Introductory remarks

miércoles, 25 de marzo de 2026 14:07

What are extremal and near-extremal black holes?

In the context of black holes, extremality is a limit where two horizons, inner and outer, coincide. This can happen because the bh is charged or rotating (although there exist other possibilities too), with the mass M reaching a lower bound for given charge Q or spin J:

$$M \geq Q \qquad Q, J \geq 0 \text{ wlog}$$
$$M \geq \frac{J}{M} \qquad G = c = 1$$

- As M decreases for fixed Q or J, the temperature (propto surface gravity) of the black hole decreases, and it becomes zero in the extremal limit.
- The extremal limits $M = Q,\ M = J^2$, have always looked very peculiar, with qualitative differences wrt non-extremal bhs (eg Schwarzschild), and with behavior different than conventional (quantum) systems at very low temperatures -- in particular extremely large degeneracies at zero temperature. Is this because *bhs are really different*, or are we instead missing something important in their physics?
- One theme in these lectures is that bhs near extremality (ie at extremely low temperatures) *do behave as conventional quantum systems*. Although our discussion won't cover the bh unitarity puzzles, these results give us confidence that bhs comply with the laws of QM.
- Then it shouldn't be too surprising that, at very low temperatures, black holes exhibit large quantum effects -- systems in cryo-labs are interesting precisely because of that!

Not so fast! Very large Schwarzschild bhs also have very low temperatures, but they don't exhibit these large quantum effects!

- What's more surprising is that we can treat these large quantum effects on black holes (quantum gravity!) with good control, and without needing a UV quantum theory of gravity -- not any string theory or holography, although these fit in well in the discussion. *Many of the lessons we'll learn have universal validity* -- eg the Schwarzian theory we'll derive is the universal low-energy effective theory near extremality.
- Near-extremal black holes are a theoretical lab for quantum gravity at low temperatures.
- One very important lesson will be that

  the classical extremal black hole of GR textbooks is an artifact of the wrong order of limits.

  Sufficiently close to extremality, black holes are fully quantum objects: There's a sector of near-zero-temperature quantum fluctuations that are strongly coupled, dominating the physics (eg the partition function), which must be fully quantized. This is what JT gravity and the Schwarzian theory do for us.

  The *breakdown of the semiclassical approximation is parametrically large*, not just in subleading terms.

  However, don't throw to the wastebin all you've learned about classical extremal black holes just yet -- instead, revisit it more carefully.

Things we won't do:

- BPS STATES & SUSY: Extremal charged BHs feature prominently in supergravity and string theory (eg microstate counting), but we won't discuss any susy. We will focus on purely bosonic, non-(near-)supersymmetric bhs. Susy bhs are very important but have significant qualitative differences and require separate treatment.
- ROTATION: For technical simplicity we'll focus mainly on the charged RN case. Rotating bhs

(Kerr) are technically harder because of less symmetry (axial U(1) vs spherical SO(3)). Many conclusions from near-extremal RN will extend to near-extremal Kerr, but not all. Besides technical complication:

-there are differences in superradiance and (in)stability (harder to suppress in Kerr)

-near-horizon geometry in Kerr is more complicated -- less symmetric

-less symmetry implies less control over quantum effects -- won't be one-loop-exact

- p-BRANES: We won't do (near-)extremal black p-branes with p>0. Some of what we'll do applies to some near-extremal black strings (p=1), but in general, and especially when p>1, these are different animals, especially when p>1 (zero Bekenstein-Hawking entropy at extremality, different near-horizon geometry, no Schwarzian sector)
- AdS/CMT: Near-extremal bhs appear in condensed matter holography as duals to strongly coupled quantum matter at low T. Very interesting but we won't have time to discuss it.
- ASTRO-PH: Any astrophysical applications? Kerr BHs are thought to exist with spin parameter a/M ~ 0.99 (measured for some AGN). Is this close enough for the effects we'll discuss? We'll examine this -- but don't hold your breath.

What we'll do:

Recent advances: the quantum Schwarzian and its consequences for near-extremal bhs; Aretakis instabilities; tidal singularities at the horizon — all discovered/clarified in the last ~10 years. Our plan is to cover all of these.

Roadmap of the lectures

- Lecture 1: **RN solution and near-extremality**. Basic features, thermodynamics, near-extremal puzzles, near-horizon geometries.
- Lecture 2: **Effective 2d and 1d theories and quantization**: JT gravity, Schwarzian theory, one-loop partition function near extremality.
- Lecture 3: **Interaction with radiation:** emission and absorption
- Lecture 4: **Classical extremal black holes reloaded**. Aretakis instability. Tidal singularities of extremal horizons.

Notation:

Technical detail

Important result

Einstein-Maxwell theory

miércoles, 25 de marzo de 2026 15:28

Einstein - Maxwell Theory

$G = c = 1$

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - F_{\mu\nu} F^{\mu\nu} \right) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Geometric units of charge: $Q_{geom} = \sqrt{\frac{G}{4\pi c^4}}\, Q_{SI}$

$\downarrow$ units of length

Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T^{EM}_{\mu\nu} = 2 \left(F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} F^2 \right)$$

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} F^{\mu\nu} \right) = 0$$

Schwarzschild: $ds^2 = -f\, dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2$

$$f = 1 - \frac{2M}{r}$$

When charged, $T^{EM}_{\mu\nu} \neq 0$ will gravitate

Estimate: $A_t \sim Q/r \quad F_{tr} \sim Q/r^2, \quad T^{EM}_{\mu\nu} \sim \frac{Q^2}{r^4}$

Einstein: $\partial^2 g_{\mu\nu} \sim T_{\mu\nu}$

$\Rightarrow \delta g^{EM}_{\mu\nu} \sim \frac{Q^2}{r^2}$: contribution to metric (gravitational potential) from EM energy. We'll verify this

Study spherically-symmetric fields

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$$ds^2 = g_{ab}(x^c)\, dx^a dx^b + \varphi^2(x^a)\, d\Omega_2 \qquad a,b = 0,1$$

φ = area-radius (will be 2d dilaton)

(radial gauge not fixed yet)

$$\nabla_\mu F^{\mu\nu} = 0 \Rightarrow \partial_a\left(\sqrt{-g}\,\varphi^2 F^{ab}\right) = 0$$

$$\Rightarrow F^2 = -\frac{2Q^2}{\varphi^4}$$

Can always go to a gauge where

$$g_{ab}\, dx^a dx^b = g_{tt}(x)\, dt^2 + g_{rr}(x)\, dr^2 \qquad x^a = (t,r)$$

$$\sqrt{-g_{(4)}} = \sqrt{-g}\,\varphi^2\,\omega_2$$

$$A = A_t(x)\, dt \qquad F_{tr} = -\partial_r A_t \equiv E(x)$$

$$F^{tr} = \frac{1}{g} F_{tr} = \frac{1}{g} E \Rightarrow F^2 = 2F^{tr}F_{tr} = \frac{2}{g} E^2$$

$$g = \det g_{ab} = g_{tt}\, g_{rr} = \frac{1}{g^{tt} g^{rr}}$$

$$\nabla_\mu F^{\mu t} = 0 \Rightarrow \partial_r\left(\frac{1}{\sqrt{-g}}\varphi^2 E\right) = 0 \Rightarrow E = \frac{Q}{\varphi^2}\sqrt{-g}$$

$$\Rightarrow F^2 = -\frac{2Q^2}{\varphi^4}$$

Gauss: $\quad$ charge $= \frac{1}{4\pi}\int_{S^2} *F = Q$

Metric:

$$R^{(4)} = \underbrace{R}_{2D} - \frac{4}{\varphi} \nabla^2 \varphi - 2(\nabla \log \varphi)^2 + \frac{2}{\varphi^2}$$

$$\sqrt{-g_{(4)}}\, R^{(4)} = \omega_2 \sqrt{-g} \left(\varphi^2 R - 4\varphi \nabla^2 \varphi - 2(\nabla \varphi)^2 + 2 \right)$$

$$= \omega_2 \sqrt{-g} \left(\varphi^2 R + 2(\nabla\varphi)^2 + 2 - 4 \underbrace{\nabla (\varphi \nabla \varphi)}_{\text{Tot. der}} \right)$$

$$\int \omega_2 = 4\pi$$

$$I = \frac{1}{4} \int d^2x \sqrt{-g} \left(\varphi^2 R + 2(\nabla\varphi)^2 + 2 - 2\frac{Q^2}{\varphi^2} \right) + \text{bdry Terms}$$

The $(\nabla\varphi)^2$ Term can be eliminated by a Weyl Transformation: $g_{ab} = \frac{r_0}{\varphi} \bar{g}_{ab}$ $\quad r_0$: fiducial length scale (To be chosen)

$$\sqrt{-g} = \frac{r_0}{\varphi} \sqrt{-\bar{g}}$$

$$\sqrt{-g}(\nabla\varphi)^2 = \sqrt{-\bar{g}}(\bar{\nabla}\varphi)^2$$

$$\sqrt{-g} R = \sqrt{-\bar{g}} \left(\bar{R} + \frac{1}{\varphi} \bar{\nabla}^2 \varphi - \frac{1}{\varphi^2} (\bar{\nabla}\varphi)^2 \right]$$

$$\varphi^2 \sqrt{-g} R = \sqrt{-\bar{g}} \left(\varphi^2 \bar{R} + \underbrace{\varphi \bar{\nabla}^2 \varphi}_{\nabla(\bar{\varphi}\bar{\nabla}\varphi) - (\bar{\nabla}\varphi)^2} - (\bar{\nabla}\varphi)^2 \right)$$

$$= \sqrt{-\bar{g}} \left(\varphi^2 \bar{R} - 2(\bar{\nabla}\varphi)^2 \right) + \text{Tot-der}$$

$$\sqrt{-g} \left(\varphi^2 R + 2(\nabla\varphi)^2 \right) = \sqrt{-\bar{g}}\, \varphi^2 \bar{R} + \text{Total derivative}$$

$$I = \frac{1}{4}\int d^2x \sqrt{-\bar g}\left(\varphi^2 \bar R + \frac{2r_0}{\varphi} - \frac{2Q^2}{\varphi^3} r_0\right)$$

$$ds^2_{(4)} = \frac{r_0}{\varphi}\,\bar g_{ab}\,dx^a dx^b + \varphi^2 d\Omega_2$$

Often rename $\phi = \varphi^2$:

$$ds^2_{(4)} = \phi^{-1/2}\,\bar g_{ab}(x^c)\,dx^a dx^b + \phi\, d\Omega_2 \qquad a,b = 0,1$$

$$\phi = \phi(x^a)$$

$$\text{Maxwell} \Rightarrow \quad F^2 = -\frac{2Q^2}{\phi^2}$$

$$I = \frac{1}{4}\int d^2x \sqrt{-\bar g}\,\left(\phi \bar R + 2U_Q(\phi)\right)$$

$$U_Q(\phi) = r_0\left(\frac{1}{\phi^{1/2}} - \frac{Q^2}{\phi^{3/2}}\right)$$

Equations of motion:

$$\frac{\delta I}{\delta \phi} = 0 \quad \Rightarrow \quad \bar R = -2U_Q'(\phi)$$

$$\frac{\delta I}{\delta \bar g^{ab}} = 0 \quad \Rightarrow \quad \nabla_a\nabla_b\phi - \bar g_{ab}\nabla^2\phi + \bar g_{ab}U_Q(\phi) = 0$$

$$\delta\sqrt{-\bar g} = -\frac{1}{2}\sqrt{-\bar g}\,\bar g_{ab}\,\delta\bar g^{ab}\,, \qquad \bar R_{ab} = \frac{1}{2}\bar g_{ab}\bar R \ \ (\text{identically})$$

$$\phi\,\delta\bar R_{ab}\,\bar g^{ab} = -\nabla_a\nabla_b\phi\ \delta\bar g^{ab} + \bar g_{ab}\nabla^2\phi\ \delta\bar g^{ab}$$

Solve for static solutions

$\phi = \phi(r)$

|

$\phi = \phi(r)$

$ds^2_{(2)} = \frac{\phi^{1/2}}{r_0}\left(-f(r)\,dt^2 + \frac{dr^2}{g(r)}\right)$: can always choose this

Fix radial gauge to $\phi(r) = r^2$

Then

tt Einstein equation: $\frac{d}{dr}\left(r(1-g)\right) = \frac{Q^2}{r^2}$

$\Rightarrow g = 1 + \frac{c}{r} + \frac{Q^2}{r^2}$: $C = -2M$ M = mass

rr Einstein equation solved by

$f = c_2\, g$ $c_2 = 1$ by scaling t

$$\Rightarrow \boxed{\begin{aligned} ds^2 &= -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r}} + r^2\, d\Omega_{(2)} \\ A_t &= -\frac{Q}{r} \end{aligned}}$$

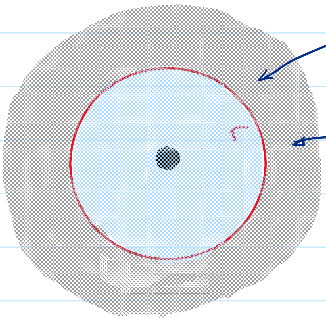
Reissner-Nordstrom solution

25 March 2026 16:49

$-g_{tt} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$: electric field of the charge seems to have a repulsive gravitational effect (even on neutral particles). To understand this, let us first compute the mass of the solution.

M is the total mass (energy) in the entire space, out to infinity. Some of it is due to the energy of the electromagnetic field, which spreads out through all of the spacetime.

At finite r, we can consider the amount of mass/energy that is creating a gravitational attraction on a unit mass placed at that radius



does not gravitate inside r

Energy $= \frac{Q^2}{2r}$

effective mass inside r:

$$M_{eff}(r) = M - \frac{Q^2}{2r}$$

$$-(g_{tt}+1) = \frac{2M_{eff}(r)}{r}$$

"repulsion": There's less energy of the EM field in a sphere of radius r than there's in the entire space, so $M_{eff}(r) < M$

This can be formalized in terms of the "Komar mass"

It's a purely gravitational effect of the energy of the electric field created by a charge, and therefore it affects all particles, even neutral ones.
It is independent of the electric attraction or repulsion that a charged particle would experience (observe that it is $\propto Q^2$ and therefore the same for $Q \gtrless 0$).

<u>Singularity</u>: $r=0$ is a curvature singularity : $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\rho\nu\sigma}R^{\mu\rho\nu\sigma} \xrightarrow{r\to 0} \infty$
It is a timelike singularity whenever $Q \neq 0$, as we will see.

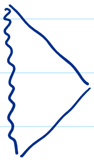
<u>Horizons</u> : look for zeroes of g_{tt} (only if static!) : $f=0 \Leftrightarrow r^2 - 2Mr + Q^2 = 0$

$$r = r_\pm = M \pm \sqrt{M^2 - Q^2}$$

We'll discuss separately the three cases $M < Q$, $M > Q$, $M = Q$

• $\boxed{M < Q}$: no horizon at real r.

$r = 0$: naked singularity (timelike)



• $\boxed{M > Q}$: there are metric singularities at $r = r_\pm$

Eddington-Finkelstein coordinates

$$dr_* = \frac{dr}{f} \qquad r_* = \int \frac{dr}{f}$$

$$v = t + r_* \qquad (v, r, \theta, \phi)$$

$$ds^2 = -f\,dv^2 + 2\,dr\,dv + r^2 d\Omega \quad : \text{ regular at } r = r_\pm$$

The same coordinates are valid on both horizons (we will see, though, that the complete structure is more complicated than it looks now).

$r = r_\pm$: Killing horizons of $\xi = \frac{\partial}{\partial v}$

r_+ : outer horizon

r_- : inner horizon

$$\kappa_\pm = \frac{1}{2} f'(r_\pm) = \pm \frac{\sqrt{M^2 - Q^2}}{r_\pm^2}$$

$\kappa_- < 0$ indicates repulsion : one has accelerate to get to it

$r > r_+$: AF region

$r_- < r < r_+$: r timelike, t spacelike

$0 < r < r_-$: r spacelike, t timelike

$r = 0$: timelike singularity

• $\boxed{M = Q}$: extremal RN black hole

$$1 - \frac{2M}{r} + \frac{M^2}{r^2} = \left(1 - \frac{M}{r}\right)^2 \qquad r_+ = r_- = M = Q \quad : \text{double root}$$

$\kappa_+ = 0$: degenerate horizon

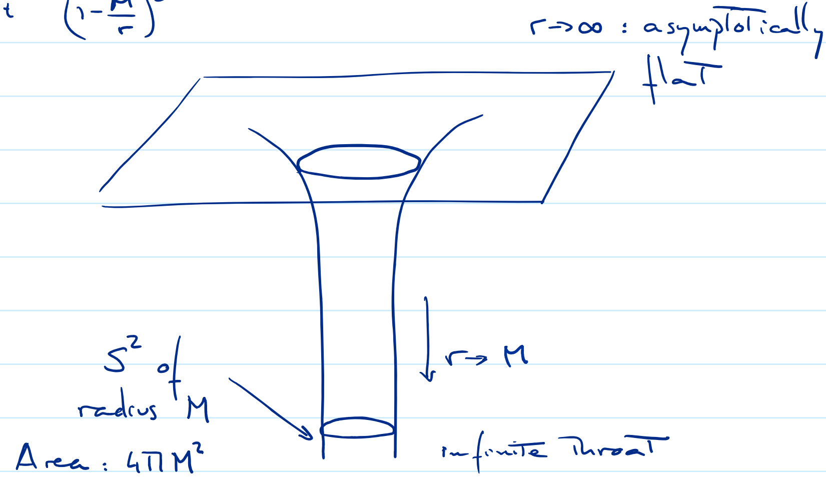
$r = r_+$: event horizon and Cauchy horizon

$r = 0$: Timelike singularity

$r = r_+$: at infinite spacelike distance

$$\int_{r>M}^{r_+ = M} \sqrt{g_{rr}}\, dr = \int_{r>M}^{M} \frac{r\, dr}{r-M} \ : \text{diverges at } r = M$$

$$ds^2\Big|_t = \frac{dr^2}{\left(1-\frac{M}{r}\right)^2} + r^2 d\Omega$$



Thermodynamics

miércoles, 25 de marzo de 2026 16:57

$$T = \frac{1}{4\pi} f'(r_+) = \frac{r_+ - r_-}{4\pi r_+^2} = \frac{\sqrt{M^2 - Q^2}}{2\pi r_+^2}$$

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$

$$M = \frac{r_+ + r_-}{2} \qquad Q = \sqrt{r_+ r_-}$$

$$\Phi_H = -A_t\Big|_H = \frac{Q}{r_+} = \sqrt{\frac{r_-}{r_+}} \quad : \text{electric potential}$$

$$S_{BH} = \frac{A}{4G} = \pi r_+^2$$

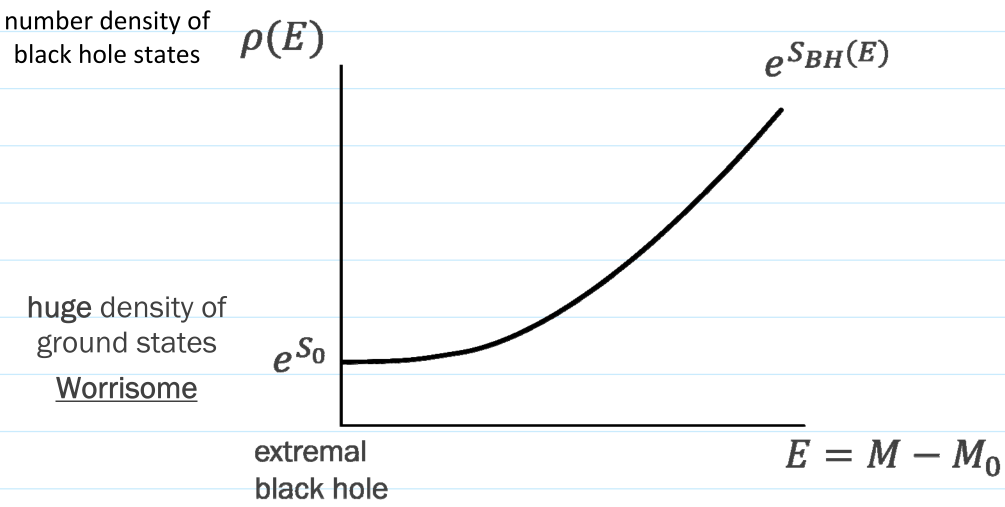
Verify 1^{st} law: $dM = T\, dS + \Phi_H\, dQ$

$$\boxed{\begin{aligned} &\partial_{r_+} M = \tfrac{1}{2} \\ &\partial_{r_+} S = 2\pi r_+ , \quad T\partial_+ S = \tfrac{1}{2}\left(1 - \tfrac{r_-}{r_+}\right) \\ &\partial_{r_+} Q = \tfrac{1}{2}\sqrt{\tfrac{r_-}{r_+}} , \quad \Phi_H \partial_+ Q = \tfrac{1}{2}\tfrac{r_-}{r_+} \\ &\partial_+ M = T\partial_+ S + \Phi_H \partial_+ Q \\ &\partial_- M = T\partial_- S + \Phi_H \partial_- Q \end{aligned}}$$

At extremality $r_+ = r_-$ $\quad M = Q , \quad T = 0 , \quad \Phi_H = 1$

$$S_{ext} = \pi Q^2 \neq 0$$

<u>Puzzle</u>:



<u>Near extremality</u>:

$$M^2 = Q^2 + \varepsilon^2 \qquad \varepsilon \ll Q \qquad M = Q + \frac{1}{2}\frac{\varepsilon^2}{Q}$$

$$r_\pm = Q \pm \varepsilon$$

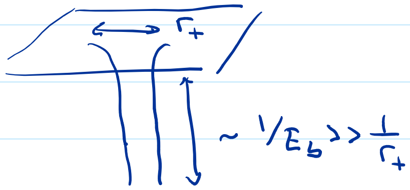
$$T = \frac{\varepsilon}{2\pi Q^2} + O(\varepsilon^2) \Rightarrow \varepsilon = 2\pi Q^2 T$$

$\Rightarrow$ $E = M - M_{ext} = 2\pi^2 Q^3 T^2 + O(T^3)$: energy above extremality

$$\boxed{E = 2\pi^2 \frac{T^2}{E_b}}$$

$E_b = \frac{G}{Q^3} = \left(\frac{l_{pl}}{r_+}\right)^2 \frac{1}{r_+} = \frac{\pi}{S_0}\frac{1}{r_+} \ll \frac{1}{r_+}$: extremely small scale

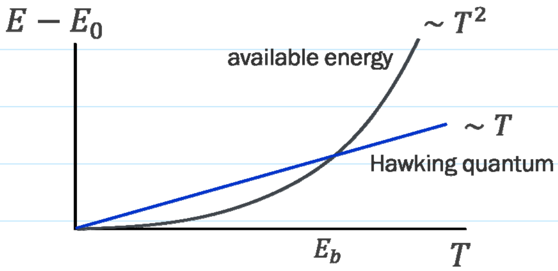
It is due to the very long throat:
when the length $\frac{1}{T} > \frac{1}{E_b}$
there'll be new phenomena.



This is very different from Schwarzschild $M \sim 1/T$.

The result is generically valid for bhs near extremality, with E_b scaling inversely w/ powers of charges.

<u>Another puzzle:</u>



$T \lesssim E_b$: too little energy available to emit a single quantum

Thermodynamics shouldn't apply: emitting a single quantum will be a large change
Looks like semiclassical QFT in curved space shouldn't apply: Hawking radiation won't be thermal

Charge emission: superradiance and pair production

miércoles, 25 de marzo de 2026 18:53

Assume charged field with $q > m$

Charge superradiance:

$$D_\mu D^\mu \phi = 0 \qquad D_\mu = \nabla_\mu - iqA_\mu$$

Electric current

$$J_\mu = iq\left(\phi (D_\mu \phi)^* - \phi^* D_\mu \phi\right)$$

Conserved $\nabla_\mu J^\mu = 0$ when field equation satisfied

Static black hole: $\xi = \partial_t$ generator of horizon

Flux across horizon is

$$F = -\xi^\mu J_\mu$$

Fourier: $\phi(t,x) = \phi_\omega(x)\, e^{-i\omega t}$

$$\Rightarrow F = 2q(\omega + qA_t)\, |\phi_\omega|^2$$

With $\Phi_H = -A_t\big|_H$: electric potential at horizon

Then, if $\boxed{\omega - q\Phi_H < 0 \quad \Rightarrow \quad F < 0}$

A charge wave scattered off the bh w/ $\omega < q\Phi_H$ will emerge with larger amplitude and larger charge than incoming: charge superradiance.

Energy and charge are being extracted from the black hole..

energy and charge are being extracted from the black hole.

For field w/ mass m: $\omega \leq m$, so charge superradiance is possible if $m < q\Phi_H$

At extremality $\Phi_H = 1$ is maximum.

Superradiance only if $q > m$

Classical superradiance: stimulated emission

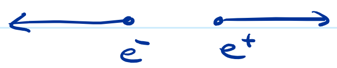
Spontaneous superradiant emission (quantum) is possible if $\omega < q\Phi_H$

only for bosonic fields

Schwinger pair production:

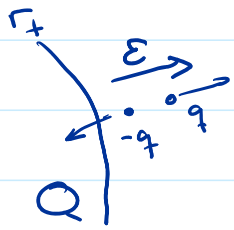
$\mathcal{E}$: electric field strength

$$\Gamma_{e^+e^-} \propto e^{-\frac{\pi m^2}{q\mathcal{E}}}$$



Near the horizon of a black hole

$$\mathcal{E} = \frac{Q}{r_+^2}$$



black hole will discharge

Close to extremality $\mathcal{E} \simeq \frac{1}{Q}$: small for very large bh (Coulomb)

$$\Gamma \sim e^{-\pi \frac{m^2}{q} Q}$$

$$\Gamma \sim e^{-\pi \frac{m^2}{q} Q}$$

This will be important if $Q \lesssim \frac{1}{\pi} \frac{q}{m^2} = Q_d$

$$Q_d = \frac{1}{\pi} \frac{q}{m^2} = \frac{1}{\pi} q \left(\frac{M_{Pl}}{m}\right)^2$$

For an electron $q \sim \frac{1}{10}$ $\quad \frac{m}{M_{Pl}} \sim 10^{-22}$

$$\Rightarrow \quad Q_d \sim 10^{+44} q \quad !$$

The bh mass would be

$$M \sim 10^{43} M_{Pl} \sim 10^{38} g \sim 10^5 M_\odot \quad : \text{supermassive}$$

- A bh smaller than this discharge extremely quickly, because the field at the horizon will be sufficiently large.
- Magnetically charged bhs can be larger since monopole mass will be much larger.
- This effect occurs for bosons and fermions, so it is not the same as superradiance.
- Both are part of one-loop effects of charged fields in the presence of the charged bh, but they're distinct phenomena.

Near-horizon geometries

25 March 2026 19:31

$$M^2 = Q^2 + \lambda^2 \mu$$

$$r_\pm = Q \pm \lambda\sqrt{\mu}$$

We'll take $\lambda \to 0$ keeping μ fixed and rescale coordinates:

$r = Q + \lambda\rho$ $\quad dr = \lambda d\rho$ $\quad$ keep ρ fixed

rescaled near-horizon distances

$t = \tau/\lambda$: redshifted near-horizon time.

Conjugate energies rescaled by λ

In RN:

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2$$

$$f = \frac{(r-r_+)(r-r_-)}{r^2} \to \frac{\lambda^2}{Q^2}(\rho^2 - \mu)$$

$$ds^2 \to \underbrace{-\frac{1}{Q^2}(\rho^2-\mu)\,d\tau^2 + Q^2\frac{d\rho^2}{\rho^2-\mu}}_{ds_2^2} + (Q+\lambda\rho)^2 d\Omega_2$$

We'll want to retain this

$$A_t = -\frac{Q}{r} \to -\frac{1}{Q}(Q-\lambda\rho)$$ (won't use this much, but important for charged fields)

$$\boxed{ds_2^2 = -\frac{1}{Q^2}(\rho^2-\mu)\,d\tau^2 + Q^2\frac{d\rho^2}{\rho^2-\mu}}$$

AdS_2 w/ radius Q for all values of μ

Constant curvature $R = -\frac{2}{Q^2} = -\frac{2}{L^2}$

$L = Q = AdS_2$ radius

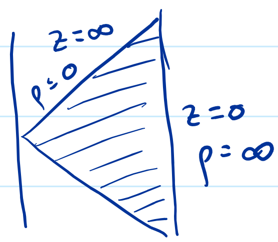
$AdS_2 \times S^2$: Bertotti-Robinson universe

(S^2 radius $= Q$)

$\mu = 0$: extremal $\quad ds_2^2 = -\frac{\rho^2}{Q^2}d\tau^2 + \frac{Q^2}{\rho^2}d\rho^2 \qquad \rho = \frac{Q^2}{z}$

$$= Q^2\,\frac{-d\tau^2 + dz^2}{z^2} \quad : \text{Poincare } AdS_2$$

AdS_2



When $\mu \neq 0$ it can be rescaled to $|\mu| = 1$

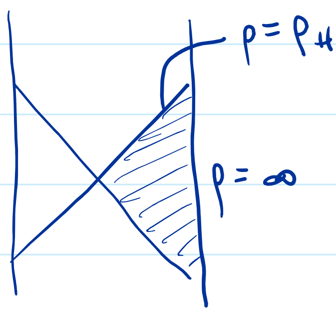
$\rho \to \sqrt{|\mu|}\,\rho \qquad \mu \to \operatorname{sgn}(\mu)$

$\tau \to \frac{1}{\sqrt{|\mu|}}\,\tau$

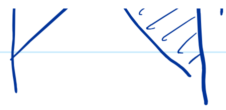
$\mu > 0$: near-extremal $\quad \rho_H = \sqrt{\mu}$

Rindler-AdS_2 aka "Thermal AdS_2"

Finite Temperature:



Finite temperature:

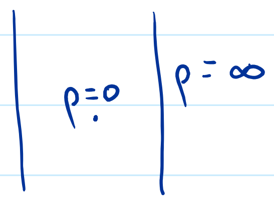


$$\varepsilon^2 = \lambda\sqrt{\mu}$$

$$T = \frac{\lambda\sqrt{\mu}}{2\pi Q^2} \xrightarrow[t\to\tau]{\text{rescale}} T = \frac{\sqrt{\mu}}{2\pi Q^2} \qquad \left(\begin{array}{l}\text{infinitesimal}\\ \text{from outside}\\ \text{throat}\end{array}\right)$$

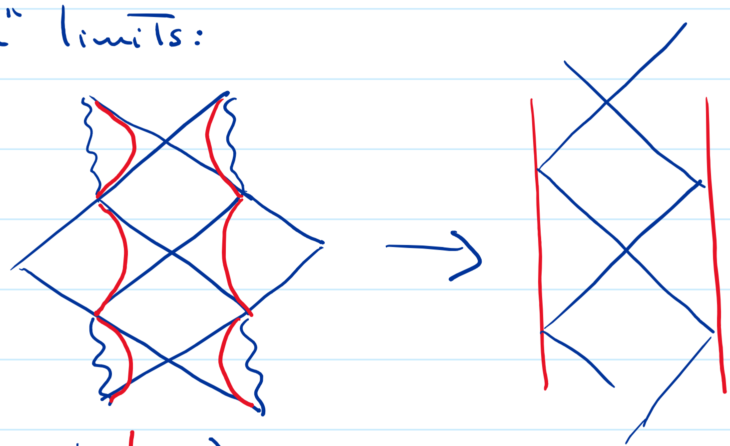
$\mu < 0$: $Q^2 > M^2$! no horizon

$$ds^2 = -\frac{1}{Q^2}\left(\rho^2 + |\mu|\right) d\tau^2 + Q^2 \frac{d\rho^2}{\rho^2 + |\mu|} \quad : \text{global } AdS_2$$

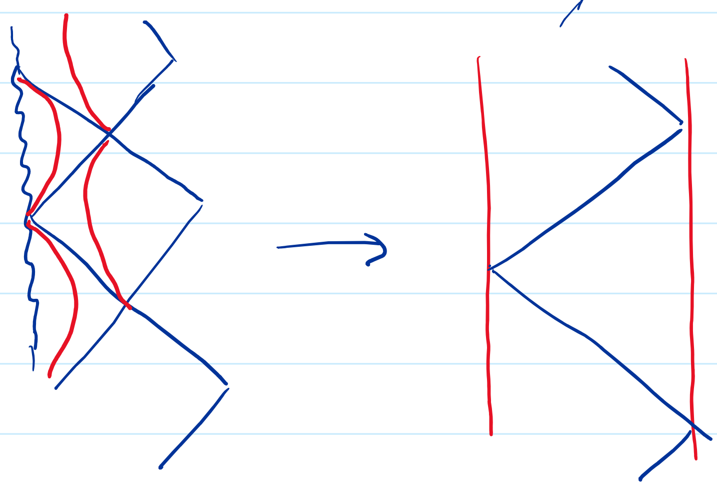


"Near-horizon" limits:

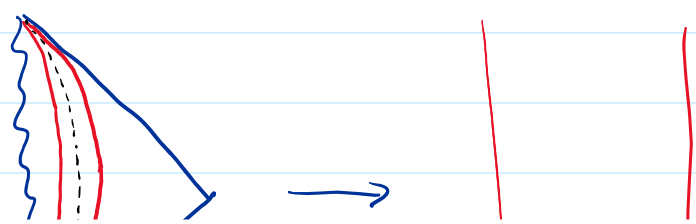
Non-extremal

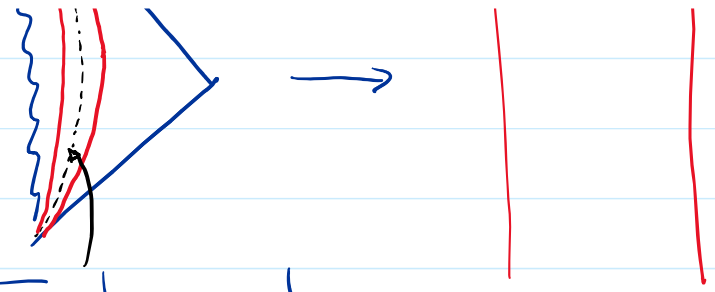


Extremal



Horizonless





centered around

$r = Q/M \Rightarrow \rho = 0$

$\hookrightarrow$ where $f' = 0$: "zero force"

Locally all these AdS_2 metrics are equivalent under diffeos.

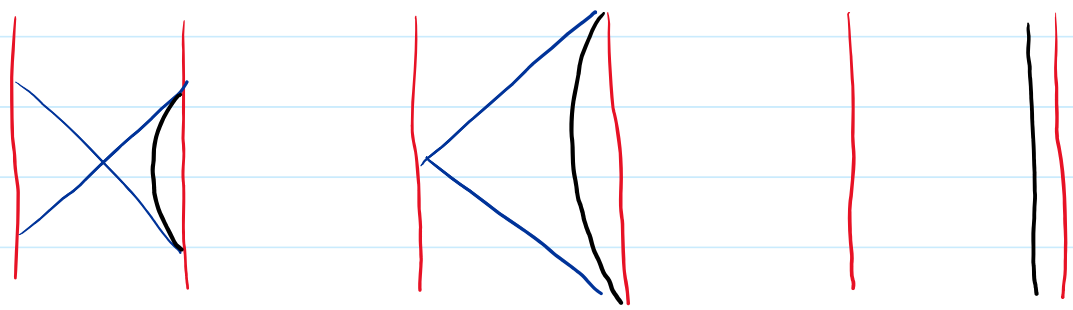
But the first correction in λ in the size of S^2 distinguishes them:

We had $\varphi^2 \, d\Omega^2 \Rightarrow \varphi = Q + \lambda\rho$

$= \varphi_0 + \lambda\, \Phi(\rho)$: dilaton

$\Phi(\rho) = \rho$ is formally the same in the three cases, but the lines (surfaces) of constant ρ are different in each case:

$$\Phi = \rho = \text{const}$$



The non-zero dilaton breaks the diff equivalence

The non-zero dilaton breaks the diff equivalence between these solutions, making them physically different.