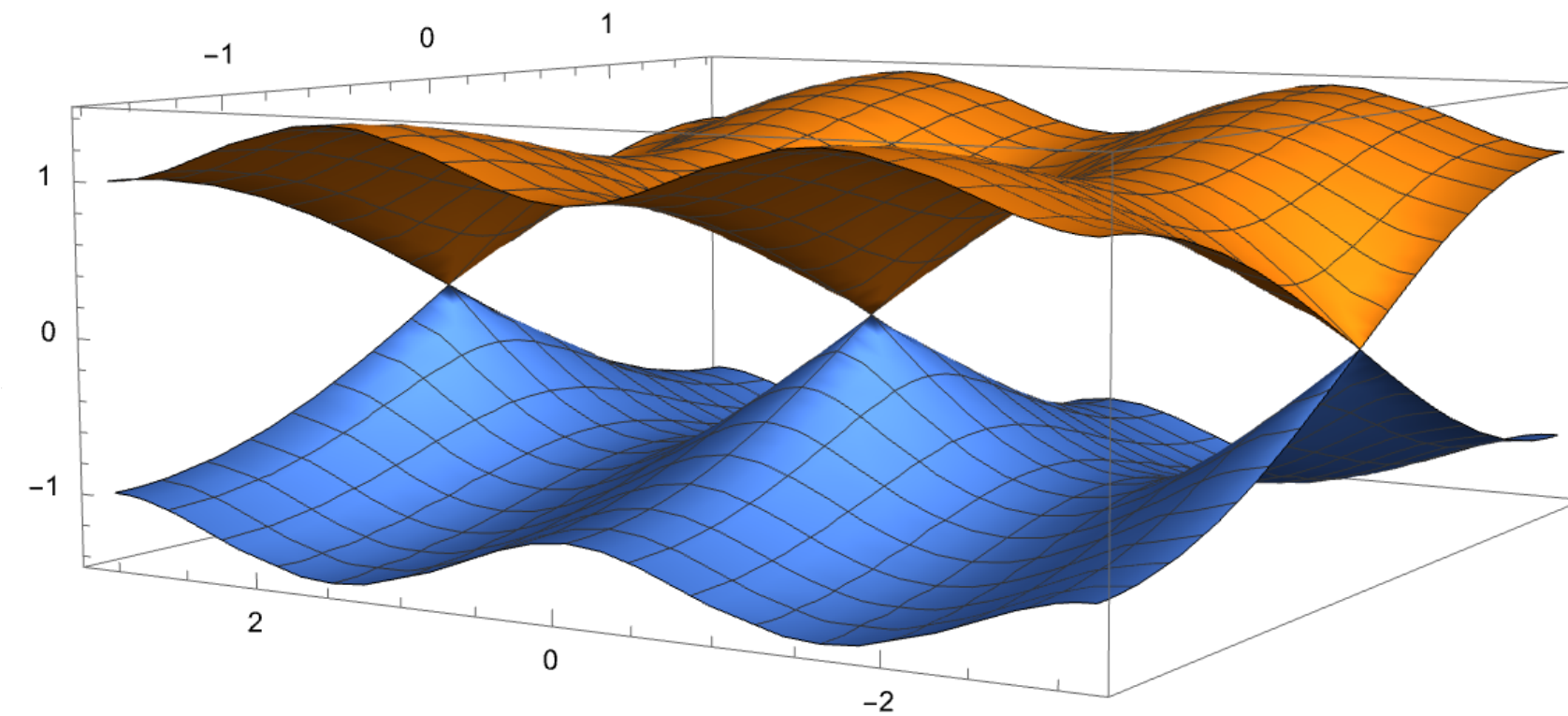
$$\mathcal{H}_{\pi\text{-flux}}^{\text{QSL}} = J \sum_{\langle i,j \rangle} \psi_i^\dagger e_{ij} U_{ij} \psi_j$$

$$e_{i,i\pm\hat{x}} = \pm i \quad e_{i,i\pm\hat{y}} = \pm i(-1)^x$$

$$U_{ij} \sim e^{-i \sum_{a=1}^3 (\mathbf{r}_i - \mathbf{r}_j) \cdot \mathbf{A}_{(i+j)/2}^a} \frac{\sigma^a}{2} \in \text{SU}(2) \quad \text{link field}$$

SU(2) gauge invariance!

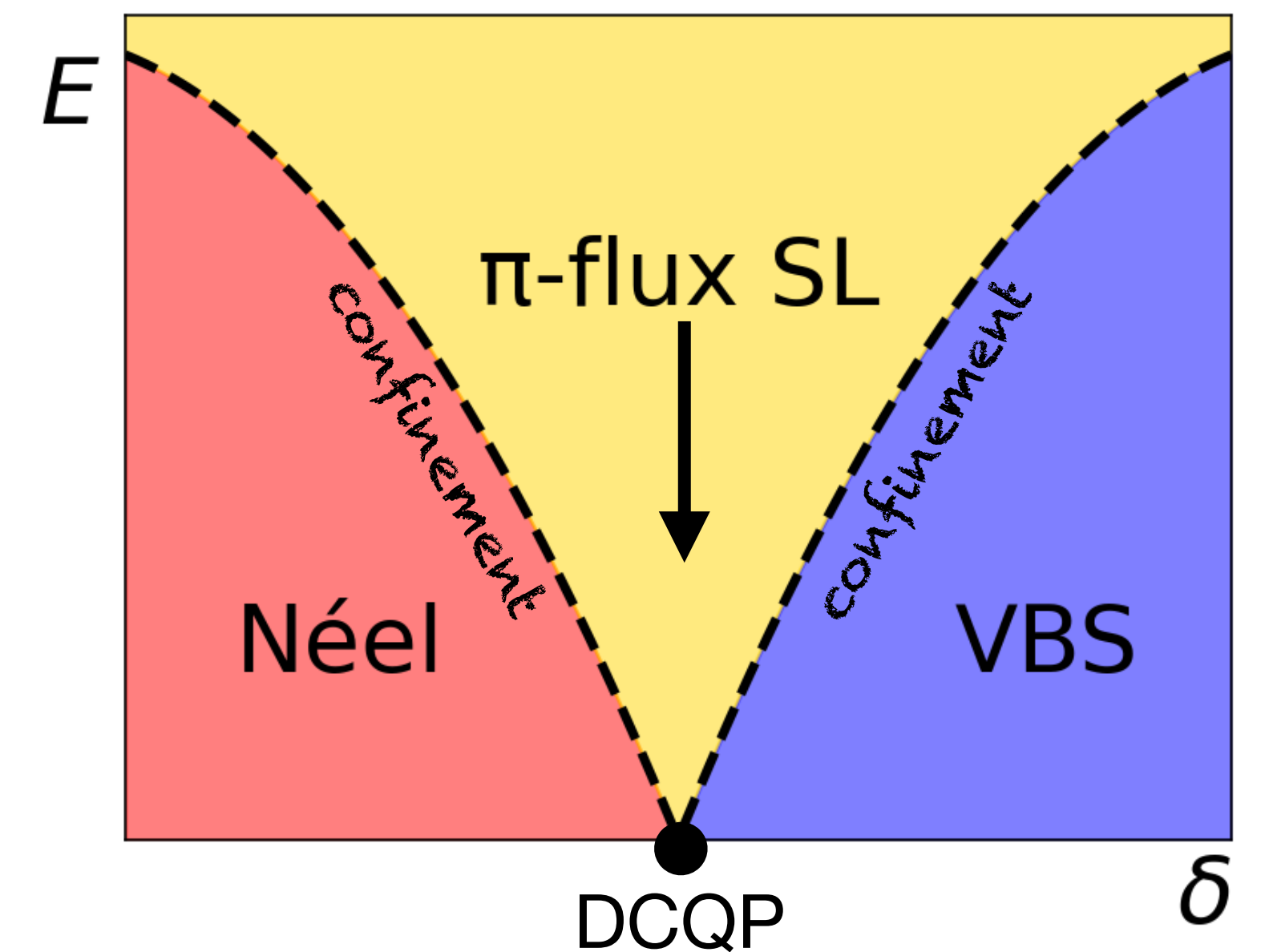
$$E_k^\pm = \pm 2J \sqrt{(\sin k_x)^2 + (\sin k_y)^2}$$



2 Dirac cones

Stable at intermediate energy scales,
eventually unstable to Néel or VBS order

C. Wang *et al.*, Phys. Rev. X **7**,
031051 (2017)



Bosonic Chargons Theory

Christos *et al.*, PNAS **120**, e2302701120 (2023)

Decouple $-J_{\perp} \sum_i \mathbf{S}_{1,i} \cdot \mathbf{S}_{2,i} \Rightarrow$ complex boson field $(B_{1,i}, B_{2,i})^T$

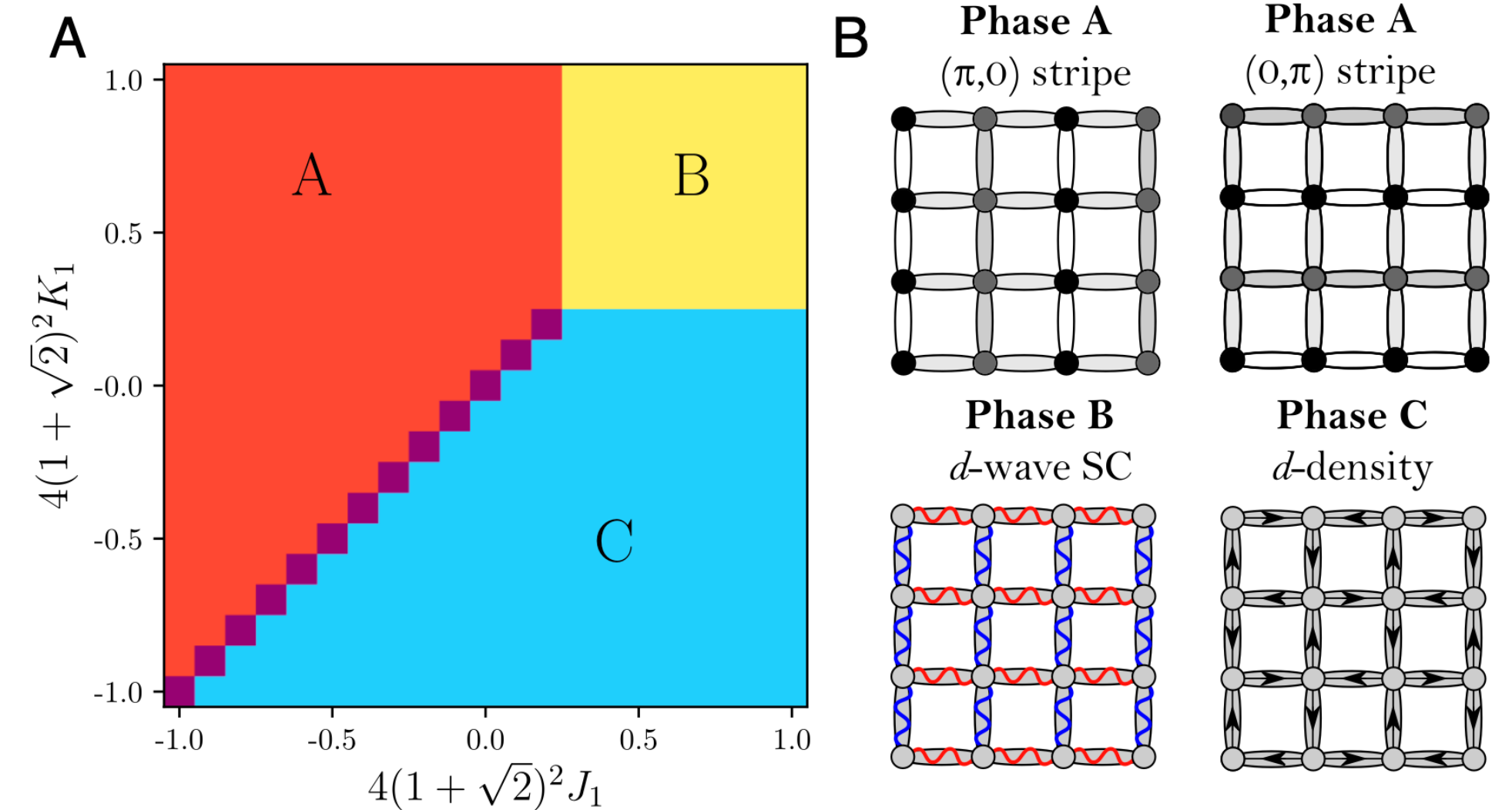
$$B_1 \sim f_{1,\sigma}^{\dagger} f_{2,\sigma}, \quad B_2 \sim f_{1,\uparrow}^{\dagger} f_{2,\downarrow}^{\dagger} - f_{1,\downarrow}^{\dagger} f_{2,\uparrow}^{\dagger}$$

- EM charge e (f_1 has charge e in FL* phase)
- no **SU(2) spin**
- $\in$ fundamental of **SU(2)** $\Rightarrow$ **π -flux** background

Gauge invariant bilinears

e_{ij} : π -flux hoppings

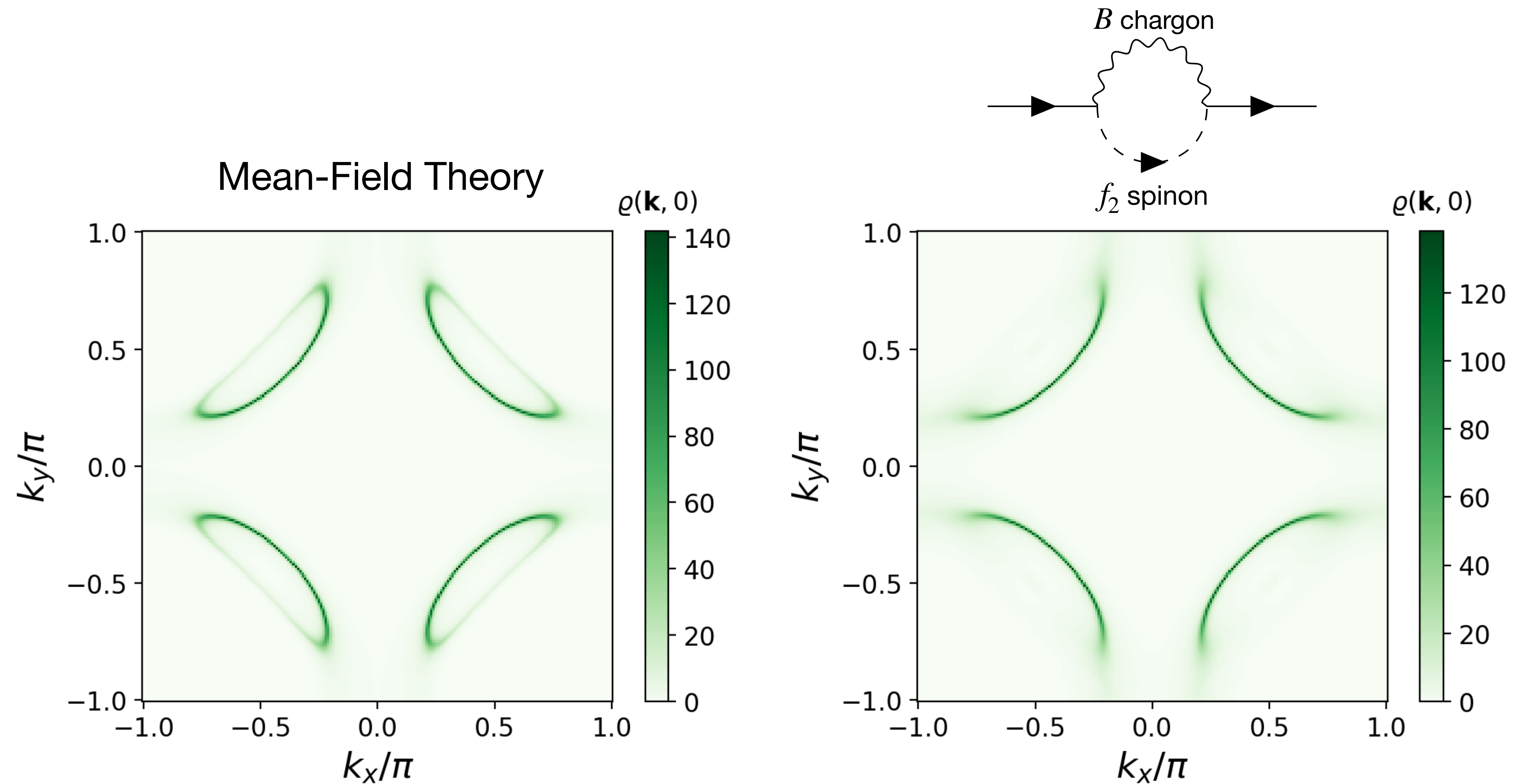
- $\rho_i = B_i^{\dagger} B_i \sim c_i^{\dagger} c_i \Rightarrow$ charge density OP
- $Q_{ij} = \text{Re} \left\{ B_i^{\dagger} e_{ij} U_{ij} B_j \right\} \sim c_i^{\dagger} c_j + c_j^{\dagger} c_i \Rightarrow$ bond density OP
- $J_{ij} = \text{Im} \left\{ B_i^{\dagger} e_{ij} U_{ij} B_j \right\} \sim i(c_i^{\dagger} c_j - c_j^{\dagger} c_i) \Rightarrow$ current density OP
- $\Delta_{ij} = B_i e_{ij} [i\sigma^2] U_{ij} B_j \sim c_i [i\sigma^2] c_j \Rightarrow$ superconducting OP



Christos *et al.*, PNAS (2023)

condensation of $B_i \Rightarrow$ confines the SU(2) gauge field + additional orders

Beyond Mean-Field Theory



Pandey, Christos, [PMB](#), Shanker, Nikolaenko, Sharma, Sachdev, arXiv:2507.05336 (2025)

Thermal Bosonic Chargons

Pandey, Christos, [PMB](#), Shanker, Nikolaenko, Sharma, Sachdev, arXiv:2507.05336 (2025)

Consider effective classical theory for the chargons at finite temperature T

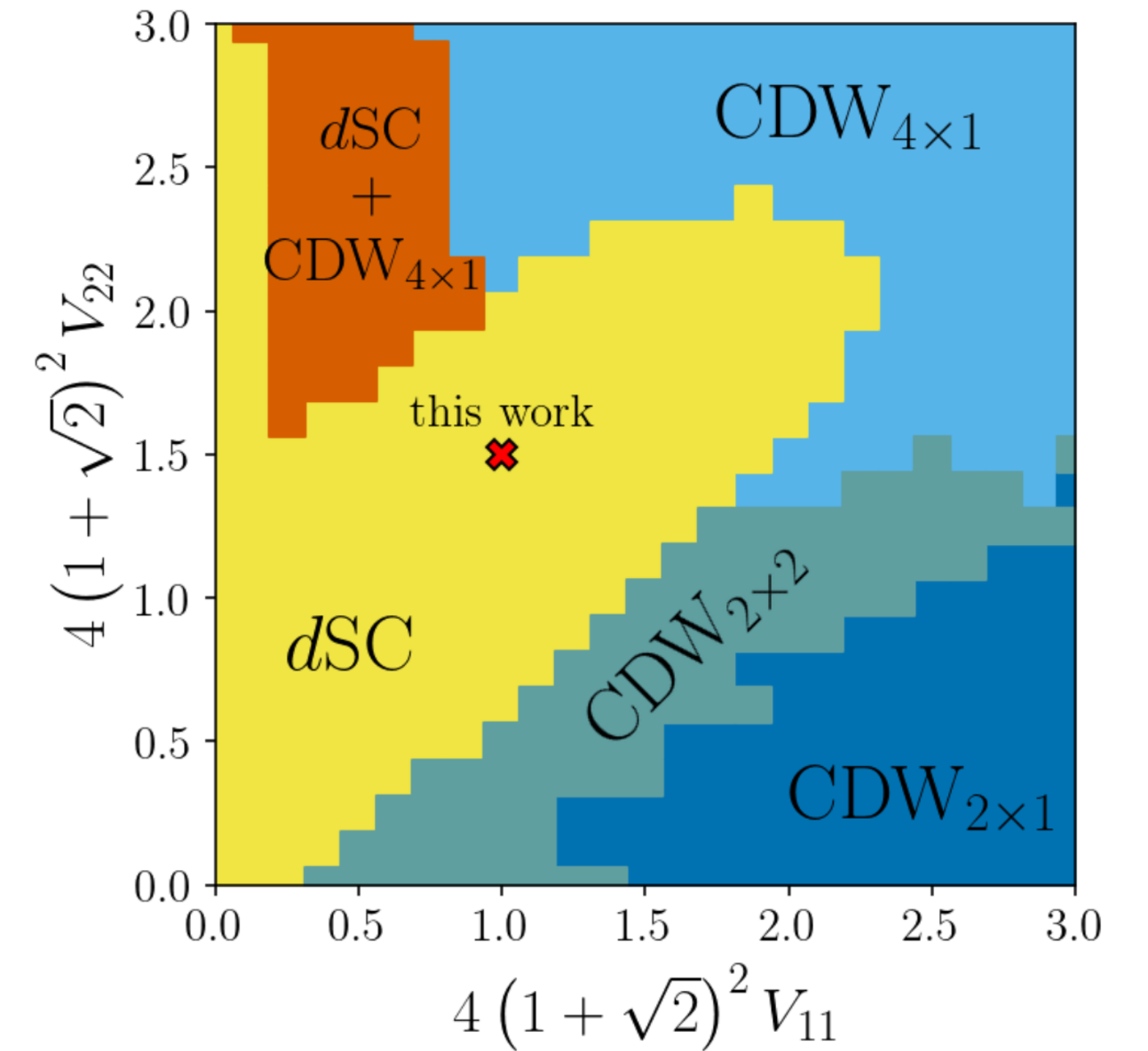
SU(2) gauge field

π -flux hoppings

$$\mathcal{E}_2[B, U] = \kappa \sum_{\square} \left[1 - \frac{1}{2} \text{ReTr} \prod_{i,j \in \square} U_{ij} \right] + r \sum_i B_i^\dagger B_i - iw \sum_{\langle i,j \rangle} B_i^\dagger e_{ij} U_{ij} B_j$$

$$\begin{aligned} \mathcal{E}_4[B, U] = & \frac{u}{2} \sum_i \rho_i^2 + V_1 \sum_i \rho_i (\rho_{i+\hat{x}} + \rho_{i+\hat{y}}) + g \sum_{\langle i,j \rangle} |\Delta_{ij}|^2 + J_1 \sum_{\langle i,j \rangle} Q_{ij}^2 \\ & + K_1 \sum_{\langle ij \rangle} J_{ij}^2 + V_{11} \sum_i \rho_i (\rho_{i+\hat{x}+\hat{y}} + \rho_{i+\hat{x}-\hat{y}}) \\ & + V_{22} \sum_i \rho_i (\rho_{i+2\hat{x}+2\hat{y}} + \rho_{i+2\hat{x}-2\hat{y}}) \end{aligned}$$

MF phase diagram



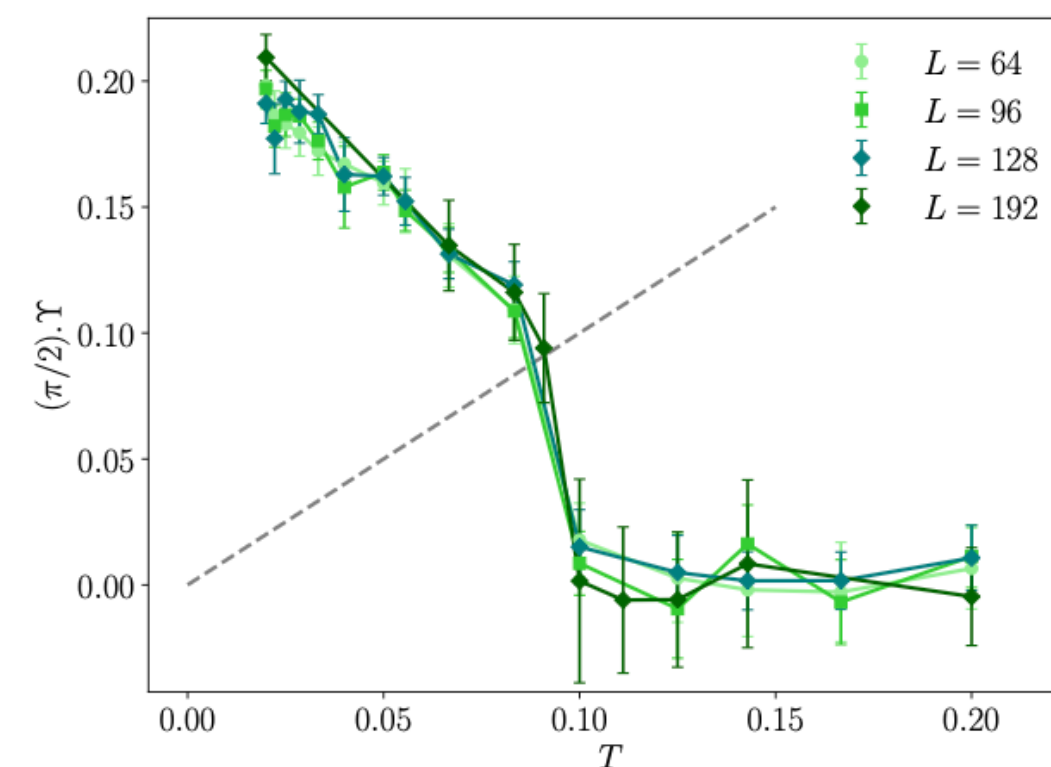
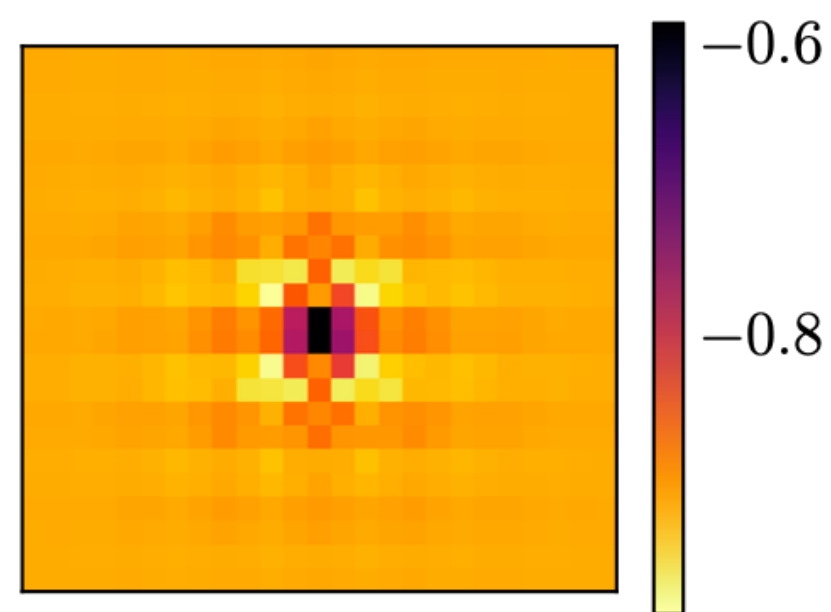
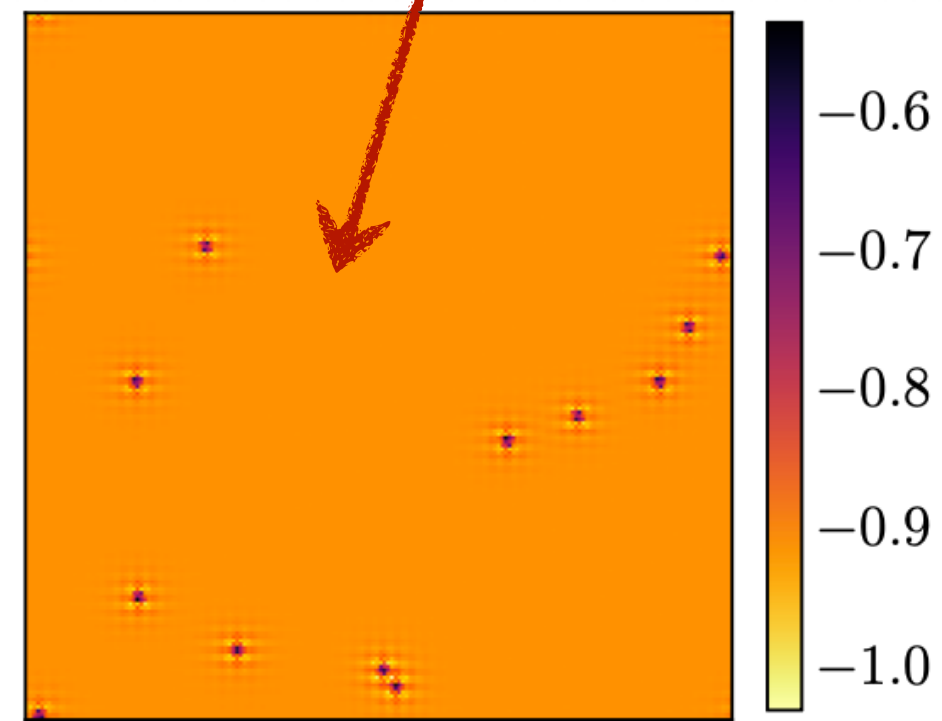
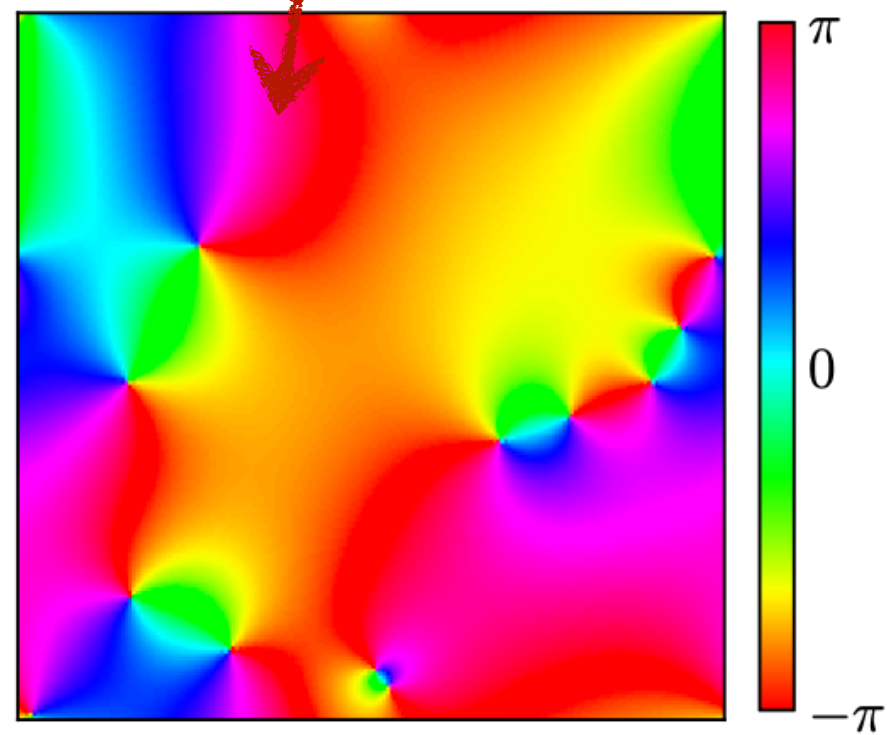
$$r = -0.732, w = 0.40, u = 0, V_1 = 0, g = 0.02, \\ J_1 = K_1 = 2/(1+\sqrt{2})^2$$

KT transition

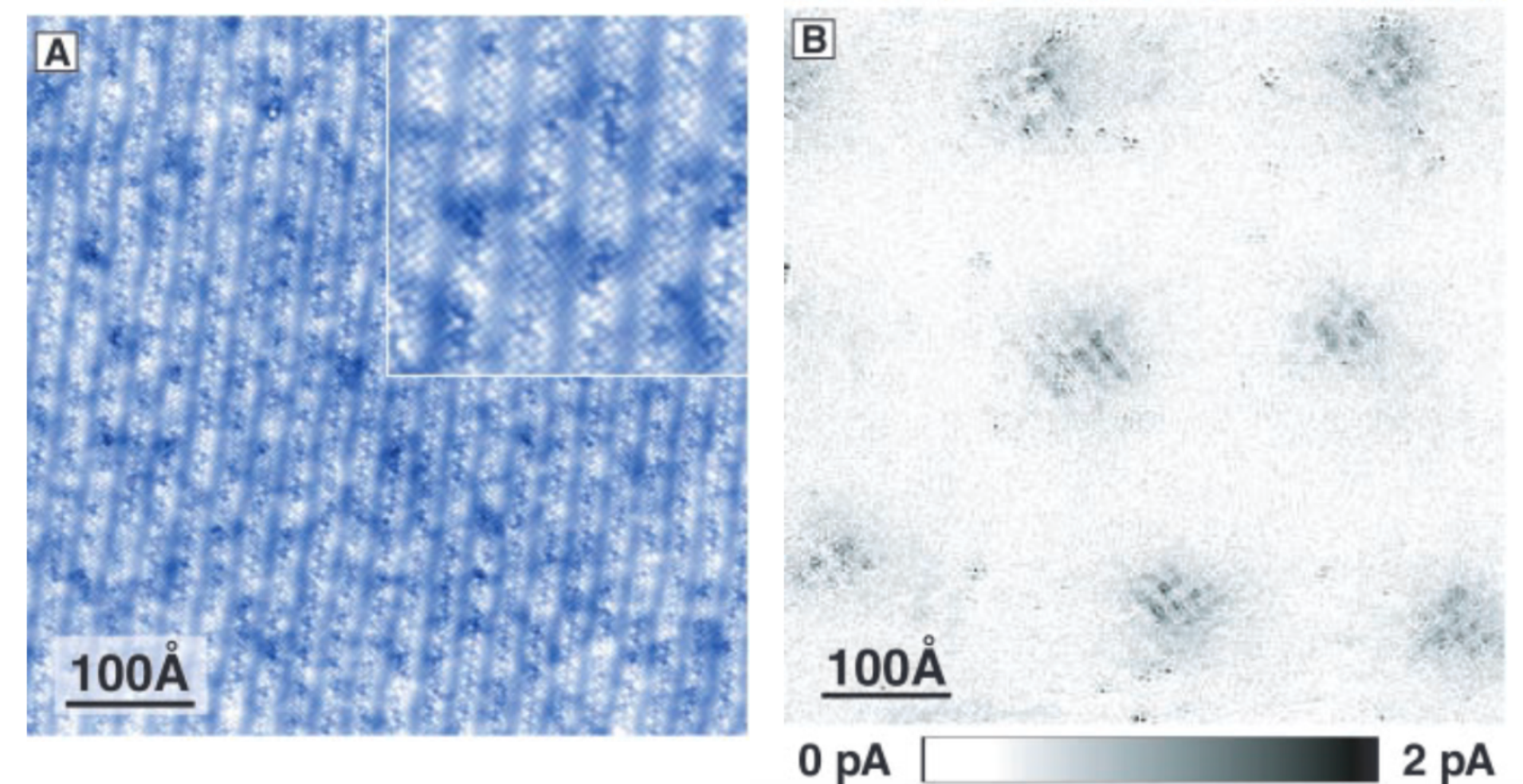
Pandey, Christos, [PMB](#), Shanker, Nikolaenko, Sharma, Sachdev, arXiv:2507.05336 (2025)

$$\arg \Delta_{ij} = \arg \left\{ B_i \epsilon U_{ij} B_j \right\}$$

$$Q_{ij} = \text{Re} \left\{ B_i^\dagger e_{ij} U_{ij} B_j \right\}$$



- charge- e OP **but** $\frac{h}{2e}$ vortices
- period-4 CDW in the vortex cores



Hoffman *et al.*, Science **295**, 466 (2002)

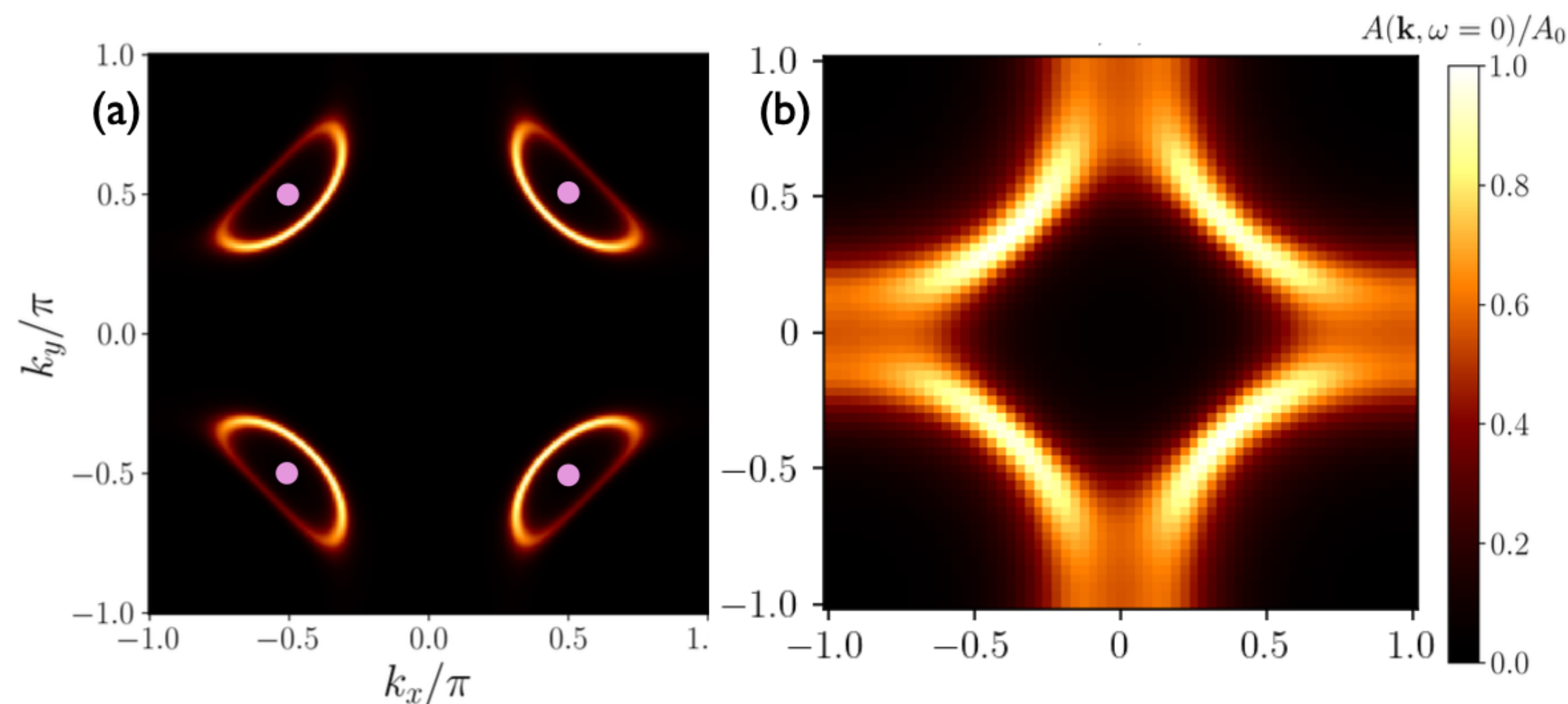
Fermi Surface topology

Pandey, Christos, [PMB](#), Shanker, Nikolaenko, Sharma, Sachdev, arXiv:2507.05336 (2025)

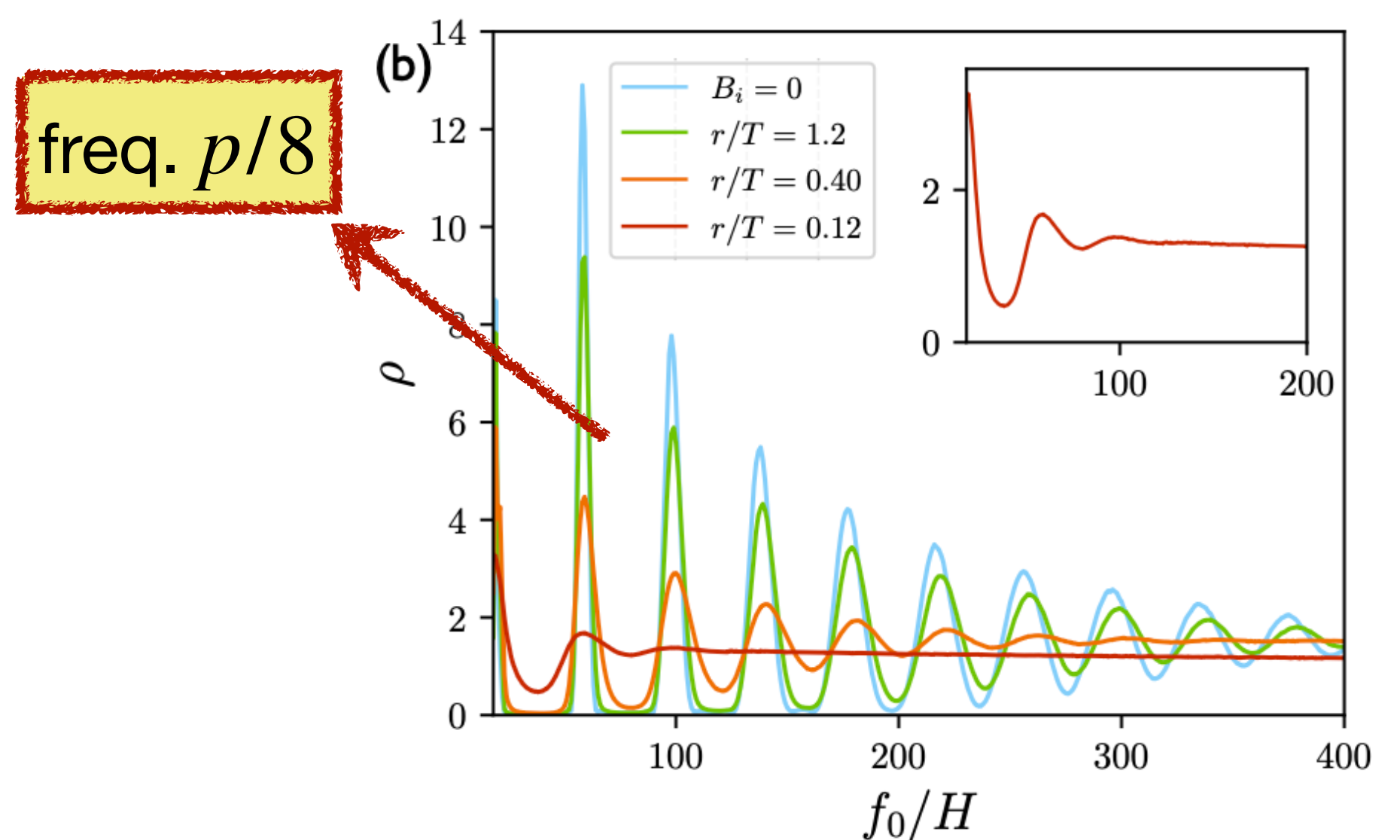
$$\mathcal{H} = - \sum_{i,j} t_{ij}^c c_i^\dagger c_j - \sum_{i,j} t_{ij}^f f_{1,i}^\dagger f_{1,j} + iJ \sum_{\langle i,j \rangle} f_{2,i}^\dagger e_{ij} U_{ij} f_{2,j} + g \sum_i \left[B_{1,i} f_{2,i}^\dagger f_{1,i} - B_{2,i} f_{2,i} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} f_{1,i} + \text{H.c.} \right]$$

"Born-Oppenheimer" approx. $\rightarrow$ sample U & B according to $e^{-\beta(\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U])}$ then average

MF vs BO spectral function



Quantum oscillations



Dynamical Spin Structure Factor

Nikolaenko, [PMB](#), Sachdev, arXiv:2511.03792 (2025)

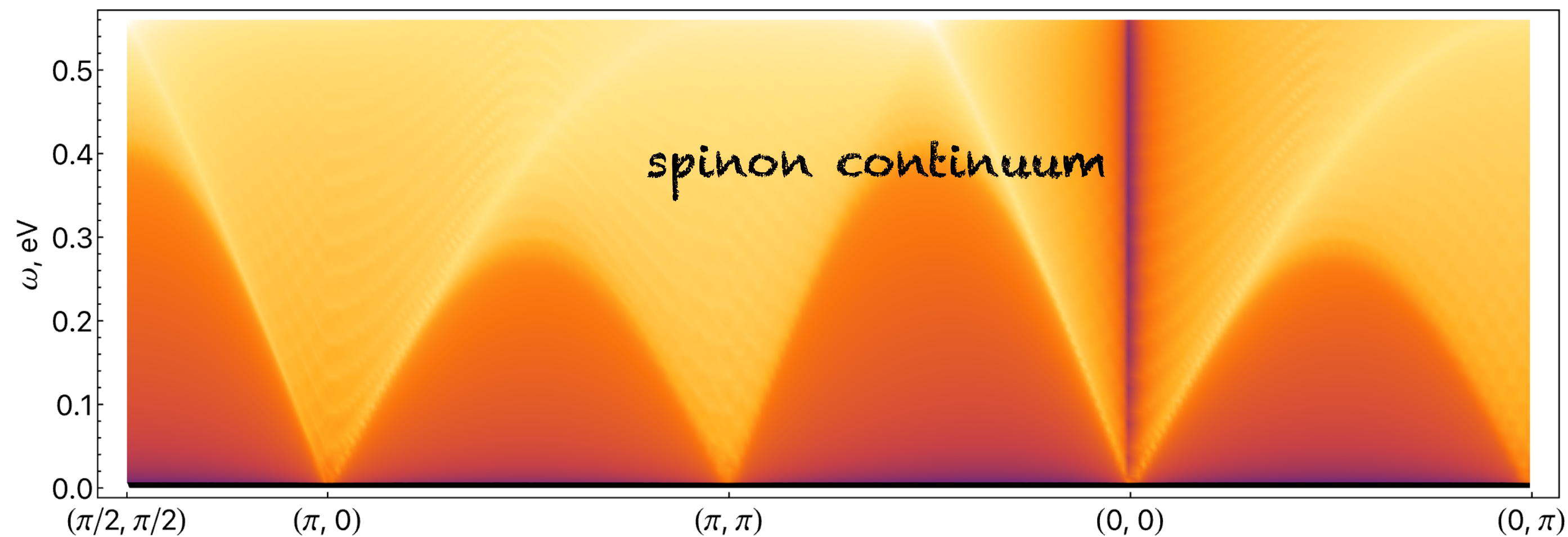
π -flux SL

$$\mathcal{H} = w \sum_{\langle i,j \rangle} (1 + \dots) \psi_i^\dagger e_{ij} \psi_j$$

bare π -flux susc.



$$\chi_{\text{spin}}(q, \omega) = \chi_0(q, \omega)$$



Dynamical Spin Structure Factor

Nikolaenko, [PMB](#), Sachdev, arXiv:2511.03792 (2025)

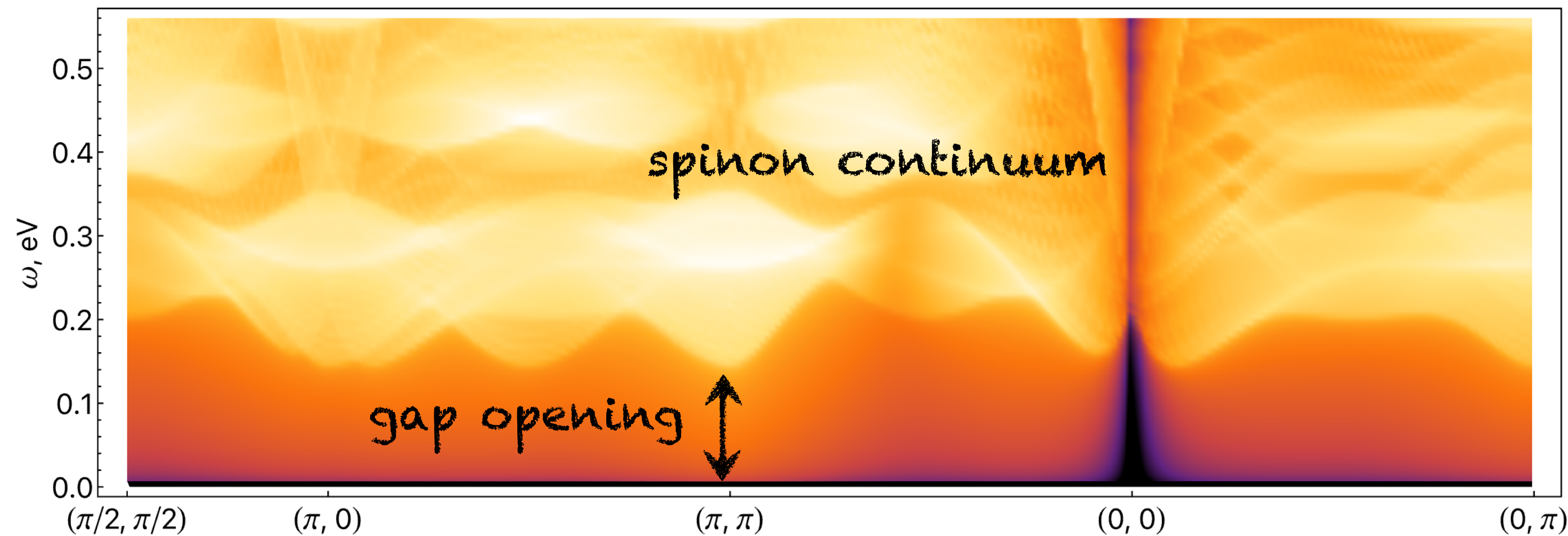
π -flux SL + period-4 CDW

$$\mathcal{H} = w \sum_{\langle i,j \rangle} (1 + Q_{ij}) \psi_i^\dagger e_{ij} \psi_j$$

bare π -flux + CDW susc.



$$\chi_{\text{spin}}(q, \omega) = \chi_0(q, \omega)$$



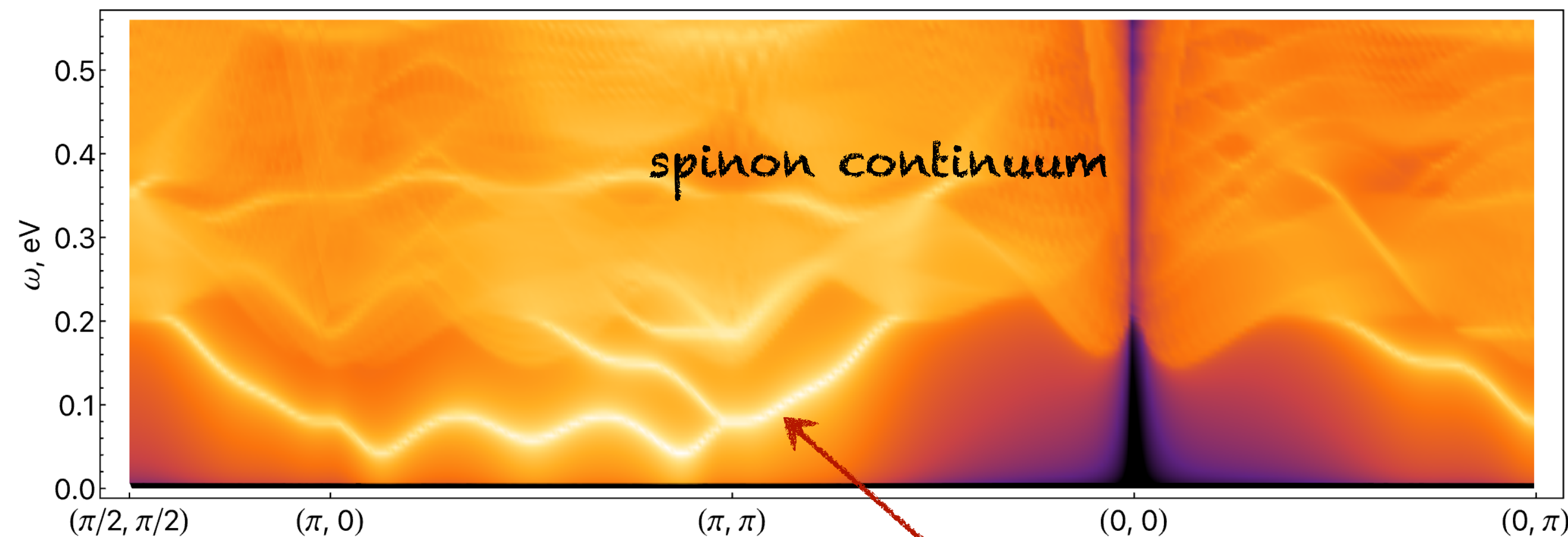
Dynamical Spin Structure Factor

Nikolaenko, [PMB](#), Sachdev, arXiv:2511.03792 (2025)

π -flux SL + period-4 CDW

$$\mathcal{H} = w \sum_{\langle i,j \rangle} (1 + Q_{ij}) \psi_i^\dagger e_{ij} \psi_j + \sum_{ij} J_{ij} (1 + Q_{ij}) (f_i^\dagger \boldsymbol{\sigma} f_i) \cdot (f_j^\dagger \boldsymbol{\sigma} f_j)$$

$$-\text{Im}\chi_{\text{spin}}(q, \omega)$$

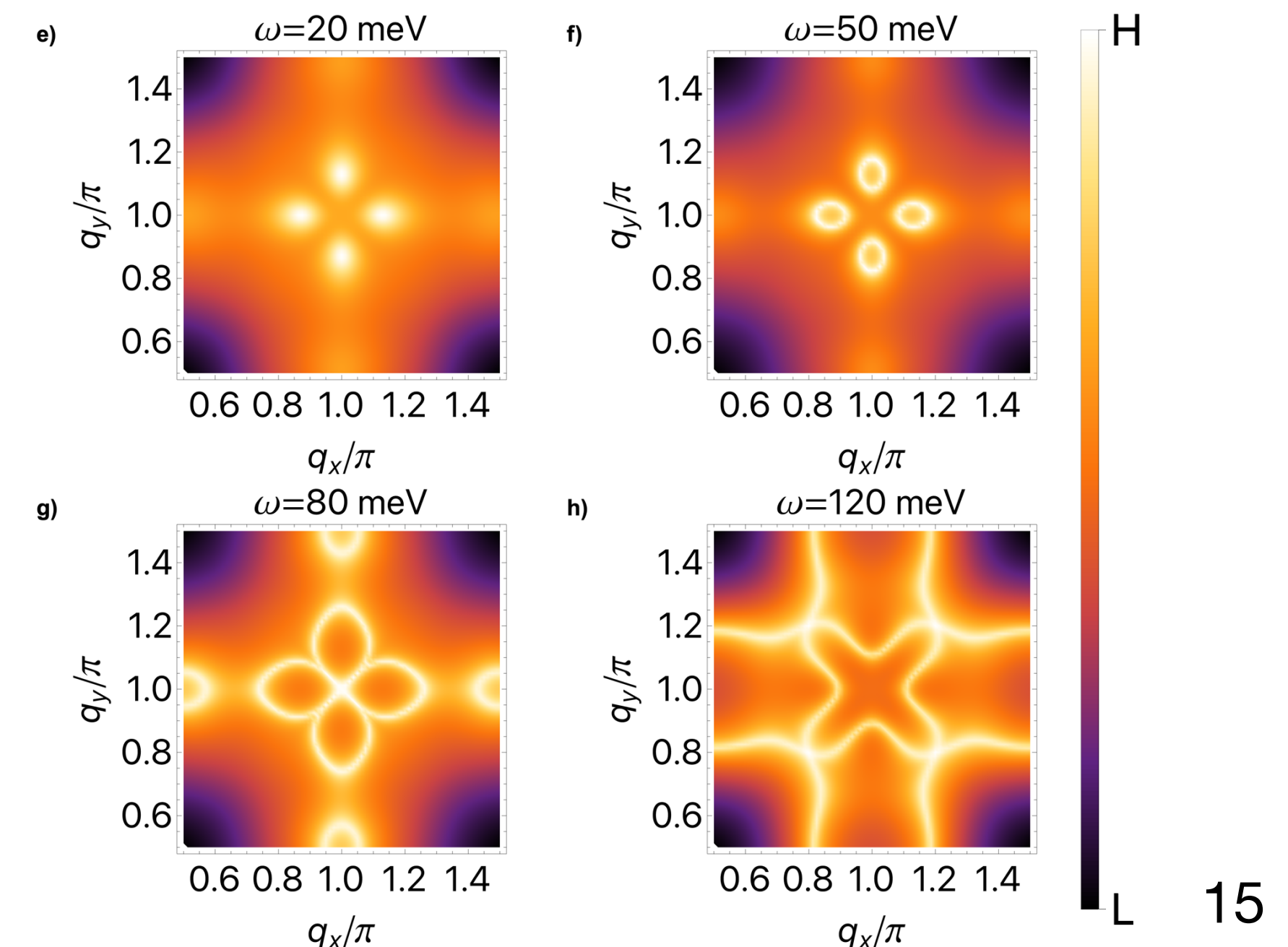


bare π -flux + CDW susc.

$$\chi_{\text{spin}}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - J(q)\chi_0(q, \omega)}$$

Willsher & Knolle, arXiv:2503.13831 (2025)

Hourglass spectrum



Dynamical Spin Structure Factor

Nikolaenko, [PMB](#), Sachdev, arXiv:2511.03792 (2025)

π -flux SL + period-4 CDW

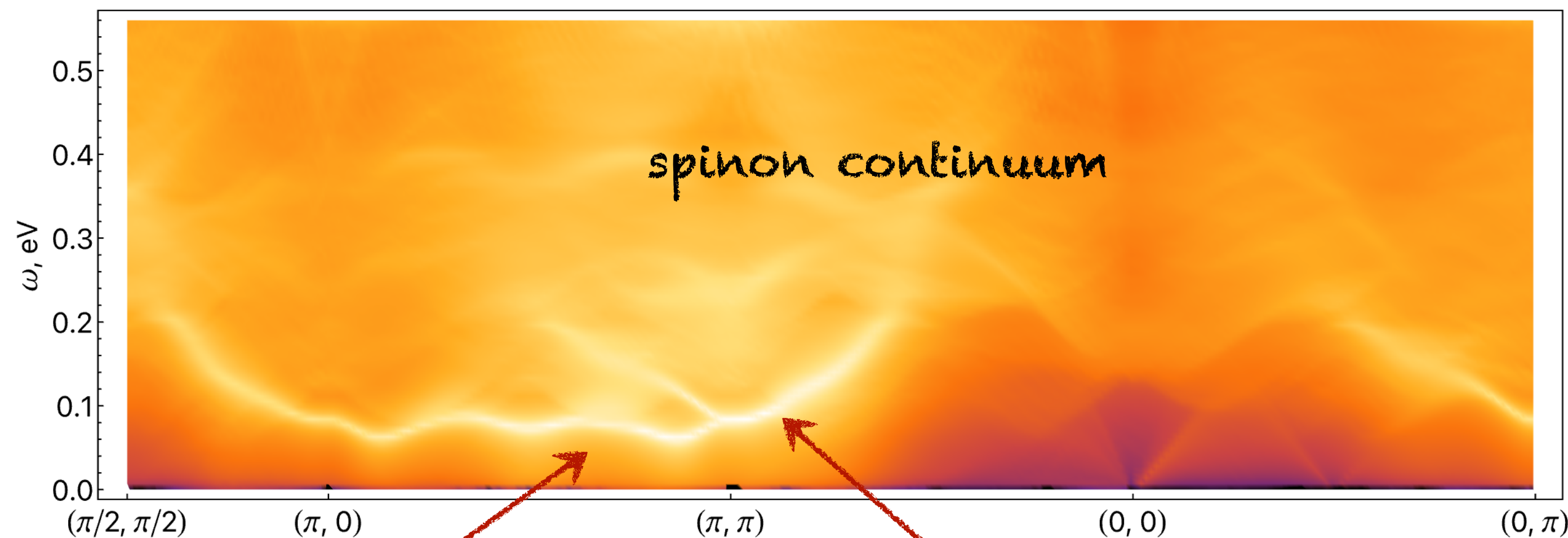
$$\mathcal{H} = w \sum_{\langle i,j \rangle} (1 + Q_{ij}) \psi_i^\dagger e_{ij} \psi_j + \sum_{ij} J_{ij} (1 + Q_{ij}) (f_i^\dagger \boldsymbol{\sigma} f_i) \cdot (f_j^\dagger \boldsymbol{\sigma} f_j) + \text{coupling to electrons}$$

bare π -flux + CDW +
electrons susc.

$$\chi_{\text{spin}}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - J(q)\chi_0(q, \omega)}$$

Willsher & Knolle, arXiv:2503.13831 (2025)

$-\text{Im}\chi_{\text{spin}}(q, \omega)$

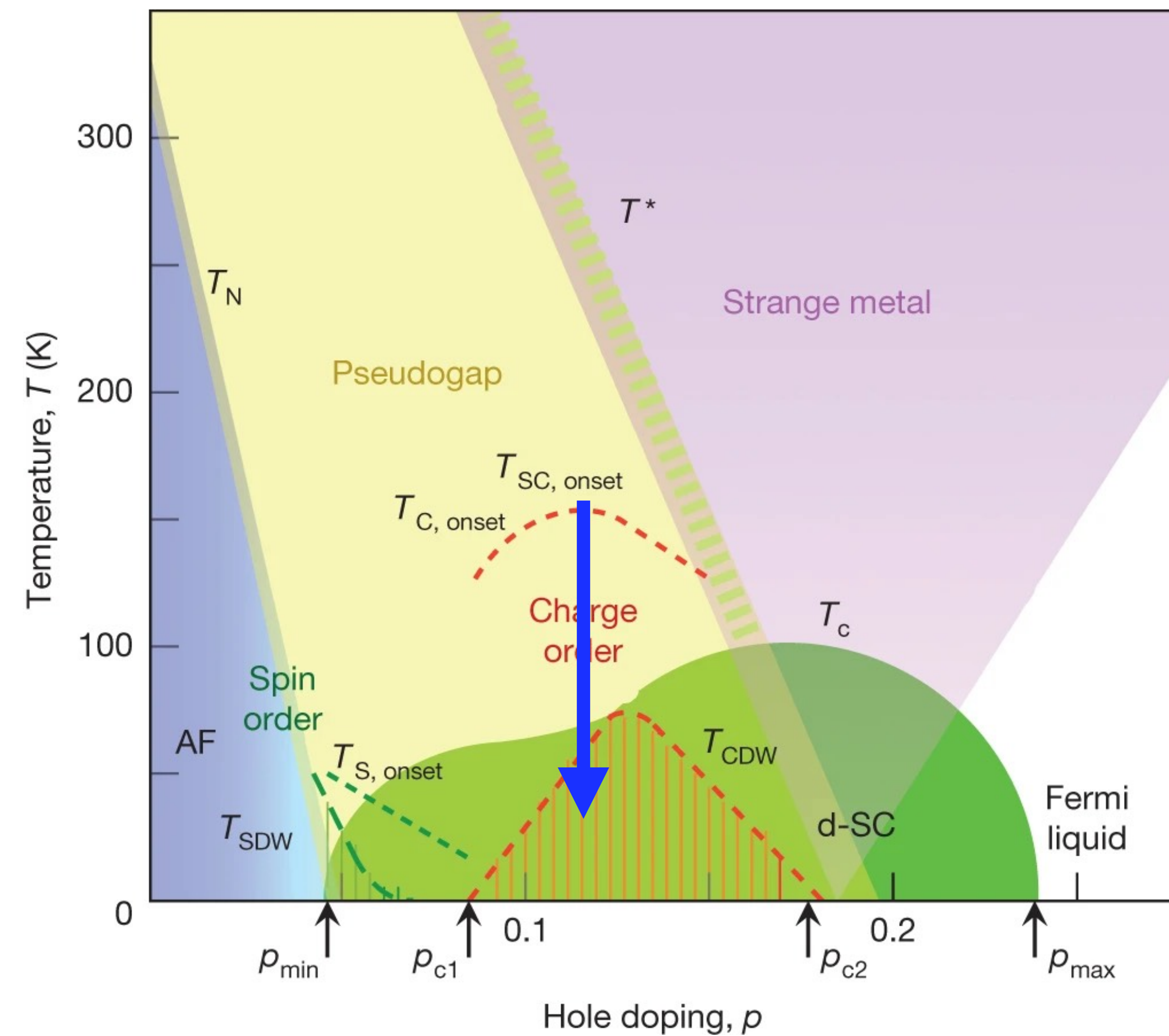


broadening of the
bound state dispersions

$S=1$ 2-spinon bound states

Low Temperature Measurements

Low Temperature Measurements

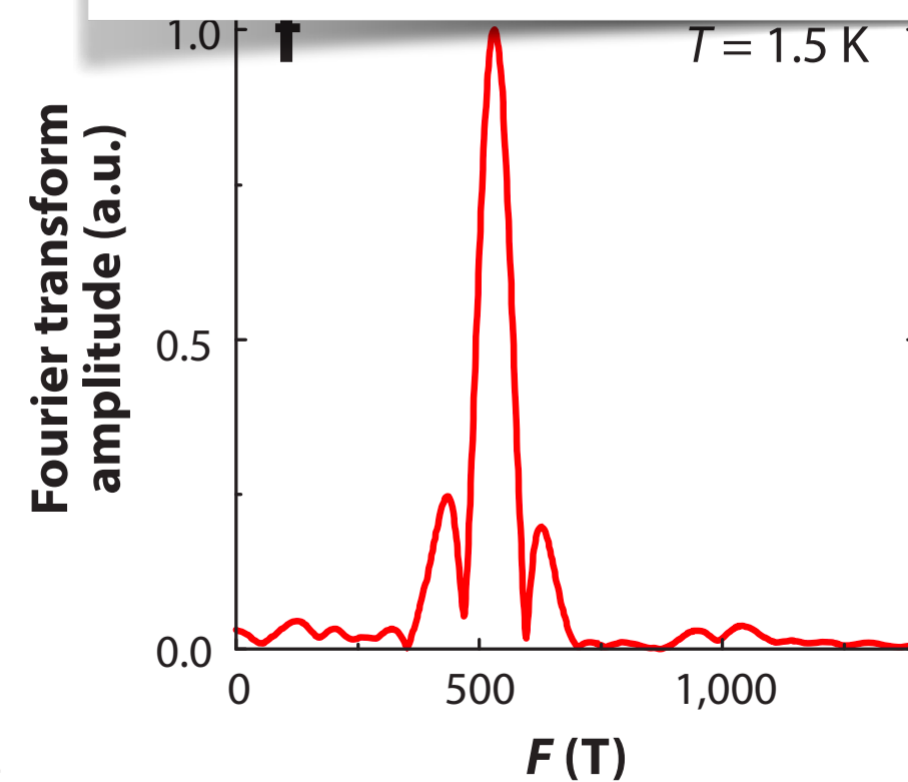
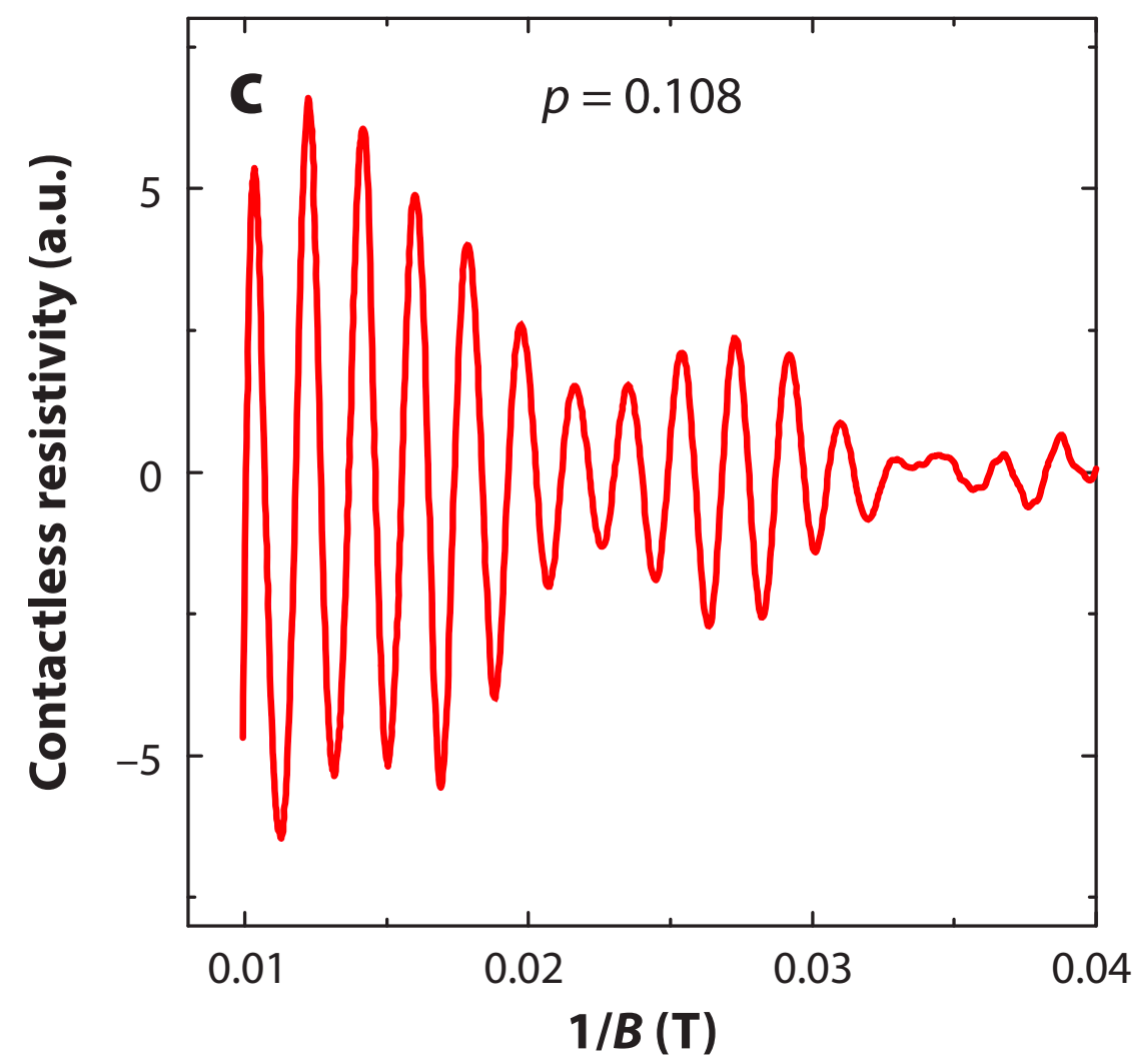


Keimer *et al.*, Nature **518**, 179 (2015)

Low Temperature Measurements

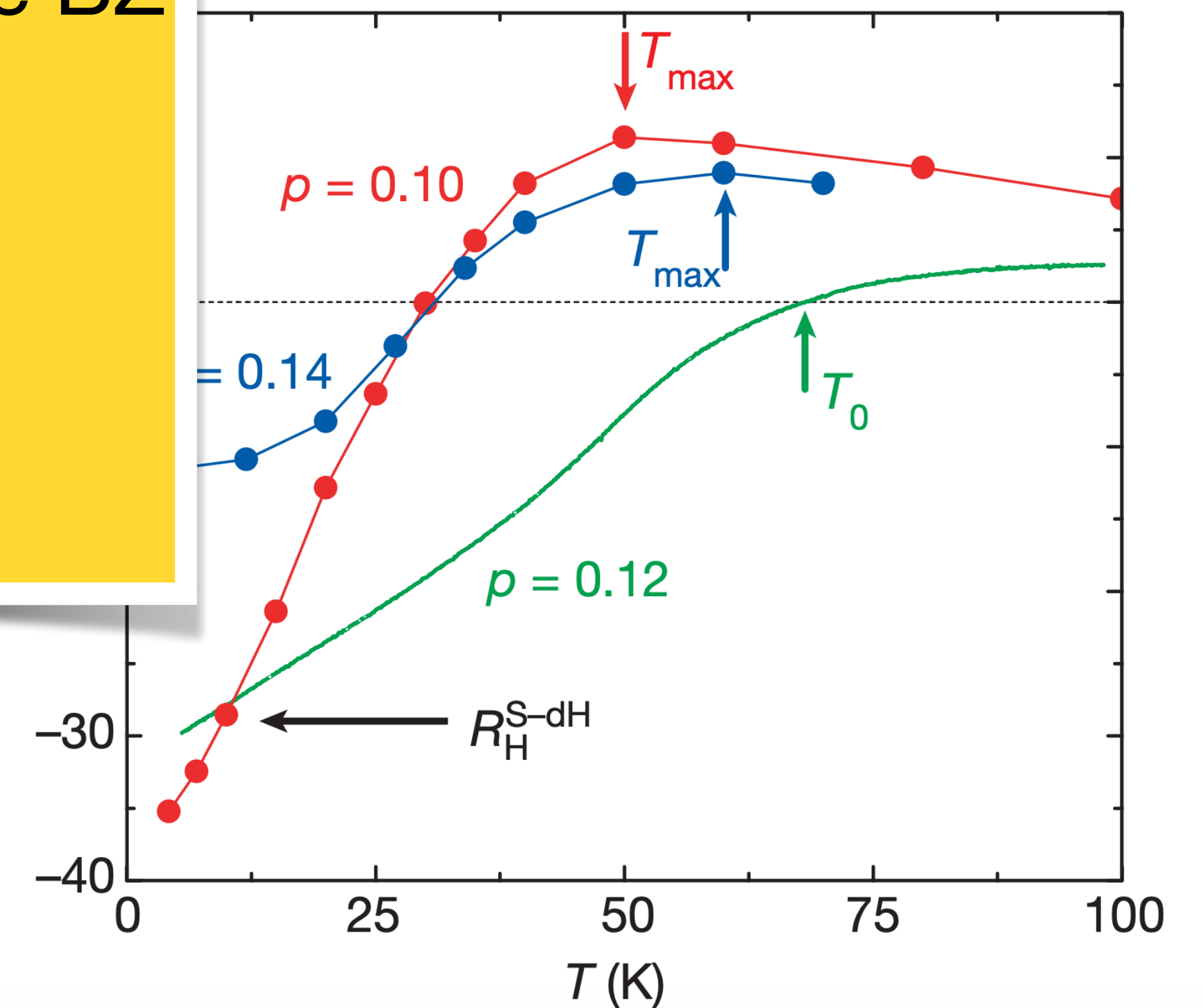
Quantum Oscillations

- oscillation frequency (in T section $f_H = \hbar A_{\perp} / (2\pi e)$)
- underdoped regime: sing frequency corresponding area



- Fermi Surface $\sim 2\%$ of the BZ
- Electron Pockets
- CDW order
- + FL* above T_{CDW}

Hall Resistance



LeBoeuf, *et al.*, Nature **450**, 533 (2007)

CDW* vs CDW

CDW*

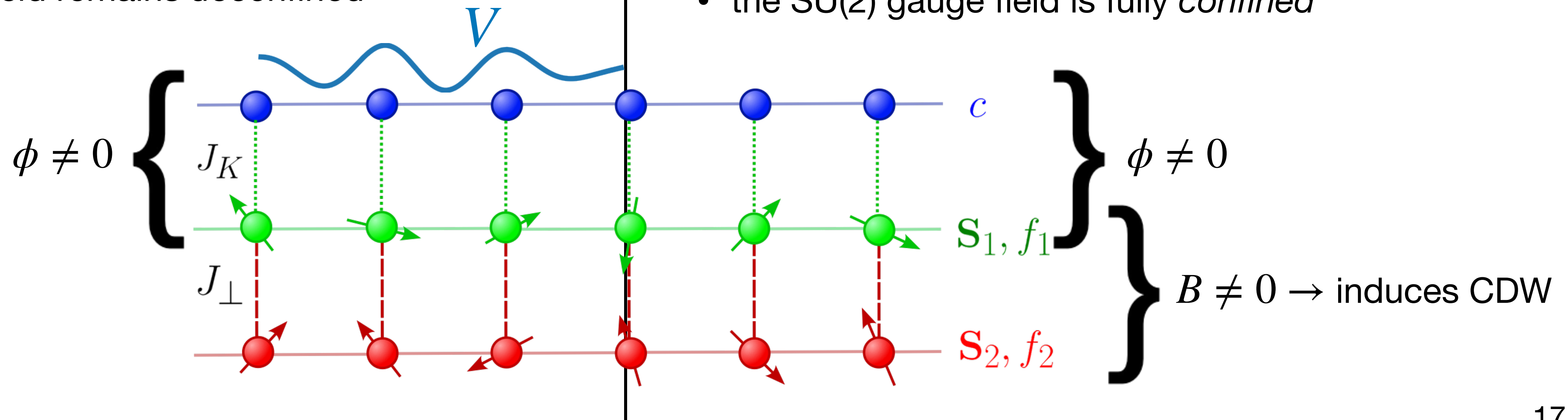
nonuniform density + deconfined SU(2) gauge field

- add modulated potential $V \sum_i [\cos(Qx) + \cos(Qy)] c_i^\dagger c_i$ to the physical electrons
- the SU(2) gauge field remains *deconfined*

CDW

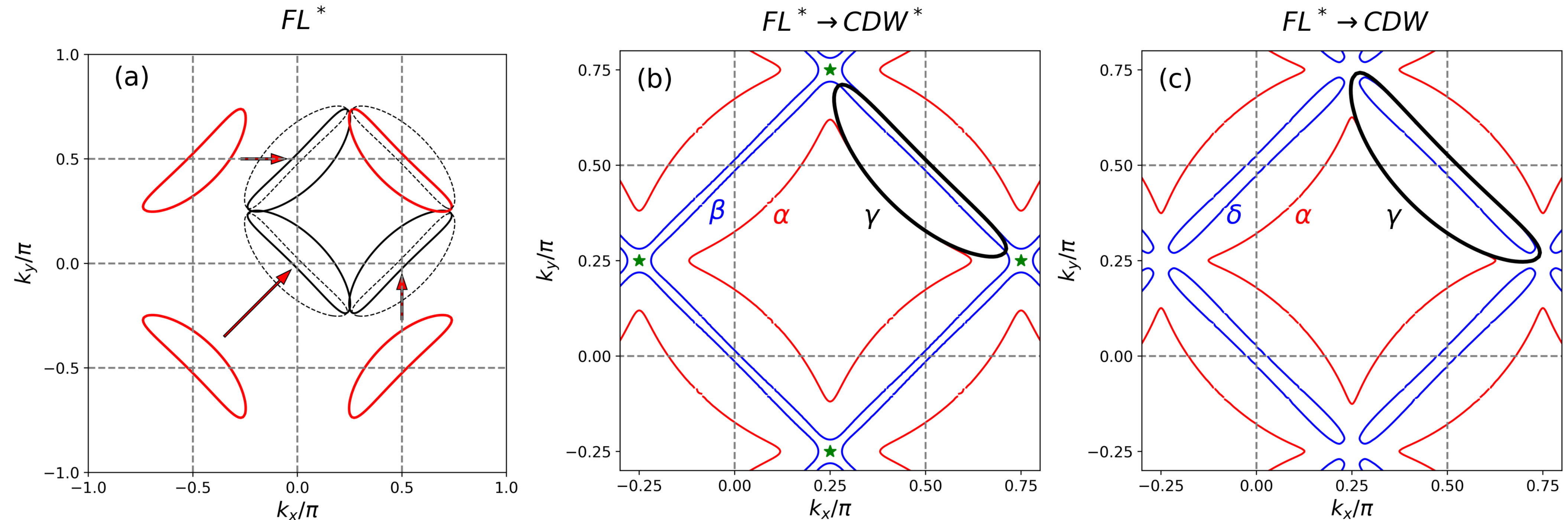
nonuniform density + confined SU(2) gauge field

- condensation of B_i such that
 $\rho_i = B_i^\dagger B_i = |b|^2 \left(1 - [(-1)^x + (-1)^y]/2\right) \cos^2(qx) \cos^2(qy)$,
 $J_{ij} = \Delta_{ij} = 0$
- the SU(2) gauge field is fully *confined*



Fermi surfaces in the CDW phase

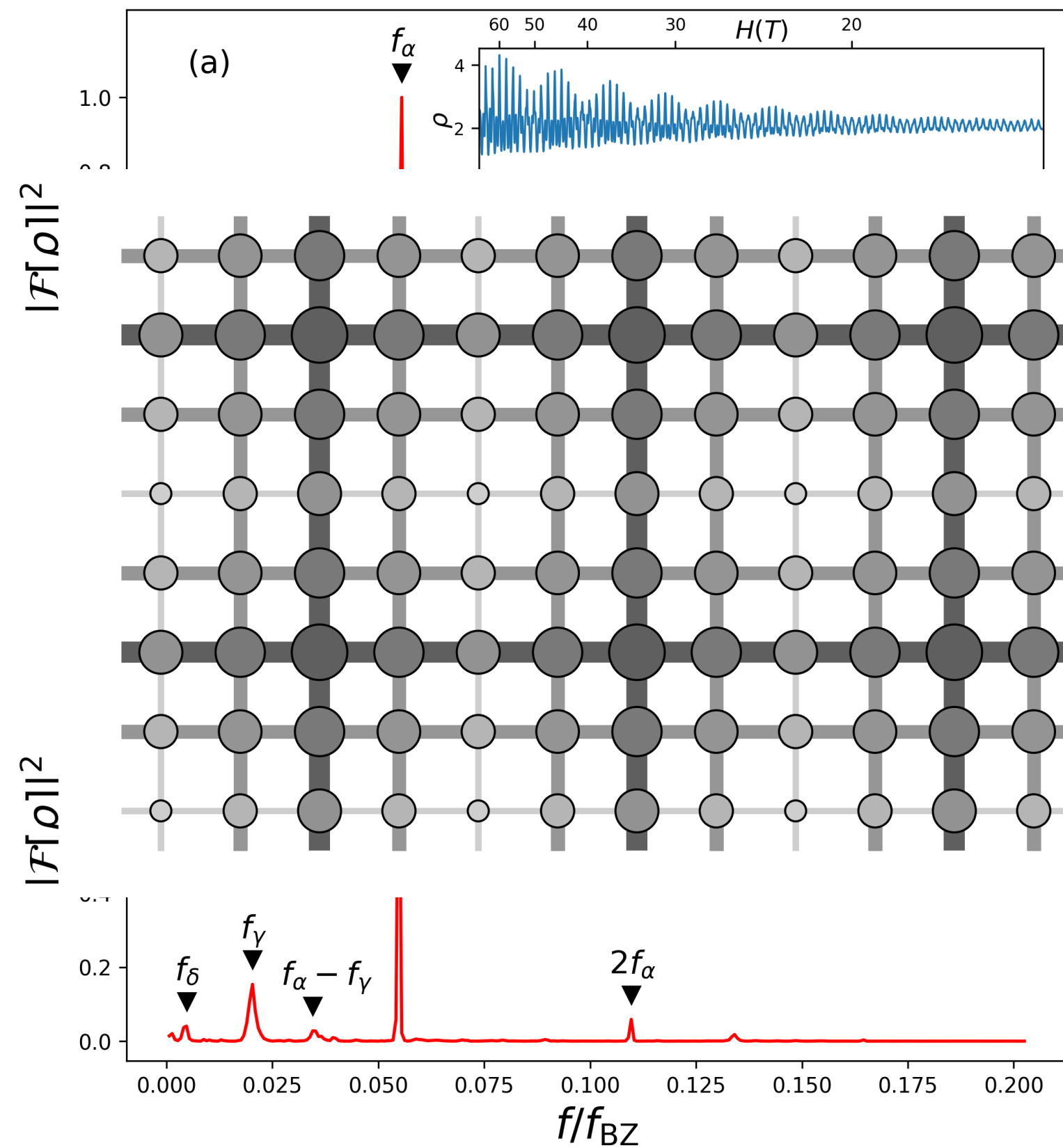
[PMB, Christos, Sachdev, PNAS 121 \(50\), e2418633121 \(2024\)](#)



- CDW^* gives two electron pockets, one made by the physical electrons, one made by the f_1 -spinons
- the condensation of B_i hybridizes the larger electron pocket with its shifted copies and gives 2 small, elongated hole pockets

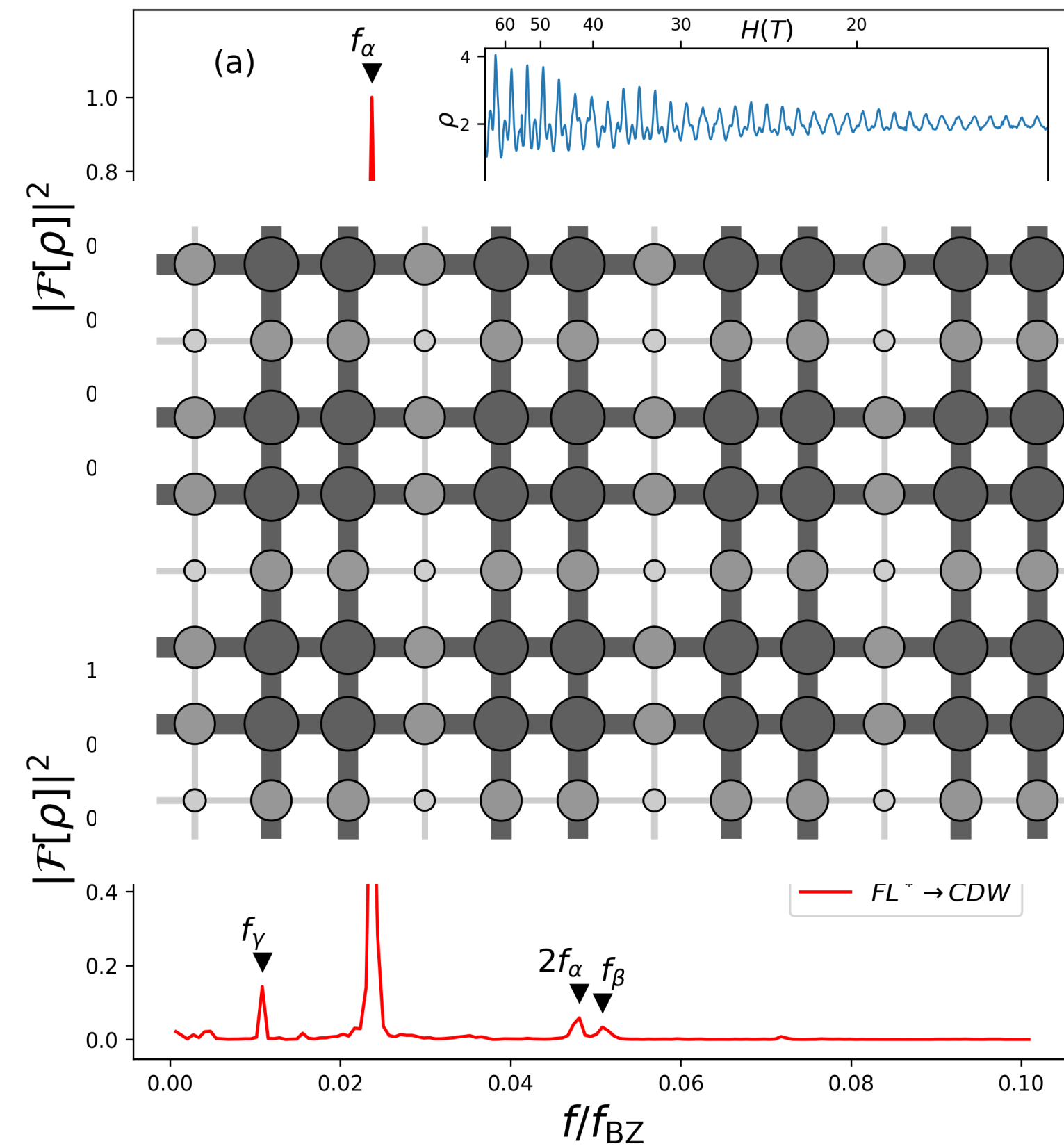
Quantum Oscillations in the DOS

Period-4 b-CDW*/CDW



$$f_\alpha \sim 1500\text{T}, \quad f_\beta \sim 3300\text{T}$$

Period-6 b-CDW*/CDW



$$f_\alpha \sim 650\text{T}, \quad f_\beta \sim 1400\text{T}$$

- the CDW* phase has a significant peak at the β -pocket frequency
- the CDW phase has a single significant contribution at the α -pocket frequency
- the period-6 CDW has oscillations frequencies close to the experimental values of 500 T to 800 T.

Excess Specific Heat

PHYSICAL REVIEW B **102**, 014506 (2020)

High density of states in the pseudogap phase of the cuprate superconductor $\text{HgBa}_2\text{CuO}_{4+\delta}$ from low-temperature normal-state specific heat

C. Girod ^{1,2} A. Legros,^{2,3} A. Forget,³ D. Colson,³ C. Marcenat ⁴ A. Demuer,⁵ D. LeBoeuf,⁵ L. Taillefer,^{2,6,*} and T. Klein^{1,†}

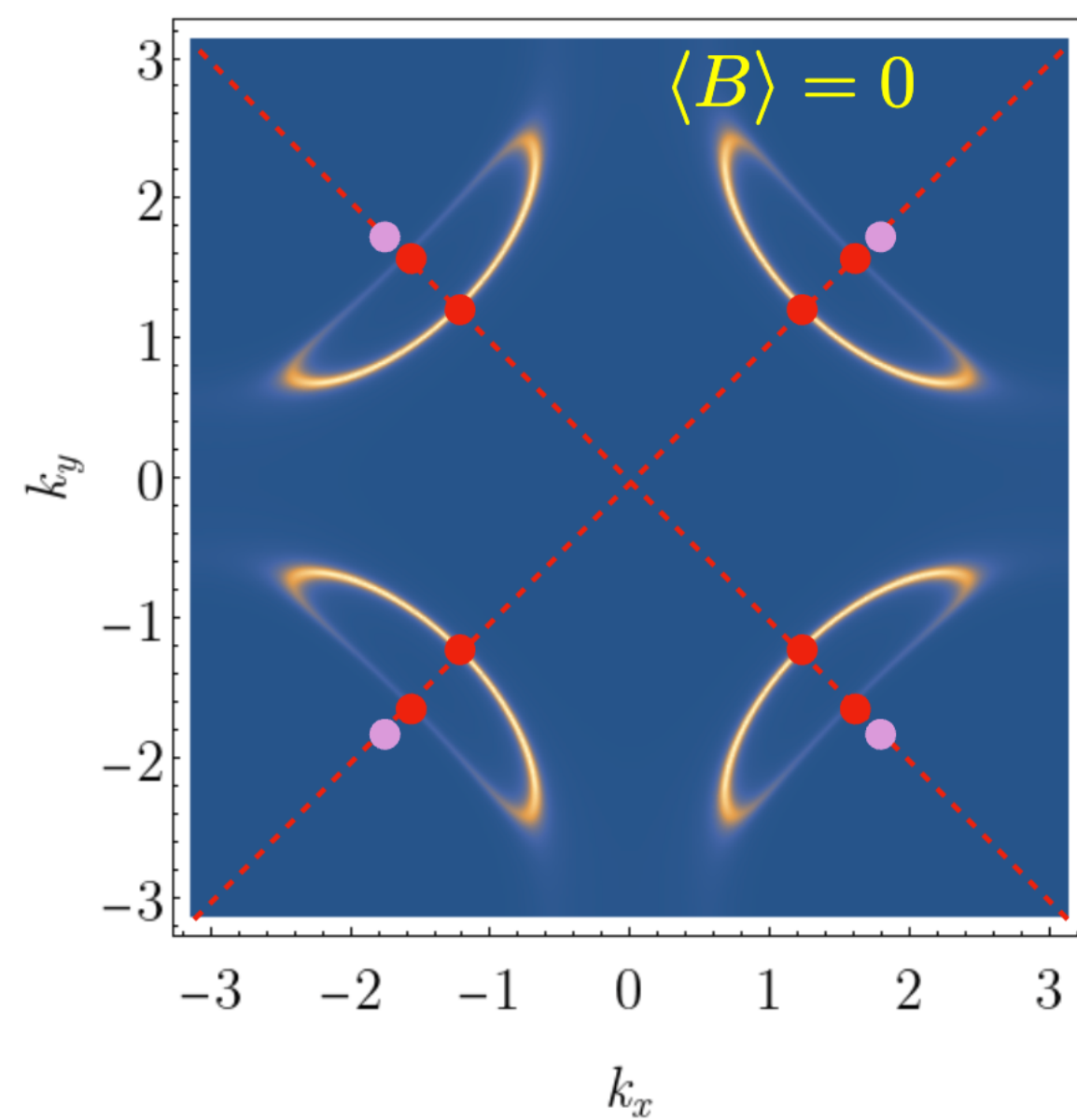
Second, the high γ value implies that the Fermi surface must consist of more than the single electronlike pocket detected by quantum oscillations in $\text{HgBa}_2\text{CuO}_{4+\delta}$ at $p \simeq 0.09$, whose effective mass $m^* = 2.7m_0$ yields only $\gamma = 4.0 \text{ mJ/K}^2 \text{ mol}$. This missing mass imposes a revision of the current scenario for how pseudogap and charge order, respectively, transform and reconstruct the Fermi surface of cuprates.

The elongated hole pockets might account for the excess specific heat

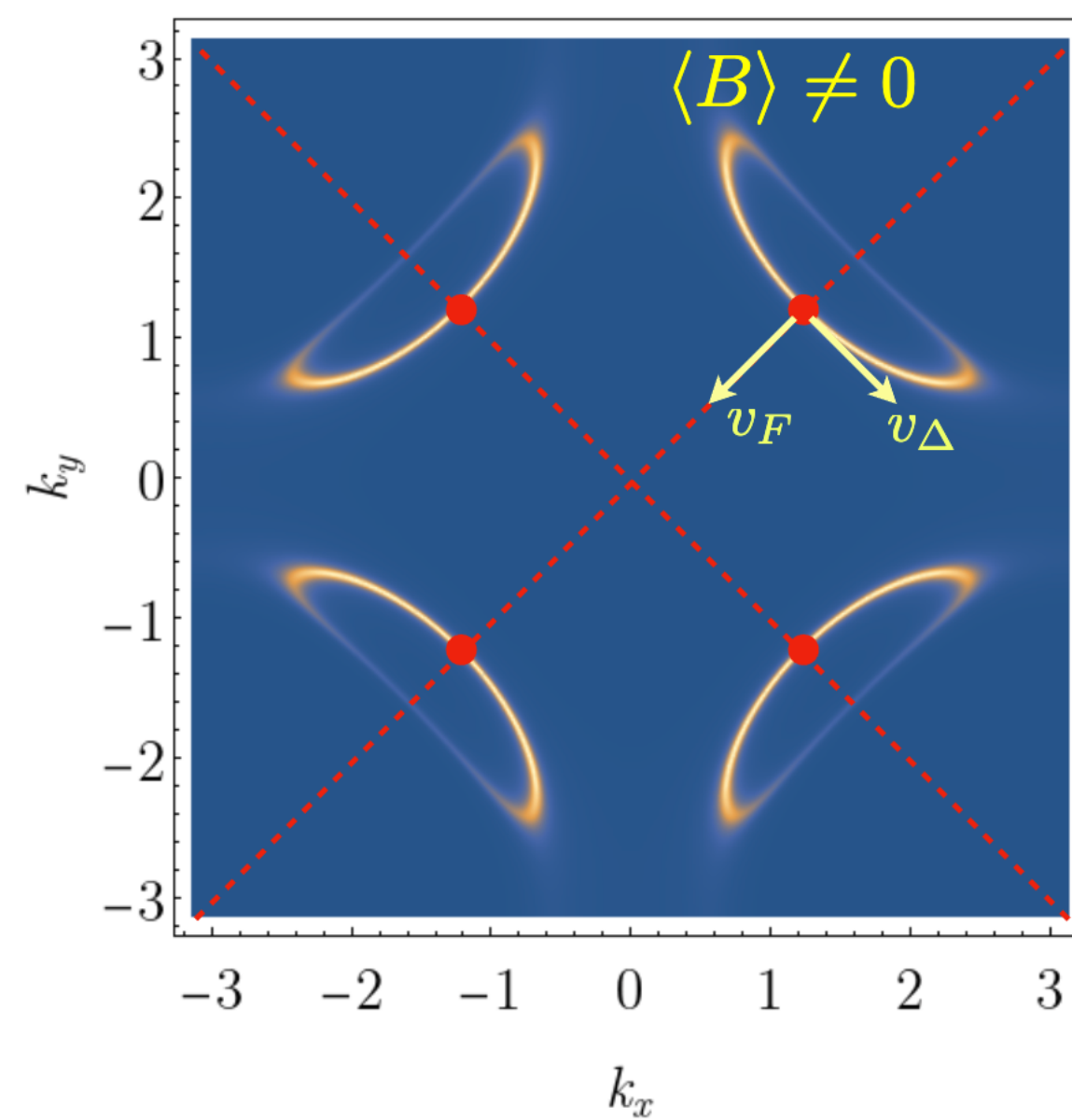
d-SC phase

Christos, Sachdev, NPQ Quantum Materials 9, 4 (2024)

(A) $FL^* \rightarrow d\text{-}SC^*$



(B) $FL^* \rightarrow d\text{-}SC$



Spin liquid nodes hybridize with
backsides for $FL^* \rightarrow d\text{-}SC$

Summary

- FL* is a good candidate '**mother state**' for the low-temperature physics of **underdoped cuprates**
- Ancilla Layer Model has a mean-field **FL*** phase
- FL* can eventually **confine** and form **Néel/VBS** or can be Higgsed down to form a **d-wave superconductor/ CDW**
- FL* can explain the presence of **Fermi arcs** in photoemission experiments and of **small Fermi surfaces** in QO experiments
- Dynamical Spin structure factor displays spinon bound states forming an "**hourglass**" spectrum
- Spin liquid is crucial to obtain the **FS topology** seen in QO experiments in the **CDW phase** and 4 nodes in the **d-SC phase**

Review article

PMB, Christos, Nikolaenko, Patel, Sachdev, arXiv:2508.20164 (2025)

Critical quantum liquids and the cuprate high temperature superconductors

Thank you!