Friedmann equations, Dark matter, Dark Energy

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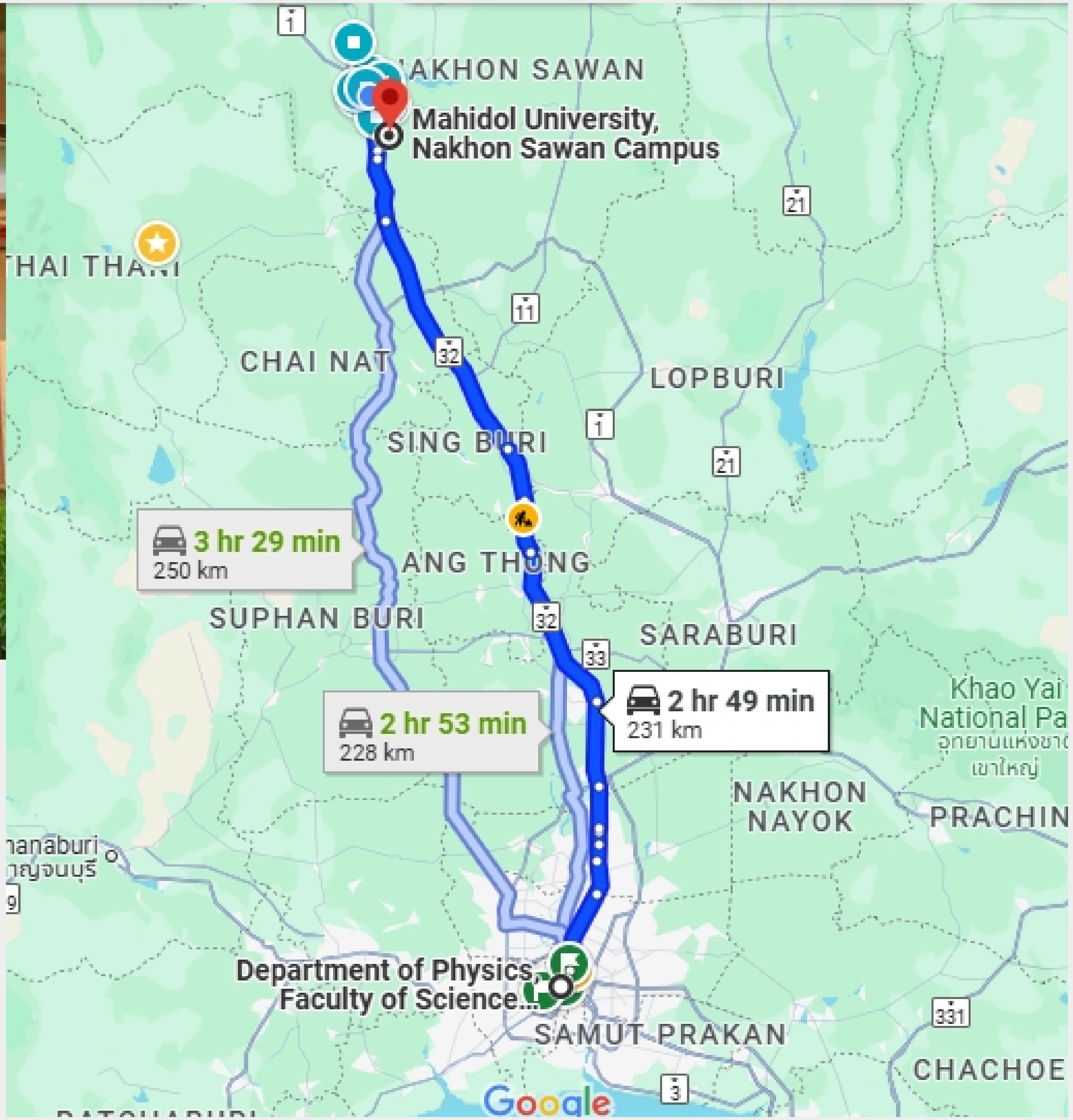
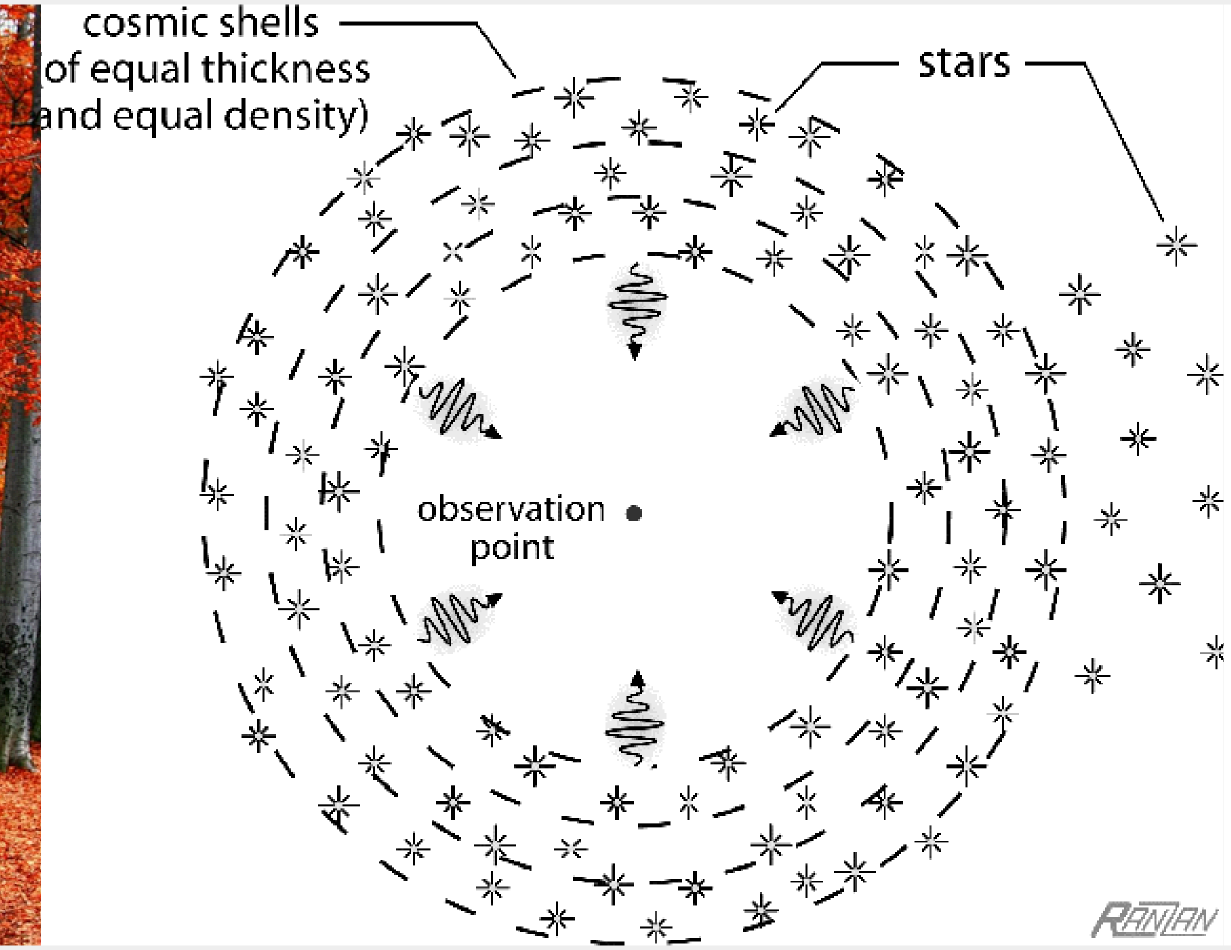


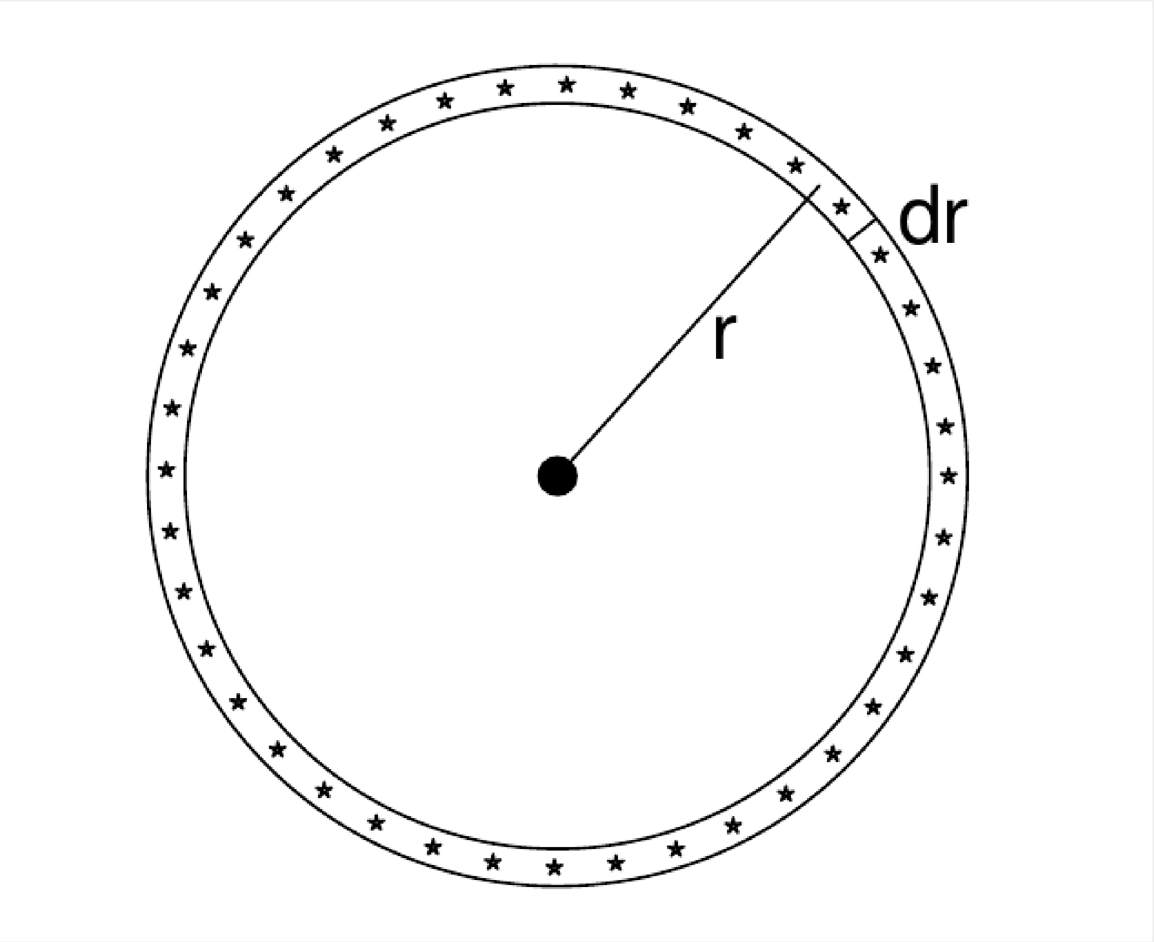
Theoretical Cosmology, High Energy Theory and Mathematical Physics and Physics of the Complex Systems

- 20th March 2023 Inaugurated.
- 6 faculty members.
- 1 post doctoral fellow
- 4 PhD Students

Olber Paradox

If our universe is infinitely old and infinite in size then why isn't the night sky uniformly bright?





$$f(r) = \frac{L}{4\pi r^2}, \;\; dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 \, dr = \frac{nL}{4\pi} dr$$

$$J = \int_0^\infty J(r)\, dr = \frac{L}{4\pi} \int_0^\infty dr = \infty$$

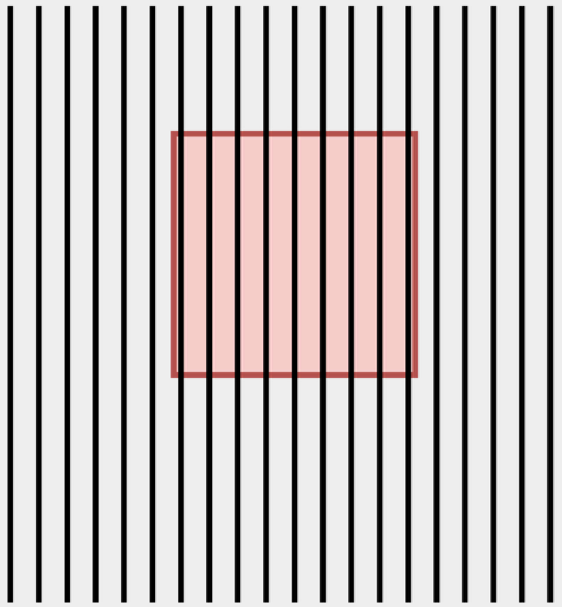
The night sky must be as bright as the surface of the sun.

Resolution of Olber Paradox

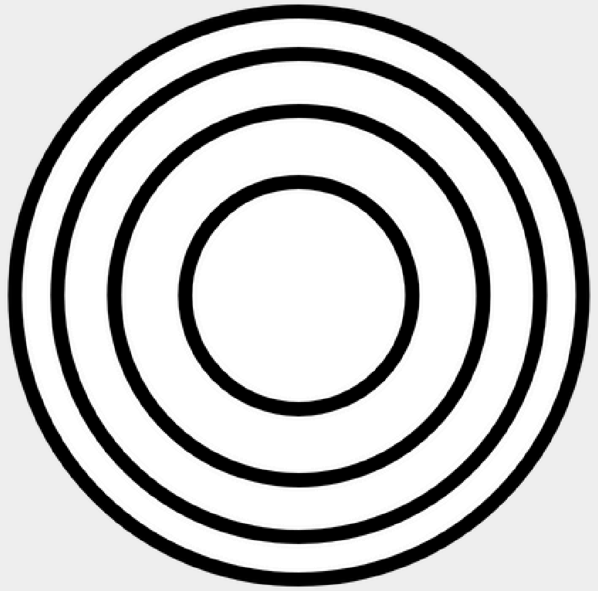
- The universe has a finite age.
- The lights from star after a finite distance has not has time to reach us. This distance is called horizon distance.

Cosmological Principle

On large scale the universe is homogeneous and isotropic. In another words viewing on large scale the property of the universe is same for all observer. There is no preferred place or direction in the universe. The Universe is homogeneous and isotropic on scales > 100 Mpc.   1 pc ≈ 3.261563777 ly



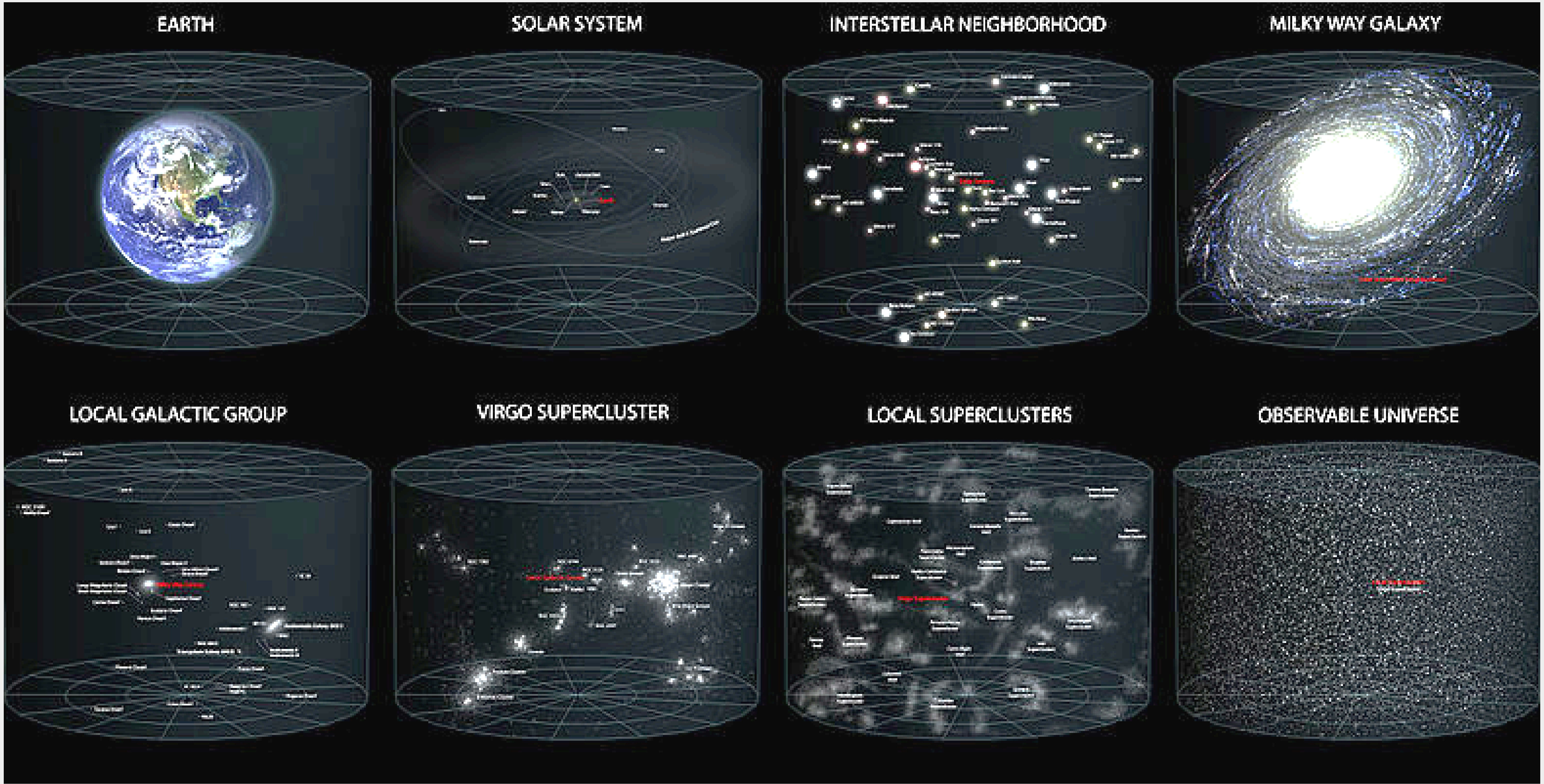
Anisotropic but homogenious



Isotropic but inhomogeneous

- First clearly asserted by Isaac Newton in his Philosophiæ Naturalis Principia Mathematica (1687).
- A corollary to the cosmological principle is that the laws of physics are universal.
- It implies that all parts of space are causally connected at some time in the past (although they may no longer be connected today). Thus, the whole Universe appeared at a single moment of time, a Creation.

Cosmological Principle



The Four Fundamental Forces of Nature

1. **Gravity** – The weakest but most far-reaching force; governs planetary motion and the structures in the universe.
2. **Electromagnetic Force** – Responsible for light, electricity, and magnetism; holds atoms and molecules together.
3. **Strong Nuclear Force** – Binds protons and neutrons in the atomic nucleus; strongest of all forces.
4. **Weak Nuclear Force** – Governs radioactive decay and nuclear fusion in stars.

Why Gravity is Key to Understanding the Cosmos?

- It shapes galaxies, stars, and planets.
- Explains black holes, dark matter, and cosmic expansion.
- Supports Einstein’s General Relativity and space-time curvature.

Newton Vs Einstein

Newton's law of gravity: Every object in this universe has a property which we called gravitational mass. There is a force acting on each object due to the presence of other objects which is proportional to the gravitational mass of the objects and inversely proportional to the square of the distance.

$$F = -\frac{GM_g m_g}{r^2}$$

Newton's Second Law: Every object in this universe has an property which we may called inertial mass that relates the force acting on the object and the acceleration of the object.

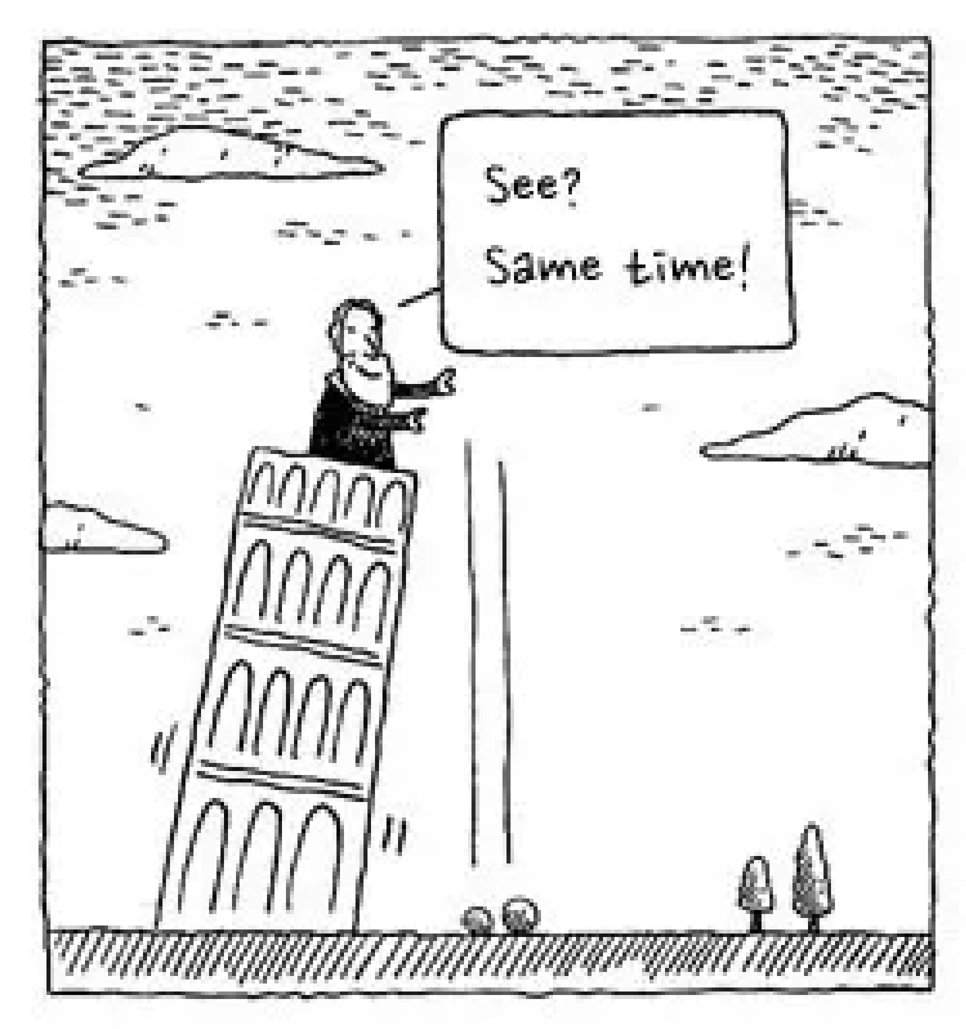
$$F = m_i a$$

Newton Vs Einstein

If there is an object on which gravitational force is acting we can write:

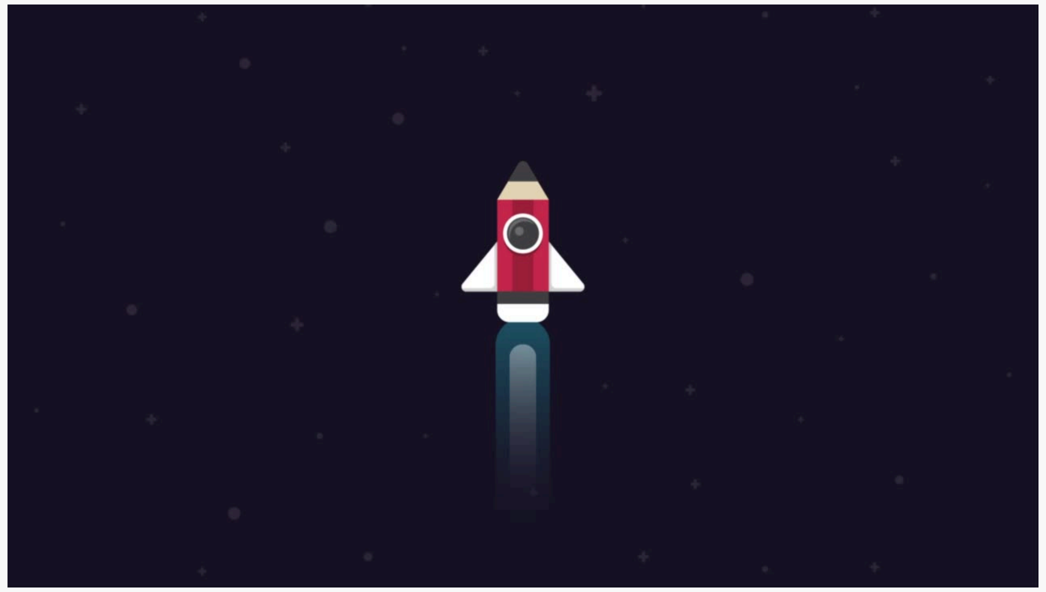
$$a = -\frac{GM_g}{r^2}\left(\frac{m_g}{m_i}\right)$$

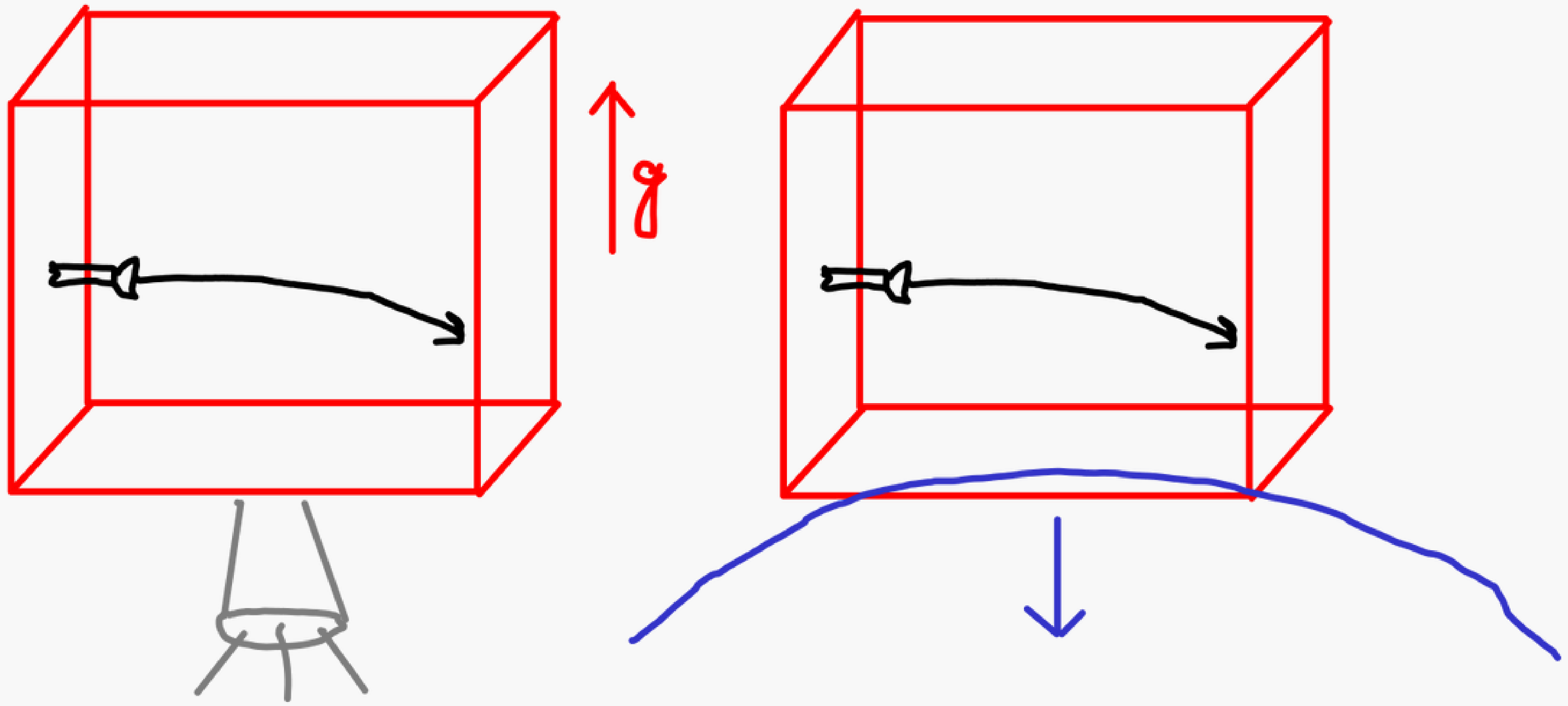
$$m_g = m_i$$



From modern experiments that the inertial and gravitational masses are the same to within one part in billion .

Equivalence Principle & General Relativity





General Relativity

The Way of Newton:

Mass tells gravity how to exert a force $(F = -GMm/r^2)$,

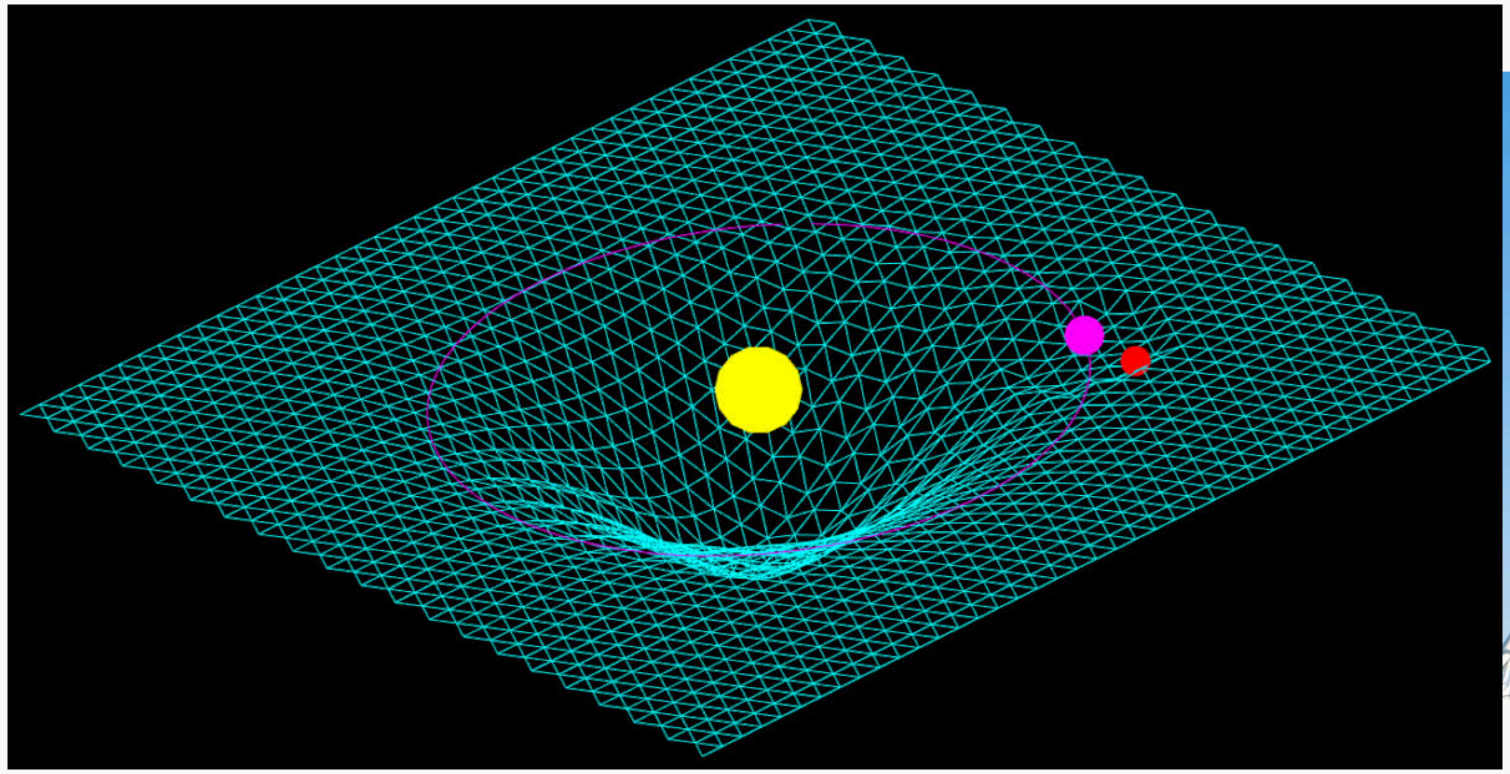
Force tells mass how to accelerate $(F = ma)$.

The Way of Einstein:

Mass-energy tells space-time how to curve,

Curved space-time tells mass-energy how to move.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$



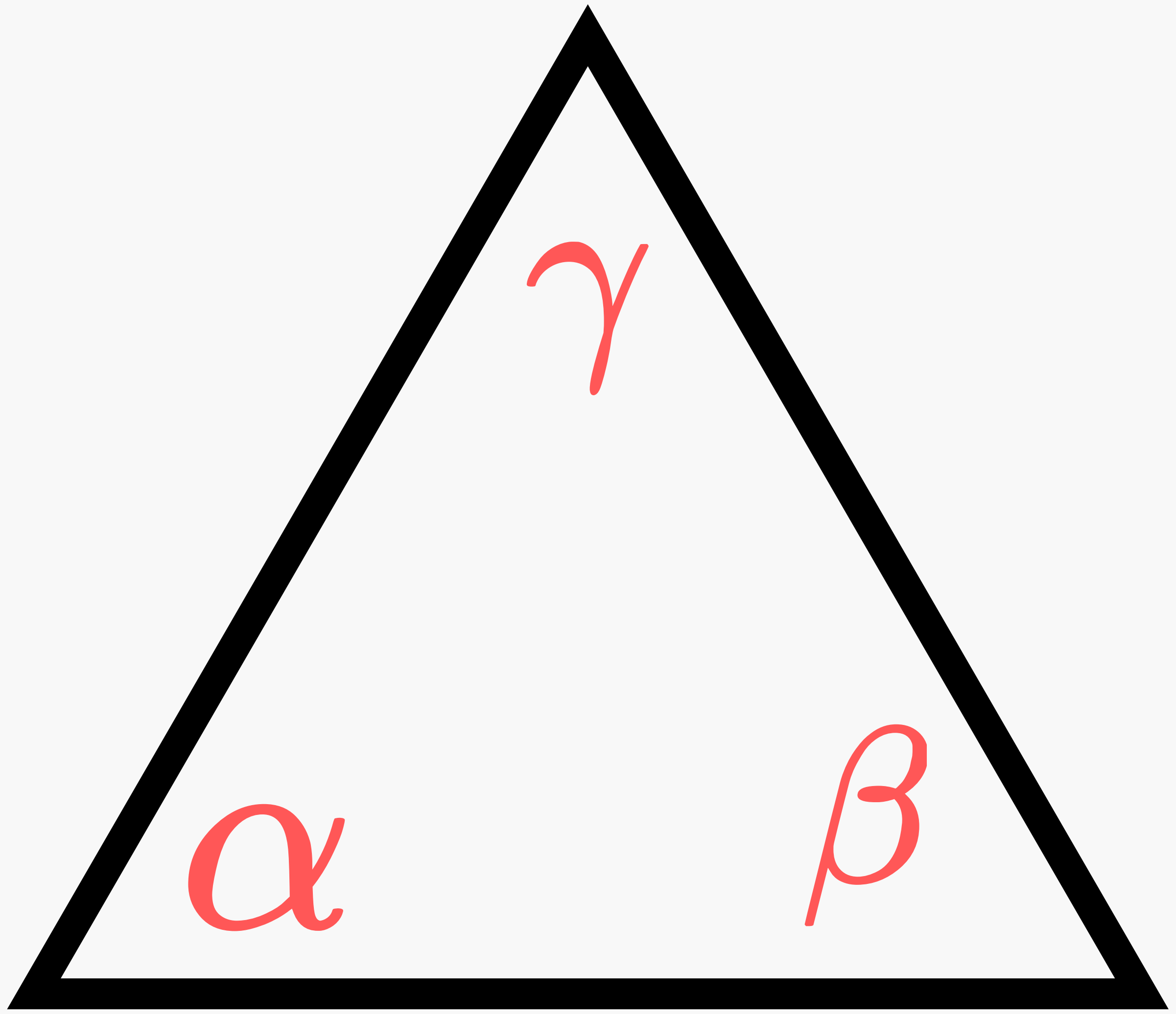
Geometry

“Gravitation is the study of geometry”

Geometry is built on assumptions: Near the beginning of the first book of the Elements, Euclid gives five postulates.

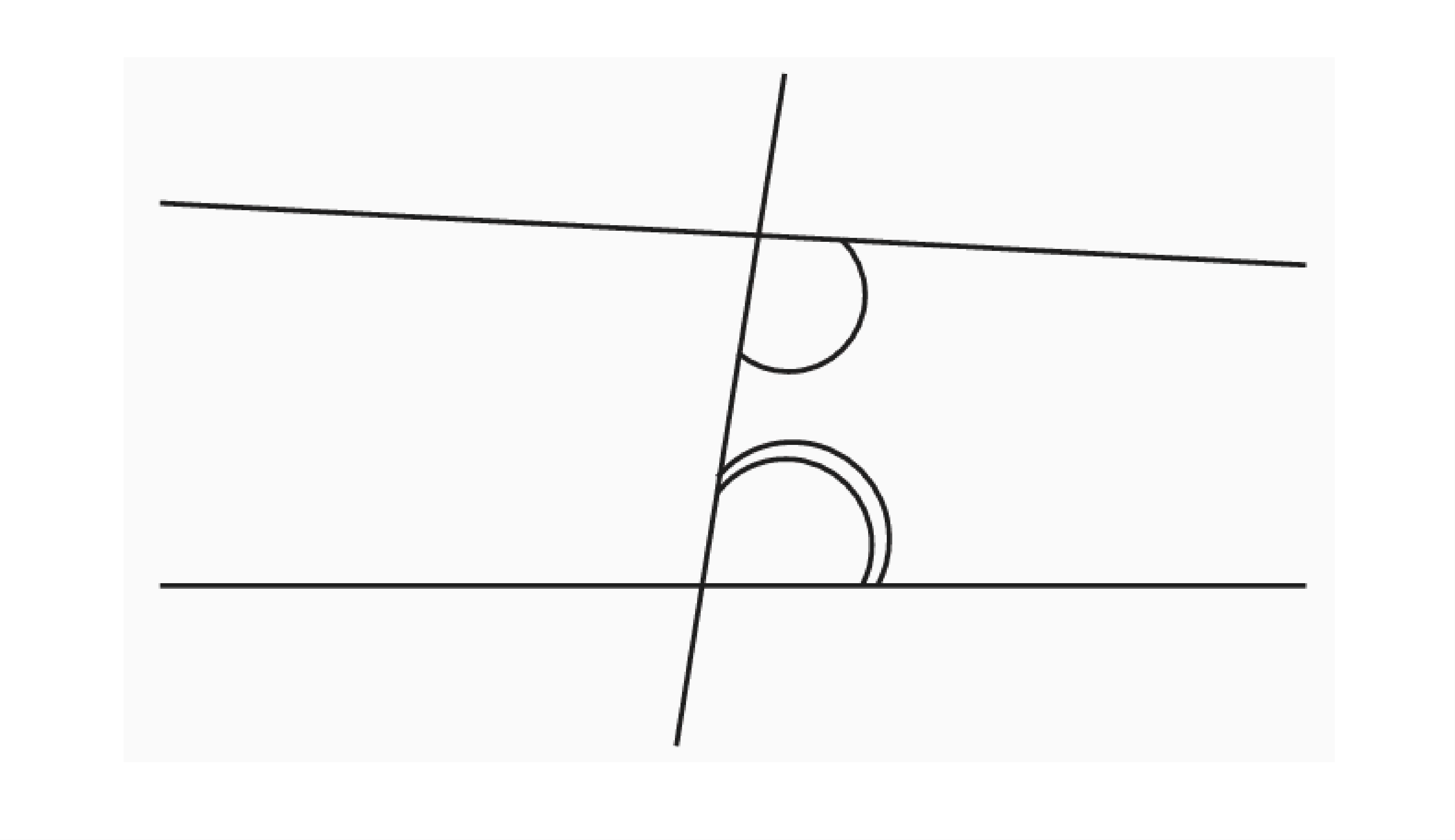
1. To draw a straight line from any point to any point.
2. To produce (extend) a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance (radius).
4. That all right angles are equal to one another.
5. If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

From these axioms Euclid deduced hundreds of theorems which tell us a lot about Euclidean geometry. For example,



$$\measuredangle\alpha + \measuredangle\beta + \measuredangle\gamma = \pi$$

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

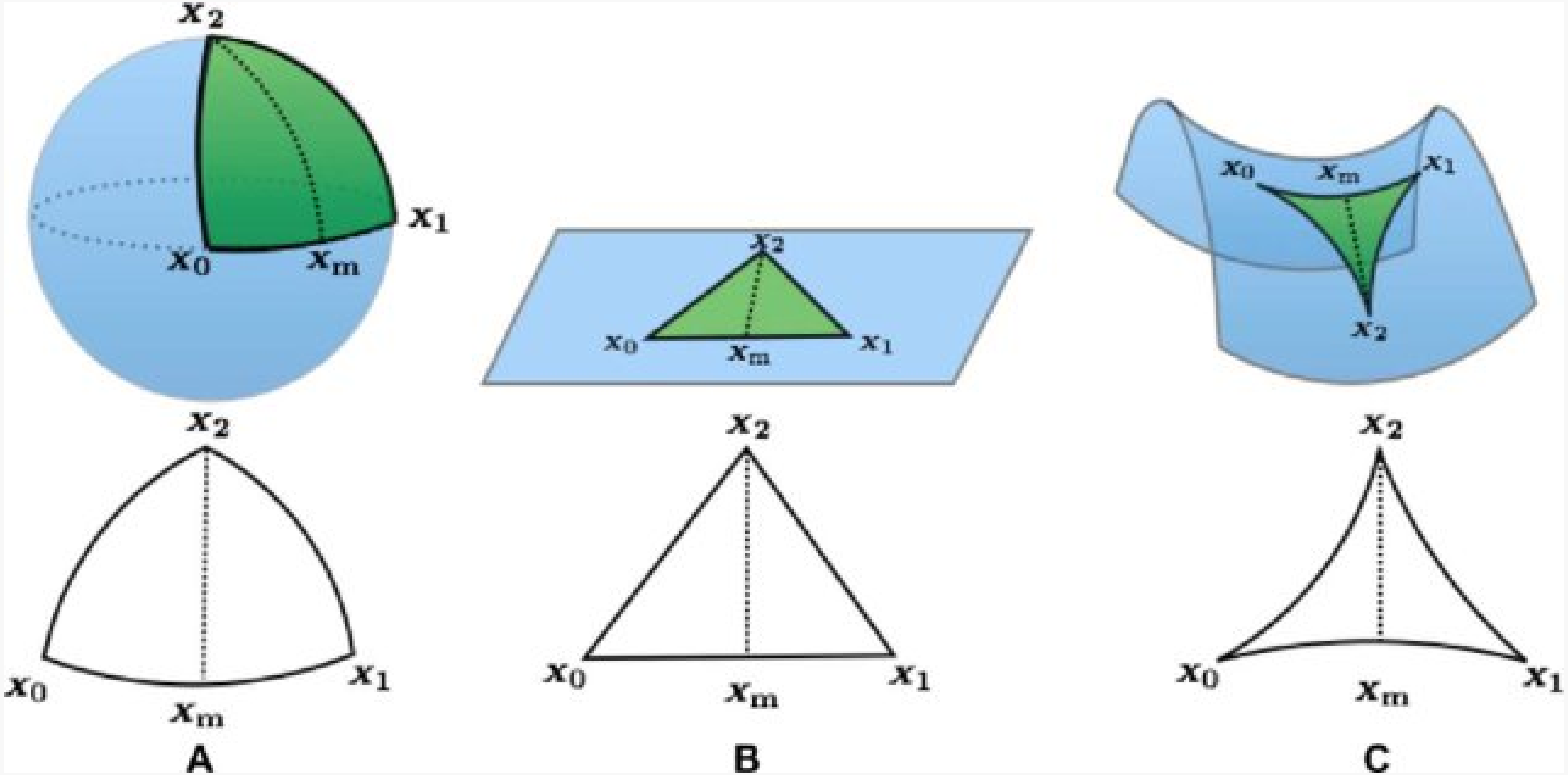


- People tried hard to prove that this follows from the other axioms of Euclid and that this is not truly an independent axiom.
- The study of this question, on the other hand, led to the birth of other, non-Euclidean geometries which satisfy all the axioms of Euclid except the one on parallel lines.

Positively Curved

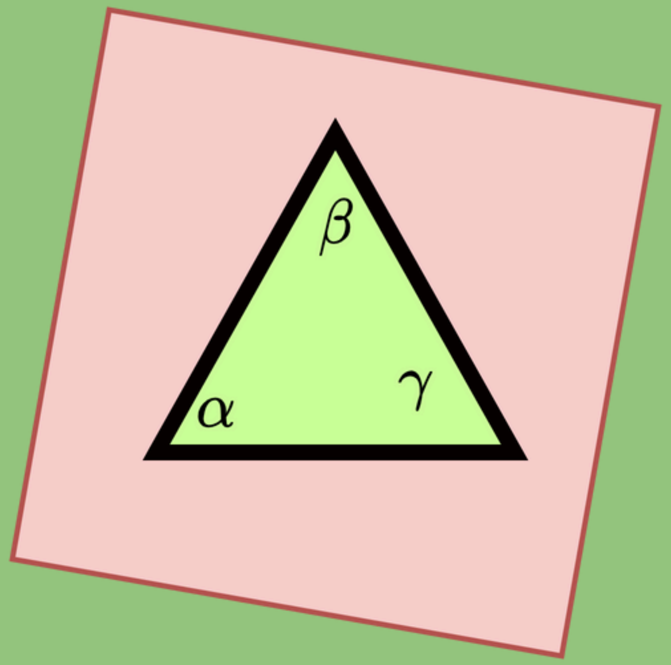
Flat

Negatively Curved



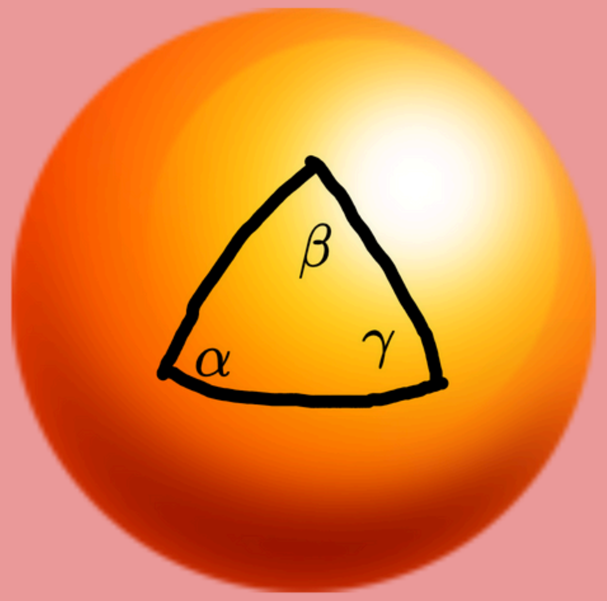
Curvature of the Universe

Flat



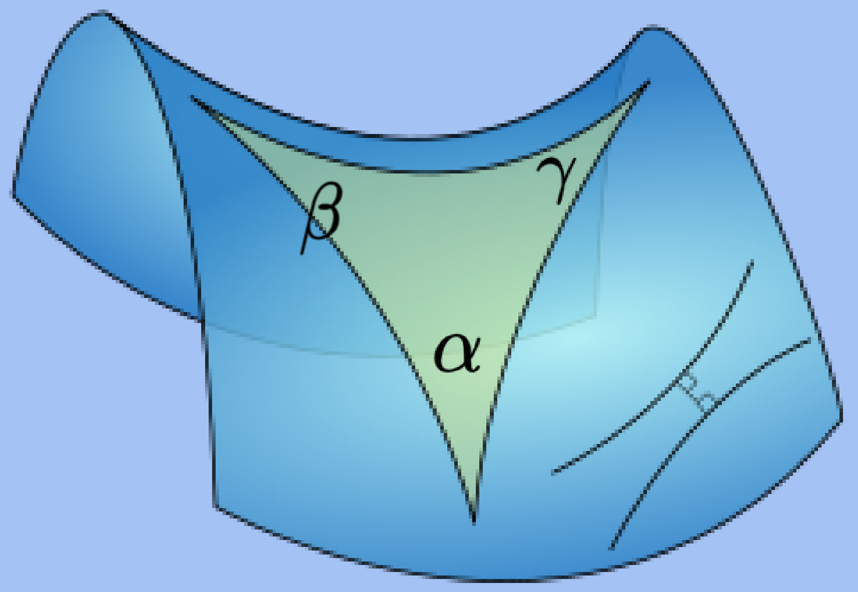
$$\alpha + \beta + \gamma = \pi$$

Positively Curved



$$\alpha + \beta + \gamma = \pi + A/R^2$$

Negatively Curved



$$\alpha + \beta + \gamma = \pi - A/R^2$$

FRW Metric

"What form can the metric of space-time assume if the universe is spatially homogeneous and isotropic at all time, and if distances are allowed to expand (or contract) as a function of time?"

Friedmann-Robertson–Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$K \longrightarrow +1$ (Positively Curved)

$K \longrightarrow 0$ (Flat)

$K \longrightarrow +1$ (Negatively Curved)

Einstein-Hilbert Action

$$S = (1/16\pi G)\int d^4x\sqrt{-g}R + S_{matter}$$

$$\delta S = 0$$

$$\delta(\sqrt{-g}R) = \sqrt{-g}(R_{\mu\nu} - ½g_{\mu\nu}R)\delta g^{\mu\nu} + \text{boundary terms}$$

$$\delta S_{matter} = -½\int d^4x\sqrt{} - gT_{\mu\nu}\delta g^{\mu\nu}$$

$$R_{\mu\nu} - ½g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

Friedmann Equations

FLRW Metric: $$ds^2 = -c^2dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

Christoffel Symbols: $$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$

Ricci Tensor: $$R_{\mu\nu} = \partial_{\alpha}\Gamma^{\alpha}_{\mu\nu} - \partial_{\nu}\Gamma^{\alpha}_{\mu\alpha} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu} - \Gamma^{\alpha}_{\nu\beta}\Gamma^{\beta}_{\alpha\mu}$$

Ricci Scalar: $$R = g^{\mu\nu} R_{\mu\nu}$$

Einstein Tensor: $$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$ $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Friedmann Equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Friedmann Equations

Hubble Parameter
Density Parameter
Curvature Constant

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

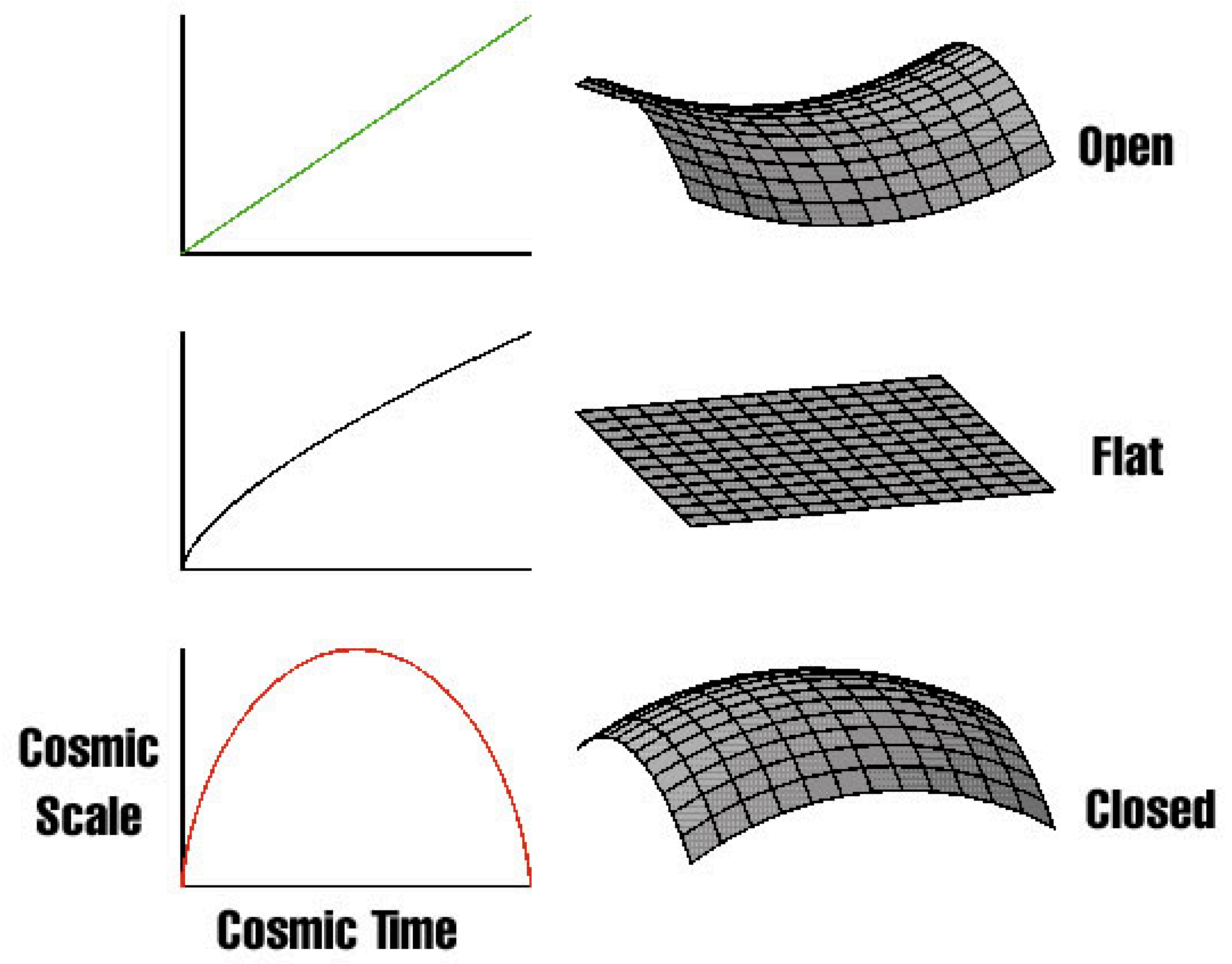
Scale Factor
Cosmological Constant

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

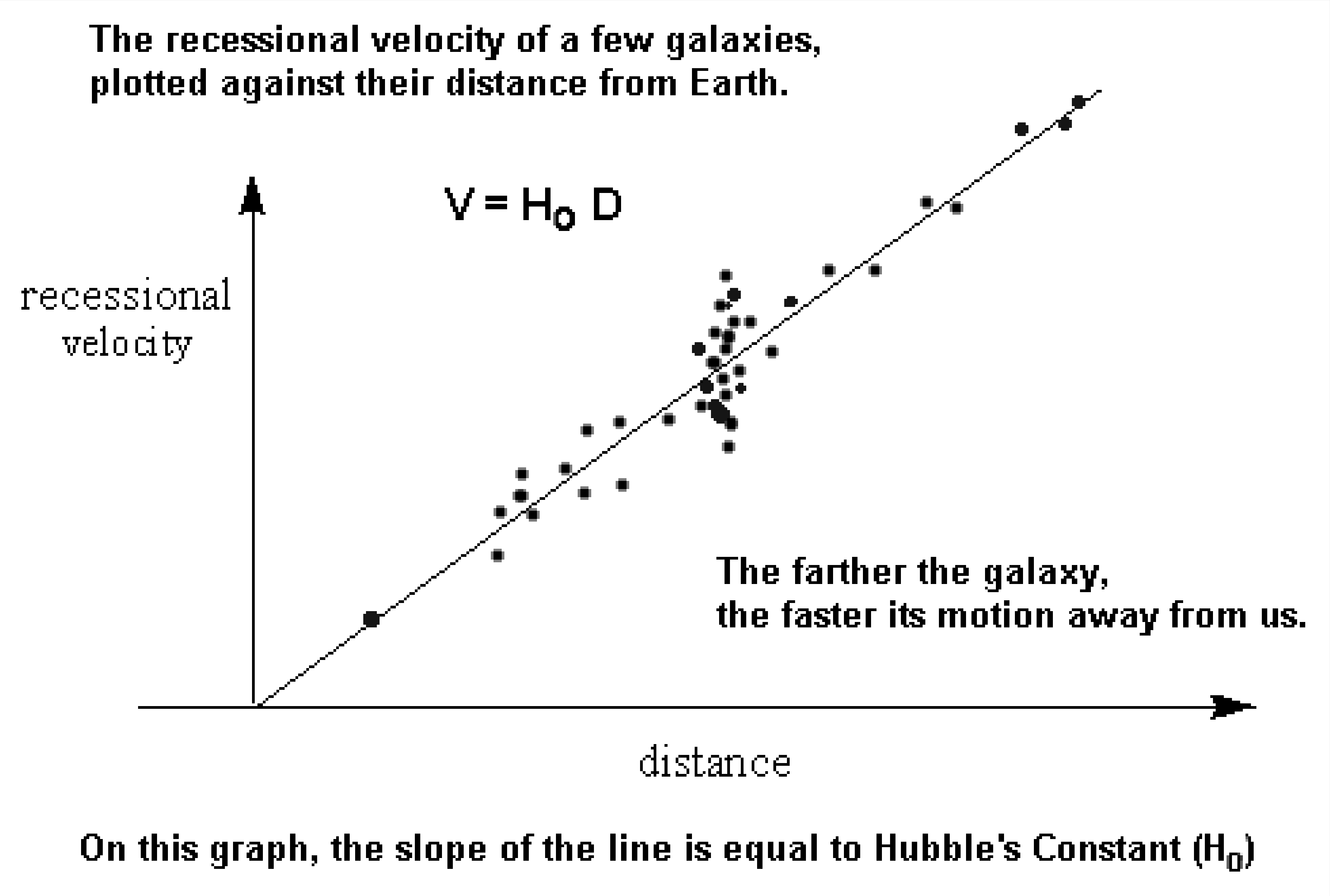
Pressure
EOS

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0 \qquad p = w\rho$$

Evolution the Universe with Curvature



Hubble's Law


The recessional velocity of a few galaxies,
plotted against their distance from Earth.
V = H0 D
recessional
velocity
The farther the galaxy,
the faster its motion away from us.
distance
On this graph, the slope of the line is equal to Hubble's Constant (H0)

The Universe Is Expanding

If we look at the universe from the earth almost everything is receding from us. Is it violating the Cosmological Principle?

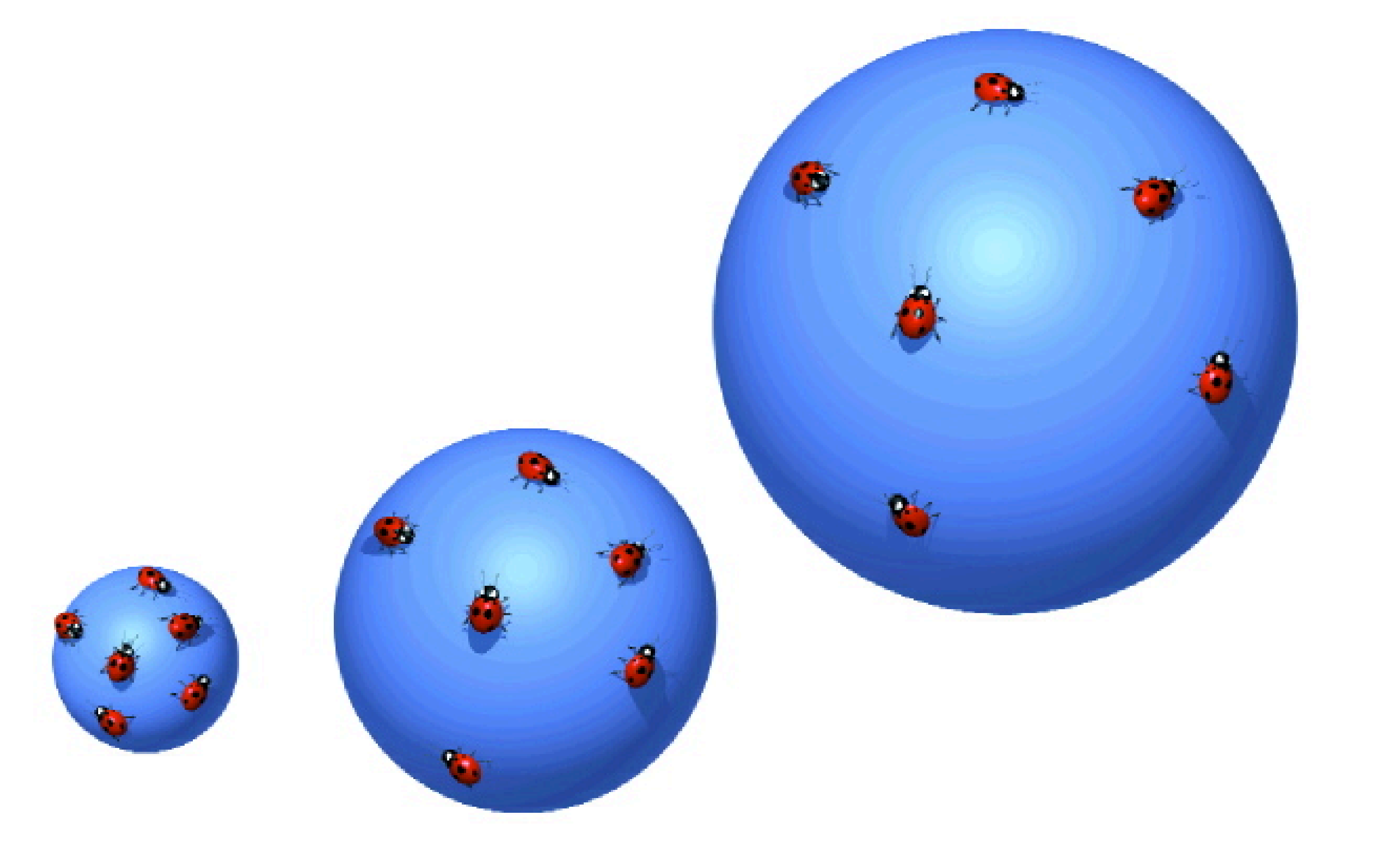


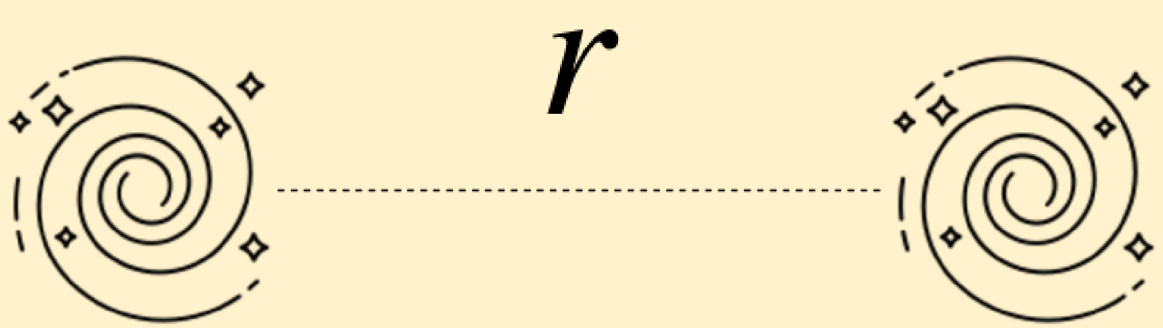
Image Link: https://www.nap.edu/jhp/oneuniverse/images/motion_2.jpg

What is the centre of this expansion?

Can the velocity of galaxies be more than the light?

Hubble Time

All galaxies are flying away from each other. Which means once upon a time they were close together. Consider two galaxies separated by a distance,



$$v = H_0 r$$

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1} ,$$

$$t_H \equiv \frac{1}{H_0} = \frac{1}{67.8 (\mathrm{km/s})/\mathrm{Mpc}} = 4.55 \cdot 10^{17} \mathrm{s} = 14.4 \text{ billion years.}$$

This is called Hubble time. So in past around Hubble time ago all Galaxies were clumped together giving rise to a very hot and dense universe in a very small volume. This is the BIG BANG model of the Universe.

Hubble time naturally provides a scale for the distance.

Hubble Radius $c/H_0 = 4300 \pm 400 \, \mathrm{Mpc}$

Equation of State

In cosmology we generally consider the pressure is related to the density. $p \equiv p(\rho),\ p = w\rho$

Matter

Any non-relativistic component which exerts zero pressure, $p = 0, w = 0$

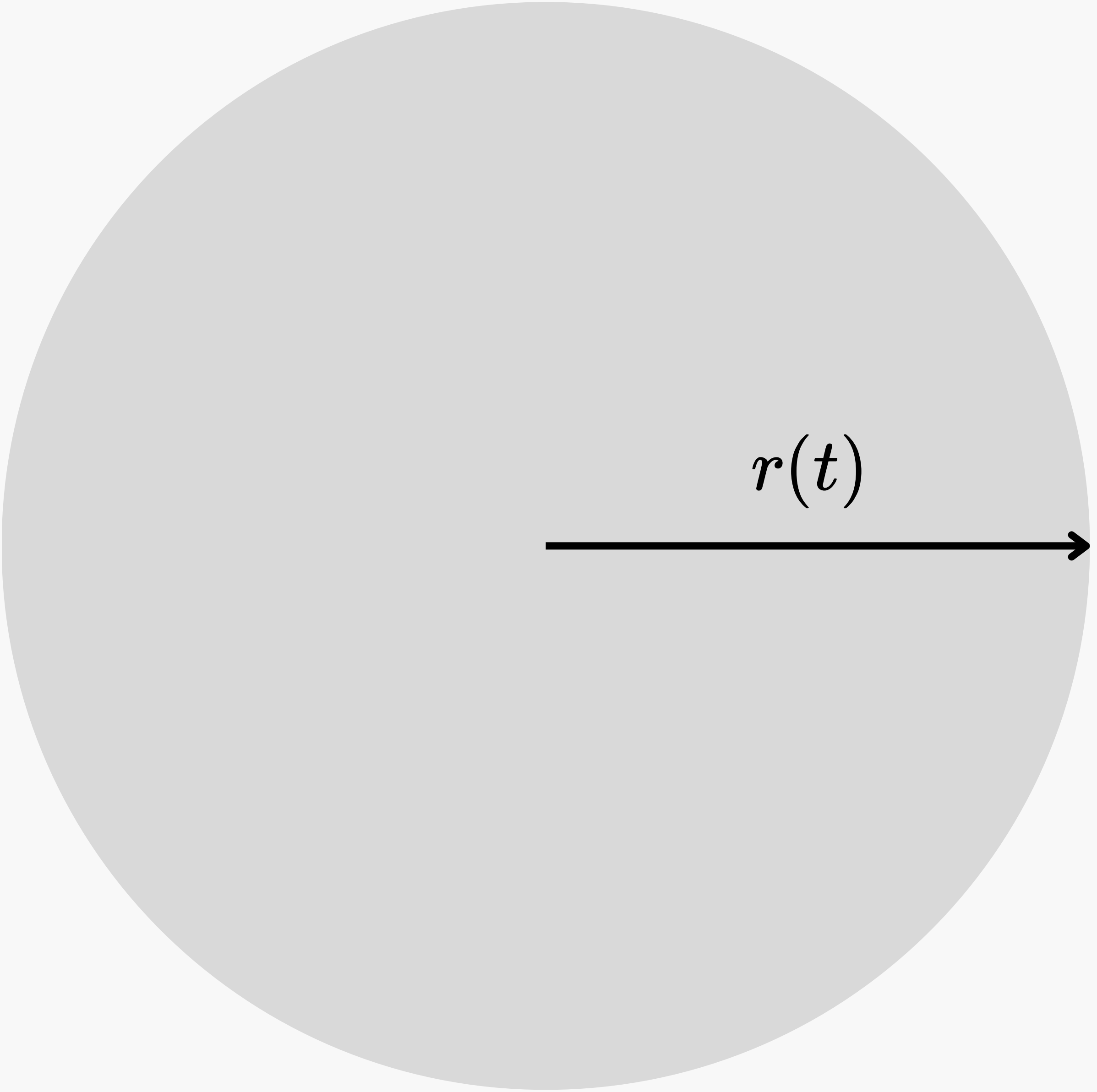
Radiation

Any relativistic component will have radiation pressure.

$$p = \frac{1}{3}\rho$$

$$w = 1/3$$

Derivation of the Friedmann Equations from Newtonian Gravity



A sphere of radius $R_s(t)$
and mass M_s contracting
or expanding under its own gravity.

$$F = -\frac{GM_s m}{R_s(t)^2}$$

$$\frac{d^2 R_s}{dt^2} = -\frac{GM_s}{R_s(t)^2}$$

$$\frac{d^2R_s}{dt^2} = -\frac{GM_s}{R_s(t)^2}$$

Multiply each side of the equation by $\frac{dR_s}{dt}$ and integrate to find

$$\frac{1}{2}\left(\frac{dR_s}{dt}\right)^2 = \frac{GM_s(t)}{R_s(t)} + U$$

where U is a constant of integration. The above equation simply states that the sum of the kinetic energy per unit mass,

$$E_{\text{kin}} = \frac{1}{2}\left(\frac{dR_s}{dt}\right)^2$$

and the gravitational potential energy per unit mass,

$$E_{\text{pot}} = \frac{GM_s(t)}{R_s(t)}$$

is constant for a bit of matter at the surface of a sphere,

as the sphere expands or contracts under its own gravitational influence.

Since the mass of the sphere is constant as it expands or contracts, we may write

$$M_s = \frac{4}{3}\pi\rho(t)R_s^3(t)$$

Since the expansion is isotropic about the sphere's center, we may write the radius $R_s(t)$ in the form

$$R_s(t) = a(t)r$$

In terms of $\rho(t)$ and $a(t)$, the energy conservation equation can be rewritten in the form

$$\frac{1}{2}r_s^2\dot{a}^2 = \frac{4\pi}{3}Gr_s^2\rho(t)a(t)^2 + U$$

Dividing each side of the above equation by $r_s^2 a(t)^2$ yields the equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2}\frac{1}{a(t)^2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{\kappa c^2}{a(t)^2}$$

The above equation gives the Friedmann equation in its Newtonian form.

The Continuity Equation

From the first law of Thermodynamics:

$$dQ = dE + PdV$$

If we consider the expansion of the Universe is an adibatic process, then

$$\dot{E} + P\dot{V} = 0$$

For concreteness, consider a sphere of comoving radius r_s expanding along with the universal expansion, so that its proper radius is $R_s(t) = a(t)r_s$.

The volume of the sphere is

$$V(t) = \frac{4\pi}{3} r_s^3 a(t)^3$$

so the rate of change of the sphere's volume is

$$\dot{V} = \frac{4\pi}{3} r_s^3 (3a^2\dot{a}) = V\left(3\frac{\dot{a}}{a}\right)$$

The internal energy of the sphere is

$$E(t) = V(t)\epsilon(t)$$

so the rate of change of the sphere's internal energy is

$$\dot{E} = V\dot{\epsilon} + \dot{V}\epsilon = V\left(\dot{\epsilon} + \frac{\dot{a}}{a}3\epsilon\right)$$

The Continuity Equation

$$V\left(\dot{\varepsilon}+3\frac{\dot{a}}{a}\varepsilon+3\frac{\dot{a}}{a}P\right)=0$$

$$\dot{\varepsilon}+3\frac{\dot{a}}{a}(\varepsilon+P)=0$$

To recap, we now have three key equations which describe how the universe expands.

There's the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3c^2}\epsilon-\frac{\kappa c^2}{R_0^2a^2}$$

the fluid equation,

$$\dot{\epsilon}+3\frac{\dot{a}}{a}(\epsilon+P)=0$$

and the acceleration equation,

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3c^2}(\epsilon+3P)$$

Fundamental Cosmological Parameters

The Expansion Rate: $H_0 = \left[\frac{\dot{a}(t)}{a(t)}\right]_{a=a_0}$

The Friedmann Constraint Equation

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_{tot}a^2 - \kappa$$

The critical density: $\rho_c(t) = \frac{3H^2}{8\pi G}$

The Density Parameter: $\Omega(t) = \frac{\rho}{\rho_c}$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} = H^2\Omega - \frac{k}{a^2}$$

$$\Omega - 1 = \frac{k}{a^2H^2} \quad \Omega_k \equiv -\frac{k}{a^2H^2} \quad \Omega + \Omega_k = 1$$

Density Parameters For Different Component of the Universe

$$\Omega_r^{(0)} = \frac{8\pi G\rho_r^{(0)}}{3H_0^2}, \quad \Omega_m^{(0)} = \frac{8\pi G\rho_m^{(0)}}{3H_0^2}, \quad \Omega_{DE}^{(0)} = \frac{8\pi G\rho_{DE}^{(0)}}{3H_0^2}, \quad \Omega_K^{(0)} = -\frac{K}{(a_0H_0)^2}.$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

$$w \equiv P/\rho$$

$$\rho \propto a^{-3(1+w)}, \quad a \propto (t - t_i)^{2/(3(1+w))}$$

In order to accelerate the universe

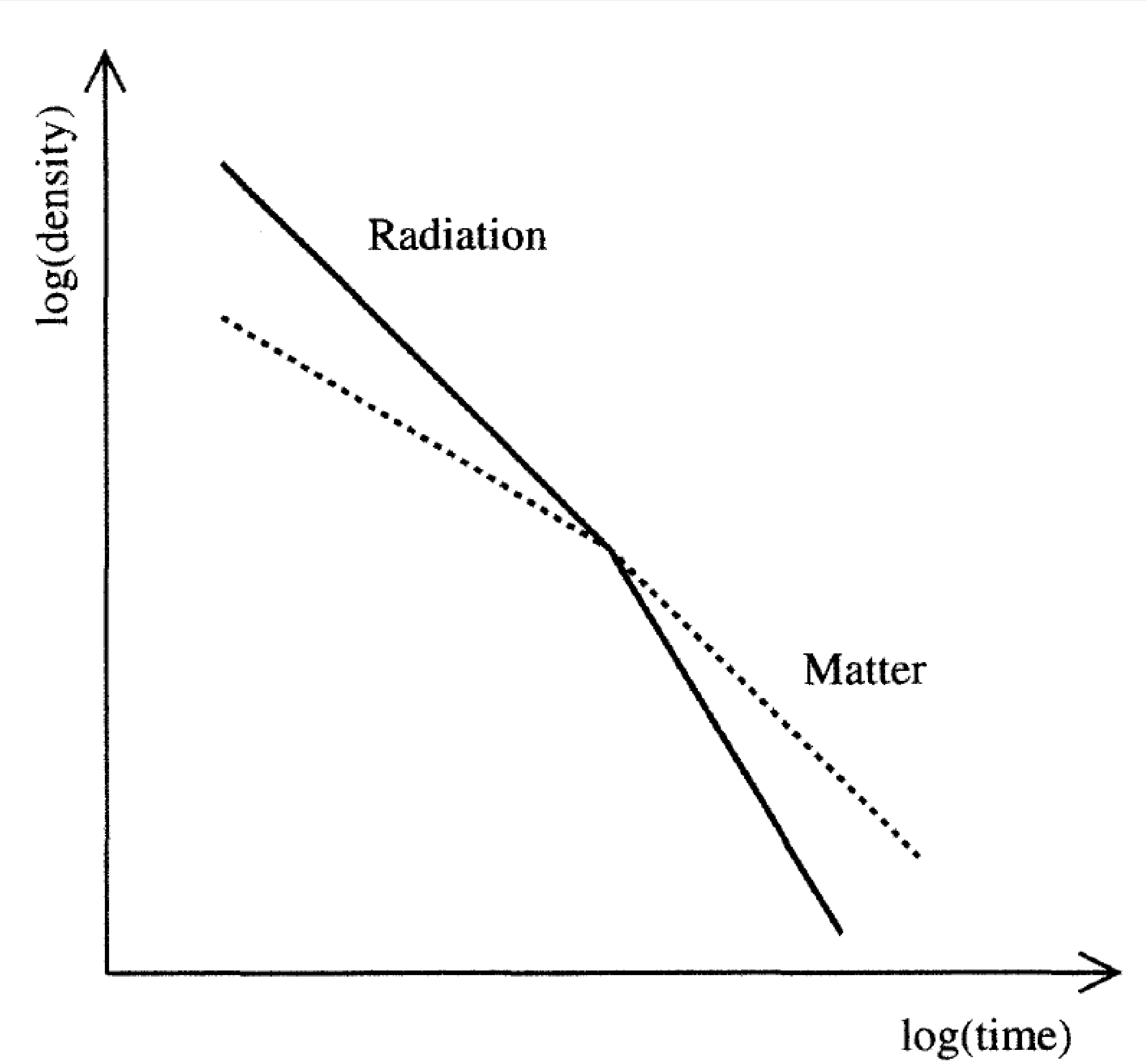
$$P < -\rho/3 \quad \rightarrow \quad w < -1/3$$

A Mixture

$$\rho_{\rm mat} \propto \frac{1}{a^3} \quad ; \quad \rho_{\rm rad} \propto \frac{1}{a^4}$$

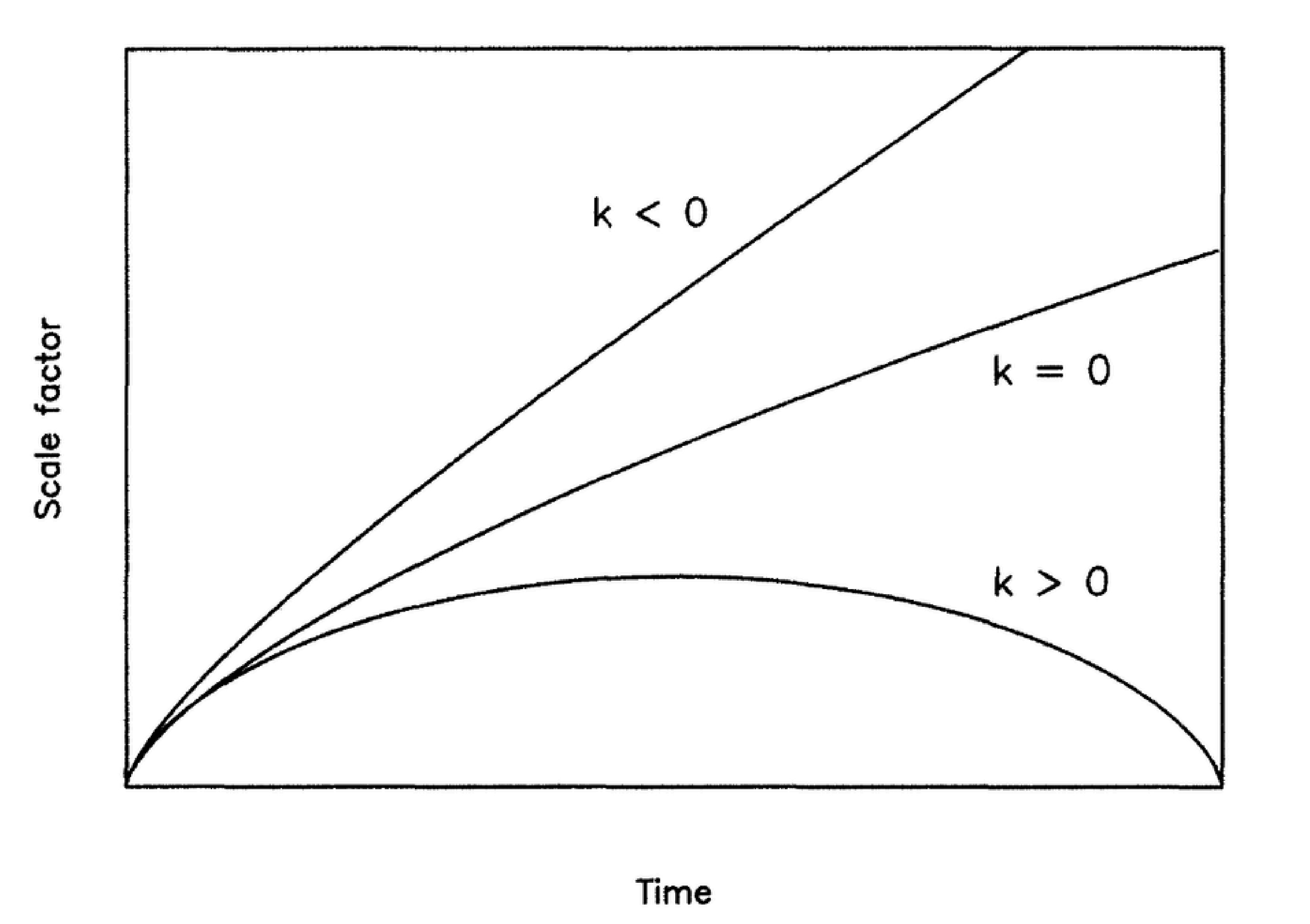
$$a(t) \propto t^{1/2} \quad ; \quad \rho_{\rm mat} \propto \frac{1}{a^3} \propto \frac{1}{t^{3/2}}$$

$$a(t) \propto t^{2/3} \quad ; \quad \rho_{\rm rad} \propto \frac{1}{a^4} \propto \frac{1}{t^{8/3}}$$



Evolution with Curvature

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$



Cosmological Distances

- Co-moving Distance.
- Luminosity Distance
- Angular Diameter Distance

FRW Metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Setting $r = \sin\chi$ $(K = +1)$, $r = \chi$ $(K = 0)$, and $r = \sinh\chi$ $(K = -1)$

$$d\sigma^2 = d\chi^2 + (f_K(\chi))^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

where

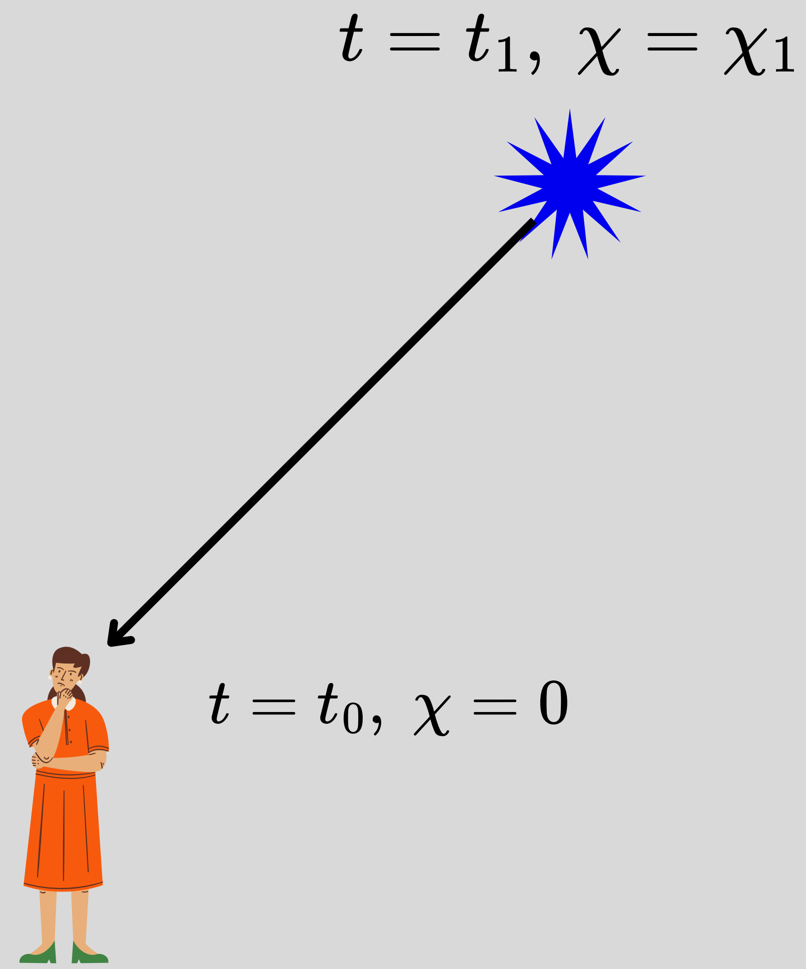
$$f_K(\chi) = \begin{cases} \sin\chi & (K = +1), \\ \chi & (K = 0), \\ \sinh\chi & (K = -1). \end{cases}$$

The function $f_K(\chi)$ can be written in a unified way:

$$f_K(\chi) = \frac{1}{\sqrt{-K}}\sinh(\sqrt{-K}\chi)$$

where the case of the flat universe is recovered by taking the limit $K \to 0$.

Co-moving Distance



Light travels along the geodesic

$$ds^2 = -c^2 dt^2 + a(t)^2 d\chi^2 = 0$$

Integrating the equation, $d\chi = -\dfrac{c\,dz}{a(z)}$, the following distance is found

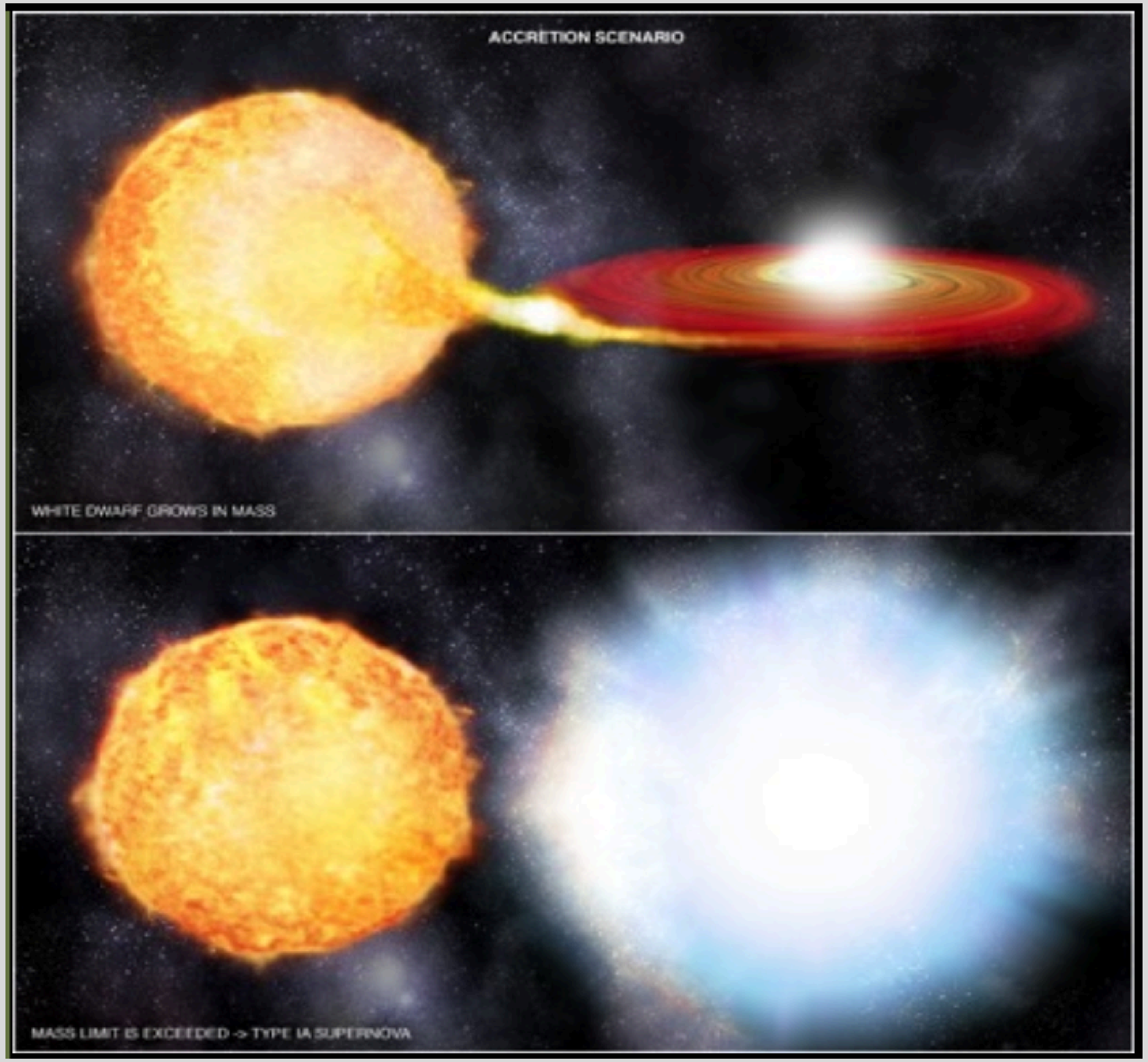
$$\chi_1 = \int_0^z \frac{c}{a(z)}\,dz = -\int_0^z \frac{dz}{[H(z)t]}$$

$$dc = \frac{c}{a_0 H_0} \int_0^z \left[\frac{d\bar{z}}{E(\bar{z})}\right]$$

$$\text{where } E(z) \equiv \frac{H(z)}{H_0}$$

Luminosity Distance

Luminocity distance is used for type Ia Supernova observation



Luminosity Distance

The Luminosity distance

$$d_L^2 \equiv \frac{L_s}{4\pi\mathcal{F}}$$

where L_s is the absolute luminosity of a source and $\mathcal{F}$ is an observed flux.

Note that the observed luminosity L_0 (detected at $\chi = 0$ and $z = 0$)

is different from the absolute luminosity L_s of the source

(emitted at the comoving distance χ with the redshift z).

The flux $\mathcal{F}$ is defined by $\mathcal{F} = \dfrac{L_0}{S},$ where $S = 4\pi(a_0 f_K(\chi))^2$ is the area of a sphere at $z = 0$.

Then the luminosity distance yields

$$d_L^2 = (a_0 f_K(\chi))^2 \frac{L_s}{L_0}$$

We need now to derive the ratio $\dfrac{L_s}{L_0}$.

Luminosity Distance

$$L_s = \Delta E_1 / \Delta t_1$$

$$L_s 0 = \Delta E_0 / \Delta t_0$$

From: $c = \lambda / \Delta t$

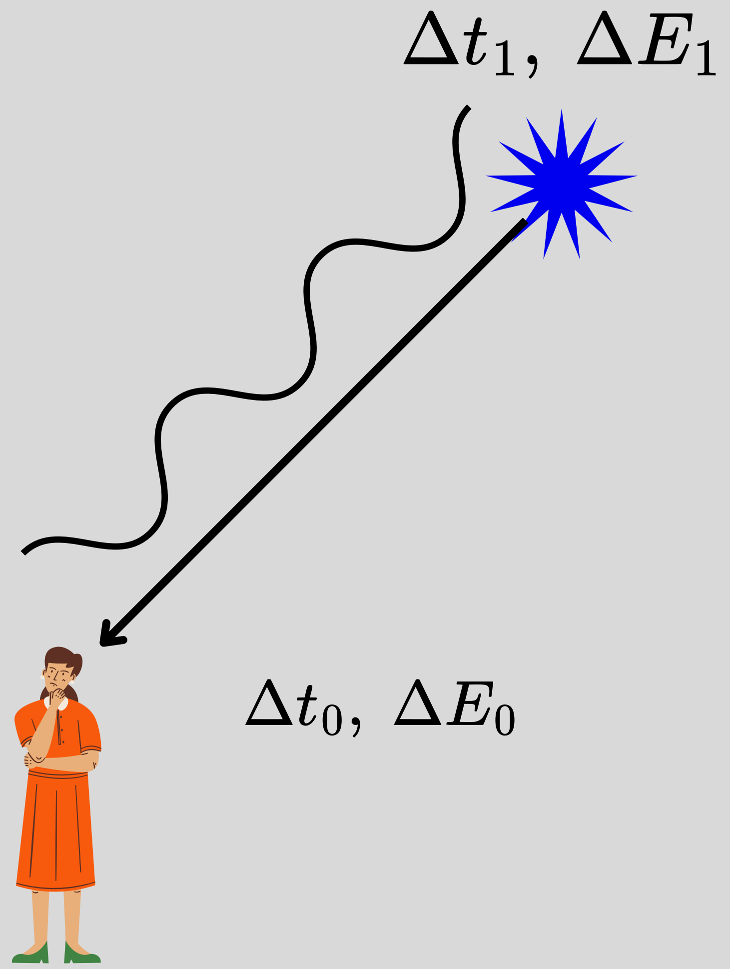
$$\frac{\Delta E_1}{\Delta E_0} = \frac{\lambda_0}{\lambda_1} = 1 + z$$

$$d_L = a_0 f_K(\chi)(1+z)$$

Recall: $\chi = d_c = \dfrac{c}{a_0 H_0} \displaystyle\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}$

$$d_L = \frac{c(1+z)}{H_0 \sqrt{\Omega_K^{(0)}}} \sinh\left(\sqrt{\Omega_K^{(0)}} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} \right)$$

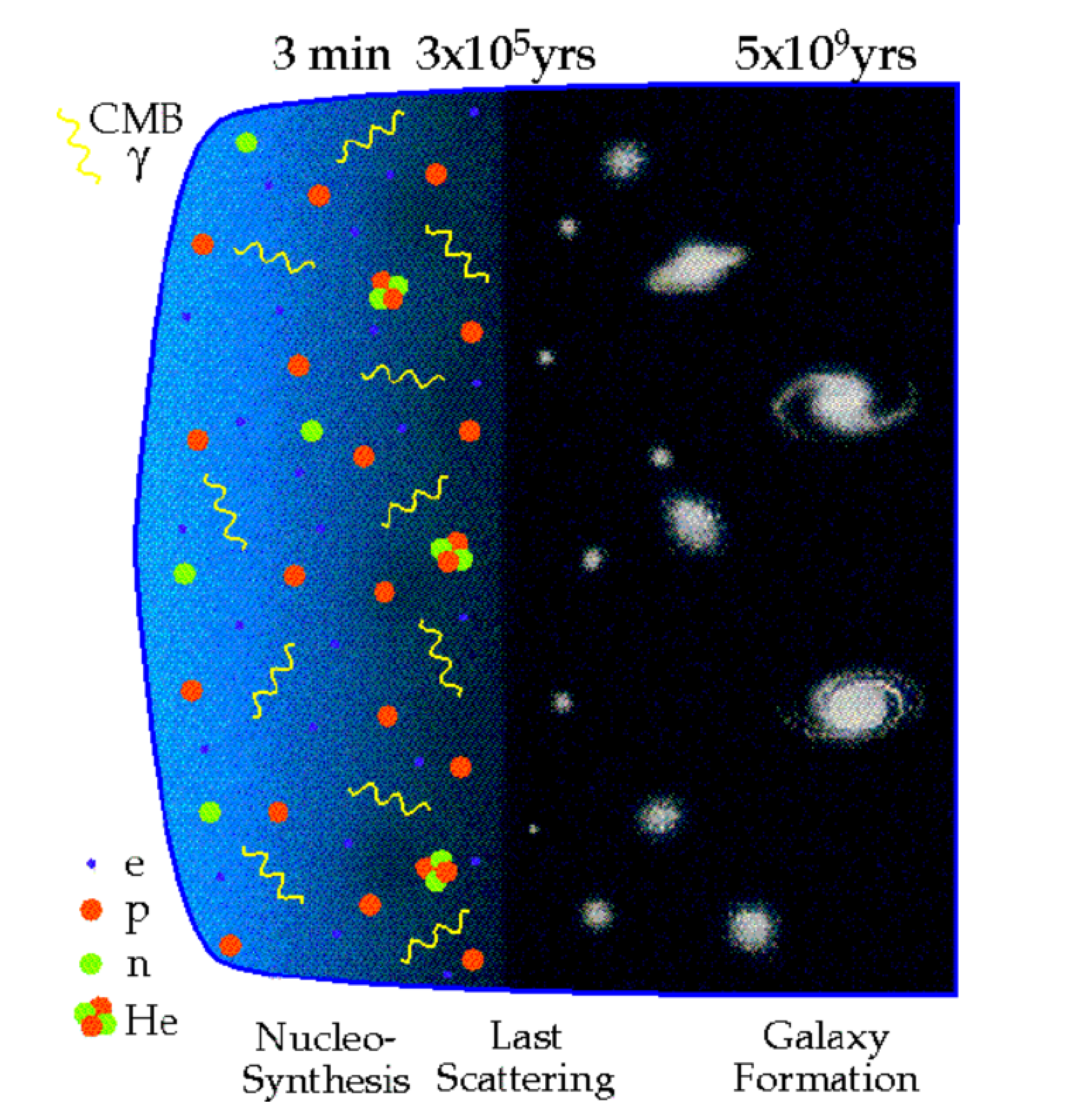
where $\Omega_K^{(0)} = -\dfrac{Kc^2}{(a_0 H_0)^2}$

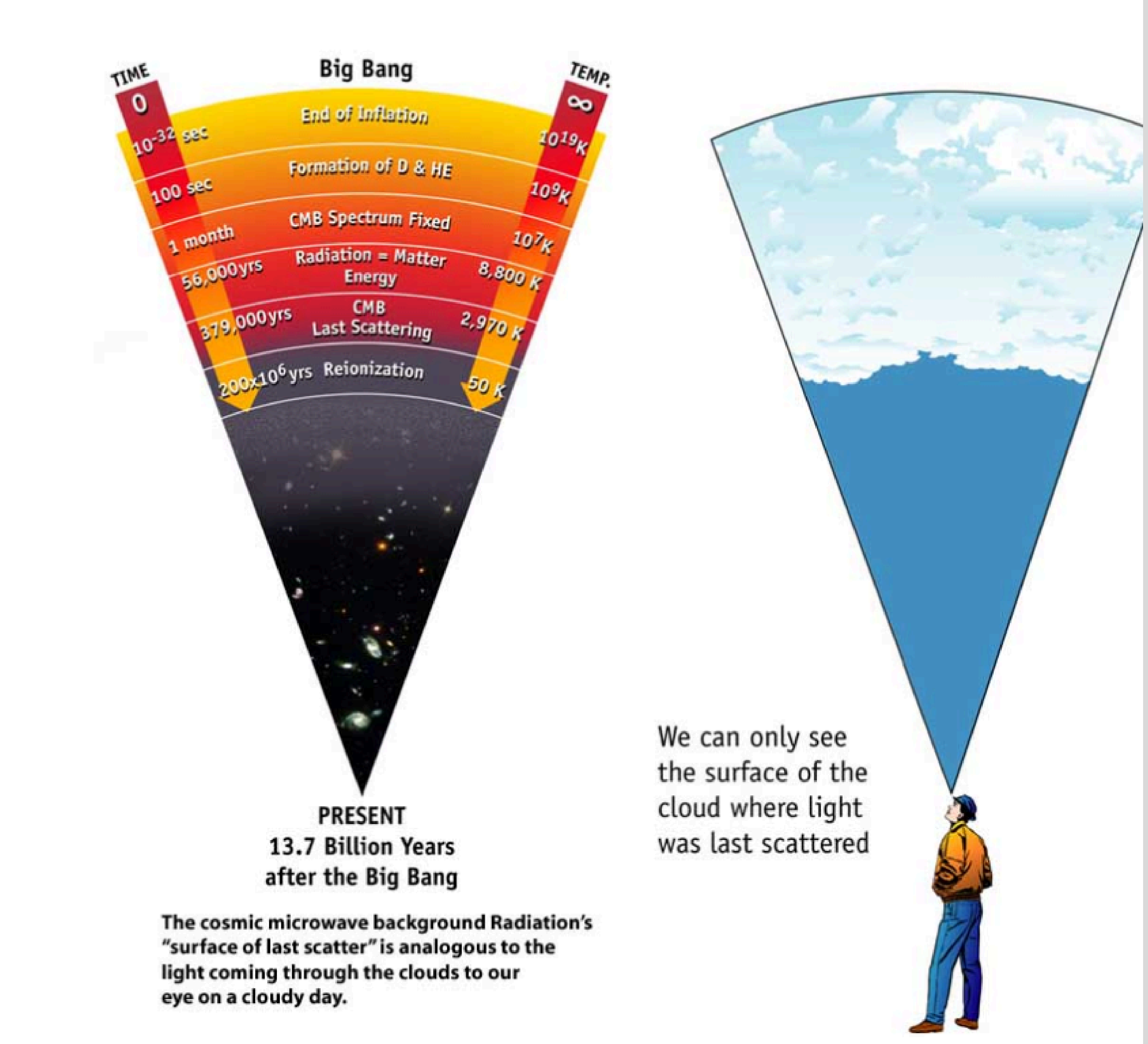


$$f_K(\chi) = \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}\chi)$$

Angular Diameter Distance

This distance often used for observation of CMB anisotropy.





Angular Diameter Distance

$$d_A = \frac{\Delta x}{\Delta \theta}$$

$\Delta x \Rightarrow$ Actual size of the object.

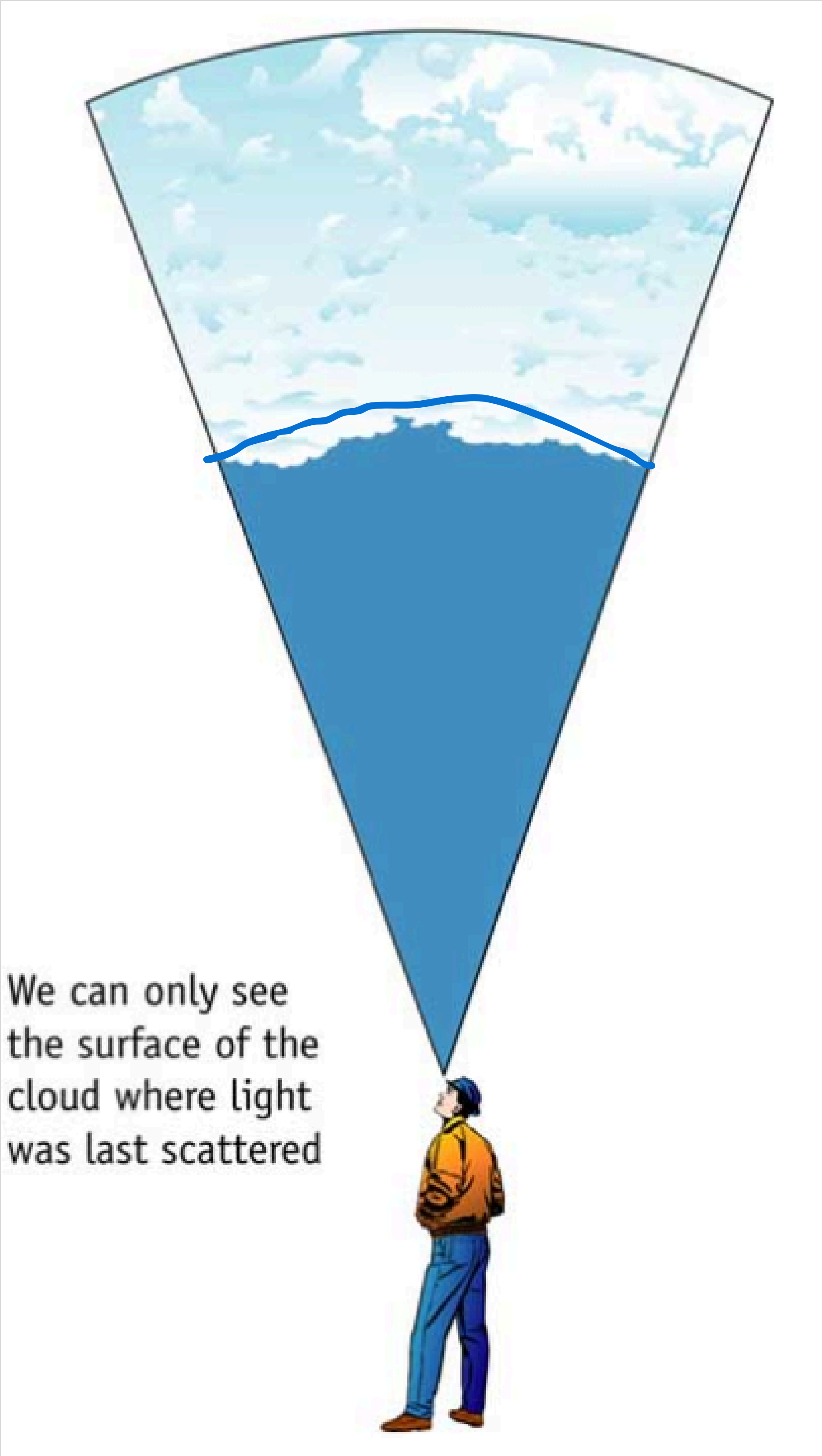
$\Delta \theta \Rightarrow$ Angle substants by the object.

$$\Delta x = a(t_1) f_K(\chi) \Delta \theta$$

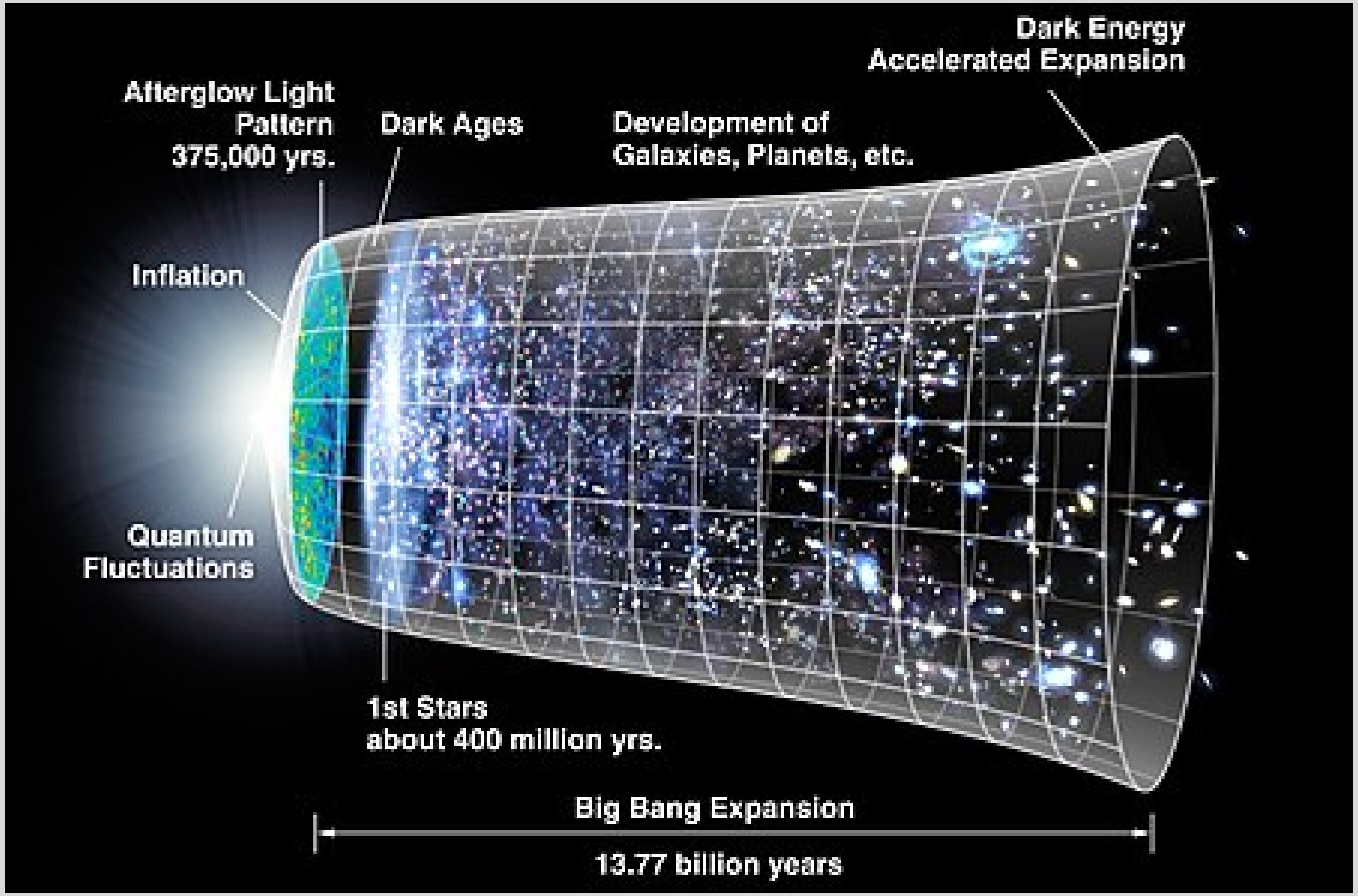
Hence the diameter distance is

$$d_A = a(t_1) f_K(\chi) = \frac{a_0 f_K(\chi)}{1+z} = \frac{1}{1+z} \cdot \frac{c}{H_0 \sqrt{\Omega_K^{(0)}}} \sinh\left(\sqrt{\Omega_K^{(0)}} \int_0^z \frac{d\bar{z}}{E(\bar{z})}\right)$$

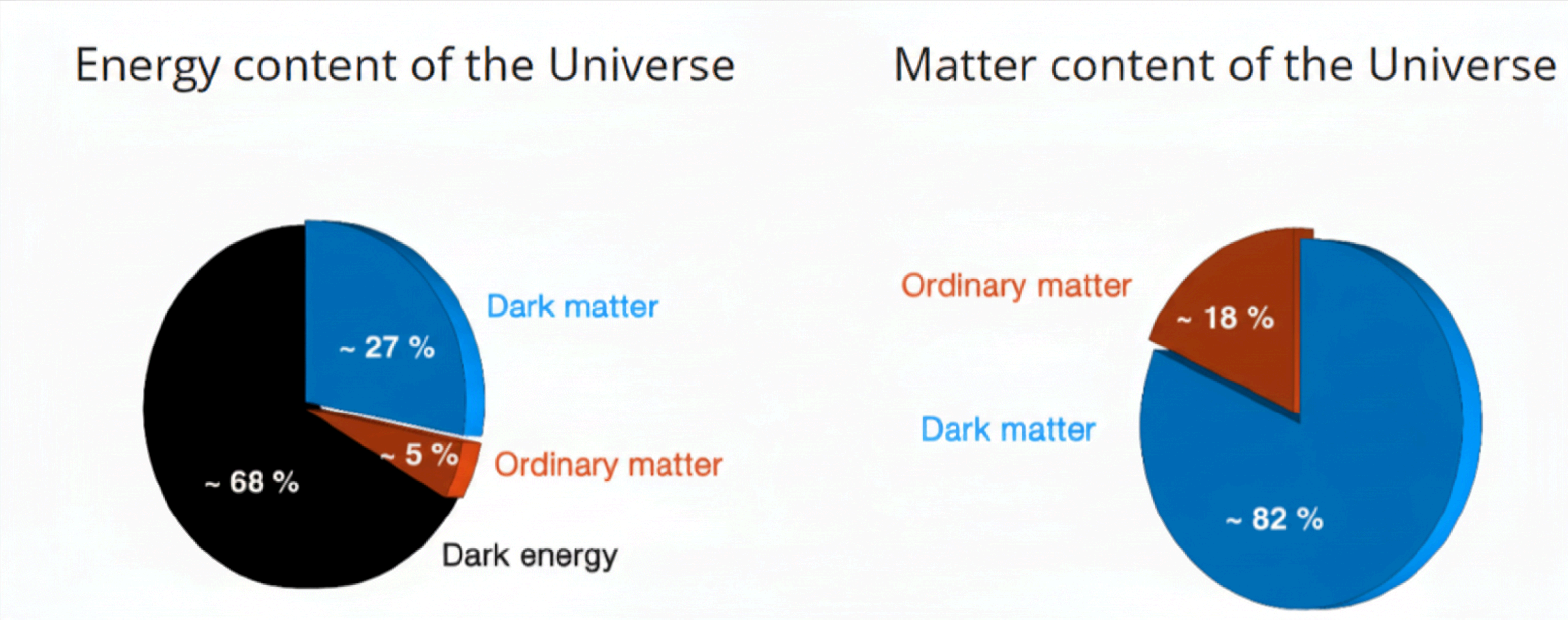
$$d_A = \frac{d_L}{(1+z)^2}$$



Accelerating Universe



Standard Model of Cosmology ($\Lambda\, CDM$)



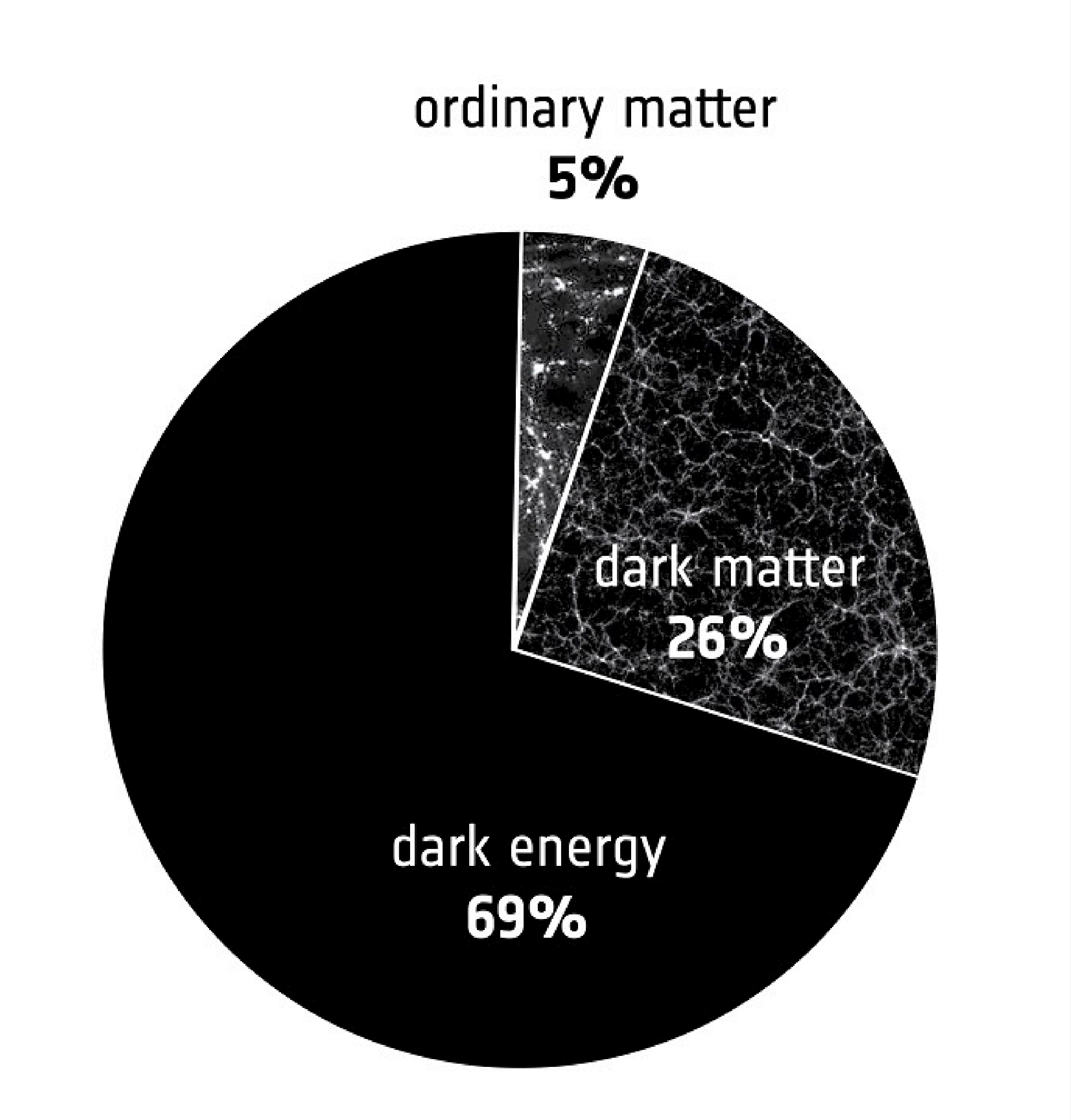
Components of The Universe

Component	Symbol	Rest mass energy (MeV)	Charge
Proton	p	938.3	+1
Neutron	n	939.6	0
Electron	e^{-1}	0.511	-1
Neutrino	ν_e, ν_μ, ν_τ	?	0
Photon	γ	0	0
Dark Matter	?	?	0
Dark Energy	?	?	0

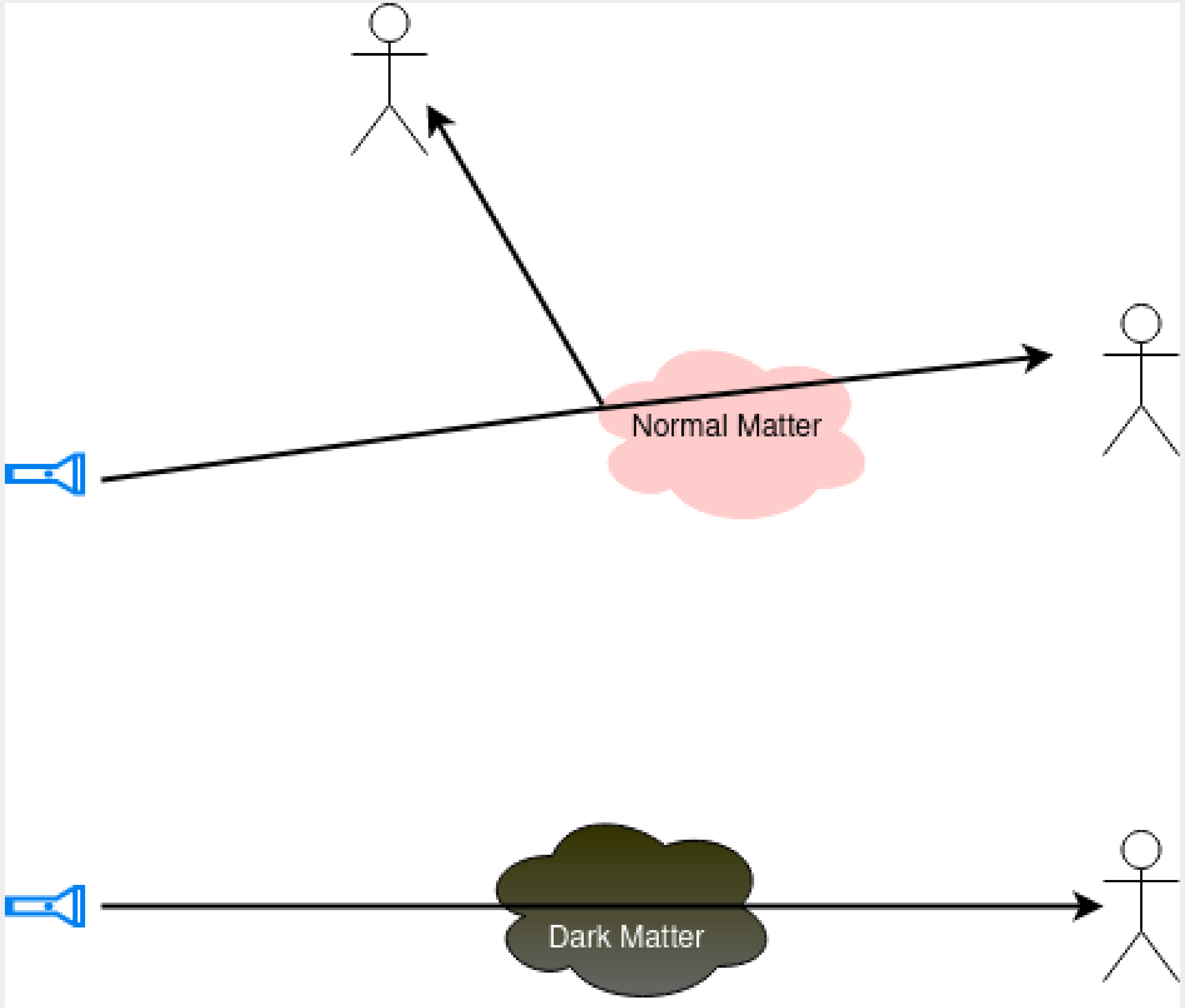
Non-relativistic components

Relativistic component

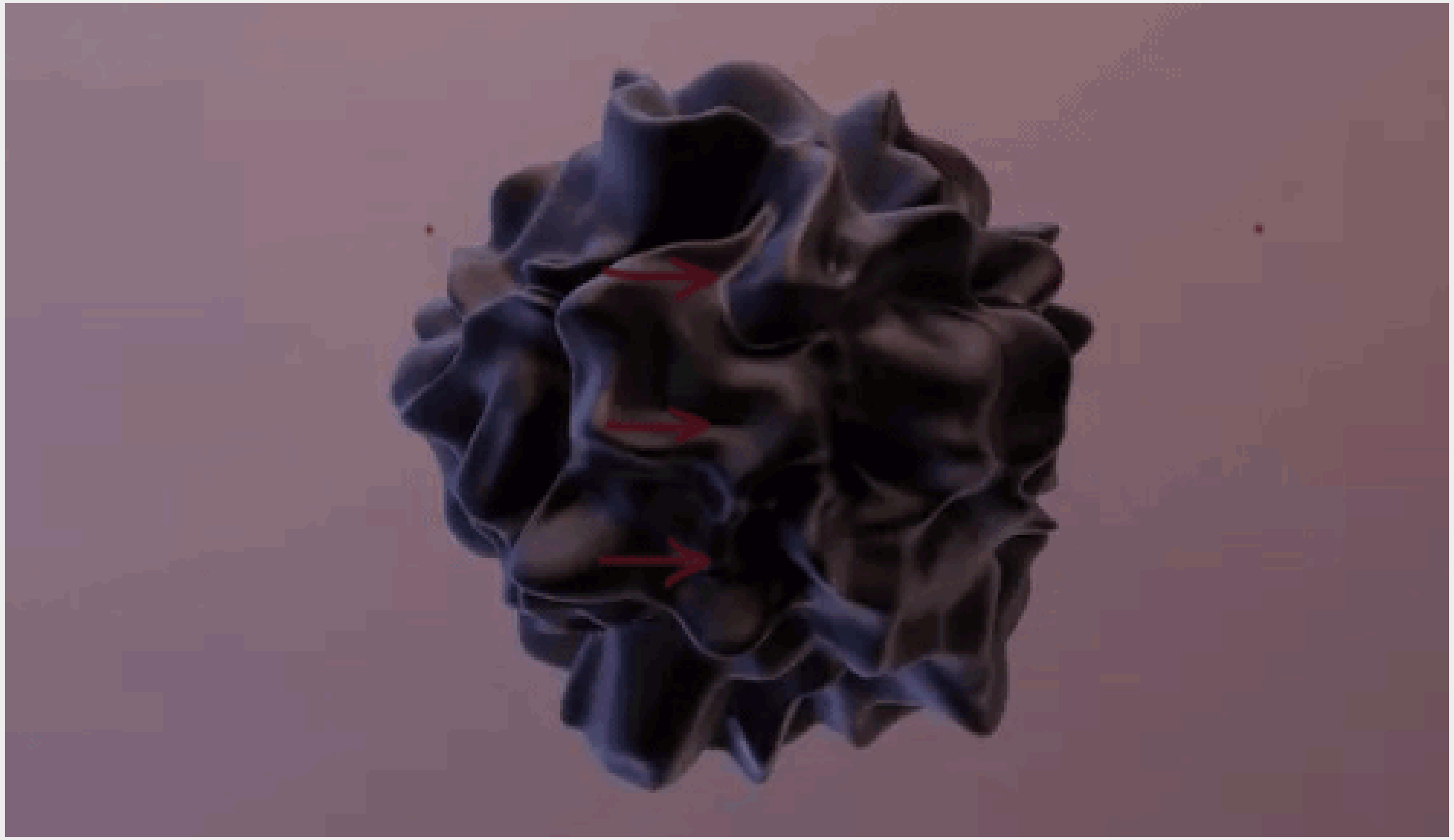
Dark Component



Dark Matter

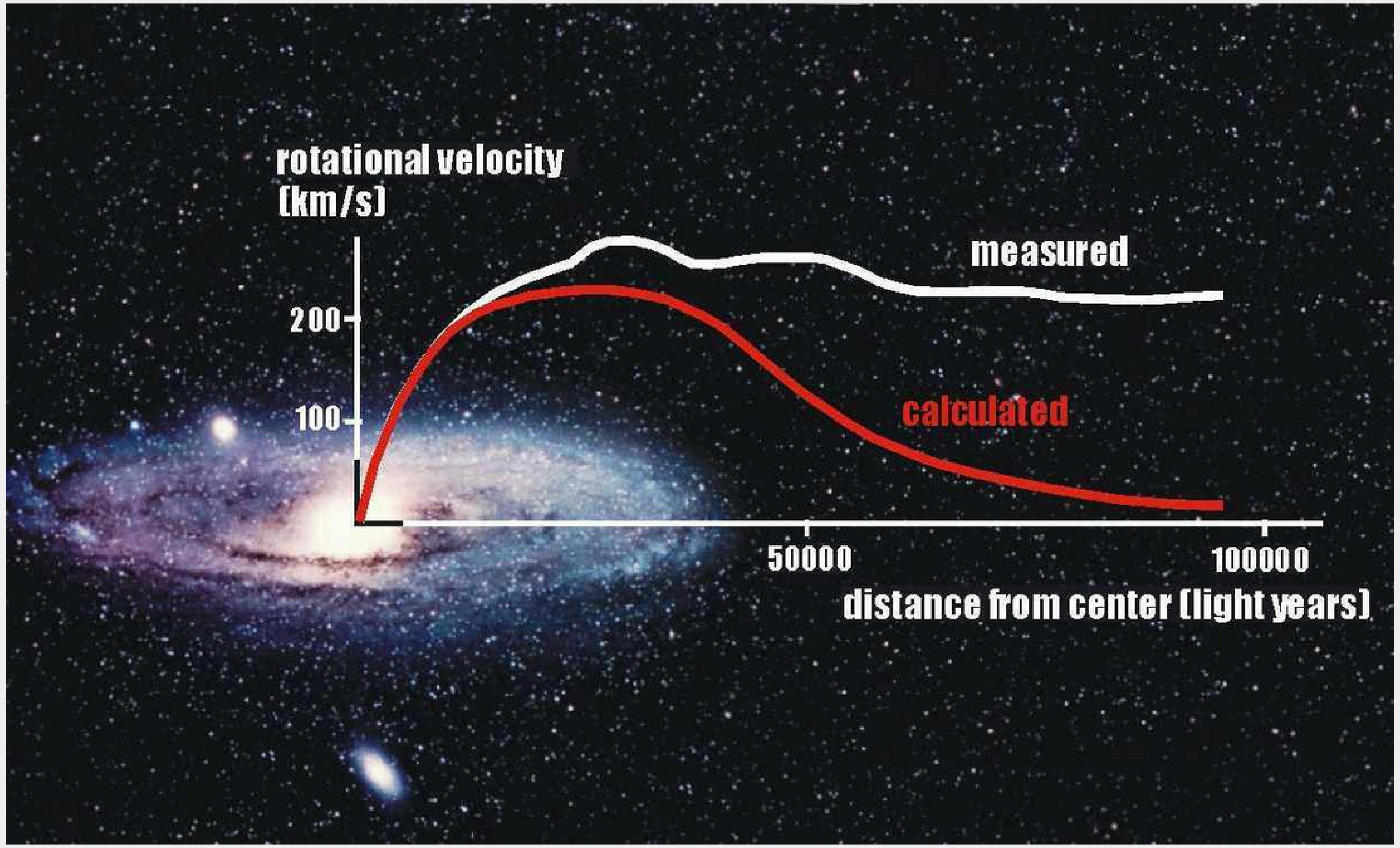


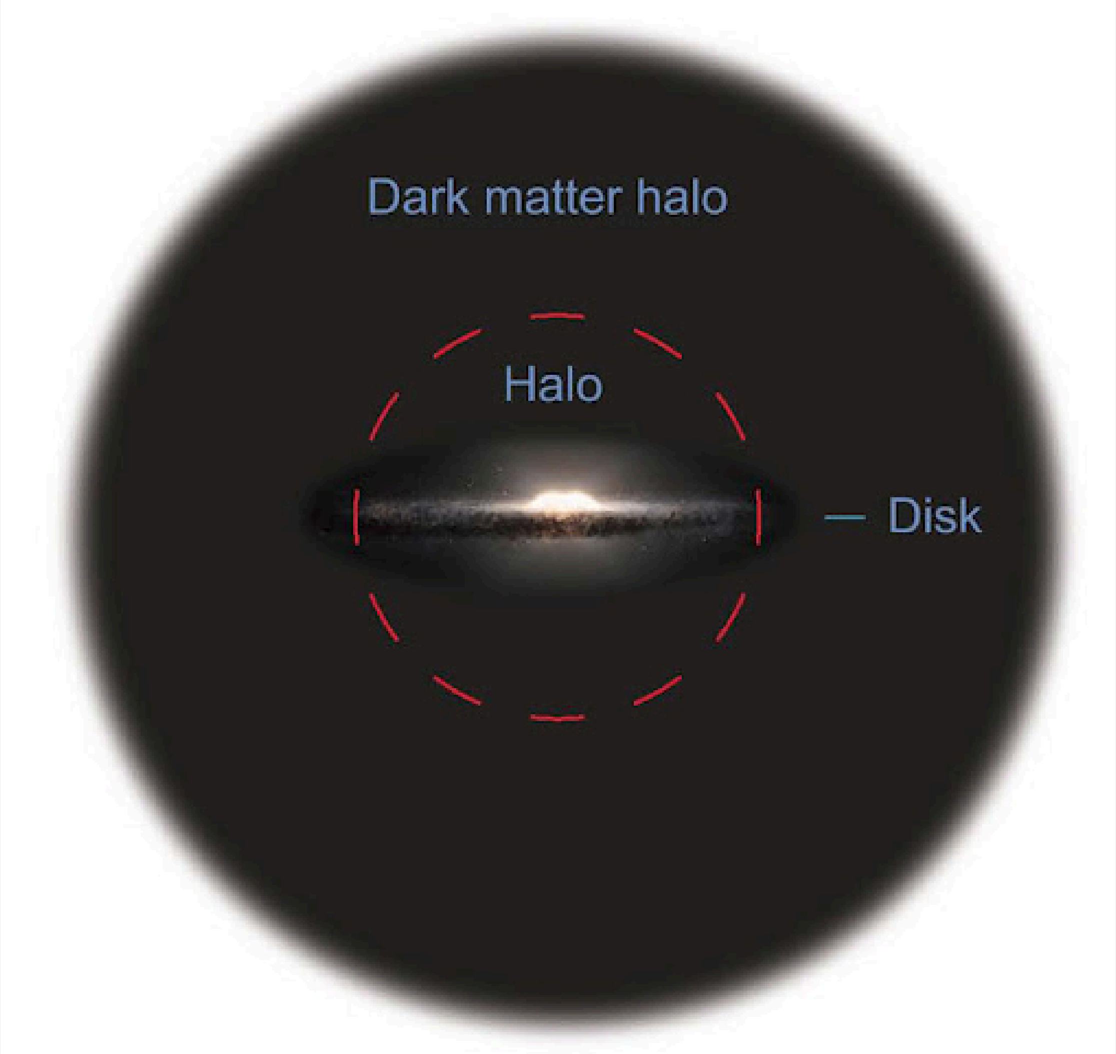
Dark Matter



Dark Matter

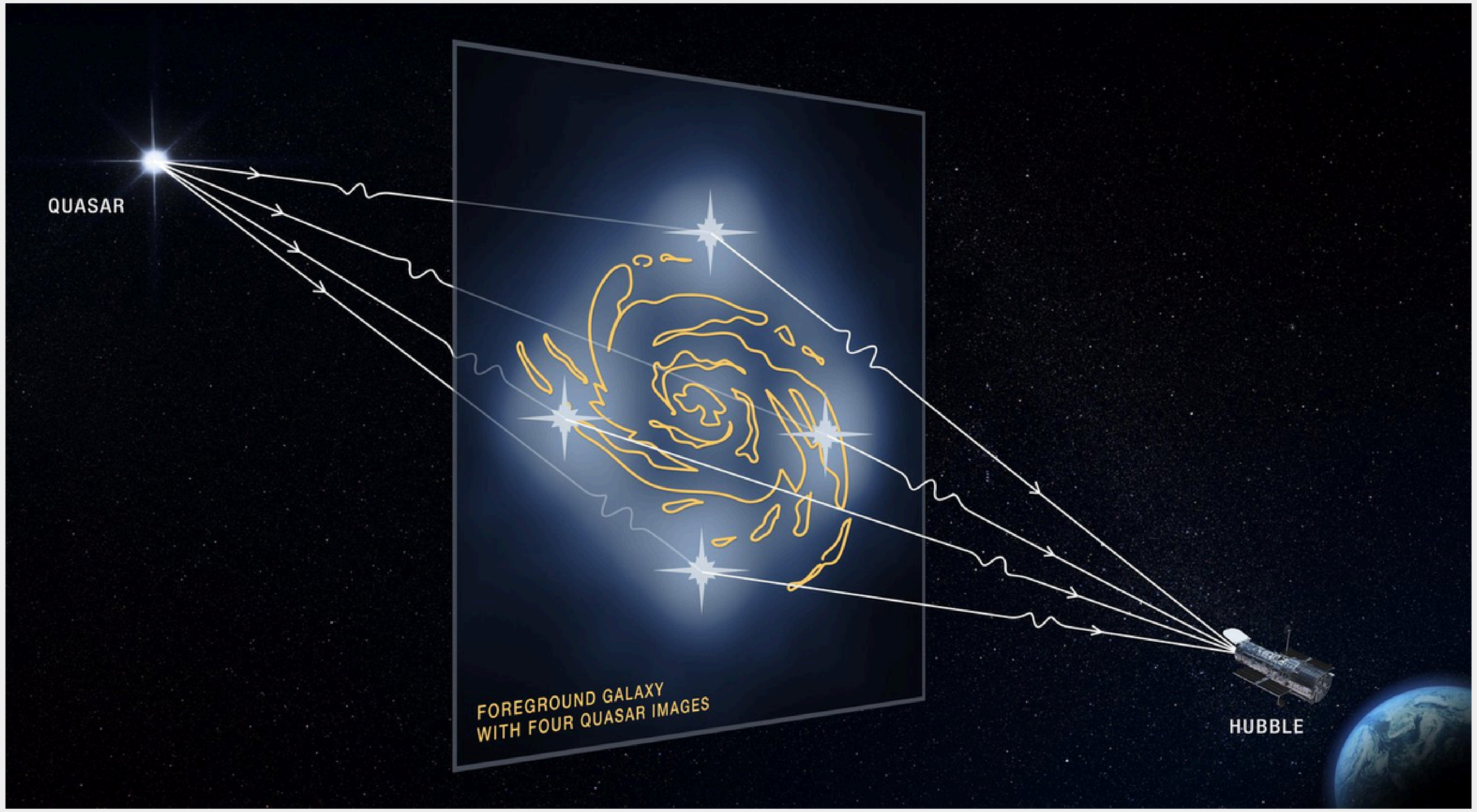
Dark matter detection from velocity rotation curve



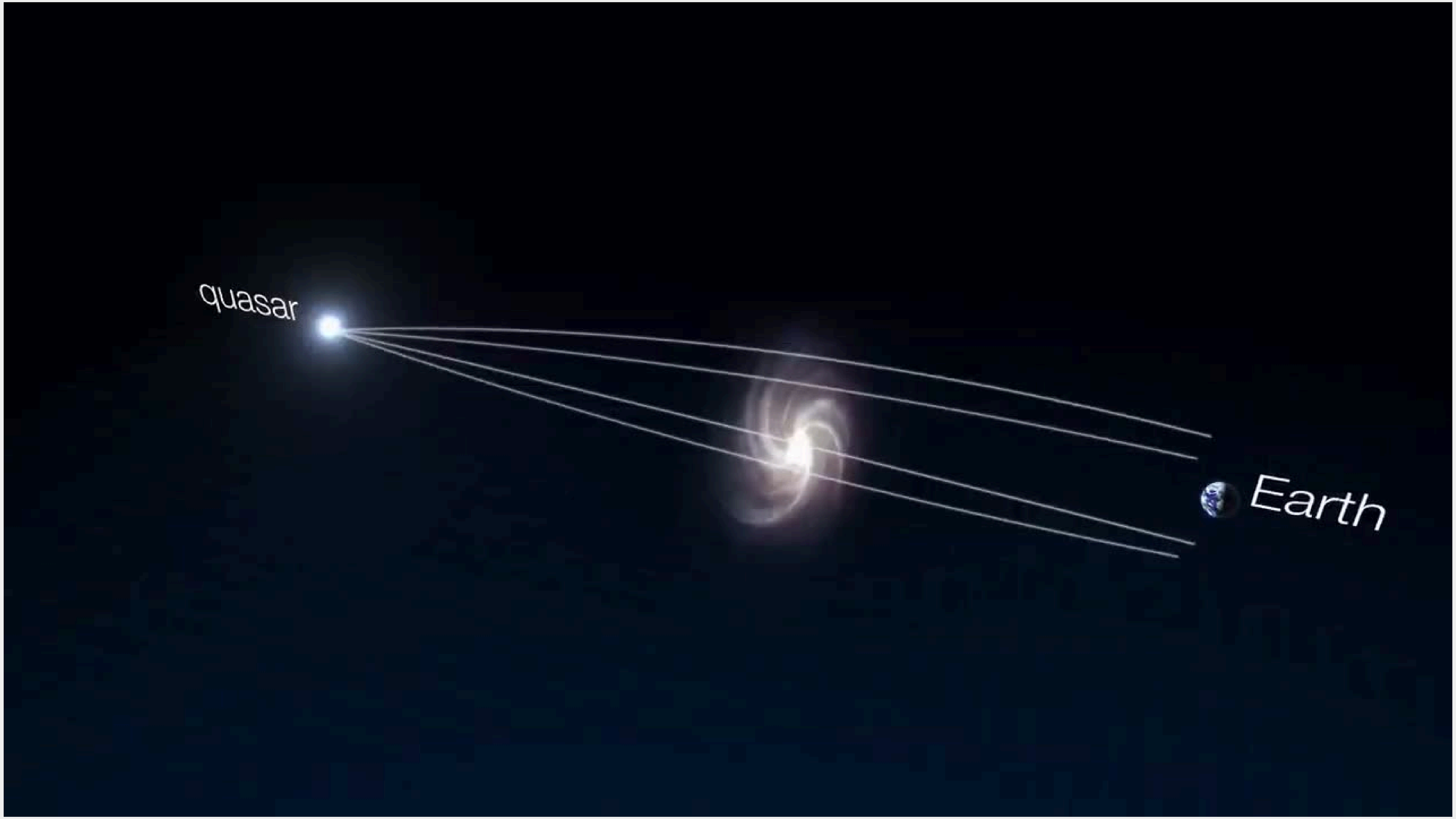


Milky Way model

DM detection from gravitational lensing



DM detection from gravitational lensing



DM detection from gravitational lensing

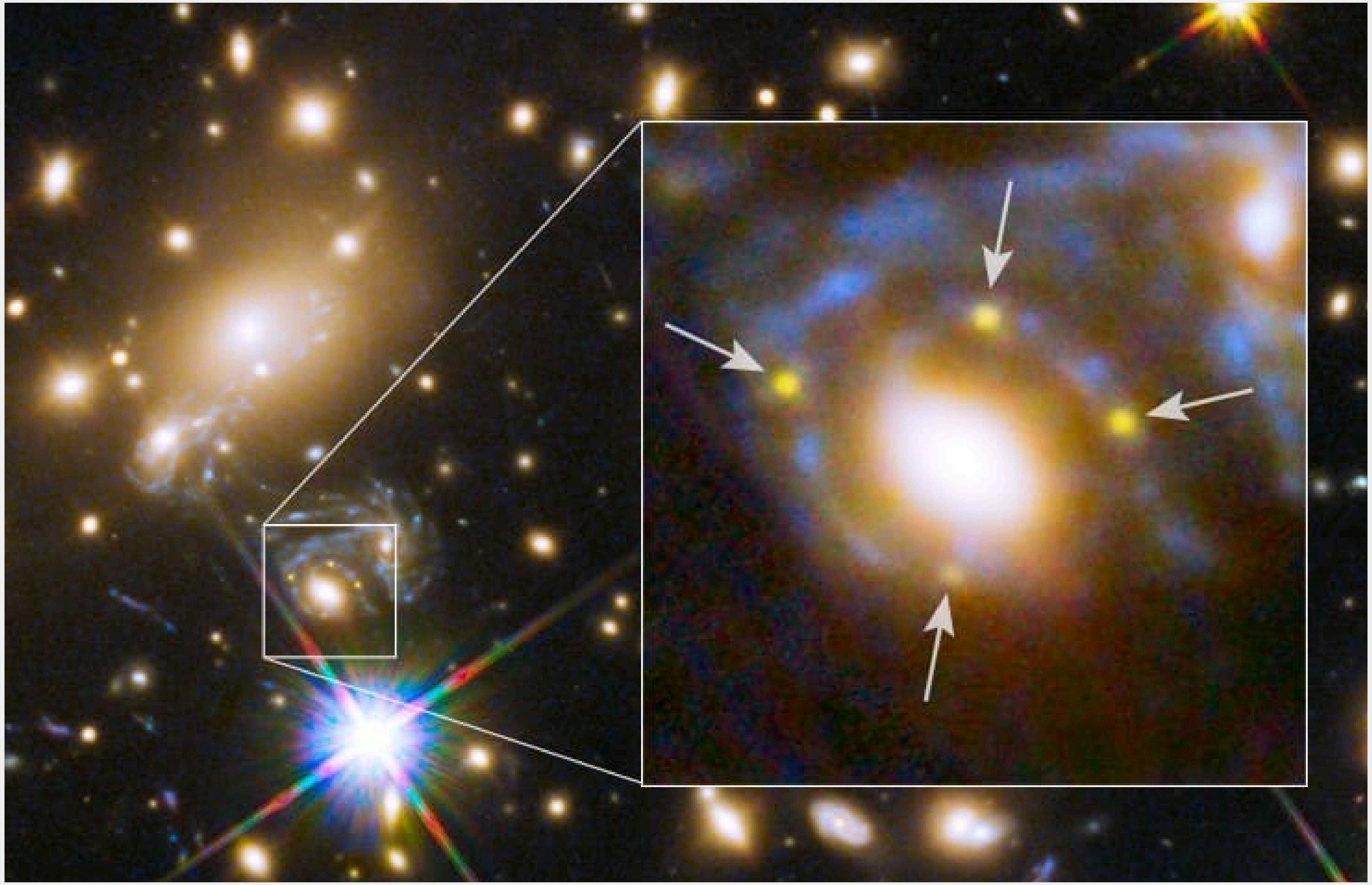


Image credit NASA/ESA/Massey

Accelerating Universe

The accelerated expansion was discovered during 1998, by two independent projects, the Supernova Cosmology Project and the High-Z Supernova Search Team, which both used distant type Ia supernovae to measure the acceleration

Evidences for acceleration

- The age of the universe
- Supernovae observations
- Cosmic Microwave background
- Baryon acoustic oscillation

The age of the Universe

If the DE is not considered the age of the universe will be less than the oldest star.

The inverse of the Hubble H_0 can roughly measure the age of the universe.

$$E(z) = \left[\Omega_r^{(0)}(1+z)^4 + \Omega_m^{(0)}(1+z)^3 + \Omega_{\mathrm{DE}}^{(0)}(1+z)^{3(1+w_{\mathrm{DE}})} + \Omega_K^{(0)}(1+z)^2\right]^{1/2}.$$

If the EOS of the dark energy is considered to be constant:

$$\rho_{DE} = \rho_{DE}^{(0)}(1+z)^{3(1+w_{DE})}; dt = -dz/[H(1+z)]$$

The age of the Universe $\Rightarrow t_0 = \int_0^\infty \frac{dz}{E(z)(1+z)}$

By neglecting the radiatioan and for the cosmological cosntant $\Rightarrow t_0 = H_0^{-1}\int_1^\infty \frac{dx}{[\Omega_m^{(0)}x^3 + \Omega_{DE}^{(0)} + \Omega_k^{(0)}x^2]^{1/2}}$

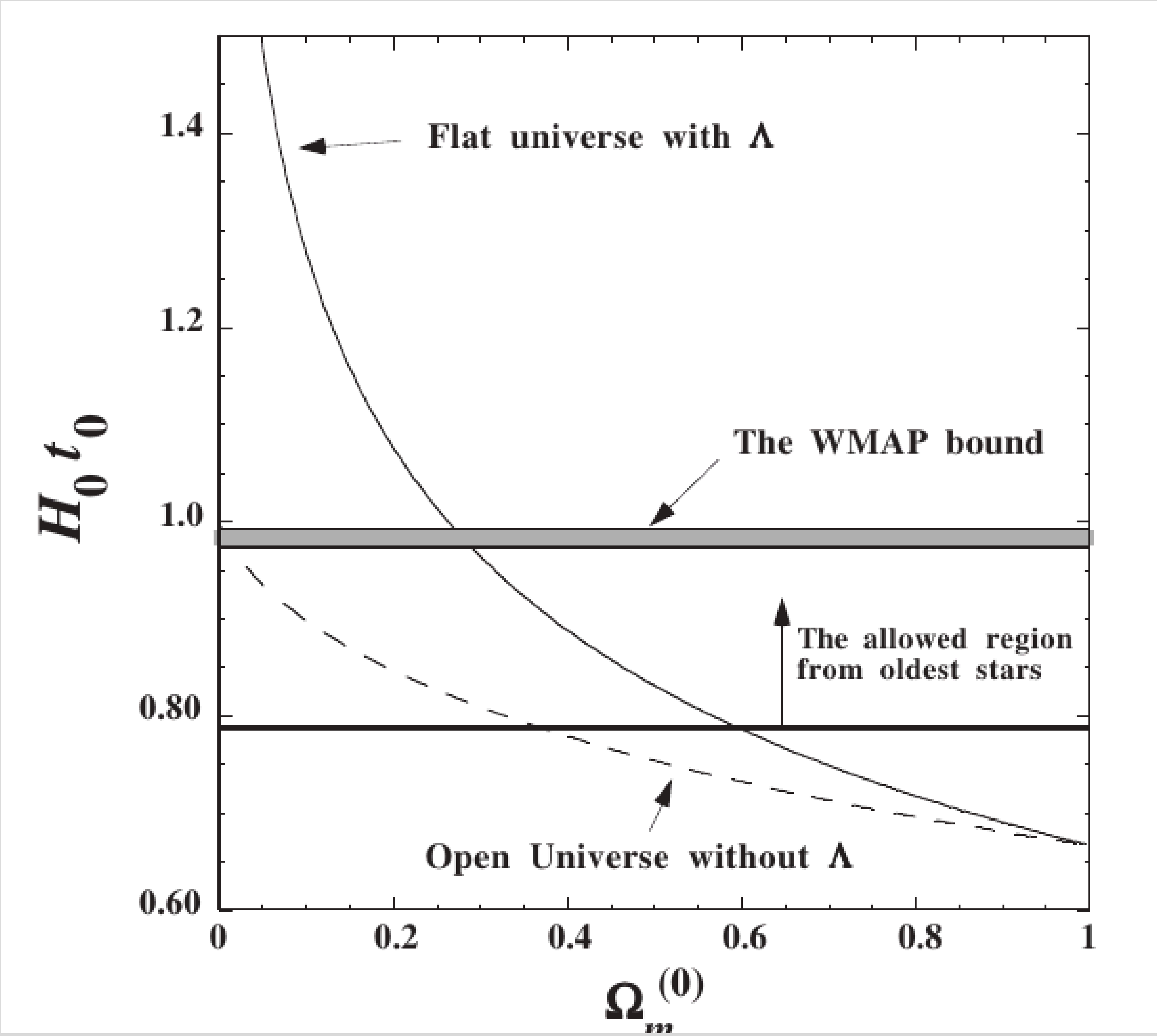
For a Flat Universe: $\Rightarrow t_0 = \frac{H_0^{-1}}{3\sqrt{1-\Omega_m^{(0)}}}\ln\left(\frac{1+\sqrt{1-\Omega_m^{(0)}}}{1-\sqrt{1-\Omega_m^{(0)}}}\right)$

When $\Omega_{DE}^{(0)} \to 0$ the age of the universe redues to: $\Rightarrow t_0 = \frac{2}{3}H_0^{-1}$

In the absence of dark energy, the age of the universe 8.2 Gyr to 10.2 Gyr.

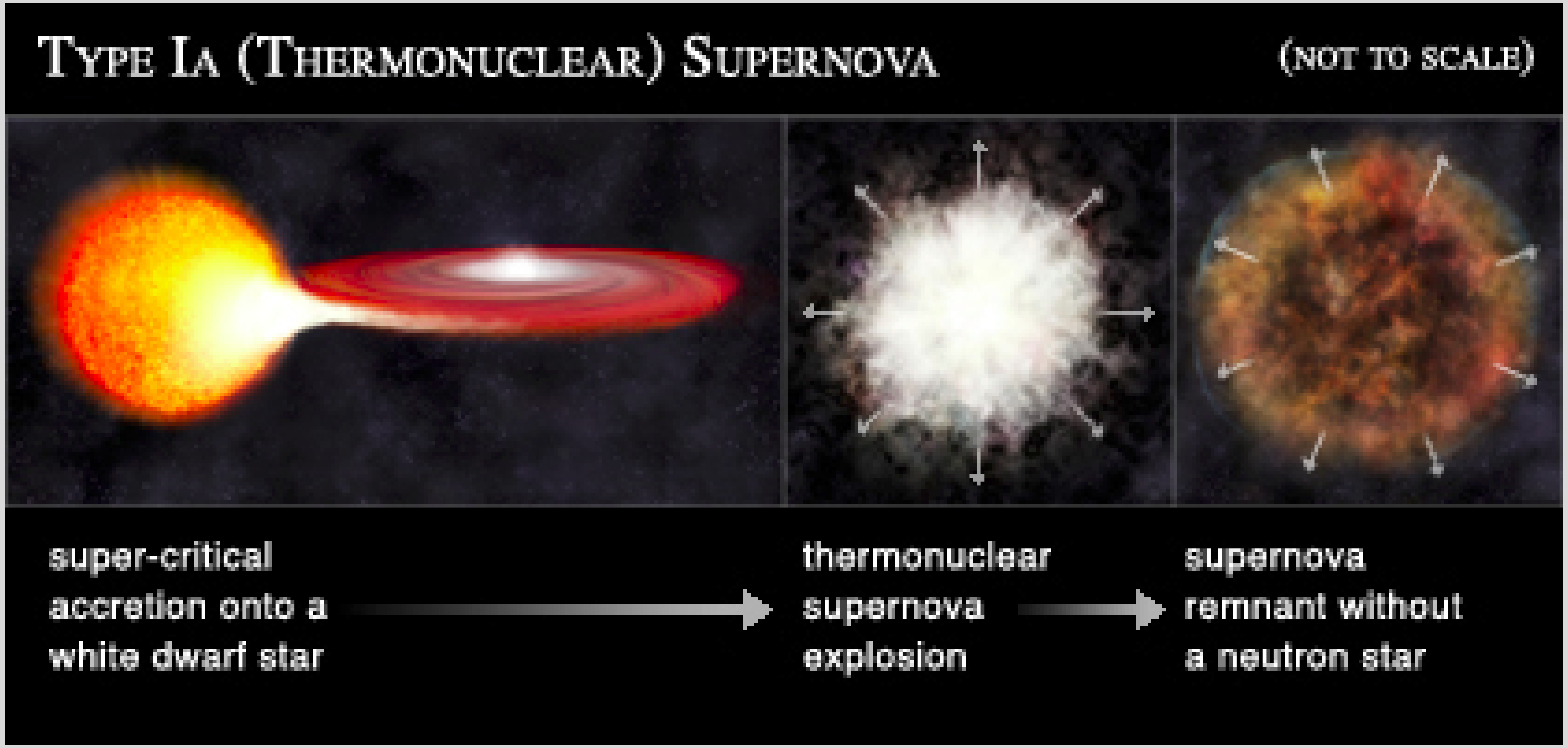
The age of the globular cluster in the Milky is more than 11 Gyr.

The age of the Universe



Type IA Supernova

Type Ia supernovae are thermonuclear explosions of white dwarf stars in binary systems, triggered when the white dwarf accretes enough mass to exceed the Chandrasekhar limit. These events produce a consistent peak luminosity, making them reliable "standard candles" for measuring cosmic distances. Their discovery led to the groundbreaking realization that the universe's expansion is accelerating, providing key evidence for the existence of dark energy.



In 1998 Riess et al. [High-redshift Supernova Search Team (HSST)] and Perlmutter et al. [Supernova Cosmology Project (SCP)] independently reported the late-time cosmic acceleration

Supernovae observations

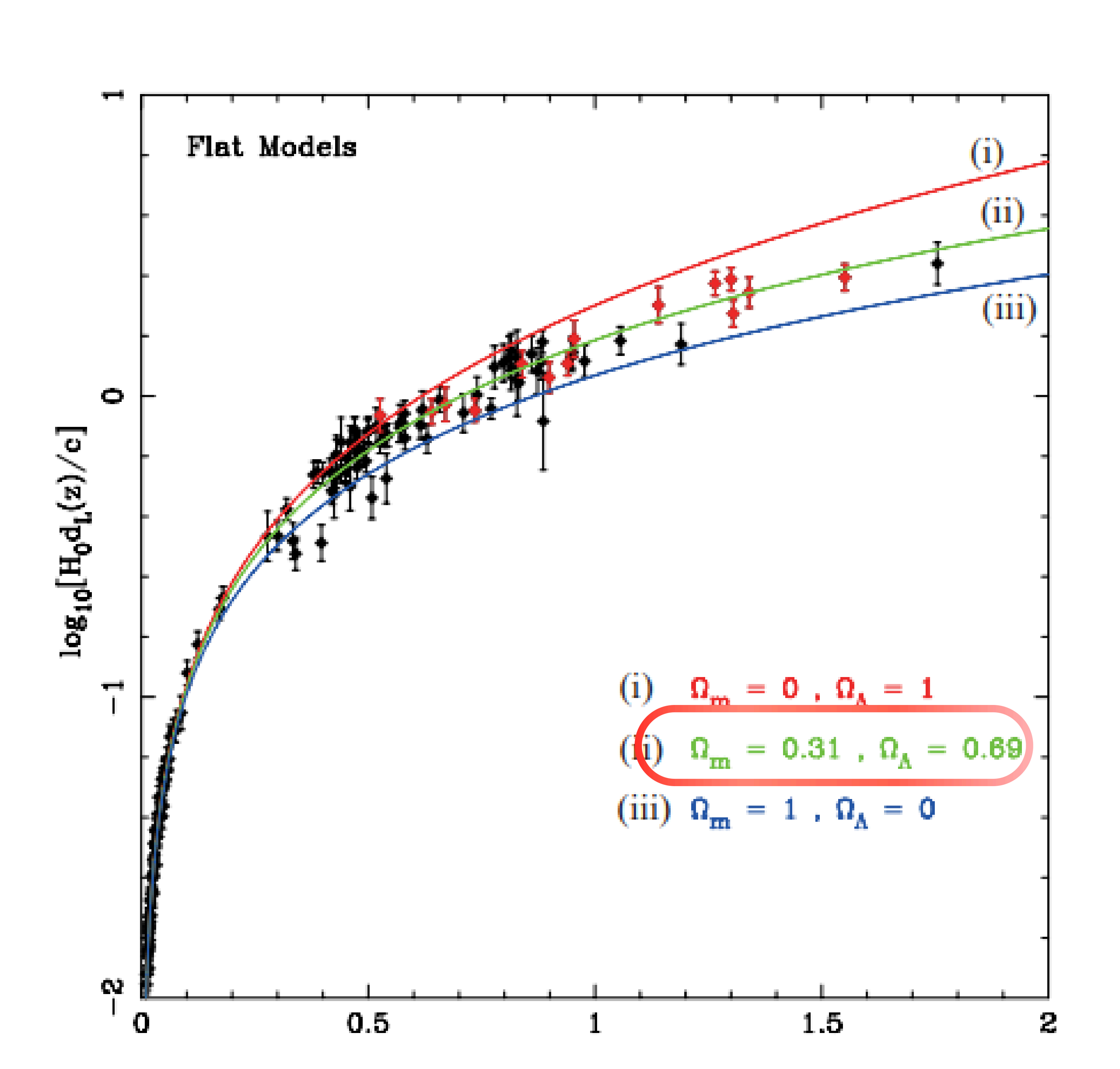
The relation between the apparent magnitude and the absolute magnitude of the supernova

$$m - M = 5\log_{10} d_L + 25.$$

$$\text{The Luminosity Distance} \Rightarrow d_L(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{d\tilde{z}}{[(1-\Omega_{DE}^{(0)})(1+\tilde{z})^3 + \Omega_{DE}^{(0)}]^{1/2}}$$

$$\text{Luminosity Distance at } z << 1: \Rightarrow d_L(z) = \frac{c}{H_0}\left[z + \frac{1}{4}(1 - 3w_{DE}\Omega_{DE}^{(0)} + \Omega_K^{(0)})z^2 + \mathcal{O}(z^3)\right]$$

Supernovae observations



Most Updated SNIa Compilations

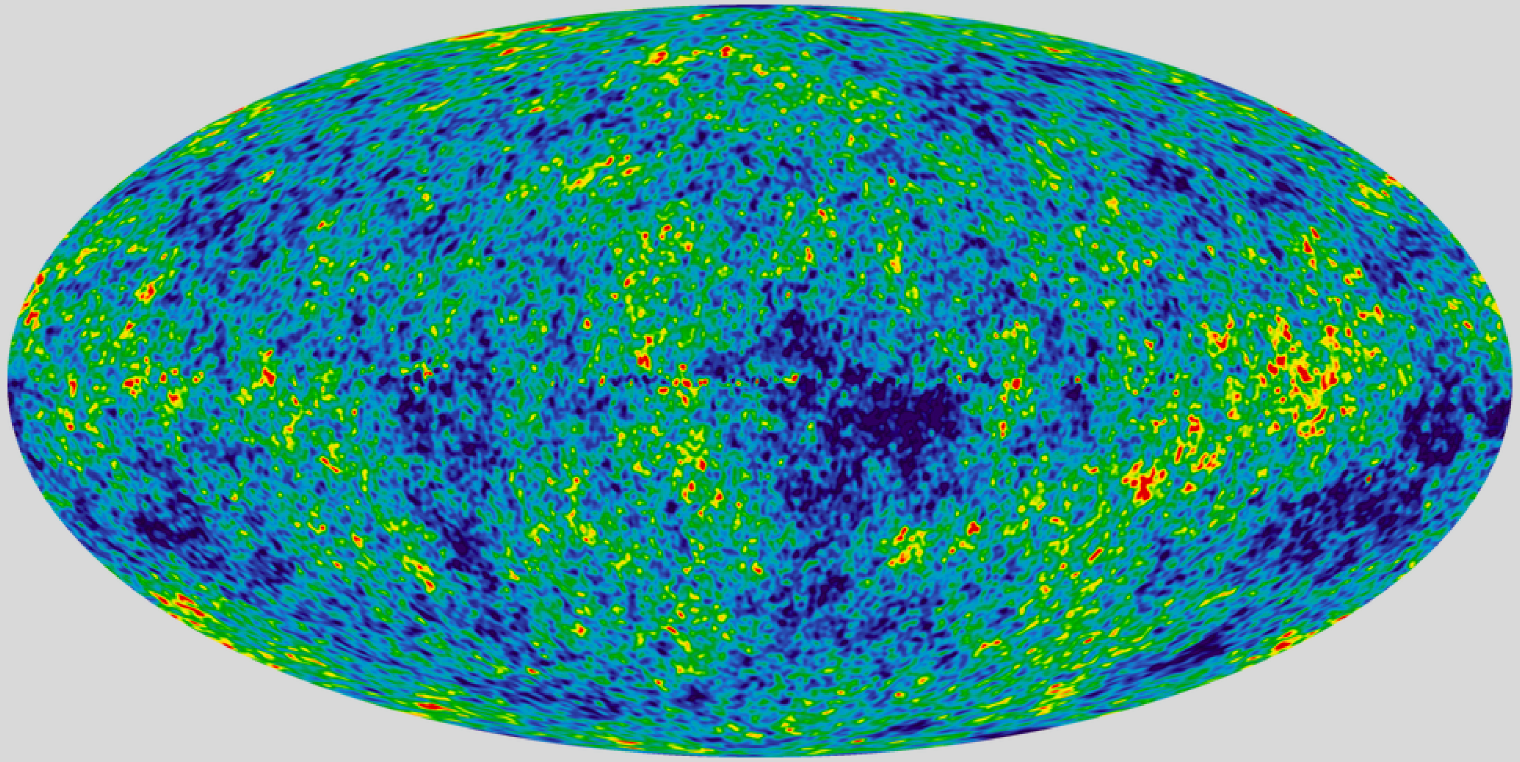
Pantheon+ and Pantheon+ SH0ES

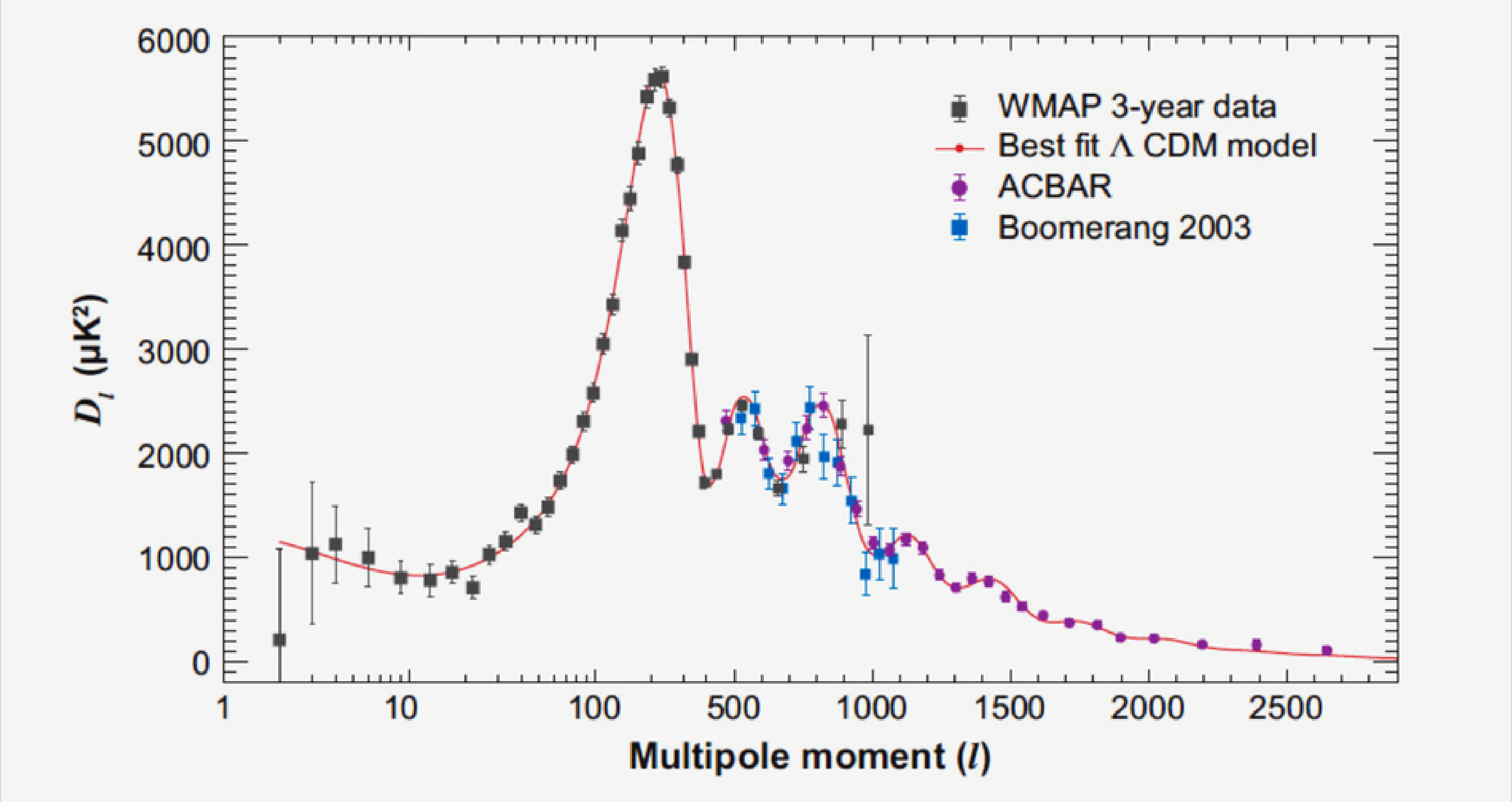
- Pantheon+ includes 2285 unique SNe Ia, with detailed metadata (coordinates, redshifts in various frames, host info) and is designed for traceability and reproducibility in cosmological analyses 7.
- Pantheon+ SH0ES is a subset used for Hubble constant (H_0) measurements, cross-calibrated with Cepheid distances.

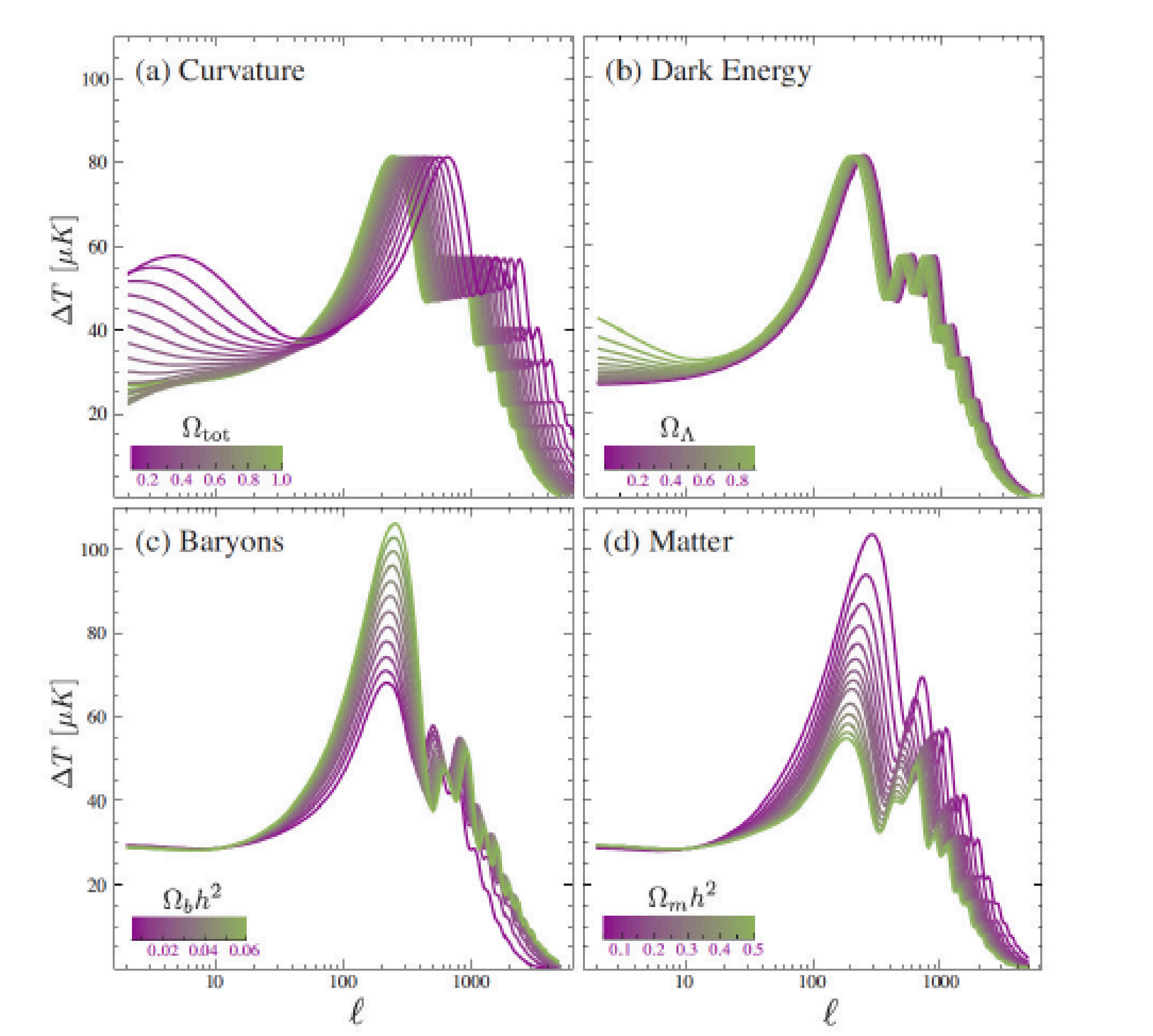
DES 5-Year (DESy5)

- The DES 5-year SN sample includes 1635 photometrically classified SNe Ia (0.10 < z < 1.13) and 194 low-z SNe Ia (0.025 < z < 0.10) for cosmology. It uses machine learning for classification and host galaxy spectroscopy for redshifts. This dataset provides the largest high-z SN Ia sample and the tightest cosmological constraints to date.

CMB and Cosmological Parameters



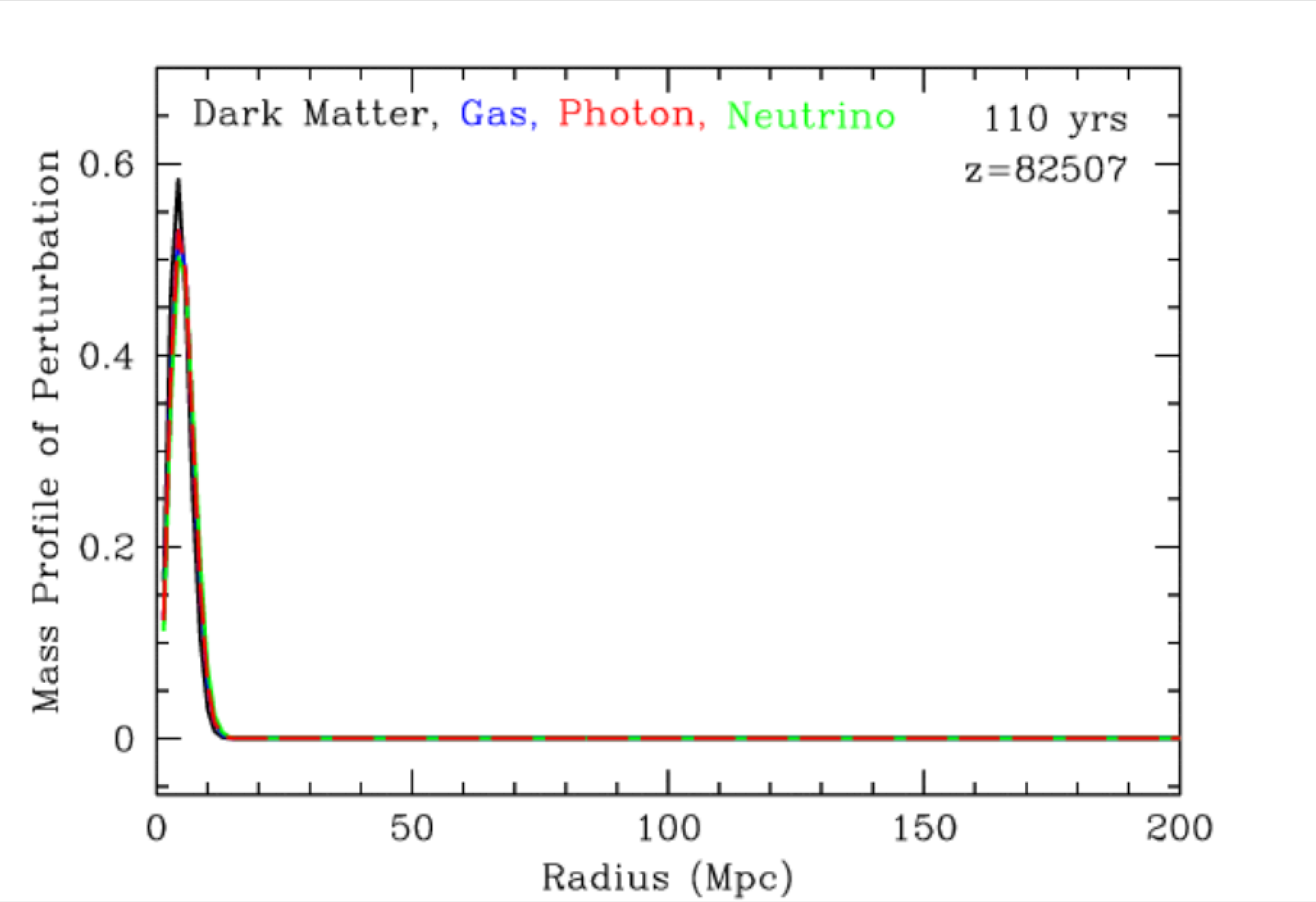


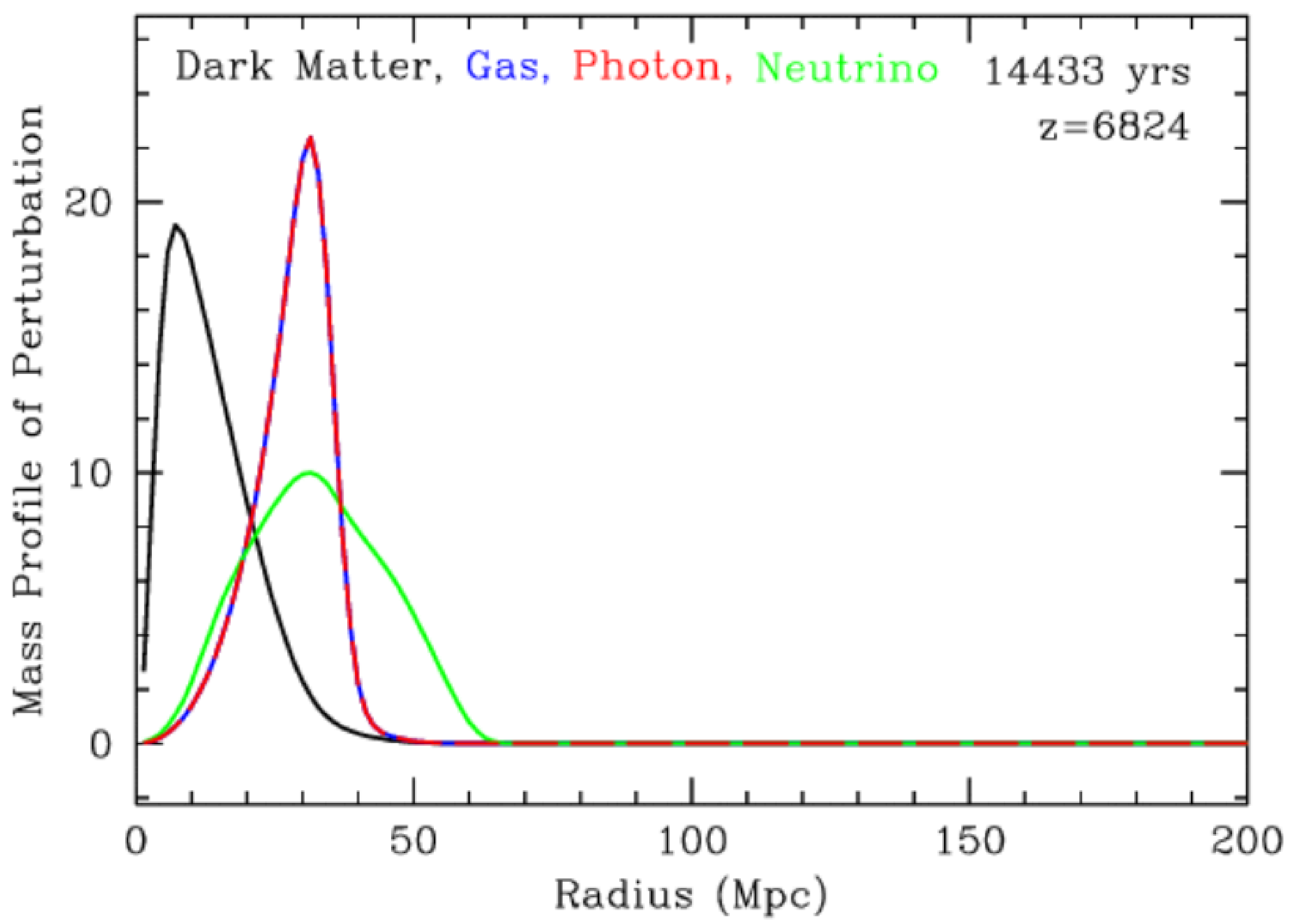


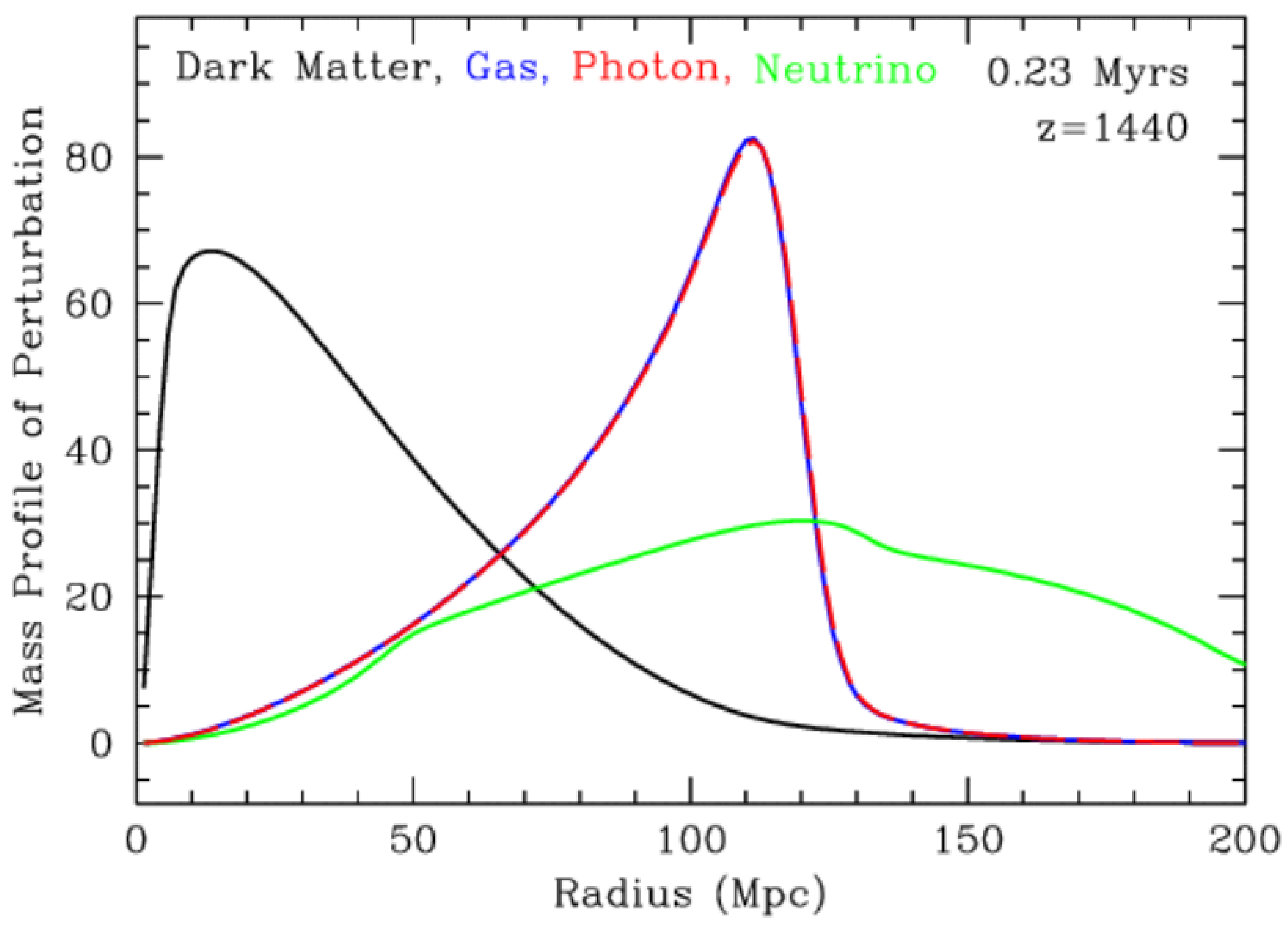
Taken from Frieman,Turner & Huterer(2008).

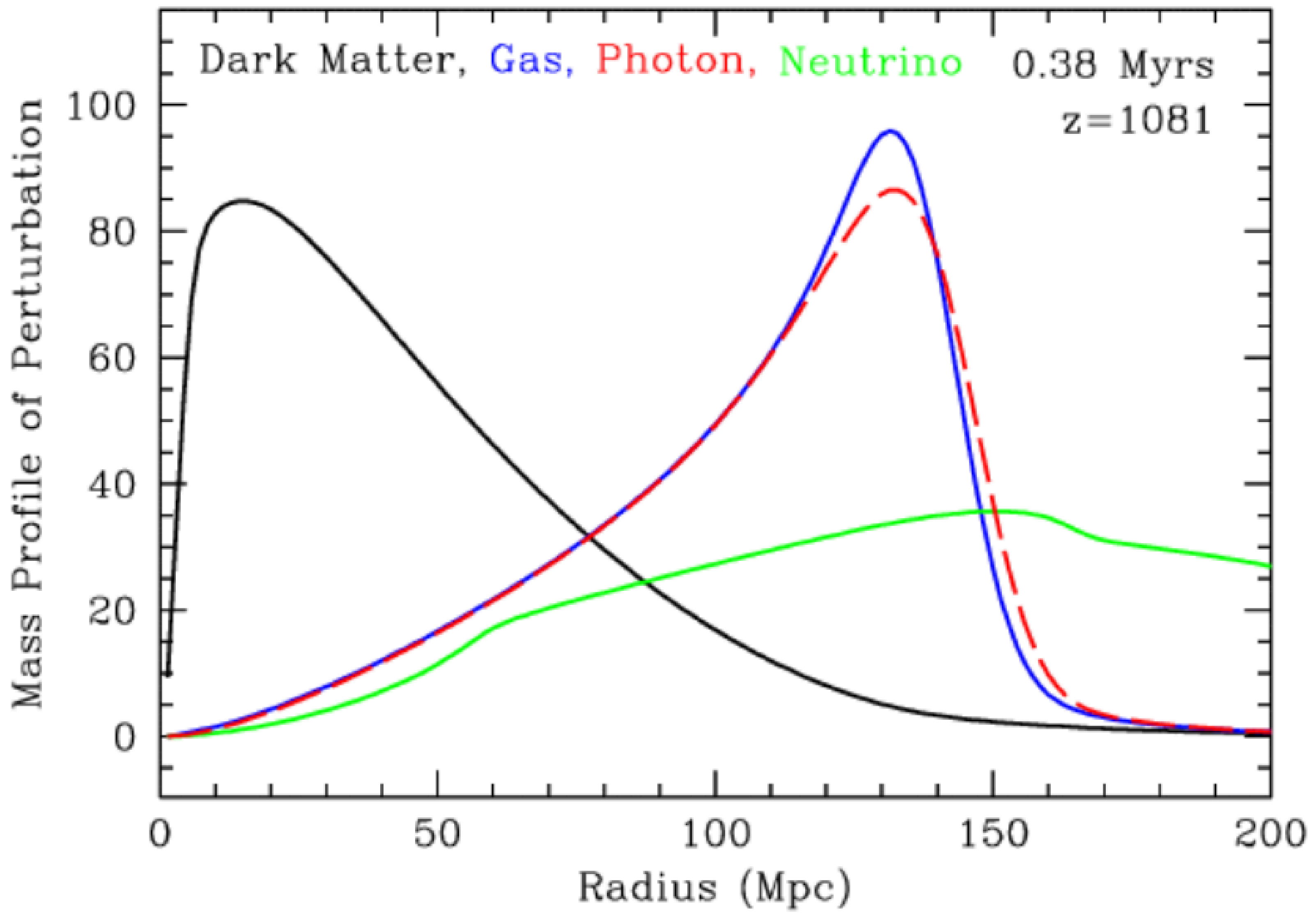
Baryon acoustic oscillations (BAO)

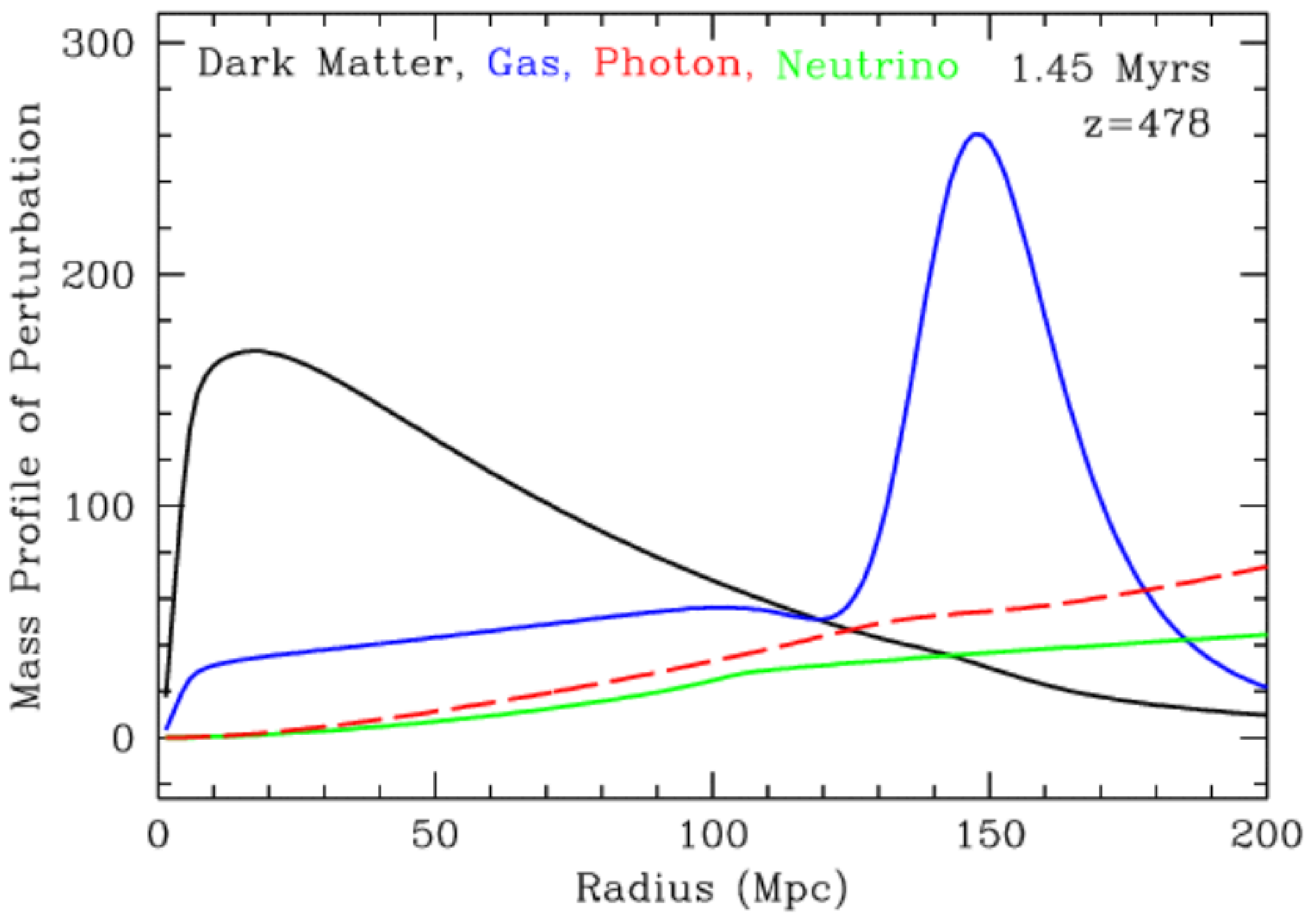
- BAO provides a standard ruler for the universe.
- These are the fluctuations in the visible matter density distribution originated due to oscillation of the acoustic wave in the primordial plasma.
- The very early universe consists of baryon, electron and photon plasma also the dark matter.
- Due to the fluctuation in the density field of the primordial plasma overdensity and under density regions form. The over density regions attracted more matter towards it.
- This process gave rise to an enormous outward pressure due to the photon and matter interaction. Gravity and this outward force started an oscillation of the plasma analogous to the sound wave.
- In the meantime the universe was expanding and the particles were losing energy. When the universe was around 379,000 years old recombination happens and the universe becomes transparent to the photon and leaving behind shells of baryonic matter.


Dark Matter, Gas, Photon, Neutrino
110 yrs
z=82507
Mass Profile of Perturbation
0
0.2
0.4
0.6
0
50
100
150
200
Radius (Mpc)

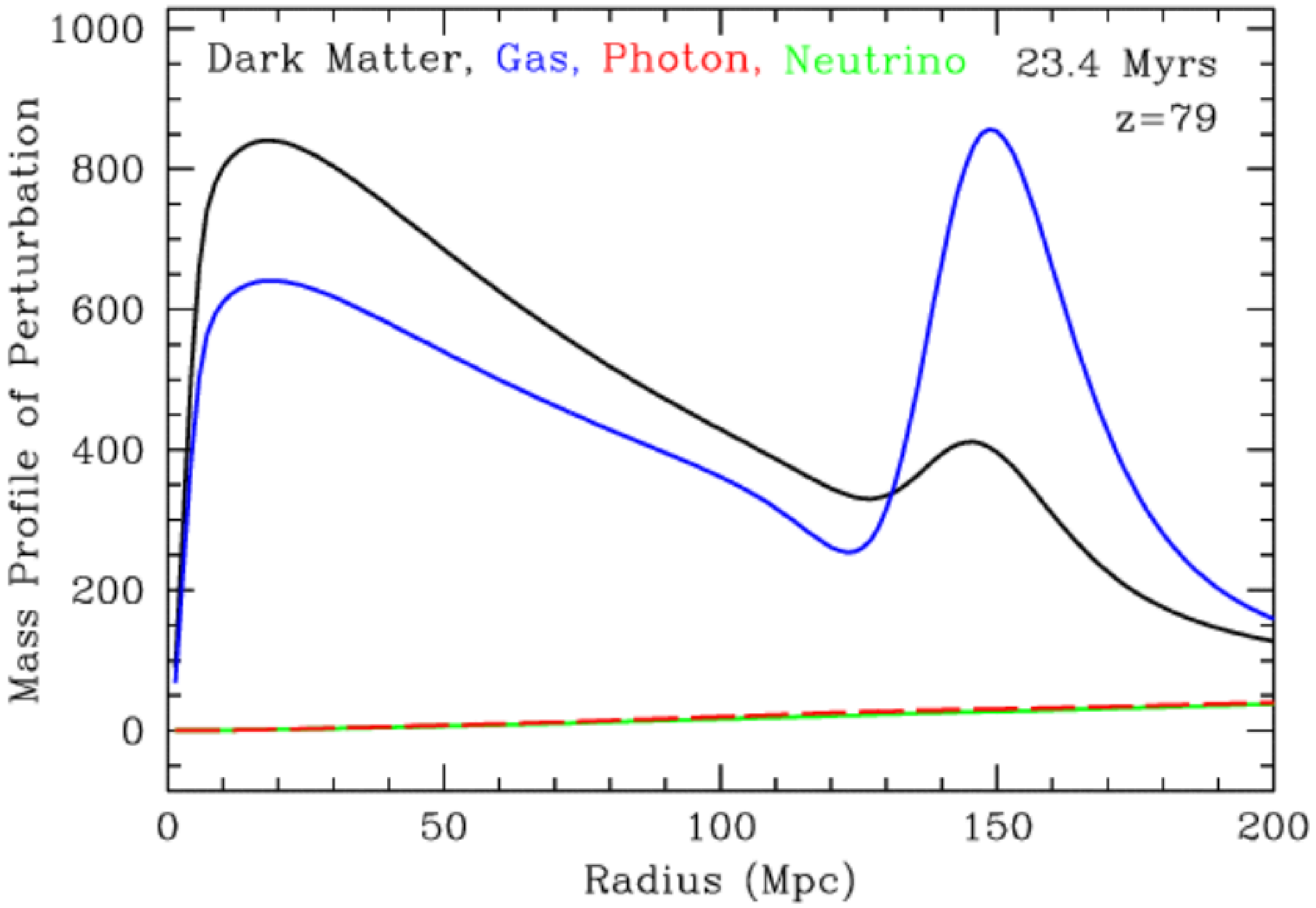

Dark Matter, Gas, Photon, Neutrino
14433 yrs
z=6824
Mass Profile of Perturbation
Radius (Mpc)
0
10
20
0
50
100
150
200

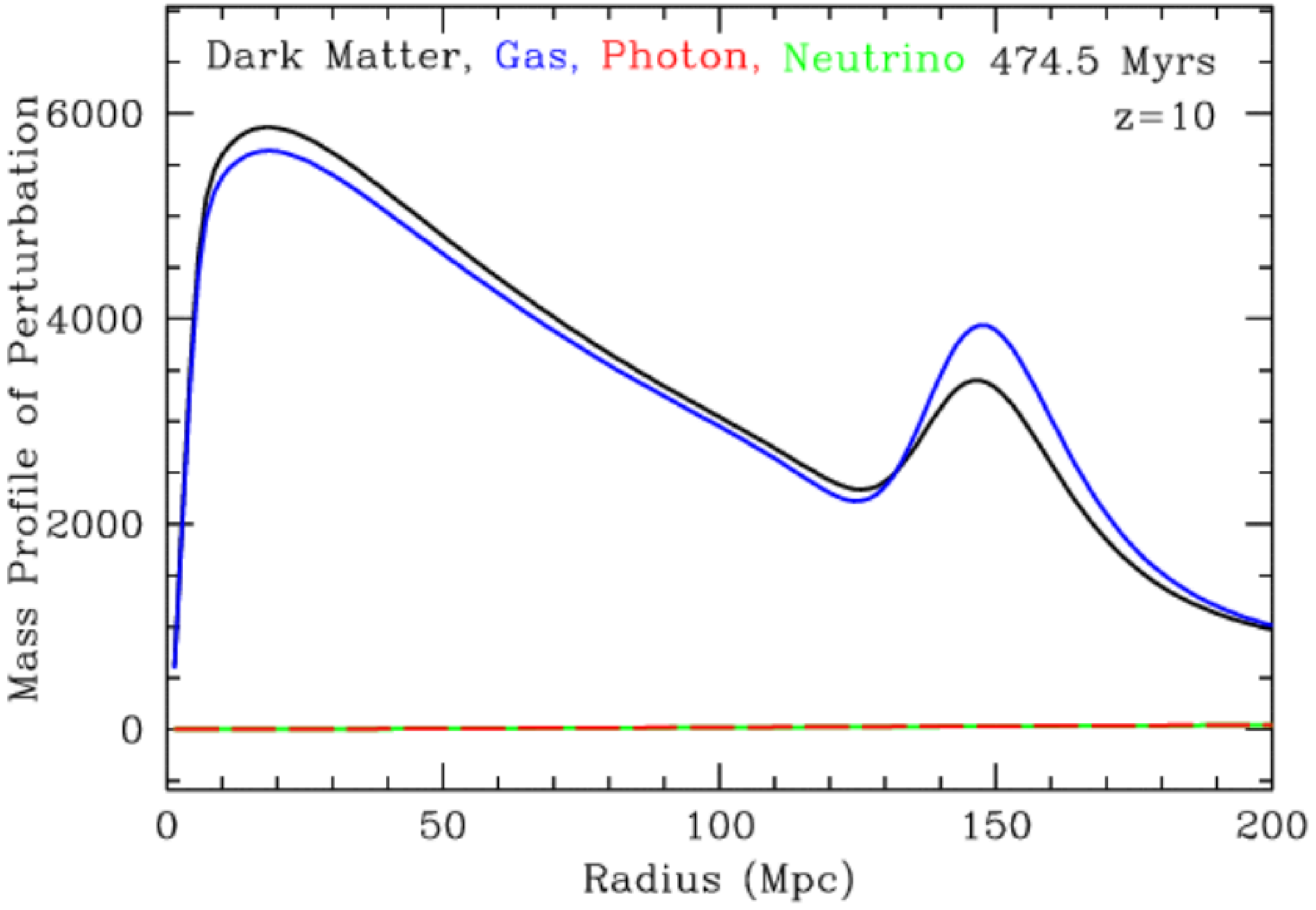

Dark Matter, Gas, Photon, Neutrino
0.23 Myrs
z=1440
Mass Profile of Perturbation
Radius (Mpc)
0
20
40
60
80
0
50
100
150
200

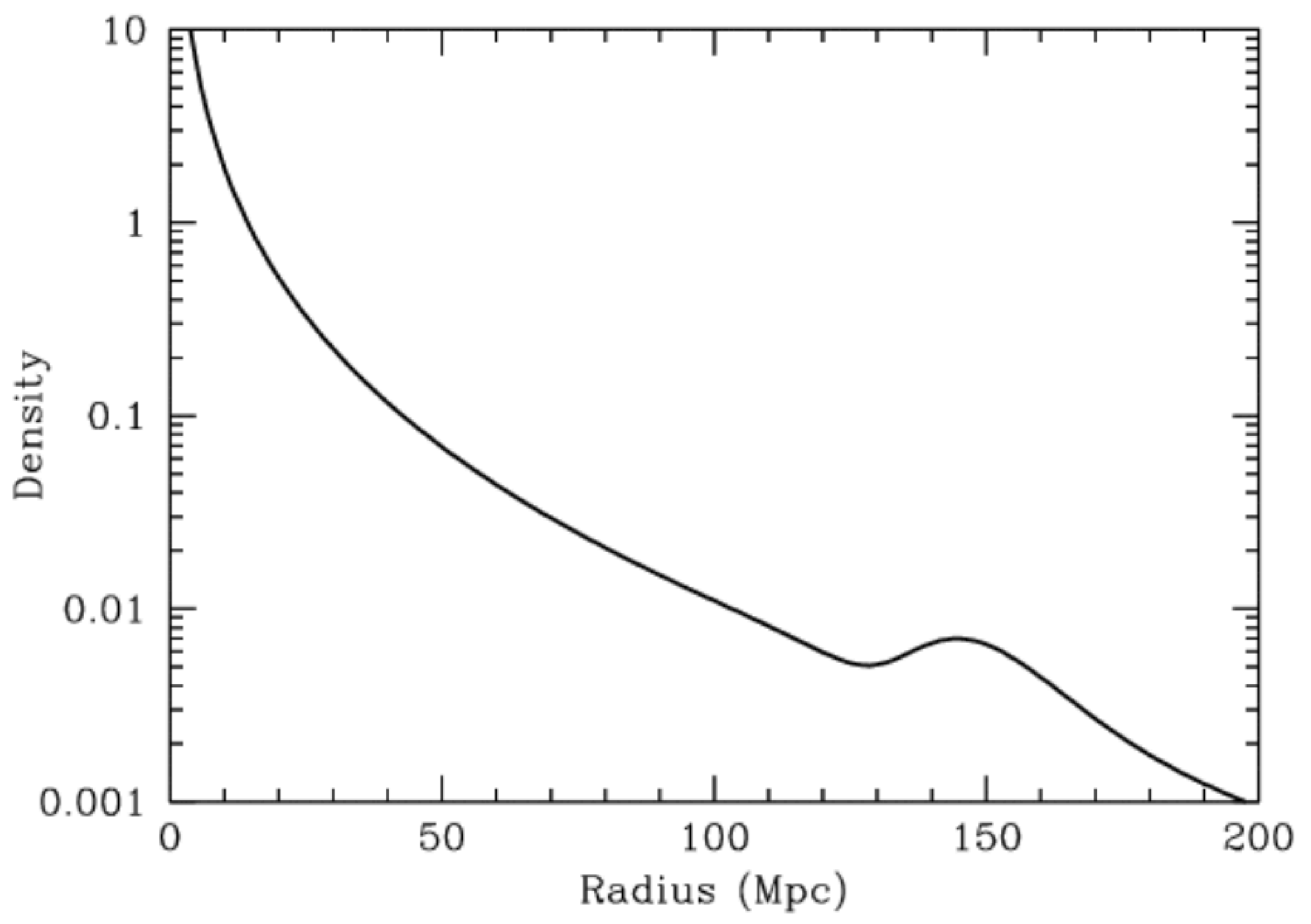

Dark Matter, Gas, Photon, Neutrino
0.38 Myrs
z=1081
Mass Profile of Perturbation
Radius (Mpc)
0
20
40
60
80
100
0
50
100
150
200

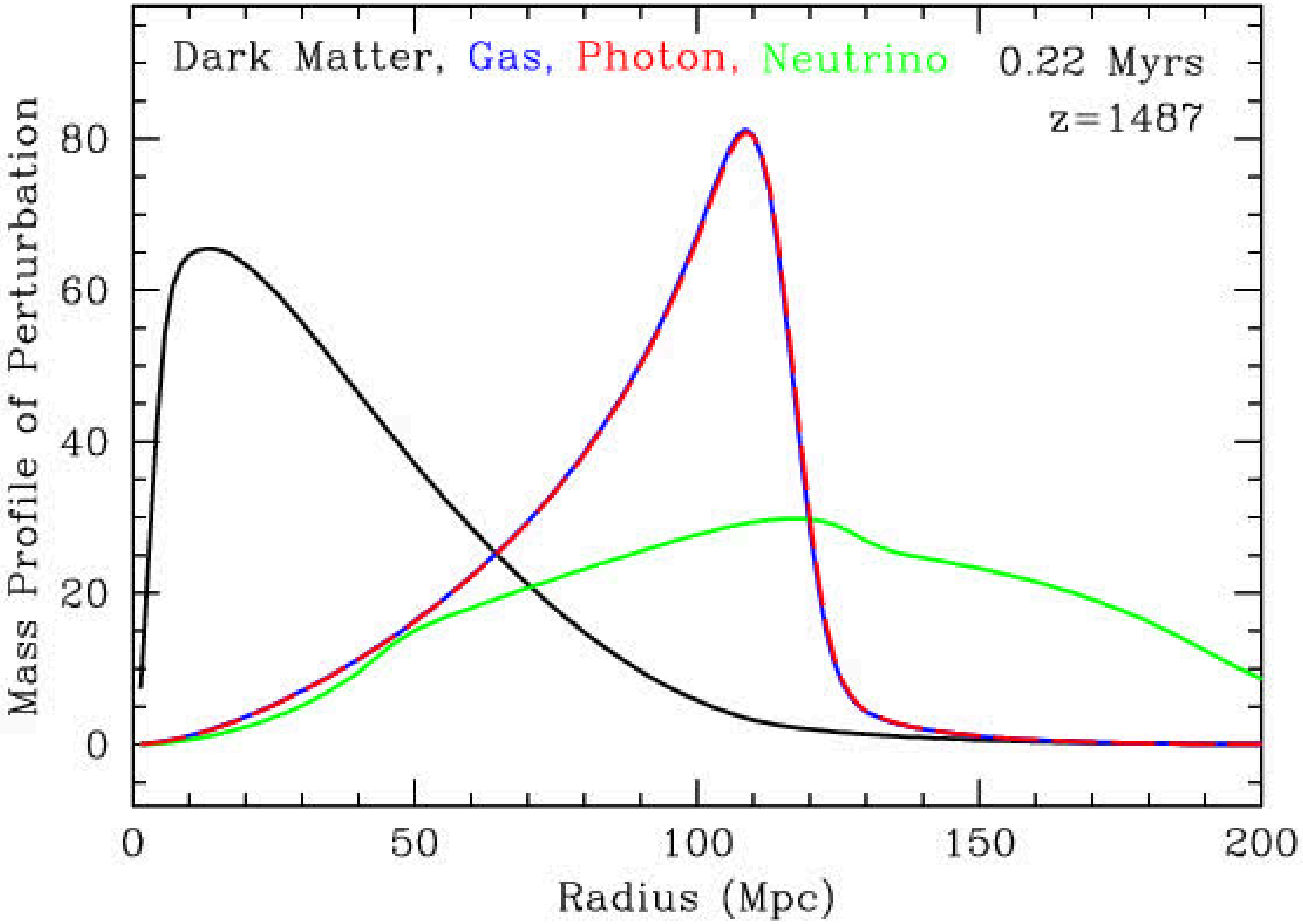


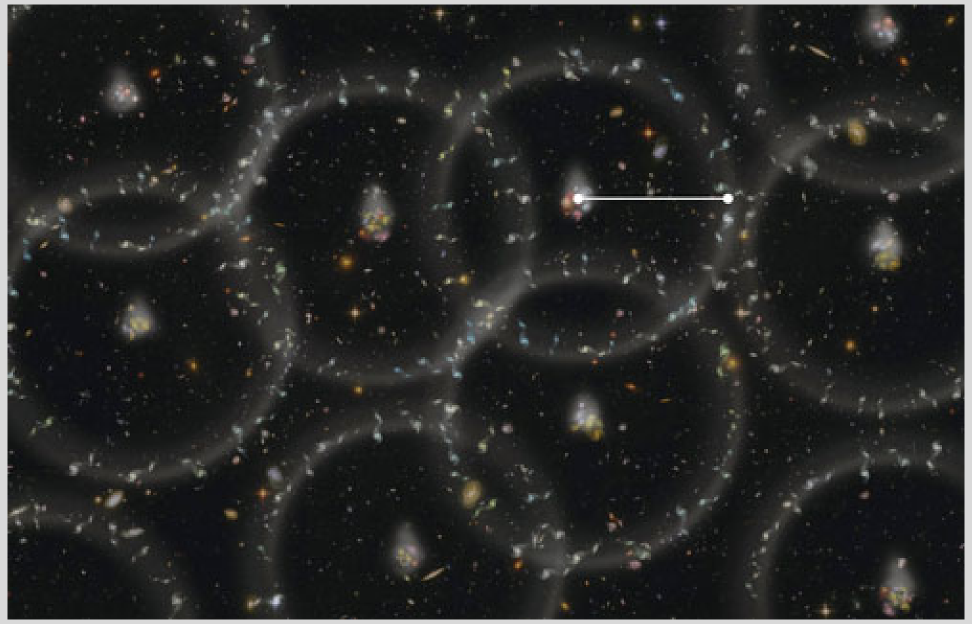
Dark Matter, Gas, Photon, Neutrino
1.45 Myrs
z=478
Mass Profile of Perturbation
Radius (Mpc)
0
100
200
300
0
50
100
150
200


Dark Matter, Gas, Photon, Neutrino
23.4 Myrs
z=79
Mass Profile of Perturbation
Radius (Mpc)
1000
800
600
400
200
0
0
50
100
150
200

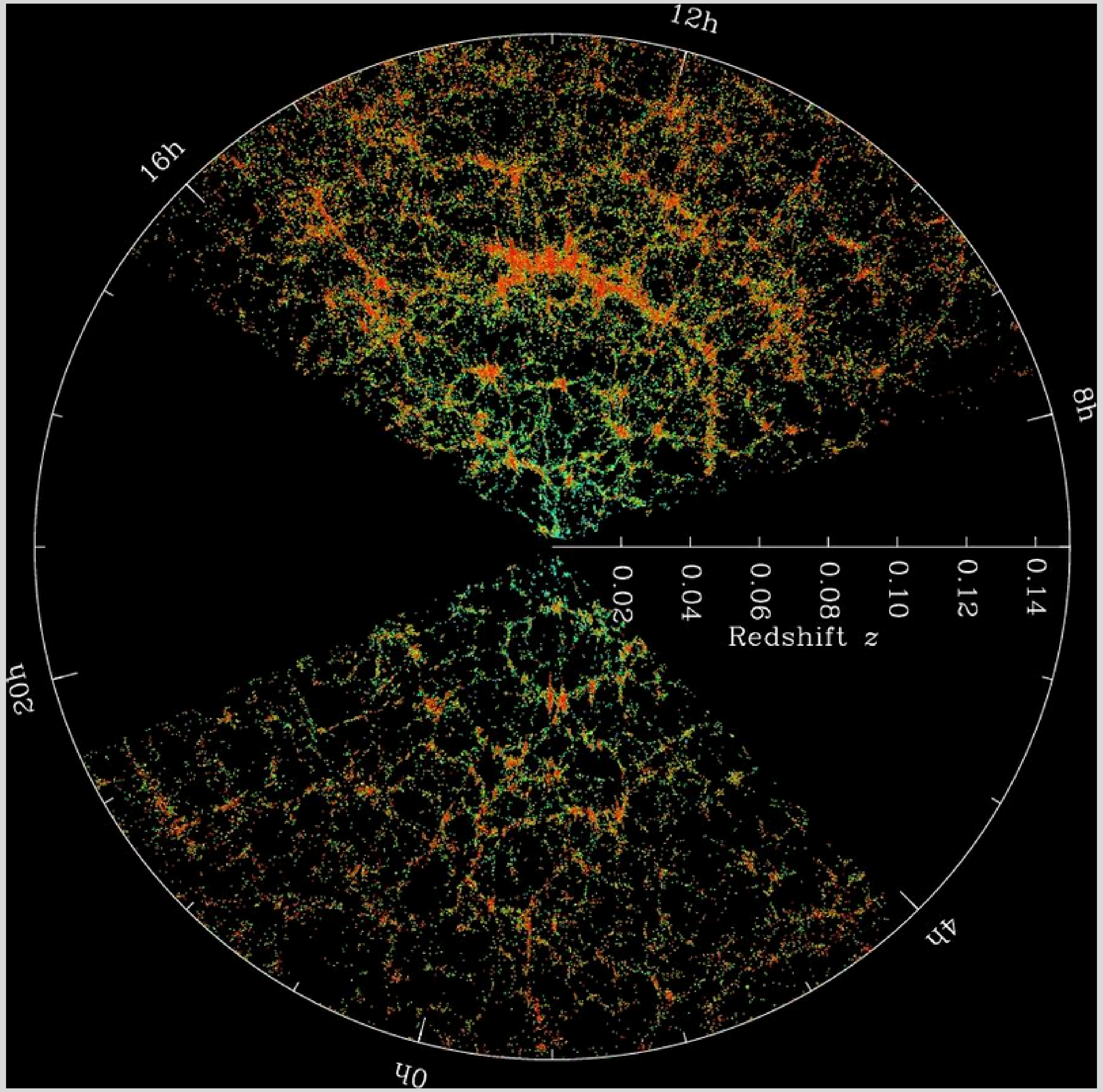

Dark Matter, Gas, Photon, Neutrino 474.5 Myrs
z=10
Mass Profile of Perturbation
6000
4000
2000
0
0
50
100
150
200
Radius (Mpc)


10
1
0.1
0.01
0.001
Density
0
50
100
150
200
Radius (Mpc)

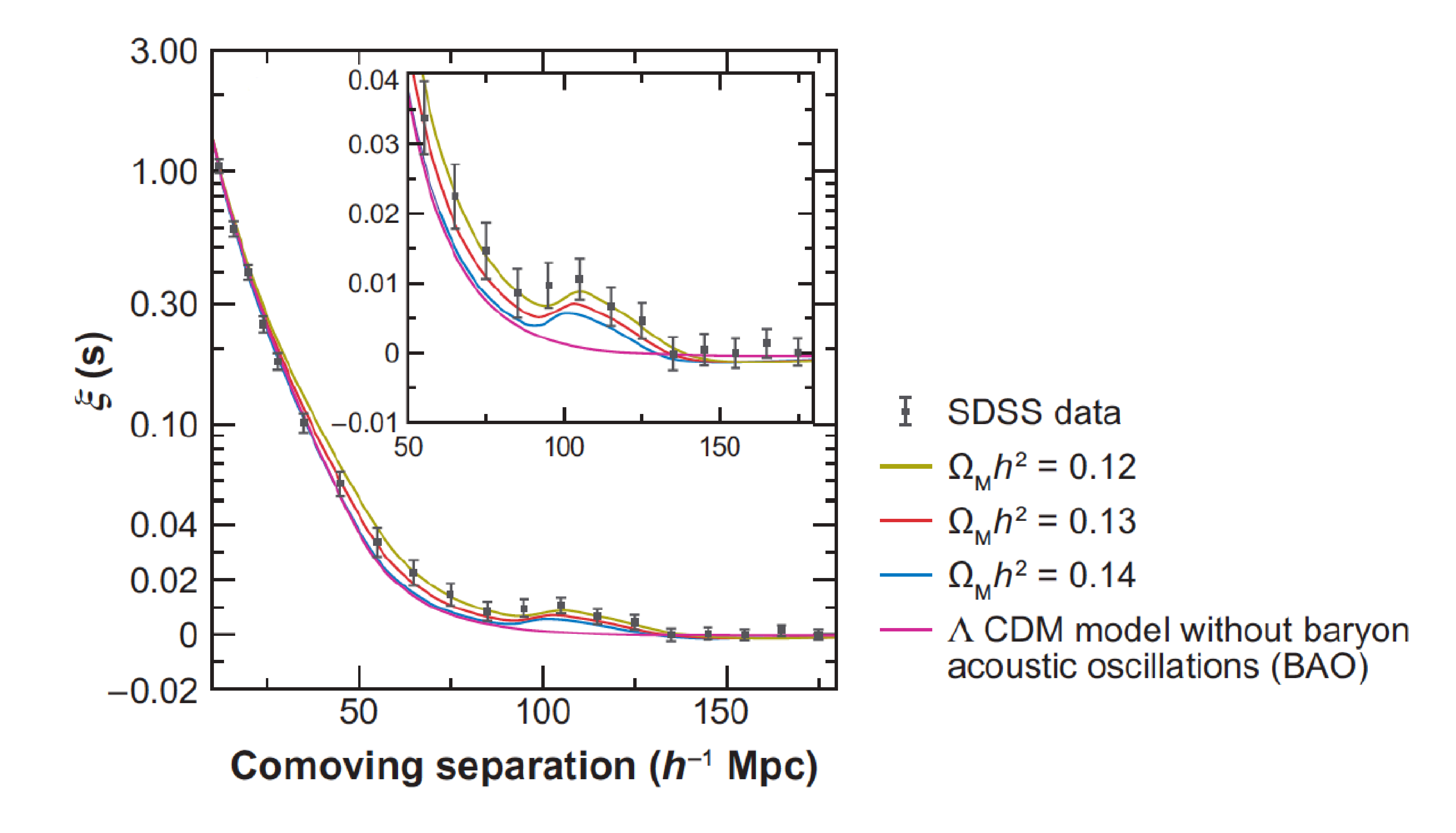

Dark Matter, Gas, Photon, Neutrino
0.22 Myrs
z=1487
Mass Profile of Perturbation
0
20
40
60
80
0
50
100
150
200
Radius (Mpc)



Measurements of the BAO signature have been carried out by Eisenstein et al. (2005) for luminous red galaxies of the Sloan Digital Sky Survey (SDSS).



The SDSS map of the Universe. Each dot is a galaxy; the color bar shows the local density.
Image Source:
https://www.darkenergysurvey.org/supporting-science/large-scale-structure/



Large Scale Structure

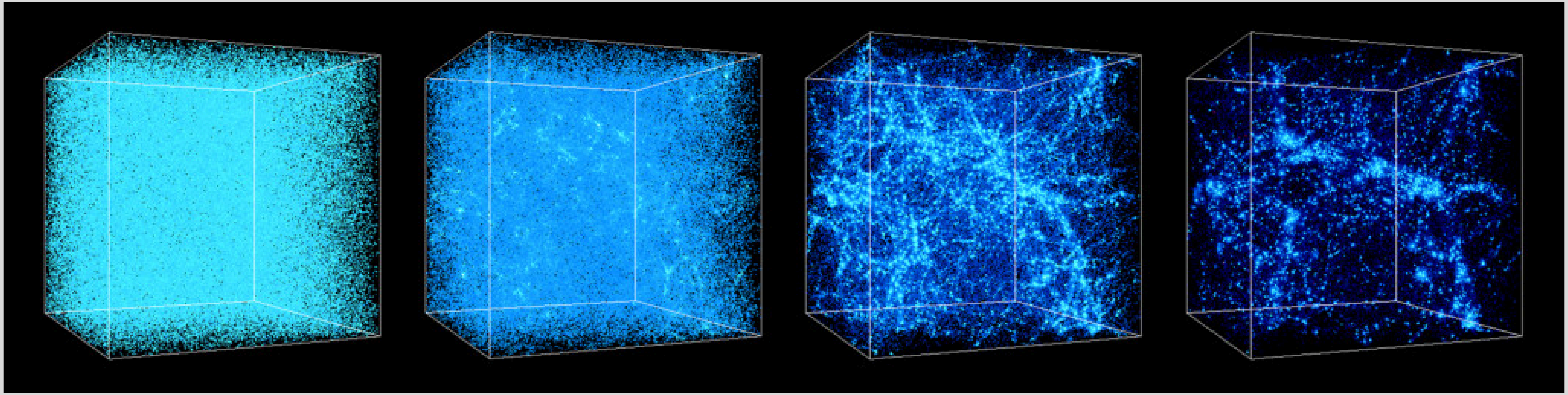
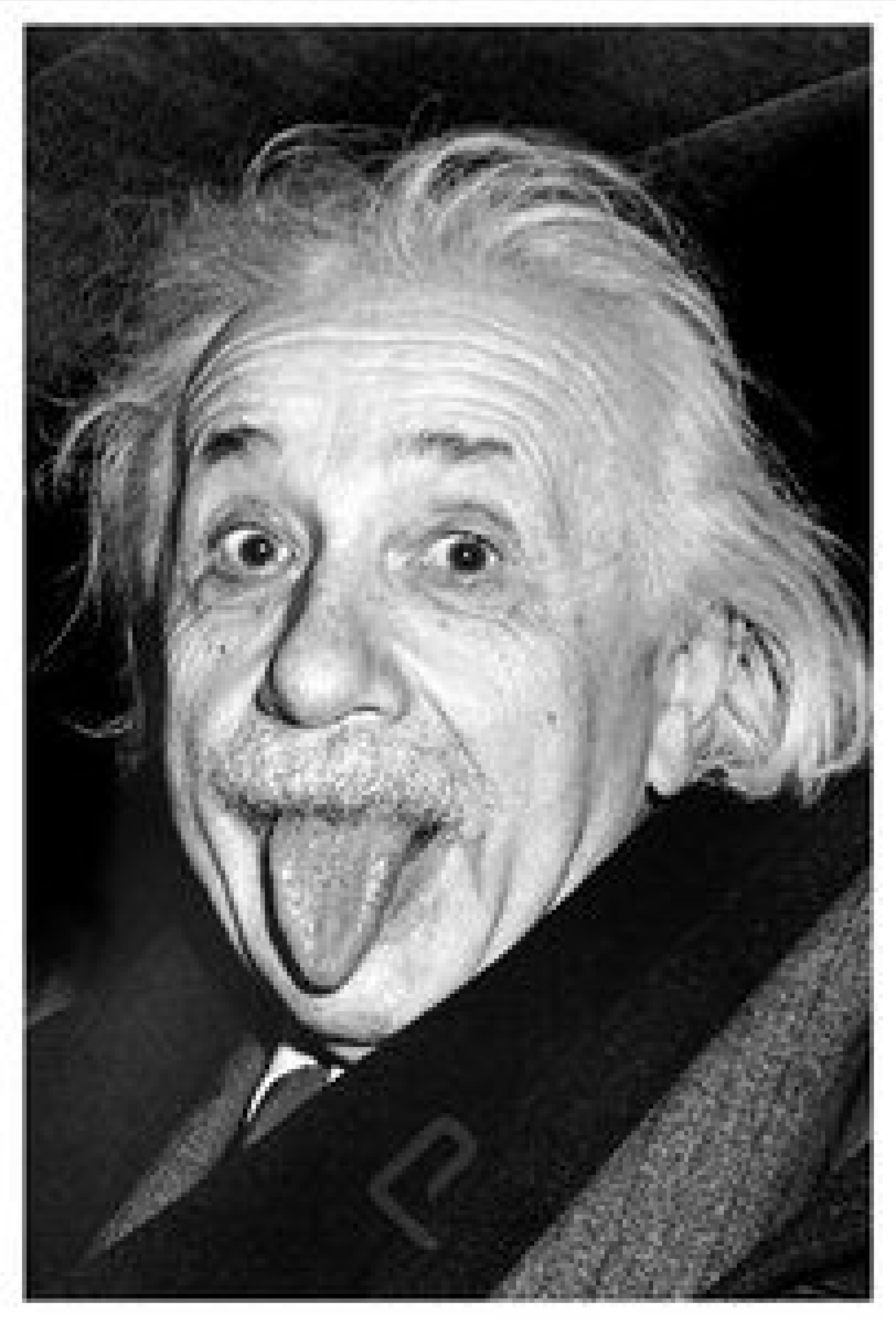


Image Source: https://www.darkenergysurvey.org/supporting-science/large-scale-structure/

- Studying LSS tells us the strength of gravity in the universe as we can measure galaxies at different distances correspond to different times in the universe's history.
- Over time, gravity is attracting more and more matter together, clustering the universe further and further.
- Most theoretical models of dark energy predict the slow down this process of gravity creating large structures.
- Studying the growth of large scale structure across time gives us information about gravity, dark energy, and how each may be changing as the Universe evolves with time.

Cosmological Constant



First introduced by Einstein in 1917 to achieve a static universe by counterbalancing the attraction of the gravity. He abandoned the idea in 1931 after Hubble's discovery of the accelerated expansion of the universe.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

The accelerated expansion was discovered during 1998, by two independent projects, the Supernova Cosmology Project and the High-Z Supernova Search Team, which both used distant type Ia supernovae to measure the acceleration. After this discovery cosmological constant becomes an active research topic.

Cosmological Constant

Before the discovery of the expanding universe by Edwin Hubble in 1929, it was widely believed that the universe was static and eternal

To counter this and preserve a static solution, Einstein introduced an additional term, $\Lambda g_{\mu\nu}$, into his field equations.

The modified Einstein field equation is then written as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Here, Λ is known as the cosmological constant.

This equation can be derived from the Einstein–Hilbert action with an additional Λ term:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(R - 2\Lambda) + S_m$$

Varying this action with respect to the metric tensor $g^{\mu\nu}$ gives:

$$\delta S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} - 8\pi G T_{\mu\nu}\right) \delta g^{\mu\nu}$$

Setting $\delta S = 0$ for all variations $\delta g^{\mu\nu}$ yields the Einstein field equations with the cosmological constant.

After Hubble's discovery that the universe is expanding, Einstein reportedly called the introduction of Λ his “greatest blunder”, though the cosmological constant later regained importance in explaining dark energy and the accelerated expansion of the universe.

Cosmological Constant

Friedmann equations with the cosmological constant:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

For a pressureless ($p = 0$) and static universe:

$$\rho = \frac{\Lambda}{4\pi G}, \qquad \frac{K}{a^2} = \Lambda$$

In this case, the expansion rate $H = 0$, meaning the universe is static. However, this static solution is unstable:

$$\frac{\Lambda}{3} > \frac{4\pi G\rho}{3} \quad \Rightarrow \quad \text{the universe will expand,}$$

$$\frac{\Lambda}{3} < \frac{4\pi G\rho}{3} \quad \Rightarrow \quad \text{the universe will collapse.}$$

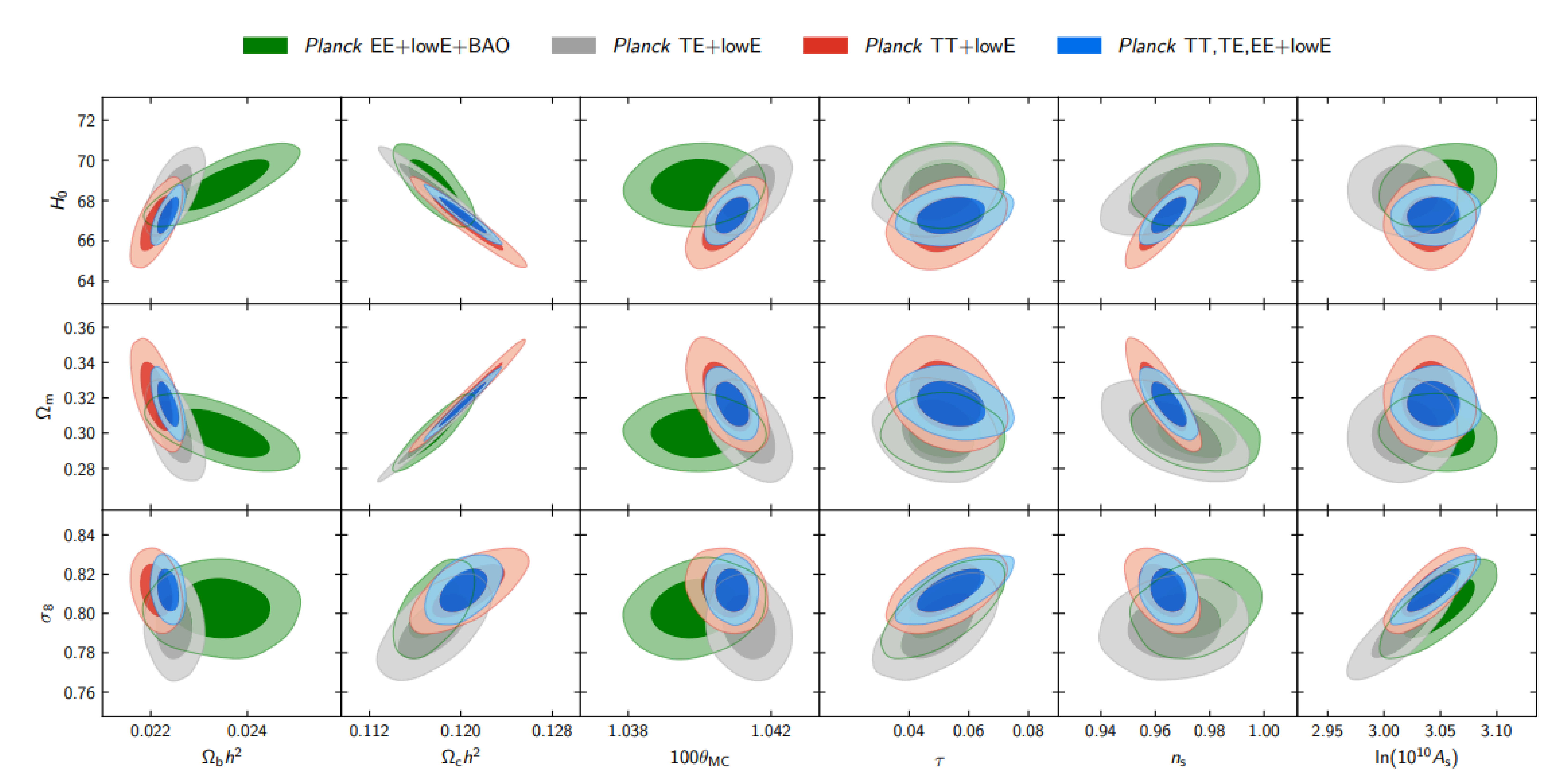
ΛCDM as the concordance model

- The name "concordance" reflects the remarkable agreement between the model and multiple independent lines of observational evidence.
- Different astrophysical measurements, such as those from the CMB, large-scale structure, and Type Ia supernovae, all yield consistent results when interpreted through the framework of the ΛCDM model.
- This consistency makes the model the standard for cosmological research and for comparing results from different studies.

CMB data Planck 2028

Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	−0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	−0.3	0.1198 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	−0.2	1.04089 ± 0.00031
τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	−0.1	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	−0.3	3.043 ± 0.014
n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	+0.2	0.9652 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	−0.3	0.1428 ± 0.0011
H_0 [km s^{-1} Mpc^{-1}]	67.32	67.36 ± 0.54	67.39 ± 0.54	+0.1	67.37 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	−0.2	0.3147 ± 0.0074
Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	+0.4	13.801 ± 0.024
σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	−0.3	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ . . .	0.8331	0.832 ± 0.013	0.828 ± 0.013	−0.3	0.830 ± 0.013
z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	−0.1	7.64 ± 0.74
$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	−0.1	1.04108 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	+0.6	147.18 ± 0.29

Planck Collaboration: Planck 2018 results. VI.



Planck Collaboration: Planck 2018 results. VI.

Though ΛCDM model is the best model of the universe we till now have, it suffers from challenges coming from both theory and observations.

Cosmological Constant Problem

$$\Lambda \approx H_0^2 = (2.1332h \times 10^{-42}\,\mathrm{GeV})^2$$

$$\rho_\Lambda \approx \frac{m_{\mathrm{pl}}^2 \Lambda}{8\pi} \approx 10^{-47}\,\mathrm{GeV}^4 \approx 10^{-123} m_{\mathrm{pl}}^4$$

However, theoretical estimates based on quantum field theory predict a vacuum energy density of

$$\rho_\Lambda^{(\mathrm{theory})} \approx 10^{74}\,\mathrm{GeV}^4,$$

which is roughly 10^{121} times larger than the observed value.

This enormous discrepancy between theory and observation is known as the

"Cosmological Constant Problem,"

and it remains one of the biggest unsolved puzzles in modern physics.

Cosmic Coincidence Problem

In the standard cosmological model, the energy density of matter (ρ_m) decreases as the universe expands:

$$\rho_m \propto a^{-3}.$$

The energy density associated with the cosmological constant (dark energy) remains constant:

$$\rho_\Lambda = \text{constant}.$$

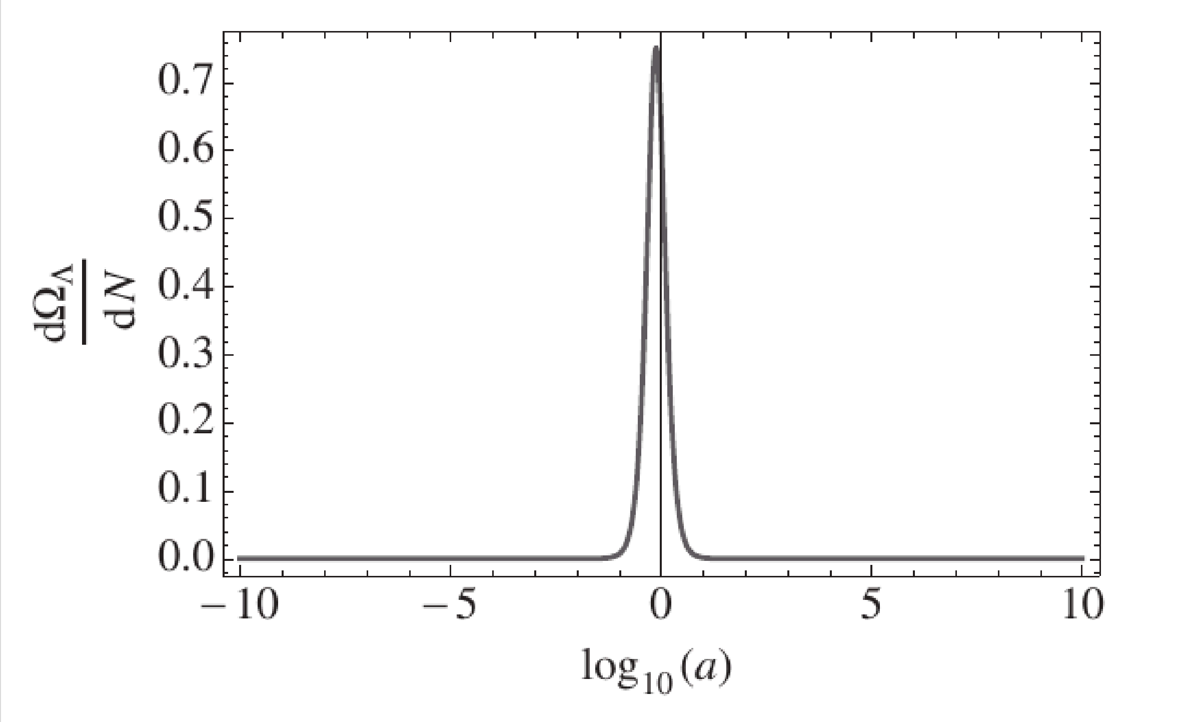
Over cosmic time, ρ_m decreases rapidly while ρ_Λ stays fixed.

This means that their ratio $\frac{\rho_m}{\rho_\Lambda}$ changes by many orders of magnitude throughout cosmic history.

Yet, observations show that today ρ_m and ρ_Λ are of the same order of magnitude today:

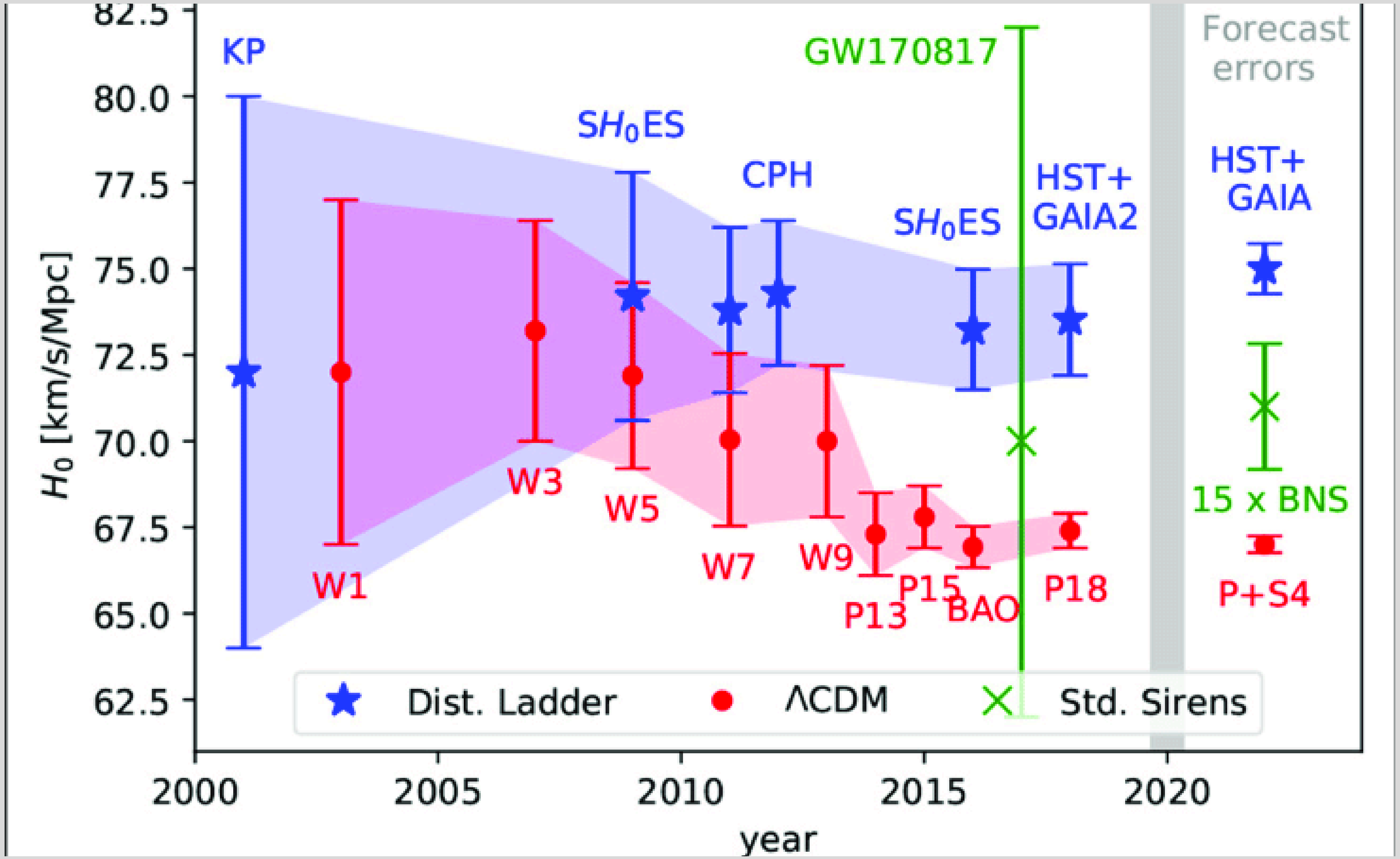
$$\rho_m \sim \rho_\Lambda.$$

This remarkable coincidence raises the question:



Hubble Tension

arXiv: 2008.11284



CMB Planck data together with BAO, BBN, and DES have constraint the Hubble parameter to be H0 (67.0 – 68.5)km/s/Mpc. On the other hand, cosmic distance ladder and time delay measurement like those reported by SHOES and HOLiCOW collaborations have reported H0 = (74.03 ± 1.42)km/s/Mpc and H0 = (73.3 +1.7 −1.8)km/s/Mpc respectively by observing the local Universe.

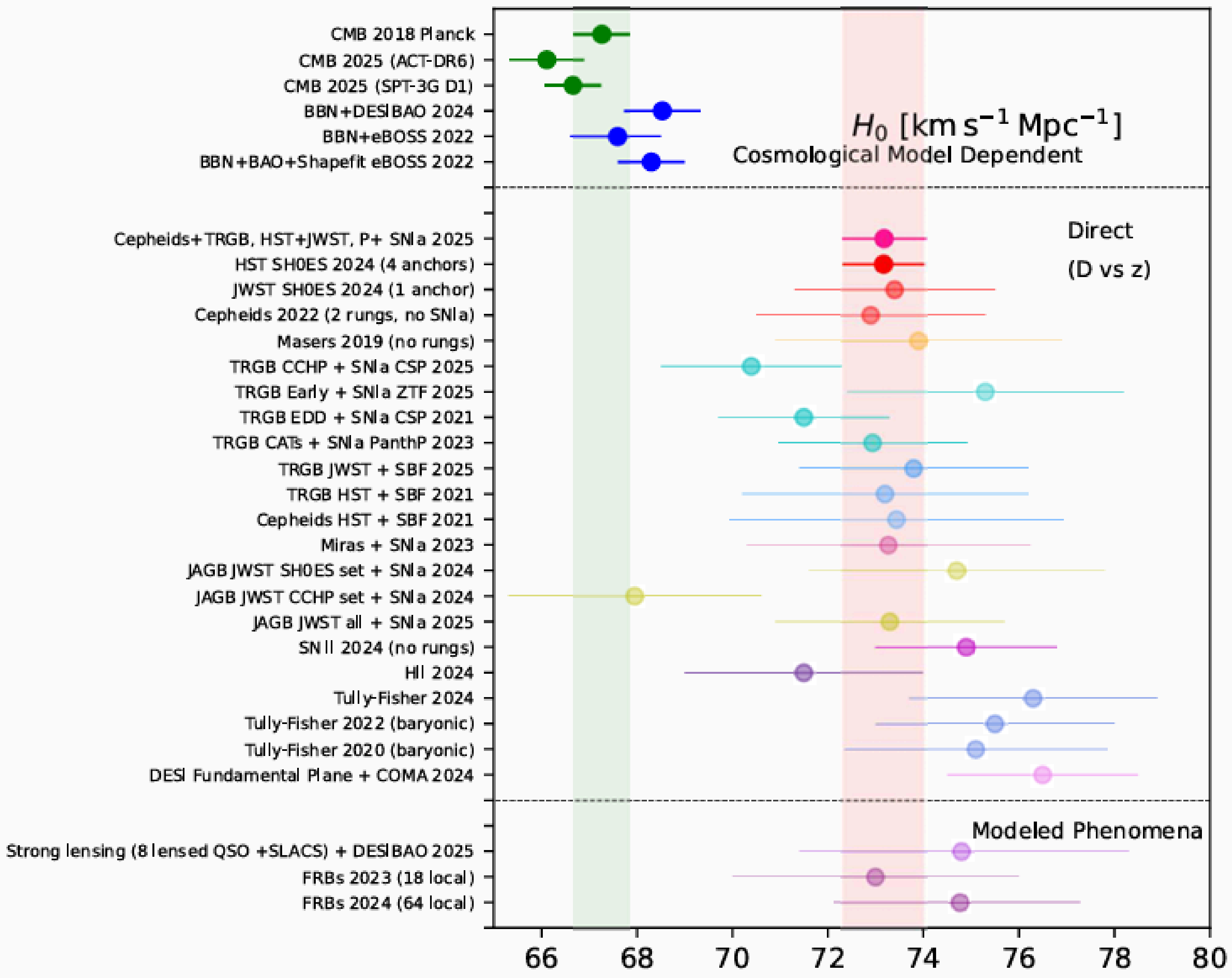


FIG. 1: *Recent determinations of the Hubble constant H_0 from a variety of methods. Local distance-ladder approaches, including Cepheid- and TRGB-calibrated Type Ia supernovae, surface-brightness fluctuations, Type II supernovae, the Tully–Fisher relation, Mira variables, carbon stars, strong-lensing time-delay cosmography, fast radio bursts, the DESI fundamental plane with the Coma cluster, and maser distances, consistently favor $H_0 \simeq 71$–77 km s^{-1} Mpc^{-1}. In contrast, early-Universe inferences from the CMB and BAO within ΛCDM yield lower values, around $H_0 \simeq 66$–68 km s^{-1} Mpc^{-1}. Updated from [1].*

2509.25288

The Perfect Host: *JWST* Cepheid Observations in a Background-Free SN Ia Host Confirm No Bias in Hubble-Constant Measurements

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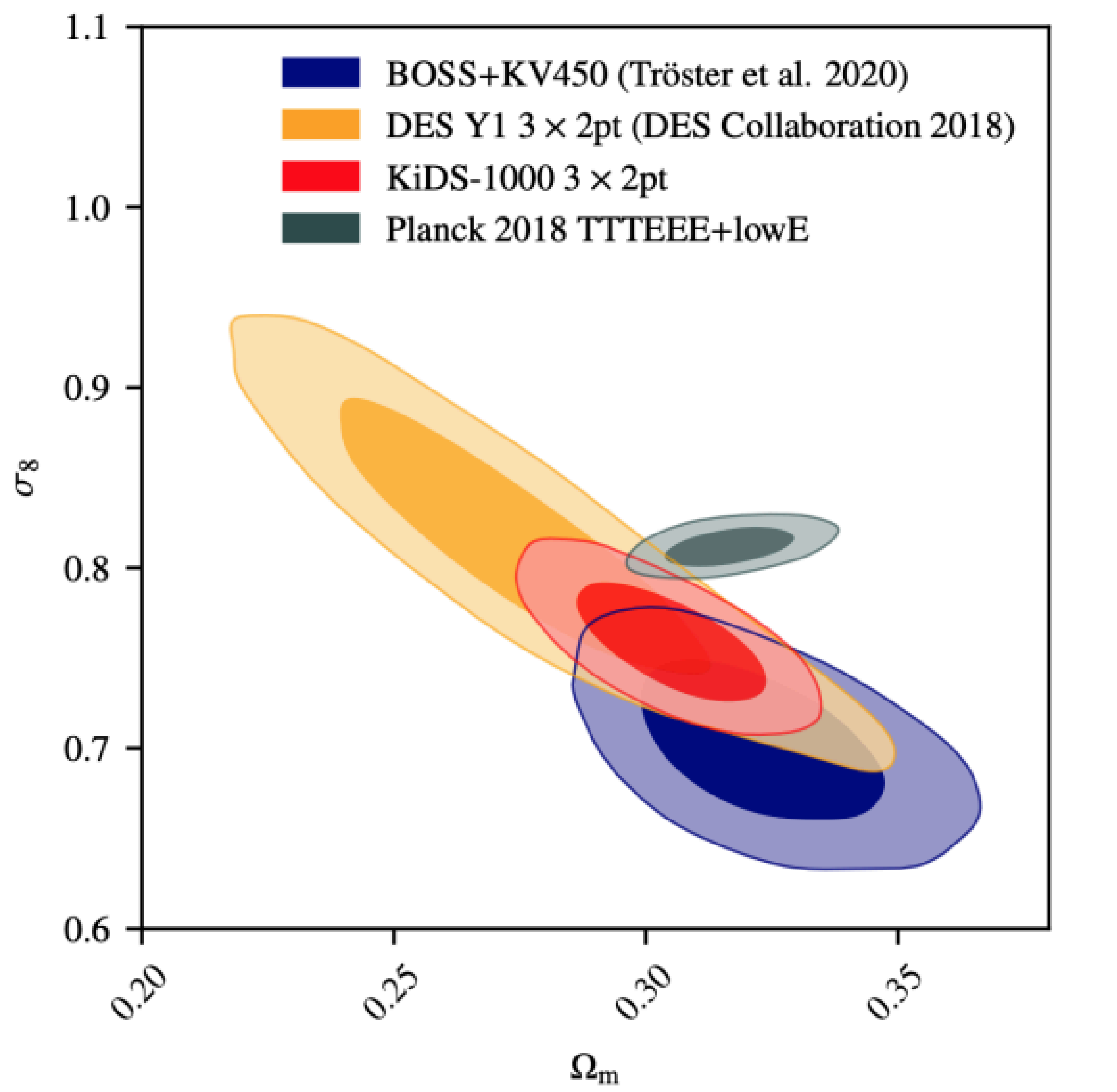
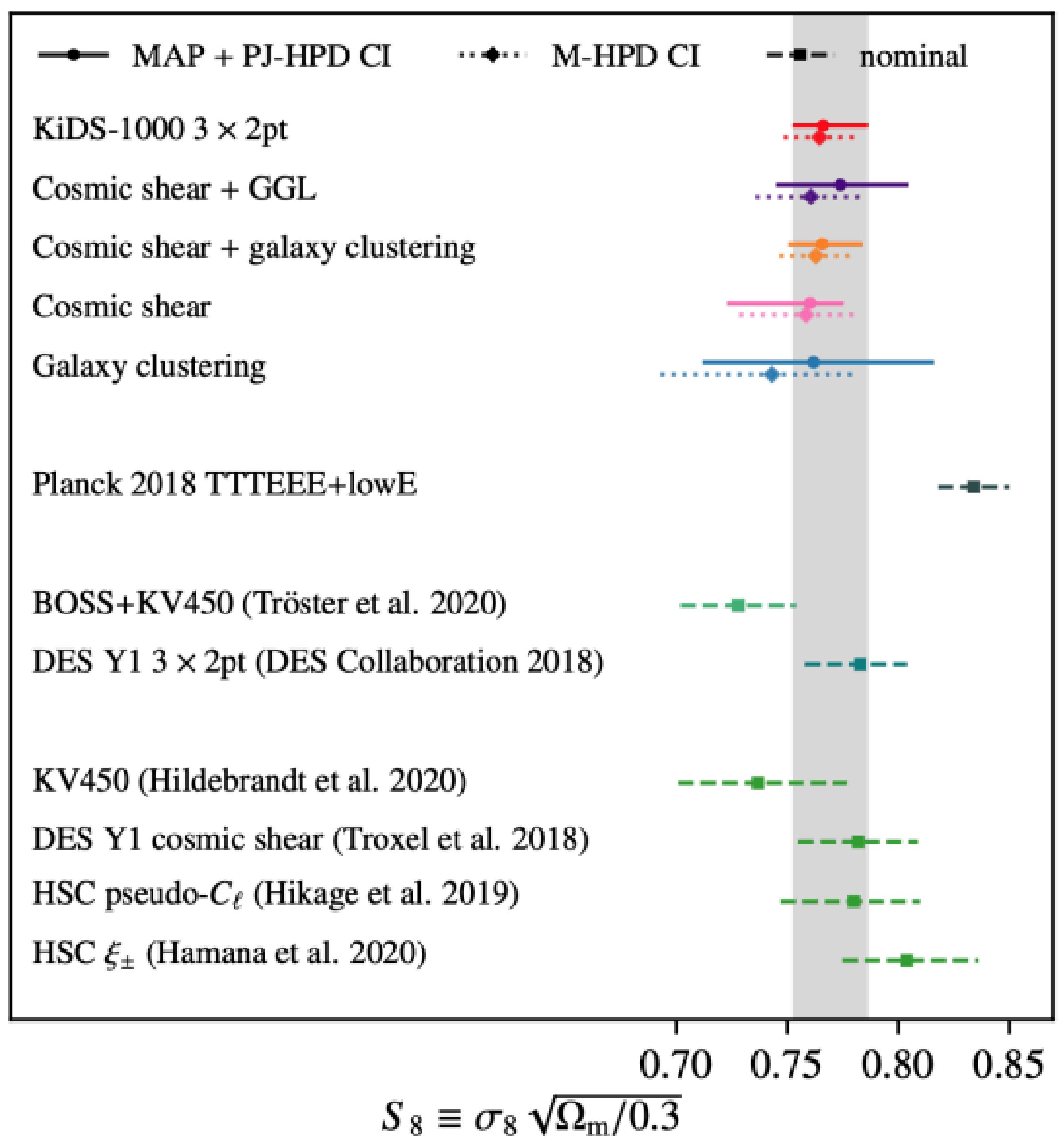
ABSTRACT

Cycle 1 *JWST* observations of Cepheids in SN Ia hosts resolved their red-giant-dominated NIR backgrounds, sharply reducing crowding and showing that photometric bias in lower-resolution *HST* data does not account for the Hubble tension. We present Cycle 2 *JWST* observations of > 100 Cepheids in NGC 3447, a unique system that pushes this test to the limit by transitioning from low to no background contamination. NGC 3447, an SN Ia host at $D \sim 25$ Mpc, is an interacting pair comprising (i) a spiral with mixed stellar populations, typical of H_0 calibrators, and (ii) a young, star-forming companion (NGC 3447A) devoid of old stars and hence stellar crowding—a rare "perfect host" for testing photometric bias. We detect ~ 60 long-period Cepheids in each, enabling a "three-way comparison" across *HST*, *JWST*, and background-free conditions. We find no component-to-component offset ($\sigma < 0.03$ mag; a calibration independent test), and a 50% reduction in scatter to ~ 0.12 mag in the background-free case, the tightest seen for any SN Ia host. Across Cycles 1-2 we also measure Cepheids in all SH0ES hosts observed by *JWST* (19 hosts of 24 SN Ia; $> 50\%$ of the sample) and find no evidence of bias relative to *HST* photometry, including for the most crowded, distant hosts. These observations constitute the most rigorous test yet of Cepheid distances and provide strong evidence for their reliability. Combining *JWST* Cepheid measurements in 19 hosts (24 SNe Ia) with *HST* data (37 hosts, 42 SNe Ia) yields $H_0 = 73.49 \pm 0.93$ km s^{-1} Mpc^{-1}. Including 35 TRGB-based calibrations (from *HST* and *JWST*) totals 55 SNe Ia and gives $H_0 = 73.18 \pm 0.88$ km s^{-1} Mpc^{-1} – $\sim 6\sigma$ above the ΛCDM+CMB expectation.

σ_8 Tension

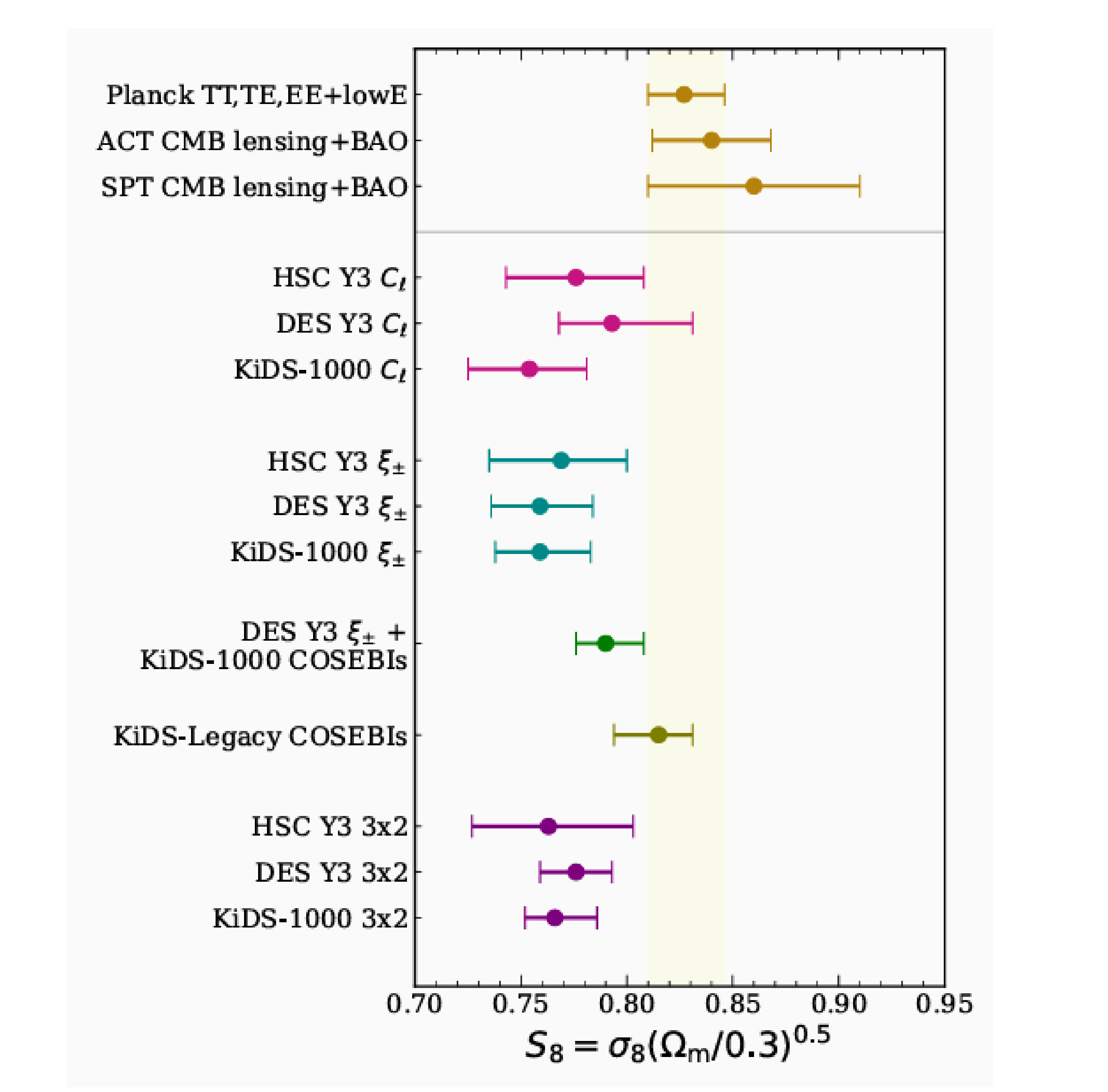




Apart from the Hubble tension, another tension between the Planck data with the weak lensing and the redshift surveys has been reported.

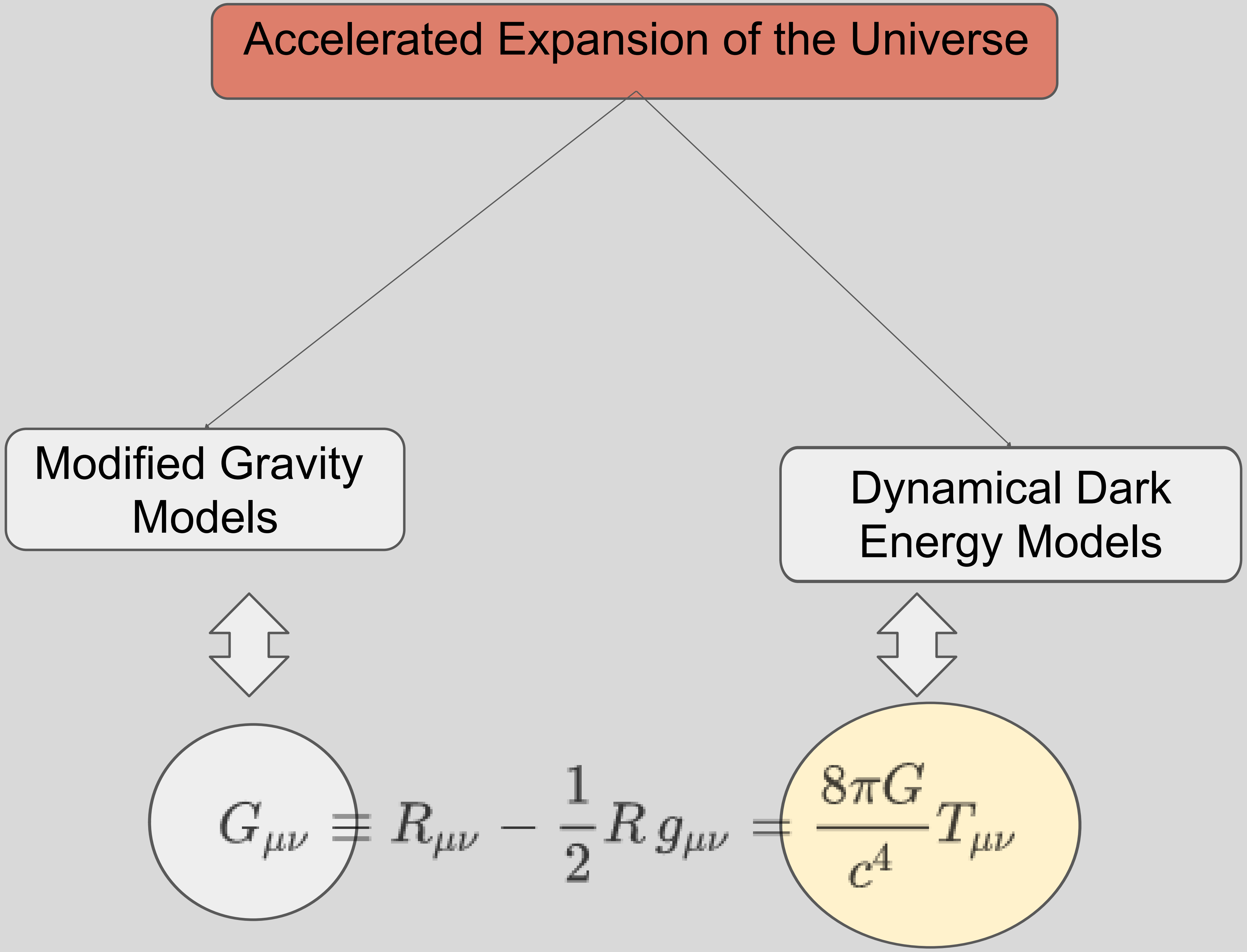
arXiv:2008.11285

Current Status



The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics 2504.01669

Alternative Approaches



Dynamical Dark Energy

$$\Omega_r^{(0)} + \Omega_m^{(0)} + \Omega_{DE}^{(0)} + \Omega_K^{(0)} = 1$$

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_{DE}) - \frac{K}{a^2}$$

$$H^2(z) = H_0^2 \left[\Omega_r^{(0)}(1+z)^4 + \Omega_m^{(0)}(1+z)^3 + \Omega_{DE}^{(0)} \exp\left\{3\int_0^z \frac{1+w_{DE}}{1+\tilde{z}}\, d\tilde{z}\right\} + \Omega_K^{(0)}(1+z)^2\right]$$

$$E(z) = \frac{c}{H_0}\left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}$$

$$w_{DE}(z) = \frac{(1+z)(E^2(z))' - 3E^2(z)}{3\left[E^2(z) - \Omega_m^{(0)}(1+z)^3\right]}$$

Strategy to Construct Dynamical Dark Energy Model

- Parametrization of different cosmological parameters, like equation of state, Hubble Parameter, different density parameters etc.
- Consideration of different scalar fields to be as the candidate of the dark energy. Like quintessence, phantom, K-essence etc.
- Considering model independent cosmographic approaches by constructing kinematic parameters.

The Chevallier–Polarski–Linder (CPL) parametrization

A Taylor Series Expansion of the EOS of the dark energy around z=0.

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$

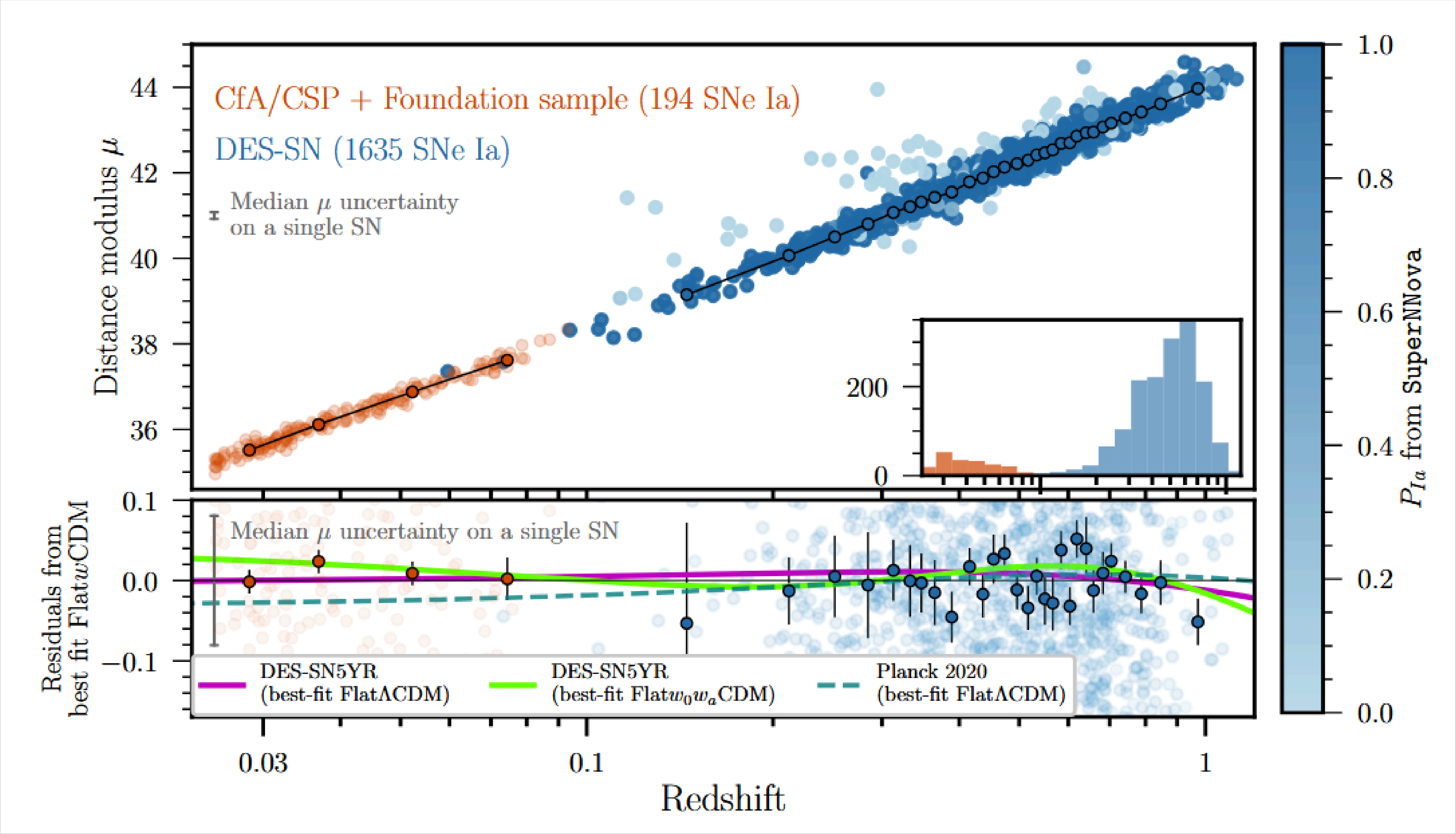
$$w_0 = -1, w_a = 0 \Rightarrow \text{Gives us back cosmological constant.}$$

$$w_a = 0 \Rightarrow \text{Dark energy with constant EOS but not Cosmological Constant}$$

$$w_0 \neq 0, w_a \neq 0 \Rightarrow w_0 - w_a \text{ dark energy model.}$$

The Dark Energy Survey

- Cosmology Results With   1500 New High-redshift Type Ia Supernovae Using The Full 5-year Dataset.
- It constructed the Hubble-diagram with sample includes 1635 supernovae, of which 1499 have a machine-learning probability of being a Type Ia greater than 50%.

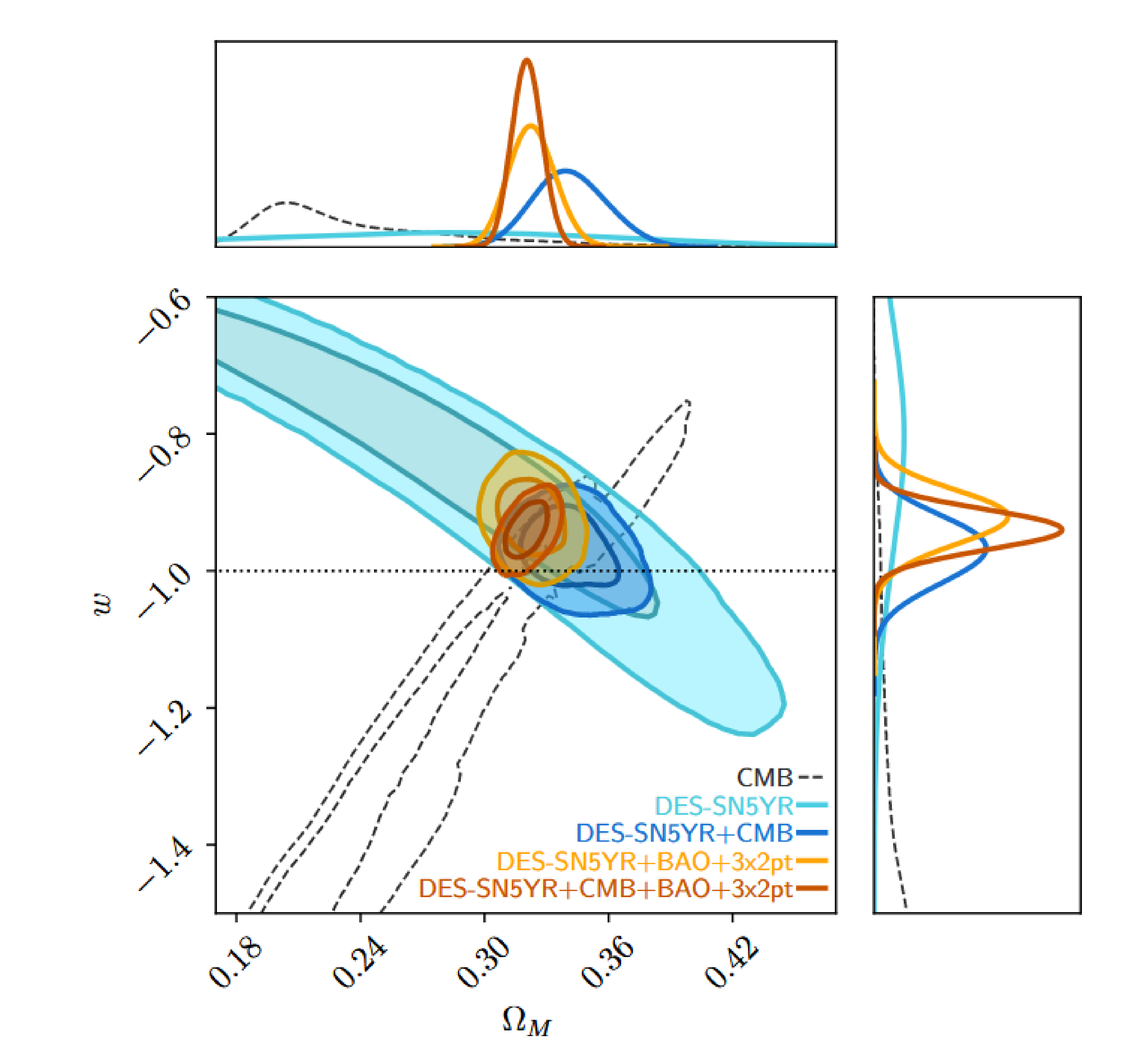


2401.02929

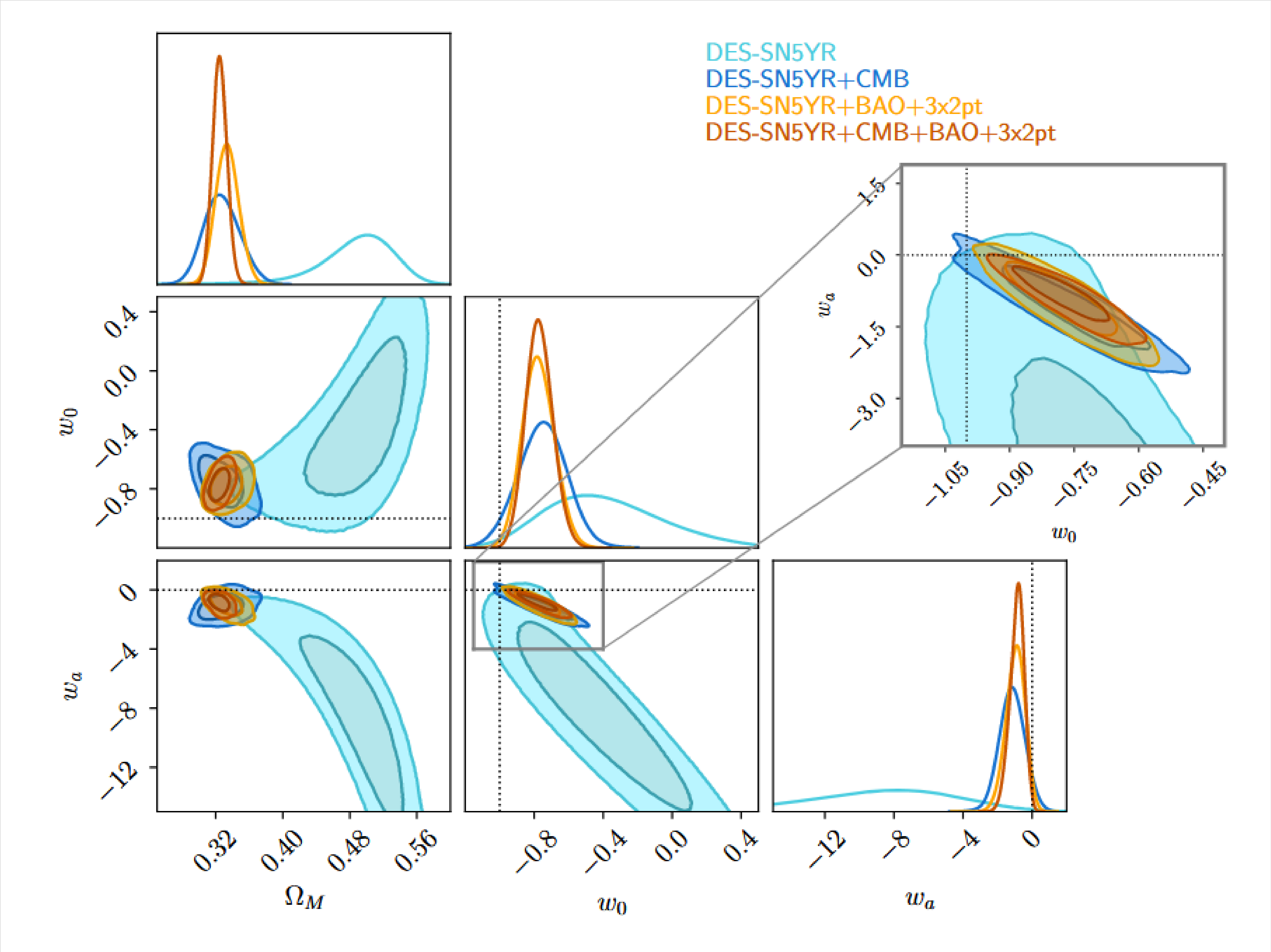
The Dark Energy Survey

Cosmological Model	Friedmann Equation: $\mathbf{E(z) = H(z)/H_0} =$	**Fit Parameters Θ**
Flat-ΛCDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)\right]^{1/2}$	Ω_M
ΛCDM	$\left[\Omega_M(1+z)^3 + \Omega_\Lambda + (1-\Omega_M-\Omega_\Lambda)(1+z)^2\right]^{1/2}$	Ω_M, Ω_Λ
Flat-wCDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{3(1+w)}\right]^{1/2}$	Ω_M, w
Flat-$w_0 w_a$CDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{3(1+w_0+w_a)} e^{-3w_a z/(1+z)}\right]^{1/2}$	Ω_M, w_0, w_a

The Dark Energy Survey



The Dark Energy Survey



The Dark Energy Survey

	Ω_M	Ω_K	w_0	w_a	χ^2/dof
DES-SN5YR (no external priors)					
Flat-ΛCDM	0.352 ± 0.017	-	-	-	1649/1734=0.951
ΛCDM	$0.291^{+0.063}_{-0.065}$	0.16 ± 0.16	-	-	1648/1733=0.951
Flat-wCDM	$0.264^{+0.074}_{-0.096}$	-	$-0.80^{+0.14}_{-0.16}$	-	1648/1733=0.951
Flat-w_0w_aCDM	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]=0.948

DES-SN5YR - [illegible]					
Flat-ΛCDM	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]=0.952
ΛCDM	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]=0.950
Flat-wCDM	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]=0.951
Flat-w_0w_aCDM	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]=0.951

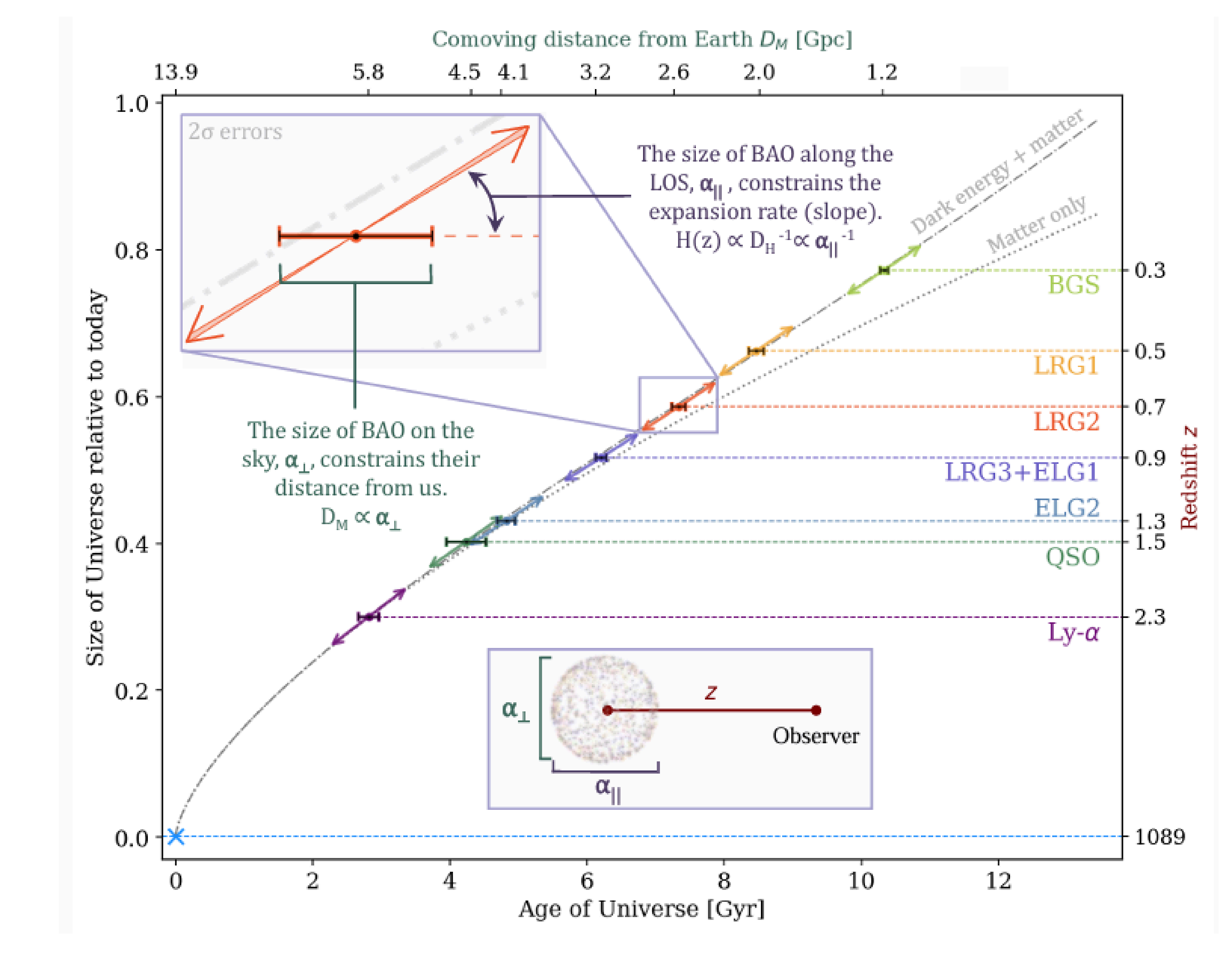
5.1.2. *Is dark energy a cosmological constant?*

As seen in Sec 4.1, a cosmological constant is a good fit to our data, but not the best fit. Our best fit equation of state parameter is slightly (more than 1σ) higher than the cosmological constant value of $w = -1$ (both for SNe alone and in combination with Planck or BAO+3×2pt). Our result agrees with the recent result from the UNION3 compilation analyzed with the UNITY framework (Rubin et al. 2023) (which appeared while this paper was under internal review). The Pantheon+ result (Brout et al. 2022a) is within 1σ of $w = -1$, but also on the high side ($w = -0.90 \pm 0.14$).

DESI 2025 DR2

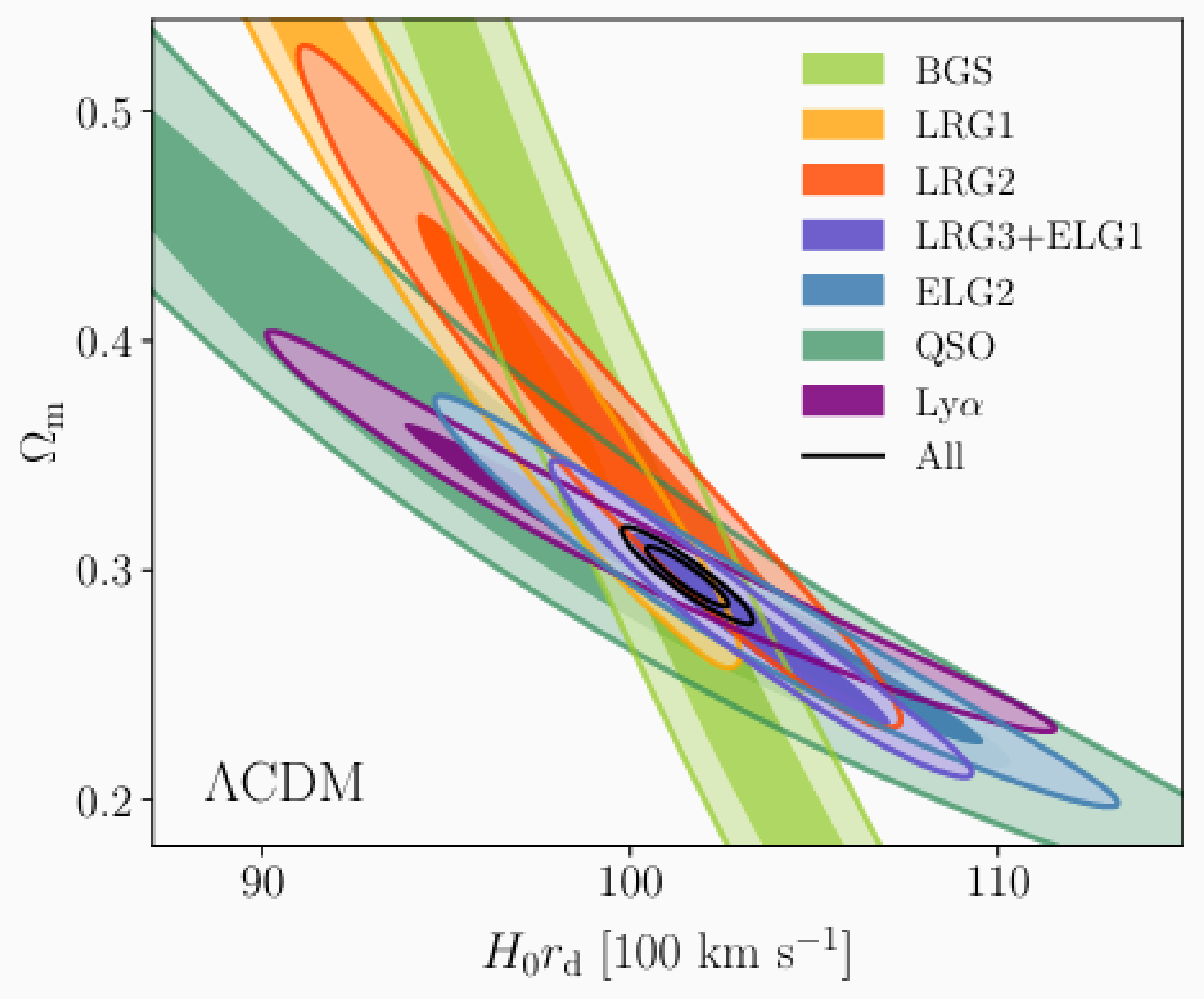
Present baryon acoustic oscillation (BAO) measurements from more than 14 million galaxies and quasars drawn from the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2), based on three years of operation.

2503.14738



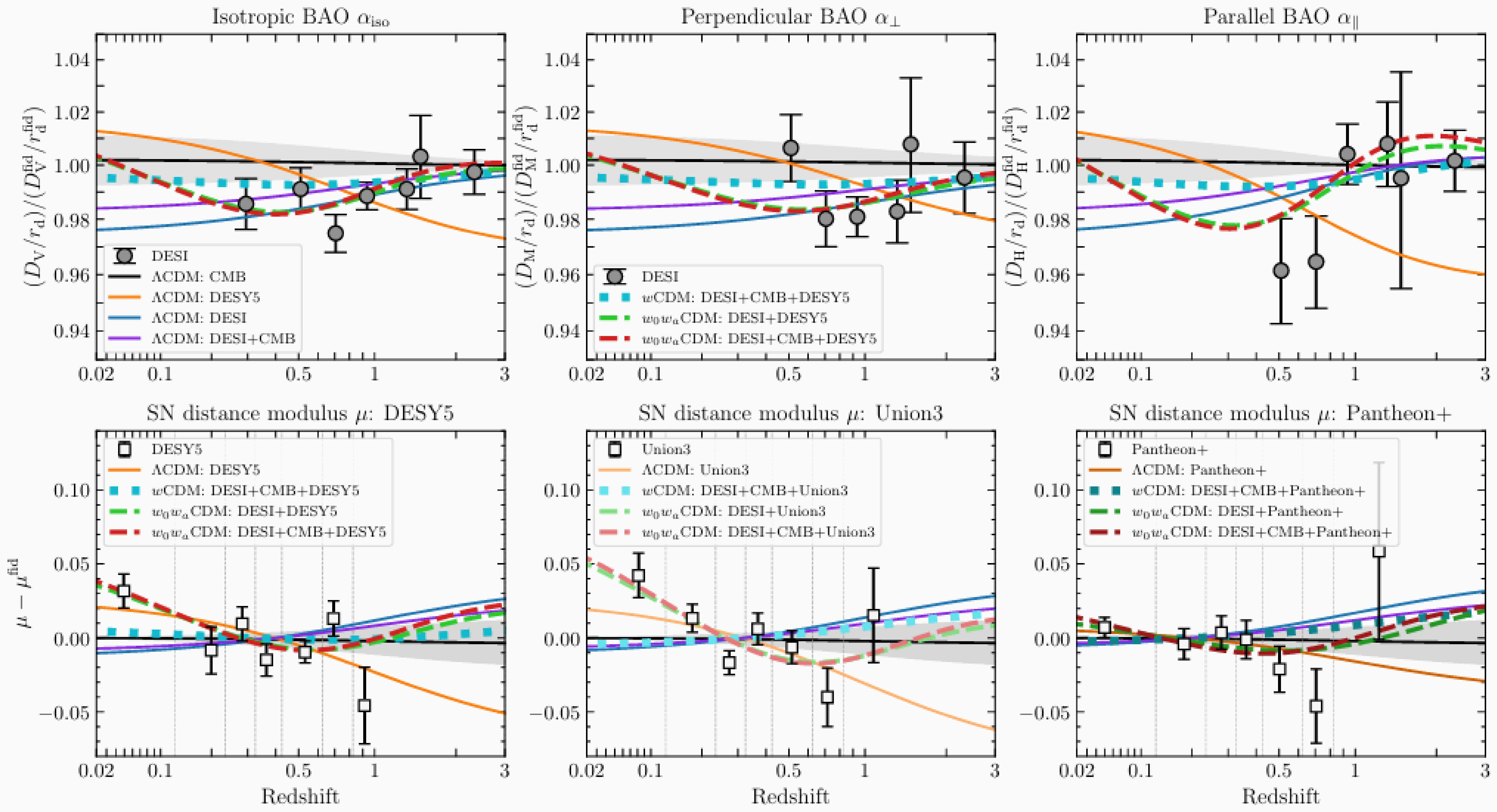
Tracer Type	Redshift Range	Number of Objects	Purpose
BGS (Bright Galaxy Sample)	0.1–0.4	~1.2 million	Low-redshift BAO
LRG (Luminous Red Galaxies)	0.4–1.1	~4.5 million	Intermediate redshift
ELG (Emission Line Galaxies)	0.8–1.6	~6.5 million	High-redshift structure
QSO (Quasars)	0.8–3.5	~2 million	Tracers and Lyα forest

Tracer	$z_{\rm eff}$	$\alpha_{\rm iso}$	$\alpha_{\rm AP}$	$D_{\rm V}/r_{\rm d}$	$D_{\rm M}/D_{\rm H}$	$r_{\rm V,M/H}$	$D_{\rm M}/r_{\rm d}$	$D_{\rm H}/r_{\rm d}$	$r_{\rm M,H}$
BGS	0.295	0.9857 ± 0.0093	—	7.942 ± 0.075	—	—	—	—	—
LRG1	0.510	0.9911 ± 0.0077	0.9555 ± 0.0261	12.720 ± 0.099	0.622 ± 0.017	0.050	13.588 ± 0.167	21.863 ± 0.425	−0.459
LRG2	0.706	0.9749 ± 0.0067	0.9842 ± 0.0227	16.050 ± 0.110	0.892 ± 0.021	−0.018	17.351 ± 0.177	19.455 ± 0.330	−0.404
LRG3+ELG1	0.934	0.9886 ± 0.0046	1.0237 ± 0.0157	19.721 ± 0.091	1.223 ± 0.019	0.056	21.576 ± 0.152	17.641 ± 0.193	−0.416
ELG2	1.321	0.9911 ± 0.0071	1.0257 ± 0.0237	24.252 ± 0.174	1.948 ± 0.045	0.202	27.601 ± 0.318	14.176 ± 0.221	−0.434
QSO	1.484	1.0032 ± 0.0153	0.9885 ± 0.0564	26.055 ± 0.398	2.386 ± 0.136	0.044	30.512 ± 0.760	12.817 ± 0.516	−0.500
Lya	2.330	0.9971 ± 0.0082	1.0071 ± 0.0216	31.267 ± 0.256	4.518 ± 0.097	0.574	38.988 ± 0.531	8.632 ± 0.101	−0.431
LRG3	0.922	0.9936 ± 0.0053	0.9996 ± 0.0172	19.656 ± 0.105	1.232 ± 0.021	0.106	21.648 ± 0.178	17.577 ± 0.213	−0.406
ELG1	0.955	0.9888 ± 0.0091	1.0574 ± 0.0291	20.008 ± 0.183	1.220 ± 0.033	0.420	21.707 ± 0.335	17.803 ± 0.297	−0.462

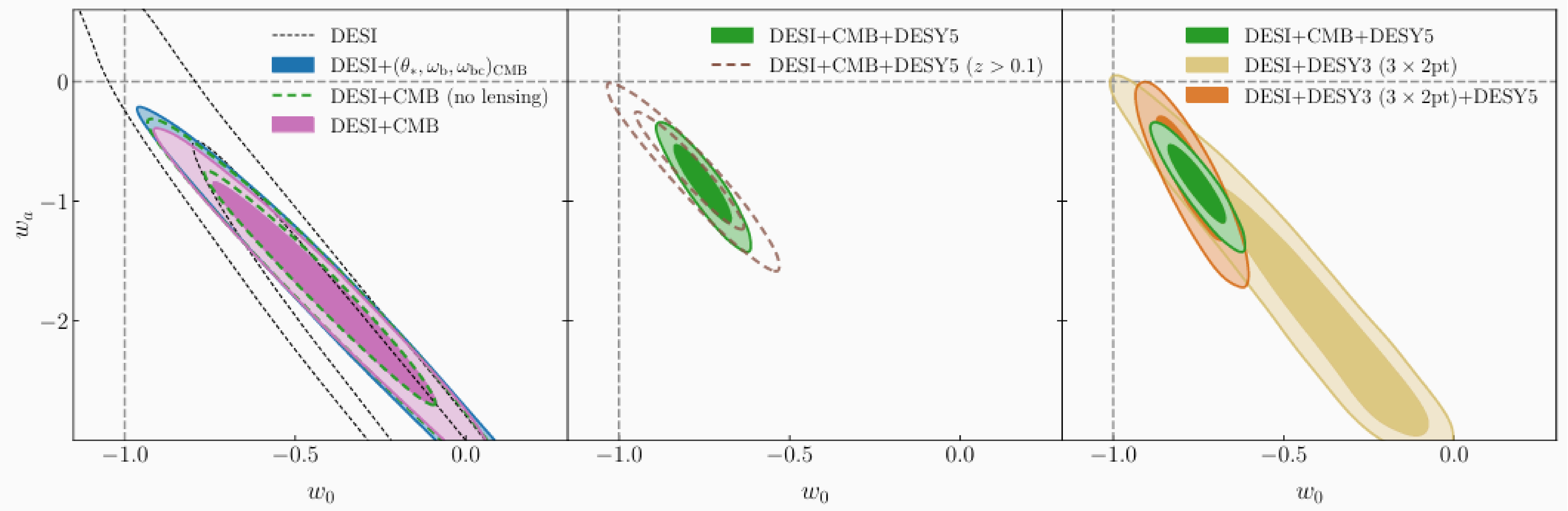

BGS
LRG1
LRG2
LRG3+ELG1
ELG2
QSO
Lyα
All
ΛCDM
0.5
0.4
0.3
0.2
Ω_m
90
100
110
$H_0 r_d$ [100 km s^{-1}]

Model/Dataset	Ω_m	H_0 [km s^{-1} Mpc^{-1}]	$10^3\Omega_K$	w or w_0	w_a
ΛCDM					
CMB	0.3169 ± 0.0065	67.14 ± 0.47	—	—	—
DESI	0.2975 ± 0.0086	—	—	—	—
DESI+BBN	0.2977 ± 0.0086	68.51 ± 0.58	—	—	—
DESI+BBN+θ_*	0.2967 ± 0.0045	68.45 ± 0.47	—	—	—
DESI+CMB	0.3027 ± 0.0036	68.17 ± 0.28	—	—	—
ΛCDM+Ω_K					
CMB	$0.354^{+0.020}_{-0.023}$	63.3 ± 2.1	$-10.7^{+6.4}_{-5.3}$	—	—
DESI	0.293 ± 0.012	—	25 ± 41	—	—
DESI+CMB	0.3034 ± 0.0037	68.50 ± 0.33	2.3 ± 1.1	—	—
***w*CDM**					
CMB	$0.203^{+0.017}_{-0.060}$	85^{+10}_{-6}	—	$-1.55^{+0.17}_{-0.37}$	—
DESI	0.2969 ± 0.0089	—	—	-0.916 ± 0.078	—
DESI+Pantheon+	0.2976 ± 0.0087	—	—	-0.914 ± 0.040	—
DESI+Union3	0.2973 ± 0.0091	—	—	-0.866 ± 0.052	—
DESI+DESY5	0.2977 ± 0.0091	—	—	-0.872 ± 0.039	—
DESI+CMB	0.2927 ± 0.0073	69.51 ± 0.92	—	-1.055 ± 0.036	—
DESI+CMB+Pantheon+	0.3047 ± 0.0051	67.97 ± 0.57	—	-0.995 ± 0.023	—
DESI+CMB+Union3	0.3044 ± 0.0059	68.01 ± 0.68	—	-0.997 ± 0.027	—
DESI+CMB+DESY5	0.3098 ± 0.0050	67.34 ± 0.54	—	-0.971 ± 0.021	—
w_0w_aCDM					
CMB	$0.220^{+0.019}_{-0.078}$	83^{+20}_{-6}	—	$-1.23^{+0.44}_{-0.61}$	< -0.504
DESI	$0.352^{+0.041}_{-0.018}$	—	—	$-0.48^{+0.35}_{-0.17}$	< -1.34
DESI+Pantheon+	$0.298^{+0.025}_{-0.011}$	—	—	$-0.888^{+0.055}_{-0.064}$	-0.17 ± 0.46
DESI+Union3	$0.328^{+0.019}_{-0.014}$	—	—	-0.70 ± 0.11	-0.99 ± 0.57
DESI+DESY5	$0.319^{+0.017}_{-0.011}$	—	—	$-0.781^{+0.067}_{-0.076}$	-0.72 ± 0.47
DESI+$(\theta_*, \omega_b, \omega_{bc})_{CMB}$	0.353 ± 0.022	$63.7^{+1.7}_{-2.2}$	—	-0.43 ± 0.22	-1.72 ± 0.64
DESI+CMB (no lensing)	0.352 ± 0.021	$63.7^{+1.7}_{-2.1}$	—	-0.43 ± 0.21	-1.70 ± 0.60
DESI+CMB	0.353 ± 0.021	$63.6^{+1.6}_{-2.1}$	—	-0.42 ± 0.21	-1.75 ± 0.58
DESI+CMB+Pantheon+	0.3114 ± 0.0057	67.51 ± 0.59	—	-0.838 ± 0.055	$-0.62^{+0.22}_{-0.19}$
DESI+CMB+Union3	0.3275 ± 0.0086	65.91 ± 0.84	—	-0.667 ± 0.088	$-1.09^{+0.31}_{-0.27}$
DESI+CMB+DESY5	0.3191 ± 0.0056	66.74 ± 0.56	—	-0.752 ± 0.057	$-0.86^{+0.23}_{-0.20}$
DESI+DESY3 (3×2pt)+Pantheon+	0.3140 ± 0.0091	—	—	-0.870 ± 0.061	$-0.46^{+0.33}_{-0.29}$
DESI+DESY3 (3×2pt)+Union3	0.333 ± 0.012	—	—	-0.68 ± 0.11	$-1.09^{+0.48}_{-0.39}$
DESI+DESY3 (3×2pt)+DESY5	0.3239 ± 0.0092	—	—	-0.771 ± 0.068	$-0.82^{+0.38}_{-0.32}$
w_0w_aCDM+Ω_K					
DESI	$0.357^{+0.041}_{-0.030}$	—	-2 ± 56	$-0.45^{+0.33}_{-0.17}$	< -1.43
DESI+CMB+Pantheon+	0.3117 ± 0.0056	67.62 ± 0.60	1.1 ± 1.3	-0.853 ± 0.057	-0.54 ± 0.22
DESI+CMB+Union3	0.3273 ± 0.0086	65.98 ± 0.86	0.6 ± 1.3	-0.678 ± 0.092	$-1.03^{+0.33}_{-0.29}$
DESI+CMB+DESY5	0.3193 ± 0.0056	66.82 ± 0.58	0.8 ± 1.3	-0.762 ± 0.060	-0.81 ± 0.24

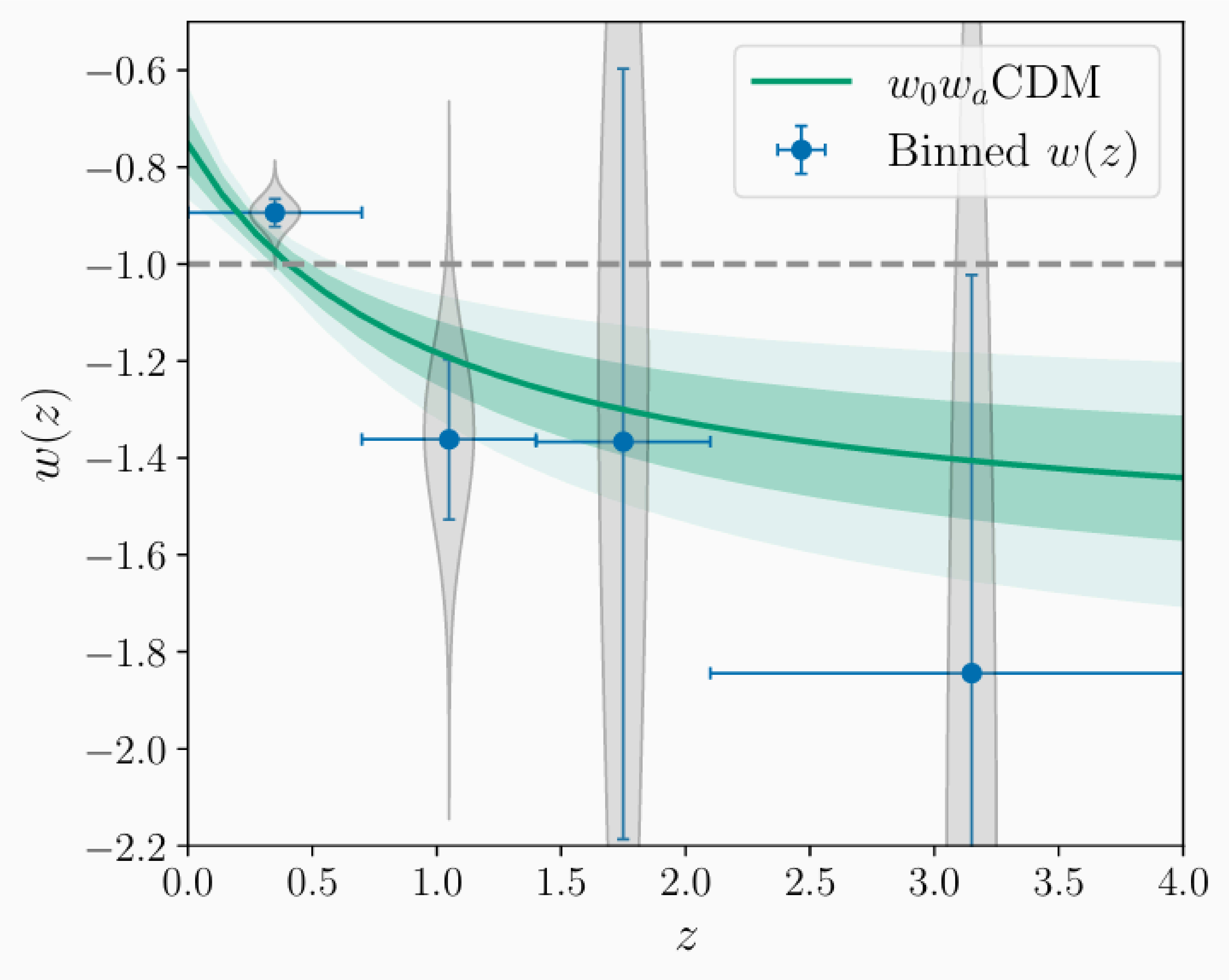
Datasets	$\Delta\chi^2_{MAP}$	Significance	Δ(DIC)
DESI	-4.7	1.7σ	-0.8
DESI+$(\theta_*, \omega_b, \omega_{bc})_{CMB}$	-8.0	2.4σ	-4.4
DESI+CMB (no lensing)	-9.7	2.7σ	-5.9
DESI+CMB	-12.5	3.1σ	-8.7
DESI+Pantheon+	-4.9	1.7σ	-0.7
DESI+Union3	-10.1	2.7σ	-6.0
DESI+DESY5	-13.6	3.3σ	-9.3
DESI+DESY3 (3×2pt)	-7.3	2.2σ	-2.8
DESI+DESY3 (3×2pt)+DESY5	-13.8	3.3σ	-9.1
DESI+CMB+Pantheon+	-10.7	2.8σ	-6.8
DESI+CMB+Union3	-17.4	3.8σ	-13.5
DESI+CMB+DESY5	-21.0	4.2σ	-17.2



Isotropic BAO $\alpha_{\rm iso}$
Perpendicular BAO $\alpha_\perp$
Parallel BAO $\alpha_\parallel$
$(D_V/r_d)/(D_V^{\rm fid}/r_d^{\rm fid})$
$(D_M/r_d)/(D_M^{\rm fid}/r_d^{\rm fid})$
$(D_H/r_d)/(D_H^{\rm fid}/r_d^{\rm fid})$
DESI
ΛCDM: CMB
ΛCDM: DESY5
ΛCDM: DESI
ΛCDM: DESI+CMB
wCDM: DESI+CMB+DESY5
w_0w_aCDM: DESI+DESY5
w_0w_aCDM: DESI+CMB+DESY5
SN distance modulus μ: DESY5
SN distance modulus μ: Union3
SN distance modulus μ: Pantheon+
$\mu - \mu^{\rm fid}$
DESY5
Union3
ΛCDM: Union3
wCDM: DESI+CMB+Union3
w_0w_aCDM: DESI+Union3
w_0w_aCDM: DESI+CMB+Union3
Pantheon+
ΛCDM: Pantheon+
wCDM: DESI+CMB+Pantheon+
w_0w_aCDM: DESI+Pantheon+
w_0w_aCDM: DESI+CMB+Pantheon+
Redshift

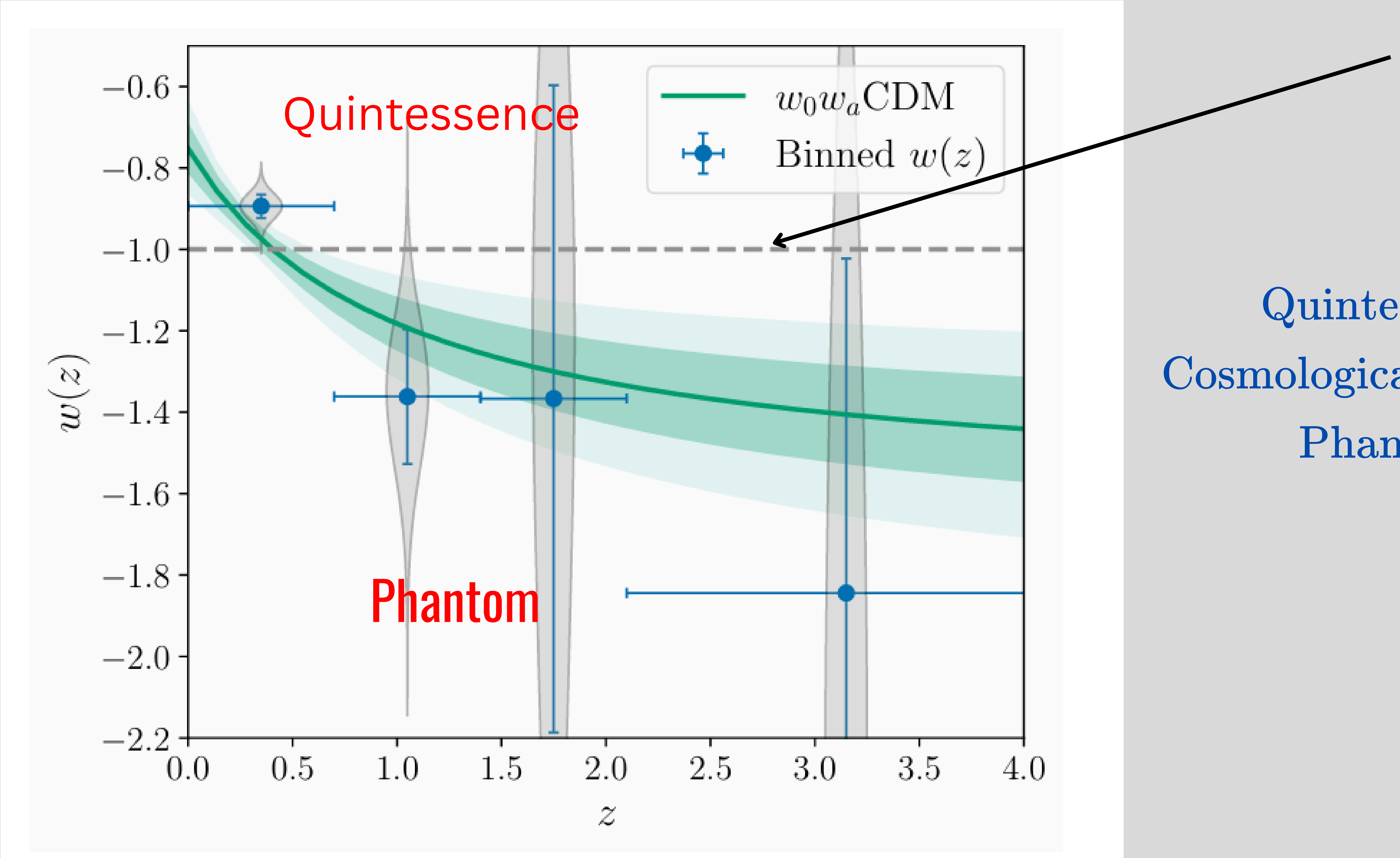


DESI
DESI+$(\theta_*, \omega_b, \omega_{bc})_{CMB}$
DESI+CMB (no lensing)
DESI+CMB
DESI+CMB+DESY5
DESI+CMB+DESY5 ($z > 0.1$)
DESI+CMB+DESY5
DESI+DESY3 (3×2pt)
DESI+DESY3 (3×2pt)+DESY5
w_a
0
−1
−2
−1.0
−0.5
0.0
w_0



$w_0 w_a$CDM
Binned $w(z)$
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
−2.2
$w(z)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
z

Phantom Barrier Crossing



Quintessence: $w > -1$

Cosmological Constant: $w = -1$

Phantom: $w < -1$

Is phantom barrier crossing an artifact of the parametrization or a real physical phenomenon?

The community is devided.

A No-Go Theorem

$$\delta' = -(1+w)(\theta - 3\Phi') - 3\mathcal{H}(c_s^2 - w)\delta$$

$$\theta' = -\mathcal{H}(1-3w)\theta - \frac{w'}{1+w}\theta + k^2\left(\frac{c_s^2\delta}{1+w} + \Psi\right)$$

Density Contrast: $\delta \equiv \frac{\delta\rho}{\rho}$

Velocity Perturbation: $\theta \equiv \frac{ik^j \delta T^0_j}{\rho + p}$

$$c_a^2 \equiv c_s^2\Big|_{\text{adiabatic}} = \frac{p'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

At the phantom barrier crossing w=-1 , the dark energy perturbation diverge. With a single degree of freedoom phantom barrier crossing is prohibited.

Hubble Tension Revisited

Kamionkowski, Marc, and Adam G. Riess. "The Hubble tension and early dark energy." Annual Review of Nuclear and Particle Science 73.1 (2023): 153-180.

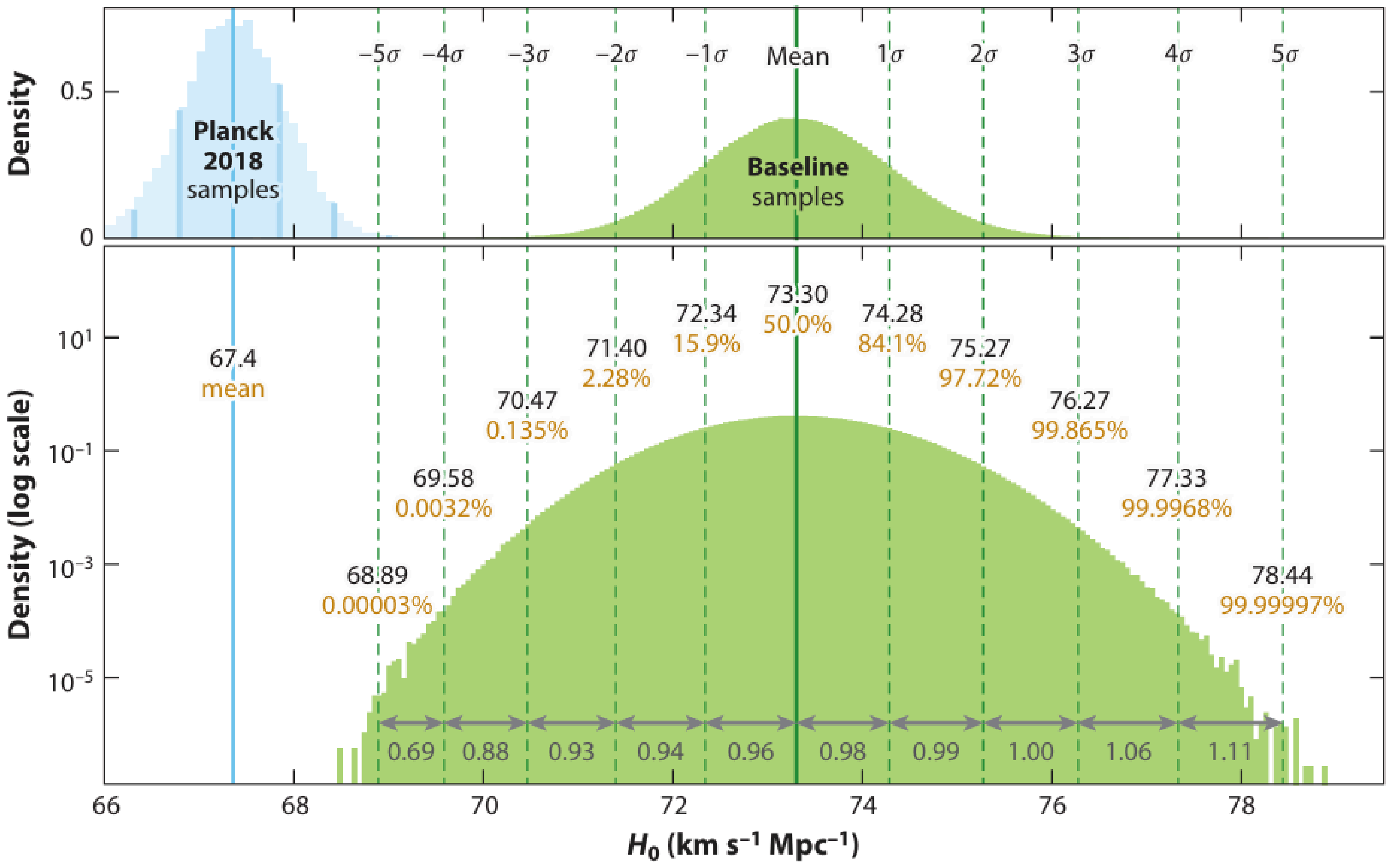


Figure 1

Extended Markov chain Monte Carlo sampling of the posterior for the Hubble constant H_0 to measure out to the 5σ confidence level. The top panel shows the probability density for the baseline from the Supernovae and H_0 for the Equation of State (SH0ES) Collaboration and from the Planck Collaboration chains (4). The bottom panel shows the log of the probability density to improve the ability to see the tails. Figure adapted from Reference 9 (CC BY 4.0).

Early Time or Late Time Modification

Sound horizon at decoupling

$$r_s = \int_{z_s}^{\infty} \frac{c_s(z)}{H(z)}\, dz = \frac{c}{\sqrt{3}H_s} \int_{z_s}^{\infty} \frac{dz}{\left[\frac{\rho(z)}{\rho(z_s)}\right]^{1/2} (1+R)^{1/2}}$$

$$R = \left(\frac{3}{4}\right)\left(\frac{\omega_b}{\omega_\gamma}\right)\frac{1}{1+z}$$

Hubble parameter at last scattering

$$H_{\mathrm{ls}} = 100\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1}\cdot\omega_r^{1/2}(1+z_{\mathrm{ls}})^2\sqrt{1+\frac{\omega_m}{\omega_r}\cdot\frac{1}{1+z_{\mathrm{ls}}}}$$

Angular diameter distance

$$D_A = \frac{c}{H_0}\int_0^{z_{\mathrm{ls}}} \frac{dz}{\left[\frac{\rho(z)}{\rho_0}\right]^{1/2}}$$

Hubble constant in terms of sound horizon and angular scale

$$H_0 = \sqrt{3} H_{ls} \theta_s \frac{\int_0^{z_{\rm ls}} dz \left[\frac{\rho(z)}{\rho_0}\right]^{-1/2}}{\int_{z_{\rm ls}}^{\infty} dz \left[\frac{\rho(z)}{\rho(z_{\rm ls})}\right]^{-1/2} (1+R)^{-1/2}}$$

Late time solution

Early time solution

Early Time Solution

Annu. Rev. Nucl. Part. Sci. 2023. 73:153–80

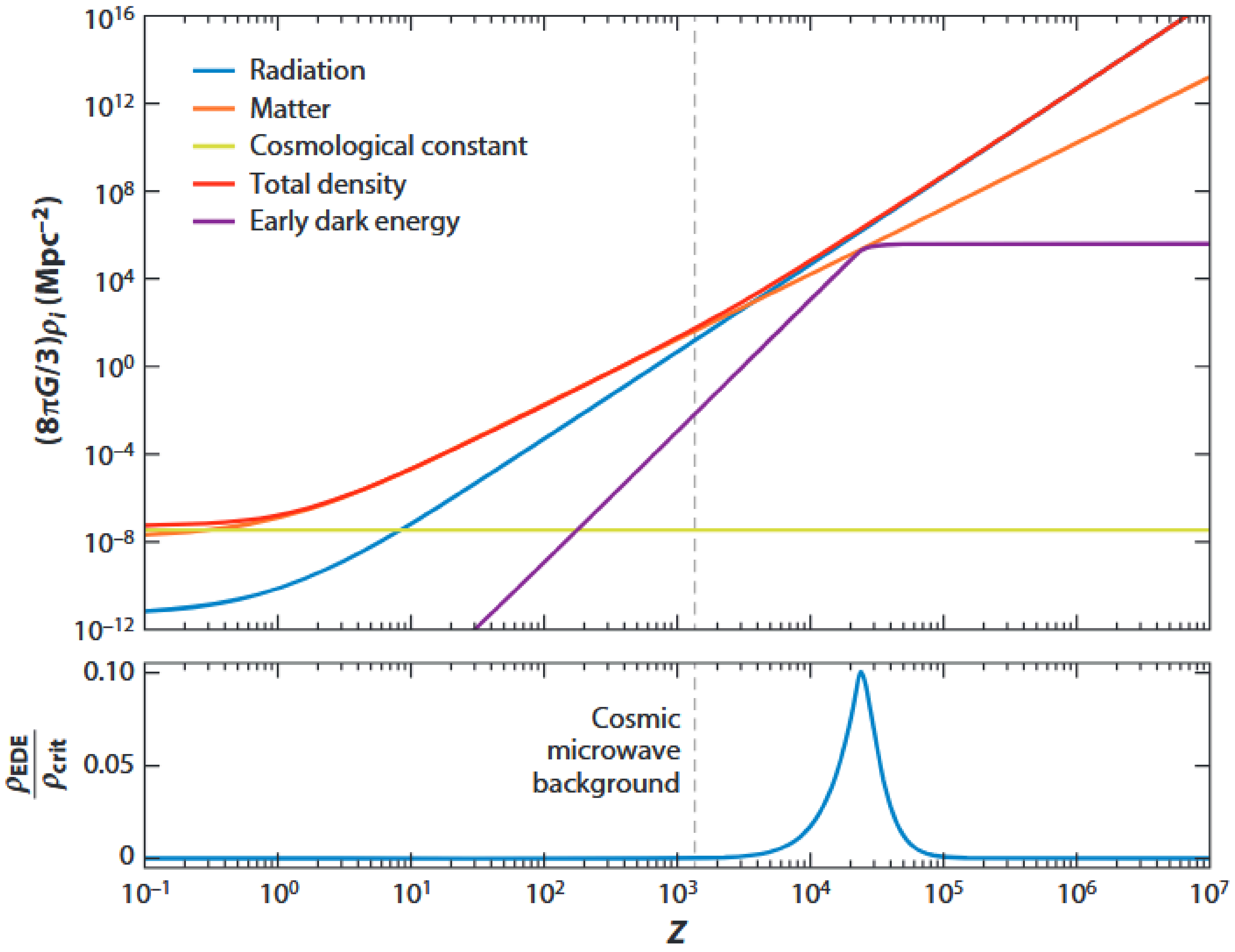
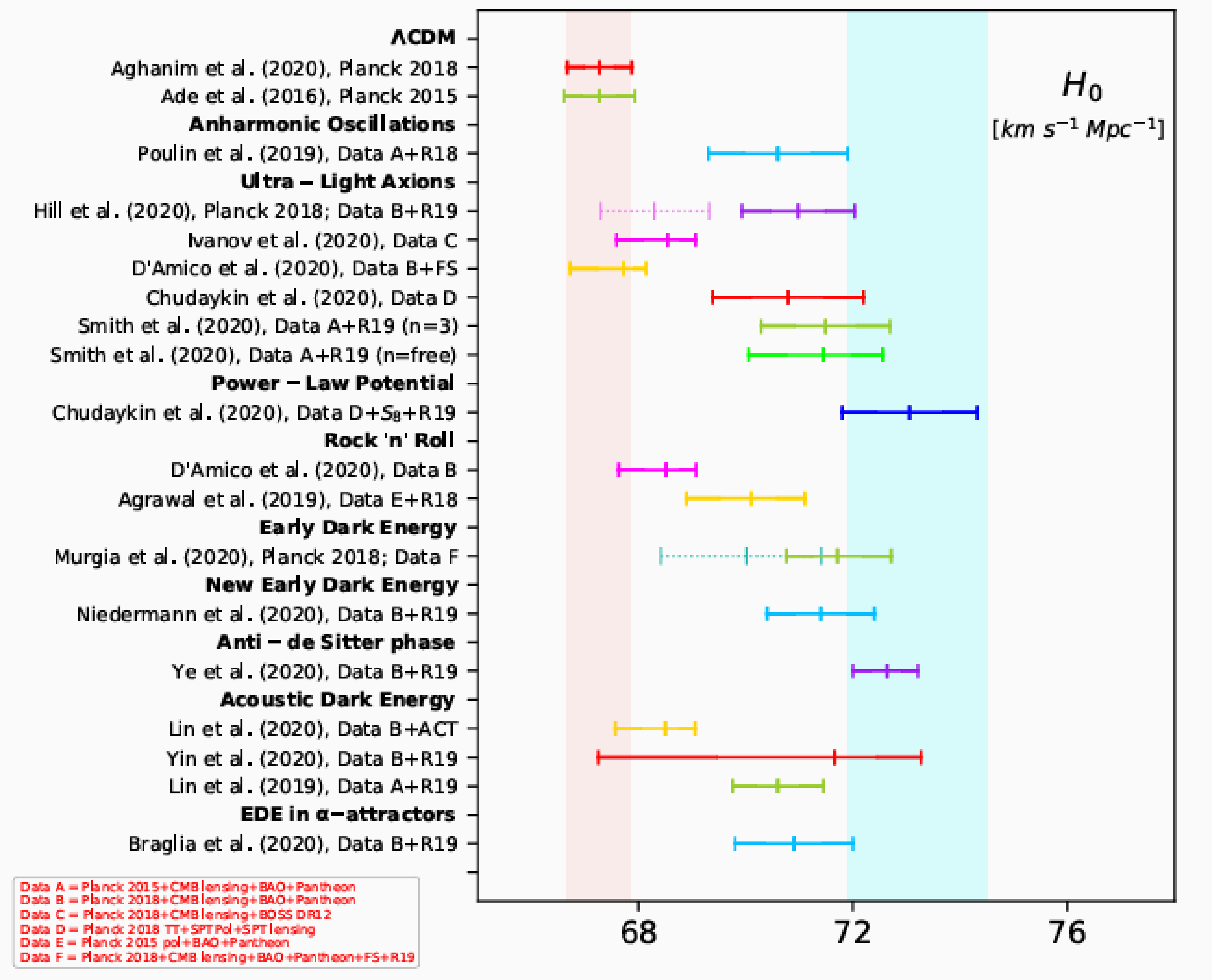
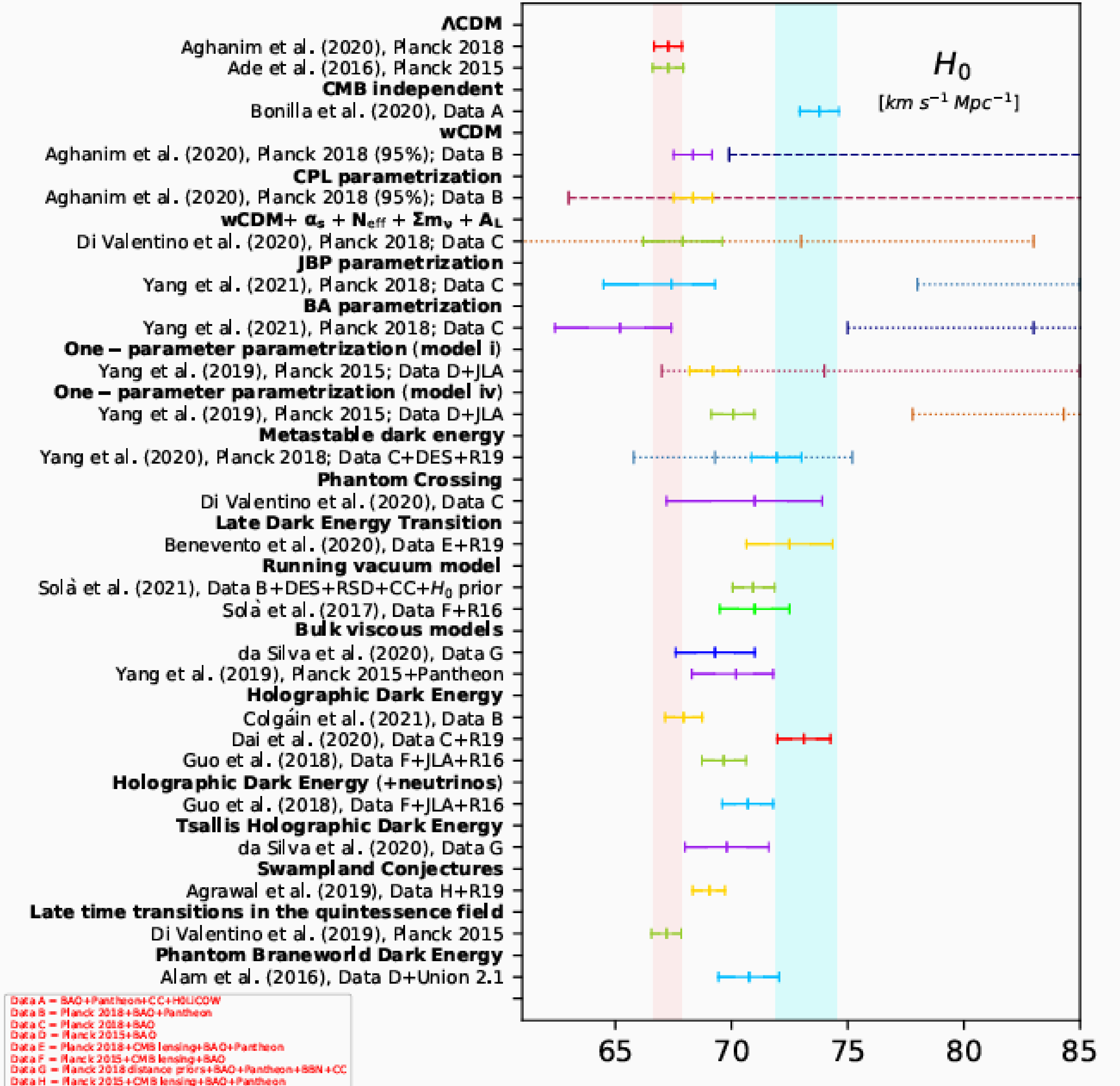


Figure 6

The evolution of the energy densities of radiation, nonrelativistic matter (baryons and cold dark matter), and the cosmological constant as a function of redshift (so time increases to the left, with the big bang far off to the right and today off to the left). Also shown is the energy density postulated for early dark energy (EDE). The bottom panel shows the fractional contribution of EDE to the total energy density. The EDE curves are schematic—the key point is that it contributes ~10% a bit before recombination but is otherwise dynamically unimportant. Figure courtesy of T. Karwal.

Di Valentino, Eleonora, Olga Mena, Supriya Pan, Luca Visinelli, Weiqiang Yang, Alessandro Melchiorri, David F. Mota, Adam G. Riess, and Joseph Silk. "In the realm of the Hubble tension—a review of solutions." Classical and Quantum Gravity 38, no. 15 (2021): 153001.





ΛCDM
Aghanim et al. (2020), Planck 2018
Ade et al. (2016), Planck 2015
CMB independent
Bonilla et al. (2020), Data A
wCDM
Aghanim et al. (2020), Planck 2018 (95%); Data B
CPL parametrization
Aghanim et al. (2020), Planck 2018 (95%); Data B
wCDM+ α_s + N_{eff} + Σm_ν + A_L
Di Valentino et al. (2020), Planck 2018; Data C
JBP parametrization
Yang et al. (2021), Planck 2018; Data C
BA parametrization
Yang et al. (2021), Planck 2018; Data C
One − parameter parametrization (model i)
Yang et al. (2019), Planck 2015; Data D+JLA
One − parameter parametrization (model iv)
Yang et al. (2019), Planck 2015; Data D+JLA
Metastable dark energy
Yang et al. (2020), Planck 2018; Data C+DES+R19
Phantom Crossing
Di Valentino et al. (2020), Data C
Late Dark Energy Transition
Benevento et al. (2020), Data E+R19
Running vacuum model
Solà et al. (2021), Data B+DES+RSD+CC+H_0 prior
Solà et al. (2017), Data F+R16
Bulk viscous models
da Silva et al. (2020), Data G
Yang et al. (2019), Planck 2015+Pantheon
Holographic Dark Energy
Colgáin et al. (2021), Data B
Dai et al. (2020), Data C+R19
Guo et al. (2018), Data F+JLA+R16
Holographic Dark Energy (+neutrinos)
Guo et al. (2018), Data F+JLA+R16
Tsallis Holographic Dark Energy
da Silva et al. (2020), Data G
Swampland Conjectures
Agrawal et al. (2019), Data H+R19
Late time transitions in the quintessence field
Di Valentino et al. (2019), Planck 2015
Phantom Braneworld Dark Energy
Alam et al. (2016), Data D+Union 2.1
H_0
[$km\ s^{-1}\ Mpc^{-1}$]
65
70
75
80
85
Data A = BAO+Pantheon+CC+H0LiCOW
Data B = Planck 2018+BAO+Pantheon
Data C = Planck 2018+BAO
Data D = Planck 2015+BAO
Data E = Planck 2018+CMB lensing+BAO+Pantheon
Data F = Planck 2015+CMB lensing+BAO
Data G = Planck 2018 distance priors+BAO+Pantheon+BBN+CC
Data H = Planck 2015+CMB lensing+BAO+Pantheon

The H_0 Olympics: A fair ranking of proposed models

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Schöneberg, N., Abellán, G. F., Sánchez, A. P., Witte, S. J., Poulin, V., & Lesgourgues, J. (2022). The H0 Olympics: A fair ranking of proposed models. Physics Reports, 984, 1-55.

1. Gaussian Tension (GT)

$$\frac{\bar{x}_{\mathcal{D}} - \bar{x}_{\mathrm{SH0ES}}}{(\sigma^2_{\mathcal{D}} + \sigma^2_{\mathrm{SH0ES}})^{1/2}}$$

2. χ^2 difference

$$\Delta\chi^2 = \chi^2_{\min,\mathcal{D}+\mathrm{SH0ES}} - \chi^2_{\min,\mathcal{D}}$$

3. Akaike Information Criterium (AIC)

$$\Delta\mathrm{AIC} = \chi^2_{\min,\mathcal{M}} - \chi^2_{\min,\Lambda\mathrm{CDM}} + 2(N_{\mathcal{M}} - N_{\Lambda\mathrm{CDM}})$$

Model	$\Delta N_{\rm param}$	M_B	Gaussian Tension	$Q_{\rm DMAP}$ Tension		$\Delta\chi^2$	ΔAIC		Finalist
ΛCDM	0	-19.416 ± 0.012	4.4σ	4.5σ	✗	0.00	0.00	✗	✗
$\Delta N_{\rm ur}$	1	-19.395 ± 0.019	3.6σ	3.8σ	✗	-6.10	-4.10	✗	✗
SIDR	1	-19.385 ± 0.024	3.2σ	3.3σ	✗	-9.57	-7.57	✓	✓ (3)
mixed DR	2	-19.413 ± 0.036	3.3σ	3.4σ	✗	-8.83	-4.83	✗	✗
DR-DM	2	-19.388 ± 0.026	3.2σ	3.1σ	✗	-8.92	-4.92	✗	✗
SIν+DR	3	$-19.440^{+0.037}_{-0.039}$	3.8σ	3.9σ	✗	-4.98	1.02	✗	✗
Majoron	3	$-19.380^{+0.027}_{-0.021}$	3.0σ	2.9σ	✓	-15.49	-9.49	✓	✓ (2)
primordial B	1	$-19.390^{+0.018}_{-0.024}$	3.5σ	3.5σ	✗	-11.42	-9.42	✓	✓ (3)
varying m_e	1	-19.391 ± 0.034	2.9σ	2.9σ	✓	-12.27	-10.27	✓	✓ (1)
varying m_e+Ω_k	2	-19.368 ± 0.048	2.0σ	1.9σ	✓	-17.26	-13.26	✓	✓ (1)
EDE	3	$-19.390^{+0.016}_{-0.035}$	3.6σ	1.6σ	✓	-21.98	-15.98	✓	✓ (2)
NEDE	3	$-19.380^{+0.023}_{-0.040}$	3.1σ	1.9σ	✓	-18.93	-12.93	✓	✓ (2)
EMG	3	$-19.397^{+0.017}_{-0.023}$	3.7σ	2.3σ	✓	-18.56	-12.56	✓	✓ (2)
CPL	2	-19.400 ± 0.020	3.7σ	4.1σ	✗	-4.94	-0.94	✗	✗
PEDE	0	-19.349 ± 0.013	2.7σ	2.8σ	✓	2.24	2.24	✗	✗
GPEDE	1	-19.400 ± 0.022	3.6σ	4.6σ	✗	-0.45	1.55	✗	✗
DM $\to$ DR+WDM	2	-19.420 ± 0.012	4.5σ	4.5σ	✗	-0.19	3.81	✗	✗
DM $\to$ DR	2	-19.410 ± 0.011	4.3σ	4.5σ	✗	-0.53	3.47	✗	✗

Table 1: Test of the models based on dataset $\mathcal{D}_{\rm baseline}$ (Planck 2018 + BAO + Pantheon), using the direct measurement of M_b by SH0ES for the quantification of the tension (3rd column) or the computation of the AIC (5th column). Eight models pass at least one of these three tests at the 3σ level.

Modified Gravity Models

f(R) gravity:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, f(R) + S_m(g_{\mu\nu}, \Psi_m)$$

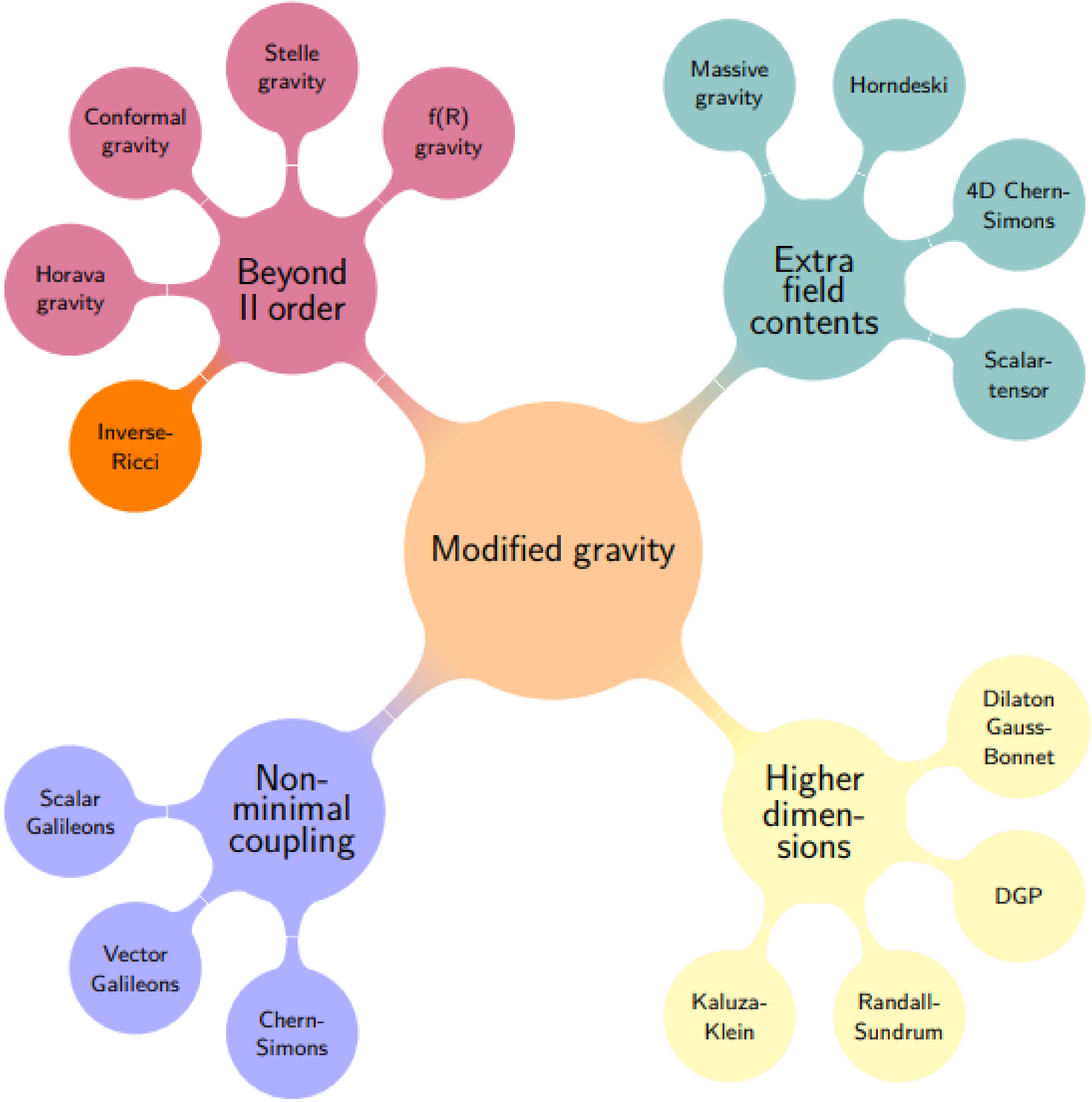
Scalar-tensor theories:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(\varphi, R) - \frac{1}{2} \zeta(\varphi)(\nabla\varphi)^2 \right] + S_m(g_{\mu\nu}, \Psi_m),$$

Gauss–Bonnet dark energy models:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) - f(\phi) R_{GB}^2 \right] + S_m(g_{\mu\nu}, \Psi_m),$$

$$R_{GB}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$


Modified gravity
Beyond II order
Stelle gravity
Conformal gravity
f(R) gravity
Horava gravity
Inverse-Ricci
Extra field contents
Massive gravity
Horndeski
4D Chern-Simons
Scalar-tensor
Non-minimal coupling
Scalar Galileons
Vector Galileons
Chern-Simons
Higher dimensions
Dilaton Gauss-Bonnet
DGP
Kaluza-Klein
Randall-Sundrum

Current Status


Thank
You