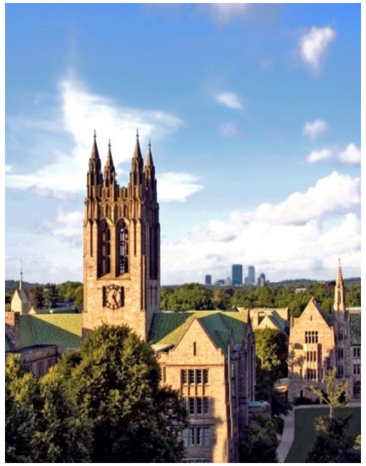
Hyperdeterminants wavefunctions

Ying Ran

(Boston College)

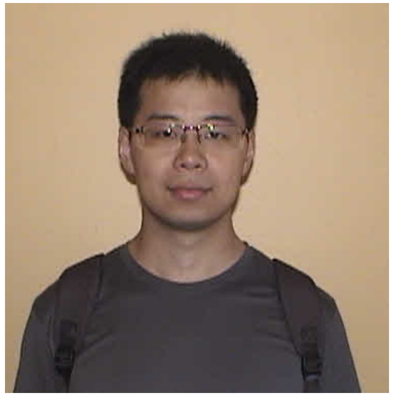


ICTP program “Quantum Matter”, Dec 2025

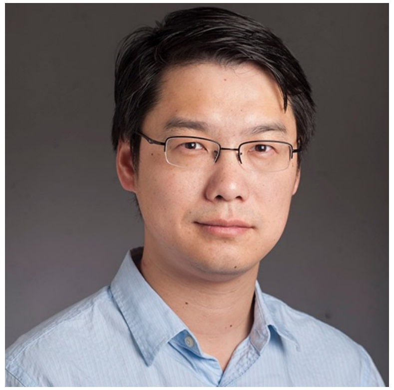
Acknowledgement:

- **Collaborators:**

Guan-Lin Lin
BC



Xiaodong Hu



Di Xiao

Univ. of Washington

Reference: Phys. Rev. B 109, 245125 (2024)

Plan

- Motivation
- Main Claim
- Benchmark Results
- Ongoing/Future directions

Fractional quantum anomalous Hall (FQAH) effects

- Theoretically, FQAH states were proposed about a decade ago:

Basic idea:

FQH: partially filled Flat Landau Level + Interactions

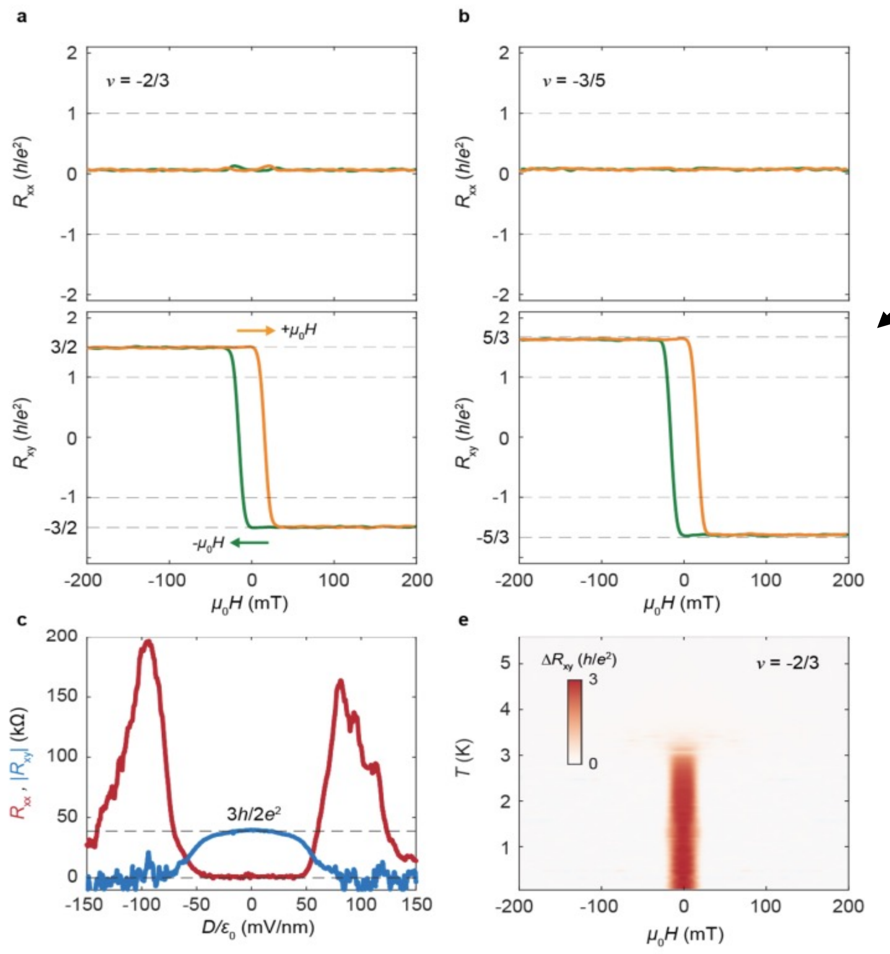
FQAH: partially filled nearly flat Chern band + Interactions

Sheng et.al, Nat. Comm. 2011
Neupert et.al, PRL 2011
Tang et.al, PRL 2011
Regnault et.al, PRX 2011
Xiao et.al., Nat. Comm. 2011
.....

Fractional Quantum Hall states
in the absence of a magnetic field

Fractional quantum anomalous Hall (FQAH) effects

- FQAH states have been observed in experimental moiré systems



Cai et.al., Nature 2023 (MoTe2)
Park et.al., Science 2023 (MoTe2)
Zeng et. al., Nature 2023 (MoTe2)
Lu et.al., Nature 2024 (Graphene)
....

Fractional Quantum Hall states
in the absence of a magnetic field

Why FQAH states are interesting?

- Practical Reasons

No B-field: new experiments can be done (e.g., heterostructure with SC)

Larger energy scale:

$$\Delta_g \sim \frac{e^2}{\varepsilon \cdot l_B} \Rightarrow \Delta_g \sim \frac{e^2}{\varepsilon \cdot a}$$

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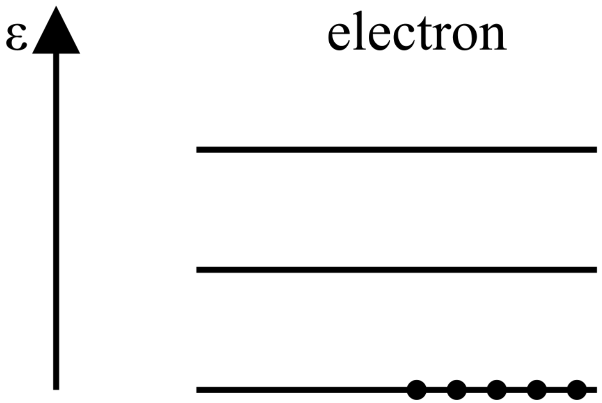
- Conceptual Reasons

More tunability: Richer phase diagrams

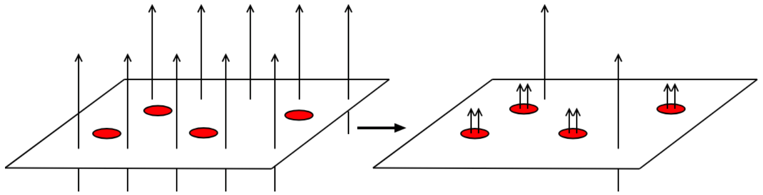
Potentially new physical regime/phases far from FQH (this talk)

New FQAH physics far from FQH

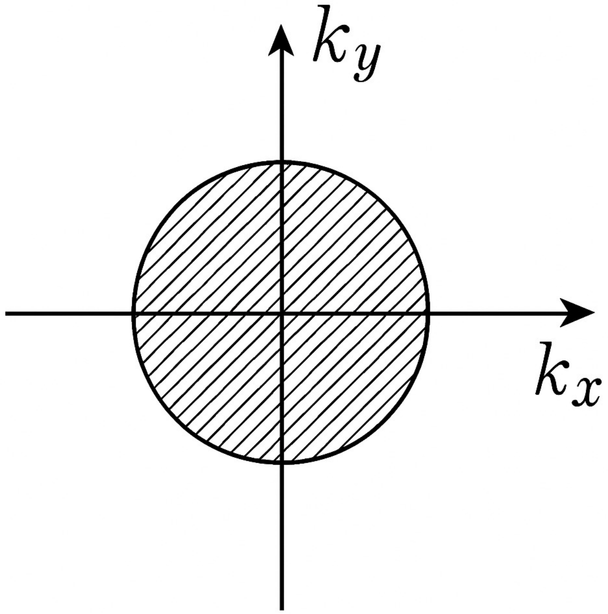
- Example: Composite Fermi Liquid at $\nu = \frac{1}{2}$ (Halperin-Lee-Read, Haldane-Pasquier…)



Jain 1989



Fermi surface of charge-neutral composite fermion



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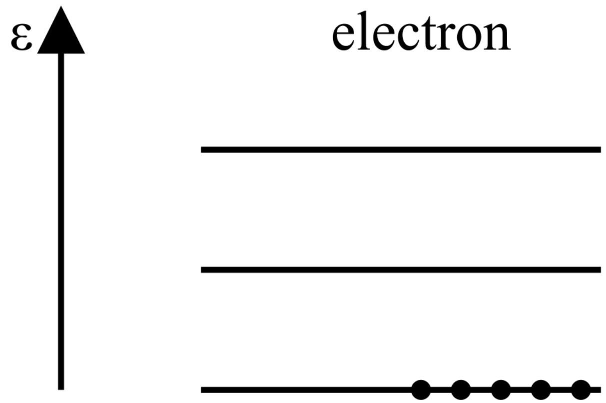

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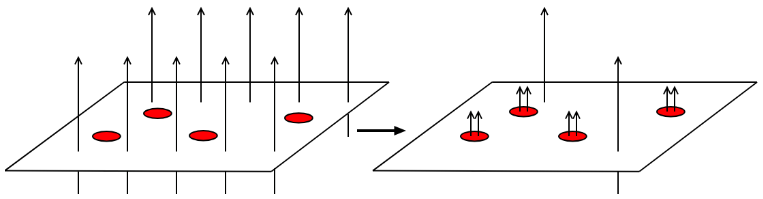

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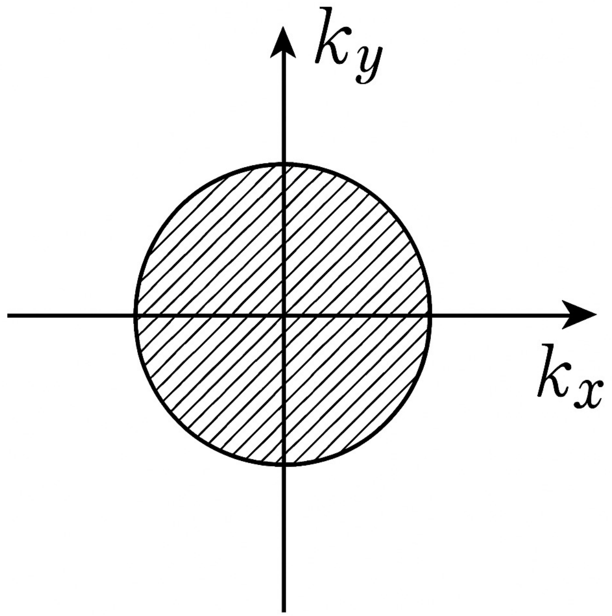

What could happen in a FQAH system?

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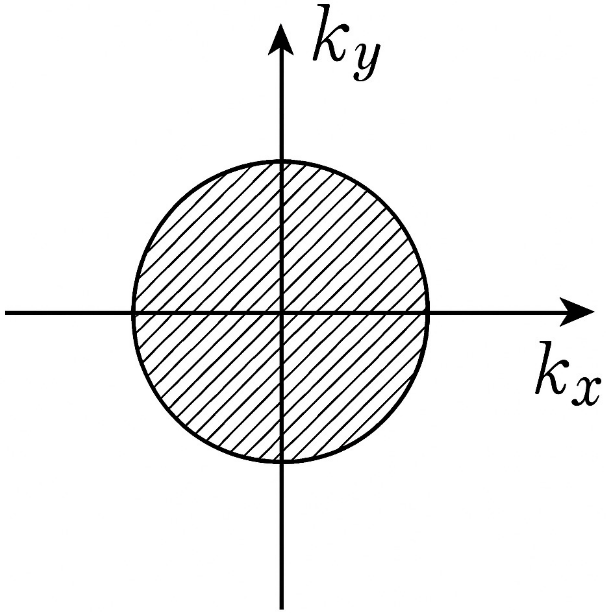
FQH



Galilean invariance:
No Brillouin Zone, CF FS must be circular

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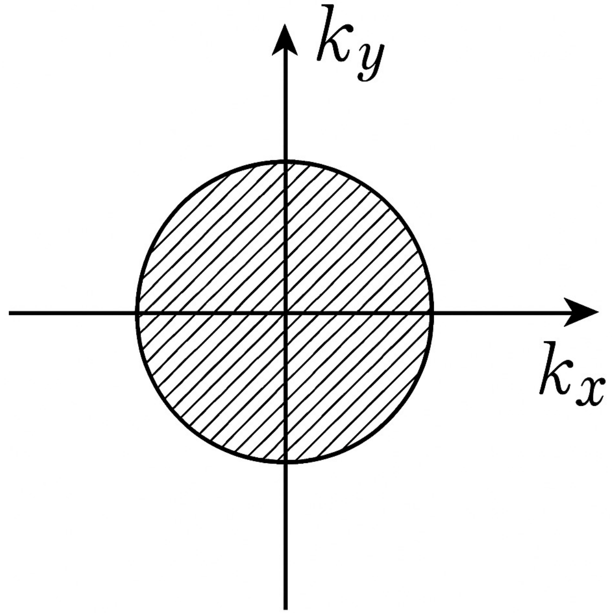
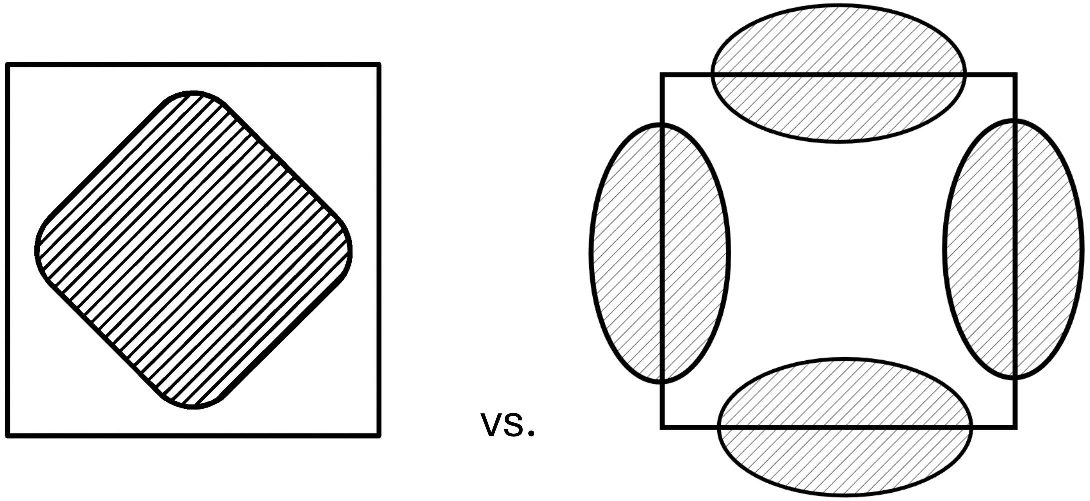


FQAH



Galilean invariance:
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Lattice symmetry:
Brillouin Zone, possible different CF FS topology

About "Mapping" between FQH and FQAH

- It was well-known that the Bloch states in the C=1 band can be mapped into the lowest Landau level, preserving all crystalline symmetries. (Jian & Qi…)
- Namely, there is no problem that the Hilbert spaces on the two sides can be mapped for C=1 band.

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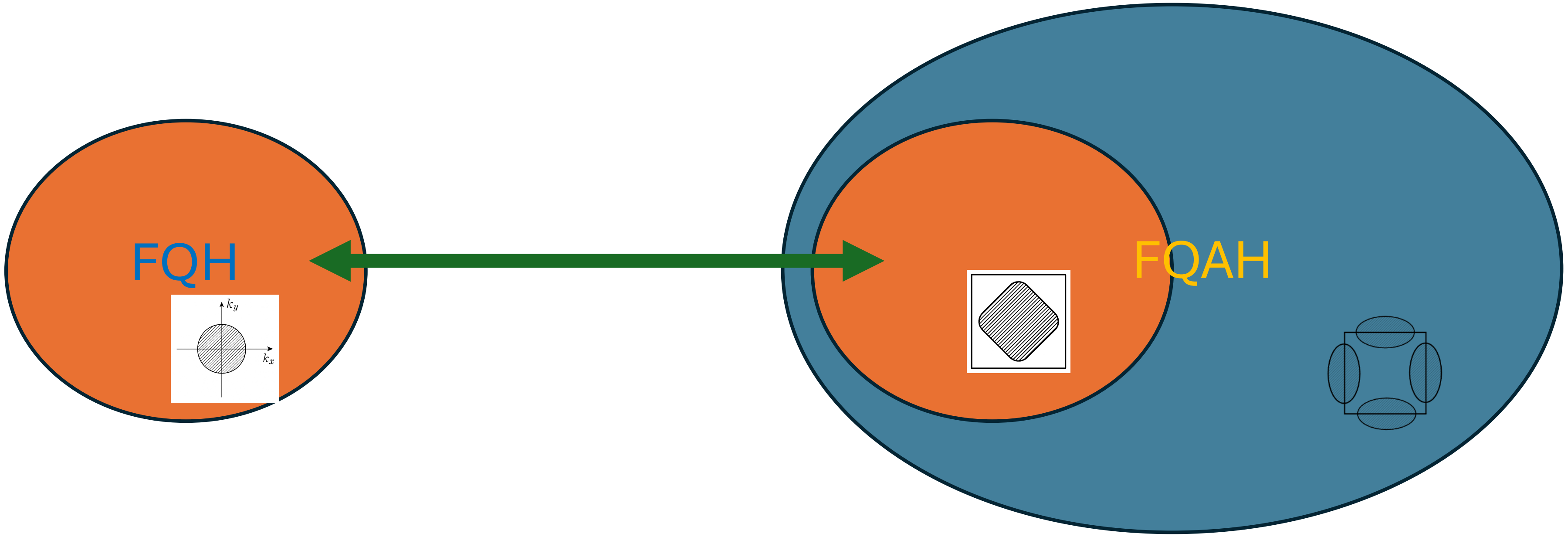

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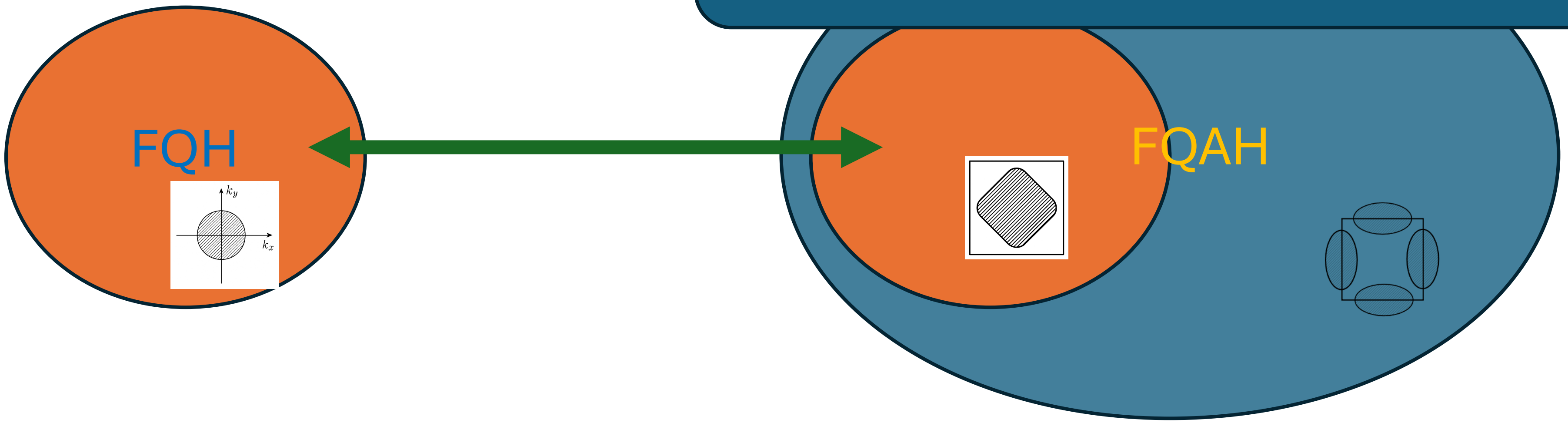
In a microscopic model, do we have ANY theoretical tool to tell which Fermi surface is realized?



Currently available theoretical tools

- Effective field theories: not microscopic

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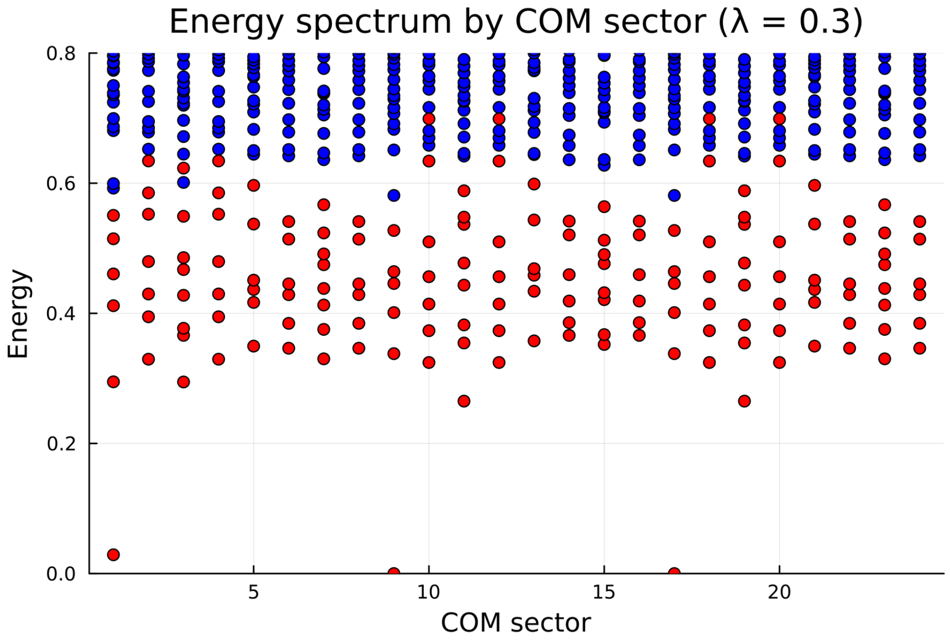
- Brute force numerics: exact-diagonalization and DMRG



(1) Small system sizes
(2) No intuitive picture
(no access to fractionalized d.o.f., e.g. composite fermion)

What we want

- A microscopic theory capable of describing FQAH physics far from FQH.

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- We really want to write down many-body wavefunctions for FQAH states.

- But, how to generalize these FQH wavefunctions? It must be continuously tunable...

The main claim

- All the general (FQH or FQAH) composite fermion states (and many others) are Hyperdeterminant (Hdet) wavefunctions.

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- There are efficient ways to (approximately) simulate these wavefunctions,

➔ (1) accurate microscopics

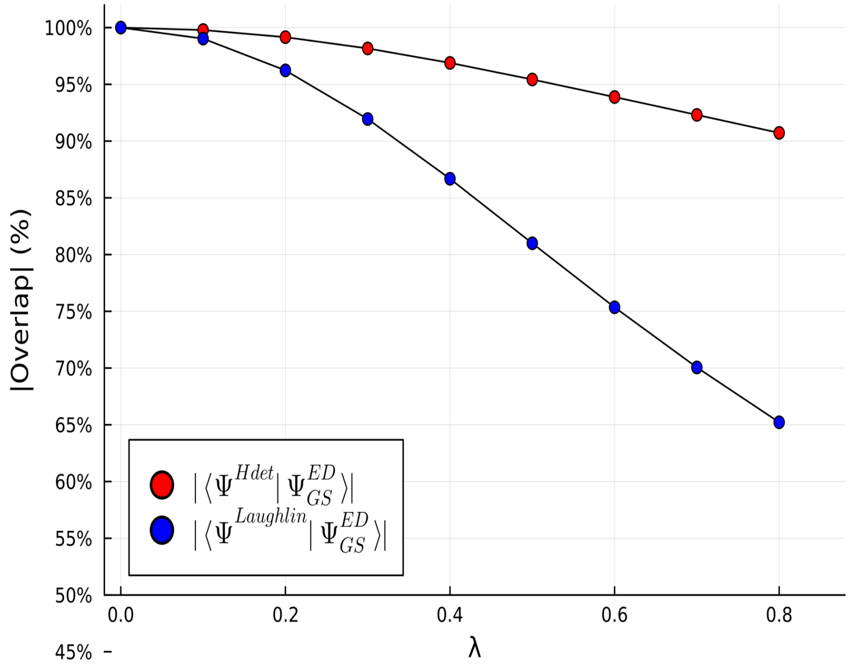
(2) direct access to the composite fermion band structure

(The generalization of states with pairing, e.g., Pfaffian state will be HyperPfaffian wavefunctions)

Preview: what Hdet theory can do

- Accurate microscopic variational wavefunction

0^{th}-order-optimized Hdet already performs well



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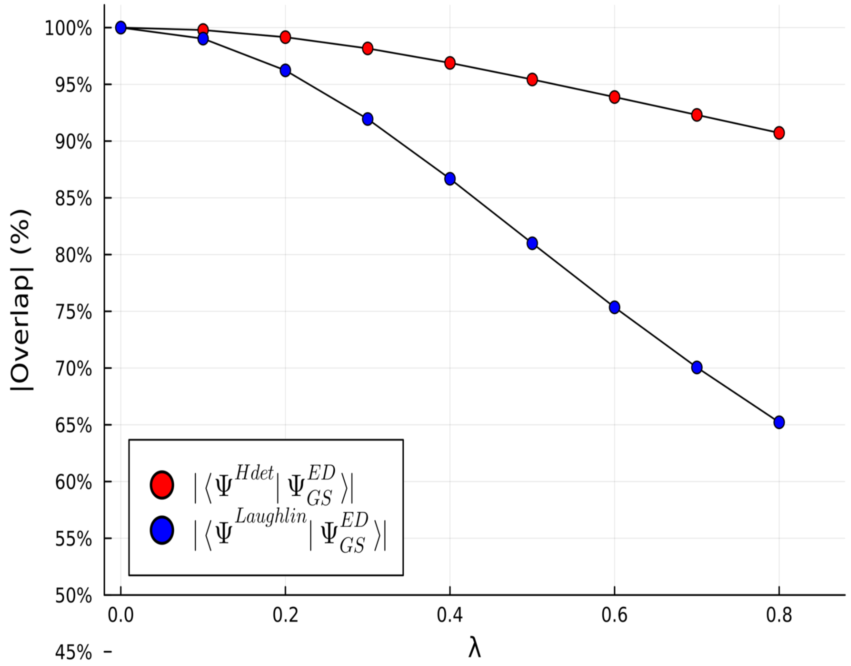
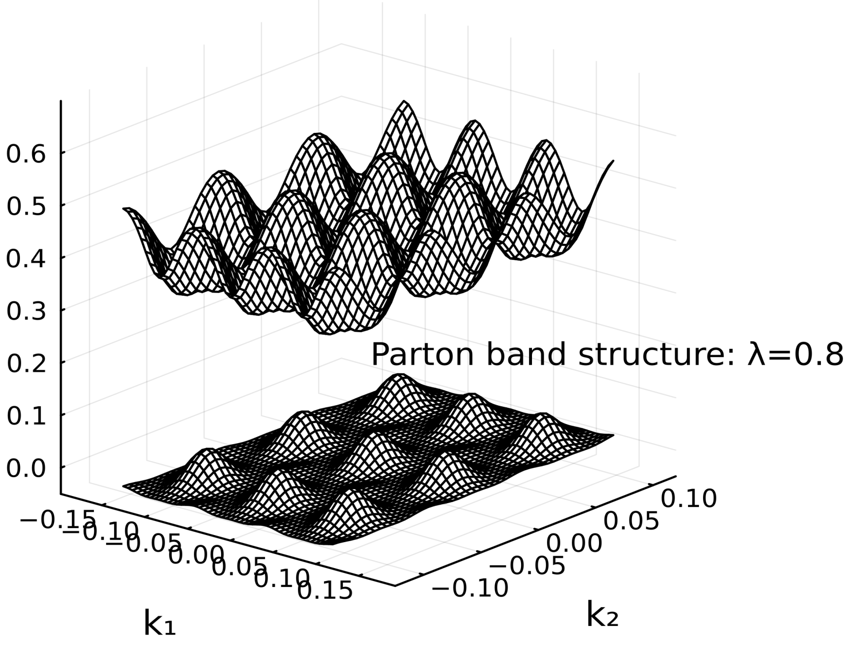


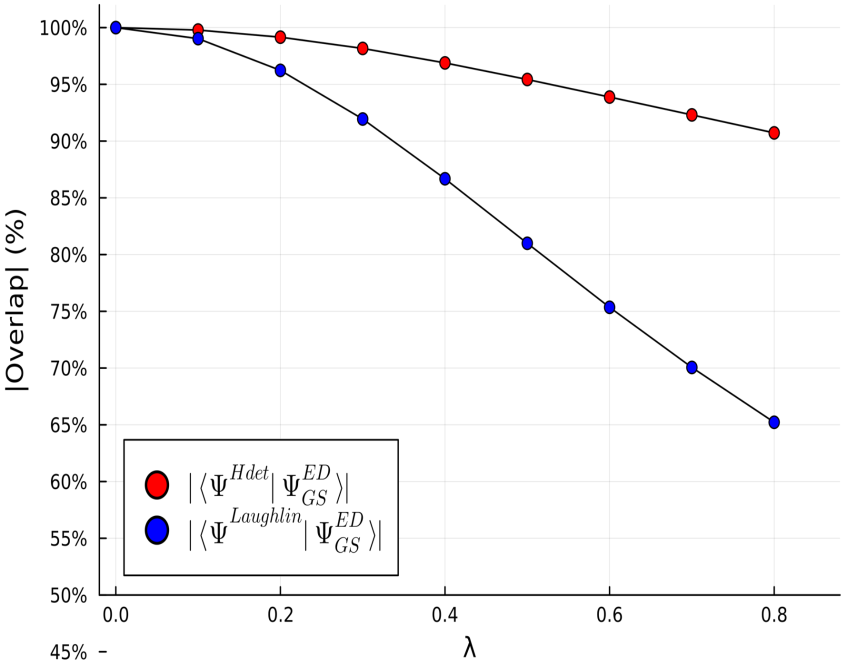


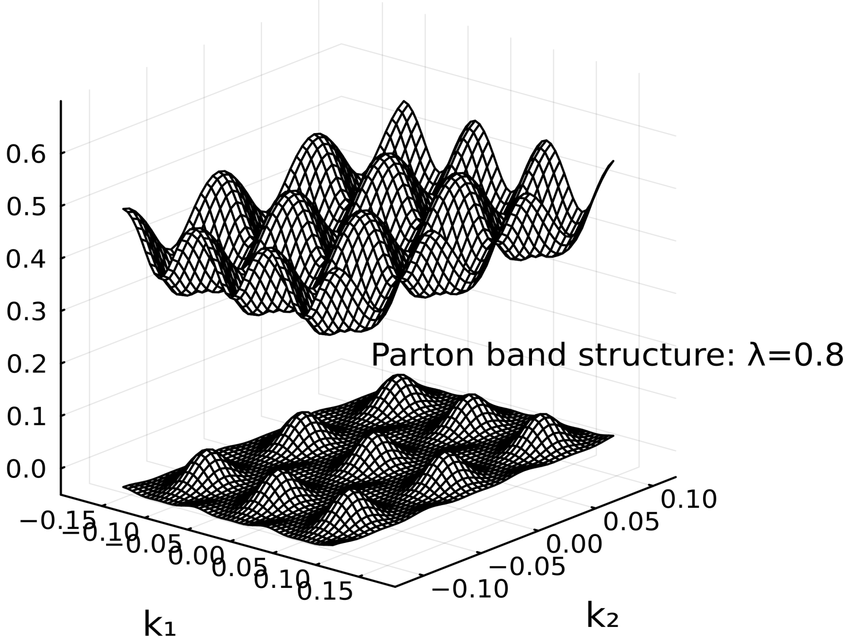
Composite fermion = parton $e \sim f^{(1)} \cdot f^{(2)} \cdot f^{(3)}$

Preview: what Hdet theory can do

- Accurate microscopic variational wavefunction
- Direct access to the fractionalized d.o.f. (e.g., composite fermion band structure)
- Applicable in the general context of correlated electron systems (including QSL)

0^{th}-order-optimized Hdet already performs well





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Hdet: Mathemathical definition

$$\det(A_{ij}) \equiv \sum_{p\in S_N} (-1)^p A_{1p(1)} \cdot A_{2p(2)} \cdots A_{Np(N)}$$

$$\mathrm{Hdet}(T_{ijk}) \equiv \sum_{P,Q\in S_N} (-1)^P \cdot (-1)^Q \cdot T_{1P(1)Q(1)} T_{2P(2)Q(2)} \cdots T_{NP(N)Q(N)}$$

$$\mathrm{Hdet}(T_{ijkl}) \equiv \sum_{P,Q,R\in S_N} (-1)^P (-1)^Q (-1)^R \, T_{1P(1)Q(1)R(1)} \cdot T_{2P(2)Q(2)R(2)} \cdots T_{NP(N)Q(N)R(N)}$$

Hdet: as a many-body wavefunction

- Slater-determinant as a many-body wavefunction

$$A_{ij} = \langle \psi_i^{(e)} | \phi_j^{(e)} \rangle$$

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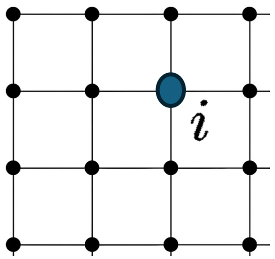
Electron's single-particle orbitals



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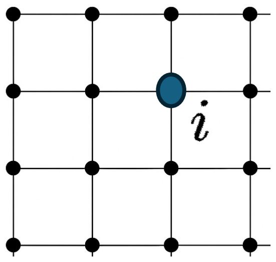
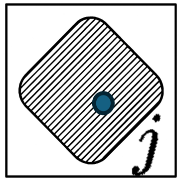
Filled states





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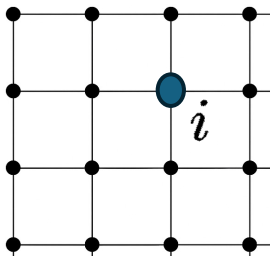

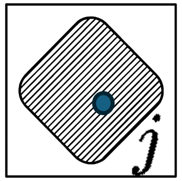

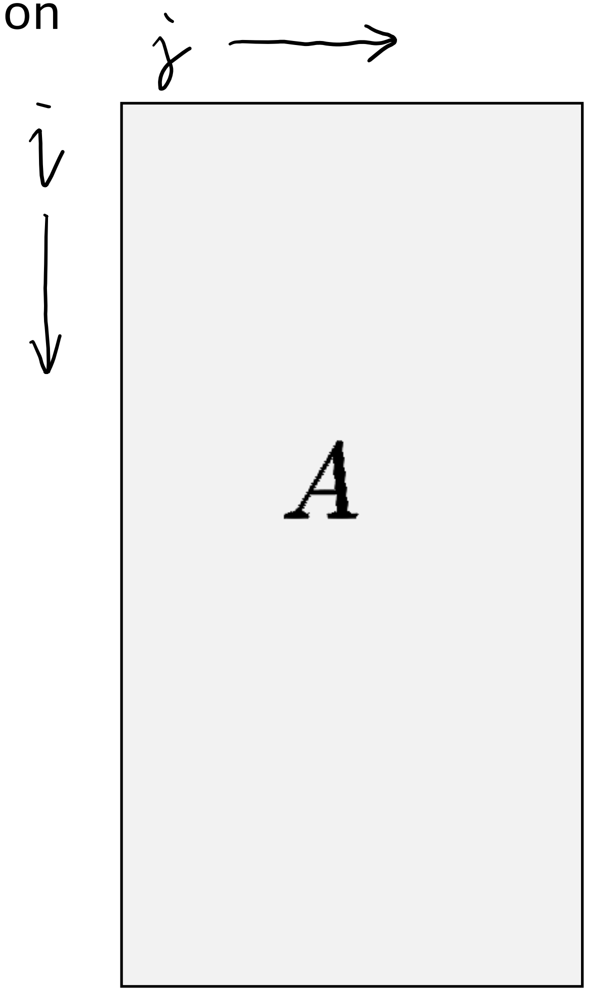

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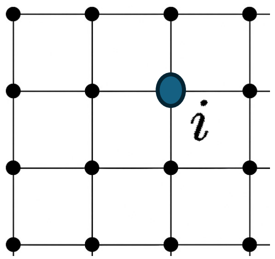

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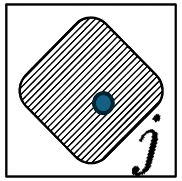
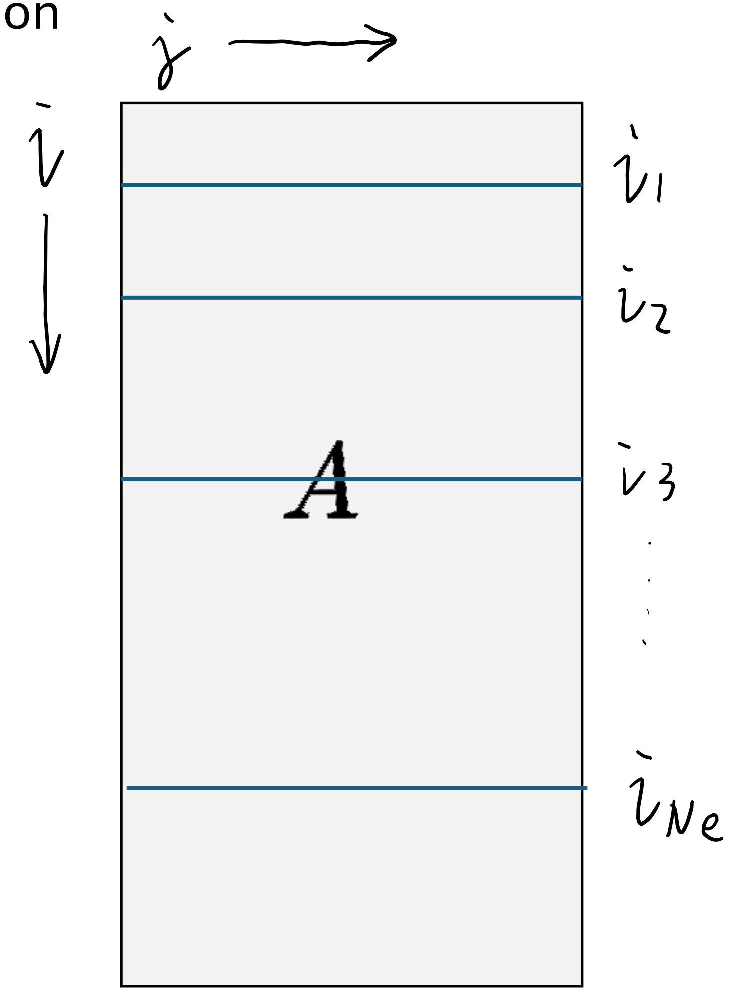


$$\left\langle \psi_{i_1}^{(e)} \psi_{i_2}^{(e)} \cdots \psi_{i_{N_e}}^{(e)} \mid \Psi \right\rangle \equiv \det\left(A_{\text{sub}}\right)$$



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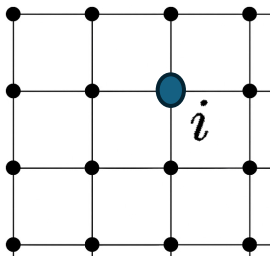

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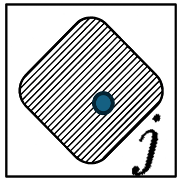

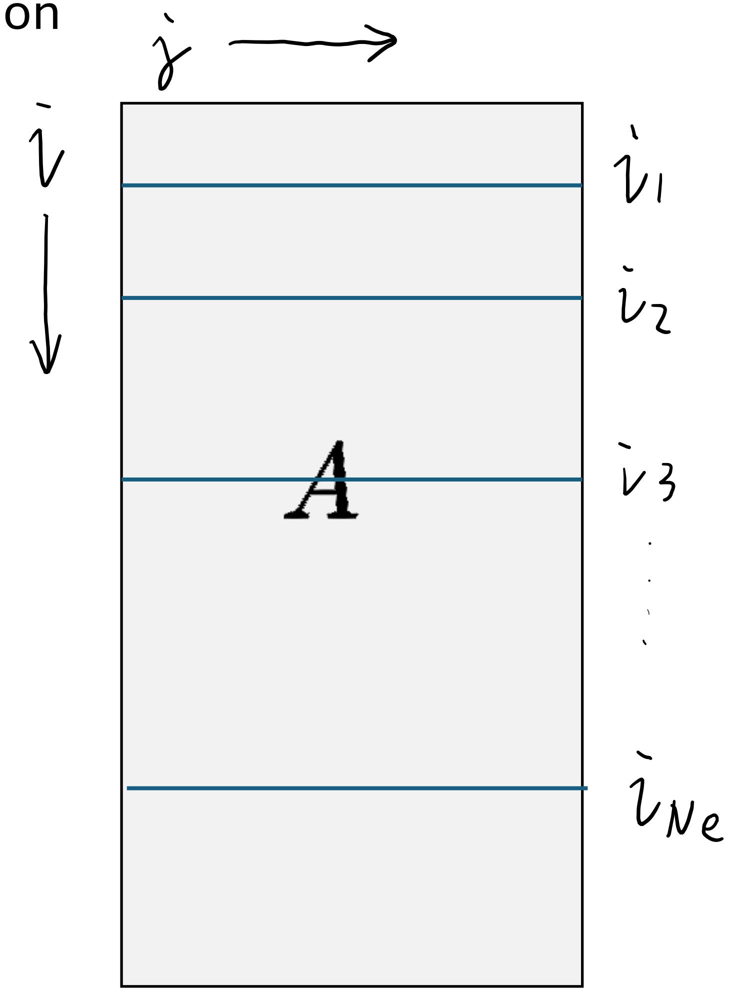

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Exponential-size many-body state captured by polynomial-size matrix

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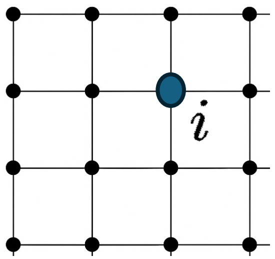

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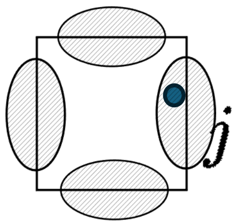

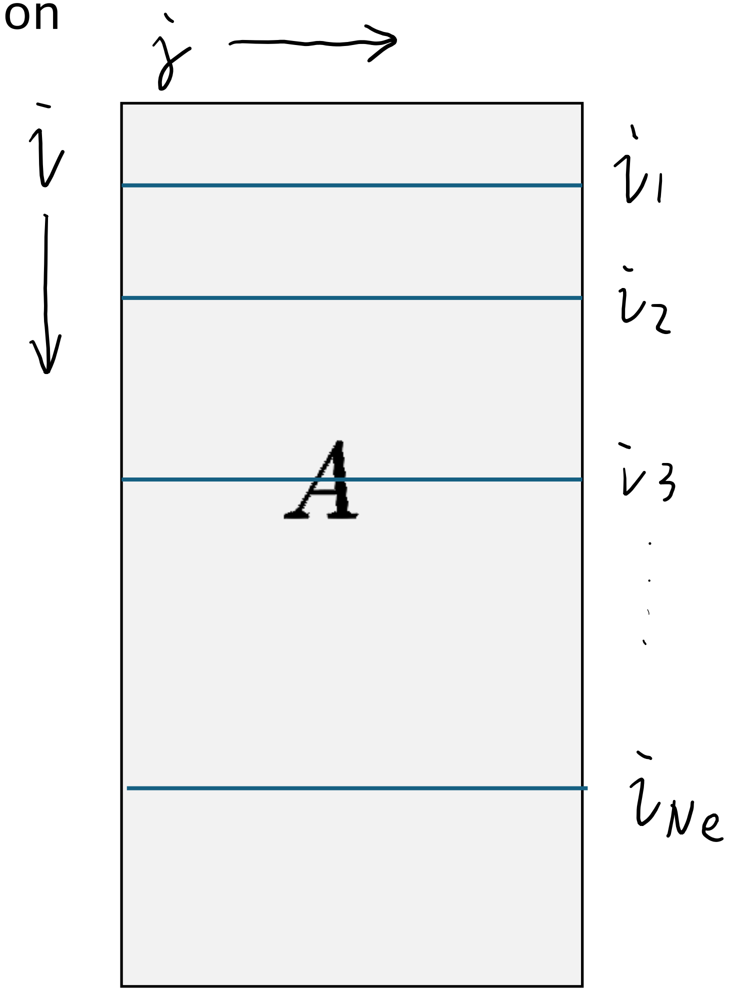

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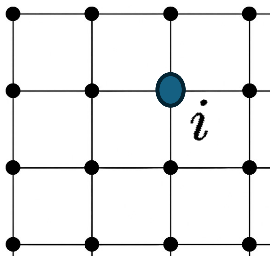

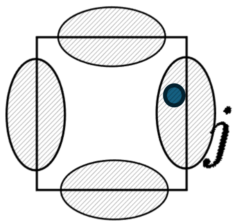

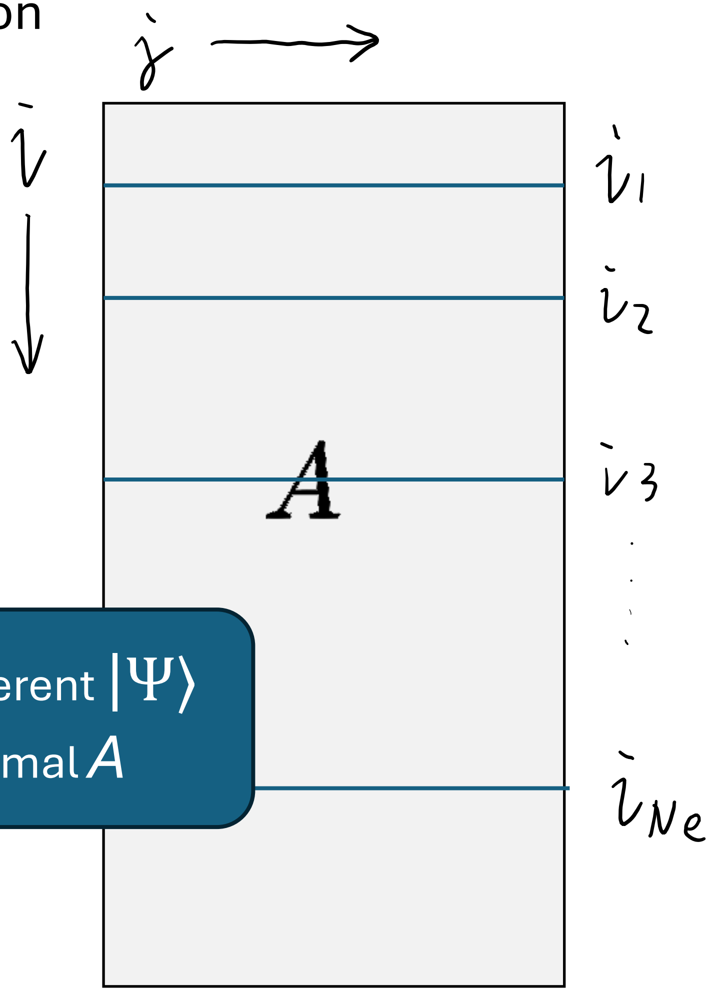

Different A ➔ Different $|\Psi\rangle$

Ground state: optimal A

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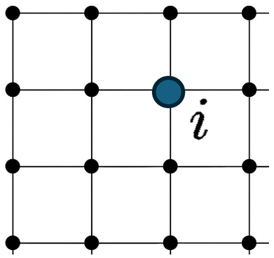

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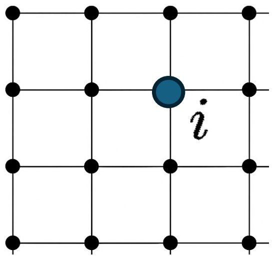
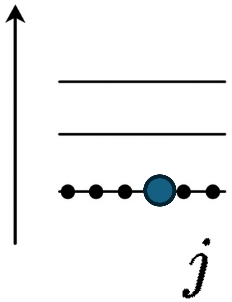
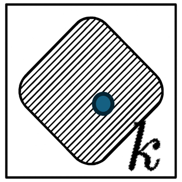
(Bosonic) Electron orbitals $\quad$ Filled parton-(1) states $\quad$ Filled parton-(2) states







Hdet: as a many-body wavefunction

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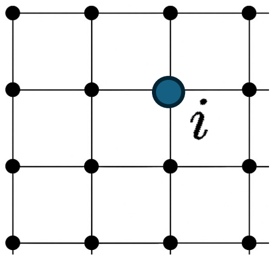

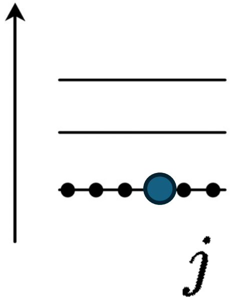

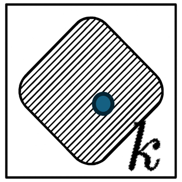

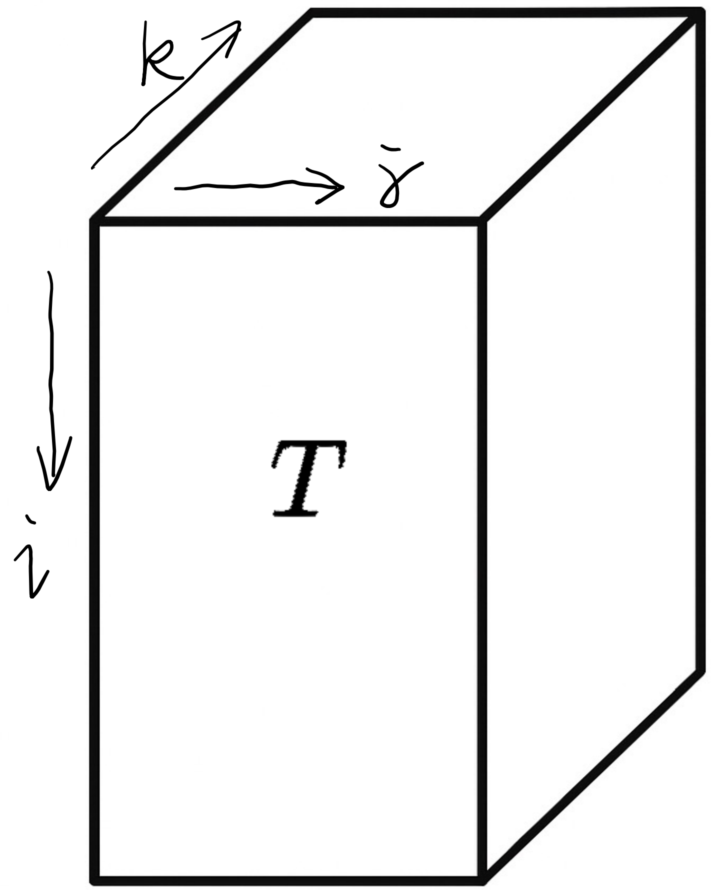

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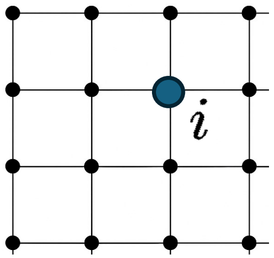

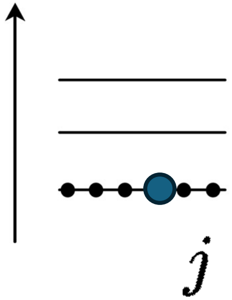

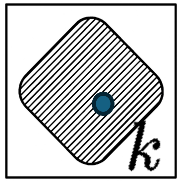

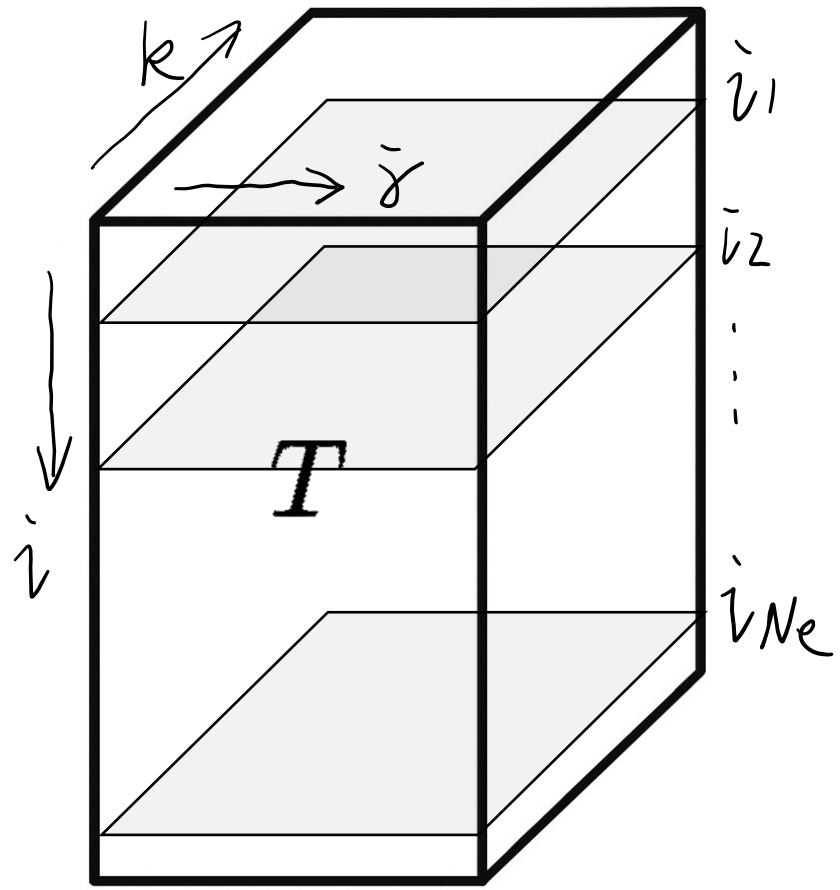

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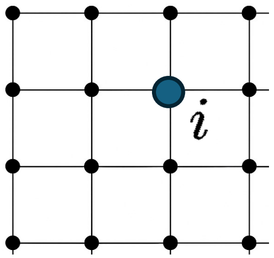

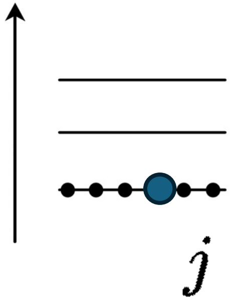

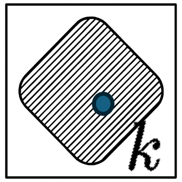

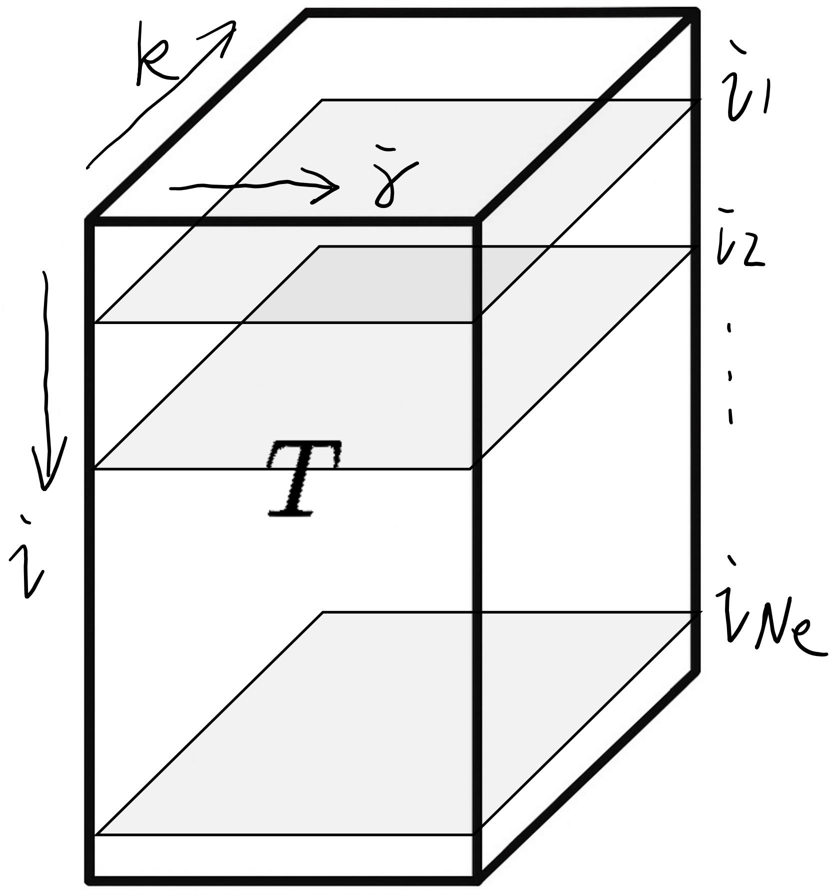

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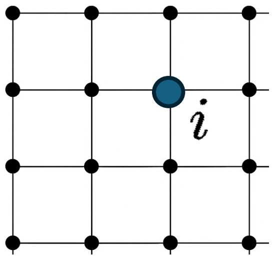

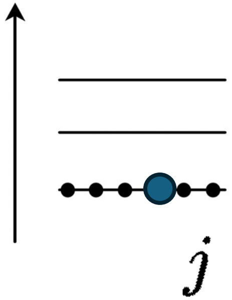

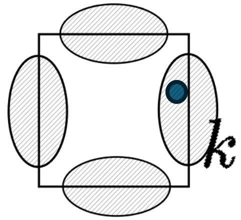

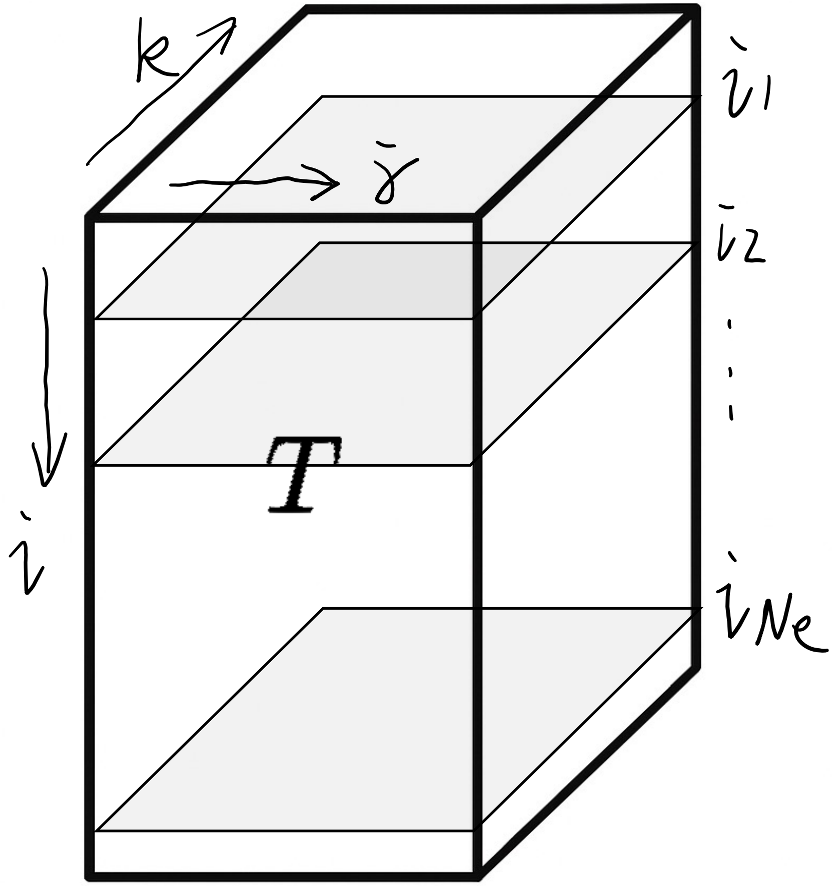

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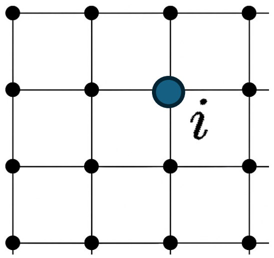

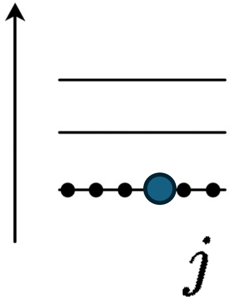

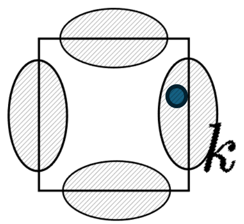

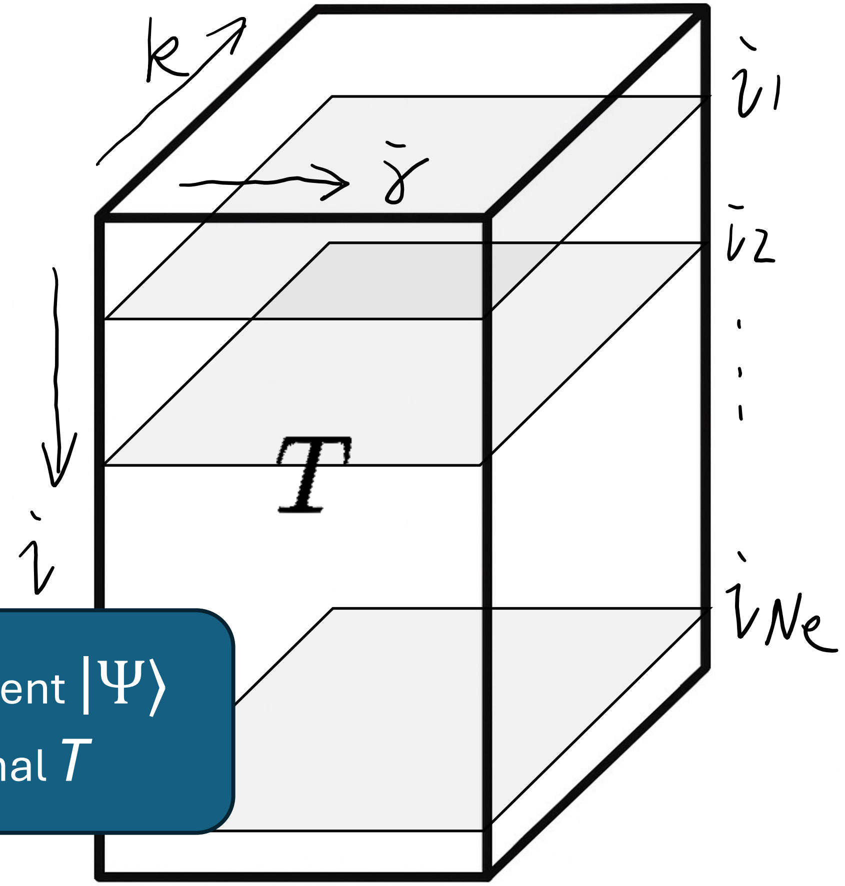

Different T → Different $|\Psi\rangle$

Ground state: optimal T

$$\left\langle \psi_{i_1}^{(e)} \psi_{i_2}^{(e)} \cdots \psi_{i_{N_e}}^{(e)} \mid \Psi \right\rangle \equiv \mathrm{Hdet}\left(T_{\mathrm{sub}}\right)$$

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Hdet and the product of determinants

$$e \sim f^{(1)} \cdot f^{(2)} \qquad T_{ijk} = \langle \psi_i^{(e)} | \phi_j^{(1)} \rangle | \phi_k^{(2)} \rangle$$

Hdet and the product of determinants

- A very special case: $e \sim f^{(1)} \cdot f^{(2)}$ $\quad T_{ijk} = \langle \psi_i^{(e)} | \phi_j^{(1)} \rangle | \phi_k^{(2)} \rangle$

If $T_{ijk} = A_{ij} \cdot B_{ik}$, then $\mathrm{Hdet}(T) = \det(A) \cdot \det(B)$

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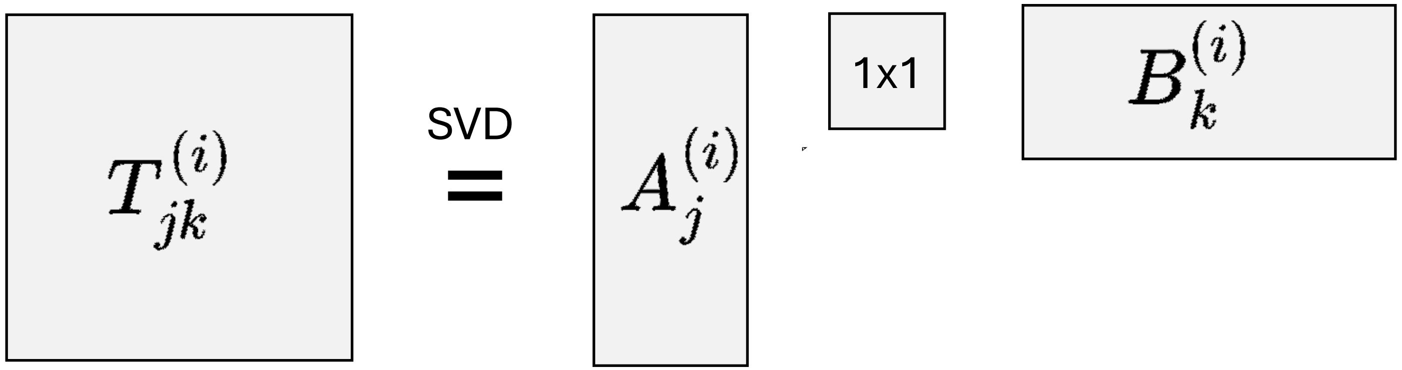

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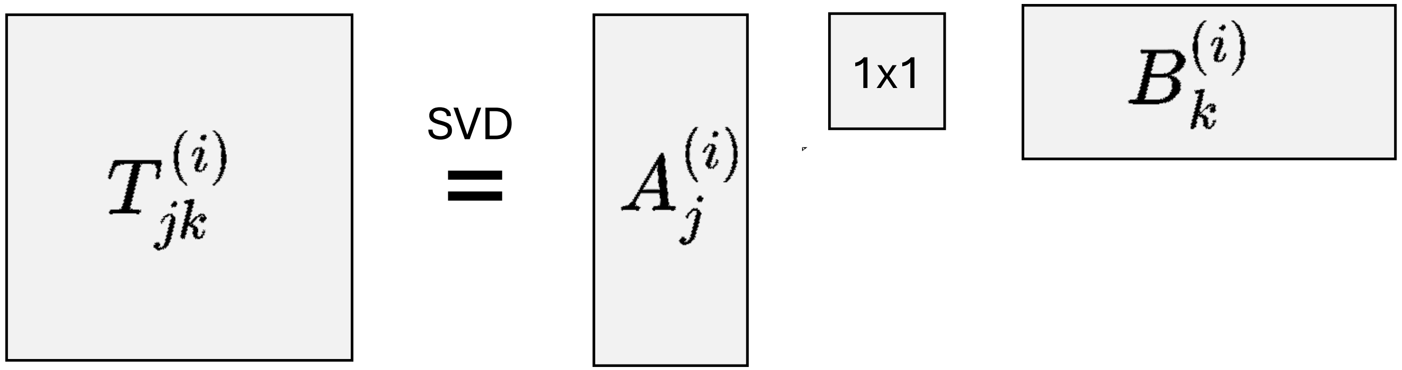

Recall: Laughlin wavefunction is indeed a product of Slater determinants

$$\prod_{i<j} (z_i - z_j)^2 e^{-\sum_i |z_i|^2 / 4l_B^2}$$

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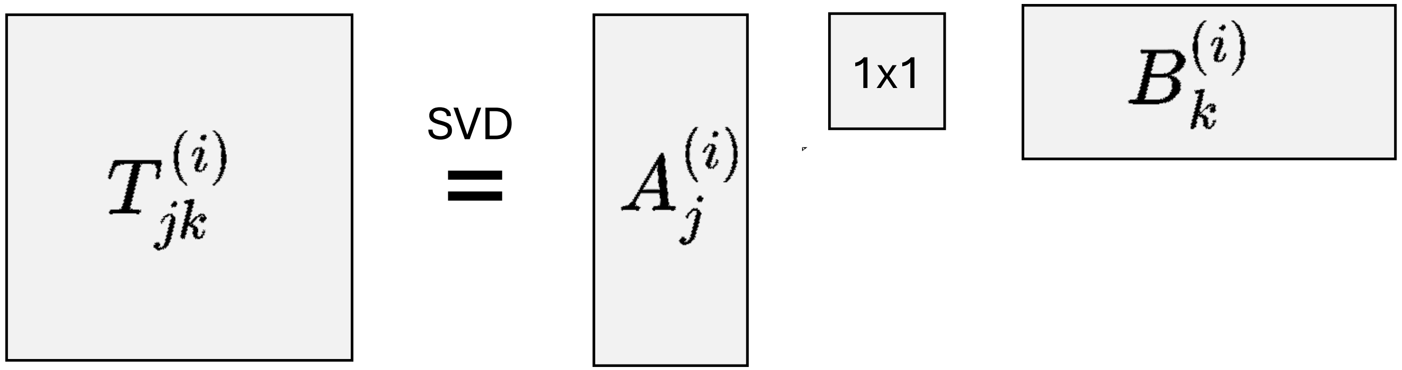
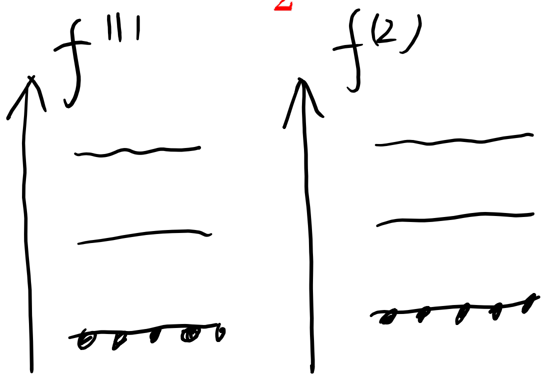


$\nu = \frac{1}{2}$



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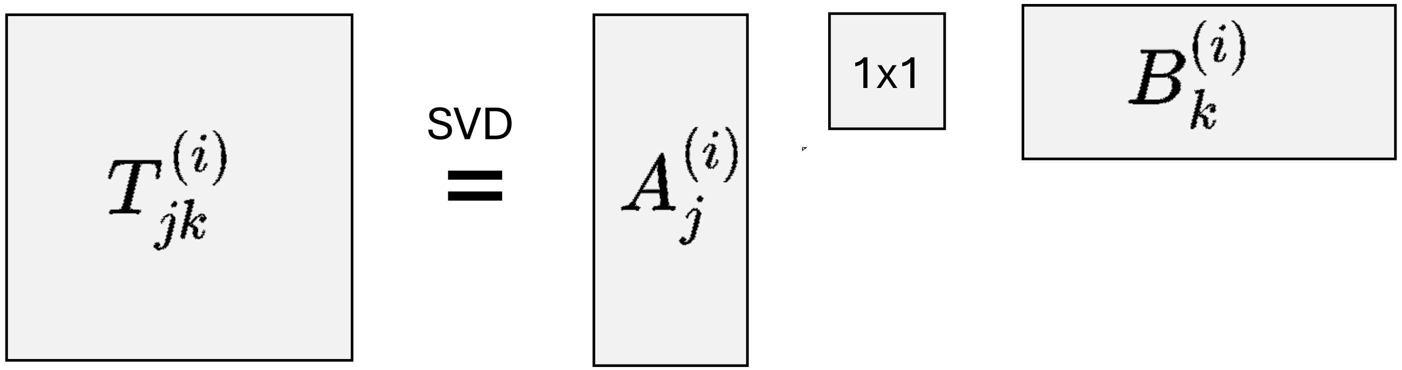

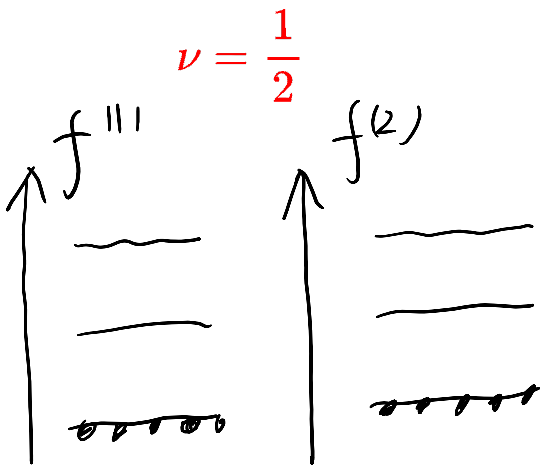

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In QSL context, Abrikosov fermion construction also leads to QSL states that are products of determinants: $\vec{S} = f_\alpha^+ \frac{\vec{\sigma}_{\alpha\beta}}{2} f_\beta$

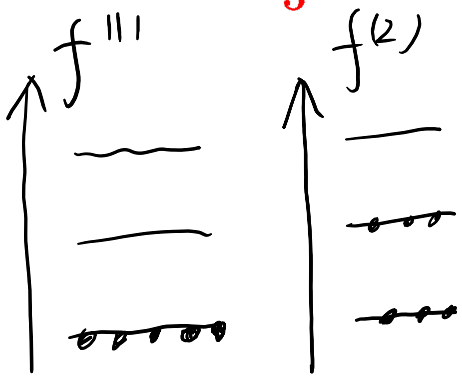
Hdet and the product of determinants

How about the next one in the Jain's sequence in FQH?

$$T_{ijk} = \langle \psi_i^{(e)} | \phi_j^{(1)} \rangle | \phi_k^{(2)} \rangle$$

$\text{et}(B)$

$$\nu = \frac{2}{3}$$



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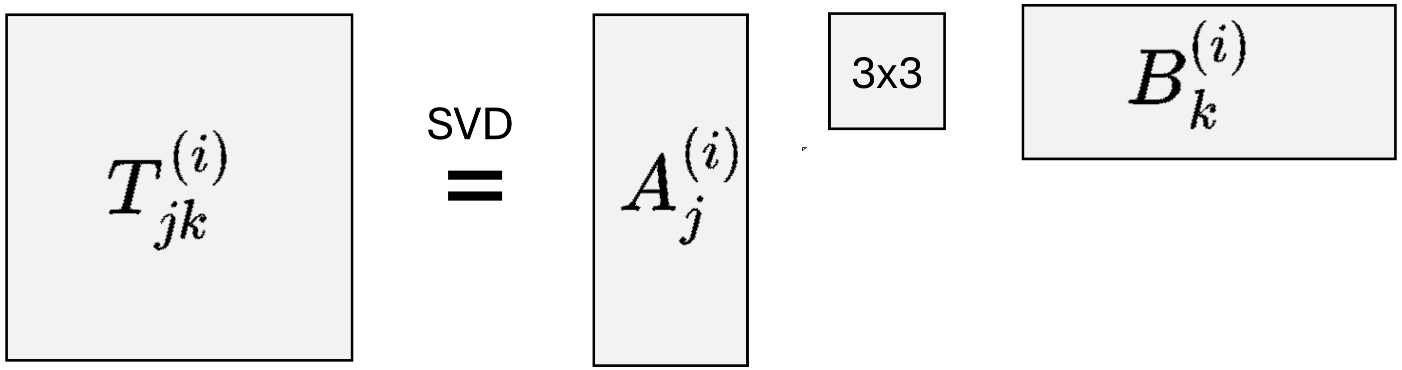
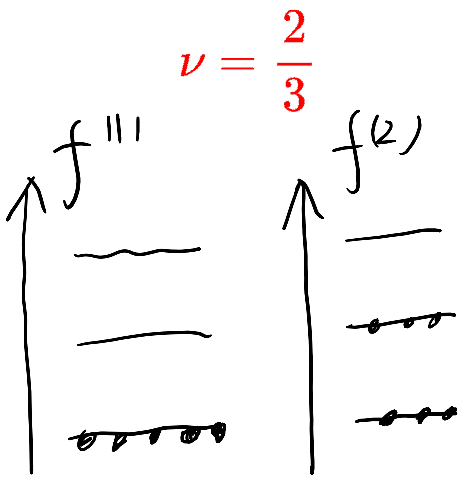
- For $\nu = \frac{2}{3}$ Bosonic FQH state, the best electron orbital-*i* you can find gives:





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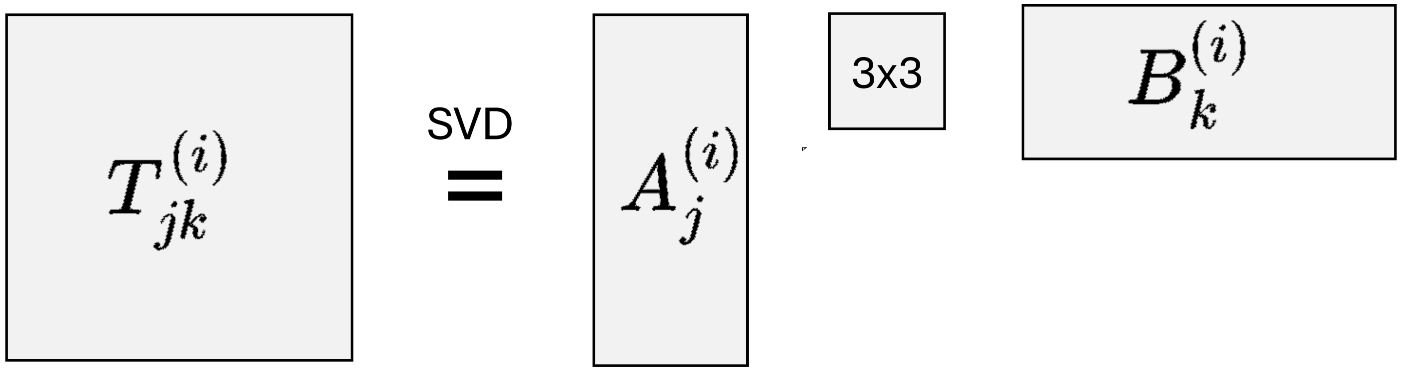

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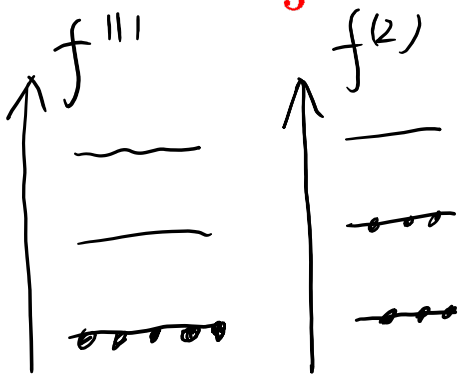

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Generally computing Hdet is NP-hard!

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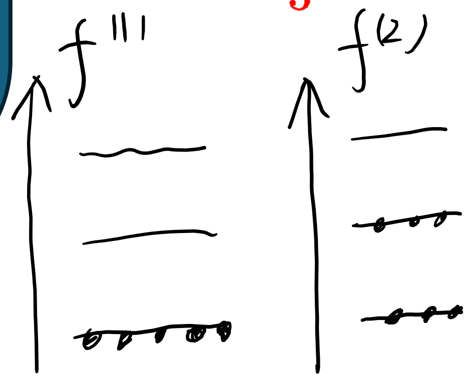

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But we have developed ways to simulate it, with approximation improvable order-by-order. (Projective-expansion)

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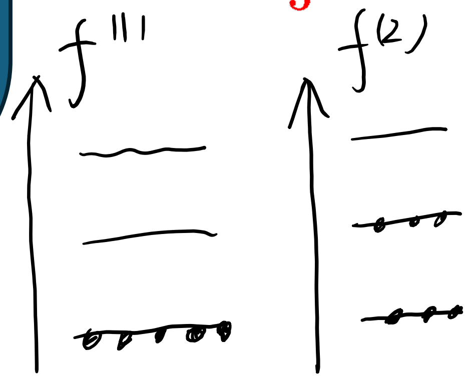

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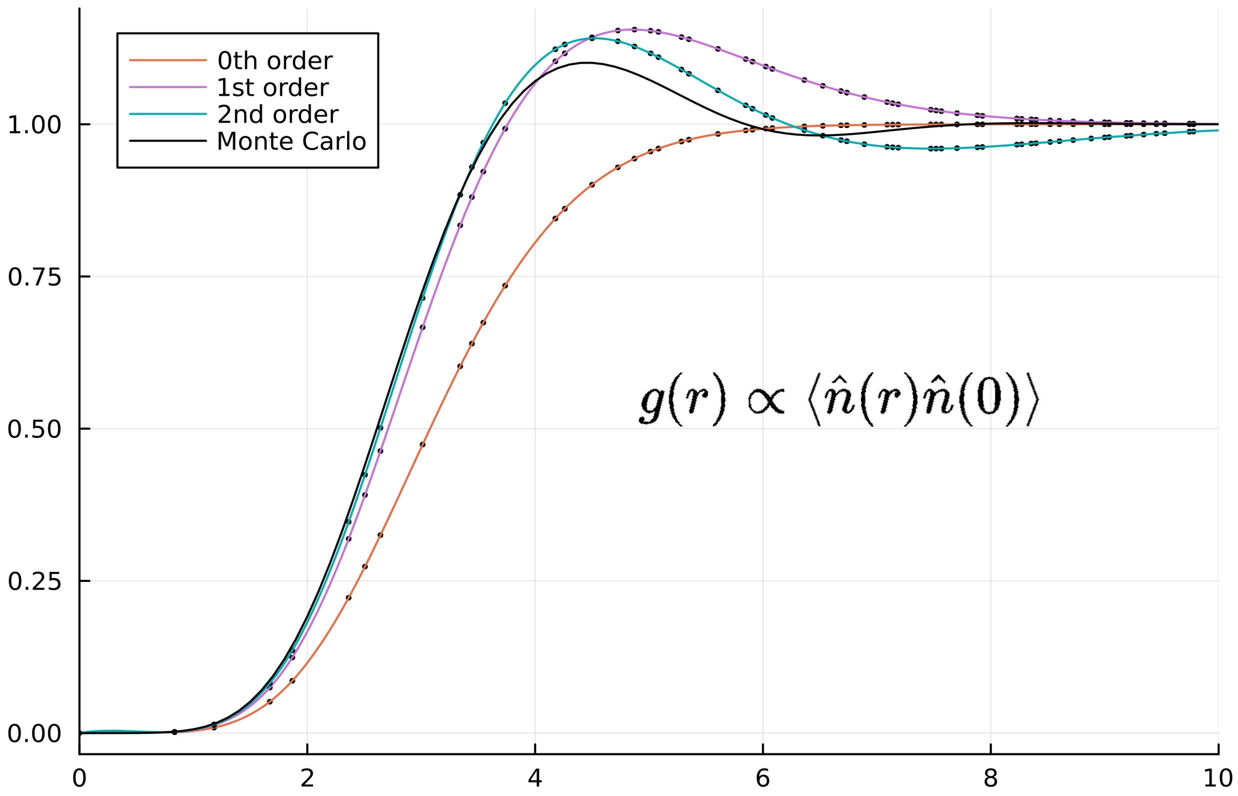
Projective expansion: Benchmark Results

- We develop a technique to simulate Hdet wavefunctions, with approximation improvable order-by-order.



The pair-correlation function for 1/3 Laughlin state

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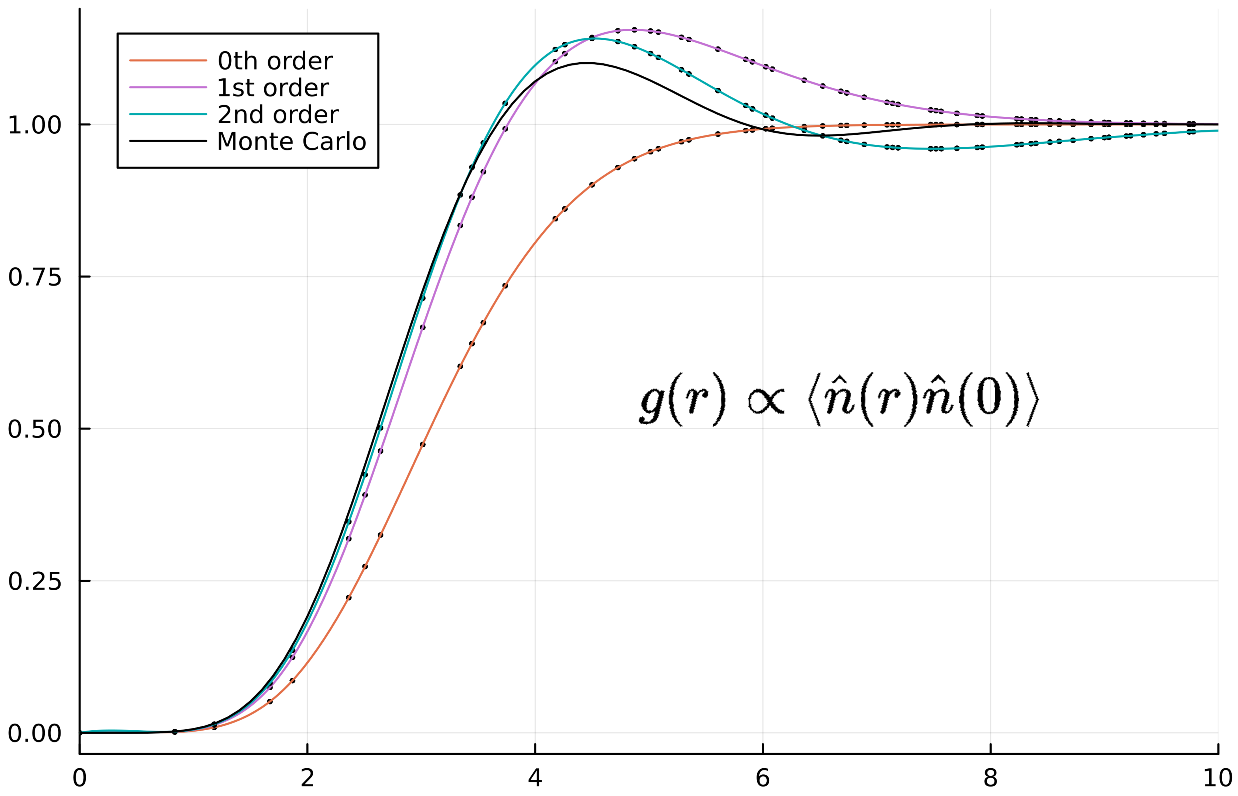

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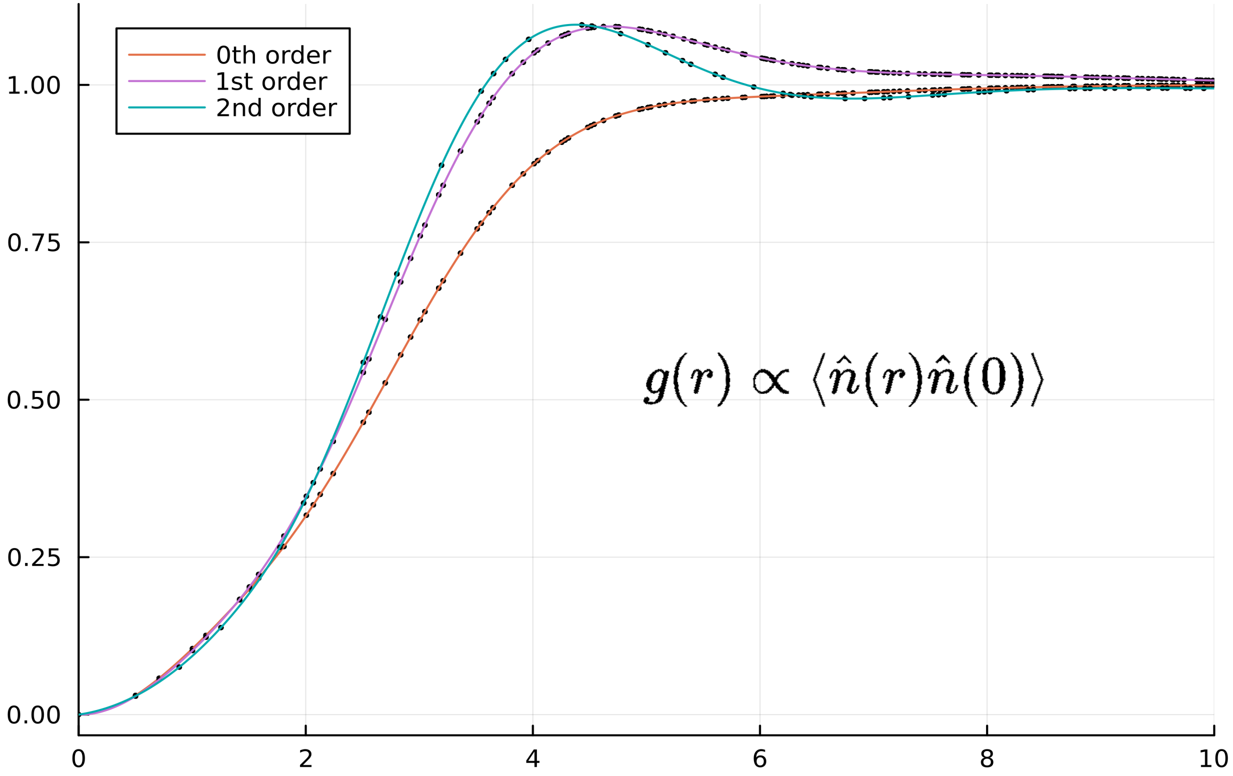

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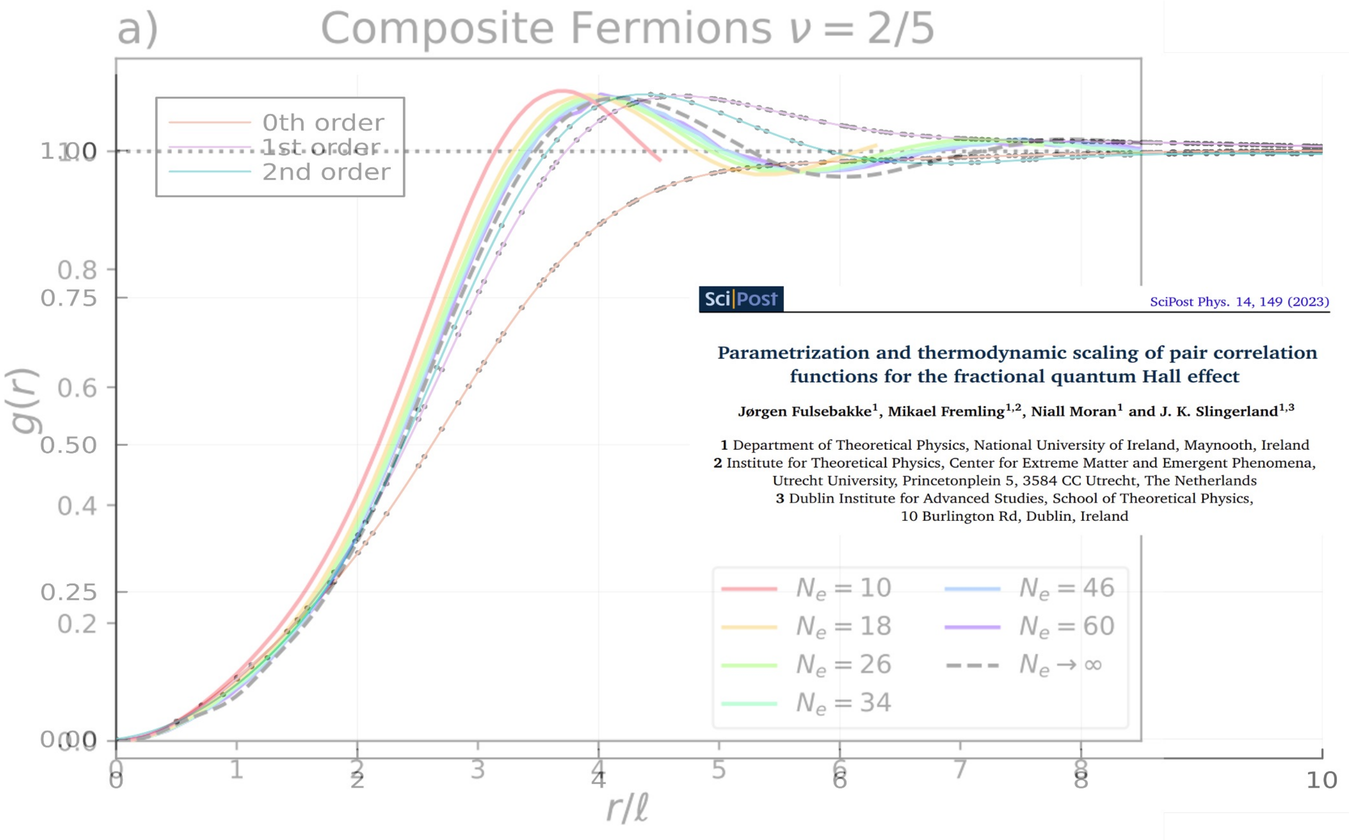

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Projective expansion: Benchmark Results

- Square lattice periodic potential in lowest Landau level

$$H = \lambda \cdot H_K + V$$

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$$H_K = P_{LLL} \sum_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} P_{LLL}$$

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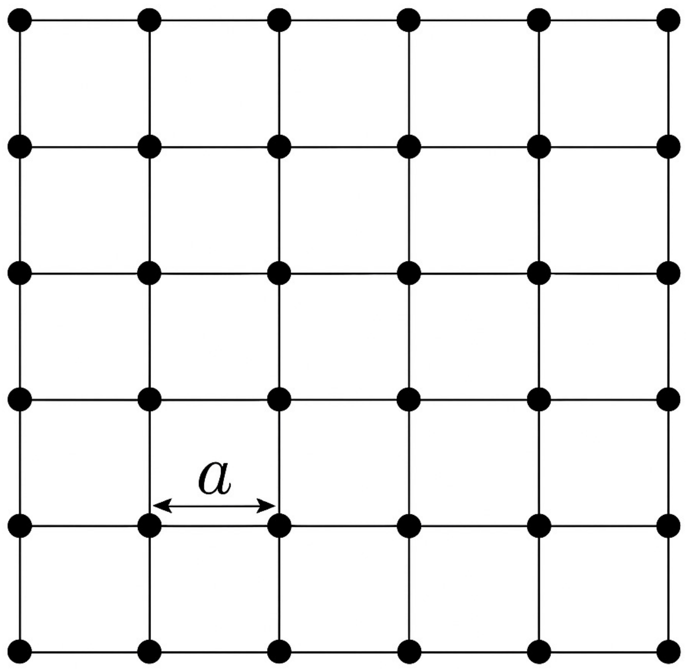

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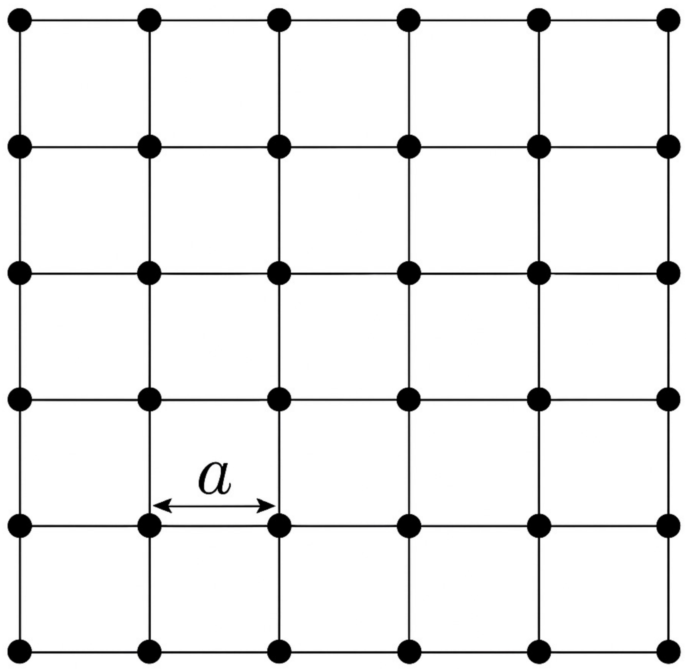

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(1) $V = V_1 \sum_{i<j} \hat{P}_{ij}^{(m_{\text{rel}}=1)}$

Haldane pseudopotential

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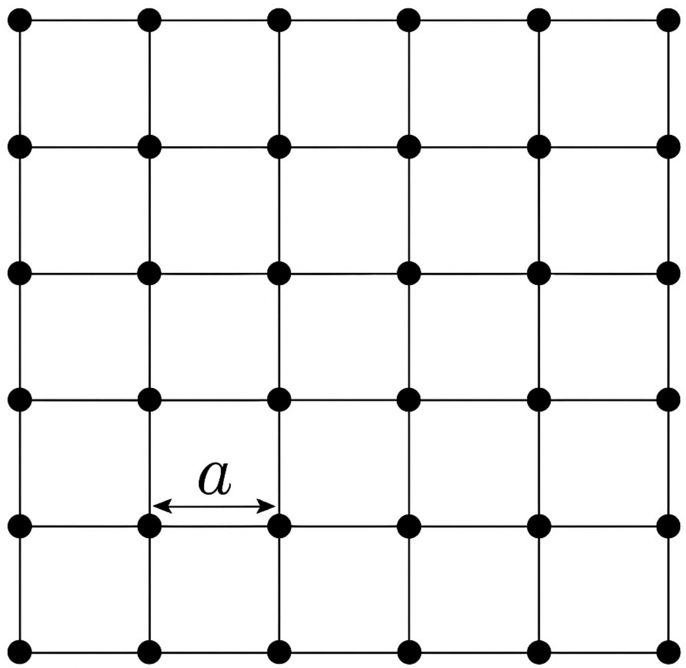

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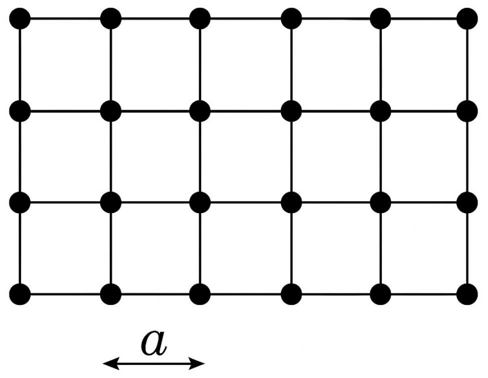
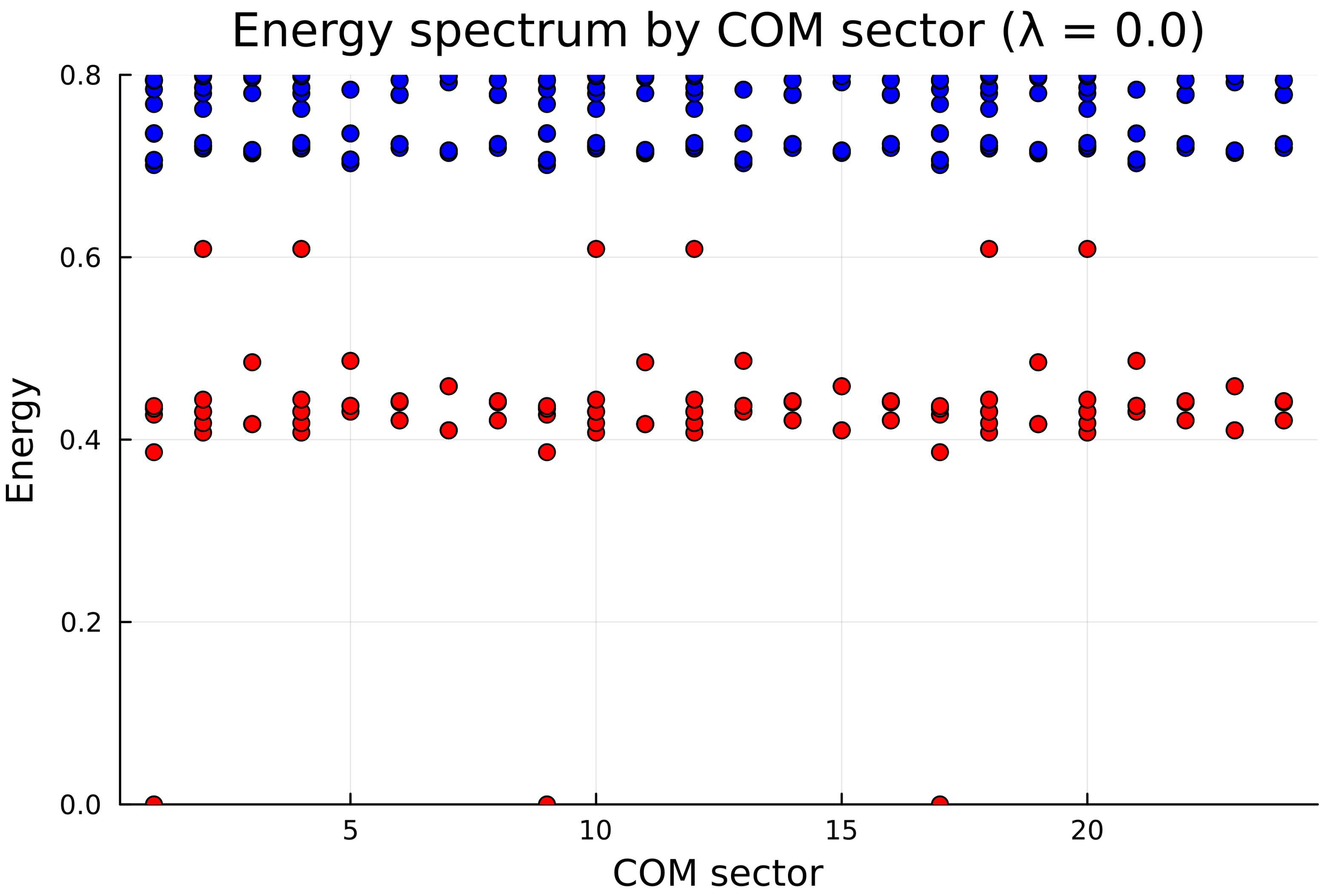


6x4 exact diagonalization



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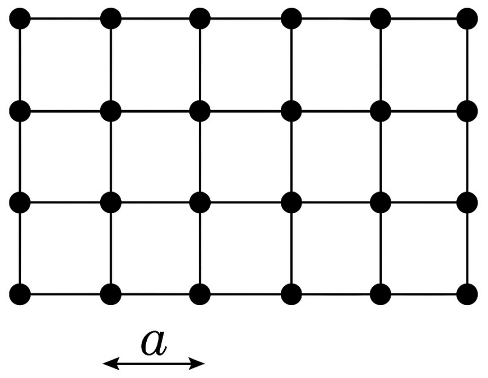
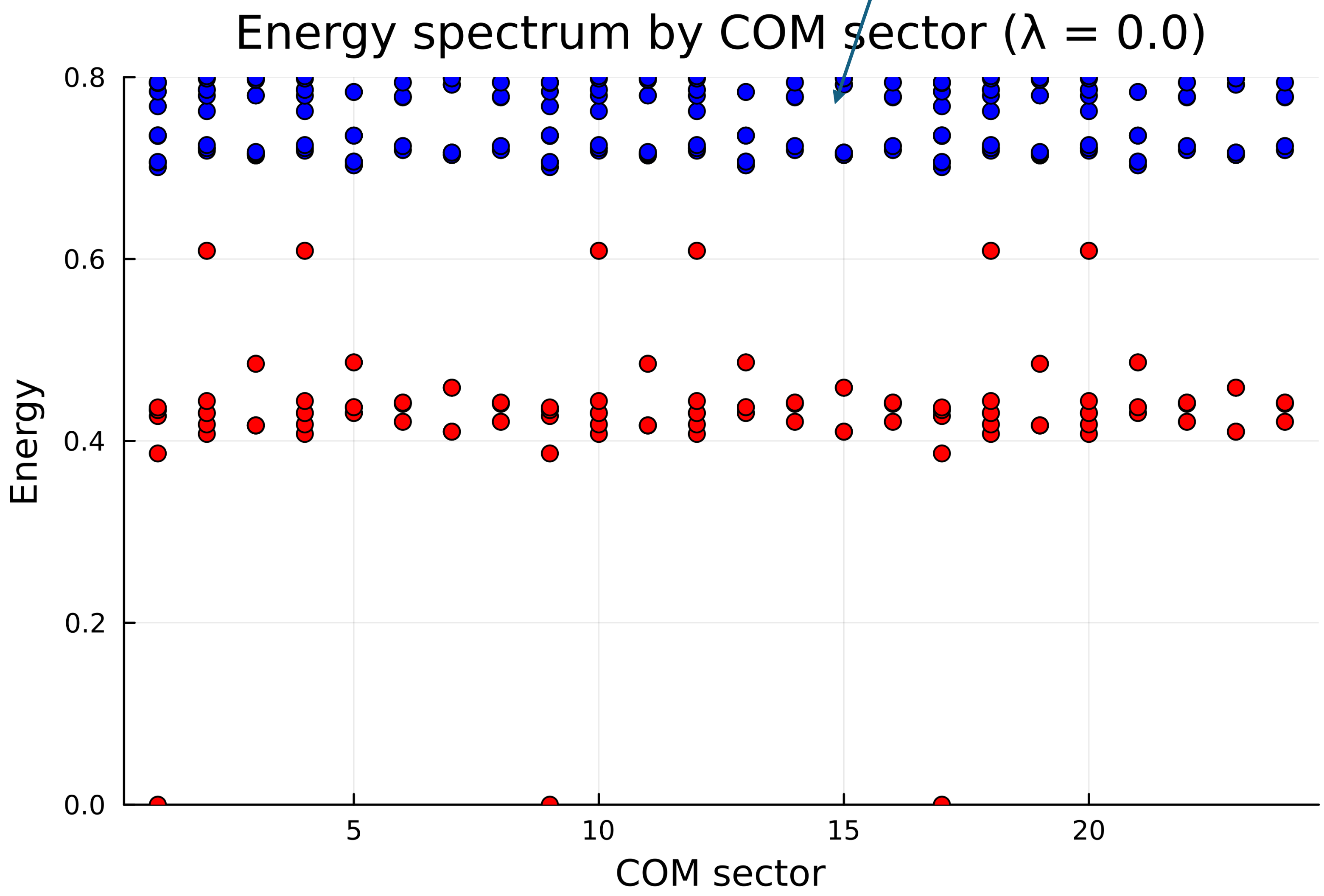
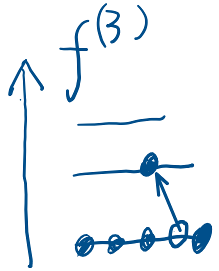


6x4 exact diagonalization

Blue: parton particle-hole continuum





Haldane V1=1 pseudopotential

$$H = \lambda \cdot H_K + V$$

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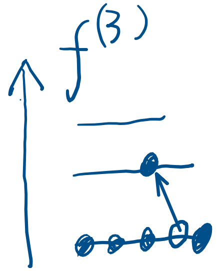

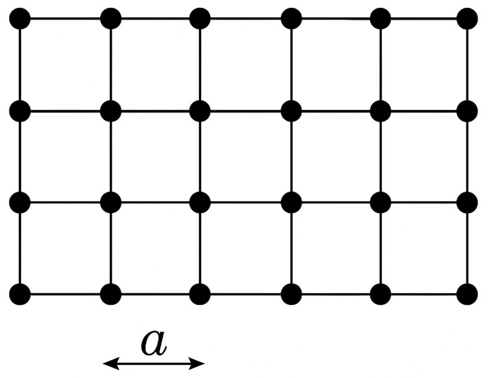

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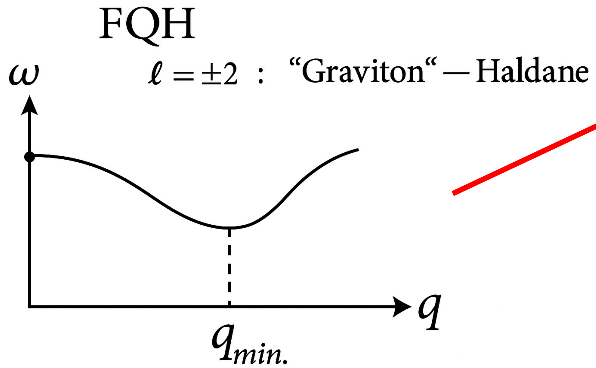

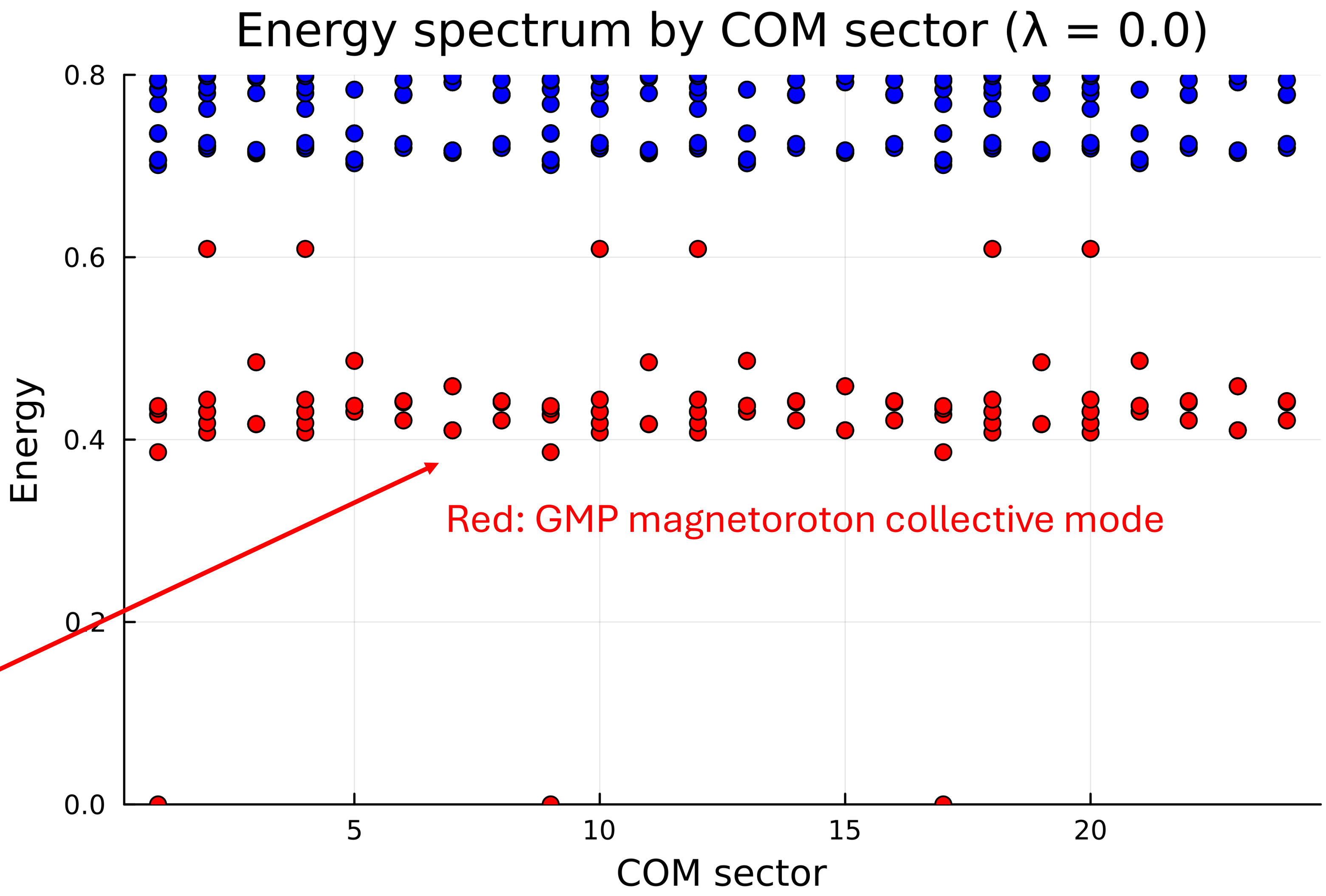

Red: GMP magnetoroton collective mode

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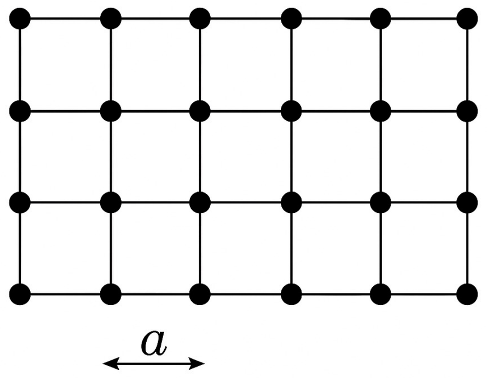

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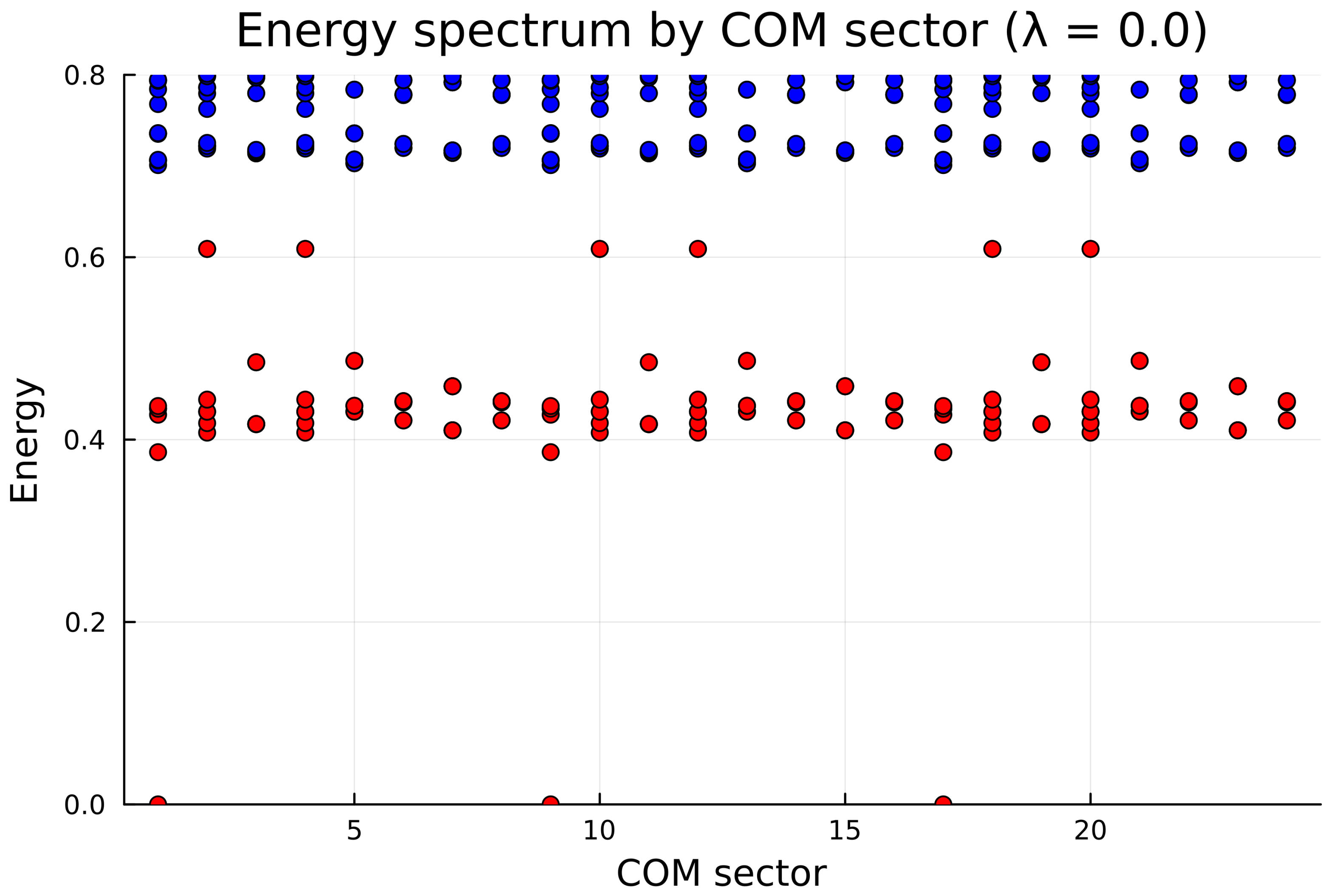

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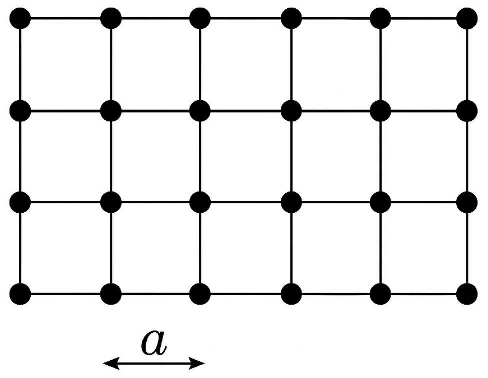

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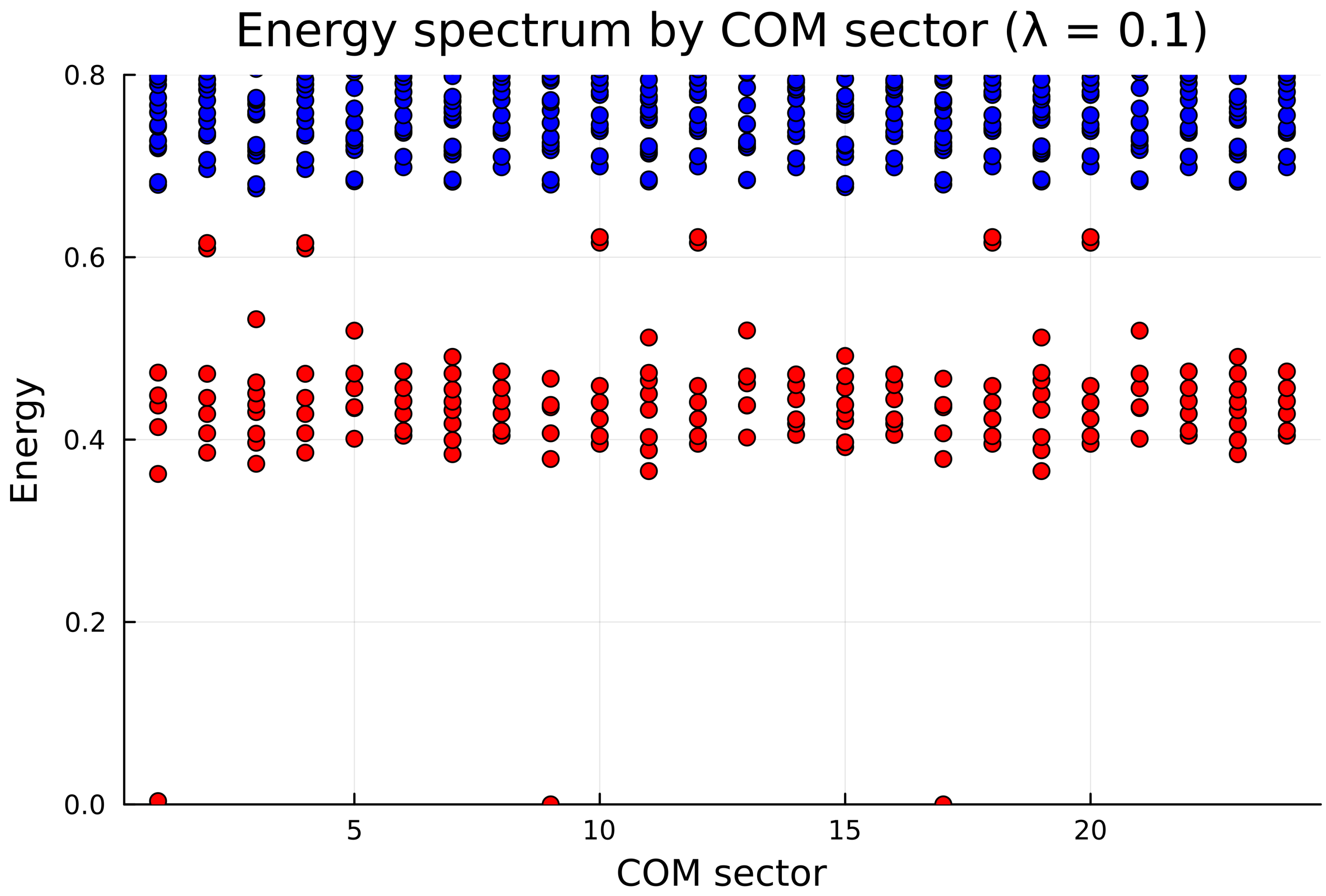

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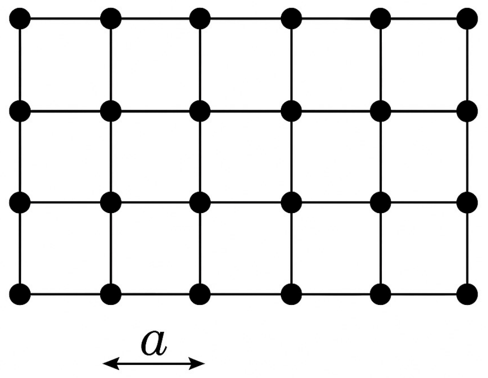

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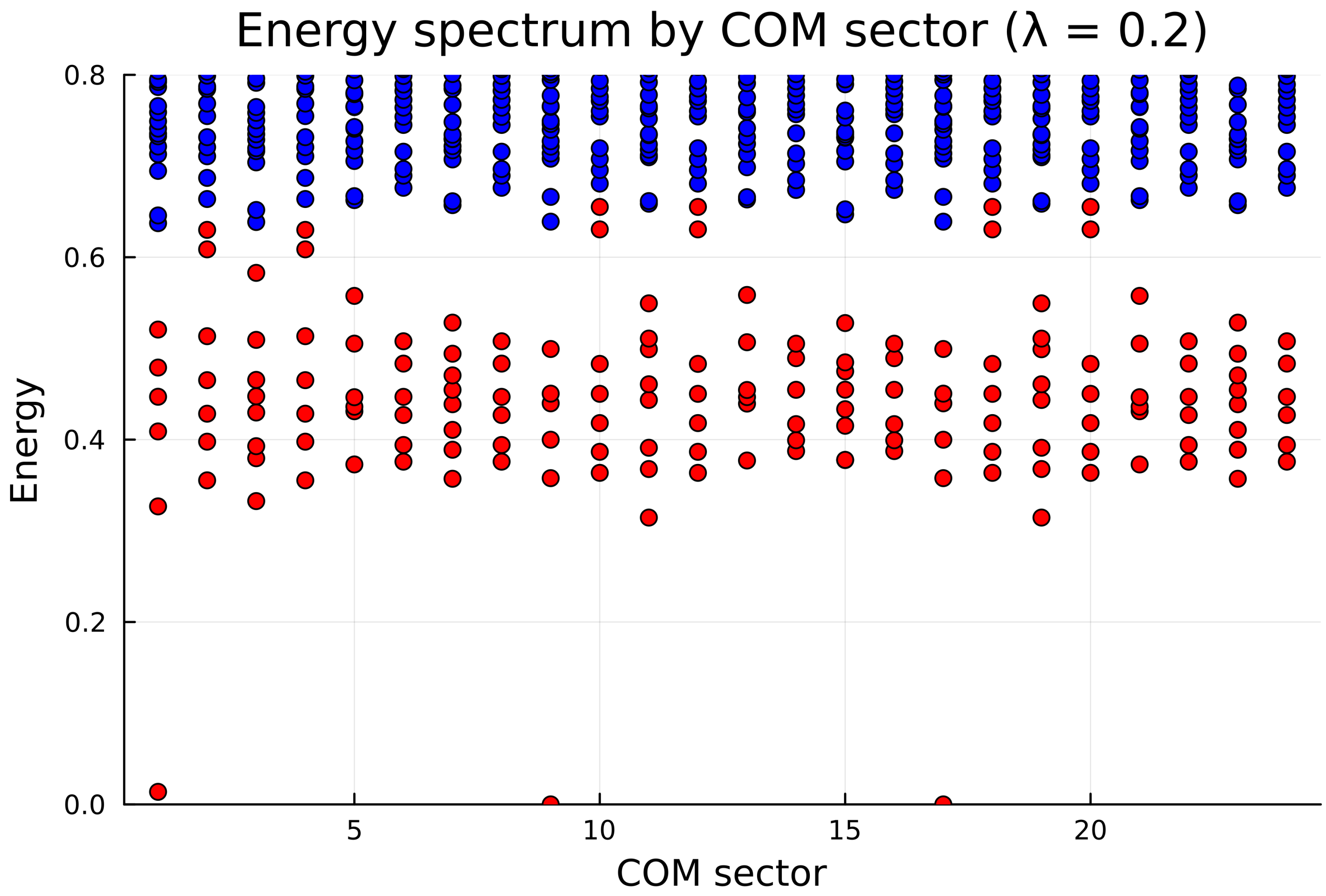

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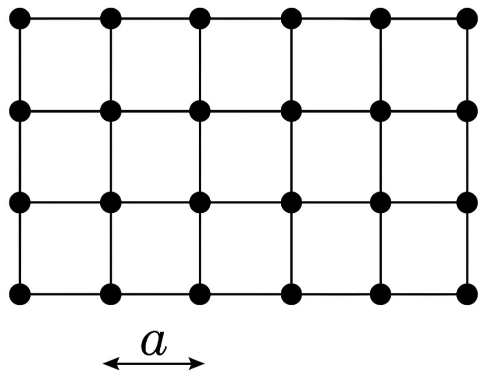

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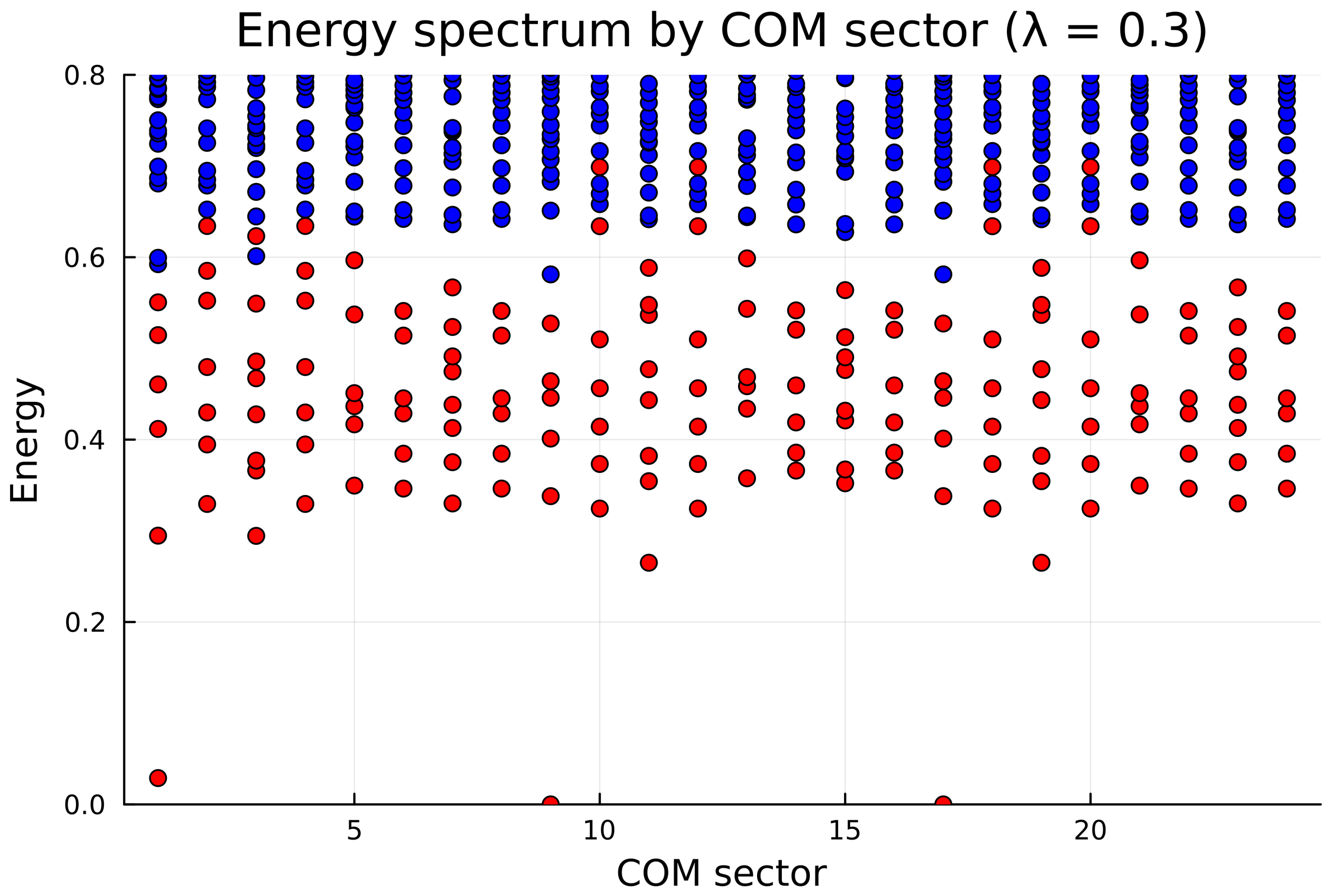

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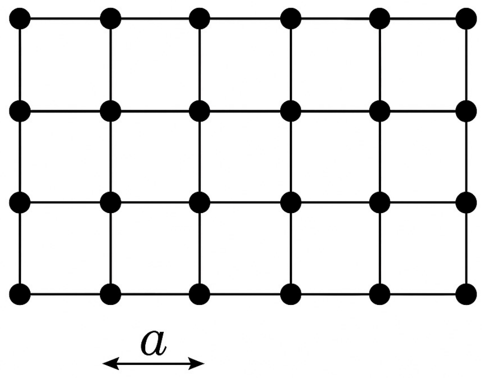

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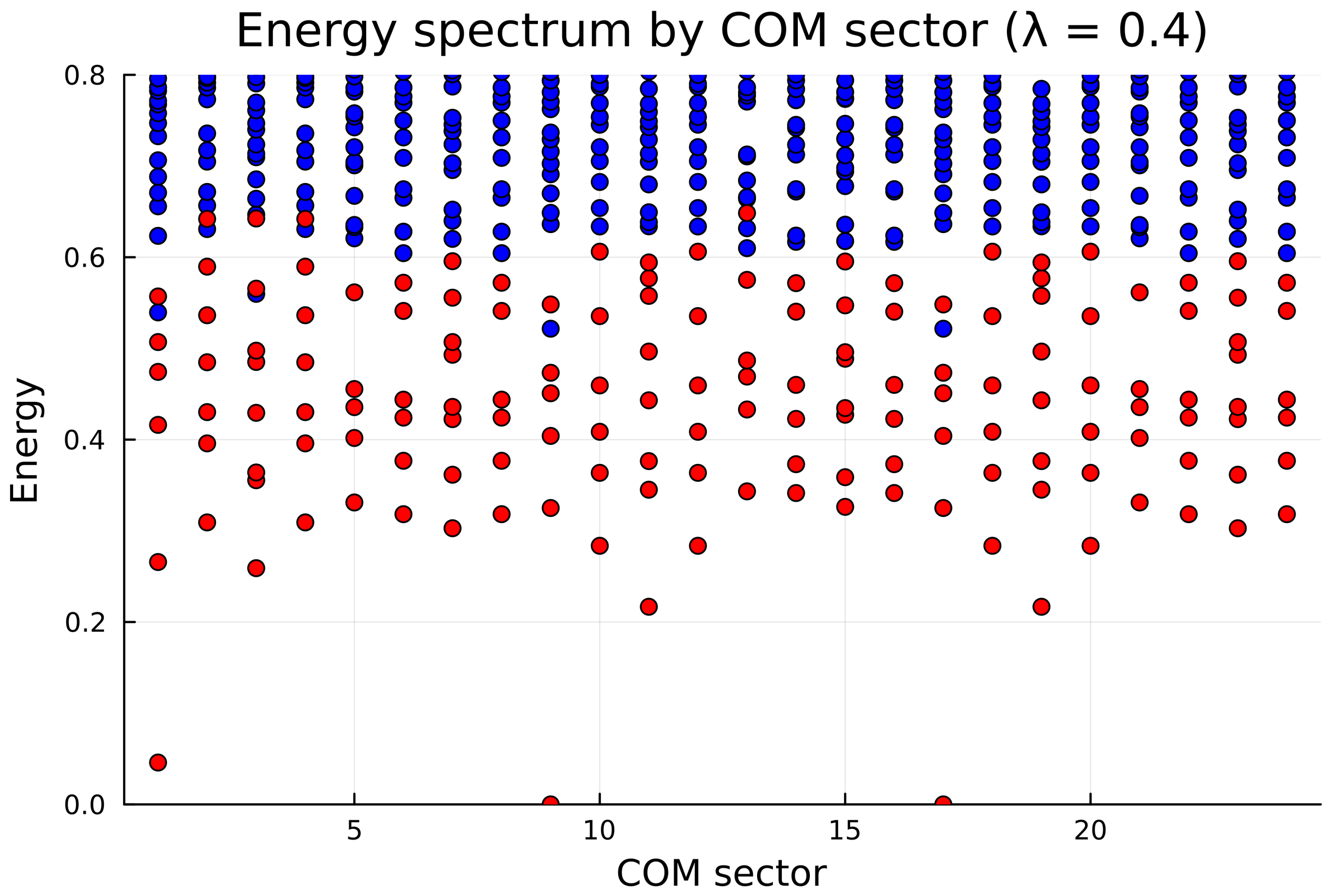

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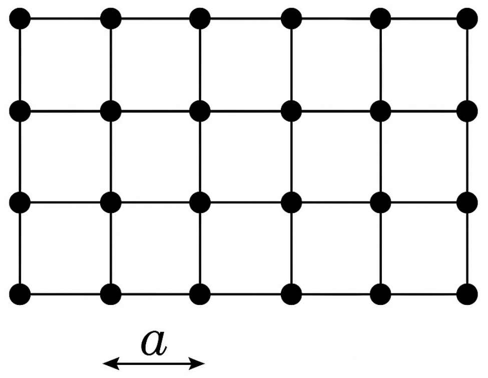

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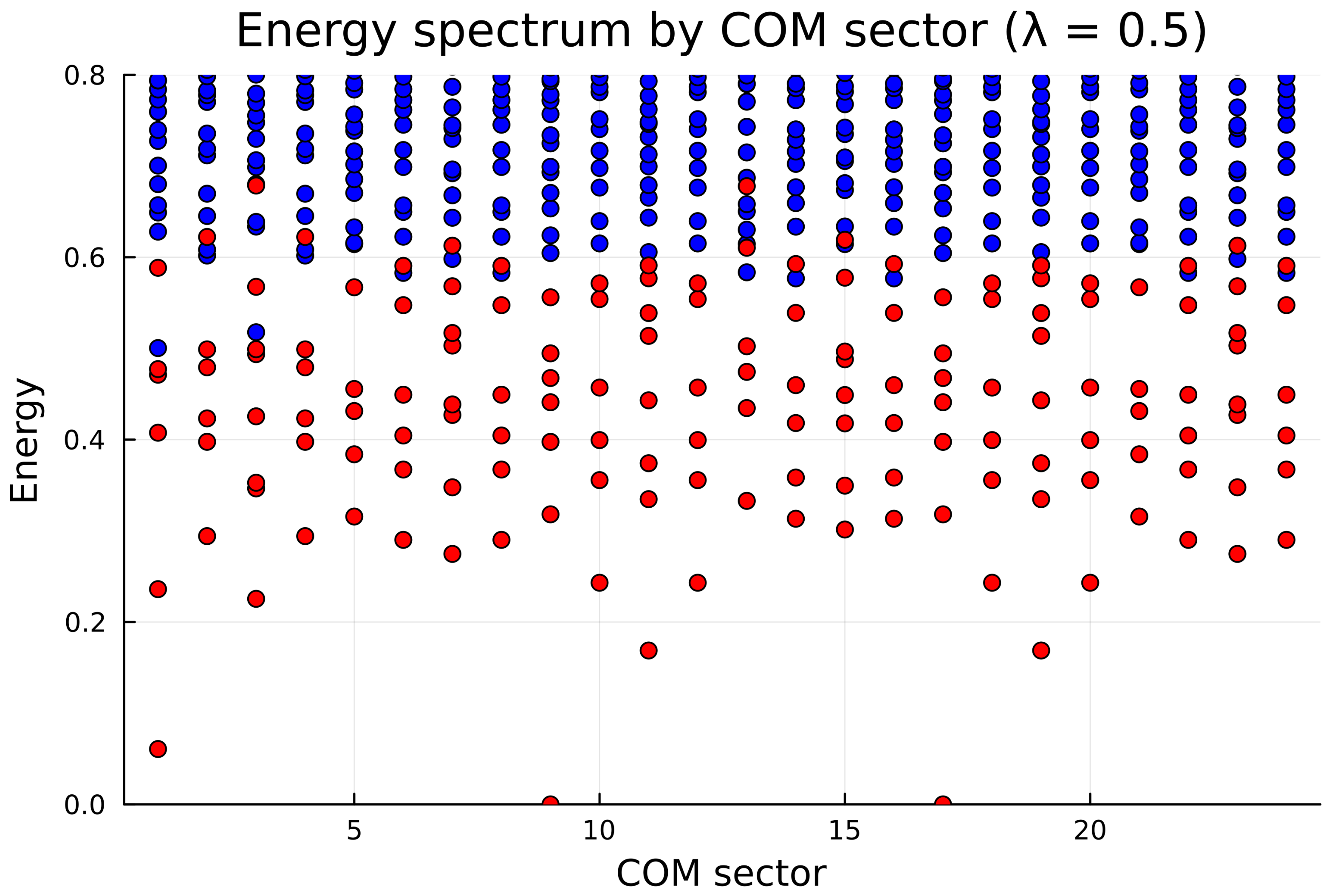

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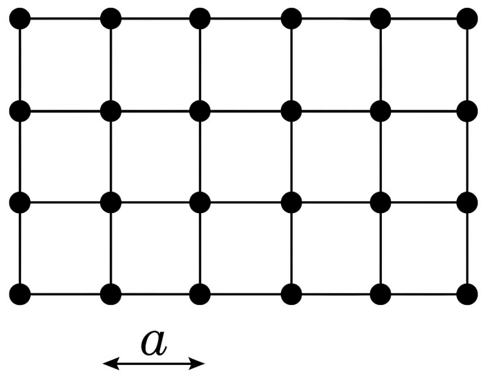

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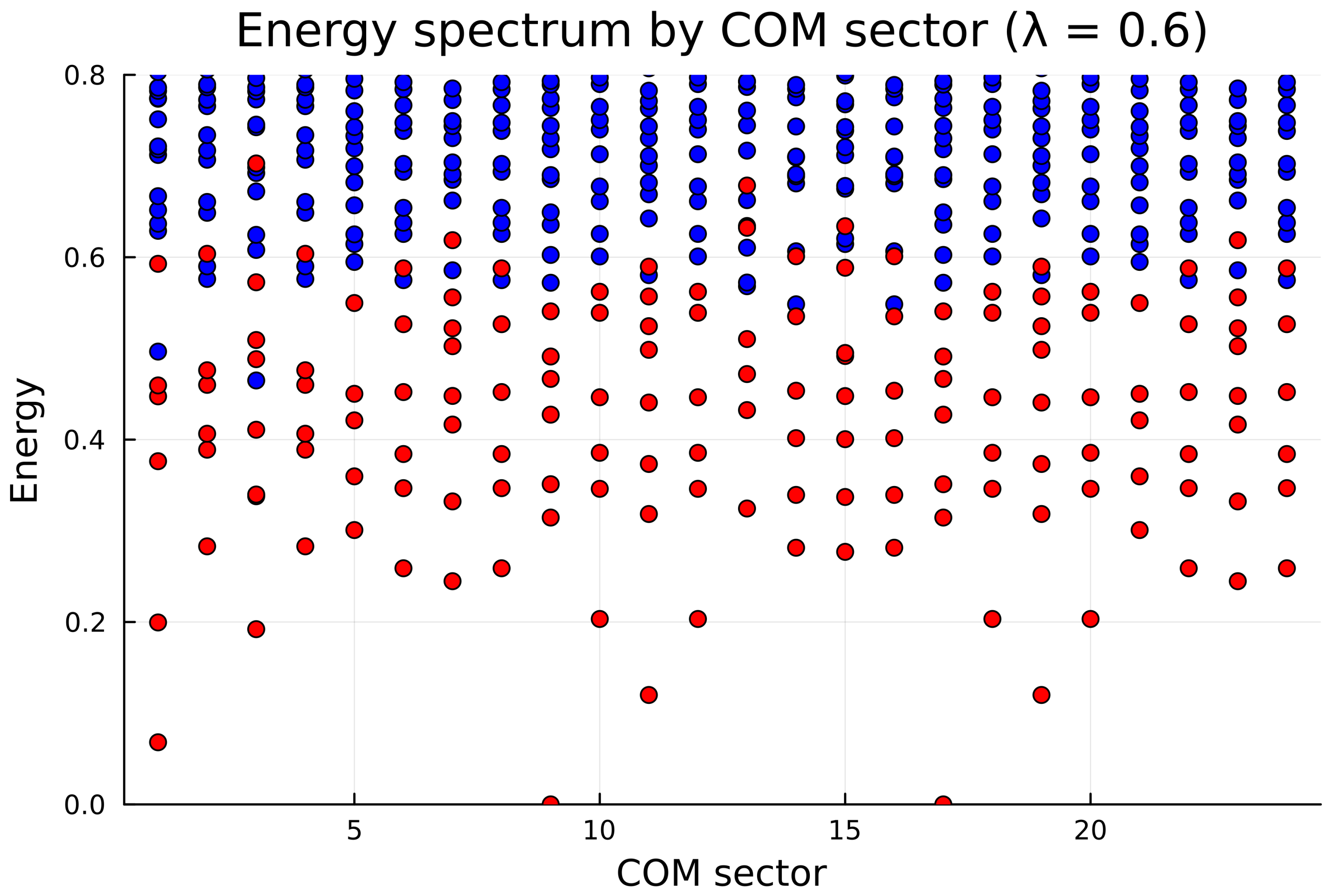

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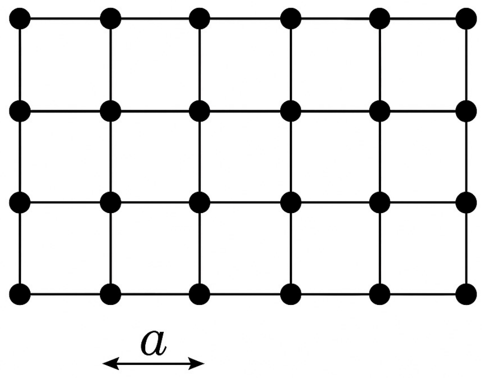

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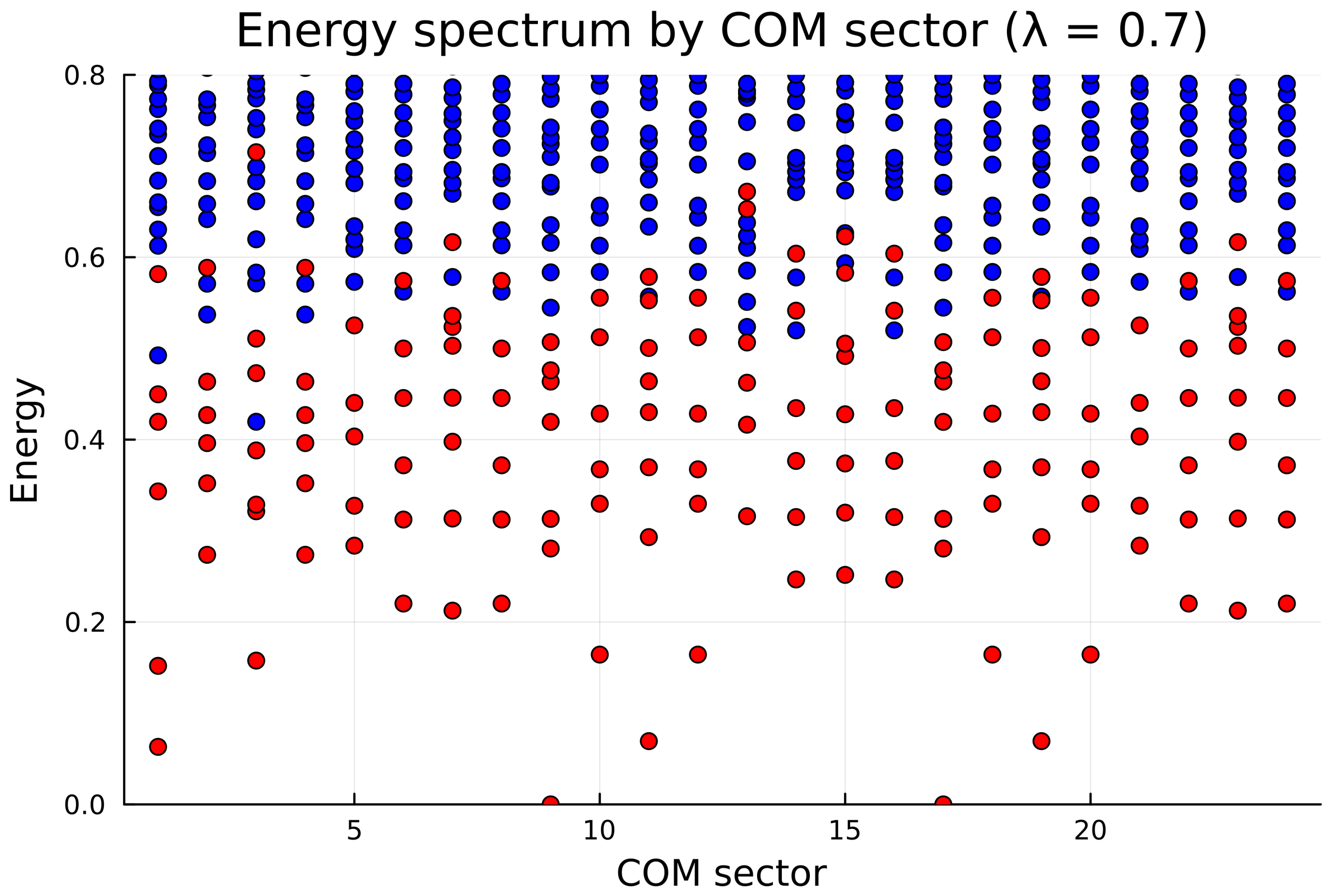

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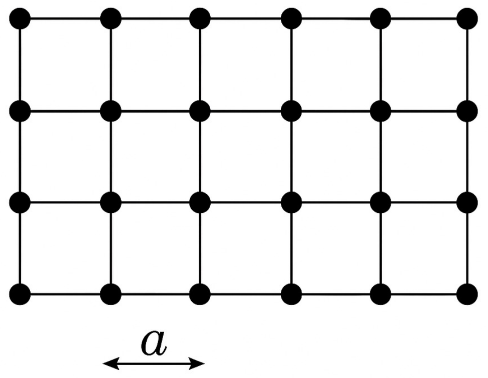

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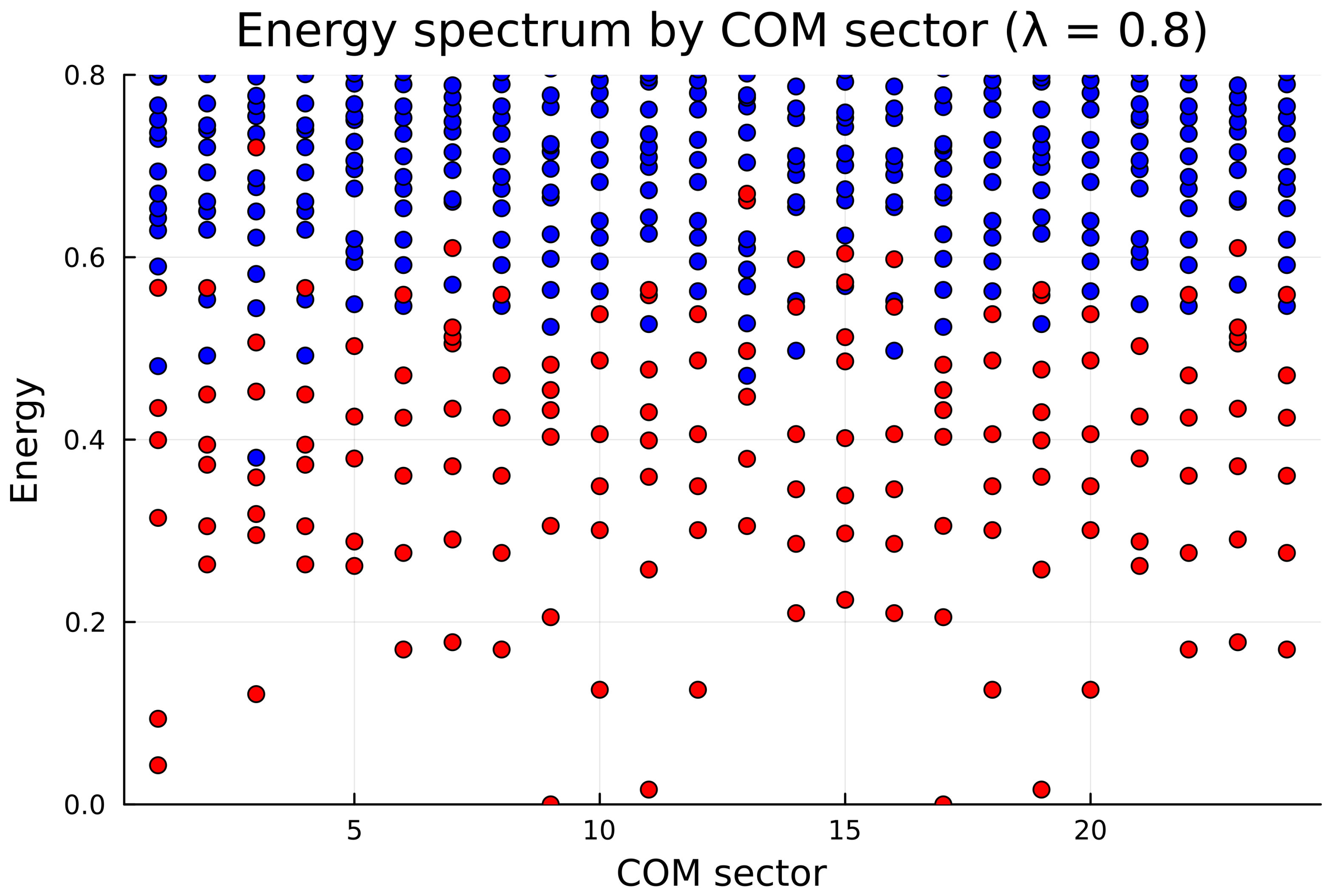

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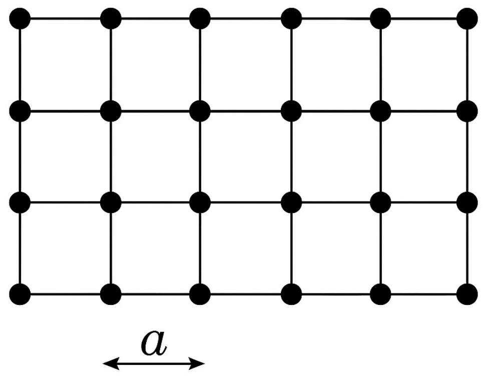

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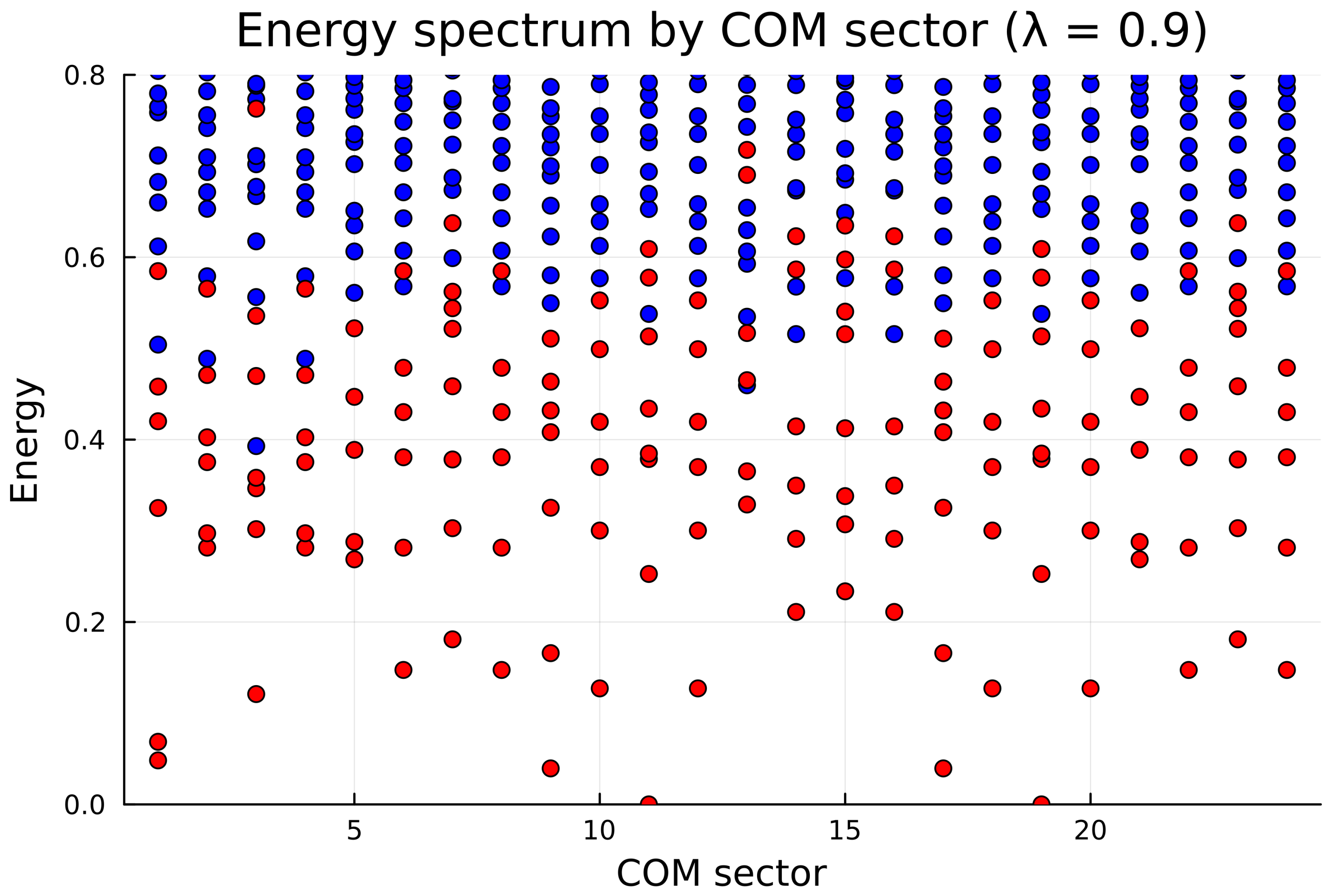

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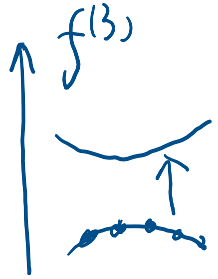
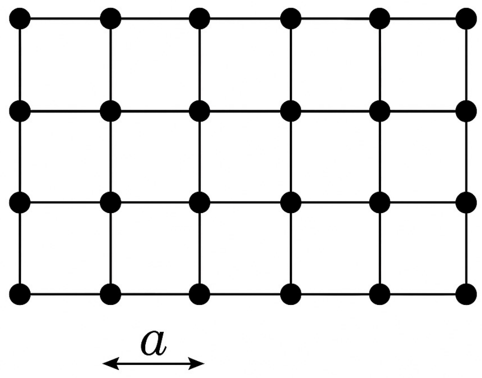
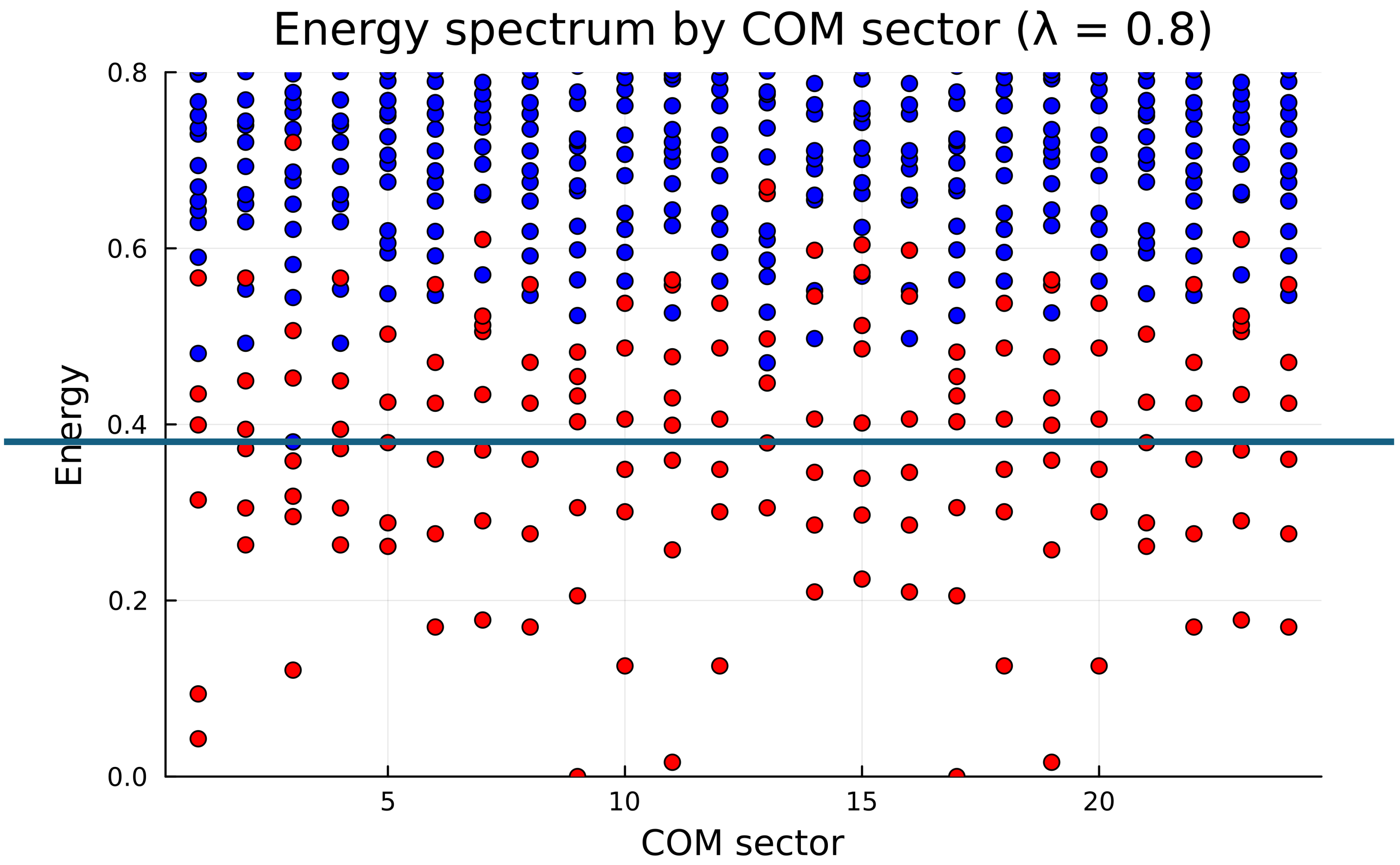
Parton particle-hole gap





6x4 exact diagonalization

Magnetoroton condense



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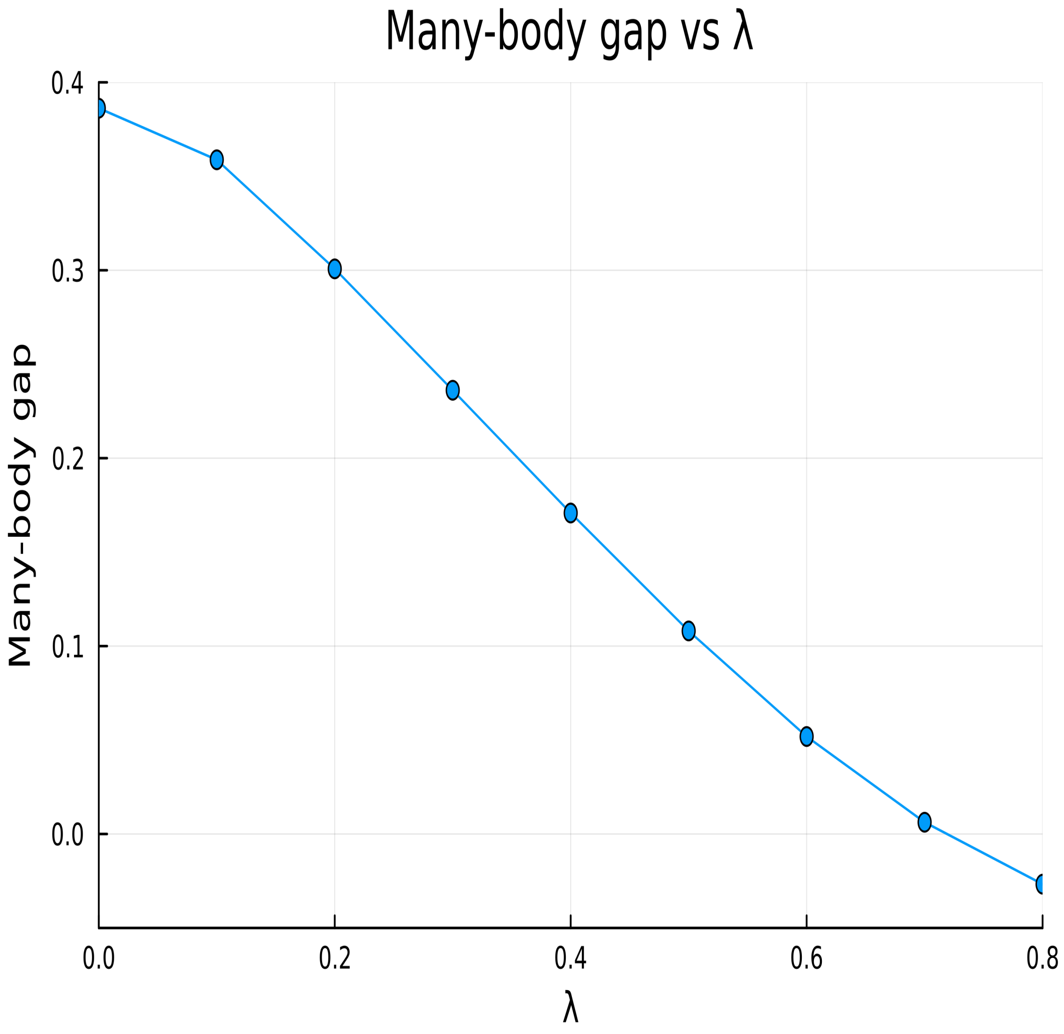

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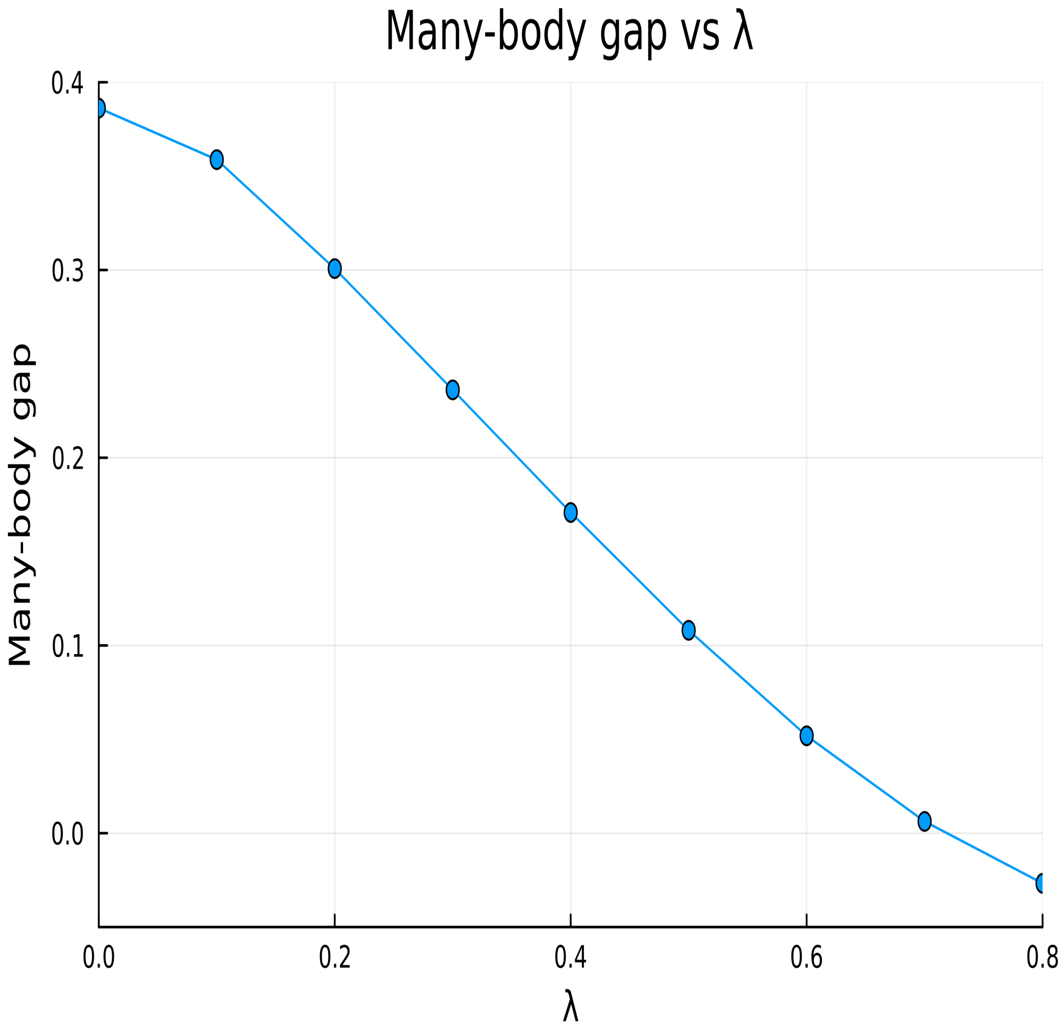
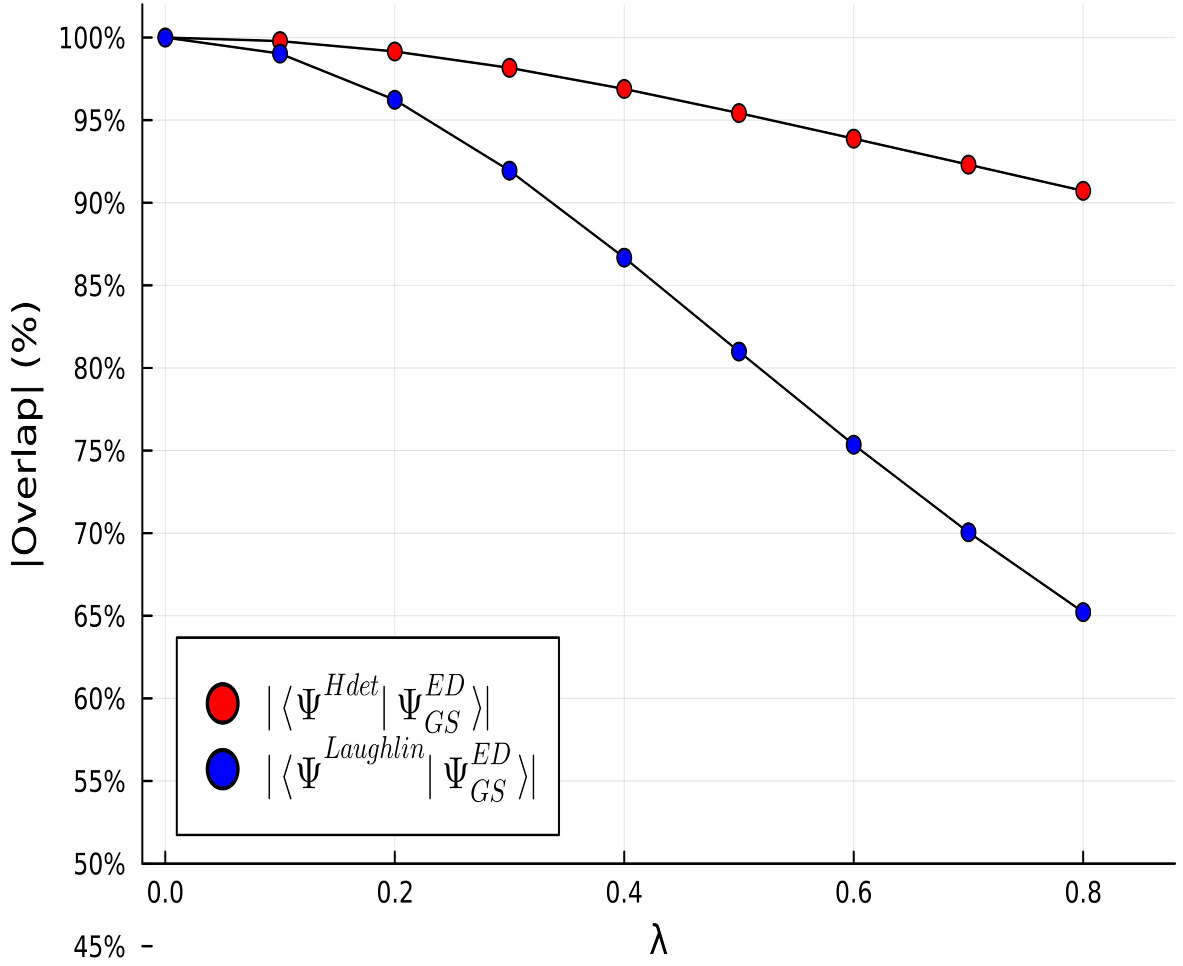
0th-order-optimized Hdet already performs well





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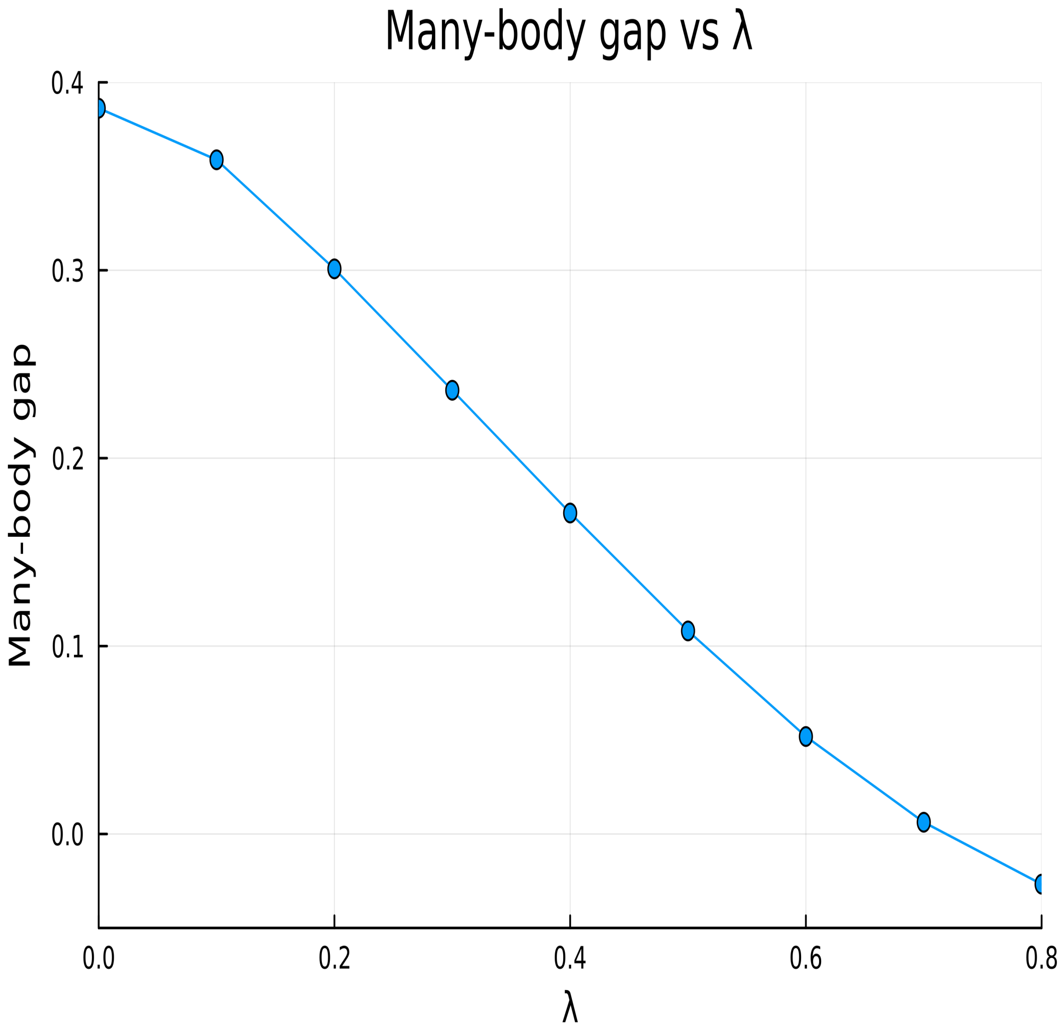
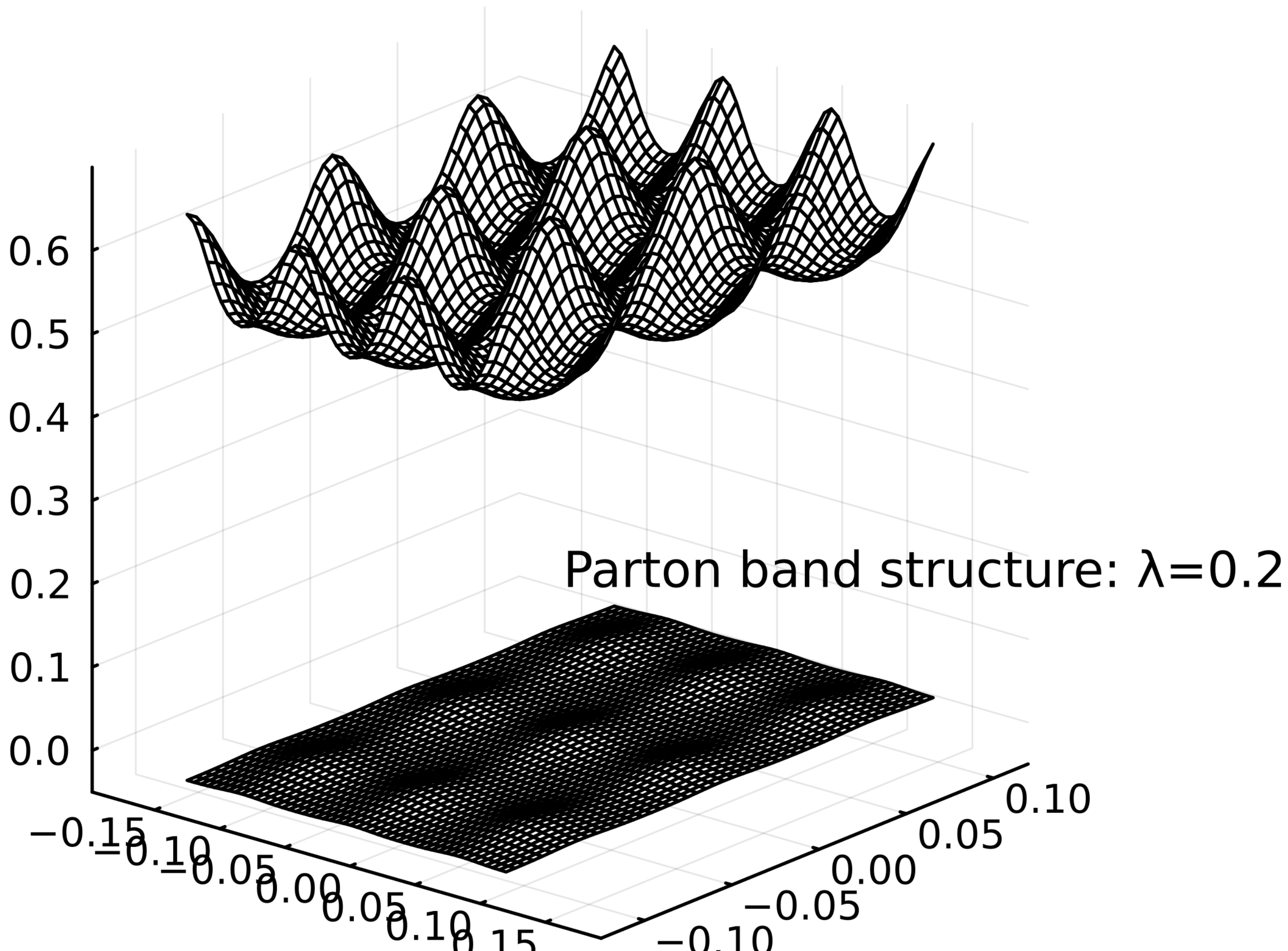
Also provide parton's bandstructure!





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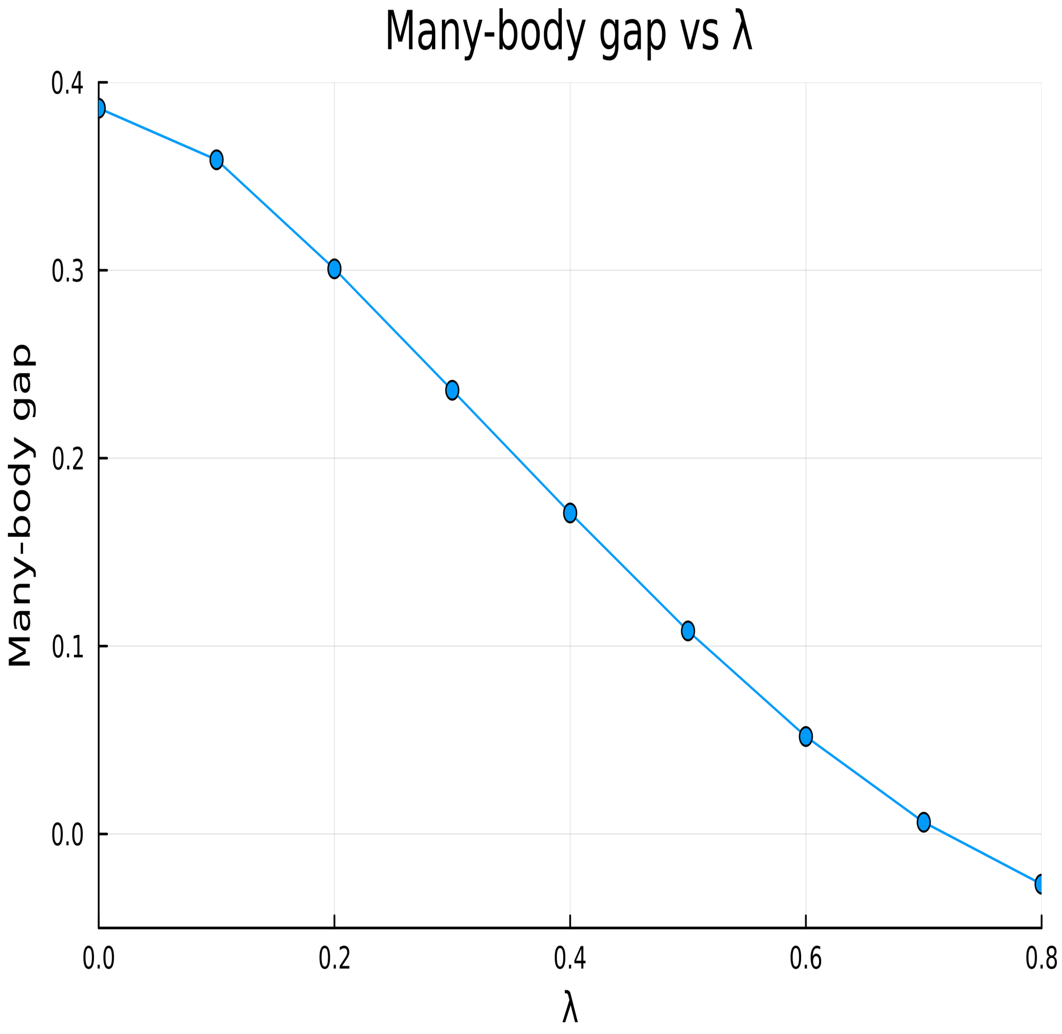

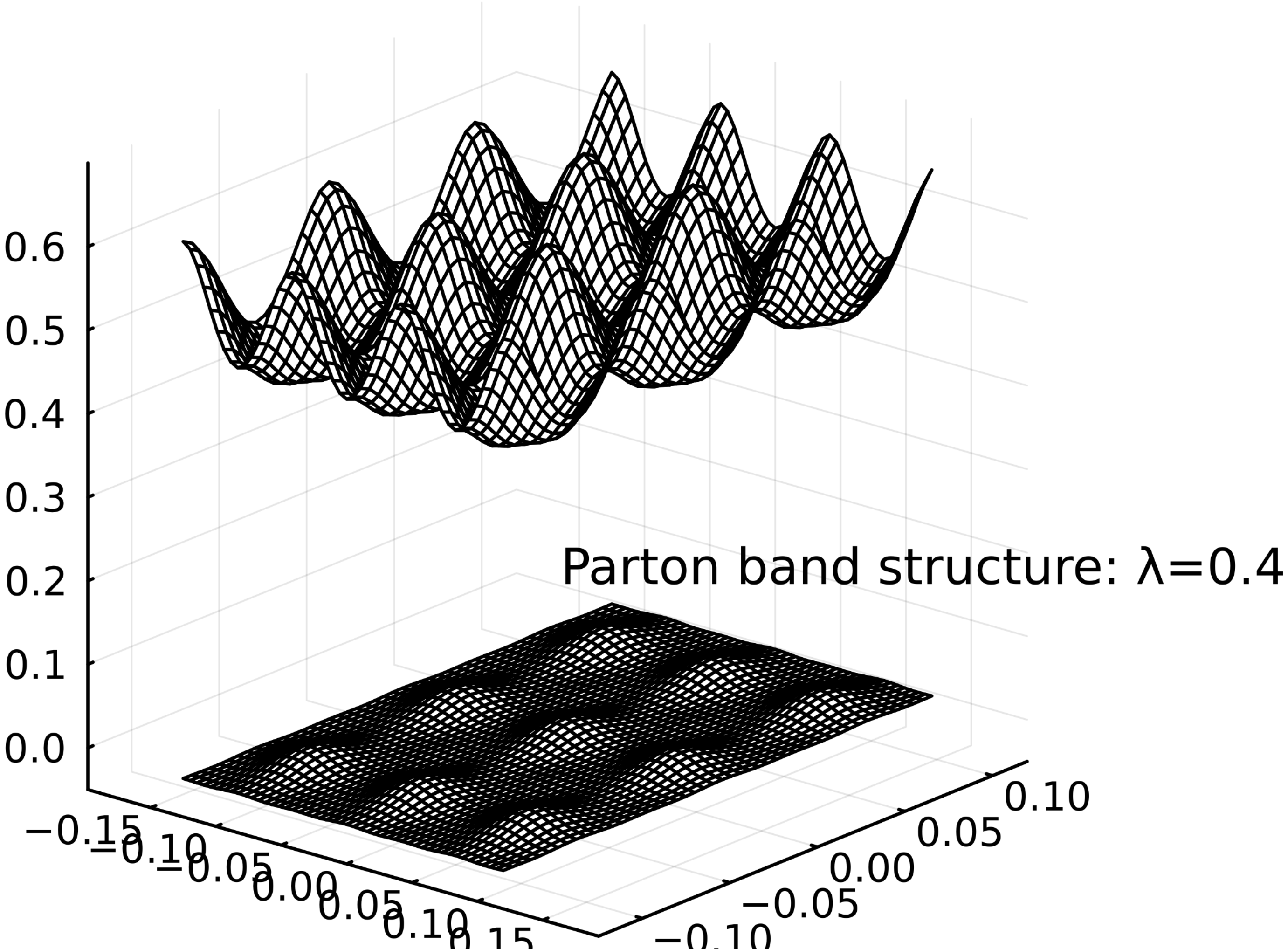

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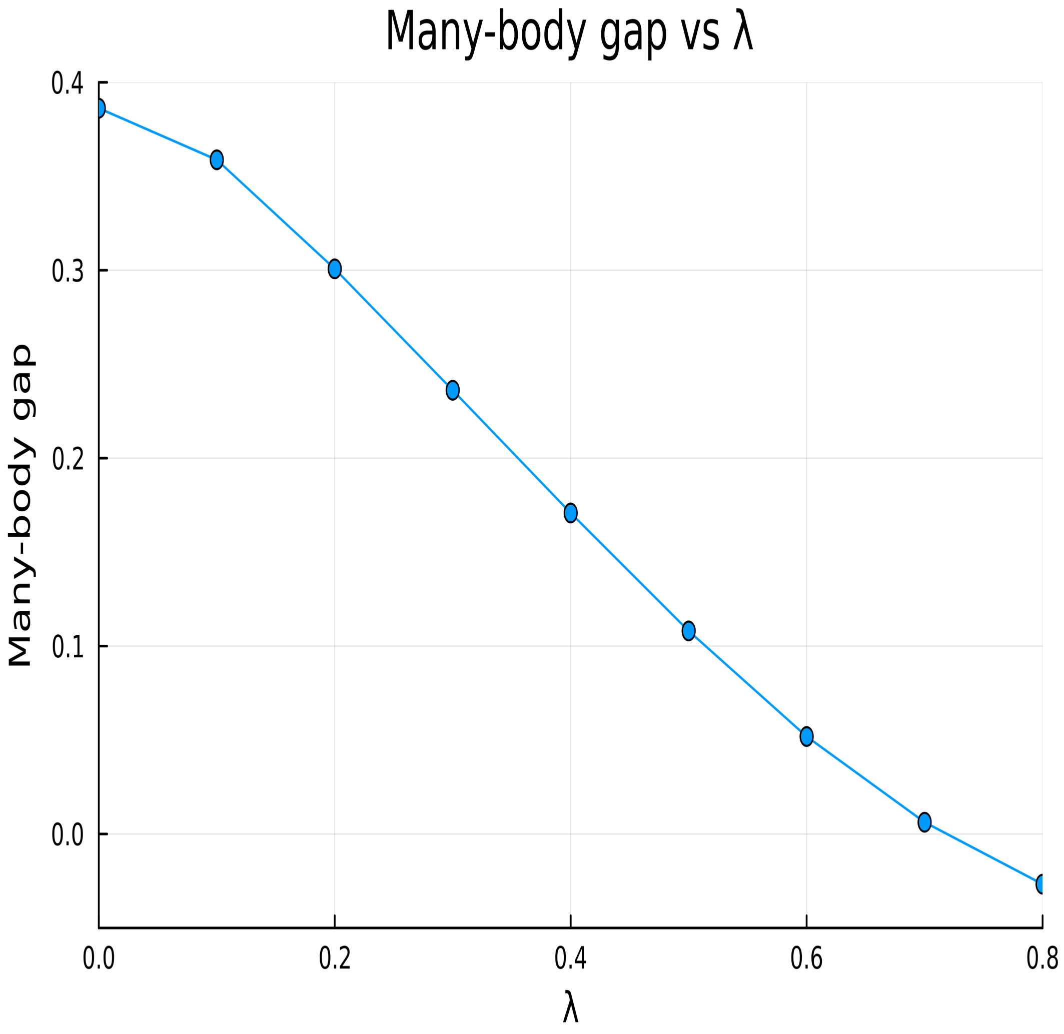

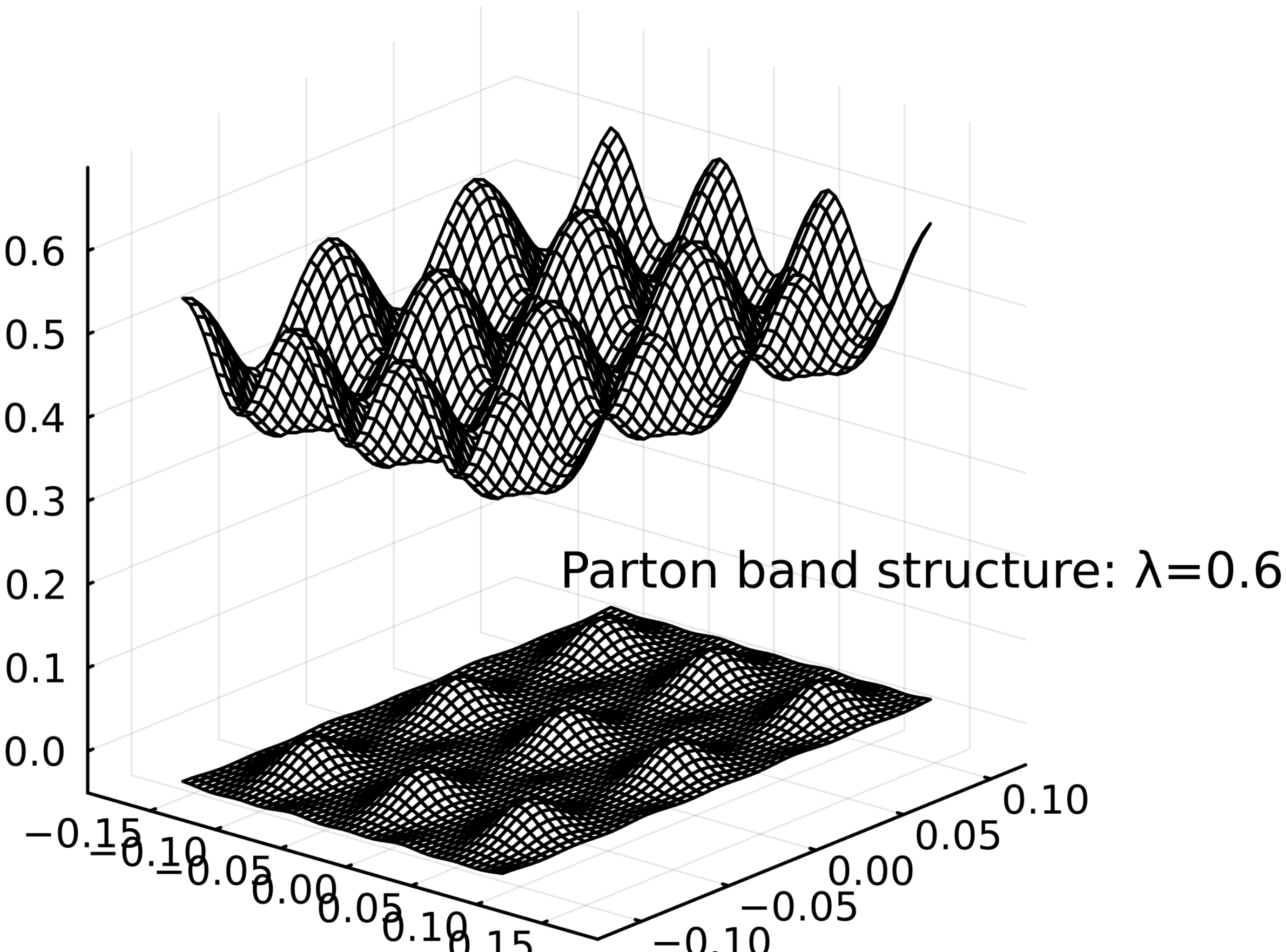

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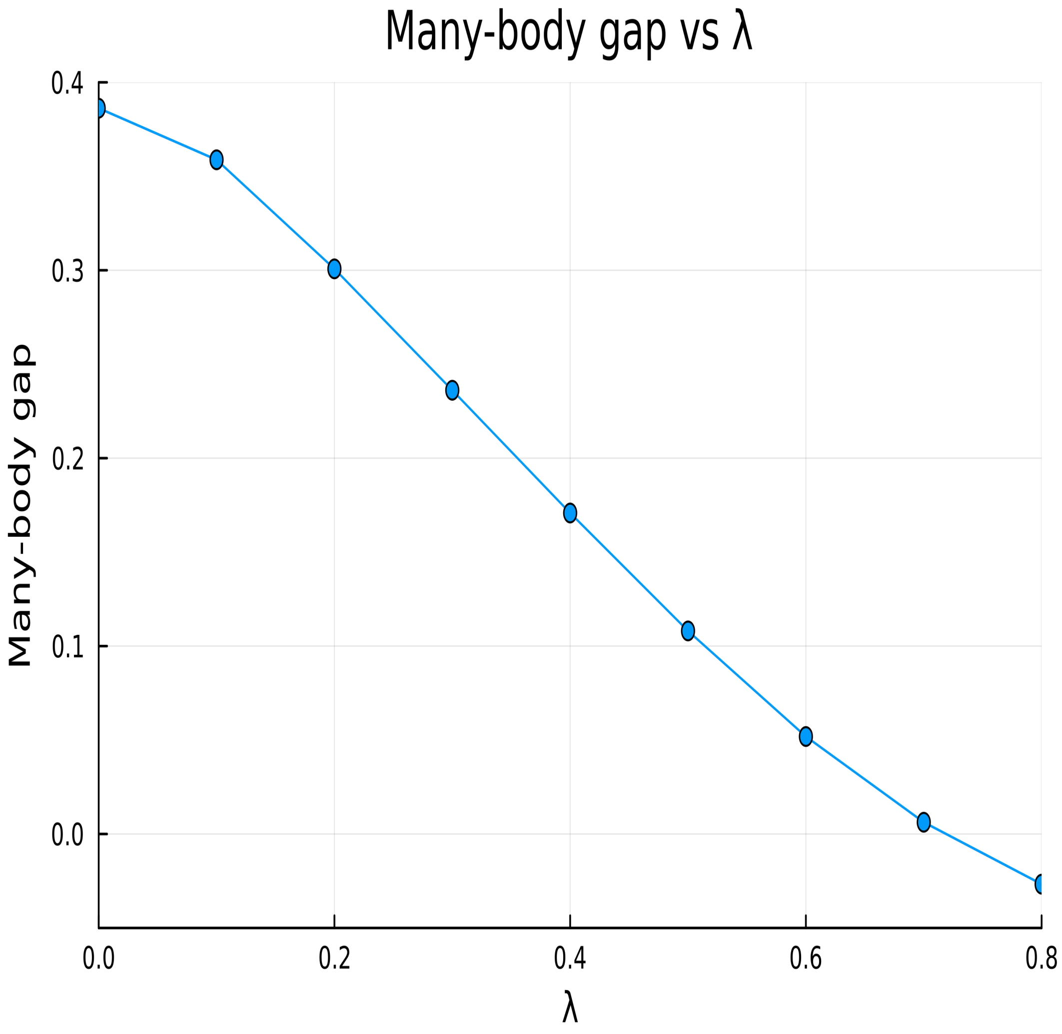

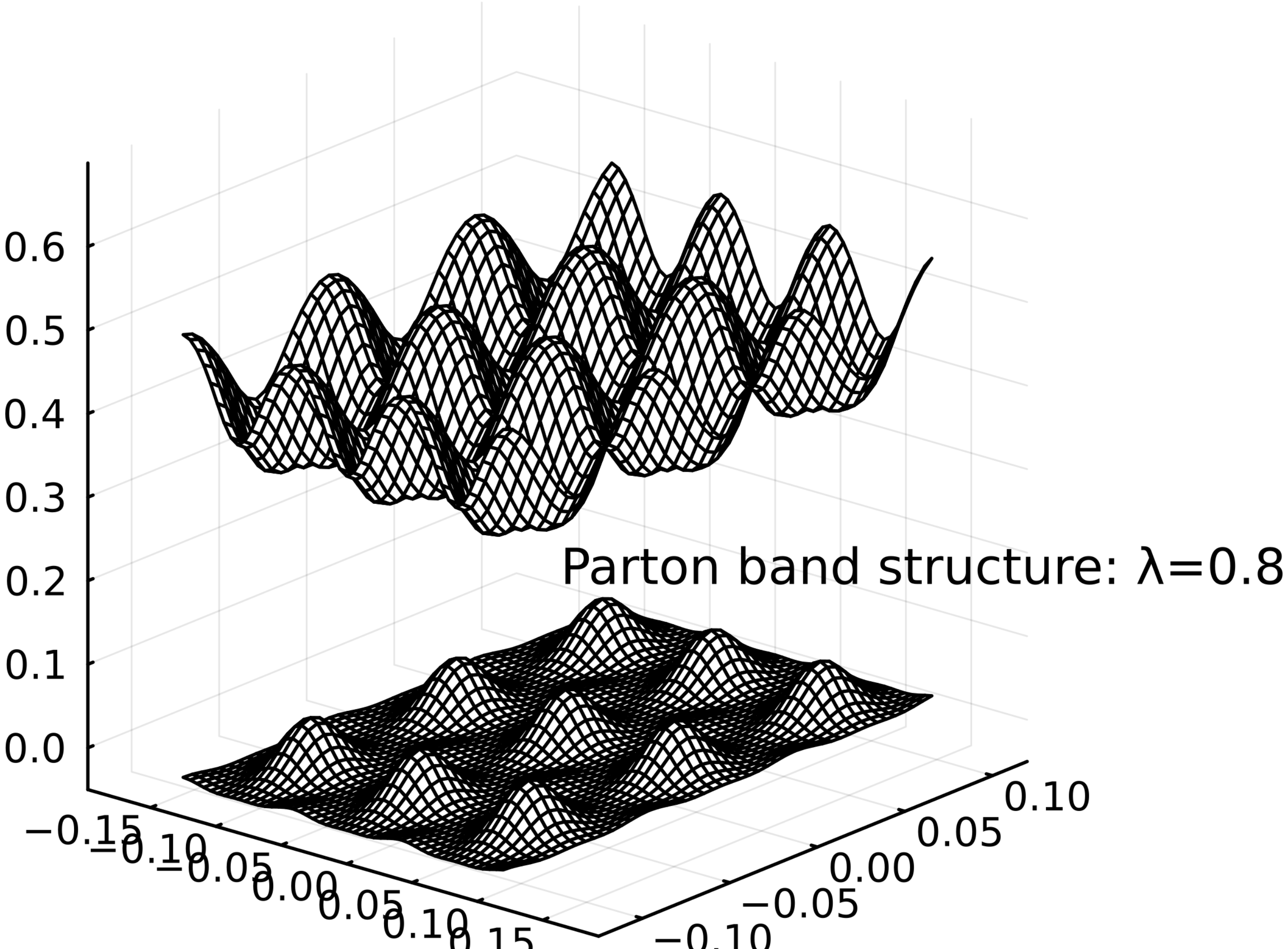

Bare Coulomb interaction

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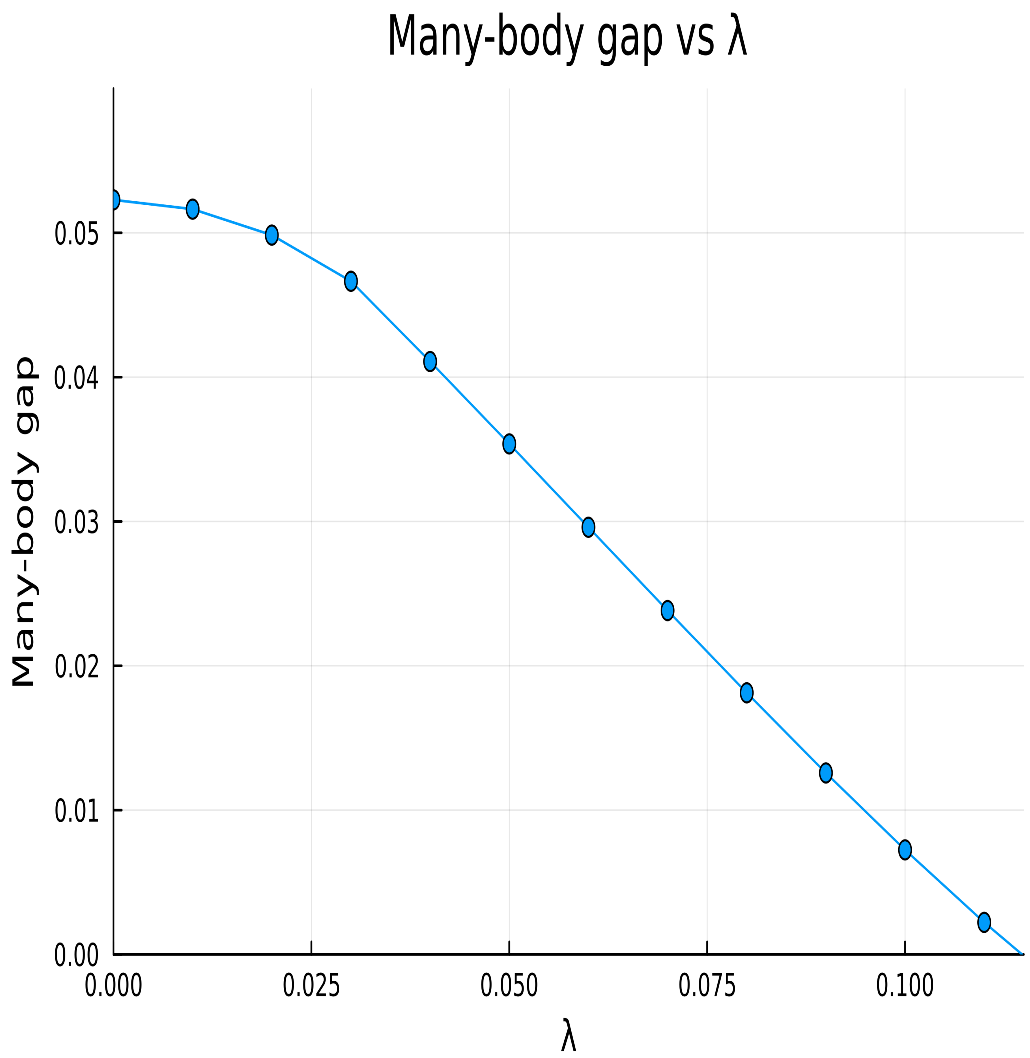

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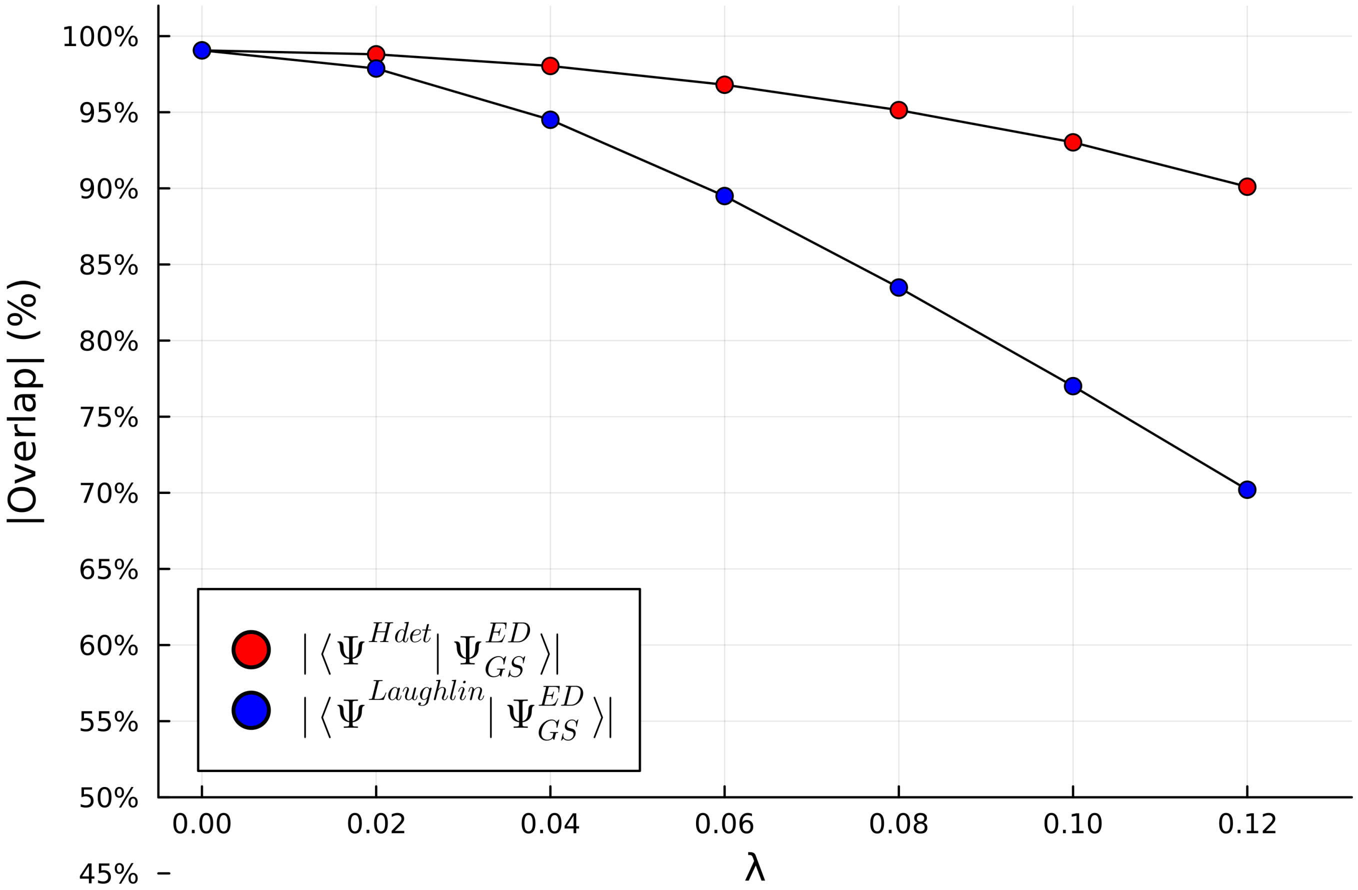
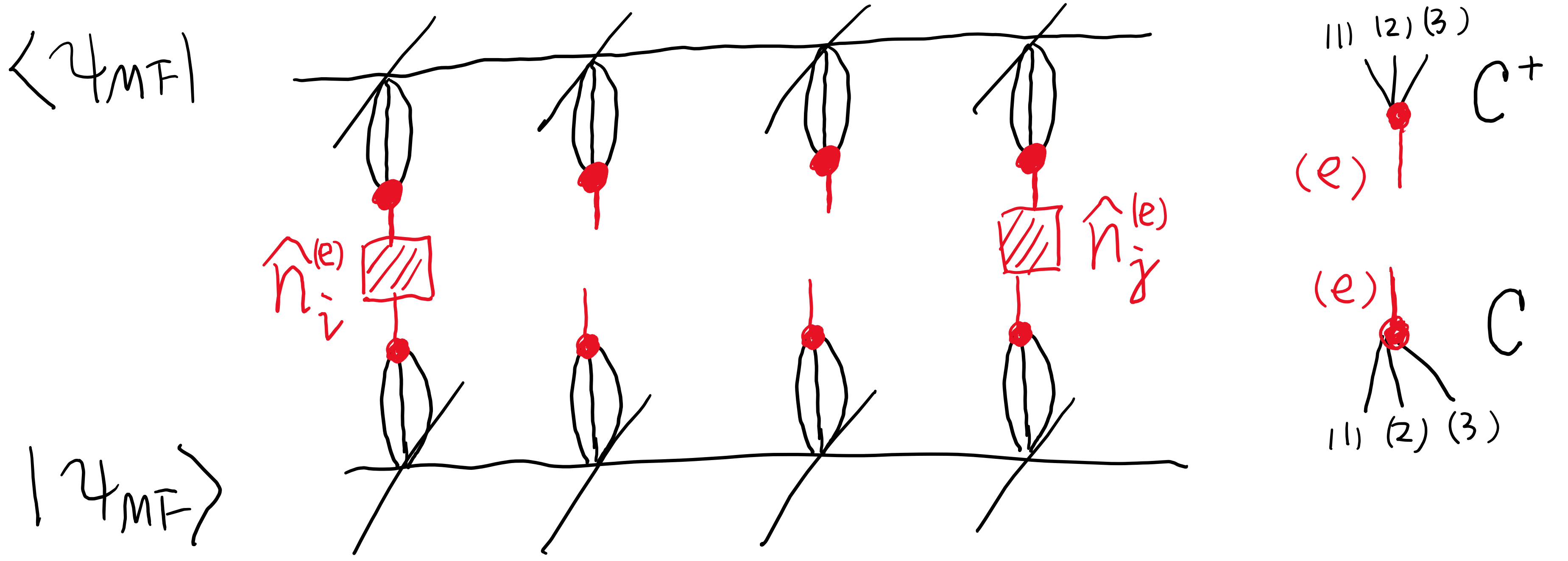


Projective expansion: How it works

- The tensor T in Hdet has a local structure. Consequently, the Hdet wavefunction can be viewed as a (grassmann) tensor-network



--- Local fusion tensor

Projective expansion: How it works

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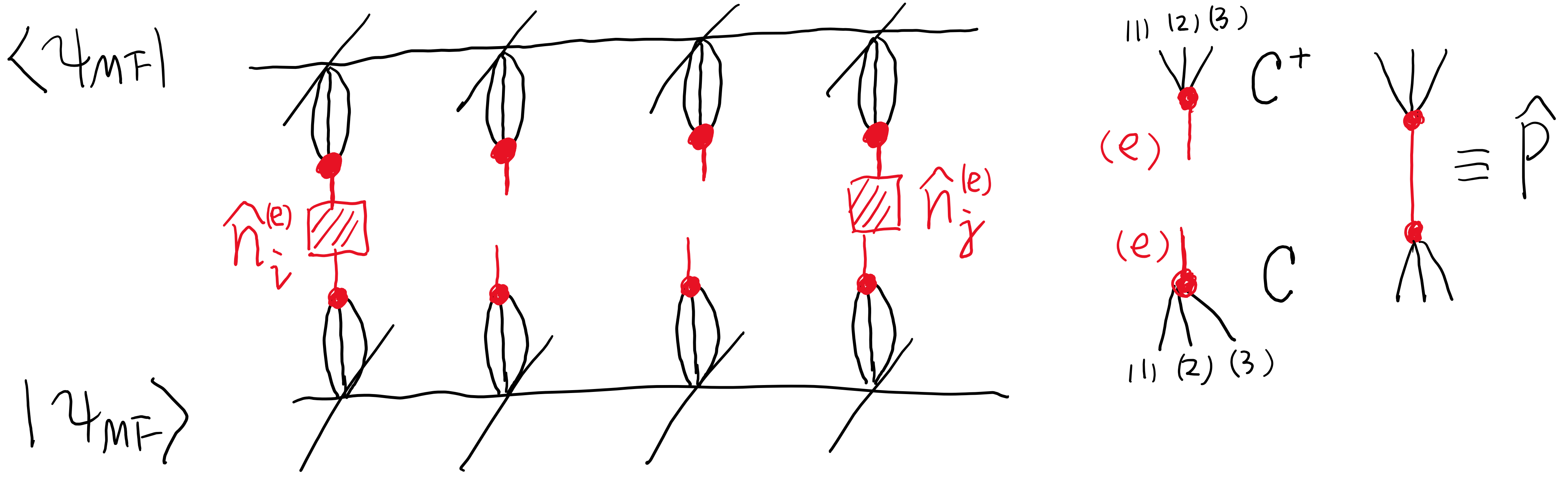

$$\langle \hat{O} \rangle = \frac{\langle \psi_{MF} | \hat{O} \cdot \prod_i \hat{P}_i | \psi_{MF} \rangle}{\langle \psi_{MF} | \prod_i \hat{P}_i | \psi_{MF} \rangle}$$

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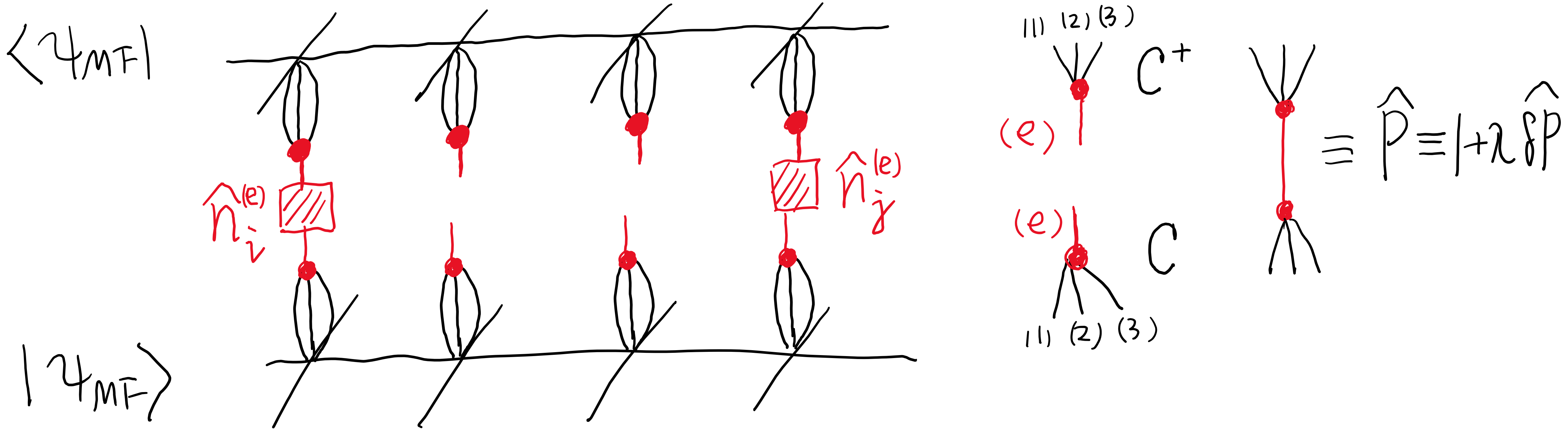

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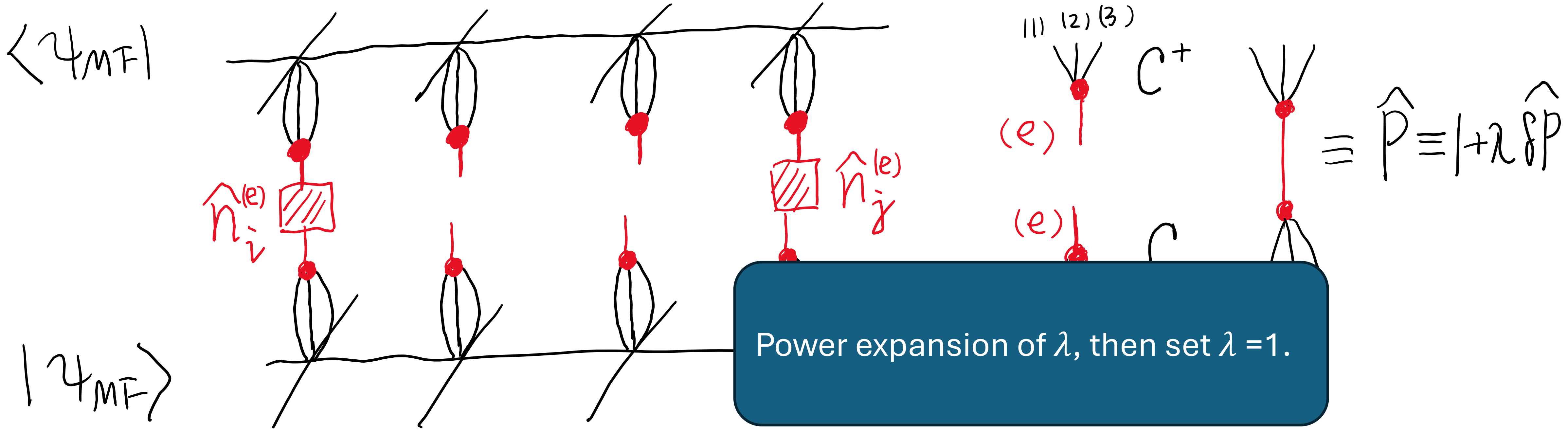

Power expansion of λ, then set λ =1.

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Ongoing/Future directions: Explore FQAH phase diagram

- Parton band inversion ➔ quantum phase transition

e.g. Bosonic ½-filled FQAH ⇔ Superfluid (Barkeshli-McGreevy 2011)

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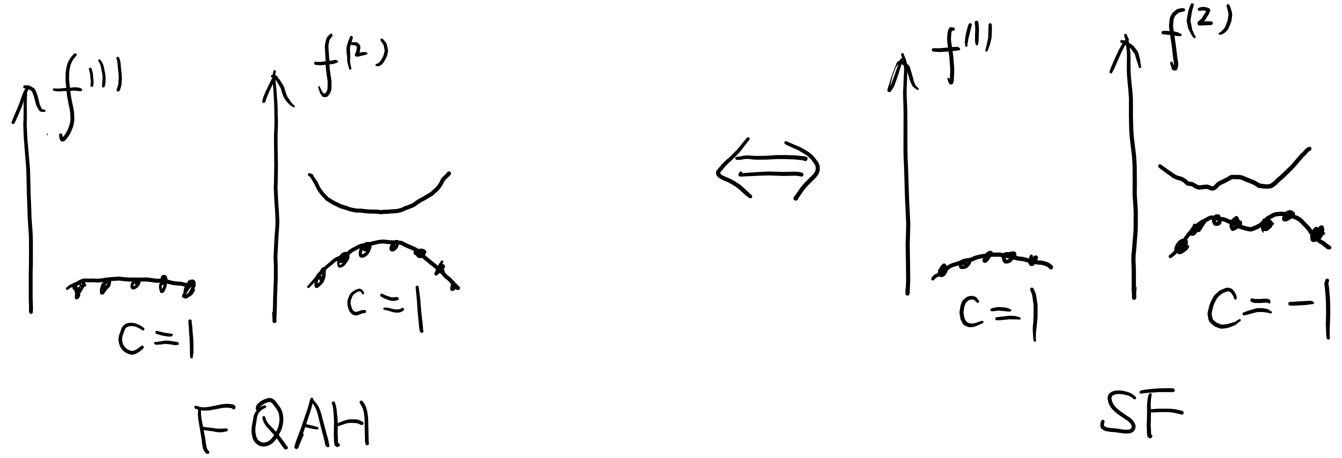

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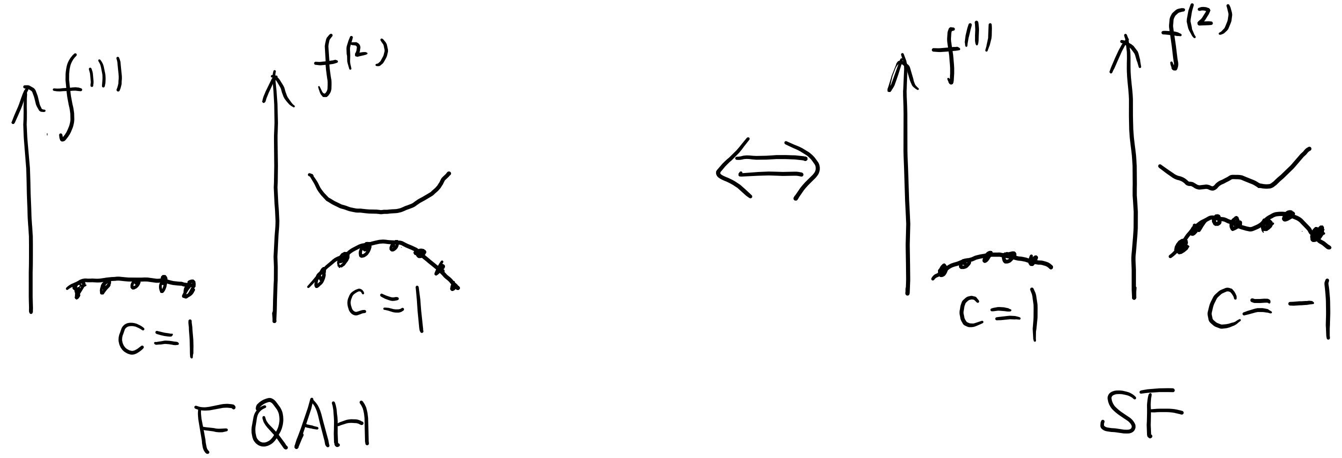

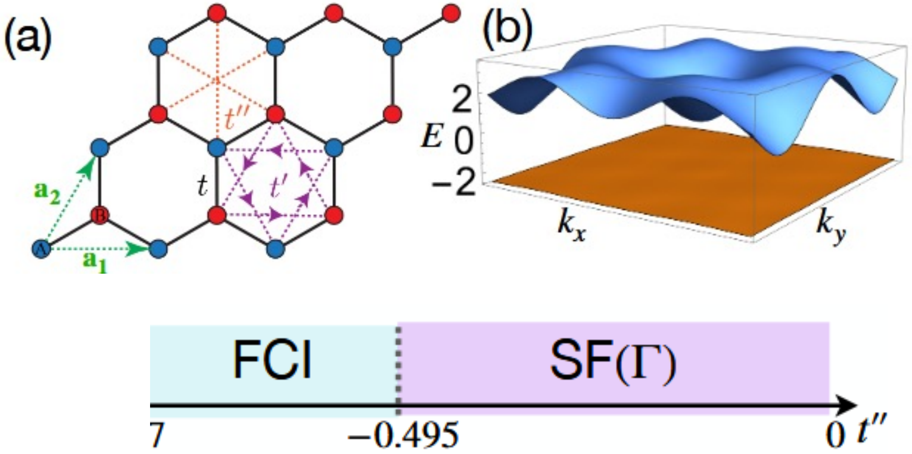

Recent DMRG sees the continuous phase transition
But has no access to parton bandstructure

Continuous Transition between Bosonic Fractional Chern Insulator and Superfluid

Hongyu Lu[1], Han-Qing Wu[2], Bin-Bin Chen[1,*], and Zi Yang Meng[1,†]

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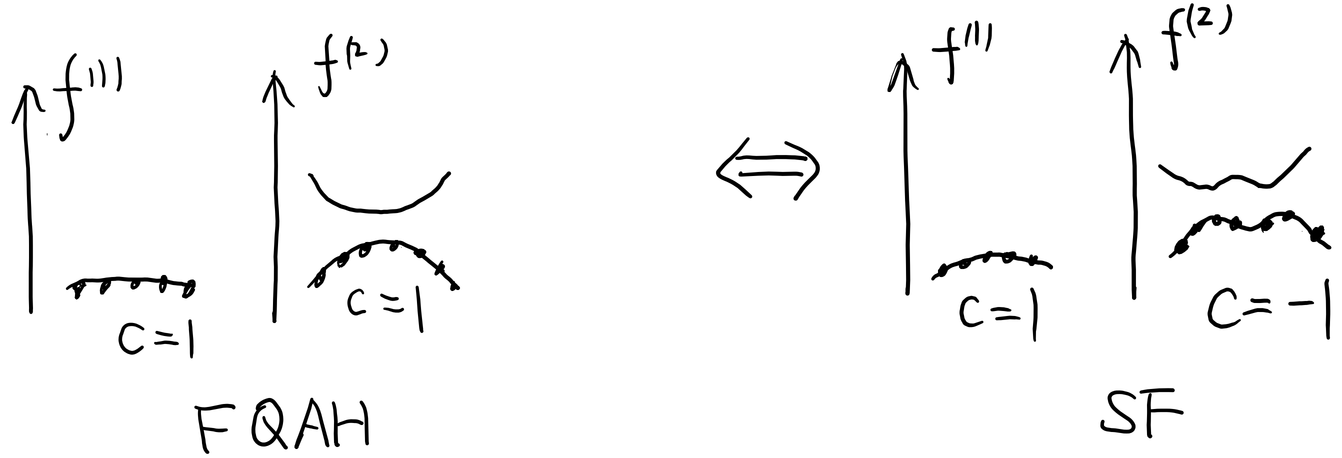

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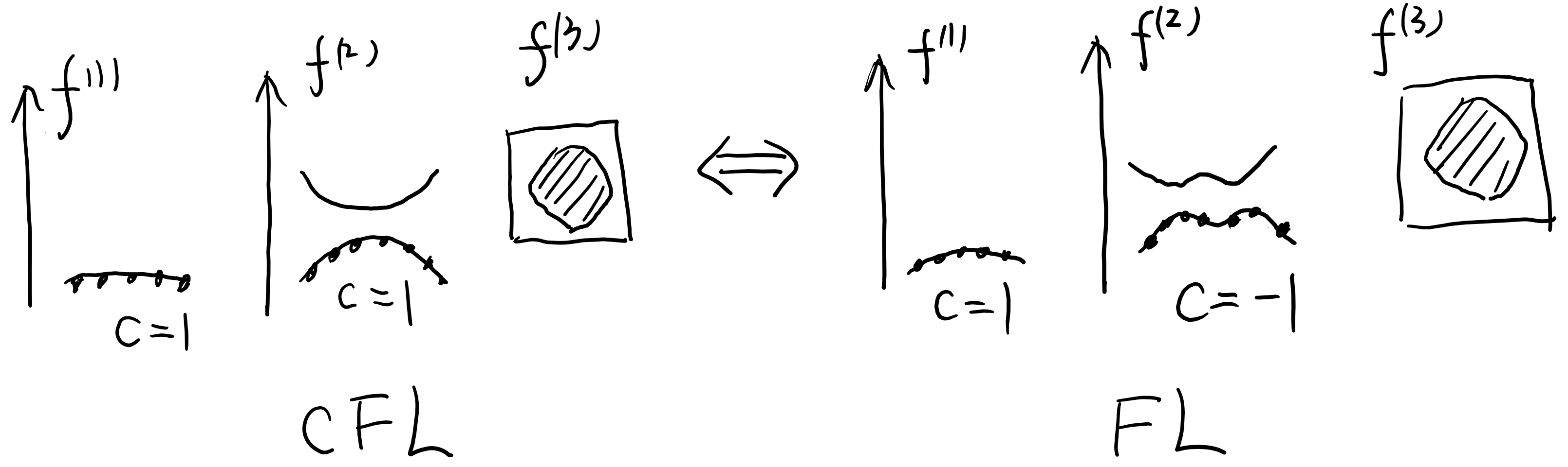
e.g. Composite Fermi liquid ⇔ Fermi liquid (Barkeshli-McGreevy)



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e.g. Bosonic ½-filled FQAH ⇔ Superfluid

Fermionic ½-filled CFL ⇔ Fermi liquid

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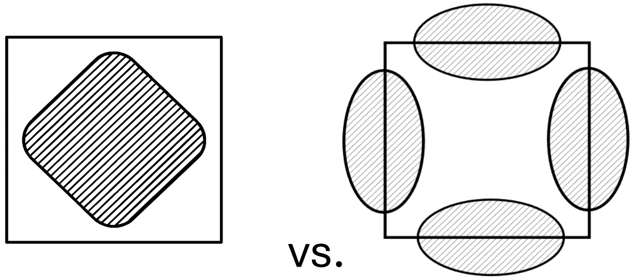


- Pairing and nonabelian states

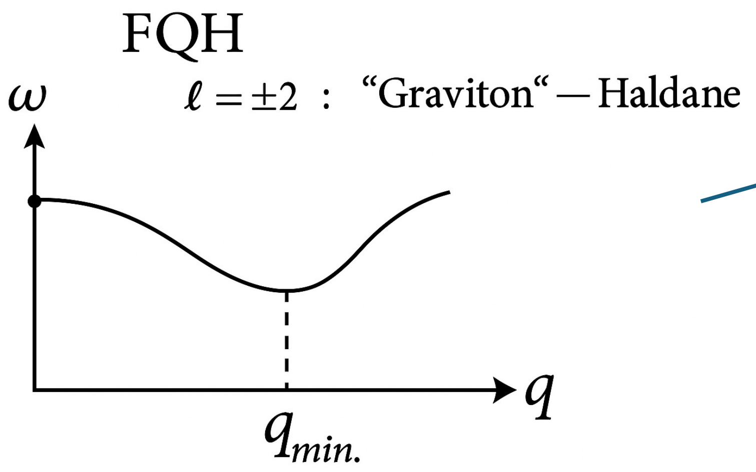
Ongoing/Future directions: Dynamical Hdet

- Based on the Projective expansion, one can compute the time-evolution of Hdet wavefunctions. Namely, we can really write down:

$$S = \int dt \langle \mathrm{Hdet}[T(t)] | i\partial_t - \hat{H} | \mathrm{Hdet}[T(t)] \rangle$$

- This calculation will reveal the collective modes

Magnetoroton (Girvn-McDonald-Platzman 1986, Haldane 2011)



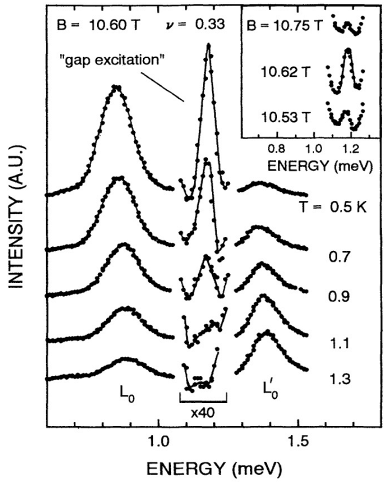


FIG. 1. Temperature dependence of inelastic light scattering spectra of a low-lying excitation of the FQHE at $\nu = \frac{1}{3}$. The single quantum well has density $n = 8.5 \times 10^{10}$ cm^{-2}. The inset shows the B dependence of the 0.5 K spectra. The light scattering peak, labeled "gap excitation," is interpreted as a $q = 0$ collective gap excitation. The bands labeled L_0 and L_0' comprise the characteristic doublets of intrinsic photoluminescence. The temperature dependence of the L_0 and L_0' intensities is due to the optical anomaly at $\nu = \frac{1}{3}$.

Pinczuk et.al, PRL 1993

Thank you!