On differences of perfect powers and prime powers

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In 2004, Mihăilescu [1] proved that the only consecutive perfect powers are 8 and 9. Despite many attempts to generalise this conjecture to perfect powers with arbitrary difference D, not much more is known today.

Given a squarefree integer $1 \leq C_1 \leq 20$ and a prime $2 \leq q < 25$, we will present a methodology that allows us to resolve the following Diophantine equation

$$C_1 x^2 + q^\alpha = y^n,$$

therefore determining which integers with squarefree part C_1 are the difference of a perfect power and a $q-$power.

This methodology combines the modular method popularised after the proof of Fermat's Last Theorem with an improved Thue–Mahler solver and new estimates on lower bounds on linear forms in three logarithms.

[1] P. Mihăilescu, Primary cyclotomic units and a proof of Catalan's conjecture, *J. Reine Angew. Math.* **572**, 2004, 167–195.