References

domingo, 29 de marzo de 2026 10:55

A few references to work that we discuss in the first lectures (there are many more -- I apologize to the authors that I'm not referencing):

Ilieasiu+Turiaci "The statistical mechanics of near extremal black holes"
JHEP 05 (2021) 145
e-Print: 2003.02860 [hep-th]

Brown+Iliesiu+Penington+Usatyuk "The evaporation of charged black holes"
JHEP 01 (2026) 109
e-Print: 2411.03447 [hep-th]

RE "Quantum cross-section of near-extremal black holes"
JHEP 04 (2025) 122
e-Print: 2501.17470 [hep-th]

For reviews:

Mertens+Turiaci "Solvable models of quantum black holes: a review on Jackiw–Teitelboim gravity"
Living Rev.Rel. 26 (2023) 1, 4
e-Print: 2210.10846 [hep-th]

Turiaci "New insights on near-extremal black holes"
e-Print: 2307.10423 [hep-th]

Near-extremal black holes and JT gravity

miércoles, 25 de marzo de 2026 20:15

Return to s-wave reduction of Einstein-Maxwell
(recall this is a consistent classical truncation,
but for quantum fluctuations one must do better)

$$I = \frac{1}{4}\int d^2x\,\sqrt{-g}\,\left(\phi R + 2U_Q(\phi)\right)$$ (dropping bars $\bar{g}_{ab} \to g_{ab}$)

$$U_Q(\phi) = r_0\left(\frac{1}{\phi^{1/2}} - \frac{Q^2}{\phi^{3/2}}\right)$$ We'll choose $r_0 = Q$

Now expand around extremality:

$$\phi = Q^2 + \Phi(x) \qquad |\Phi| \ll Q^2$$

$$\phi R = Q^2 R + \Phi(x) R$$

$$2U_Q(\phi) = \frac{2}{Q^2}\Phi = \frac{2}{L^2}\Phi \qquad L \equiv Q$$

$$I = \frac{1}{4}\int d^2x\,\sqrt{-g}\left[\underbrace{Q^2 R}_{\text{Topological}} + \Phi(x)\left(R + \frac{2}{L^2}\right) + O(\Phi^2/Q^2)\right]$$

Topological $\int R = 2\pi\chi$ χ: Euler number

Jackiw-Teitelboim gravity:

$$I_{JT} = \frac{1}{4}\int d^2x\,\sqrt{-g}\,\Phi(x)\,(R+2) \qquad L = 1$$

Must add Gibbons-Hawking-York term to have

Must add Gibbons-Hawking-York term to have a well-defined variational problem.

- The GHY term for the effective 2D reduced theory can be derived from the 4D Einstein-Maxwell theory by matching the $AdS_2 \times S^2$ throat to the asymptotically flat "far" region that describes an extremal RN solution (since "far" the deviation from extremality is infinitesimal).

  This would give a topological boundary term

  $$\frac{1}{2}\int_{\partial M} dx \sqrt{-h}\, \Phi_0 K$$

  which would add the bdry contribution to the Euler characteristic.

- Instead, we obtain it in the usual manner for obtaining a well-defined variational pple w/ Dirichlet bc's. This yields

$$I_{JT} = \frac{1}{4}\int_M d^2x \sqrt{-g}\, \Phi(x)\,(R+2) \qquad \Phi(x)\Big|_{\partial M} = \frac{\Phi_b}{\varepsilon}$$

$$-\frac{1}{2}\int_{\partial M} dx \sqrt{-h}\, \frac{\Phi_b}{\varepsilon}(K-1) \qquad \text{so } \phi\big|_b = Q^2 + \frac{\Phi_b}{\varepsilon}$$

GHY

counterterm

We've added a <u>boundary counterterm</u>, which

We've added a <u>boundary counterterm</u>, which is local and invariant of the intrinsic bdry geometry (unlike GHY) so it doesn't affect the eoms for Dirichlet bc's. It cancels divergences from the infinite volume of AdS_2 to yield finite action, as we will see explicitly.

Equations

$R = -2 \quad : \quad AdS_2$

$$g_{ab} \Box \Phi - \nabla_a \nabla_b \Phi - g_{ab} \Phi = 0 \qquad (T^{\Phi}_{ij} = 0)$$

This can be written as $\Box \Phi = 2\Phi$ (Trace)

$$\nabla_a \nabla_b \Phi = g_{ab} \Phi$$

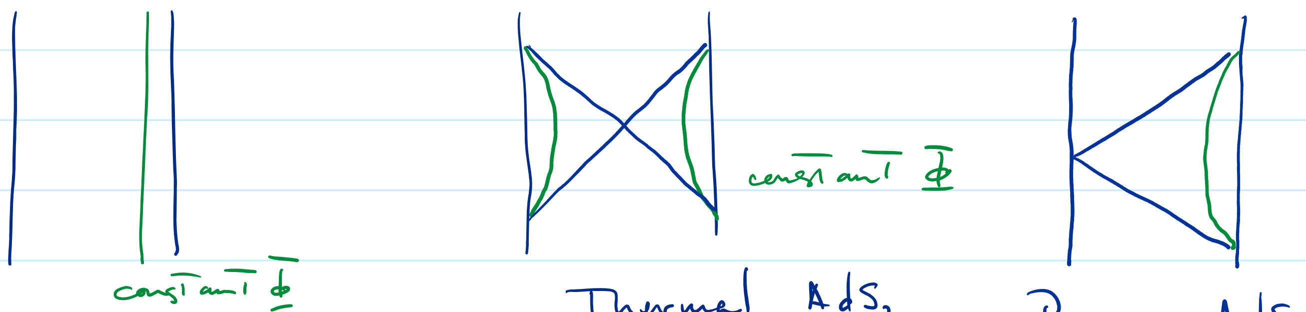
Solutions

$$ds^2 = (\rho^2 + \kappa) dt^2 + \frac{d\rho^2}{\rho^2 + \kappa}$$

$\kappa = 0, \pm 1$

$\Phi = a\rho$ (constant a) for all κ

but notice that ρ is different in each case:



global AdS_2 Thermal AdS_2 Poincare AdS_2

$\Phi \to \infty$ at the boundary $\rho \to \infty$ so we introduce a cutoff.

When we do this, we can consider fluctuations of the boundary geometry.

Near-AdS_2 and Schwarzian dynamics

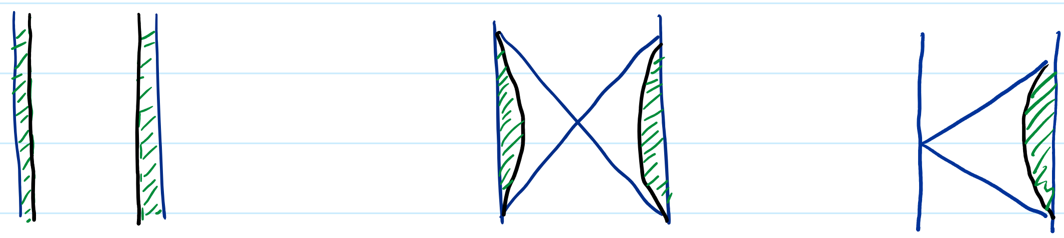
Introduce a cutoff at $\Phi =$ const $\gg 1$

$r = 1/\varepsilon$

"renormalized"

$\phi\big|_{\text{cutoff}} = \phi_b / \varepsilon$

This gives near-AdS_2 geometries which are inequivalent and which can have a dynamical boundary:



The dynamics of near-AdS_2 JT gravity is the dynamics of a "particle" (or 2) moving near the boundary of AdS_2:

Consider the solution in Poincaré AdS_2, $k=0$ $\rho = 1/2$

$$ds^2 = \frac{dt^2 + dz^2}{z^2} \qquad \Phi = 1/z$$

(actually the most general solution is $\Phi = \frac{b(t)}{z} + a(t)$)

(actually the most general solution is $\Phi = \frac{b(t)}{z} + a(t)$)

We introduce a boundary. This could be at $z = \varepsilon$, $t = T$

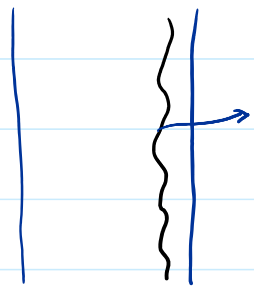
induced metric $ds^2\big|_b = h_{TT}\, dT^2 = \frac{1}{\varepsilon^2} dT^2 \qquad \varepsilon \ll 1$

T = renormalized proper time at bdry

More generally we take it to be the curve $t(T), z(T)$

with $h_{TT} = \frac{1}{\varepsilon^2}$

$$= \frac{1}{z^2(T)}\left(\dot{t}(T) + \dot{z}^2(T)\right) \Rightarrow z(T) = \varepsilon\, \dot{t}(T) + O(\varepsilon^3)$$



proper time T (renormalized)

boundary trajectory $t(T)$, $z(T)$

Compute extrinsic curvature of boundary

$$K = 1 + \varepsilon^2 \{t, T\} + O(\varepsilon^4)$$

↳ will be subtracted by counterterm

$$\{t, T\} \equiv \frac{\dddot{t}}{\dot{t}} - \frac{3}{2}\frac{\ddot{t}^2}{\dot{t}^2} \qquad \text{Schwarzian derivative}$$

Symmetric under $t \to \frac{at + b}{ct + d} \qquad SL(2, \mathbb{R})$

$$\left[\text{Then } t(T) = \frac{aT + b}{cT + d} \iff \{t, T\} = 0\right]$$

Bulk viewpoint: AdS_2 has isometry group $SL(2,\mathbb{R})$

Diffeos in the bulk: There are normalizable diffeos generated by non-normalizable vectors $x^a \to x^a + \xi^a(x)$ hence they are large diffeos that are "dynamical." *

At the boundary, there generate reparametrizations of the boundary. They correspond to the Schwarzian modes

*They are s-waves and hence not dynamically propagating, but they yield non-trivial equations with distinct physical solutions. In the path integral, these modes must be integrated, while the $SL(2,\mathbb{R})$ must be divided: $\int \frac{\mathrm{Diff}(S^1)}{\mathrm{Vol}(SL(2,\mathbb{R}))}$

Gravitational action: bulk term vanishes: $R+2=0$

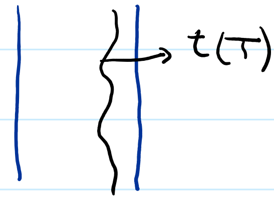
In the quantum theory one can also set $R+2=0$, by integrating Φ in the path integral along a contour in the imaginary time direction, which yields $\delta(R+2)$

$$I_{JT}\Big|_{\text{on-shell}} = \frac{1}{8\pi G}\int_{\partial M} d\tau \sqrt{|h|}\, \Phi\,(K-1)$$

$$= \frac{\phi_b}{8\pi G} \int_{\partial M} dT \, \{t, T\}$$: Schwarzian action for The boundary particle

This is finite when $\varepsilon \to 0$

$t(T)$: diffeomorphisms of boundary = Trajectory of bdry particle



In 4D, it can be viewed as a collective motion of The mouth of The Throat, as glued To The far region

To do Thermodynamics, we go To Euclidean Time:

$T \to -i\tau$ periodic $\tau \sim \tau + \beta$

$$I_{JT} = -\frac{\phi_b}{8\pi G} \int_0^\beta d\tau \{t, \tau\}$$

Equations of motion obtained by varying $t \to t + \delta t$

use $\int d\tau \, \delta \{t, \tau\} = -\int d\tau \frac{\frac{d}{d\tau}\{t, \tau\}}{\dot{t}} \delta t$

$\Rightarrow \frac{d}{d\tau}\{t, \tau\} = 0$: $\{t, \tau\}$ is conserved (by $SL(2,\mathbb{R})$ symmetry) (by $SL(2,\mathbb{R})$ symmetry)

Solved by $t(\tau) = \tan \frac{\pi\tau}{\beta}$: Thermal AdS_2

If we now compute the action we get

$$I_{ce}^{Sch}(\beta) = -\frac{1}{4\pi G}\frac{\pi^2}{\beta}\phi_b$$

$$Z[\beta] = e^{-\beta F} \approx e^{-I_{Euc}}$$

$$= e^{-I_{ce}^{Sch}[\beta]}$$: leading semiclassical saddle point result

$\Rightarrow \beta F \approx I_E \Rightarrow F \propto T^2$ not $F \propto T$ ie not CFT_1

$$E = -\frac{\partial \log Z}{\partial \beta} \approx \partial_\beta I_{Euc} = \frac{\pi}{4}\phi_b T^2$$: as we found in near-extremal RN

$$S = -(\beta\partial_\beta - 1)\log Z \approx (\beta\partial_\beta - 1) I_{Euc}$$

$$= \frac{\pi}{2}\phi_b T$$

Quantization of Schwarzian theory

sábado, 28 de marzo de 2026 23:06

We have identified a mode in the geometry that at low temperatures has very small action:

$$I_{Schw} = -\frac{\phi_b}{8\pi G}\int_0^\beta d\tau \{t,\tau\} \qquad \tau = \beta\hat{\tau}$$

$$\{t,\tau\} = \frac{1}{\beta^2}\{t,\hat{\tau}\}$$

$$= -\frac{\phi_b}{8\pi G\beta}\underbrace{\int_0^1 d\hat{\tau}\,\{t,\hat{\tau}\}}_{\text{dimensionless}}$$

$$\sim \frac{\phi_b}{G} T \sim \frac{T}{E_b} \qquad T = 1/\beta$$

ie it becomes very light and strongly coupled: it must be fully quantized. But if we want to quantize this syetsm, the consistency of the classical truncation is not enough -- there may be other modes that are equally light, or which couple to the light modes and can't be truncated.

There are indeed other light modes: rotational modes of the S^2 are clearly very light. It is indeed easy to see that they enter at the same scale as the Schwarzian: take the Kerr-Newman solution close to extremality, and expand the energy for small angular momentum:

$$E = M(J) - M(J=0) \sim J^2/Q^3 \sim J^2 E_b$$

This is of course not exactly valid because for small J the spin must be quantized, but we can already see that the energy scale involved when J=O(1) is the same as the E_b we found.

This mode can be quantized as a two-dimensional SO(3) gauge theory coupled to JT gravity. When we work in grand-canonical ensemble with fixed angular velocity, this describes small quantum fluctuations of the spin of the black hole.

One can also consider fluctuations that add (or subtract) charge to the black hole. They also enter at the same scale, and can be quantized as a 2d U(1) gauge theory. Again, if we fix the electric potential, then the charge can undergo quantum fluctuations described by this mode.

These 2d gauge modes add to the quantum Schwarzian corrections to the partition function, with the same sign. We will not consider them here -- in this sense, we can think that we're working in a canonical ensemble of fixed temperature, and fixed spin and charge.

There are other fluctuation modes of the geometry, which break spherical symmetry. In the full 4d geometry, in general these fields couple significantly to the light modes we're considering. One might think that non-zero KK modes of the S^2 are massive and therefore can be neglected. But this argument is not valid, since the S^2 radius is the same as the AdS2 radius, and we're considering dynamics on that scale. However, it is relatively easy to see that the couplings of the massive KK modes to the effective 2d dilaton gravity are suppressed by inversepowers of the black hole size, 1/(S_0)^#. Therefore we can safely freeze them when the black hole is large.

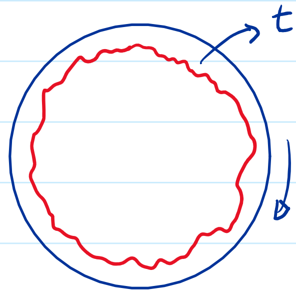
Therefore in the following we will quantize the black hole by considering only the mode that describes fluctuations in how the mouth of the throat connects to the far region.

One could attempt to compute the one-loop partition function in the full 4d geometry by analyzing the spectrum of the spin-2 fluctuation operator (the Lichnerowicz operator) for off-shell metric fluctuations h_{mn}. For near-extremal black holes, the near-horizon analysis suggests a family of nearly gapless modes, and indeed an explicit analysis shows that there are eigenvalues that scale linearly with the temperature T. These light off-shell modes of the full geometry, which are spherically symmetric modes of h_{mn} (hence necessarily off-shell!) correspond to the the Schwarzian modes of the throat region [Kolanowski+Marolf+Rakic+Rangamani+Turiaci 2024].

Quantization of the Schwarzian action

The Schwarzian action can be exactly quantized (Stanford + Witten)

We'll do perturbative quantization around the thermal solution:



$$\tan \frac{\pi}{\beta}\left(\tau + \varepsilon(\tau)\right) \quad \text{with small } \varepsilon(\tau)$$

$$' = d/d\tau$$

$$I_{Sch} = -\frac{\phi_b}{8\pi G} \int_0^\beta d\tau \left\{ f(\tau), \tau \right\} \qquad t \to f \text{ henceforth}$$

expand for $f(\tau) = \tan \frac{\pi}{\beta}\left(\tau + \varepsilon(\tau)\right)$ to quadratic order in ε

Use the Schwarzian chain rule $\{f(\phi),\tau\} = \{f,\phi\}\phi'^2 + \{\phi,\tau\}$

w/ $f(\phi) = \tan\frac{\pi}{\beta}\phi$ and $\phi(\tau) = \tau + \varepsilon(\tau)$

$$\{f,\tau\} = \frac{2\pi^2}{\beta^2}\phi'^2 + \{\phi,\tau\}$$

$$\phi'^2 = 1 + 2\varepsilon' + \varepsilon'^2 + \cdots$$

$$\{\phi,\tau\} = \frac{\varepsilon'''}{1+\varepsilon'} - \frac{3(\varepsilon'')^2}{2(1+\varepsilon')^2} = \varepsilon''' - \varepsilon'''\varepsilon' - \frac{3}{2}\varepsilon''^2 + O(\varepsilon^3)$$

$$\mathrm{Sch}\left(\tan\frac{\pi}{\beta}(\tau+\varepsilon(\tau))\right) = \frac{2\pi^2}{\beta^2} + \frac{2\pi^2}{\beta^2}\varepsilon'^2 + 2\frac{2\pi^2}{\beta^2}\varepsilon' + \varepsilon''' - \varepsilon'\varepsilon''' - \frac{3}{2}\varepsilon''^2 + O(\varepsilon^3)$$

Periodic ε: Terms linear in ε will cancel when integrated.

Use also $-\varepsilon'''\varepsilon' = \varepsilon''^2 - \underbrace{(\varepsilon''\varepsilon')'}_{\text{Tot-der : cancels}}$

Then

$$I = -\frac{\phi_b}{8\pi G}\int_0^\beta d\tau\left(\frac{2\pi^2}{\beta^2} + \frac{2\pi^2}{\beta^2}\varepsilon'^2 - \frac{1}{2}\varepsilon''^2\right)$$

Now expand ε in Fourier modes:

$$\varepsilon(\tau) = \frac{\beta}{2\pi}\sum_{n>1}\left(e^{-i\frac{2\pi}{\beta}n\tau}\varepsilon_n + e^{i\frac{2\pi}{\beta}n\tau}\varepsilon_n^*\right) \qquad \varepsilon_n^* = \varepsilon_{-n}$$

real ε

Recall the Schwarzian corresponds to reparametrizations

$\tau \to t(\tau)$, ie $\mathrm{Diff}(S^1)$

but $SL(2,\mathbb{R})$ are exact symmetries (bulk isometries).

We exclude $n=0,\pm1$ because these modes would generate infinitesimal $SL(2,\mathbb{R})$ transformations, which are symmetries of the system. Including them would be overcounting.

$$\boxed{\begin{aligned} &\text{Use} \quad \int_0^\beta d\tau\, e^{-i\frac{2\pi}{\beta}(n+m)\tau} = \delta_{nm} \\ &\text{so that} \quad \int_0^\beta d\tau\, \varepsilon'^2 = 2\beta \sum_{n\geq 1} n^2 |\varepsilon_n|^2 \\ &\qquad\qquad \int_0^\beta d\tau\, \varepsilon''^2 = \frac{8\pi^2}{\beta} \sum_{n\geq 1} n^4 |\varepsilon_n|^2 \end{aligned}}$$

Then

$$I = -\frac{1}{4\pi G}\frac{\pi^2}{\beta}\phi_b + \frac{1}{4\pi G}\phi_b\frac{2\pi^2}{\beta}\sum_{n\geq 1} n^2(n^2-1)|\varepsilon_n|^2$$

To perform the one-loop integral we need the integration measure. This can be done in several different ways very rigorously. Here we'll use several shortcuts.
The first one allows us to derive the β-dependent part of the calculation w/out knowing the precise measure.

We write

$$Z[\beta] = e^{S_0 + \frac{\phi_b}{4\pi G}\frac{\pi^2}{\beta}} \int \prod_{n=2}^{\infty} f_n\, d^2(\varepsilon_n) \exp\left[-\frac{\phi_b}{4\pi G}\frac{2\pi^2}{\beta} g_n |\varepsilon_n|^2\right]$$

with f_n and g_n non-negative functions of n we won't use for now.
The integral over ε_n is gaussian:

$$\int_{-\infty}^{+\infty} dx\, e^{-cx^2} = \sqrt{\frac{\pi}{c}}$$

Then, over $\varepsilon_n \varepsilon_n^*$ we'll find a result of the form

$$\prod_{n=2}^{\infty} h_n \beta \frac{2G}{\pi\phi_b}$$ with h_n another function we won't use

The most important part for us is

$\prod^{\infty} \beta$ (The factor $\frac{G}{\cdot}$ can be easily restored)

$$\prod_{n=2}^{\infty} \beta \qquad \text{(The factor } \frac{G}{\phi_b} \text{ can be easily restored)}$$

which we treat w/ ζ-function regularization.

Write $\log \prod_{n=2}^{\infty} \beta = \log\beta \sum_{n=2}^{\infty} 1$

Now

$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ convergent for $s>1$. But can be analytically continued for $s<1$

$$\sum_{n=1}^{\infty} 1 = \zeta(0) = -\frac{1}{2} \quad \Rightarrow \quad \sum_{n=2}^{\infty} 1 = \zeta(0) - 1 = -3/2$$

$$\Rightarrow \prod_{n=2}^{\infty} \beta = \exp\left(-\frac{3}{2}\log\beta\right) = \beta^{-3/2}$$

Then

$$Z[\beta] = A \, e^{S_0 + \frac{\phi_b}{4\pi G}\frac{\pi^2}{\beta} - \frac{3}{2}\log G\frac{\beta}{\phi_b}}$$

where A contains the (β-independent) one-loop determinant of all the modes we are not considering (and which we assume are somehow regularized), times a numerical factor from the $\prod_{n=2}^{\infty} h_n$.

We can compute this, regularizing it so we can show it is finite.

[Can jump to $\boxed{}$ below]

For computing the complete integral of the Schwarzian we need the integration measure over $\frac{\mathrm{Diff}(S^1)}{\mathrm{Vol}\, SL(2,\mathbb{R})}$

where we don't sum over the bulk isometries [illegible] $\dfrac{[\text{illegible}]}{\mathrm{Vol}\, SL(2,\mathbb{R})}$

where we don't sum over the bulk isometries that are generated by the modes $n=0,\pm 1$

The ultralocal measure

$\mathcal{D}\varepsilon = \prod_{n=0}^{\infty} d\varepsilon_n \, d\varepsilon_n^*$ would not be correct.

Simply excluding $n=0,\pm1$ gives the correct temperature dependence, but is not otherwise the right procedure.

To obtain the correct measure we take another shortcut.

We can define the measure from a quadratic norm on fluctuations

$$\|\delta\varepsilon\|^2 = \int_0^\beta d\tau \, \delta\varepsilon(\tau)\, \mathcal{O}\, \delta\varepsilon(\tau)$$

with $\mathcal{O}$ an operator that satisfies:

- local
- positive on physical modes
- its kernel is exactly the $SL(2,\mathbb{R})$ modes

$$1, \; e^{\pm i \frac{2\pi}{\beta}\tau} \qquad (\text{ie } n=0,\pm1)$$

The simplest such operator is

$$\mathcal{O} = \partial_\tau\left(\partial_\tau^2 + \frac{\beta^2}{4\pi^2}\right)$$

This induces, up to an overall constant,

$$\mathcal{D}\varepsilon = \prod_{n>1} 4\pi n (n^2-1)\, d^2\varepsilon_n$$

In more detail:

The Schwarzian Theory can be viewed as the coadjoint orbit of Diff(S^1). It carries a natural symplectic form, ie the Kirillov-Kostant form on Diff$(S^1)/SL(2,\mathbb{R})$

The operator $\mathcal{O} = \partial_\tau\left(\partial_\tau^2 + (\beta/2\pi)^2\right)$ is the linearized Kirillov-Kostant symplectic form on $SL(2,\mathbb{R})$, so the measure we wrote is its associated Liouville measure.

What we did is pick by hand the lowest order local form whose kernel is the $SL(2,\mathbb{R})$ directions, and this reproduces the exact geometric answer.

With $\mathcal{D}\varepsilon = \prod_{n=2}^{\infty} 4\pi n(n^2-1)\, d^2|\varepsilon_n|$

Then

$$Z[\beta] = e^{S_0 + \frac{\phi_b}{4\pi G}\frac{\pi^2}{\beta}} \int \prod_{n=2}^{\infty} 4\pi n(n^2-1)\, d^2|\varepsilon_n| \exp\left[\frac{\phi_b}{4\pi G}\frac{2\pi^2}{\beta} n^2(n^2-1)|\varepsilon_n|^2\right]$$

The integrals are gaussians so

$$Z(\beta) = e^{S_0 + \frac{\phi_b}{4\pi G}\frac{\pi^2}{\beta}} \prod_{n=2}^{\infty} \frac{4\pi G}{\phi_b}\frac{2\beta}{n} = \frac{1}{4\pi^2}\left(\frac{\phi_b}{4G\beta}\right)^{3/2} e^{S_0 + \frac{\phi_b}{4\pi G}\frac{\pi^2}{\beta}}$$

As before, we compute the products w/ ζ-fn regularization

$$\prod_{n>1} \frac{a}{n} = \frac{a^{-3/2}}{\sqrt{2\pi}} \qquad a = \frac{8\pi G}{\phi_b}\beta$$

$$\log \prod_{n=2}^{\infty} \frac{a}{n} = \sum_{n=2}^{\infty} (\log a - \log n)$$

$$\sum_{n=1}^{\infty} 1 = \zeta(0) = -\frac{1}{2} \quad \Rightarrow \quad \sum_{n=2}^{\infty} 1 = -\frac{1}{2} - 1 = -3/2$$

$$\sum_{n=1}^{\infty} \log n = -\zeta'(0) = \frac{1}{2}\log 2\pi = \sum_{n=2}^{\infty} \log n$$

$$\Rightarrow \log \prod_{n=2}^{\infty} \frac{a}{n} = -\frac{3}{2}\log a - \frac{1}{2}\log 2\pi \Rightarrow \prod_{n=2}^{\infty} \frac{a}{n} = \frac{a^{-3/2}}{\sqrt{2\pi}}$$

Then

$$\boxed{Z(\beta) = Z_{\text{other}} \frac{1}{4\pi^2} \left(\frac{\phi_b}{4G\beta} \right)^{3/2} e^{S_0 + \frac{\phi_b}{4\pi G} \frac{\pi^2}{\beta}}}$$

$$\log Z = S_0 + \frac{\phi_b}{4\pi G} \frac{\pi^2}{\beta} + \frac{3}{2} \log \frac{\phi_b}{4G\beta} + \cdots$$

$$E = -\partial_\beta \log Z$$

$$S = -(\beta \partial_\beta - 1) \log Z$$

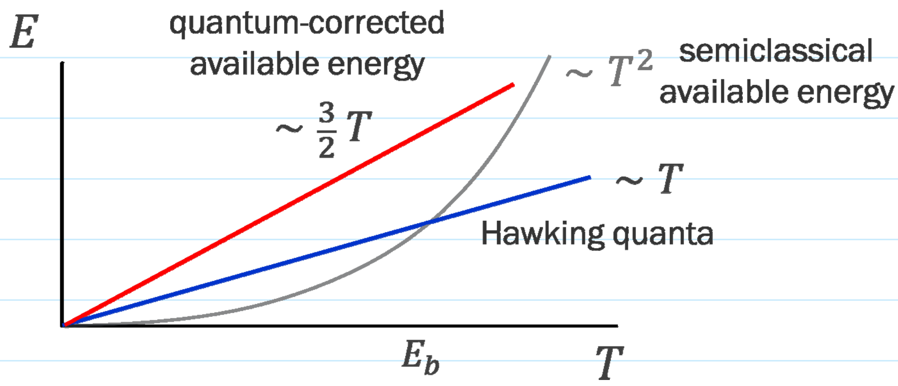
$$\Rightarrow \boxed{\begin{aligned} E &= \frac{3}{2} T + \frac{\phi_b}{4\pi G} \pi^2 T^2 \\ S &= S_0 + \frac{\phi_b}{4\pi G} 2\pi^2 T + \frac{3}{2} \log T/E_b \end{aligned}} \qquad \begin{aligned} T &= 1/\beta \\ E_b &\propto \frac{G}{\phi_b} \end{aligned}$$

This result points to the resolution of the puzzles of non-susy near-extremal black holes:

(1) large degeneracy

(2) Too little energy to radiate at temperature T

- First do (2): observe that $E = \frac{3}{2} T + O(T^2)$



Radiation emission must be reconsidered with full quantum fluctuations of the geometry. We'll do that

· Now (1). Density of states

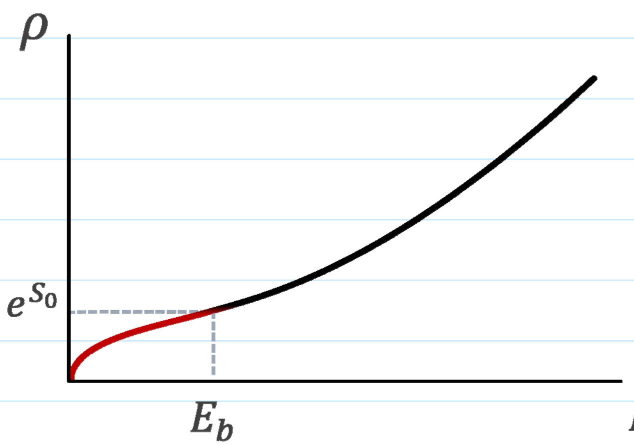
$$Z[\beta] = \int dE \, \rho(E) \, e^{-\beta E}$$

Inverse Laplace Transform:

$$\rho(E) = e^{S_0} \sinh\left(2\pi\sqrt{2E/E_b}\right)$$

$E \gg E_b$ $\quad \rho(E) \simeq e^{S_0 + 2\pi\sqrt{2E/E_b}} = e^{S_{BH}}$

$E \lesssim E_b$ $\quad \rho(E) \simeq e^{S_0} \, 2\pi\sqrt{\frac{2E}{E_b}}$



Can't trust for $E \lesssim E_b e^{-S_0}$ because other saddles

Can't trust for $E \lesssim E_b e^{-S_0}$ because other saddles may contribute at that order

- Additional remarks:

Z_{other} would yield a correction

$$\Delta S \propto \log r_+ \propto \log A_H \; :$$

fluctuations at the scale $\sim r_+$: always subleading

$$\frac{\log A_H}{A_H} \ll 1$$

Instead, for low T/E_b, $E_b \propto G/\phi_b$, we have parametrically large quantum effects.

Restoring $\hbar$:

$$S = \frac{A_H}{4G\hbar} + \underbrace{\frac{3}{2}\log T/E_b}_{\substack{\text{fluctuations}\\ \text{at the scale}\\ \text{of throat length}}} + \underbrace{c\log(A_H/G\hbar)}_{\substack{\text{fluctuations at}\\ \text{the scale of}\\ \text{throat width}}}$$

At small temperatures

$$\frac{\Delta S}{S} \propto \hbar \log T/E_b$$

Usual classical limit:

$$\boxed{\text{“First } \hbar \to 0 \text{ then } T \to 0\text{” is wrong!}}$$

When $\hbar \neq 0$, at some T we'll have $\hbar \log T = O(1)$

In the extremal limit, the classical approximation breaks down due to unavoidable large quantum breaks down due to unavoidable "large" quantum effects.