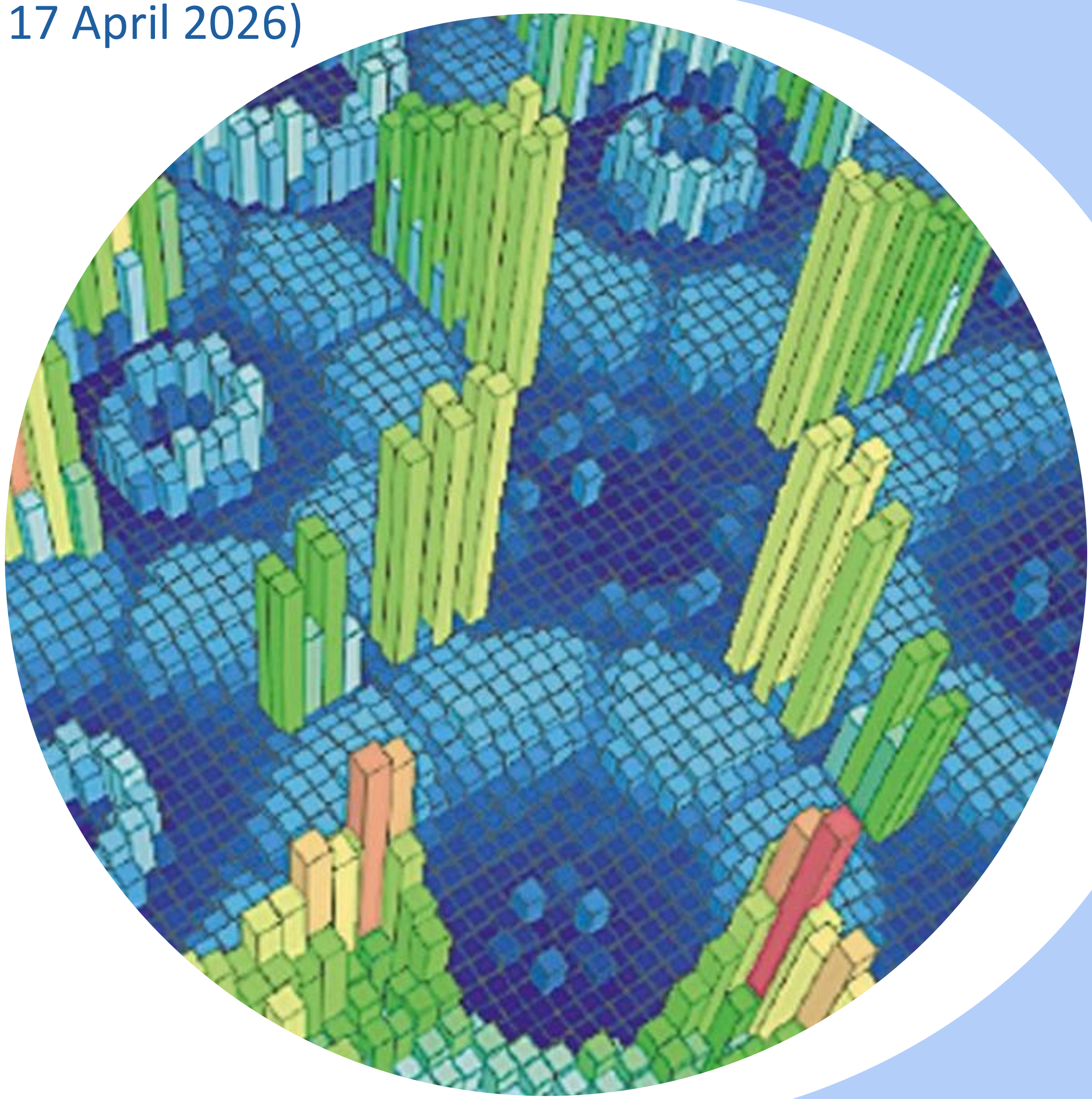
Joint IAEA-ICTP Workshop on Reactor Physics, Thermal Hydraulics and Plant Design Engineering of Small Modular Reactors' (13 - 17 April 2026)

FUNDAMENTALS OF REACTOR PHYSICS



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Presentation Layout

Part I

- High level overview of Nuclear Reactor Analysis and Design
- Rigorous and Simplified Formulations of Neutron Balance Equations
- Challenges and Approaches

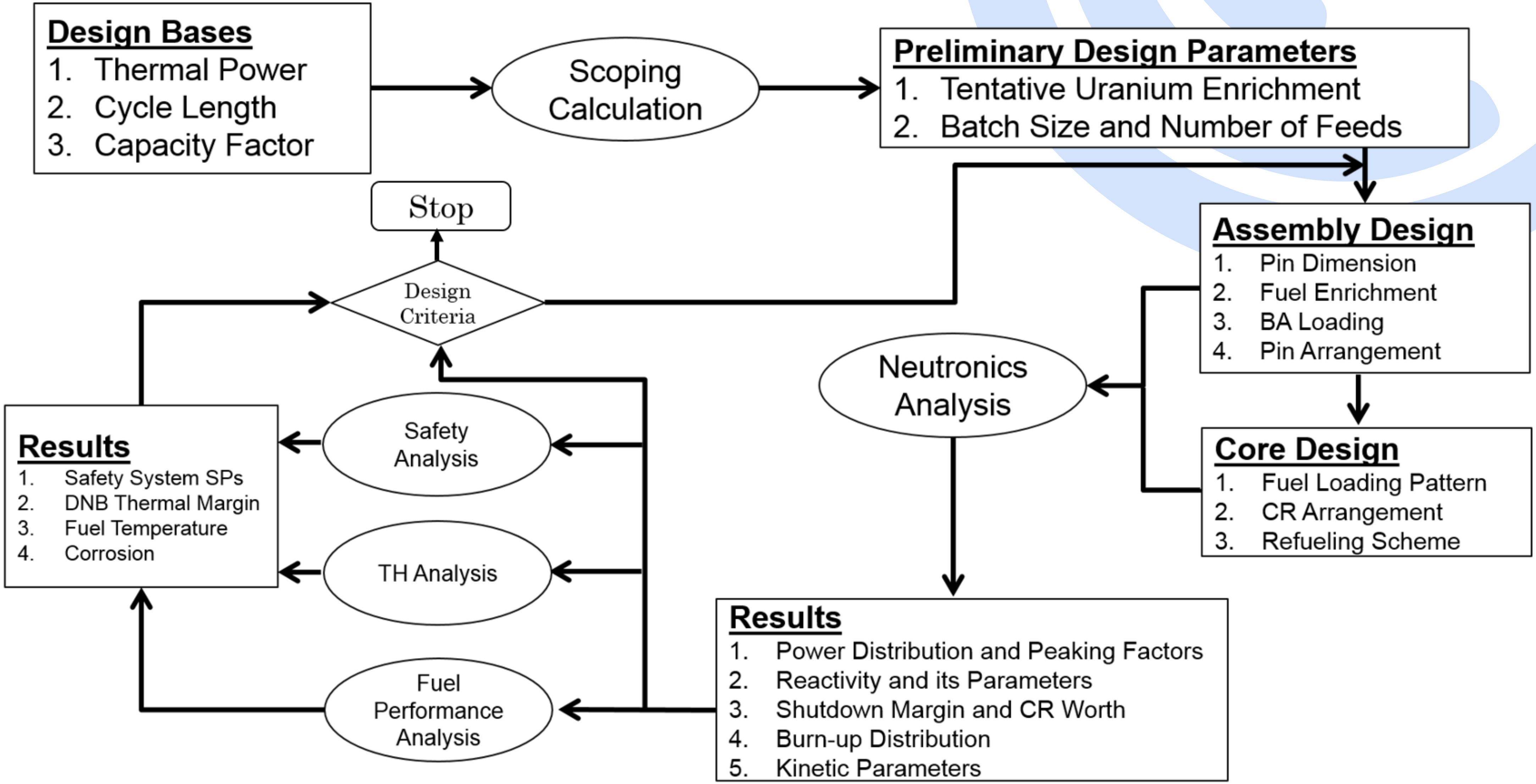
Part II

- Rigorous and Simplified Neutron Balance Equations Derivation – a mathematical perspective
- Numerical solution of 1-dimensional neutron diffusion equation

Part III

- Reactor physics fundamentals
- SMRs perspective

Nuclear Analysis and Design Procedure – Bird's eye view



Neutron Balance Formulations

Famous Boltzmann's Equation or Neutron Transport Equation

$$\frac{1}{v(E)}\frac{\partial\psi(r,E,\vec{\Omega},t)}{\partial t}^{(1)} = -\vec{\Omega}\cdot\nabla\psi(r,E,\vec{\Omega},t)^{(2)} - \Sigma_t(r,E,\vec{\Omega})\psi(r,E,\vec{\Omega},t)^{(3)}$$

$$+\chi(E)\int_{E'} dE' \int_{\Omega'} d\Omega' \nu\Sigma_f(r,E',\vec{\Omega}')\,\psi(r,E,\vec{\Omega},t)^{(4)}$$

$$+\int_{E'} dE' \int_{\Omega'} d\Omega'^{\Sigma_s}\left(r,E'\rightarrow E,\vec{\Omega}'\rightarrow\Omega\right)\psi(r,E,\vec{\Omega},t)^{(5)} + Q(r,E,\vec{\Omega},t)^{(6)} + \sum_{i=1}^{l}\chi_i\lambda_i C(r,t)^{(7)}$$

(1) → Rate of accumulation of neutrons within the element
(2) → Rate of leakage of neutrons from the element
(3) → Total loss of neutrons (absorption + scattering out of the energy group)
(4) → Total production of neutrons with fission reaction
(5) → Total production of neutron (in-scattering)
(6) → External neutron source
(7) → Delayed neutron production

Pre-Cursor Equation

$$\frac{\partial C(r,t)}{\partial t} = \sum_j \beta_i^j \int_{\Omega} d\Omega \int_E dE \;\; \nu_j(E')\Sigma_f(r,E \;\;,\vec{\Omega}\;\;)\psi(r,E,\vec{\Omega},t) - \lambda_i C_i(r,t)$$

(1) → Rate of production precursor concentration
(2) → Decay of precursor

Bateman's Equation

$$\frac{dN_n}{dt} = -\lambda_n N_n - \sigma_{abs}\phi \;\; N_n + \sigma_{c(n-1)}\phi \;\; N_{n-1} + \lambda_{n'}N_{n'}$$

Rigorous and Simplified Forms

$$\frac{1}{v(E)}\frac{\partial\psi(\vec{r},E,\vec{\Omega},t)}{\partial t}^{(1)} = -\vec{\Omega}\cdot\nabla\psi(\vec{r},E,\vec{\Omega},t)^{(2)} - \Sigma_t(\vec{r},E,\vec{\Omega})\psi(\vec{r},E,\vec{\Omega},t)^{(3)}$$
$$+\chi(E)\int_{E'} dE' \int_{\Omega'} d\Omega' \nu\Sigma_f(\vec{r},E',\vec{\Omega}')\,\psi(\vec{r},E,\vec{\Omega},t)^{(4)}$$
$$+\int_{E'} dE' \int_{\Omega'} d\Omega'^{\Sigma_s}(\vec{r},E'\to E,\vec{\Omega}'\to\Omega)\,\psi(\vec{r},E,\vec{\Omega},t)^{(5)} + Q(\vec{r},E,\vec{\Omega},t)^{(6)} + \sum_{i=1}^{l}\chi_i\lambda_i C(\vec{r},t)^{(7)}$$
$$\frac{\partial C_i(r,t)}{\partial t} = \sum_j \beta_i^j \int_{\Omega} d\Omega \int_E dE \quad \nu_j(E')\Sigma_f(r,E \quad ,\vec{\Omega} \quad)\psi(r,E,\vec{\Omega},t) - \lambda_i C_i(r,t)$$

Continuous energy NTE

$$\frac{1}{v(E)}\frac{\partial\psi_g(\vec{r},\vec{\Omega},t)}{\partial t}^{(1)} = -\vec{\Omega}\cdot\nabla\psi_g(\vec{r},\vec{\Omega},t)^{(2)} - \Sigma_t(\vec{r},\vec{\Omega})\psi_g(\vec{r},\vec{\Omega},t)^{(3)}$$
$$+\chi_g\sum_{g'}\nu\Sigma_{fg'}(\vec{r},t)\phi_g(\vec{r},t)^{(4)}$$
$$+\sum_{g'}\int_{\Omega'} d\Omega'\Sigma_{sgg'}(\vec{r},\vec{\Omega}'\to\Omega)\,\psi_{g'}(\vec{r},\vec{\Omega},t)^{(5)} + Q_g(\vec{r},\vec{\Omega},t)^{(6)} + \sum_{i=1}^{l}\chi_i\lambda_i C_i(\vec{r},t)^{(7)}$$
$$\frac{\partial C_i}{\partial t} = \beta_i\sum_{g'}\nu\Sigma_{fg'}\phi_g(\vec{r},t) - \lambda_i C_i(\vec{r},\mathrm{t})$$

Multi-group NTE

$$\frac{1}{v_g}\frac{\partial\phi_g(\vec{r},t)}{\partial t} = -D_g(\vec{r})\nabla^2\phi_g(\vec{r},t) + \Sigma_{t,g}(\vec{r})\phi_g(\vec{r},t) = \chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\overline{r,t}) +$$
$$\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r},t) + Q_g(\overline{r},t) + \sum_{i=1}^{l}\chi_i\lambda_i C(\vec{r},t)$$
$$\frac{\partial C_i}{\partial t} = \beta_i\sum_{g'}\nu\Sigma_{fg'}\phi_g(\vec{r},t) - \lambda_i C_i(\vec{r},\mathrm{t})$$

Multi-group NDE

$$\frac{dP(t)}{dt} = \frac{\rho(t)-\beta(t)}{\Lambda(t)}P(t) + \sum_{i=1}^{l}\chi_i\lambda_i C(t)$$
$$\frac{\partial C_i}{\partial t} = \frac{\beta_i(t)}{\Lambda(t)}P(t) - \lambda_i C_i(\mathrm{t})$$

Point Kinetics Equation

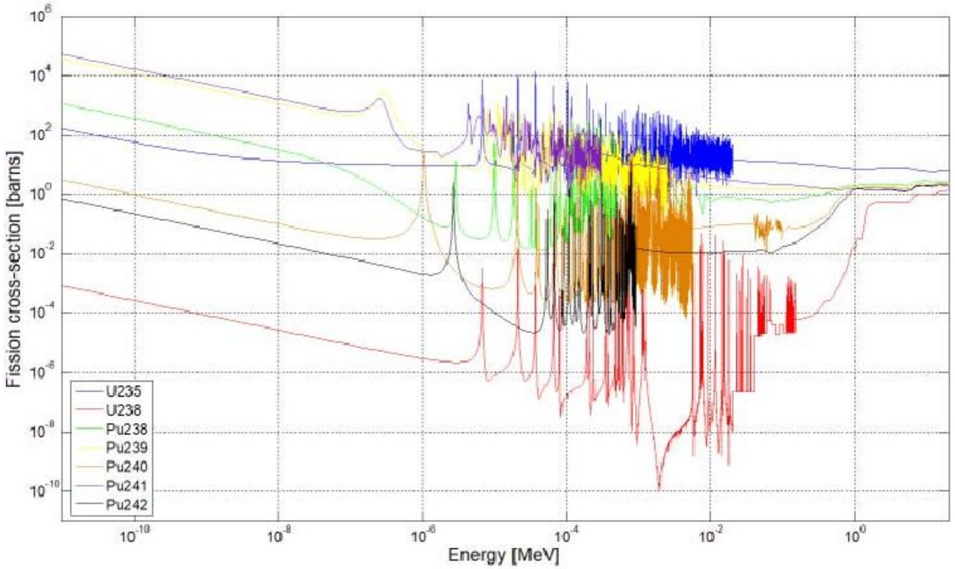
Increasing Computational Time

Decreasing Complexity

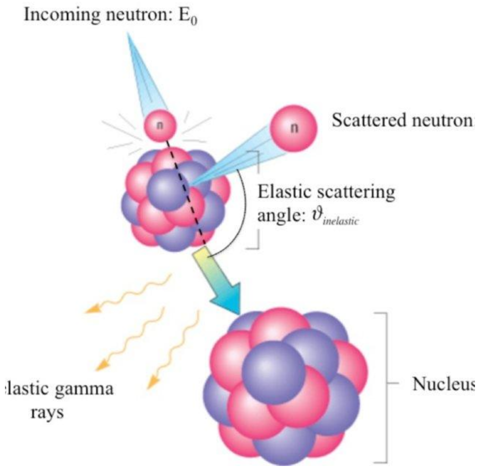
Neutronics Calculations - Challenges

Seven (7) independent variables – 3 in space (x,y,z), 2 in direction (θ,ϕ), energy and time

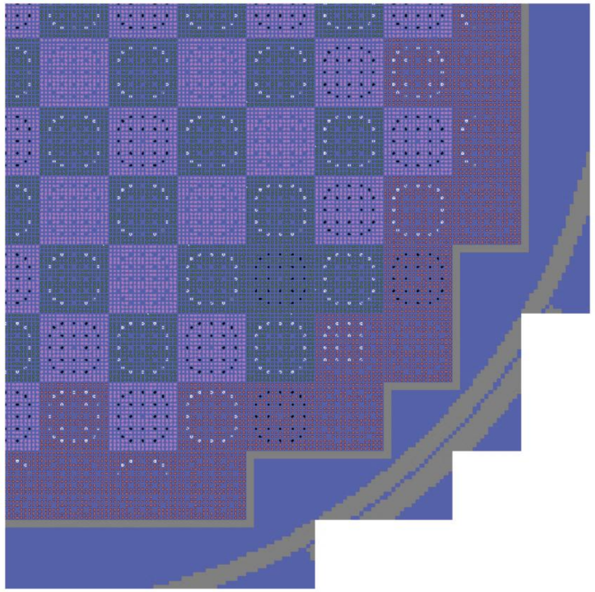
Challenges in Solution



Cross-sections energy dependent



Angular dependence



Heterogeneity in core

Neutronics Calculations – Possible Approaches

Stochastic (MC) Methods

- **High accuracy**
  - Direct simulation of particles' whole behavior
  - No discretization of variables (energy, angle, space)
- No constraints on geometry construction
- Simple parallel calculation
- **Computationally Expensive**
- Large memory

Deterministic Methods

- Accuracy depends on the methodology as well as level of complexity.
- Simple Problems require less computational power.
- Parallel calculations are possible but relatively complex.
- Fast and acceptably accurate solutions

Mathematician's point of view

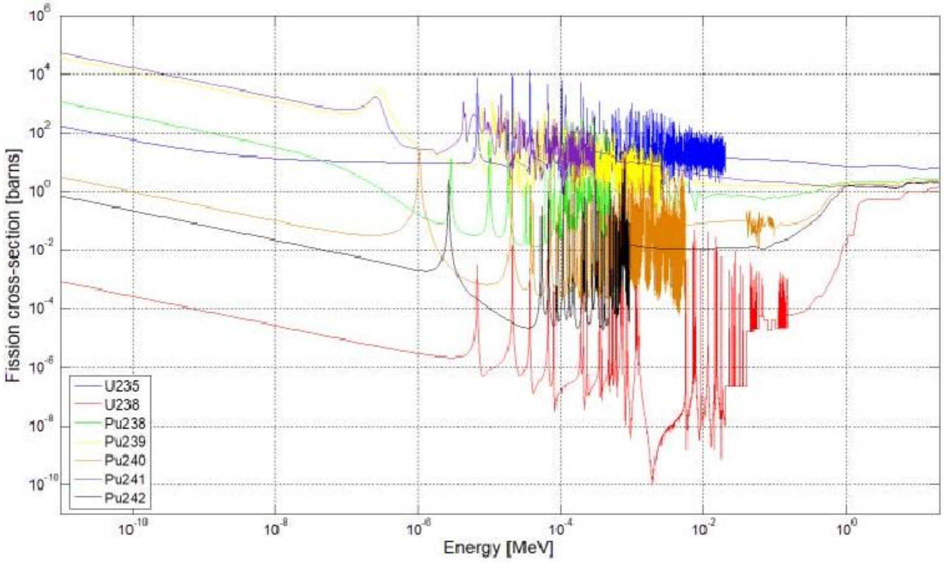
… As Wigner pointed out, **neutron transport can be analyzed from two distinct points of view, analogous to Lagrangian and Eulerian formulation of hydrodynamics**. One can either consider the particle density in a unit volume of phase space or one can focus attention on the individual particles and consider their motion. … (Laurence B. Miller, 1967)

Code user's point of view

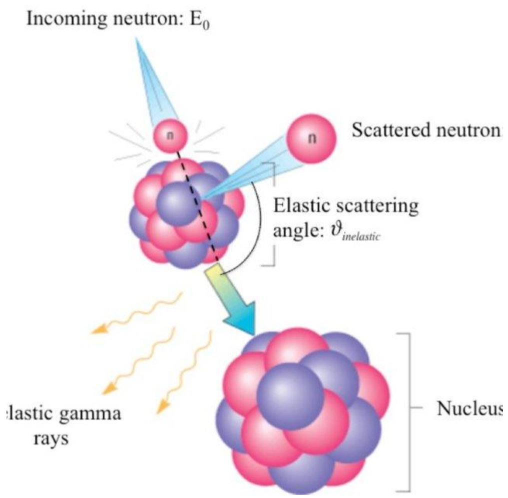
An estimate of a physical quantity calculated by the Monte Carlo method inevitably has its statistical uncertainty.
Implementation of deterministic methodology is somewhat simpler and interesting. Nevertheless, numerical solution has discretization errors.

Neutronics Calculations – Monte Carlo

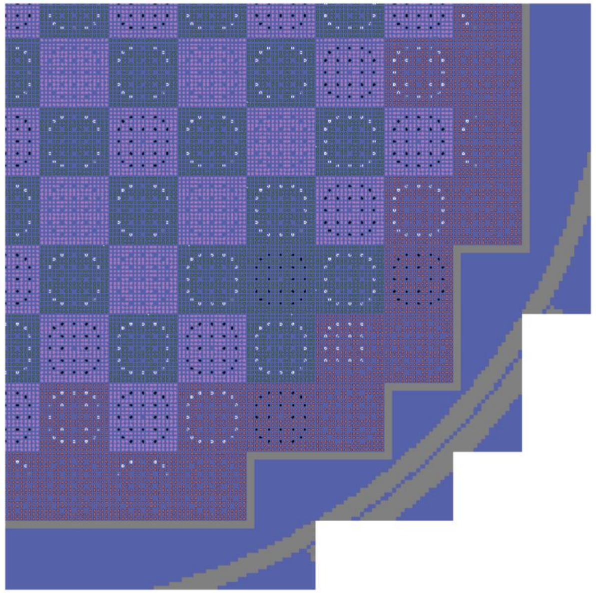
Can use point-wise energy cross-section



Can treat angular dependence without any simplification



Can modelled any heterogeneity of reactor core



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Evaluated Nuclear Data File

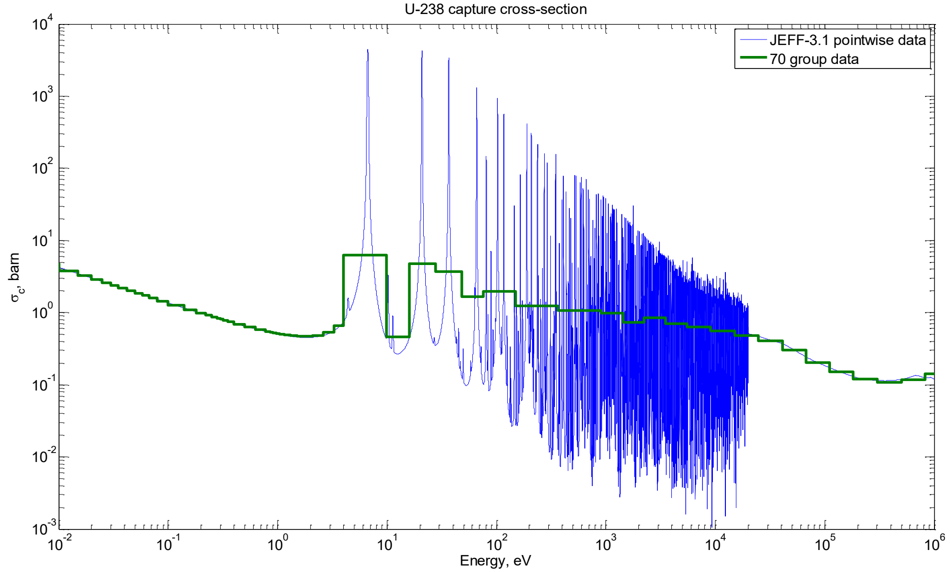
Neutron Transport and interactions are modelled explicitly

- No Truncation Errors
- Uncertainty Quantification is necessary

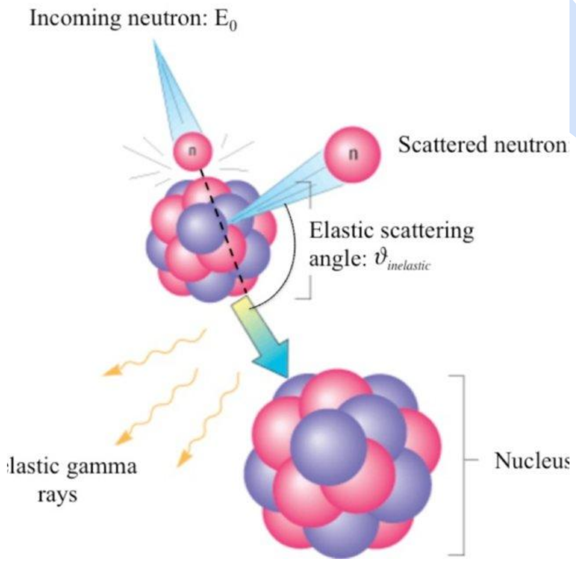
3D Modelling

Neutronics Calculations – Deterministic Methods I

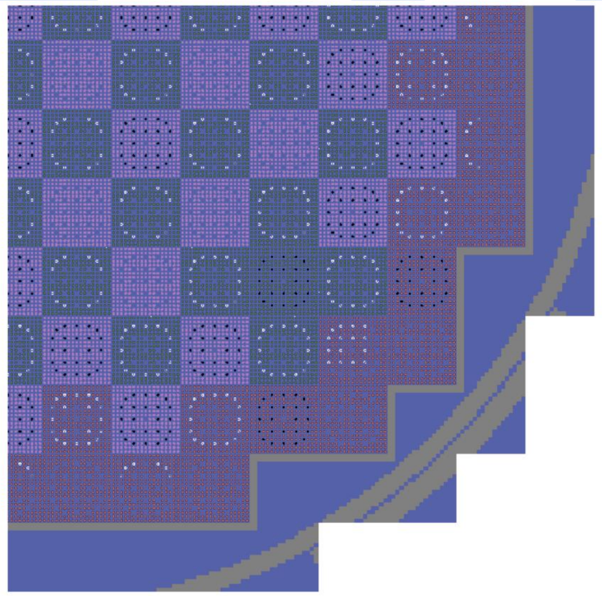
Ultra-fine group condensation



Angular discretization or ray line tracing or functional forms



Detailed Heterogeneity



Ultra fine multi-group structure

Direct whole core transport calculations

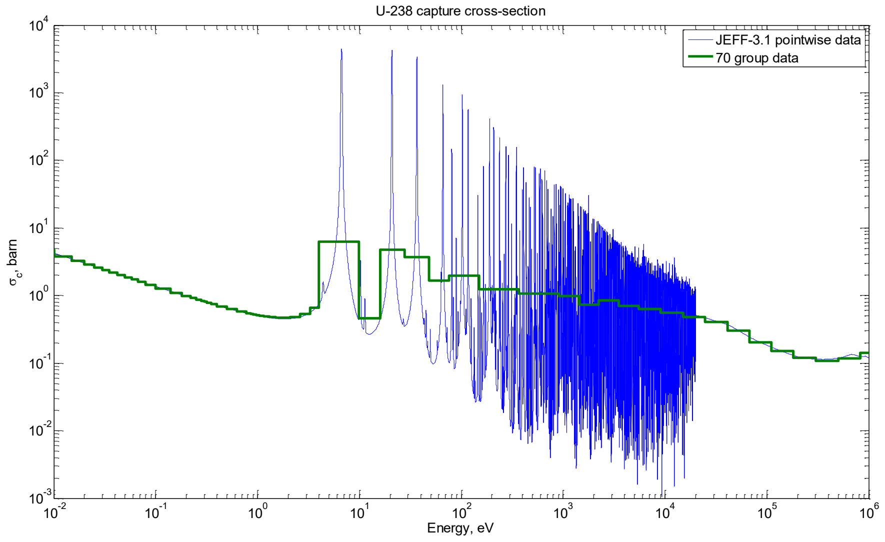
Truncation Errors

3D Modelling detailed

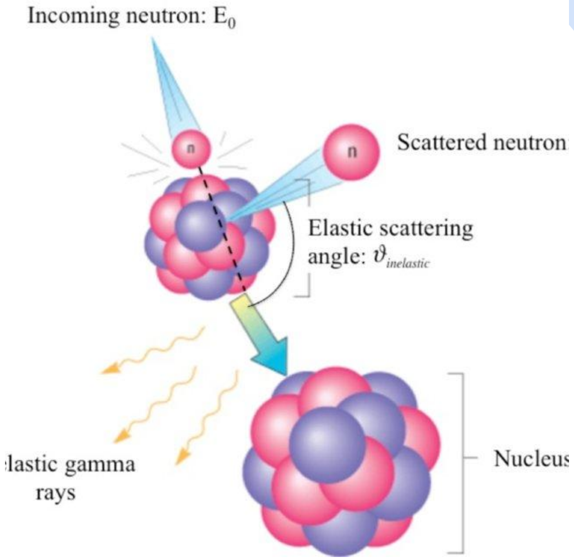
Can only be performed with 2D + 1D MOC CMFD calculations e.g., nTRACER code

Neutronics Calculations – Deterministic Methods II

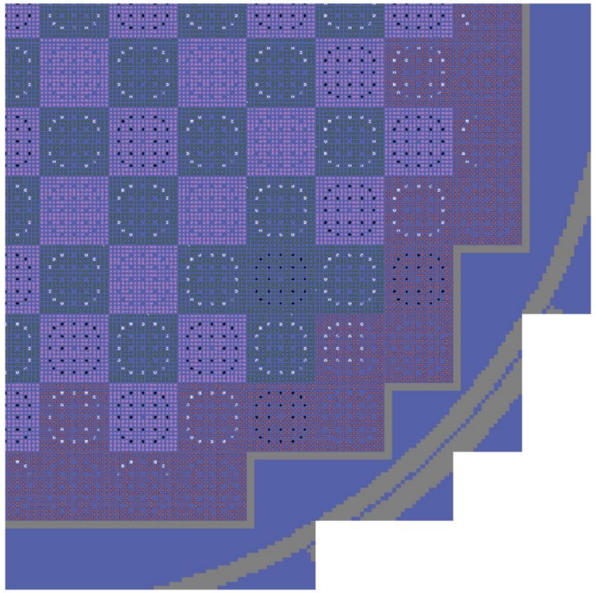
Group condensation



Angular discretization or functional forms



Not detailed Geometry Calculations



Multi-group structure

Neutron Transport Equation

Truncation Errors

Simplified Models

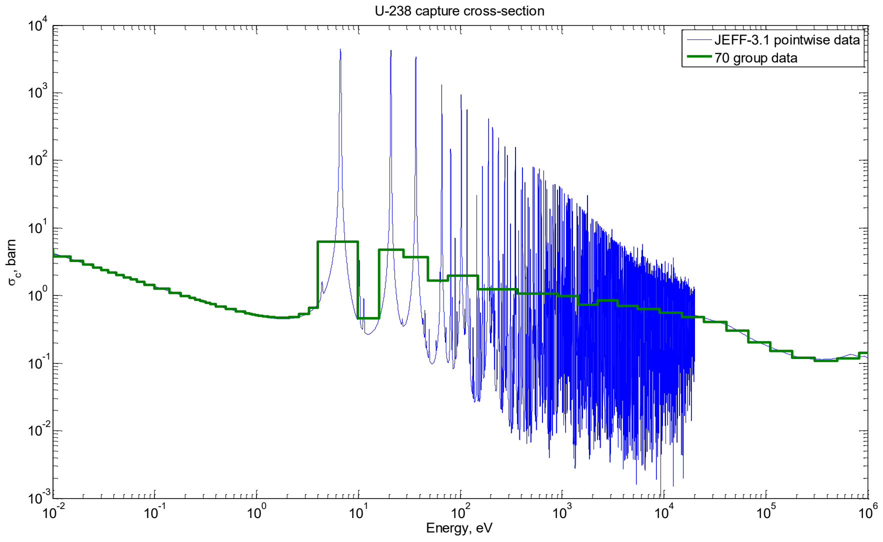
Collision Probability Method or Discrete Ordinate Method for the solution of Neutron Transport

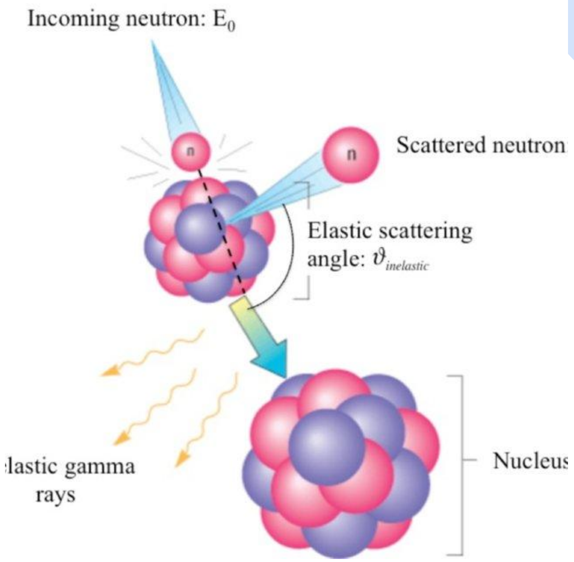
Neutronics Calculations – Deterministic Methods III

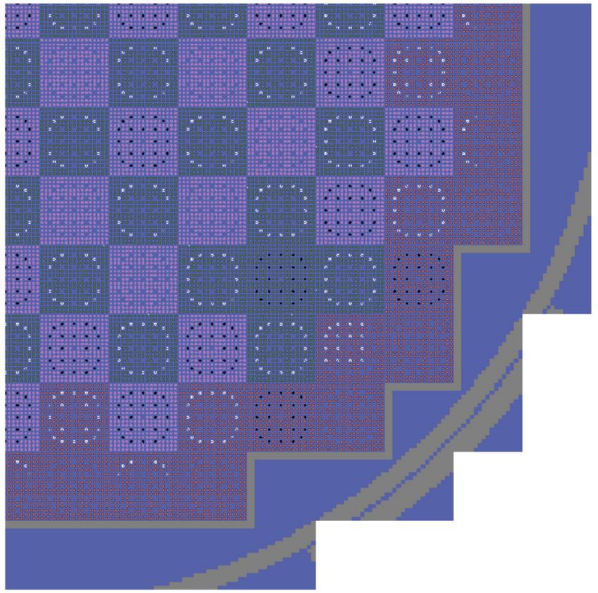
Group condensation

Angular discretization or functional forms

Not detailed Geometry Calculations







Multi-group structure

Neutron Transport Equation

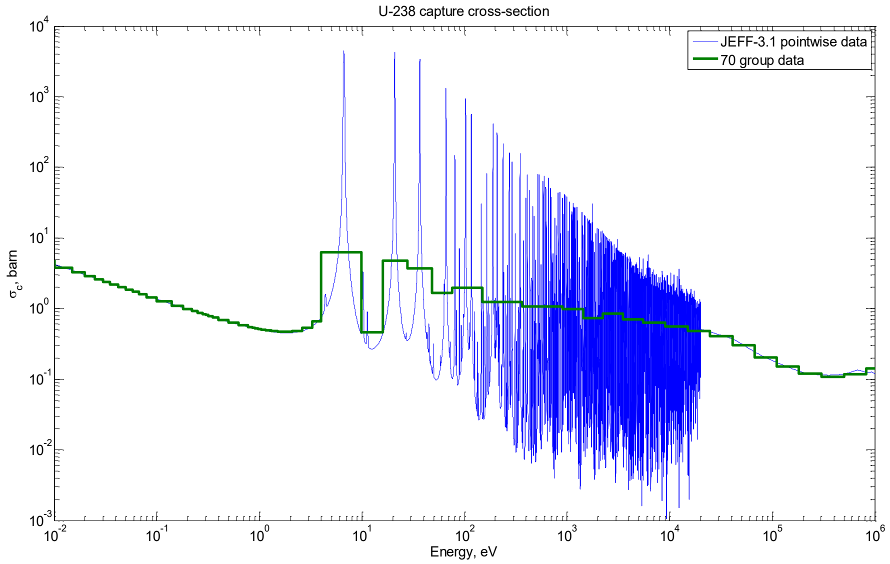
Truncation Errors

Simplified Models

Collision Probability Method or Discrete Ordinate Method for the solution of Neutron Transport

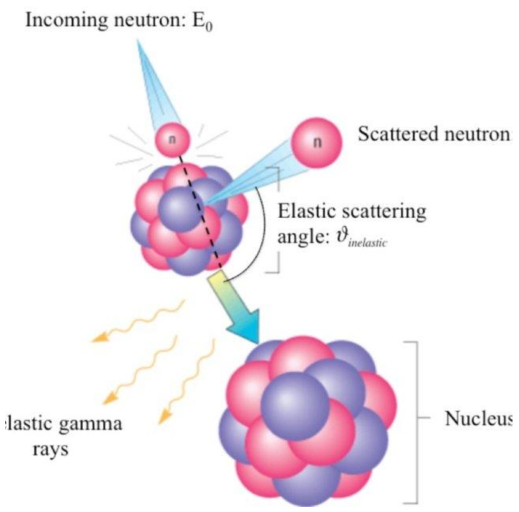
Neutronics Calculations – Deterministic Methods IV

Group condensation

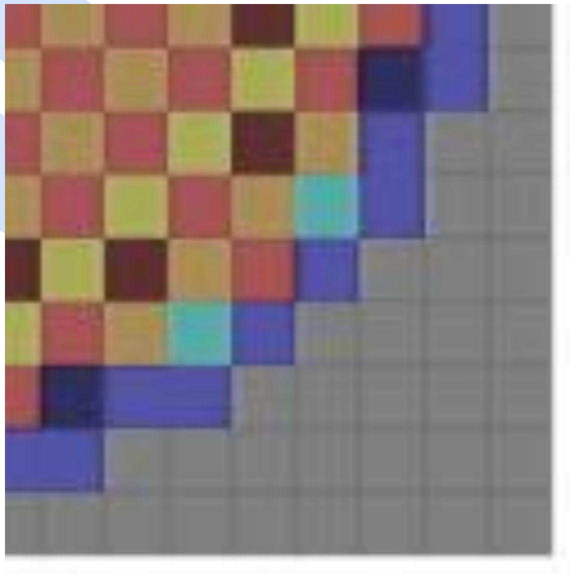


Multi-group structure

No Angular Dependence



Homogenized Geometries



3D Models but Simplified

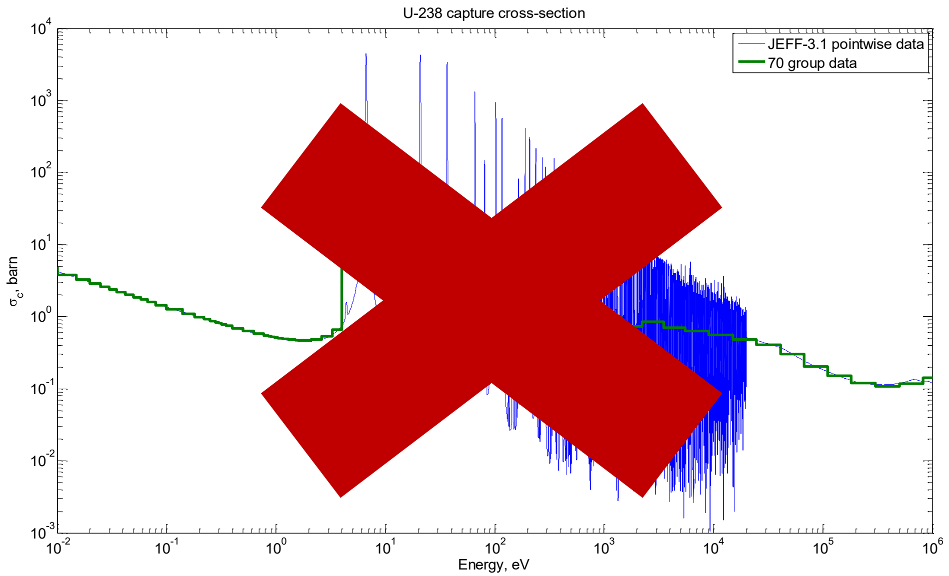
Neutron Diffusion Equation

Truncation Errors, Simplification Errors

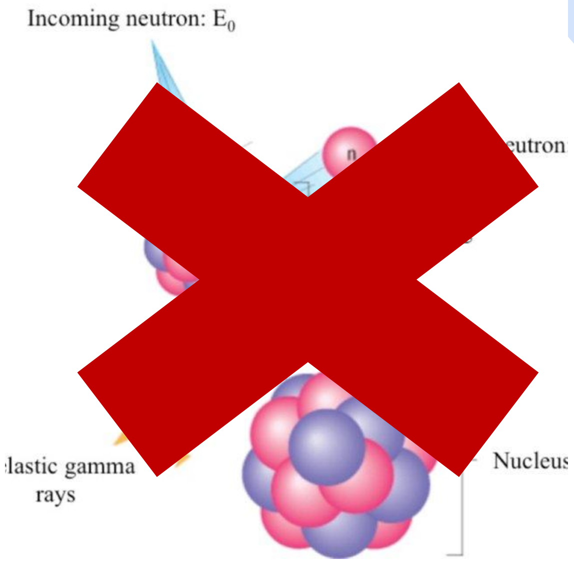
Finite Difference Methods, Finite Element Methods, Nodal Methods

Neutronics Calculations – Deterministic Methods V

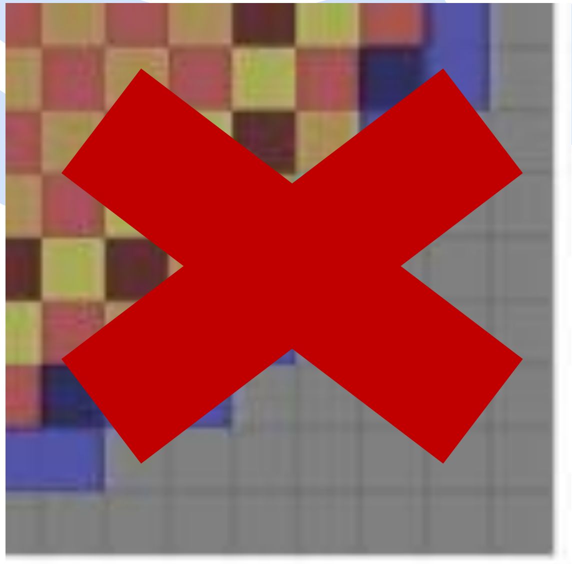
No Energy Dependence



No Angular Dependence



No Geometrical Dependence



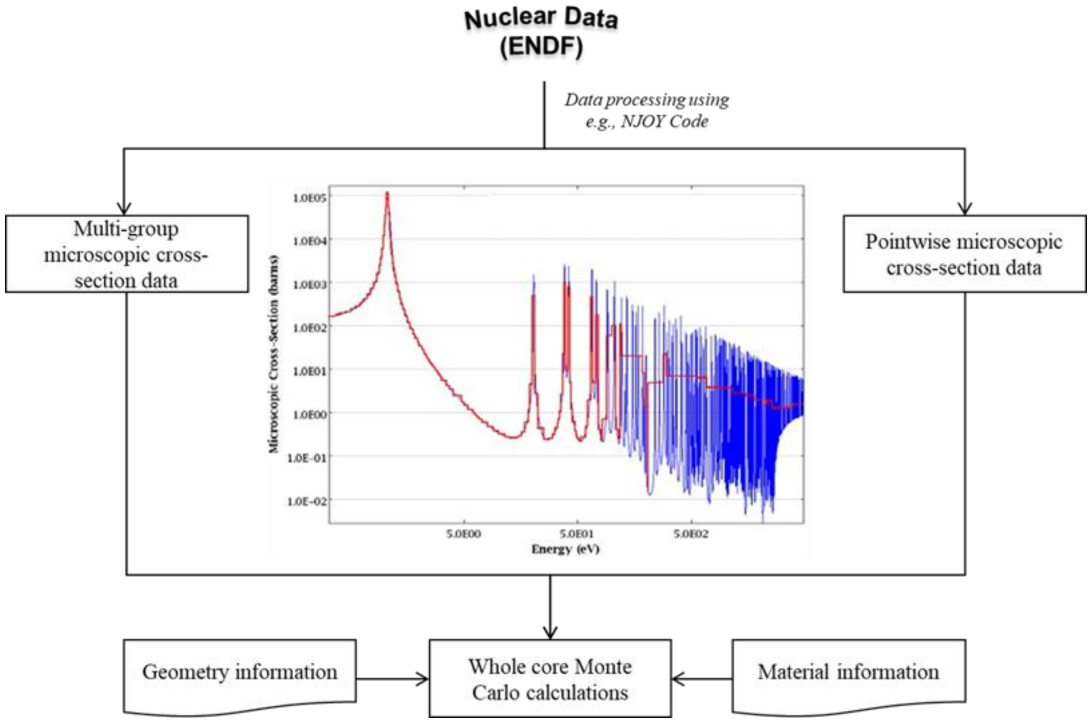
No energy structure

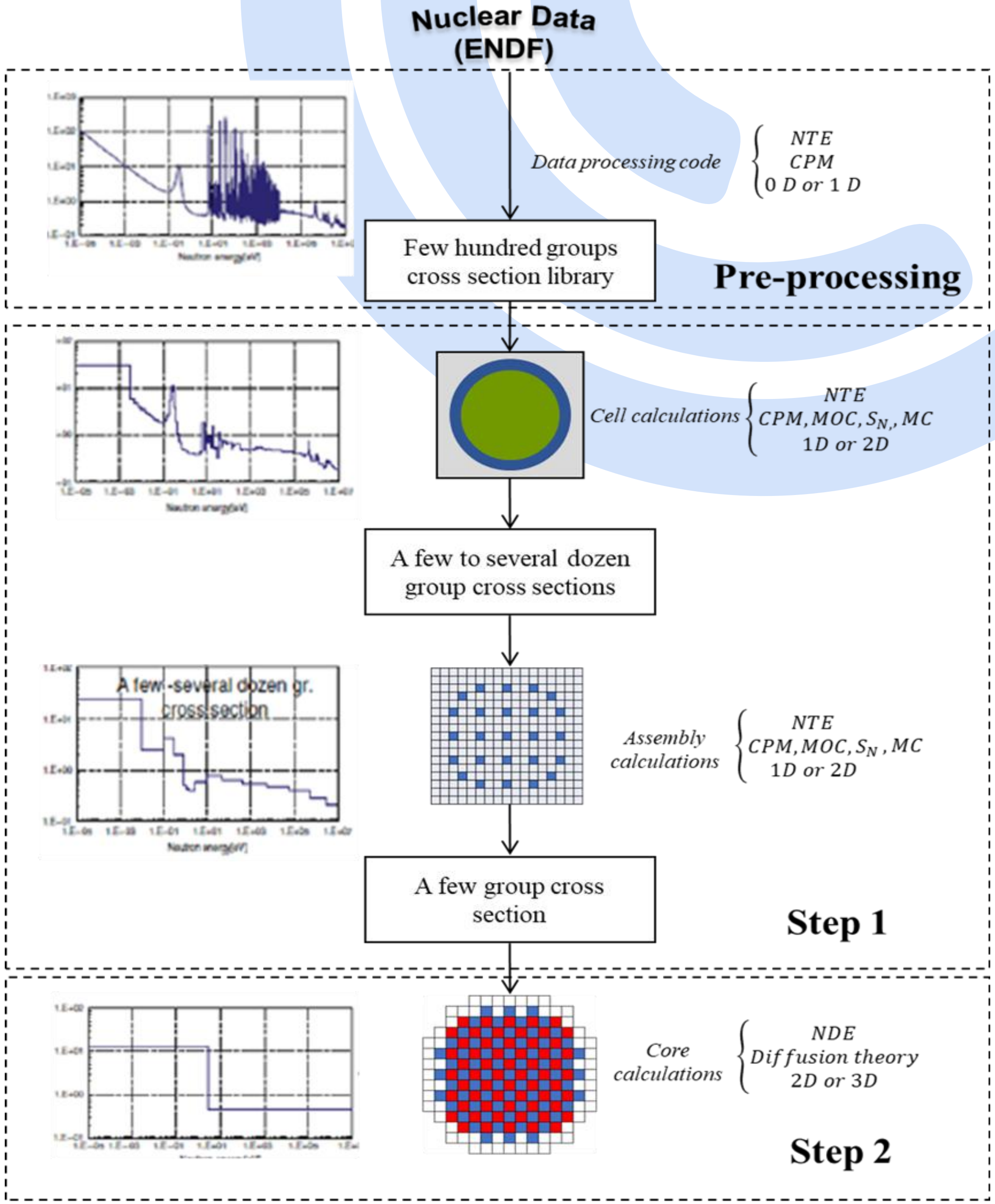
Point Kinetics Equation

Truncation Errors, Simplification Errors

Point Model

Neutronics Calculations – Computational Strategies





Rigorous to Simplified Neutron Balance

- Discretization of continuous energy variable, integrals over E replaced by sums over energy groups.
- Let's recall the continuous energy NTE

$$\vec{\Omega}\cdot\nabla\psi(r,E,\vec{\Omega})+\Sigma_t(r,E,\vec{\Omega})\psi(r,E,\vec{\Omega})=\chi(E)\int_{E'}dE'\int_{\Omega'}d\Omega'\nu\Sigma_f(r,E',\vec{\Omega}')\psi(r,E,\vec{\Omega})$$
$$+\int_{E'}dE'\int_{4\pi}d\Omega'\Sigma_s(r,E'\rightarrow E,\vec{\Omega}'\rightarrow\Omega)\psi(r,E,\vec{\Omega})$$

where all notations are standard, with the following boundary conditions:

$$\psi(r,E,\vec{\Omega})=\psi^b(r,E,\vec{\Omega});\qquad x\in\partial V,\vec{\Omega}.\vec{n}<0,\ 0<E<\infty$$

- Energy group structure is increasing group number index with decreasing energy:

$$E_{\min}=E_G<E_{G-1}<E_{G-2}\ldots<E_2<E_1=E_{\max}$$

- For each energy group:

$$\psi(r,\vec{\Omega})=\int_{E_g}^{E_{g-1}}\psi(r,E,\vec{\Omega})\,dE\rightarrow Angular\ Flux\ for\ energy\ group\ 'g'$$

Rigorous to Simplified Neutron Balance

$$\int_g dE = \int_{E_g}^{E_{g-1}} dE \rightarrow For\ each\ energy\ group\ 'g'$$

$$\int_0^{\infty} dE' = \sum_{g'=1}^{G} \int_{E_{g'-1}}^{E_{g'}} dE' \rightarrow For\ each\ energy\ group\ 'g'$$

- Integration of NTE over a group ‘g’

$$\vec{\Omega}\cdot\vec{\nabla}\int_{E_{g-1}}^{E_g} \psi(\vec{r},E,\vec{\Omega})dE + \int_{E_{g-1}}^{E_g} \Sigma_t(r,E,\vec{\Omega})\psi(r,E,\vec{\Omega})\,dE = \int_{E_{g-1}}^{E_g} dE\,\chi(E)\sum_{g'=1}^{G}\int_{E_{g'-1}}^{E_{g'}} dE'\nu(E')\Sigma_f(\vec{r},E')\int_{\Omega'} d\Omega'\psi(r,E,\vec{\Omega})$$
$$+\int_{E_{g-1}}^{E_g} dE\sum_{g'=1}^{G}\int_{E_{g'-1}}^{E_{g'}} dE'\int_{4\pi} d\Omega'\Sigma_s(r,E'\rightarrow E,\vec{\Omega}'\rightarrow\Omega)\psi(r,E',\vec{\Omega}')$$

- Please note that the above equation is still exact, no approximation has been made.
- Let’s make the assumption here; angular flux is separable:

$$\psi(\vec{r},\mathrm{E},\vec{\Omega}) = f(E)\psi(\vec{r},\vec{\Omega})$$

- where f(E) is an energy dependent shape function such that:

$$\int_{E_{g-1}}^{E_g} dE\, f(E) = 1$$

Rigorous to Simplified Neutron Balance

- The revised NTE will be as follows:

$$\vec{\Omega}\cdot\vec{\nabla}\psi(\vec{r},\vec{\Omega})+\int_{E_{g-1}}^{E_g}\Sigma_t(\vec{r},E,\vec{\Omega})\psi(r,\vec{\Omega})f(E)dE=\int_{E_{g-1}}^{E_g}dE\chi(E)\sum_{g'=1}^{G}\int_{E_{g'-1}}^{E_{g'}}dE'\nu(E')\Sigma_f(\vec{r},E')f(\mathrm{E}')\int_{\Omega'}d\Omega'\psi(r,\vec{\Omega}')$$
$$+\int_{E_{g-1}}^{E_g}dE\sum_{g'=1}^{G}\int_{E_{g'-1}}^{E_{g'}}dE'\int_{4\pi}d\Omega'\Sigma_s(r,E'\rightarrow E,\vec{\Omega}'\rightarrow\Omega)f(E')\psi(r,\vec{\Omega}')$$

- Let's define multi-group constants in the above equation:

$$\Sigma_{t,g}=\int_{E_{g-1}}^{E_g}dE\,f(E)\Sigma_t(\vec{\mathrm{r}},E,\vec{\Omega})$$

$$\Sigma_{s,g'\rightarrow g}=\int_{E_{g-1}}^{E_g}dE\int_{E_{g'-1}}^{E_{g'}}dE'\int_{\Omega'}d\Omega'\,f(E')\Sigma_s(\vec{\mathrm{r}},E'\rightarrow E,\vec{\Omega}'\rightarrow\vec{\Omega}')$$

$$\chi_g=\int_{E_{g'-1}}^{E_{g'}}dE\,\chi(E)$$

$$\nu\Sigma_{f,g}=\int_{E_{g'-1}}^{E_{g'}}dE'\,\nu(E')\Sigma_f(\vec{\mathrm{r}},E')f(E')$$

- **Multi-group transport equation** can also be written as;

$$\left[\vec{\Omega}\cdot\vec{\nabla}+\Sigma_{t,g}\right]\psi_g=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}+\sum_{g'=1}^{G}\int_{4\pi}d\Omega'\Sigma_{s,g'\rightarrow g}\psi_{g'}$$

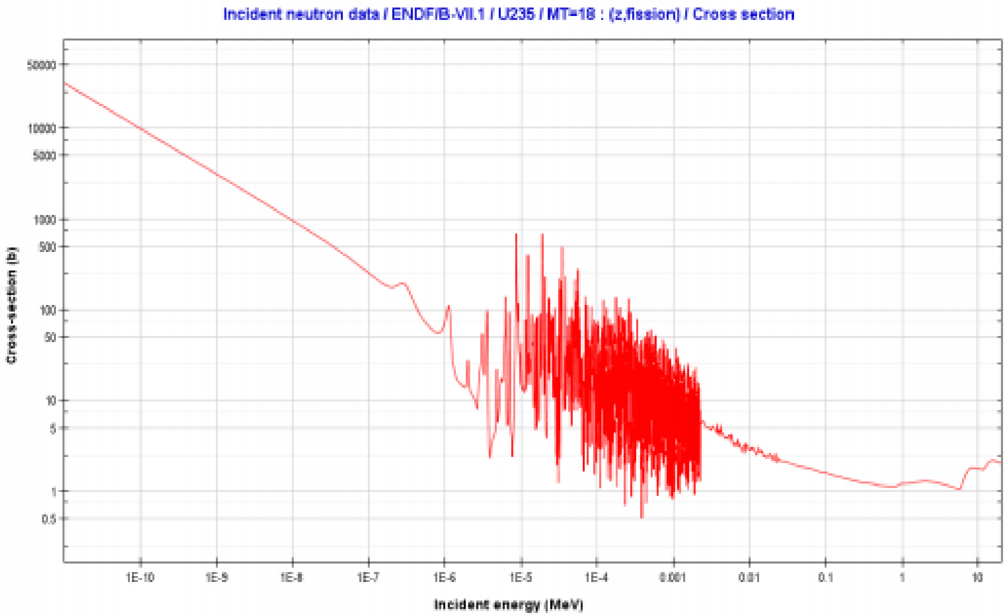
Rigorous to Simplified Neutron Balance

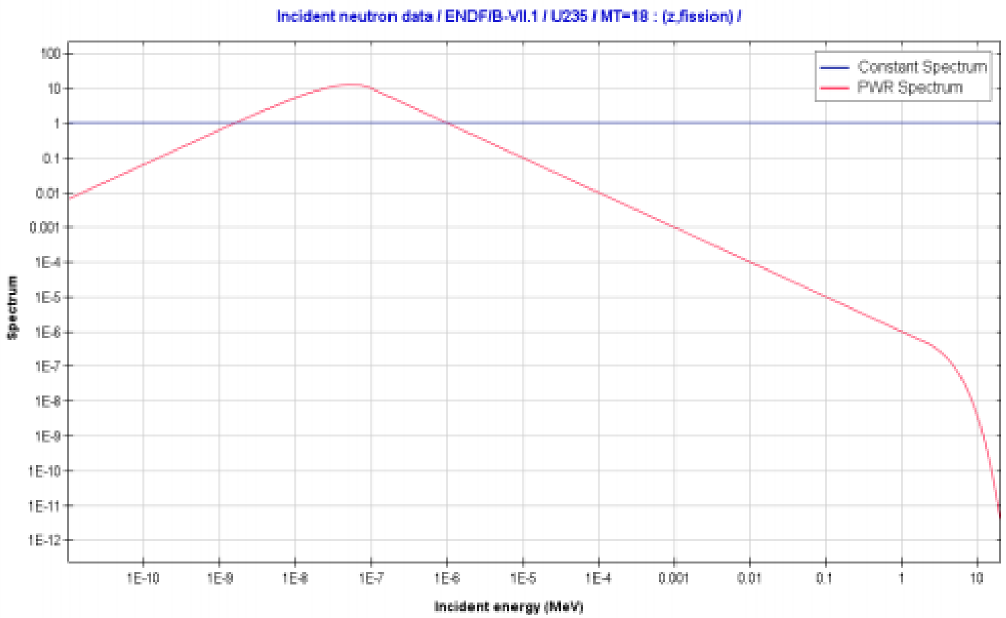
- The multi-group transport equation can be solved only if
  - *Microscopic cross-sections are known (Available from various resources)*
  - *Prior knowledge of f(E) (Usually not known!)*
- f(E) is the shaping function;
  - Since the flux is not known we usually choose f(E) for large number of energy groups and typical neutron spectrum of the reactor:

$$f(E) = \frac{1}{\Delta E}$$

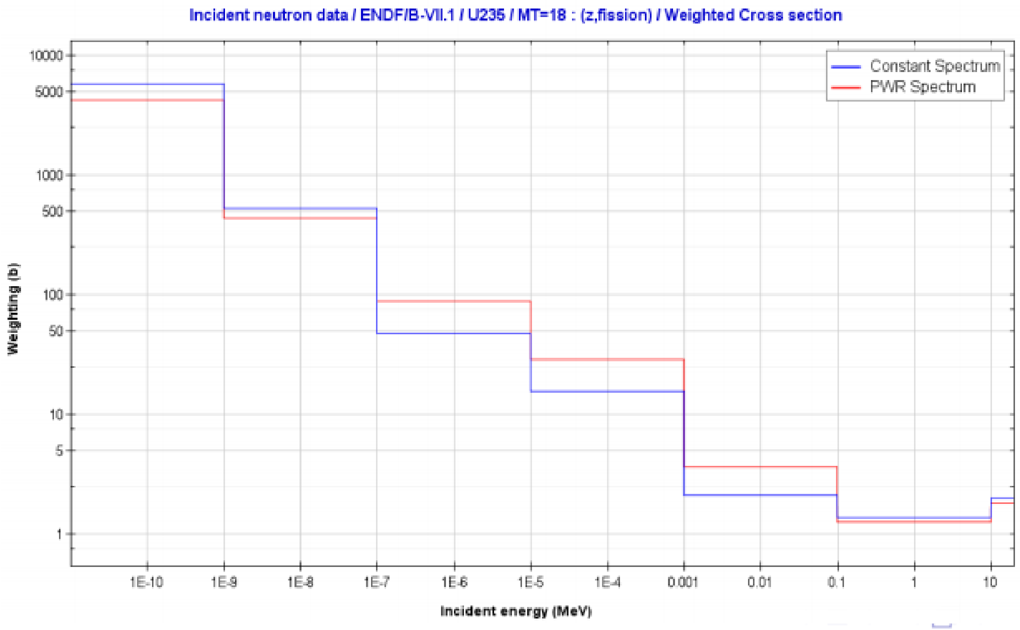
  - Will only be valid if large number of energy groups are chosen
- With the known
  - shaping function, microscopic cross-section and the material information
  - multi-group neutron transport equation can be solved to calculate the **group angular fluxes.**
- **The calculation of f(E) is still a computationally expensive task**
- *Reason to use two step neutronics analysis.*

Rigorous to Simplified Neutron Balance





Shaping function affects neutronics calculation results



Rigorous to Simplified Neutron Balance

- Let's begin from the multi-group NTE

$$\left[\vec{\Omega}\cdot\vec{\nabla}+\Sigma_{t,g}\right]\psi_g=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}+\sum_{g'=1}^{G}\int_{4\pi}d\Omega'\Sigma_{s,g'\to g}\psi_{g'}$$

- The integration over the solid angle will make the above NTE angle independent
  - 1st term

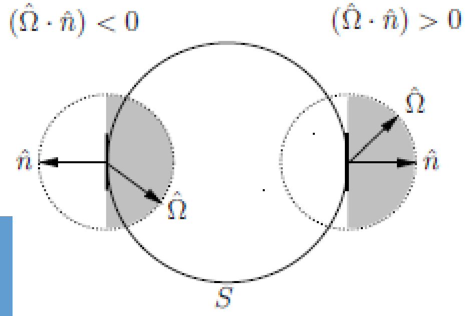
$$\int_{\Omega}\left(\vec{\Omega}\cdot\vec{\nabla}\right)\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}=\vec{\nabla}.\int_{\Omega}\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}$$

  - Remember the definition of partial currents

$$\overrightarrow{J^+}(r)=\int_{\vec{n}\cdot\vec{\Omega}>0}\vec{n}\cdot\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}\qquad\overrightarrow{J^-}(r)=\int_{\vec{n}\cdot\vec{\Omega}<0}-\vec{n}\cdot\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}$$

$$\vec{n}.\vec{J}(r)=\overrightarrow{J^+}(r)-\overrightarrow{J^-}(r)=\int_{\vec{n}\cdot\vec{\Omega}>0}\vec{n}\cdot\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}-\int_{\vec{n}\cdot\vec{\Omega}<0}-\vec{n}\cdot\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}$$

$$\vec{J}(r)=\int_{\Omega}\vec{\Omega}\psi_g(\vec{r},\vec{\Omega})d\vec{\Omega}$$



$$\boxed{\vec{\nabla}.\vec{J}_g(\vec{r})+\Sigma_{t,g}(\vec{r})\phi_g(\vec{r})=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r})}$$

Rigorous to Simplified Neutron Balance

$$\vec{\nabla}.\vec{J}_g(\vec{r})+\Sigma_{t,g}(\vec{r})\phi_g(\vec{r})=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r})$$

- Let's apply the diffusion approximation

$$\vec{J}_g(\vec{r})=-D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})$$

$$\vec{\nabla}.-D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})+\Sigma_{t,g}(\vec{r})\phi_g(\vec{r})=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r})$$

$$-D_g(\vec{r})\left(\vec{\nabla}.\vec{\nabla}\right)\phi_g(\vec{r})+\Sigma_{t,g}(\vec{r})\phi_g(\vec{r})=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r})$$

Multi-group neutron diffusion equation is:

$$-D_g(\vec{r})\nabla^2\phi_g(\vec{r})+\Sigma_{t,g}(\vec{r})\phi_g(\vec{r})=\chi_g\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r})$$

To solve this equation we need spatial group constants

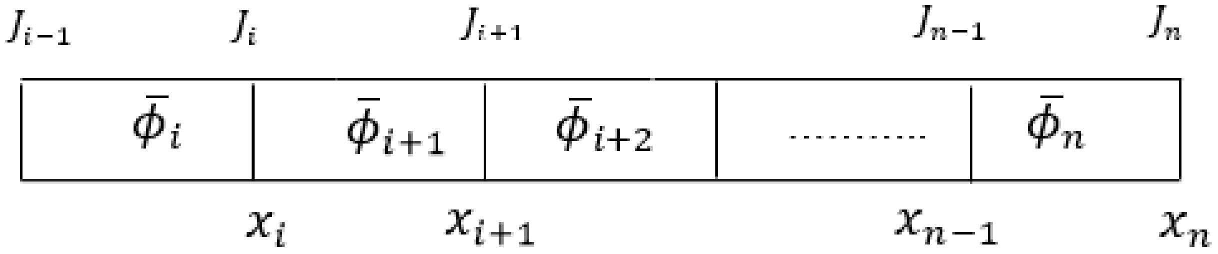
Numerical Methods to Solve NDE

- A numerical method can be chosen considering
  a. Effectiveness of the method (extendable being transient, multi-dimensions, acceleration)
  b. Flexibility of the method (different geometries, different scenarios)
  c. Error analysis on the result (Nearly always impossible or incase very difficult !!)
  d. Theoretical basis (Strongly complemented by rigorous mathematics)
  e. Solution convergence with mesh refinement (More accurate results with mesh refinement)
  f. Stability of the method (Fourier Transform)
- Solution methods can be broadly categorized in the following:
  1. **Finite difference methods (FDM) – will be discussed in detail (use point fluxes)**
  2. Flux synthesis methods (FSM) – (use point fluxes)
  3. Finite-element methods (FEM) – (use expansion coefficients)
  4. Response matrix methods (RMM) – (use partial currents)
  5. Nodal methods (NM) – (use node average fluxes)
     - Nodal expansion method – will be discussed if the time permits
     - Analytical nodal method etc.

Solution of One-Dimensional NDE using FDM

- Multi-group, 1-dimensional NDE is given as follows:

$$-\frac{d}{dx}D(x)\frac{d}{dx}\phi(x)+\Sigma_a(x)\phi(x)=\frac{1}{k}\nu\Sigma_f(x)\phi(x)$$



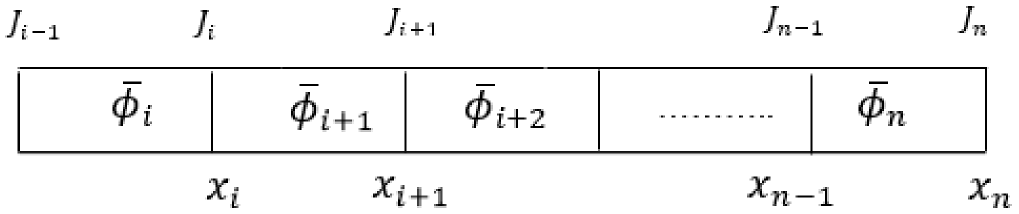
- The macroscopic cross-sections are assumed to be constant within each mesh
- We know that;

$$J(x)=-D\frac{d\phi}{dx}$$

- 1-dimensional NDE can be rewritten as;

$$\frac{d}{dx}J(x)+\Sigma_a(x)\phi(x)=\frac{1}{k}\nu\Sigma_f(x)\phi(x)$$

Solution of One-Dimensional NDE using FDM



$$\frac{d}{dx}J(x) + \Sigma_a(x)\phi(x) = \frac{1}{k}\nu\Sigma_f(x)\phi(x)$$

- Integrating the above equation in *i*

$$J_i - J_{i-1} + \int_i \Sigma_a(x)\phi(x)dx = \frac{1}{k}\int_i \nu\Sigma_f(x)\phi(x)dx$$

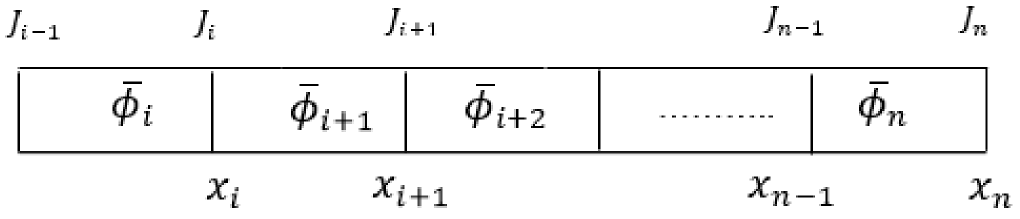
$$J_i - J_{i-1} + \Sigma_a \Delta x_i \overline{\phi_i} = \frac{1}{k}\nu\Sigma_{f_i}\Delta x_i \overline{\phi_i}$$

where $\bar{\phi}_i$ is the node average value in the above balance equation.

- *At the interface neutron current is defined as*

$$J(x_i + \epsilon) = -D_{i+1}\frac{\bar{\phi}_{i+1} - \phi(x_i)}{\frac{\Delta x_{i+1}}{2}} \quad \textit{and} \quad J(x_i - \epsilon) = -D_i\frac{\phi(x_i) - \bar{\phi}_i}{\frac{\Delta x_i}{2}}$$

Solution of One-Dimensional NDE using FDM



- Based on the neutron current continuity at the interface:

$$-D_{i+1}\frac{\bar{\phi}_{i+1}-\phi(x_i)}{\frac{\Delta x_{i+1}}{2}} = -D_i\frac{\phi(x_i)-\bar{\phi}_i}{\frac{\Delta x_i}{2}}$$

- Collecting like terms

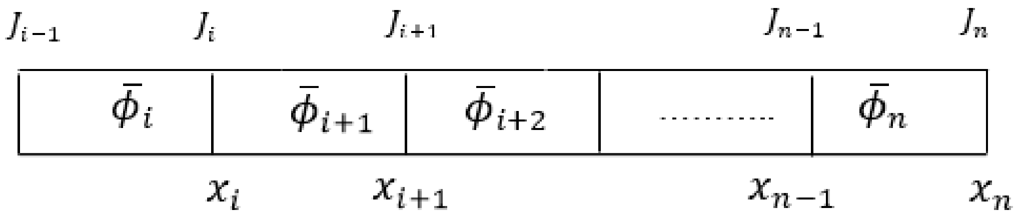
$$\phi_i\left(\frac{D_{i+1}}{\Delta x_{i+1}}+\frac{D_i}{\Delta x_i}\right) = \frac{D_{i+1}}{\Delta x_{i+1}}\bar{\phi}_{i+1}+\frac{D_i}{\Delta x_i}\bar{\phi}_i$$

- Finding interface flux in terms of average fluxes

$$\phi_i = \frac{\frac{D_{i+1}}{\Delta x_{i+1}}\bar{\phi}_{i+1}+\frac{D_i}{\Delta x_i}\bar{\phi}_i}{\left(\frac{D_{i+1}}{\Delta x_{i+1}}+\frac{D_i}{\Delta x_i}\right)}$$

Solution of One-Dimensional NDE using FDM

$$\phi_i = \frac{\dfrac{D_{i+1}}{\Delta x_{i+1}}\bar{\phi}_{i+1} + \dfrac{D_i}{\Delta x_i}\bar{\phi}_i}{\left(\dfrac{D_{i+1}}{\Delta x_{i+1}} + \dfrac{D_i}{\Delta x_i}\right)}$$



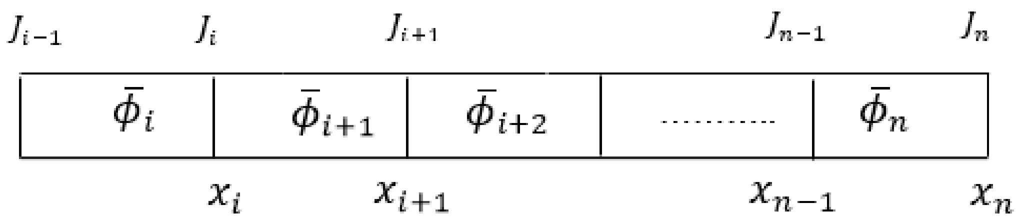
- Substituting the above equation in the neutron current at the surface equations

$$J_i = -\frac{D_{i+1}}{\dfrac{\Delta x_{i+1}}{2}}\bar{\phi}_{i+1} + \frac{D_{i+1}}{\dfrac{\Delta x_{i+1}}{2}}\,\frac{\dfrac{D_{i+1}}{\Delta x_{i+1}}\bar{\phi}_{i+1} + \dfrac{D_i}{\Delta x_i}\bar{\phi}_i}{\left(\dfrac{D_{i+1}}{\Delta x_{i+1}} + \dfrac{D_i}{\Delta x_i}\right)}$$

$$J_i = \frac{-\dfrac{D_{i+1}^2}{\Delta x_{i+1}}\bar{\phi}_{i+1} - D_{i+1}\dfrac{D_i}{\Delta x_i}\bar{\phi}_{i+1} + \dfrac{D_{i+1}^2}{\Delta x_{i+1}}\bar{\phi}_{i+1} + D_{i+1}\dfrac{D_i}{\Delta x_i}\bar{\phi}_i}{\dfrac{\Delta x_{i+1}}{2}\left(\dfrac{D_{i+1}}{\Delta x_{i+1}} + \dfrac{D_i}{\Delta x_i}\right)}$$

$$J_i = \frac{\cancel{-\dfrac{D_{i+1}^2}{\Delta x_{i+1}}\bar{\phi}_{i+1}} - D_{i+1}\dfrac{D_i}{\Delta x_i}\bar{\phi}_{i+1} + \cancel{\dfrac{D_{i+1}^2}{\Delta x_{i+1}}\bar{\phi}_{i+1}} + D_{i+1}\dfrac{D_i}{\Delta x_i}\bar{\phi}_i}{\dfrac{\Delta x_{i+1}}{2}\left(\dfrac{D_{i+1}}{\Delta x_{i+1}} + \dfrac{D_i}{\Delta x_i}\right)} = \frac{D_{i+1}\dfrac{D_i}{\cancel{\Delta x_i}}\left(\bar{\phi}_i - \bar{\phi}_{i+1}\right)}{\left(D_i\Delta x_{i+1} + D_{i+1}\Delta x_i\right)\Big/ 2\cancel{\Delta x}_i} = \frac{2D_{i+1}D_i\left(\bar{\phi}_i - \bar{\phi}_{i+1}\right)}{\left(D_i\Delta x_{i+1} + D_{i+1}\Delta x_i\right)}$$

Solution of One-Dimensional NDE using FDM



- Henceforth J_i and J_{i-1} will be

$$J_i = \frac{2D_{i+1}D_i\left(\bar{\phi}_i - \bar{\phi}_{i+1}\right)}{\left(D_i\Delta x_{i+1} + D_{i+1}\Delta x_i\right)} \qquad J_{i-1} = \frac{2D_iD_{i-1}\left(\bar{\phi}_{i-1} - \bar{\phi}_i\right)}{\left(D_{i-1}\Delta x_{i-1} + D_i\Delta x_i\right)}$$

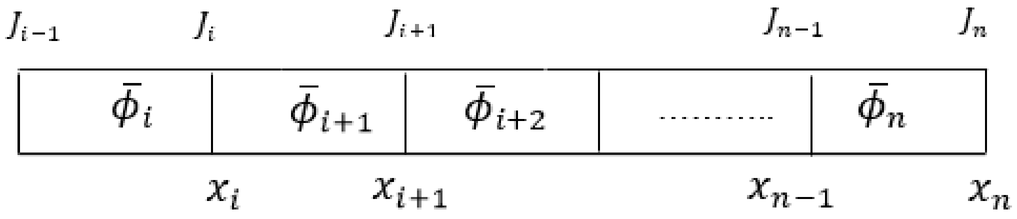
- Balance equation would be

$$\frac{2D_{i+1}D_i\left(\bar{\phi}_i - \bar{\phi}_{i+1}\right)}{\left(D_i\Delta x_{i+1} + D_{i+1}\Delta x_i\right)} - \frac{2D_iD_{i-1}\left(\bar{\phi}_{i-1} - \bar{\phi}_i\right)}{\left(D_{i-1}\Delta x_{i-1} + D_i\Delta x_i\right)} + \Sigma_{a_i}\Delta x_i\bar{\phi}_i = \frac{1}{k}\nu\Sigma_{f_i}\Delta x_i\bar{\phi}_i$$

- Rewriting;

$$\frac{2D_{i+1}D_i}{D_i\Delta x_{i+1} + D_{i+1}\Delta x_i}\bar{\phi}_i - \frac{2D_{i+1}D_i}{D_i\Delta x_{i+1} + D_{i+1}\Delta x_i}\bar{\phi}_{i+1} - \frac{2D_iD_{i-1}}{D_{i-1}\Delta x_{i-1} + D_i\Delta x_i}\bar{\phi}_{i-1} + \frac{2D_iD_{i-1}}{D_{i-1}\Delta x_{i-1} + D_i\Delta x_i}\bar{\phi}_i + \Sigma_{a_i}\Delta x_i\bar{\phi}_i = \frac{1}{k}\nu\Sigma_{f_i}\Delta x_i\bar{\phi}_i$$

Solution of One-Dimensional NDE using FDM



$$\frac{2D_{i+1}D_i}{D_i\Delta x_{i+1}+D_{i+1}\Delta x_i}\bar{\phi}_i-\frac{2D_{i+1}D_i}{D_i\Delta x_{i+1}+D_{i+1}\Delta x_i}\bar{\phi}_{i+1}-\frac{2D_iD_{i-1}}{D_{i-1}\Delta x_{i-1}+D_i\Delta x_i}\bar{\phi}_{i-1}+\frac{2D_iD_{i-1}}{D_{i-1}\Delta x_{i-1}+D_i\Delta x_i}\bar{\phi}_i+\Sigma_{a_i}\Delta x_i\bar{\phi}_i=\frac{1}{k}\nu\Sigma_{f_i}\Delta x_i\bar{\phi}_i$$

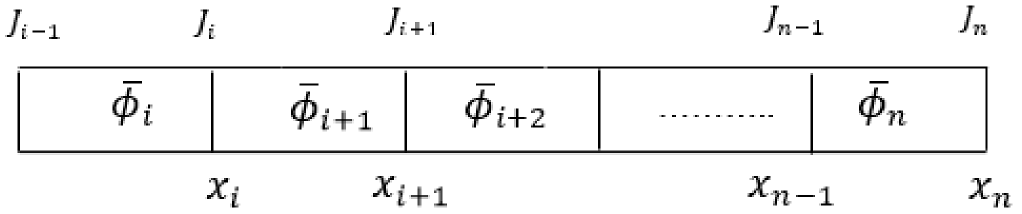
- The above is in the form of three point difference equation can be rewritten as;

$$a_i^L\bar{\phi}_{i-1}+b_i\bar{\phi}_i+a_i^R\bar{\phi}_{i+1}=\frac{1}{k}d_i\bar{\phi}_i$$

where

$$a_i^L=-\frac{2D_iD_{i-1}}{D_{i-1}\Delta x_{i-1}+D_i\Delta x_i},\quad a_i^R=-\frac{2D_{i+1}D_i}{D_i\Delta x_{i+1}+D_{i+1}\Delta x_i},\quad b_i=\Sigma_{a_i}\Delta x_i-a_i^L-a_i^R,\quad d_i=\nu\Sigma_{f_i}\Delta x_i$$

Solution of One-Dimensional NDE using FDM



- The boundaries need special treatment for the three point difference equation;

<u>Flux Zero Boundary Condition</u>

$$a_i^L \bar{\phi}_{i-1} + b_i \bar{\phi}_i + a_i^R \bar{\phi}_{i+1} = \frac{1}{k} d_i \bar{\phi}_i$$

- The three point difference equation for left hand side boundary will modified such that

$$\bar{\phi}_{i-1} = 0$$

$$b_i \bar{\phi}_i + a_i^R \bar{\phi}_{i+1} = \frac{1}{k} d_i \bar{\phi}_i$$

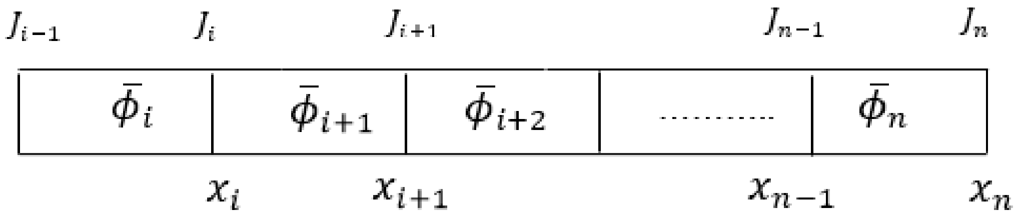
- The three point difference equation for left hand side boundary will modified such that

$$\bar{\phi}_{i+1} = 0$$

$$a_i^L \bar{\phi}_{i-1} + b_i \bar{\phi}_i = \frac{1}{k} d_i \bar{\phi}_i$$

- Try to calculate the changes in coefficients by yourself.

Solution of One-Dimensional NDE using FDM



- The boundaries need special treatment for the three point difference equation;

<u>Current Zero Boundary Condition or Reflected Boundary Condition</u>

$$a_i^L \bar{\phi}_{i-1} + b_i \bar{\phi}_i + a_i^R \bar{\phi}_{i+1} = \frac{1}{k} d_i \bar{\phi}_i$$

- The three point difference equation for the left hand side boundary will modified such that

$$\bar{\phi}_{i-1} = \bar{\phi}_i$$

$$\left(a_i^L + b_i\right) \bar{\phi}_i + a_i^R \bar{\phi}_{i+1} = \frac{1}{k} d_i \bar{\phi}_i$$

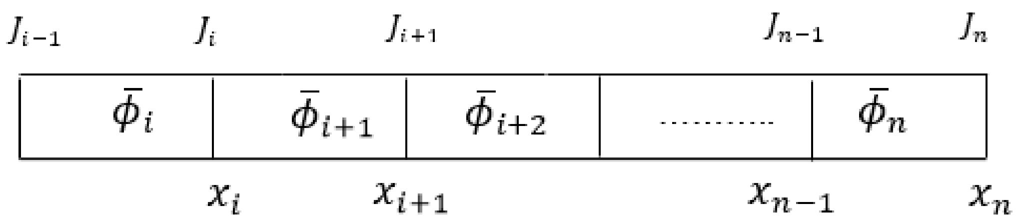
- The three point difference equation for the left hand side boundary will modified such that

$$\bar{\phi}_i = \bar{\phi}_{i+1}$$

$$a_i^L \bar{\phi}_{i-1} + \left(b_i + a_i^R\right) \bar{\phi}_i = \frac{1}{k} d_i \bar{\phi}_i$$

- Try to calculate the changes in coefficients by yourself.

Solution of One-Dimensional NDE using FDM



- Consider all the boundary conditions are known, one can solve this system of equations.
- Remember, each node has a single equation with two unknowns;
  - Flux of that particular node (Local value)
  - Neutron multiplication factor which is eigenvalue
- To solve this we need to use the power method to solve this Eigen-value problem

Global Value (Unknown)

For the 1st Node

BC → $\boxed{a_1^L \bar{\phi}_0} + b_1 \bar{\phi}_1 + \boxed{a_1^R \bar{\phi}_2} = \frac{1}{k} d_1 \bar{\phi}_1$ → Guess

For the 2nd Node

Previous Node → $\boxed{a_2^L \bar{\phi}_1} + b_2 \bar{\phi}_2 + \boxed{a_2^R \bar{\phi}_3} = \frac{1}{k} d_2 \bar{\phi}_2$ → Guess

For the 3rd Node

Previous Node → $\boxed{a_3^L \bar{\phi}_2} + b_3 \bar{\phi}_3 + \boxed{a_3^R \bar{\phi}_4} = \frac{1}{k} d_3 \bar{\phi}_3$ → Guess

⋮

For the ith Node

Previous Node → $\boxed{a_i^L \bar{\phi}_{i-1}} + b_i \bar{\phi}_i + \boxed{a_i^R \bar{\phi}_{i+1}} = \frac{1}{k} d_i \bar{\phi}_i$ → Guess

⋮

For the nth Node

Previous Node → $\boxed{a_n^L \bar{\phi}_{n-1}} + b_n \bar{\phi}_n + \boxed{a_n^R \bar{\phi}_{n+1}} = \frac{1}{k} d_n \bar{\phi}_n$ → BC

Solution of One-Dimensional NDE using FDM

Transformation to a fixed source problem

For the 1st Node

$$a_1^L\bar{\phi}_0 + b_1\bar{\phi}_1 + a_1^R\bar{\phi}_2 = \frac{1}{k}d_1\bar{\phi}_1$$

For the 2nd Node

$$a_2^L\bar{\phi}_1 + b_2\bar{\phi}_2 + a_2^R\bar{\phi}_3 = \frac{1}{k}d_2\bar{\phi}_2$$

For the 3rd Node

$$a_3^L\bar{\phi}_2 + b_3\bar{\phi}_3 + a_3^R\bar{\phi}_4 = \frac{1}{k}d_3\bar{\phi}_3$$

For the ith Node

$$a_i^L\bar{\phi}_{i-1} + b_i\bar{\phi}_i + a_i^R\bar{\phi}_{i+1} = \frac{1}{k}d_i\bar{\phi}_i$$

For the nth Node

$$a_n^L\bar{\phi}_{n-1} + b_n\bar{\phi}_n + a_n^R\bar{\phi}_{n+1} = \frac{1}{k}d_n\bar{\phi}_n$$

For the 1st Node

BC → $\boxed{a_1^L\bar{\phi}_0} + b_1\bar{\phi}_1 + \boxed{a_1^R\bar{\phi}_2} = S_1$ → Guess

For the 2nd Node

Previous Node → $\boxed{a_2^L\bar{\phi}_1} + b_2\bar{\phi}_2 + \boxed{a_2^R\bar{\phi}_3} = S_2$ → Guess

For the 3rd Node

Previous Node → $\boxed{a_3^L\bar{\phi}_2} + b_3\bar{\phi}_3 + \boxed{a_3^R\bar{\phi}_4} = S_3$ → Guess

⋮

For the ith Node

Previous Node → $\boxed{a_i^L\bar{\phi}_{i-1}} + b_i\bar{\phi}_i + \boxed{a_i^R\bar{\phi}_{i+1}} = S_i$ → Guess

⋮

For the nth Node

Previous Node → $\boxed{a_n^L\bar{\phi}_{n-1}} + b_n\bar{\phi}_n + \boxed{a_n^R\bar{\phi}_{n+1}} = S_n$ → BC

Solution of One-Dimensional NDE using FDM

- Please note that it's a fixed source not an Eigen-value problem anymore.
- Therefore, the following system of linear equations need to be solved only

$$\underline{\underline{A}}\,\underline{\bar{\phi}} = \underline{S}$$

$$\begin{bmatrix} b_1 & a_1^R & 0 & \cdots & 0 \\ a_2^L & b_2 & a_2^R & 0 & \vdots \\ 0 & a_3^L & b_3 & a_3^R & 0 \\ \vdots & \ddots & \ddots & \ddots & a_{n-1}^R \\ 0 & \cdots & 0 & a_n^L & b_n \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \bar{\phi}_3 \\ \vdots \\ \bar{\phi}_n \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ S_n \end{bmatrix}$$

- Solve this matrix using Gauss-elimination method, Gauss-Seidel or any other iterative scheme.

Solution of One-Dimensional NDE using FDM

For the 1st Node

BC → $\boxed{a_1^L\bar{\phi}_0} + b_1\bar{\phi}_1 + \boxed{a_1^R\bar{\phi}_2} = \frac{1}{k}d_1\bar{\phi}_1$

Global Value (Unknown)

Guess

For the 2nd Node

Previous Node → $\boxed{a_2^L\bar{\phi}_1} + b_2\bar{\phi}_2 + \boxed{a_2^R\bar{\phi}_3} = \frac{1}{k}d_2\bar{\phi}_2$

Guess

For the 3rd Node

Previous Node → $\boxed{a_3^L\bar{\phi}_2} + b_3\bar{\phi}_3 + \boxed{a_3^R\bar{\phi}_4} = \frac{1}{k}d_3\bar{\phi}_3$

Guess

⋮

For the ith Node

Previous Node → $\boxed{a_i^L\bar{\phi}_{i-1}} + b_i\bar{\phi}_i + \boxed{a_i^R\bar{\phi}_{i+1}} = \frac{1}{k}d_i\bar{\phi}_i$

Guess

⋮

For the nth Node

Previous Node → $\boxed{a_n^L\bar{\phi}_{n-1}} + b_n\bar{\phi}_n + \boxed{a_n^R\bar{\phi}_{n+1}} = \frac{1}{k}d_n\bar{\phi}_n$

BC

- The main idea is to suppose guess the flux and k-eff for the first iteration.
- Solve the system of linear equations as a fixed source problem as the flux and k-eff are fixed.
- After the inner iteration, we can calculate the Eigen-vector and Eigen-value using the power method.
- Power method is used to calculate the dominant Eigen-value and corresponding Eigen-vectors.

Solution of One-Dimensional NDE using FDM

- The nuclear reactor diffusion Eigen-value problem has the mathematical form of the following

$$\underline{\underline{M}}\,\underline{\bar{\phi}} = \frac{1}{k}\underline{\underline{F}}\,\underline{\bar{\phi}}$$

$$\begin{bmatrix} b_1 & a_1^R & 0 & \cdots & 0 \\ a_2^L & b_2 & a_2^R & 0 & \vdots \\ 0 & a_3^L & b_3 & a_3^R & 0 \\ \vdots & \ddots & \ddots & \ddots & a_{n-1}^R \\ 0 & \cdots & 0 & a_n^L & b_n \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \bar{\phi}_3 \\ \vdots \\ \bar{\phi}_n \end{bmatrix} = \frac{1}{k} \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & \cdots & d_3 & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & d_n \end{bmatrix} \begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \bar{\phi}_3 \\ \vdots \\ \bar{\phi}_n \end{bmatrix}$$

- Let's recall the power method to solve the Eigen-value problems; for dominant Eigen-value and corresponding Eigen-vector

$$\underline{\underline{A}}\,\underline{x} = \lambda\,\underline{x}$$

$$\lambda^{(1)}x^{(1)} = Ax^{(0)}$$
$$\lambda^{(2)}x^{(2)} = Ax^{(1)}$$
$$\vdots$$
$$\lambda^{(n)}x^{(n)} = Ax^{(n-1)}$$

$$\rightarrow$$

$$\boxed{\varepsilon_\lambda\left(10^{-6}\right) \le \frac{\left|\lambda^n - \lambda^{n-1}\right|}{\lambda^n}}$$

$$\boxed{\varepsilon_x\left(10^{-5}\right) \le max/avg\left[\frac{\left|\underline{x}^n - \underline{x}^{n-1}\right|}{\underline{x}^n}\right]}$$

Solution of One-Dimensional NDE using FDM

- For the very first iteration of power method

$$\underline{\underline{M}}\,\underline{\bar{\phi}}^{(\mathrm{Calc})} = \frac{1}{k^{(guess)}}\underline{\underline{F}}\,\underline{\bar{\phi}}^{(guess)} \Rightarrow \underline{\underline{M}}\,\underline{\bar{\phi}}^{(1)} = \frac{1}{k^{(0)}}\underline{\underline{S}}^{(0)}$$

- It is not wrong to say that the above equation is a fixed source problem and linear equation solver can be used to calculate the calculated flux. Based on this calculated flux new Source term can be calculated

$$S^{(1)} = \nu\Sigma_f\,\bar{\phi}^{(1)}$$

- In the next iteration;

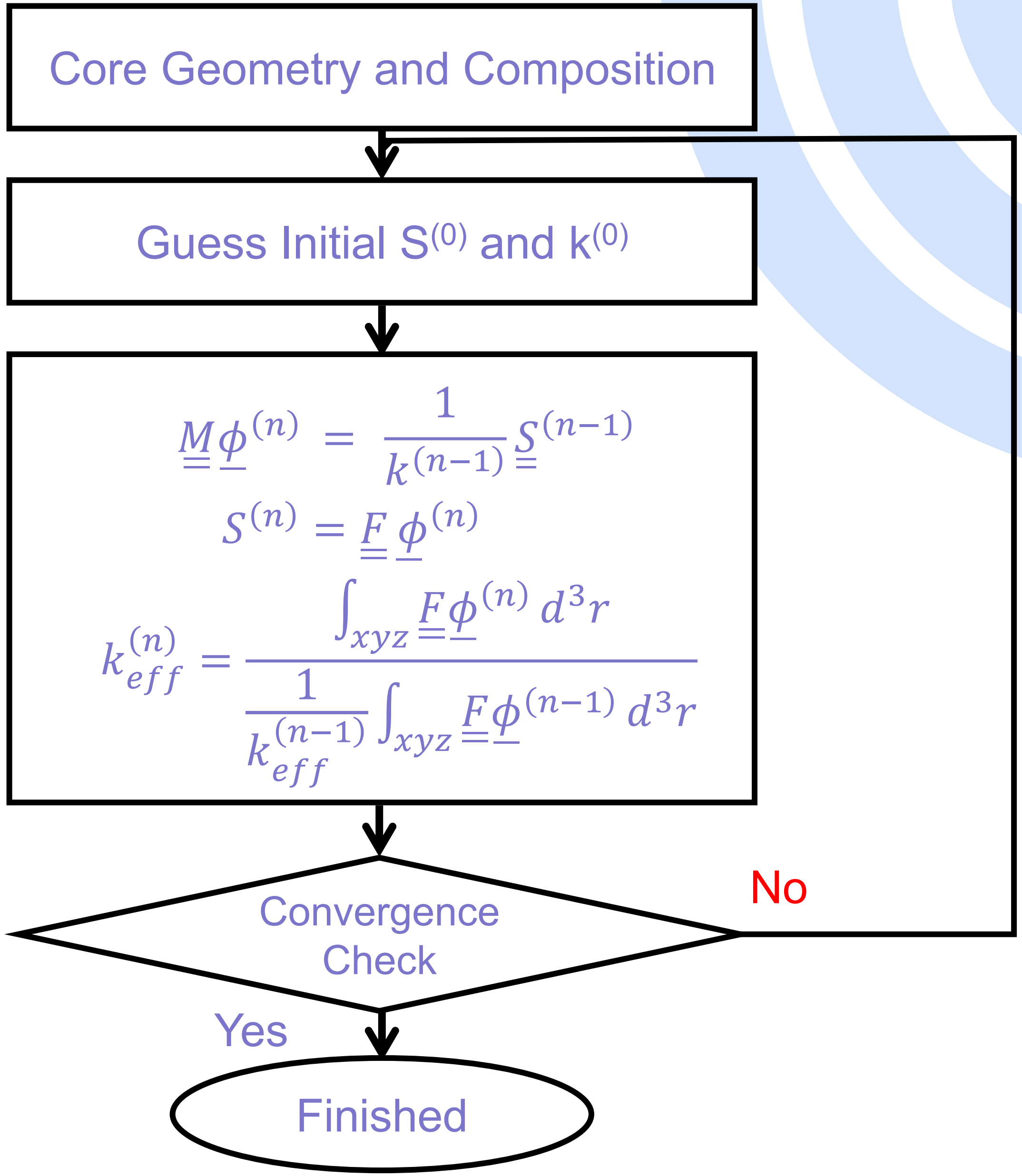
$$\underline{\underline{M}}\,\underline{\bar{\phi}}^{(2)} = \frac{1}{k^{(1)}}\underline{\underline{S}}^{(1)} \Rightarrow \underline{\underline{M}}\,\underline{\bar{\phi}}^{(n)} = \frac{1}{k^{(n-1)}}\underline{\underline{S}}^{(\mathrm{n}-1)}$$

$$k_{eff}^{(n)} = \frac{\int\limits_{xyz}\underline{\underline{F}}\,\underline{\bar{\phi}}^{(n)}d^3r}{\int\limits_{xyz}\underline{\underline{M}}\,\underline{\bar{\phi}}^{(n)}d^3r} \Rightarrow k_{eff}^{(n)} = \frac{\int\limits_{xyz}\underline{\underline{F}}\,\underline{\bar{\phi}}^{(n)}d^3r}{\dfrac{1}{k_{eff}^{(n-1)}}\int\limits_{xyz}\underline{\underline{F}}\,\underline{\bar{\phi}}^{(n-1)}d^3r}$$

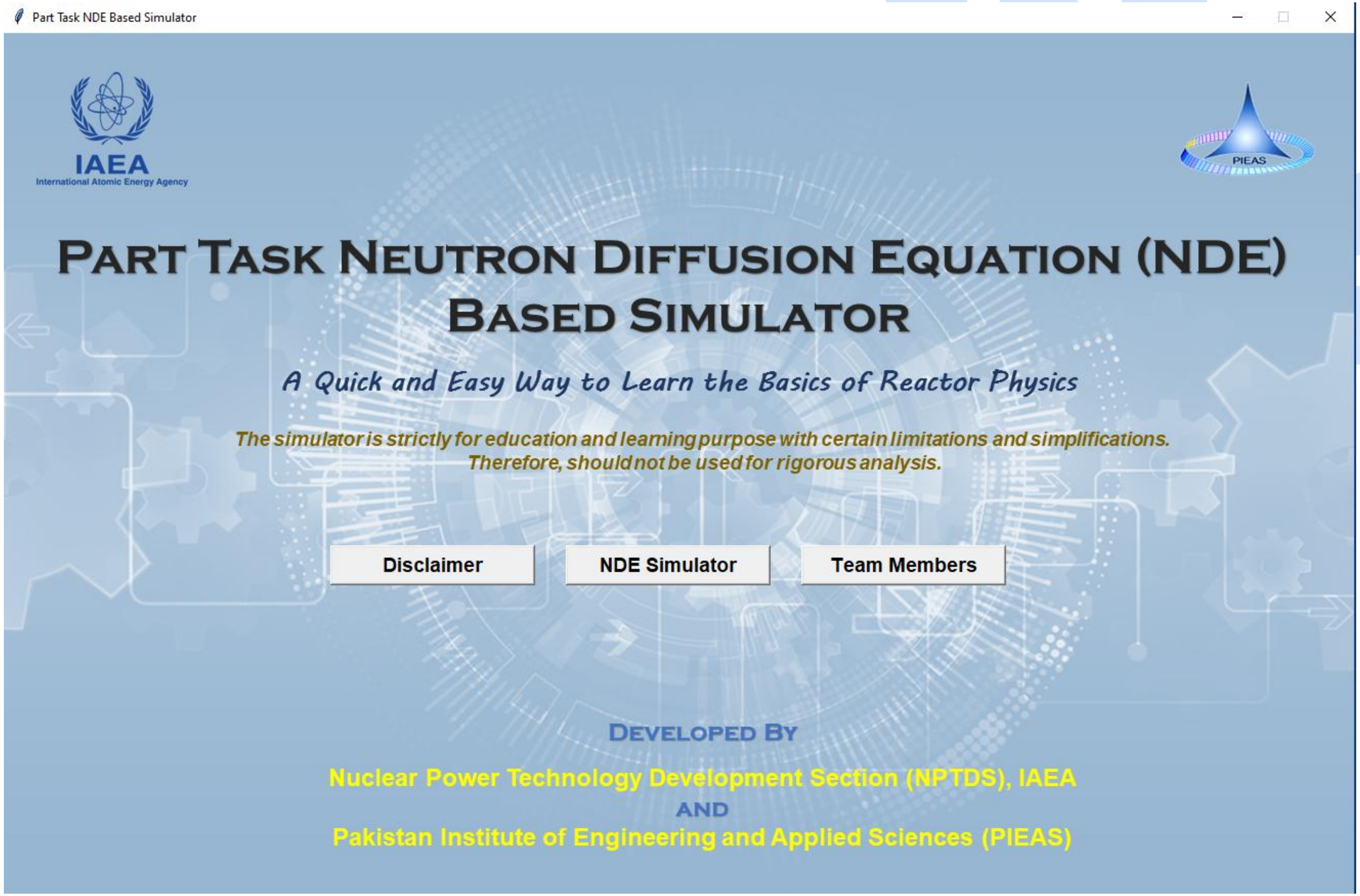
- The procedure will be repeated until;

$$\boxed{\varepsilon_{k-eff}\left(10^{-8}\right) \le \frac{\left|k_{eff}^{n} - k_{eff}^{n-1}\right|}{k_{eff}^{n}}\ ,\ \varepsilon_{\phi}\left(10^{-5}\right) \le\ max/avg\left[\frac{\left|\underline{\bar{\phi}}^{n} - \underline{\bar{\phi}}^{n-1}\right|}{\underline{\bar{\phi}}^{n}}\right]}$$

Solution of One-Dimensional NDE using FDM



IAEA Part-Task Simulator



Reactor Criticality

A critical neutron system is needed. This part discusses the factors that can affect the neutron population in the system and results in reactor's criticality

- Reactor design does incorporate all the factors that contribute: *Positively or negatively* towards the growth of neutrons in the multiplying medium.

+

1. Reproduction factor

$$\eta = \frac{No.of\ fast\ neutrons\ produced\ due\ to\ thermal\ fission}{No.of\ thermal\ neutrons\ absorbed\ in\ the\ fuel}$$

2. Fast Fission Factor

$$\varepsilon = \frac{No.of\ n's\ emitted\ fast\ fission + No.of\ n's\ emited\ by\ thermal\ fission}{No.of\ n's\ emitted\ by\ thermal\ fissions}$$

−

1. Thermal utilization factor

$$f = \frac{Thermal\ neutrons\ absorbed\ in\ fuel}{Thermal\ neutrons\ absorbed\ in\ the\ whole\ system}$$

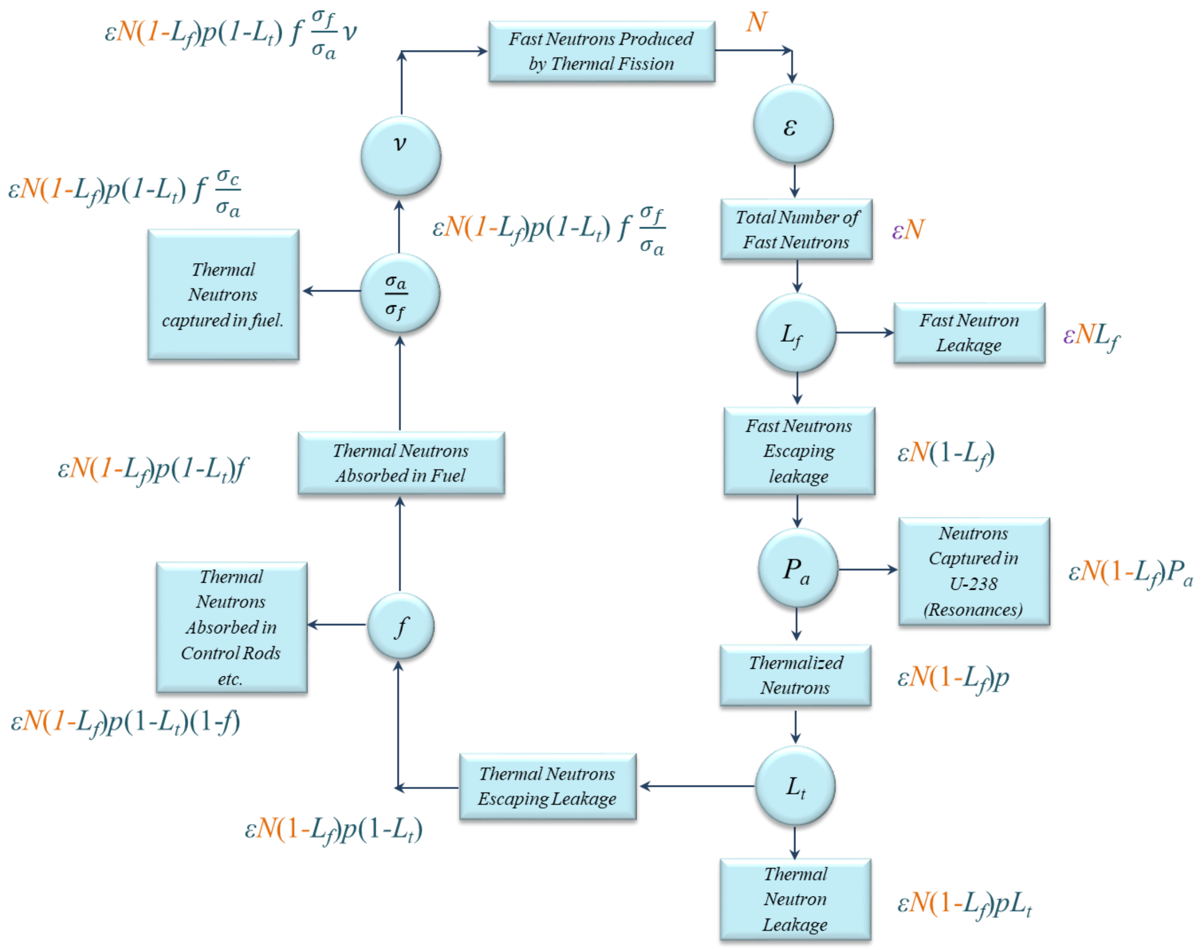
2. Resonance absorption

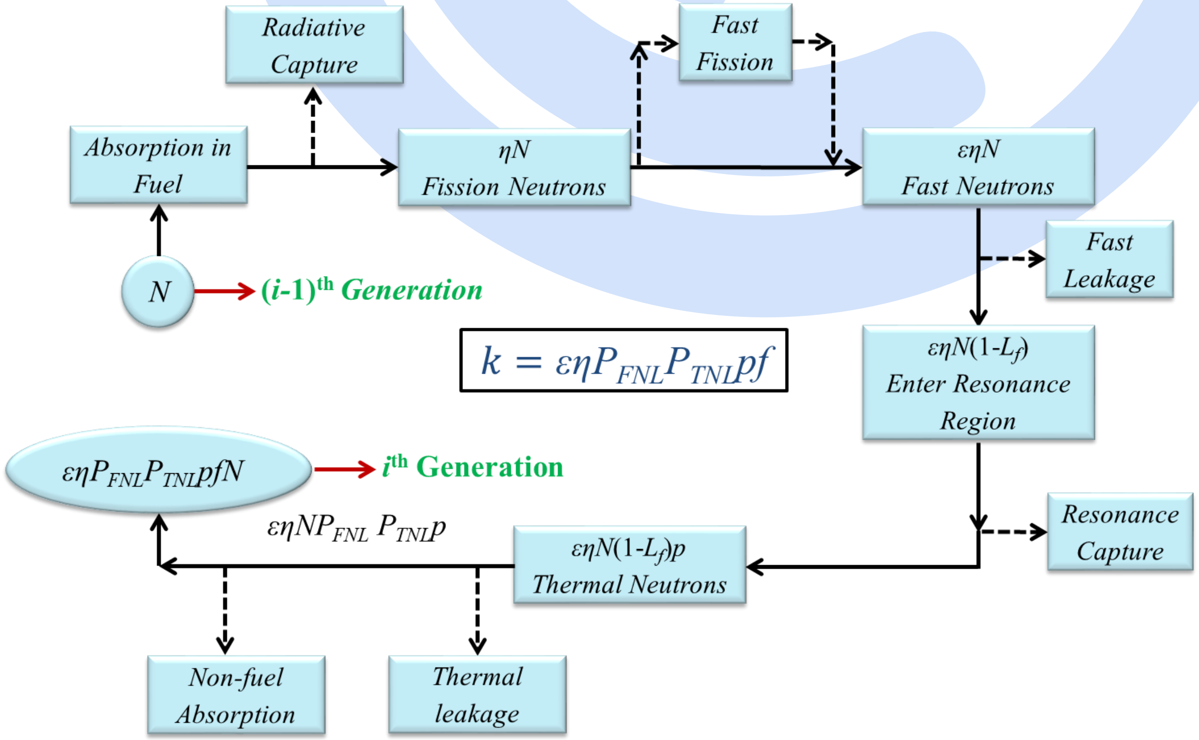
$$P_a = \frac{Fission\ neutrons\ absorbed\ in\ resonances}{Total\ fission\ neutrons}$$

3. Leakages

$$\frac{Fast\ neutrons\ or\ thermal\ neutrons\ leaked\ from\ thecore}{Total\ neutrons}$$

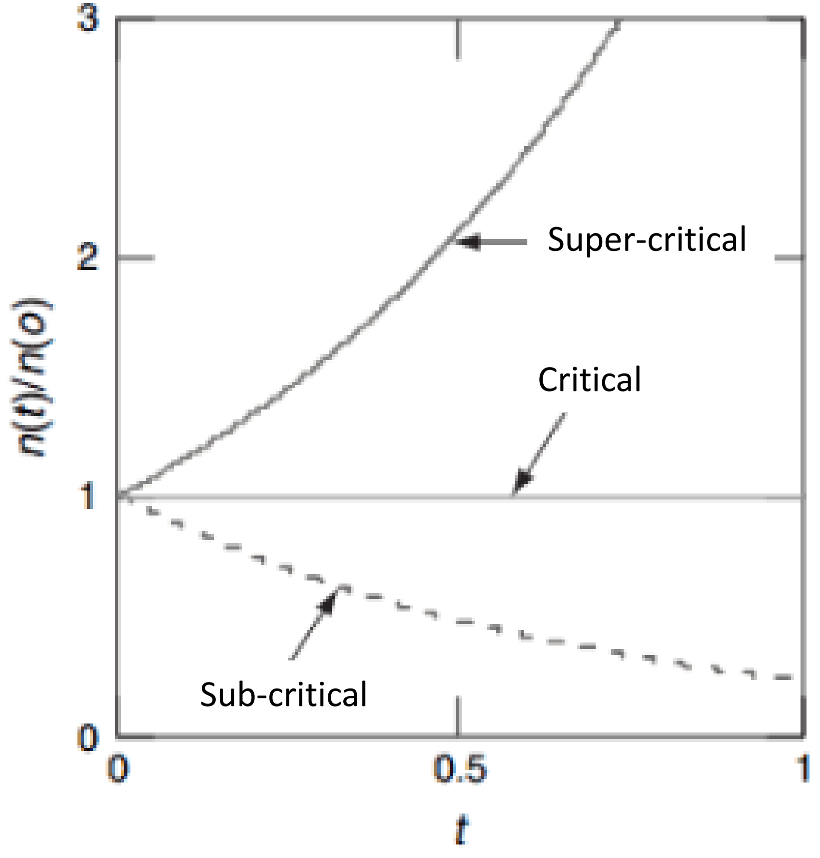
Reactor Criticality





$$k = \frac{Total\ number\ of\ neutrons\ absorbed\ in\ one\ generation\ (N_i)}{Total\ number\ of\ neutrons\ absorbed\ in\ the\ immediately\ preceeding\ generation(N_{i-1})}$$

Reactor Criticality



Super-critical

- Chain reaction is divergent
- Number of fissions grow overtime
- Degree of super-criticality is

$k_{ex}= k$-1 (**Excess Reactivity Concept**)

Critical

- Chain reaction is stable
- Number of fissions remain constant

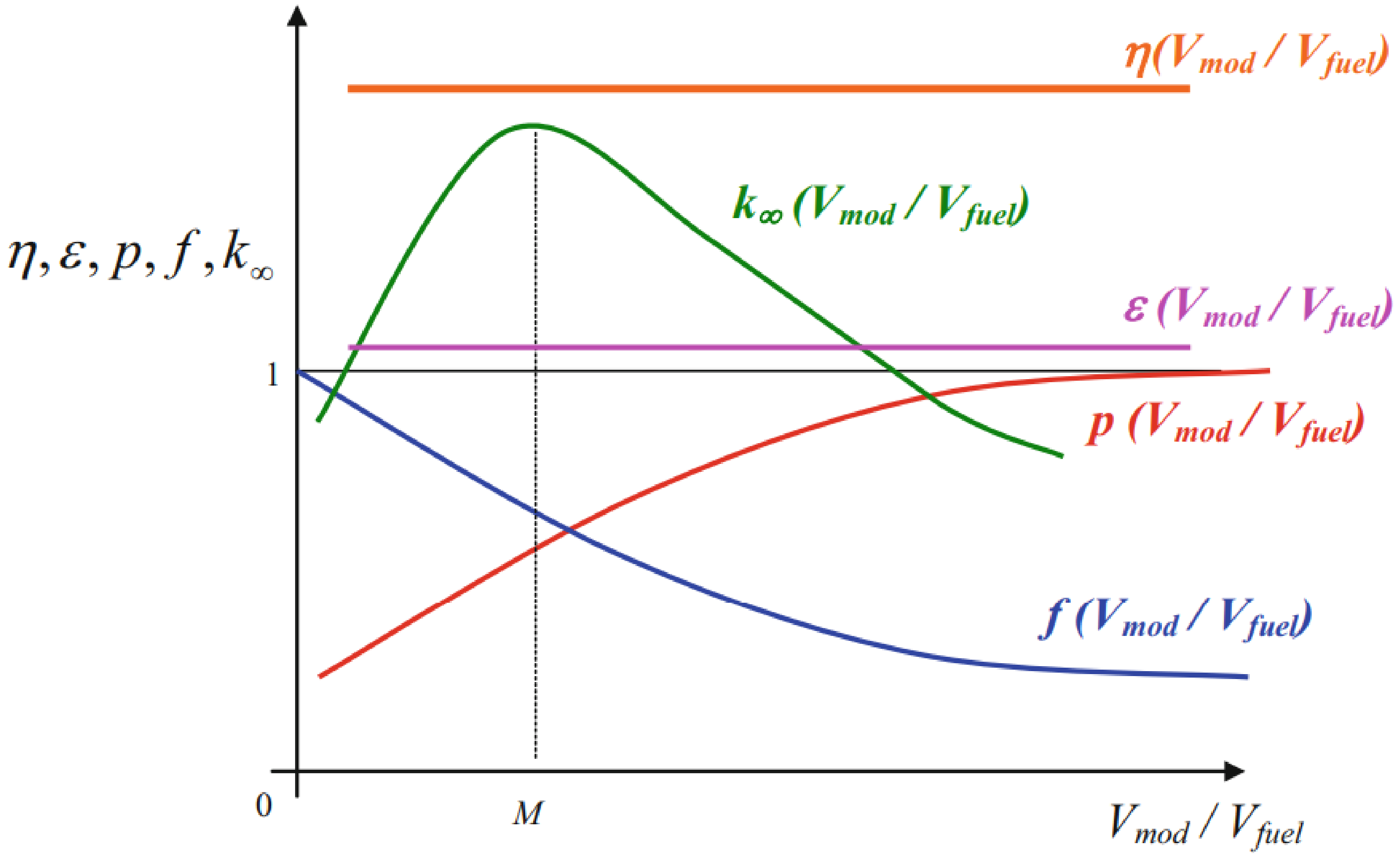
Sub-critical

- Chain reaction tends to extinction
- Number of fissions decrease
- Degree of sub-criticality is

k_{sub}= 1-k (**Shutdown Margin Concept**)

$$\rho = \frac{k_{eff} \; - 1}{k_{eff}}$$

Reactor Criticality



f	thermal utilization factor	p	escape probability factor
k_∞	infinite multiplication factor	η	fuel multiplication factor
M	optimum neutron utilization	ε	fast fission factor
V_{fuel}	fuel volume	V_{mod}	moderator volume

Reactivity Coefficients

Reactivity Coefficients

Moderator temperature coefficient of reactivity – MTC

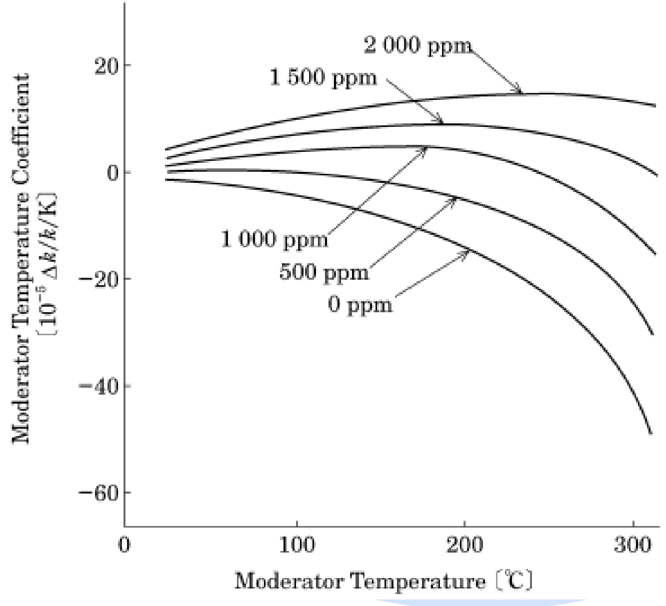
MTC is the change in reactivity per degree change in moderator temperature.

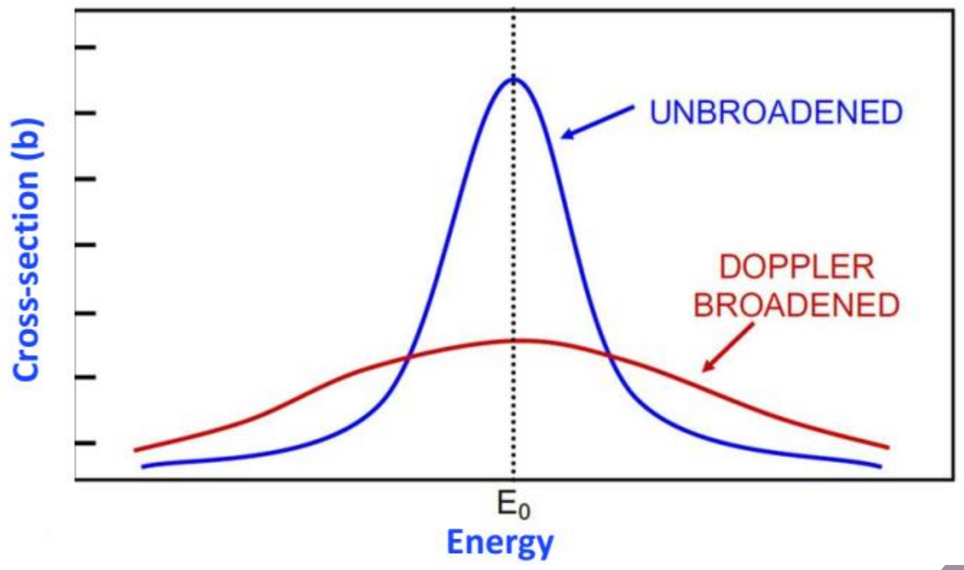
$$\alpha_M = \frac{d\rho}{dT_m}$$

Fuel temperature coefficient of reactivity – FTC

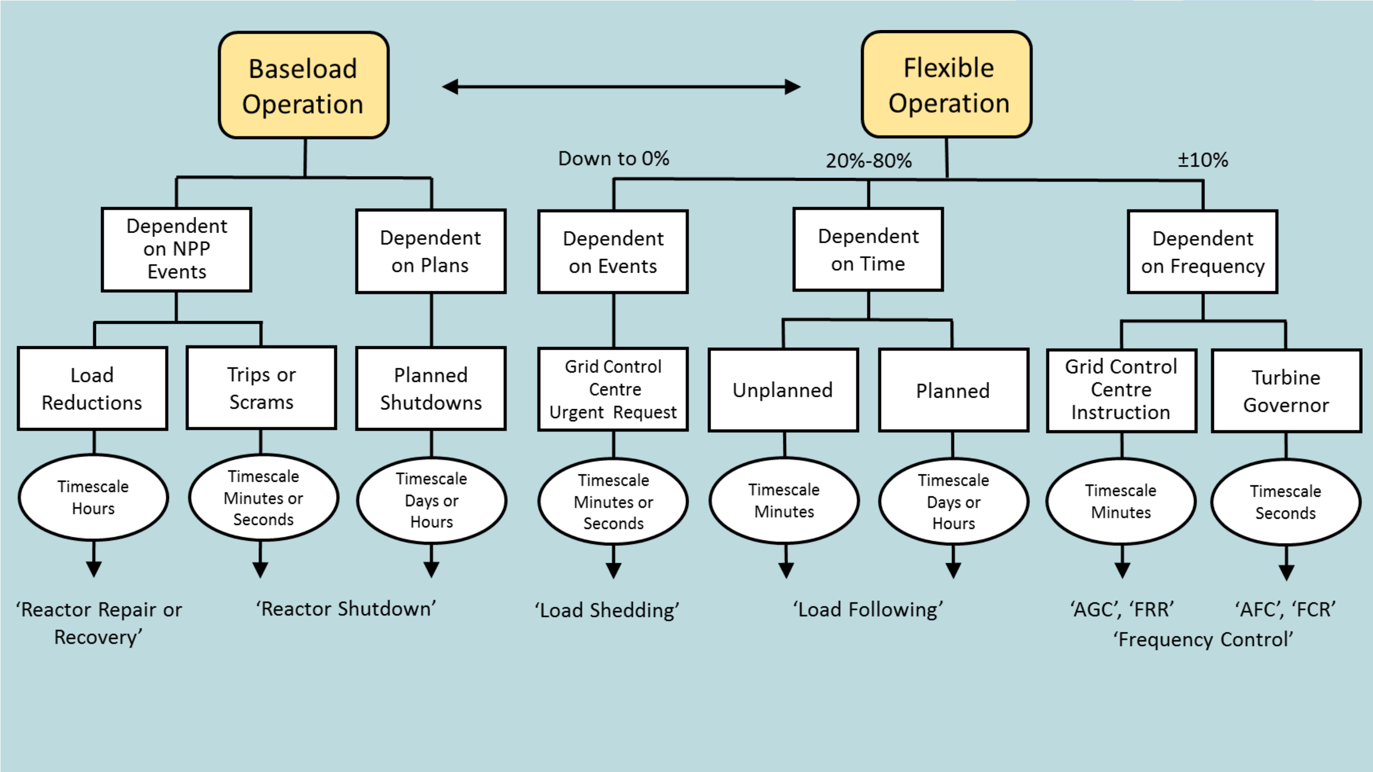
FTC is the change in reactivity per degree change in fuel temperature.

$$\alpha_F = \frac{d\rho}{dT_F}$$





Flexible Operation of SMRs



Beneficial economic reasons and less complexity of operation have made the "baseload" mode of operation preferable

SMRs: Favourable Points for Load Following

Physics of SMRs	Impact to Load-Follow Requirements
Small Core	Reduced xenon oscillations
Large number of Reactivity Control Cluster Assemblies (RCCA)	Flexible power control modes
Boron-free	• reactivity management & power change • power peaks and asymmetries? • ability to load-follow during stretch-out?
Integrated Primary Circuit	• reduced source of wear and tear • innovations to be carefully assessed
New digital I&C	high degree of power control automation

SMRs have intrinsic characteristics to address modern
load-follow requirements of manoeuvring capability of modern LWRs

Summary and Takeaway

Part I

- High level overview of Nuclear Reactor Analysis and Design
- Rigorous and Simplified Formulations of Neutron Balance Equations
- Challenges and Approaches

Part II

- Rigorous and Simplified Neutron Balance Equations Derivation – a mathematical perspective
- Numerical solution of 1-dimensional neutron diffusion equation

Part III

- Reactor physics fundamentals
- SMRs perspective

THANK YOU