Two-dimensional clock model: A cornucopia of emergence

Modern many-body physics is replete with examples of emergence wherein collective behavior of a many-body system as described via a long-wavelength or coarse-grained description hosts a higher symmetry in comparison to the bare microscopic theory. Field theoretically such emergence are typically predicated on the presence of an operator that becomes irrelevant (in the renormalization group sense) in the infrared limit, leading to either a critical point or even an entire critical phase hosting a higher symmetry than the short-wavelength microscopic theory. In this talk, I will present a comprehensive study on the frustrated J1-J2 classical q-state clock model with even $q > 4$ on a two dimensional square lattice and reveal a rich tapestry of emergent properties driven by competing interactions. The unfrustrated regime, similar to the standard clock model with $q > 4$, features an critical XY-like intermediate phase with emergent U(1) symmetry that separates the low-temperature ordered phase and the high-temperature paramagnetic phase. On the other hand, frustration stabilizes five distinct regimes: a low-temperature stripe order phase that breaks $\mathbb{Z}q \times \mathbb{Z}2$ symmetry, the high-temperature paramagnetic phase, two $\mathbb{Z}2$-broken nematic phases (one with and one without the XY-like quasi-long-range order), and finally an exotic stripe phase with emergent discrete $\mathbb{Z}q$ spin variables with shifted angle that are absent in the microscopic Hamiltonian. Interestingly, this exotic emergent $\mathbb{Z}q$ symmetry arises not from an operator becoming irrelevant but rather from a particular operator becoming relevant in the infrared limit — highlighting a nonstandard route to emergence. Using large-scale corner transfer matrix renormalization group calculations, complemented by classical Monte Carlo simulations, we map the complete phase diagram and classify all transitions — including Berezinskii–Kosterlitz–Thouless, Ising, first-order, and Landau-incompatible deconfined transitions between different phases. Finally, I will outline an effective field-theoretic framework that captures these emergent orders, symmetries and their interwoven transitions.

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