Interaction with radiation fields

domingo, 29 de marzo de 2026 15:51

Consider a field ϕ is in the presence of the near-extremal black hole in the quantum regime E, T \sim E_b. It can be absorbed or emitted by the black hole. If the field's characteristic frequency is much larger than E_b then its absorption will drive the black hole into a semiclassical regime of E, T >> E_b (obviously it can't be emitted). But at frequencies comparable to E, T (which are relevant for Hawking emission) then it'll interact with the black hole affecting its quantum levels: the radiation will affect the quantum state of the black hole, ie there will be a non-negligible backreaction. This is not accounted for in the conventional framework of QFT in curved spacetime.

Schematically, the picture we will develop is the following. We consider quantum near extremality with T, E \sim E_b. Then we can split the system into two regions: far and near.

Far region: propagation of low-frequency radiation is largely classical, without significant quantum fluctuations. We'll solve the classical wave equation (mostly massless Klein-Gordon)

Near-region: for energies E, \omega >> E_b the field propagates classically in a classical black hole throat geometry -- the AdS2 x S^2 geometries we have considered before. At the mouth of the AdS2 throat, this field can be matched to the far field solution by a standard procedure known as the method of matched asymptotics, yielding a field configuration valid over all the spacetime. In the AdS2 region, the two independent solutions of the field can be separated into a component that grows towards the mouth (the AdS2 boundary), and another that decreases:

$\rho \to \infty$

$$\phi(t,r) = \phi_0(t)\,\rho^{\Delta-1} + \dots + \phi_1(t)\,\rho^{-\Delta}$$

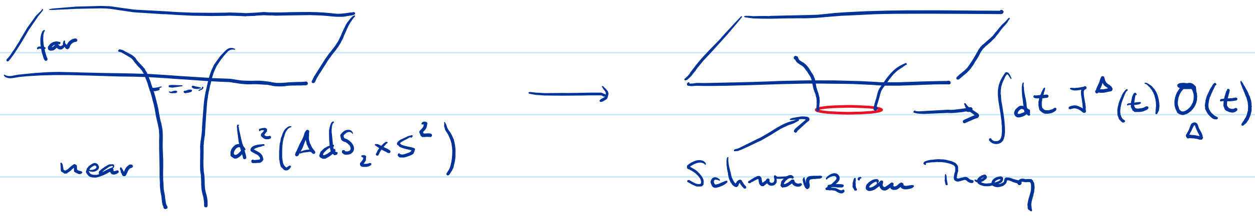
ρ = AdS_2 radial coordinate

The exponent of these two solutions is determined by field properties (eg mass and spin) and is characterized by the "conformal dimension" \Delta. These two components are what the far field "sees" of all of the field in the throat. That is, the field in the throat can be replaced by a "conformal primary" that lives at the AdS2 boundary, and has a given dimension \Delta. The growing solution is "non-normalizable", meaning that it is not dynamical, but rather acts as a source that excites a response, which corresponds to the non-growing "normalizable" component of the field. You can think of it in terms of linear response theory: a distant source is acting on the system, and the system responds by, eg, becoming excited.

$\phi_0(t)$: Time-dependent external source = $J^{\Delta}(t)$

$\phi_1(t)$: response to source excitation = $\langle O_{\Delta}(t) \rangle$

In the quantum regime with E, \omega \sim E_b we do not talk anymore about the classical throat geometry of the black hole. Instead, the black hole is characterized by an energy eigenstate |E > of the Schwarzian theory (or an ensemble of such states at finite temperature). But the picture of "source and response" still applies -- only that their relation is now governed not by the classical propagation in the throat, but by the quantum theory.



Black hole = $|E\rangle$ energy eigenstate

That is, the radiation field propagates classically in the far zone, and in the mouth region it interacts with the Schwarzian theory, acting as a source that will excite a response. Linear response theory extracts the information from the two-time correlation function (retarded Green's function)

Schematically

$$\langle O(t)\rangle_J = \left(\frac{\delta}{\delta J}\langle O(t)\rangle_J\right)\Big|_{J=0}\delta J = \left(\frac{\delta}{\delta J}\int \mathcal{D}\phi\, O(t)\, e^{-I[\phi]+i\int J O}\right)\Big|_{J=0}\delta J$$

$$= \int \mathcal{D}\phi\, O(t)\, O(0)\, e^{-iI_E}\Big)\,\delta J = \langle O(t)\, O(0)\rangle\, \delta J$$

$$\langle O(t)\, O(0)\rangle$$

This measures the probability amplitude that a perturbation induced at time t=0 generates a response at a later time t. For conformal primaries of dimension \Delta coupled to JT gravity, these 2-pt correlation functions have been computed explicitly, including when JT gravity is a quantum theory. Then, by using the fully-quantum two-point correlation functions instead of the classical ones, we solve the problem of the interaction of the field with the quantum black hole.

$\langle O(t)\, O(0)\rangle$ is classically given by $\dfrac{\phi_1(t)}{\phi_0(t)}$

Radiation emission and absorption

domingo, 29 de marzo de 2026 19:18

Hawking emission rate of energy from a black hole with Temperature β_H^{-1} is

$$\frac{dE(\omega)}{dt} = \sum_i \frac{d\omega}{2\pi} \frac{\omega P^{(i)}_{abs}(\omega)}{e^{\beta_H \omega} - 1}$$

i: angular momentum modes

$P_{abs}(\omega)$: absorption probability "greybody factor"

This has the black hole in a bath of radiation at Temperature β_H^{-1}.

We want to compute the absorption probability $P_{abs}(\omega)$ given by a classical black hole, then by a quantum black hole.

In the limit of low frequency, $\omega \ll \frac{1}{r_+}$, this can be done analytically for an arbitrary spherically symmetric black hole.

We'll do it only for the RN black hole.

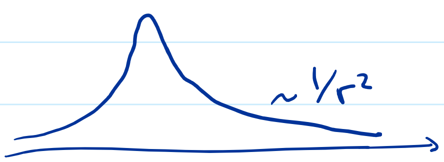
One can write the scalar wave equation in Schrödinger form, which is useful for intuition:

$$\Box \Psi = 0 \qquad \Psi = \sum_{l,m} \frac{1}{r} R_l(r) Y_l^m e^{-i\omega t}$$

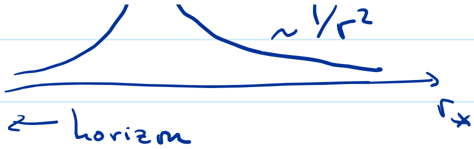
$$\Box = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g}\, g^{\mu\nu} \partial_\nu \qquad \text{Change to } r_*: \frac{dr_*}{dr} = \frac{1}{f(r)}$$

$$f(r) = \frac{(r-r_+)(r-r_-)}{r^2} \qquad \partial_r = \frac{1}{f} \partial_{r_*}$$

$$\partial^2_{r_*} R + \left[\omega^2 - f\left(\frac{l(l+1)}{r^2} + \frac{f'}{r}\right)\right] R = 0$$



$$\partial_{r_*} u + \left[\omega - f\left(\frac{\quad}{r^2} + \frac{1}{r}\right)\right] u = 0$$



Potential barrier lowest
for $\ell = 0$: dominates low-freq absorption

We'll continue the analysis not using the Schrödinger form.

Scalar field absorption in RN black hole

domingo, 29 de marzo de 2026 21:05

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2$$

$$f = \frac{(r-r_+)(r-r_-)}{r^2} \qquad \kappa = \frac{1}{2} f'(r_+)$$

$\Box \Psi = 0$ s-wave $\Psi(t, r, \Omega) = e^{-i\omega t}\,\Psi(r)$

$$f\,\partial_r\left(r^2 f\,\partial_r \Psi\right) + \omega^2 r^2 \Psi = 0$$

We assume $\omega \ll 1/r_+$

Far region is $r \gg r_+$

Near region is $r \ll 1/\omega$

They overlap where $r_+ \ll r \ll 1/\omega$

and we match the solutions there

Solve in far region: $f \simeq 1$

$$\partial_r\left(r^2 \partial_r \Psi\right) + \omega^2 r^2 \Psi = 0$$

$$\Psi_F = d_1 \frac{e^{-i\omega r}}{\omega r} + d_2 \frac{e^{i\omega r}}{\omega r}$$

ingoing outgoing

Near region $\quad f \approx \dfrac{(r-r_+)(r-r_-)}{r_+^2}$

$$\dots \sim \frac{\quad}{r_+^2}$$

Geometry becomes Rindler-$AdS_2 \times S^2$

$$\partial_r\left((r-r_+)(r-r_-)\partial_r \Psi\right) + \frac{\omega^2 r_+^2}{(r-r_+)(r-r_-)}\Psi = 0$$

This is the wave eqn in Rindler-AdS_2.

Select horizon ingoing solution

$$\Psi_N = c_1\left(\frac{r-r_+}{r-r_-}\right)^{-i\omega/2\kappa}$$

Expand this for large r: mouth of throat

$$\Psi_N \approx c_1\left(1 + i\frac{\omega r_+^2}{r}\right)$$

non-normalizable "source"

normalizable "response"

for a conformal operator in AdS_2 with dimension $\Delta = 1$

In general, $\Psi_N \xrightarrow{r\to\infty} c_1 + \frac{c_2}{r}$

but the horizon condition determines the response to a given source, ie $\frac{c_2}{c_1}$

This is equivalent to determining the 2-pt function of the operator

Now we can match this to the far-zone solution at small $r \ll 1/\omega$:

solution at small $r \ll 1/\omega$:

$$\Psi_F = -i(d_1 - d_2) + \frac{d_1 + d_2}{\omega r}$$

Choosing for convenience $c_1 = 1$ we find

$$d_1 = \frac{i}{2}\left(1 + \omega^2 r_+^2\right) \qquad d_2 = \frac{i}{2}\left(-1 + \omega^2 r_+^2\right)$$

ingoing amplitude   outgoing amplitude

This solves the scattering problem

Notice that

$|d_1|^2 - |d_2|^2 = \omega^2 r_+^2 \ll 1$: The wave couples weakly to the black hole

Ingoing & outgoing fluxes computed from current:

$$J_\mu = i\left(\Psi^* \partial_\mu \Psi - \Psi \partial_\mu \Psi^*\right)$$

Absorption probability

$$P_{abs} = \frac{\mathcal{F}_{in}^{\infty} - \mathcal{F}_{out}^{\infty}}{\mathcal{F}_{in}^{\infty}} = \frac{|d_1|^2 - |d_2|^2}{|d_1|^2}$$

$$\Rightarrow \quad \boxed{P_{abs}(\omega) = 4 r_+^2 \omega^2}$$

Can obtain now

$$\sigma_{abs} = \frac{\pi}{\omega^2} P_{abs} \quad (\text{"optical Theorem"})$$

$$\Rightarrow \quad \boxed{\sigma_{abs} = 4\pi r_+^2 = A_H}$$

In the quantum case, instead of the classical

$\frac{C_2}{C_1} = \frac{\text{"response"}}{\text{"source"}}$ we use the quantum retarded Green's function

The imaginary part of the ret-Green's fn gives the absorption probability.

Scalar wave scattering off the Reissner-Nordström black hole

We write the background in the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2\,, \qquad A = -\frac{Q}{r}dt \tag{1}$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2}\,. \tag{2}$$

The surface gravity is

$$\kappa = \frac{1}{2}f'(r_+) = \frac{r_+ - r_-}{2r_+^2}\,. \tag{3}$$

The geometry is characterized by two scales, and it is often convenient to think of them as the horizon radius r_+ and the surface gravity κ. Near extremality, $r_+ \ll 1/\kappa$.

Neutral scalar absorption

The wave equation is

$$\Box\Psi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi\right) = 0\,, \tag{4}$$

which for s-waves becomes

$$f\partial_r\left(r^2 f\partial_r\Psi\right) + \omega^2 r^2\Psi = 0\,. \tag{5}$$

We will assume $\omega \ll 1/r_+$. We might also assume the stronger condition $\omega \ll \kappa$ but this does not seem necessary—indeed we will see we can do $\kappa = 0$.

In the far zone, $r \gg 1/\omega$, we neglect the gravitational interaction between the wave and the background. Setting then $f \approx 1$, yields the wave equation

$$\partial_r\left(r^2\partial_r\Psi\right) + \omega^2 r^2\Psi = 0\,, \tag{6}$$

which is solved by

$$\Psi_F = d_1\frac{e^{-i\omega r}}{\omega r} + d_2\frac{e^{i\omega r}}{\omega r}\,. \tag{7}$$

We have conveniently normalized the amplitudes d_i to have same dimension as Ψ (which canonically is 1/length, but this will not matter since it is a classical linear field). d_1 is the amplitude of the incoming wave, d_2 of the outgoing wave. For higher partial waves we would get spherical Bessels.

Non-extremal black hole. Now solve near the horizon—this is not the solution in the entire near-zone $\omega r \ll 1$, but we need it to impose the correct boundary conditions. We expand around the horizon,

$$f = 2\kappa(r-r_+) + O(r-r_+)^2\,, \tag{8}$$

where we assume $\kappa \neq 0$. The extremal case $\kappa = 0$ will be dealt with later. The wave equation becomes

$$\partial_r\left((r-r_+)\partial_r\Psi\right) + \left(\frac{\omega}{2\kappa}\right)^2\frac{1}{r-r_+}\Psi = 0\,, \tag{9}$$

which is solved by

$$\Psi_H = b_1\left(\frac{r-r_+}{r_+-r_-}\right)^{-i\omega/(2\kappa)}\,. \tag{10}$$

We have selected the ingoing component, and again conveniently normalized the amplitude b_1 (which we will later set to one without loss of generality).

(which we will later set to one without loss of generality).

Next we solve in the near-zone $\omega r \ll 1$, but not very close to the horizon, so we can neglect $\omega^2 r^2/f$ (which we kept in (9)). Here

$$\partial_r \left(r^2 f \partial_r \Psi\right) = 0 \tag{11}$$

and then

$$\Psi_N = c_1 + c_2 \log \frac{r - r_+}{r - r_-}\,. \tag{12}$$

At this point we can match the solutions sequentially $\Psi_H \leftrightarrow \Psi_N \leftrightarrow \Psi_F$. Expanding Ψ_H for small $\omega/\kappa \ll 1$,[1] we find

$$\Psi_H \simeq b_1 \left(1 - i\frac{\omega}{2\kappa} \log \frac{r - r_+}{r_+ - r_-}\right)\,. \tag{13}$$

We match this to Ψ_N at $r \sim r_+$ by setting

$$c_1 = b_1\,, \qquad c_2 = -b_1 \frac{i\omega}{2\kappa}\,. \tag{14}$$

horizon regularity
fixes solution (up to total amplitude)

Now expand Ψ_N at large $r \gg r_+$,

$$\Psi_N \simeq c_1 - c_2 \frac{r_+ - r_-}{r}\,. \tag{15}$$

For partial waves
$\Psi \sim c_1 r^{\ell} + c_2 r^{-1-\ell}$
non normalizable "source"
$\to$ normalizable "response"

We compare this to the far-zone solution (7) at small $r \ll 1/\omega$,

$$\Psi_F \simeq -i(d_1 - d_2) + \frac{d_1 + d_2}{\omega r}\,, \tag{16}$$

$\frac{c_2}{c_1} \propto$ 2-point correlation function

and find

$$c_1 = -i(d_1 - d_2)\,, \qquad c_2(r_+ - r_-) = -\frac{d_1 + d_2}{\omega}\,. \tag{17}$$

The relations (14) and (17) allow us to find the ratio of the ingoing and outgoing amplitudes at infinity, d_1 and d_2, to the ingoing amplitude at the horizon, b_1—this is the solution to the scattering problem. Normalizing the amplitude to $b_1 = 1$ for convenience, we find

$$d_1 = \frac{i}{2}\left(1 + \omega^2 r_+^2\right)\,, \qquad d_2 = \frac{i}{2}\left(-1 + \omega^2 r_+^2\right)\,. \tag{18}$$

Then

$$|d_1|^2 - |d_2|^2 = \omega^2 r_+^2\,, \qquad |d_1|^2 = \frac{1}{4} + O(\omega r_+)^2\,. \tag{19}$$

Here we have used the expression (3) for κ.

One consequence that we extract is that the wave is weakly coupled to the black hole: the difference between ingoing and outgoing components of the wave is small $\sim (\omega r_+)^2 \ll 1$, i.e., only a small fraction of the wave is absorbed by the black hole.

Having solved the scattering problem, we proceed to compute the absorption probability. The field current is

$$J_\mu = i\left(\Psi^* \partial_\mu \Psi - \Psi \partial_\mu \Psi^*\right)\,. \tag{20}$$

Field current
$\to$ flux

For our field, this current is radial. The flux $\mathcal{F}$ across a surface is the total number of particles that pass through the surface per unit time. For a sphere of radius R, this is given by

$$\mathcal{F} = \frac{1}{\Delta t}\int_{S^2 \times \Delta t} *J = \frac{1}{\Delta t}\int_{S^2 \times \Delta t} dt\, d\omega_2 \sqrt{-h}\, n^r J_r$$

$$\mathcal{F} = \frac{1}{\Delta t}\int_{S^2\times\Delta t} *J = \frac{1}{\Delta t}\int_{S^2\times\Delta t} dt\, d\omega_2 \sqrt{-h}\, n^r J_r$$
$$= 4\pi R^2 f(R) J_r(R)\,. \tag{21}$$

[1] When this is not satisfied close enough to extremality, we can still solve the problem as we will see below. A general analysis using the tortoise radial coordinate unifies the two calculations.

We want to compute this flux at asymptotic infinity and at the horizon. The asymptotic ingoing flux is

$$\mathcal{F}_{in}^{(\infty)} = \frac{8\pi}{\omega}|d_1|^2\,, \tag{22}$$

and the difference between incoming and outgoing fluxes (total flux) is

$$\mathcal{F}_{in}^{(\infty)} - \mathcal{F}_{out}^{(\infty)} = \frac{8\pi}{\omega}\left(|d_1|^2 - |d_2|^2\right)\,. \tag{23}$$

At the horizon

$$\mathcal{F}_{in}^{(H)} = 8\pi\omega r_+^2 |b_1|^2\,. \tag{24}$$

Flux conservation for a solution of the equations ensures that (23) and (24) are the same, as we can easily verify using the values for the scattering amplitudes (19). We find the absorption probability

$$\boxed{P_{abs} = \frac{\mathcal{F}_{in}^{(\infty)} - \mathcal{F}_{out}^{(\infty)}}{\mathcal{F}_{in}^{(\infty)}} = \frac{\mathcal{F}_{in}^{(H)}}{\mathcal{F}_{in}^{(\infty)}} = 4\omega^2 r_+^2\,.} \quad = 1 - \frac{|d_2|^2}{|d_1|^2} \tag{25}$$

The absorption cross section is

$$\boxed{\sigma = \frac{\pi}{\omega^2}P_{abs} = 4\pi r_+^2\,.} \tag{26}$$

The above derivation was done for the Reissner-Nordström black hole, but little input was needed from the geometry, and it is easy to extend the calculation to a generic static, spherical black hole to show, under mild assumptions on the geometry[2], that $\sigma = A_H$.

Extremal black hole. The above calculations break down when

$$f = \left(\frac{r - r_+}{r}\right)^2 \tag{27}$$

and $\kappa = 0$.

The far-zone solution remains (7) as before, but near the horizon we set

$$f \approx \left(\frac{r - r_+}{r_+}\right)^2\,, \tag{28}$$

and the wave equation becomes

$$\partial_r\left(\left(\frac{r - r_+}{r_+}\right)^2 \partial_r\Psi\right) + \left(\frac{\omega r_+}{r - r_+}\right)^2 \Psi = 0\,. \tag{29}$$

This is the equation for field propagation in AdS_2. The solution that is ingoing at the horizon is

$$\Psi_N = c_1 e^{i\frac{\omega r_+^2}{r - r_+}}\,. \tag{30}$$

The solution seems to diverge as $r \to r_+$ (with infinite oscillations in the radial direction), but the flux across the horizon is finite, The solution seems to diverge as $r \to r_+$ (with infinite oscillations in the radial direction), but the flux across the horizon is finite,

$$\mathcal{F}_{in}^{(H)} = 8\pi\omega r_+^2 |c_1|^2 \,. \tag{31}$$

At large $r \gg r_+$ (the mouth of the AdS_2 throat), the near-field becomes

$$\Psi_N \simeq c_1 \left(1 + i\frac{\omega r_+^2}{r}\right) . \tag{32}$$

[2]Namely, the geometry must not have any length scale affecting the propagation of the field other than the horizon radius r_+ and the surface gravity κ.

This tells us that in AdS_2 the field corresponds to an operator of conformal dimension 1. Matching this solution to the far-zone field at $r \ll 1/\omega$, (16), gives

$$c_1 = -i(d_1 - d_2) \,, \qquad ic_1(\omega r_+)^2 = d_1 + d_2 \,. \tag{33}$$

Setting $c_1 = 1$ without loss of generality, the solution for d_1 and d_2 is the same as (18). When we compute the fluxes we recover the same form of the absorption rate and cross section as in (25), (26).

Notice we have been able to consistently skip having to distinguish between Ψ_H and Ψ_N—a single solution in the AdS_2 region does the job. We will see next that the same happens in the near-extremal regime where the geometry near the horizon is Rindler-AdS_2.

Near-extremal black hole. In this case, in the near-horizon region we approximate

$$f \approx \frac{(r - r_+)(r - r_-)}{r_+^2} \,, \tag{34}$$

so the wave equation becomes

$$\partial_r \left((r - r_+)(r - r_-)\partial_r \Psi\right) + \frac{\omega^2 r_+^2}{(r - r_+)(r - r_-)}\Psi = 0 \,. \tag{35}$$

This is slightly different than we used in (9), although it becomes the same if we approximate $r - r_- \simeq r_+ - r_-$. It is indeed the wave equation in Rindler-AdS_2. The ingoing solution is also slightly different than (10), namely,

$$\Psi_N = c_1 \left(\frac{r - r_+}{r - r_-}\right)^{-i\omega/(2\kappa)} \,. \tag{36}$$

At large $r \gg r_+$, this solution takes the same form as in (32), so the subsequent matching produces the same results.

4

Full quantum calculation

domingo, 29 de marzo de 2026 20:28

Far field is $\Psi(t,r) \sim C_{in}\, e^{-i\omega(t-r)} + C_{out}\, e^{-i\omega(t+r)}$

At matching mouth $\Psi(t,r) = \psi(t) + O(r^{-1})$

source $\Delta = 1$ for massless s-wave scalar

Interaction bh-radiation:

$$I = I_{\text{schwarzian}} - \int dt\, \psi(t)\, \mathcal{O}(t)$$

$\uparrow$ BH

$\psi(t) = e^{-i\omega t}\,\psi_0$

$H_{int} = e^{-i\omega t}\,\psi_0\, \mathcal{O}(t)$

This interaction drives transitions between bh states:

$|E_i\rangle \to |E_i + \omega\rangle$ absorption

$|E_i\rangle \to |E_i - \omega\rangle$ emission

We saw that the coupling is weak $(\sim (\omega r_+)^2 \ll 1)$ so we can apply Fermi's Golden rule:

$$\Gamma_{i\to f} = 2\pi \left| \langle E_f, N_f |\, \mathcal{O}\psi_0 \,| E_i, N_i\rangle \right|^2 \rho(E_f)$$

energy of bh state; number of scalar quanta; density of bh states

Matching calculation determines $|\psi_0|^2 = \langle N_\omega\rangle \dfrac{r_+^2\omega}{\pi^2}$

Absorption probability

$$\Gamma_{abs}(\omega) = \langle N_\omega\rangle \frac{2 r_0^2 \omega}{\pi} \left| \langle E_i + \omega |\, \mathcal{O}\, | E_i\rangle \right|^2 \rho(E_i + \omega)$$

Emission: $\Gamma_{emit}(\omega) = -\Gamma_{abs}(\omega)$ (Time-reversal invce)

Total absorption:

$$P_{abs}(\omega) = \frac{2\pi}{\langle N_\omega \rangle}\left(\Gamma_{abs}(\omega) - \Gamma_{emit}(\omega)\right)$$

$$\Rightarrow \boxed{\begin{aligned} P_{abs}(\omega) = 4 r_0^2 \omega \Big(& |\langle E_i+\omega | O | E_i \rangle|^2 \rho(E_i+\omega) \\ & - |E_i - \omega | O | E_i \rangle|^2 \rho(E_i+\omega) \Big) \end{aligned}}$$

We need the matrix elements

$|\langle E_f | O | E_i \rangle|^2$ for scalar primaries O coupled to the Schwarzian Theory (JT gravity)

These are Transforms of The Two-point function:

$$\langle E | O(0)\, O(\tau) | E \rangle = \int_0^\infty dE'\, \rho(E')\, e^{-\tau(E-E')} |\langle E | O | E' \rangle|^2$$

$$\boxed{\begin{aligned} \langle E | O(0)\, O(\tau) | E \rangle &= \langle E | O\, e^{H\tau}\, O\, e^{-H\tau} | E \rangle \\ &= \sum_{E'} \langle E | O\, e^{H\tau} | E' \rangle \langle E' | O\, e^{-H\tau} | E \rangle \\ &= \sum_{E'} e^{-\tau(E-E')} \langle E | O | E' \rangle \langle E' | O | E \rangle \\ &= \sum_{E'} e^{-\tau(E-E')} |\langle E | O | E' \rangle|^2 \end{aligned}}$$

$$\Rightarrow \rho(E') |\langle E | O | E' \rangle|^2 = \int_{-i\infty}^{+i\infty} d\tau\, e^{-\tau E'} \langle E | O(0)\, O(\tau) | E \rangle$$

These have been computed for primaries of any Δ.

For $\Delta = 1$ primaries These are relatively simple

For $\Delta = 1$ primaries there are relatively simple

$$|\langle E_f | \mathcal{O} | E_i \rangle|^2 = 2\pi^2 E_b \, e^{-S_0} \frac{E_f - E_i}{\cosh\left(2\pi\sqrt{\frac{2E_f}{E_b}}\right) - \cosh\left(2\pi\sqrt{\frac{2E_i}{E_b}}\right)}$$

and we already computed

$$\rho(E) = \frac{e^{S_0}}{2\pi^2 E_b} \sinh\left(2\pi\sqrt{\frac{2E}{E_b}}\right) \Theta(E)$$

From here we obtain $P_{abs}(\omega)$ for a black hole of energy E_i, and we can then obtain the Hawking emission rate, and the absorption cross section.

Classical check: for $\omega, E_i \gg E_b$ we recover the classical result $P_{abs} = 4 r_+^2 \omega^2$ ✓

and $\sigma_{abs} = 4\pi r_+^2 = A_H$ ✓

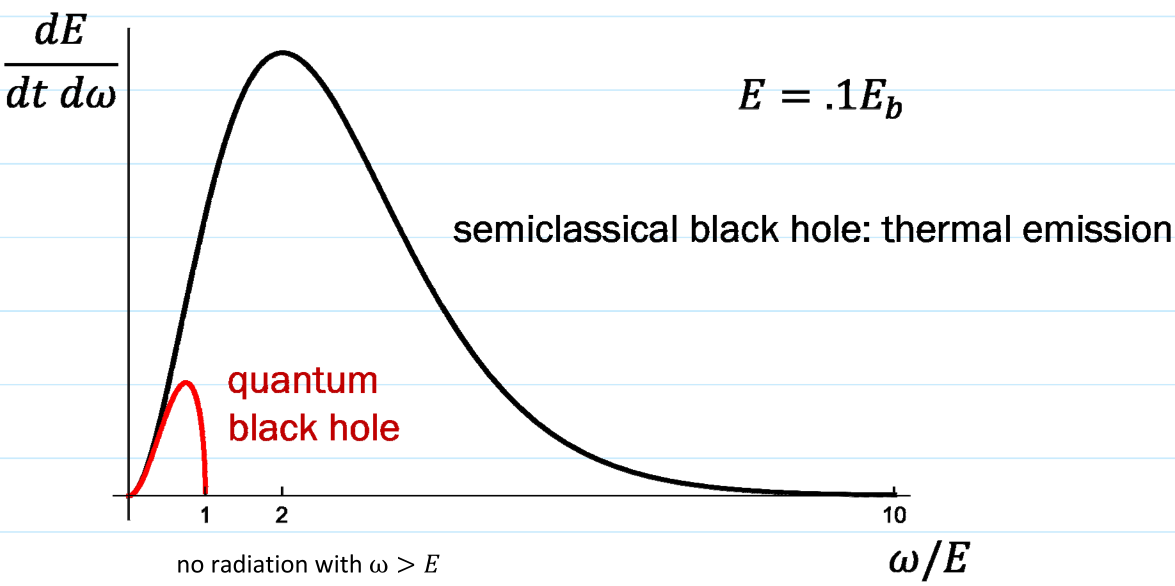
But then for $\omega, E_i < E_b$ there are large deviations from the semiclassical behavior.

Quantum black hole radiation emission and absorption

domingo, 29 de marzo de 2026 23:16

Quantum gravity modifications to Hawking radiation

Inserting the quantum form of $P_{abs}(\omega)$ we can recompute the power spectrum of the emitted radiation



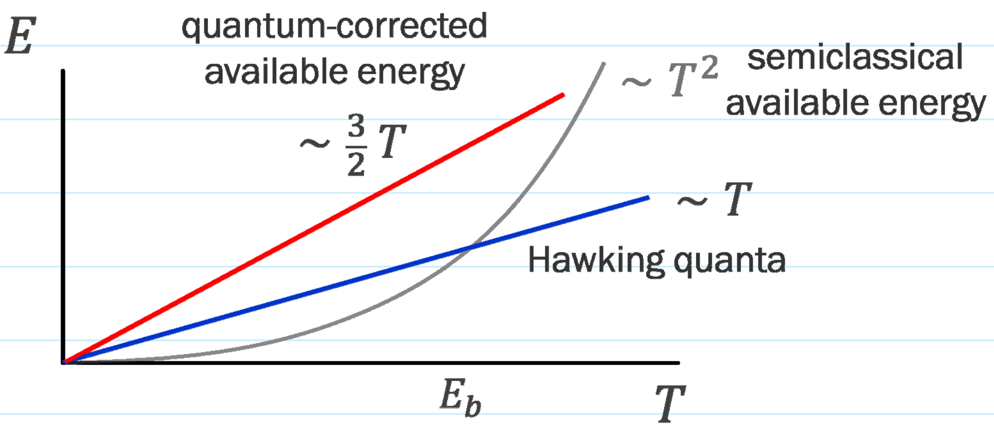
Quantum Black Hole: non-thermal emission

The reduced emission rate is naturally expected: there are fewer black states when E < E_b, and therefore fewer states that can emit radiation.

Ensemble equivalence breaks down because fluctuations are large. If we fix the temperature T, then there are large variations in the energy of the states in the ensemble, and viceversa.

Indeed, if the black hole starts in say, the canonical ensemble, then evaporation will drive the ensemble of states away from the thermal one. Anna Biggs has found that the evolution leads to a new kind of ensemble.

This result now resolves the old puzzle about the semiclassical emission near extremality



The good news is that now we have the quantum gravity tools to redo the calculations -- we don't need UV quantum gravity, but only to apply old tools in a careful manner

Quantum cross section of near-extremal black holes

This provides an in-principle manner of observing the effects of large quantum fluctuations of the geometry.

The semiclassical absorption cross-section for a scalar at low frequencies is

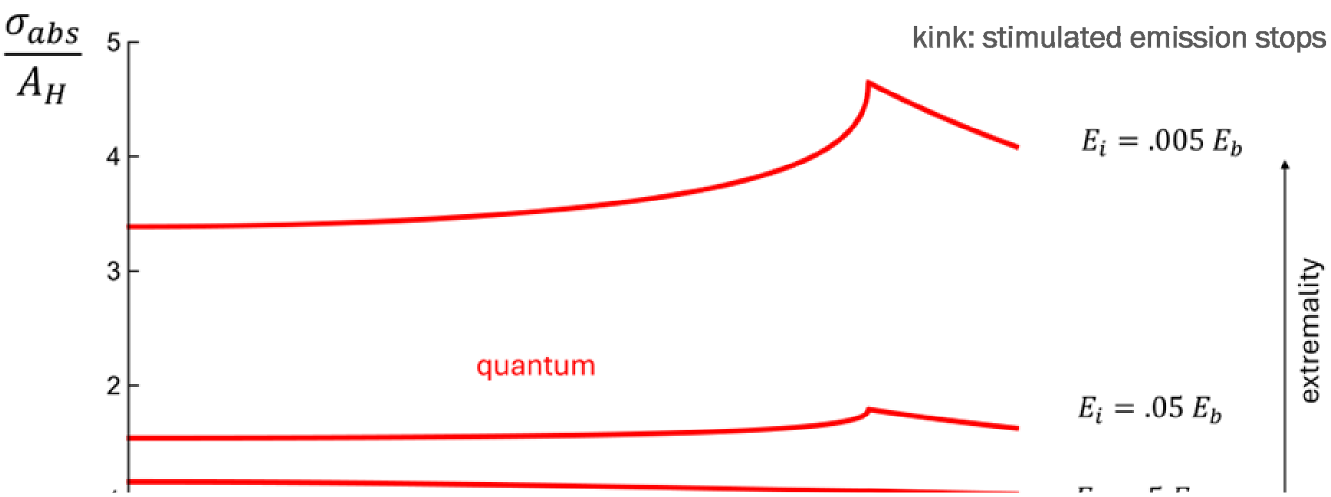
$\sigma_{abs} = A_H$

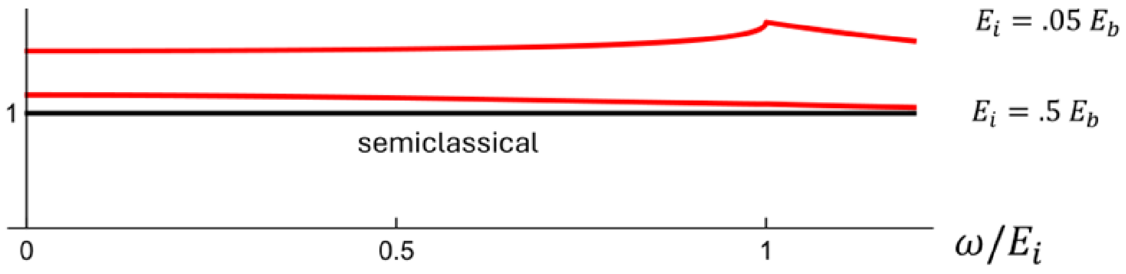
We could take the cross section as an observational method for determining the number of black hole states

$\rho(E) \sim \exp\left(\frac{\sigma_{abs}}{4G\hbar}\right)$ (semiclassical)

ie we can expect that σ_{abs} will directly measure the number of absorbing states.

Then, since quantum corrections reduce the density of states, we might expect that they should reduce σ_{abs}. However, this is not what the calculation shows: quantum effects make σ_{abs} larger. A quantum near-extremal black hole looks LARGER (to low frequency radiation) the closer it is to extremality.





The explanation is that absorption depends on two factors: the density of absorbing states, and the probability that these states make transitions between them. Quantum effects make the former smaller, but the latter larger, and when factored together, the enhanced transition rates turn out to outweigh the reduction in the number of absorbing states.