Higher orders of perturbation series in classical mechanics

Abstract: "Perturbation theory in a coupling constant is one of the oldest analytical tool in classical mechanics. It is a common knowledge that its usefulness is limited by the fact that perturbation series for non-integrable systems are only divergent ones. It means that higher-order terms of these series grow quickly making impossible any predictions of long-time behaviour for such systems. The talk is devoted to explicit calculations of higher-order terms of perturbation series for additional integrals in a few generic examples of classical models. It is shown that additional integrals defined beyond the perturbation theory have square-root singularities around all classical periodic orbits. As long-period orbits are dense additional integrals have a dense set of singularities. The form of singularity around a specific periodic orbit is determined by the monodromy matrix of this orbit. An important ingredient of this approach is the calculation of exponentially small deviations of the monodromy matrix trace from the free value. Expanding the function with prescribed singularities into perturbation series permits to find the asymptotic behaviour of higher orders of perturbation series. For the models considered the nth term of perturbation series increases as $n! / \ln(n)n$ when $n \to \infty$ but for certain systems dominant periodic orbits are rare and this asymptotic is achieved in a stepwise manner. For a relatively big interval of n coefficients increase like a power, i.e., perturbation series seems to converge, then for larger n another sizeable step appears and so on. Such behaviour may be a reminiscence of the existence of long-lived states. The obtained expressions are in a good agreement with numerical calculations."

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