SATO-TATE TYPE DISTRIBUTIONS FOR SOME FAMILIES OF GAUSSIAN HYPERGEOMETRIC FUNCTIONS

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ABSTRACT. In the 1980's, Greene introduced *hypergeometric functions over finite fields* using Jacobi sums. The framework of his theory establishes that these functions possess many properties that are analogous to those of the classical hypergeometric series studied by Gauss, Kummer and others. These functions have played important roles in the study of Apéry-style supercongruences, the Eichler-Selberg trace formula, Galois representations and zeta-functions of arithmetic varieties. In this talk, we consider three families of hypergeometric functions that arise naturally in the arithmetic of elliptic curves and $K3$ surfaces. Using the theory of harmonic Maass forms and Mock modular forms we will discuss the value distributions (over large finite fields) of these three families of functions. We will show that for two such families the limiting distribution is semicircular (i.e. $SU(2)$), whereas for the other family the distribution is the Batman distribution for the traces of the real orthogonal group O_3.

REFERENCES

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