WACQT | Wallenberg Centre for Quantum Technology

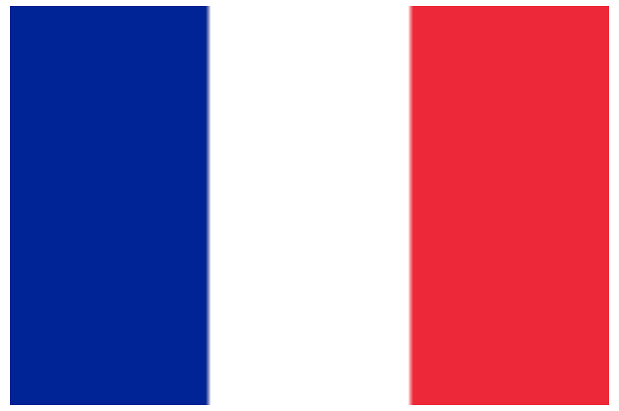
Quantum computing and quantum error correction with bosonic systems

Giulia Ferrini | WACQT

Myself

- Master: Theoretical Physics, Genova (Italy), 2008

- PhD: Theoretical physics, Grenoble (France) 2008- 2011
- Post-doc: theory of quantum computation in an experimental quantum optics lab, Paris (France) 2012-2016



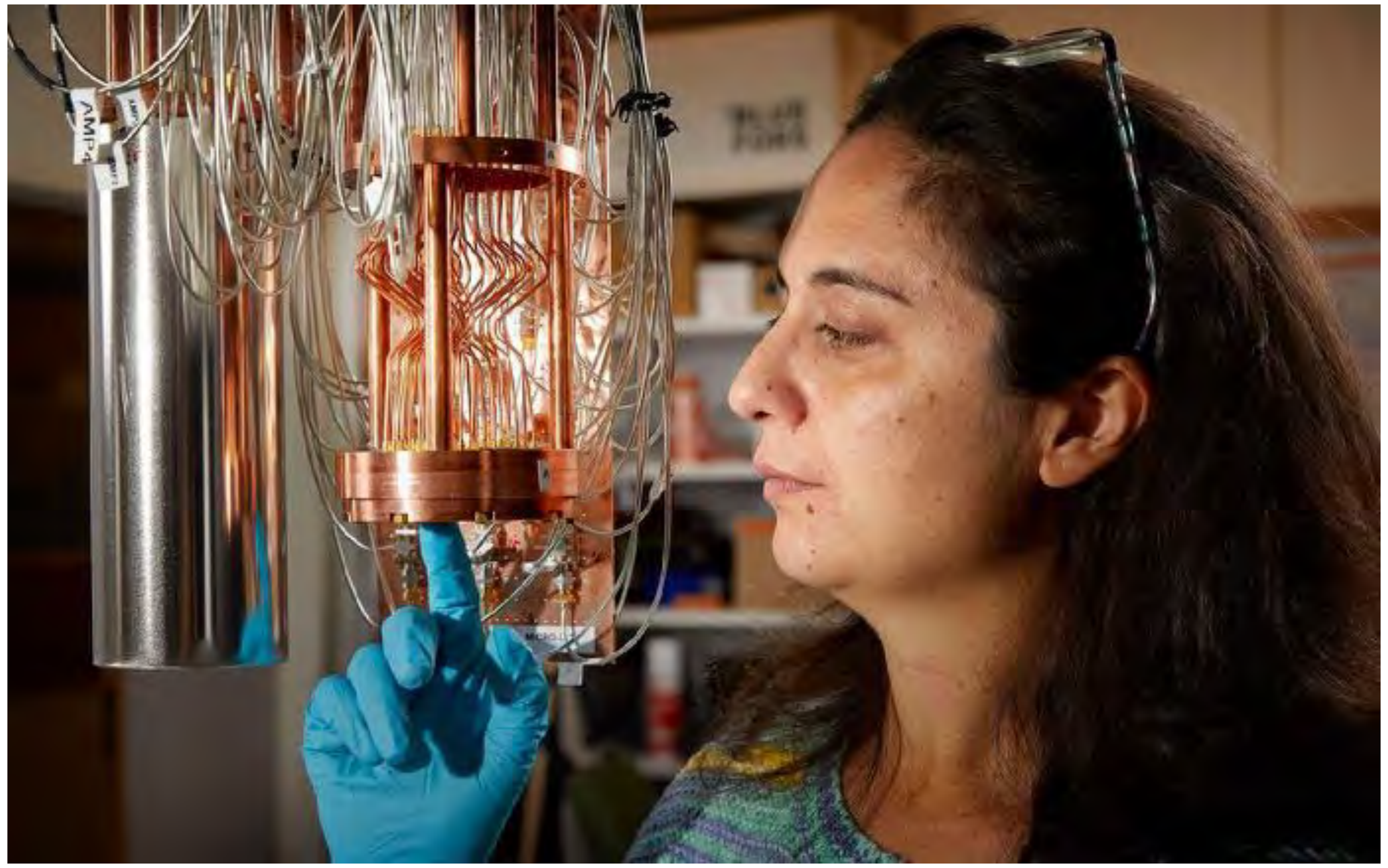
- Marie Slodowska-Curie fellowship Mainz (Germany) 2016-2017

- Assistant/Associate/ Professor at Chalmers Gothenburg (Sweden) 2018

Wallenberg Center for Quantum Technology

WACQT Wallenberg Centre for Quantum Technology

- Main goals:
  (1) To build the Swedish Quantum computer (core project);
  (2) To develop quantum technology know-how in Sweden (excellence project)
- Located in: Gothenburg, Stockholm, Lund and Linköping
- 12 years, started 1st January 2018
- Involving industry
- Funding: >150 ml euros (KAW, industry and university)
- > 200 employed researchers (about 100 at Chalmers)



WACQT PI Giovanna Tancredi and WACQT cryostat

- Part 1a: Intro to quantum computing with bosonic (continuous-variable) systems; measurement-based quantum computing with CV (45 min)
- Part 1b: Sampling models, computational hardness and simulatability (45 min)
- Part 2a: Bosonic codes for Quantum Computing: Intro + rotationally symmetric bosons codes (RSB) (45 min)
- Part 2b: Translationally symmetric bosons codes (GKP) and comparison (45 min)
- Part 3a: Research results: simulatability of GKP circuits / vacuum provides quantum advantage (45 min)
- Part 3b: Research results: a new magic measure for qubits using bosonic codes (45 min)

Ask me questions when you want!

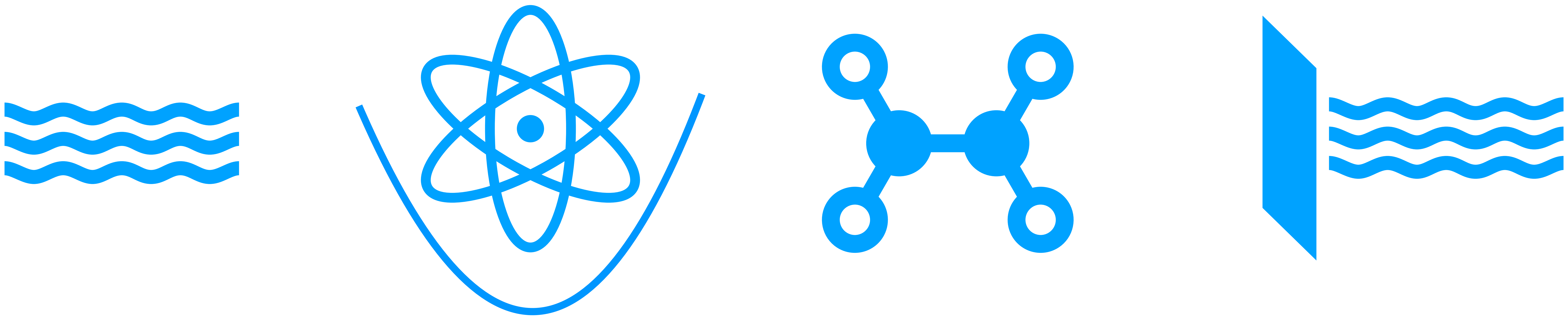
Part 1a: Introduction to quantum computing with bosonic (continuous-variable) systems; Measurement-based quantum computing with CV

Introduction

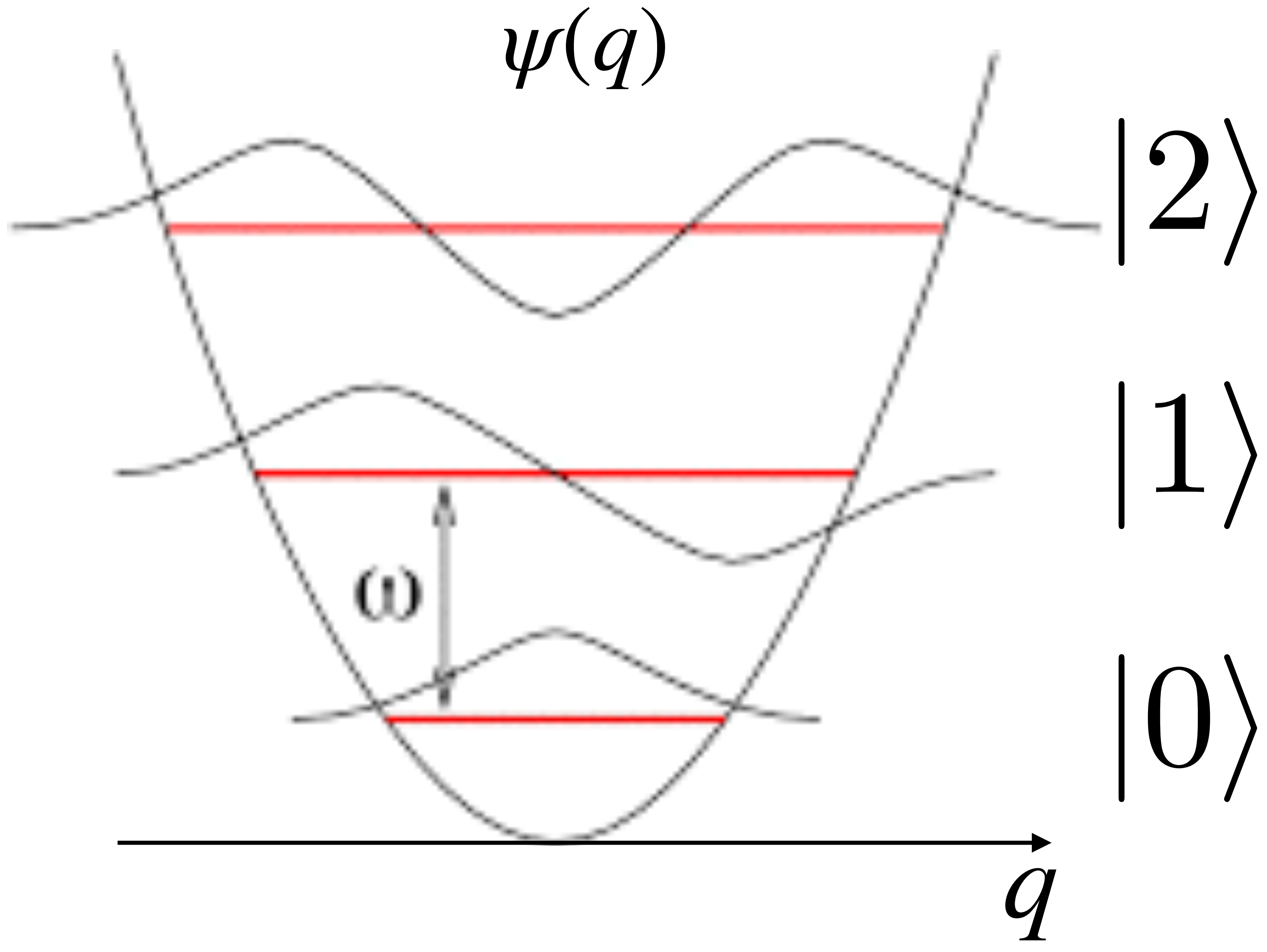
Qubits (two-level systems) are great to build quantum computers!

…but many physical systems are not described by solely two levels!

Including electromagnetic radiation, vibrations of trapped ions and molecules, oscillating mirrors…



Quantised harmonic oscillator



$$\hat{\mathcal{H}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad [\hat{a}, \hat{a}^\dagger] = 1$$

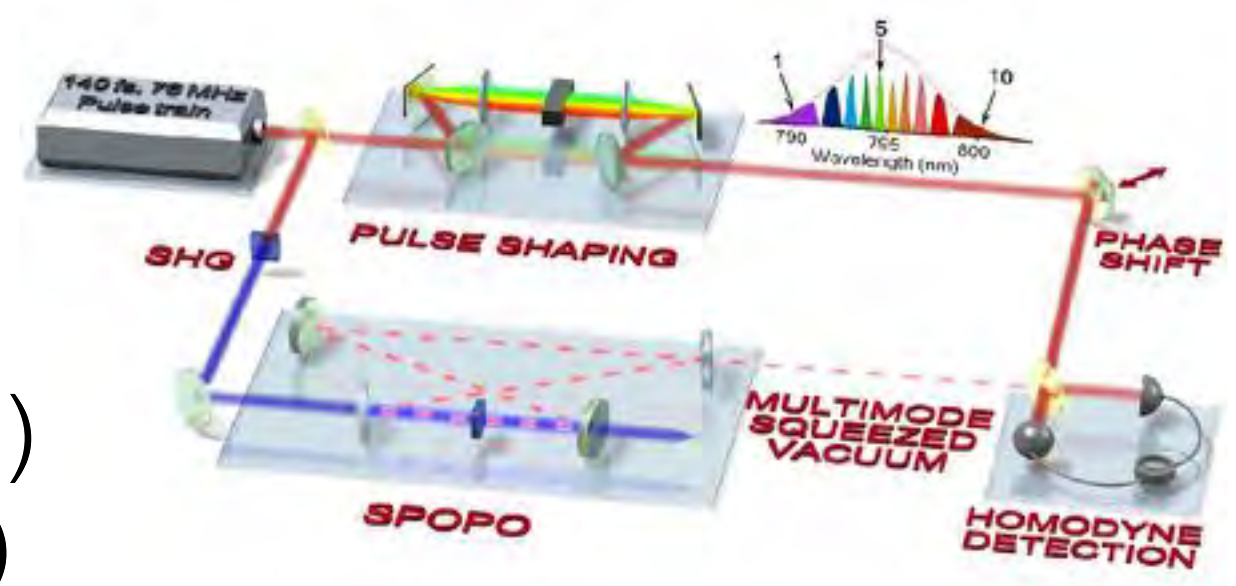
$$\hat{n} = \hat{a}^\dagger\hat{a} \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{n}\,|\,n\rangle = n\,|\,n\rangle \qquad a|n\rangle = \sqrt{n}|n-1\rangle.$$

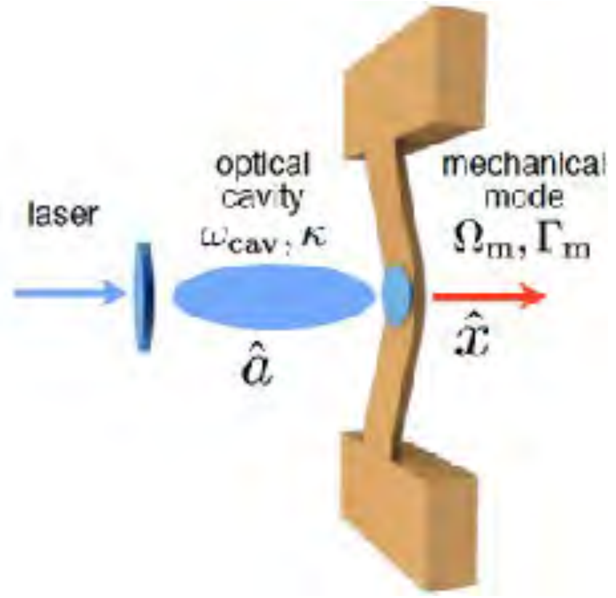
We can build a quantum computer based on the quantised harmonic oscillator!

Physical implementations (non exhaustive list)

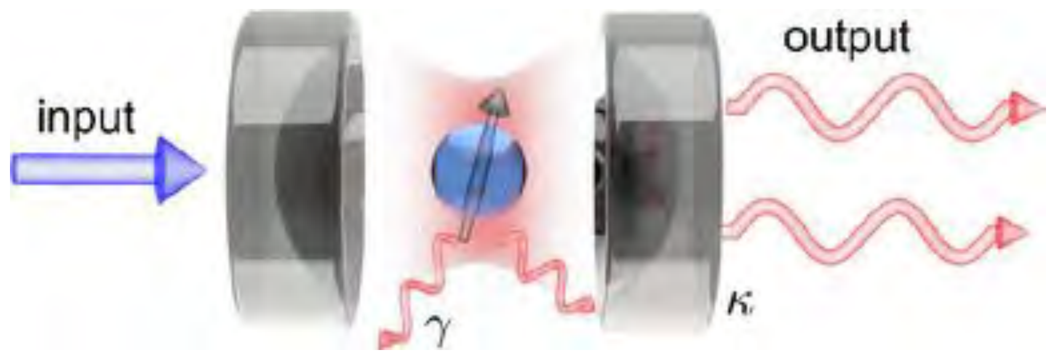
- Optical systems
  - Xanadu (Toronto)
  - Furusawa (Tokyo)
  - Silberhorn (Paderborn)
  - Pfister (Charlottesville)
  - Andersson (Lyngdby)
  - Sciarrino
  - Parigi-Treps, Laurat (Paris)
  - Pann…



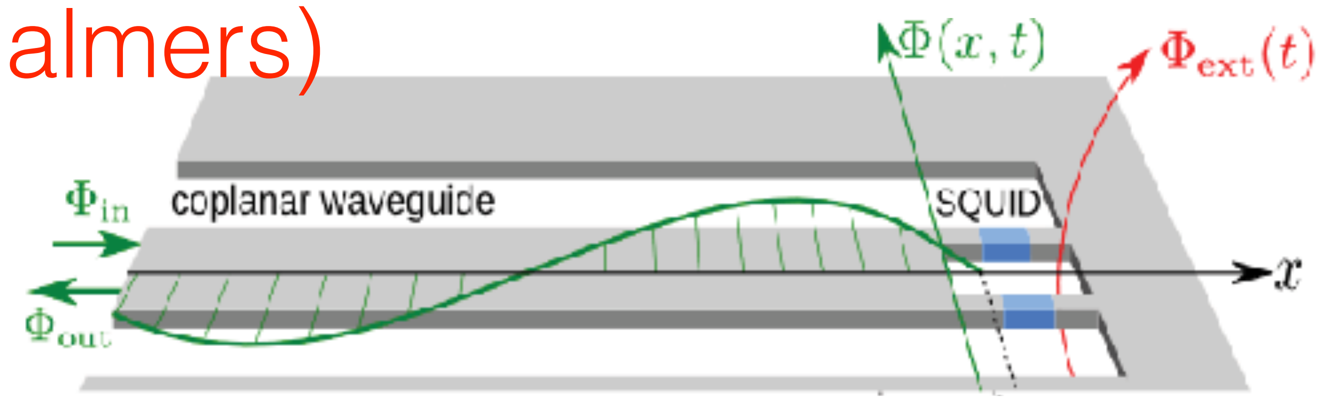
- Optomechanical devices
  - Wieczorek (Chalmers)
  - Van Laer (Chalmers)
  - Aspelmeyer (Vienna)…



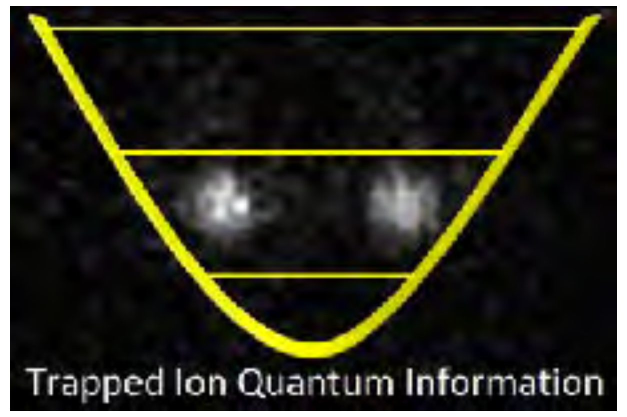
- Cavity QED
  - Alexei Ourjoumtsev…



- µw coupled to superconducting devices
  - Amazon (Pasadena)
  - Alice and Bob (Paris)
  - Gasparinetti (Chalmers)
  - Pann (Shangai)
  - Yu (Shenzen)
  - Yale group
  - Kirchmaier (Innsbruck)…



- Trapped ions
  - Home (Zurich)
  - Leibfried (NIST Boulder)
  - Matzukevic (Singapur)
  - Kim (China)
  - Slodicka (Olomouc)…

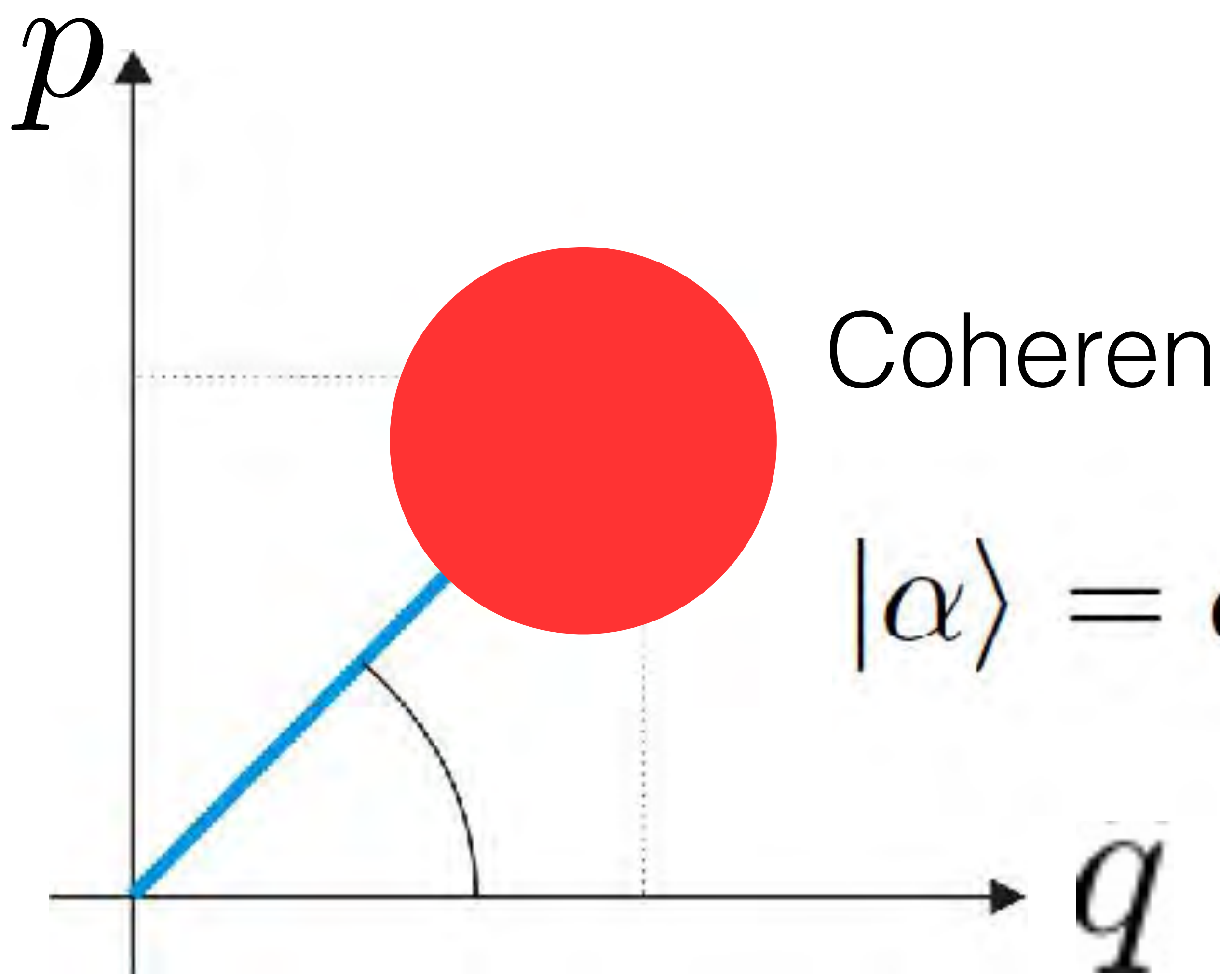


- Deterministic production of entangled states: one million entangled modes
- Good for quantum error correction through bosonic codes

Called "Continuous-Variable" because quadratures have a continuous spectrum

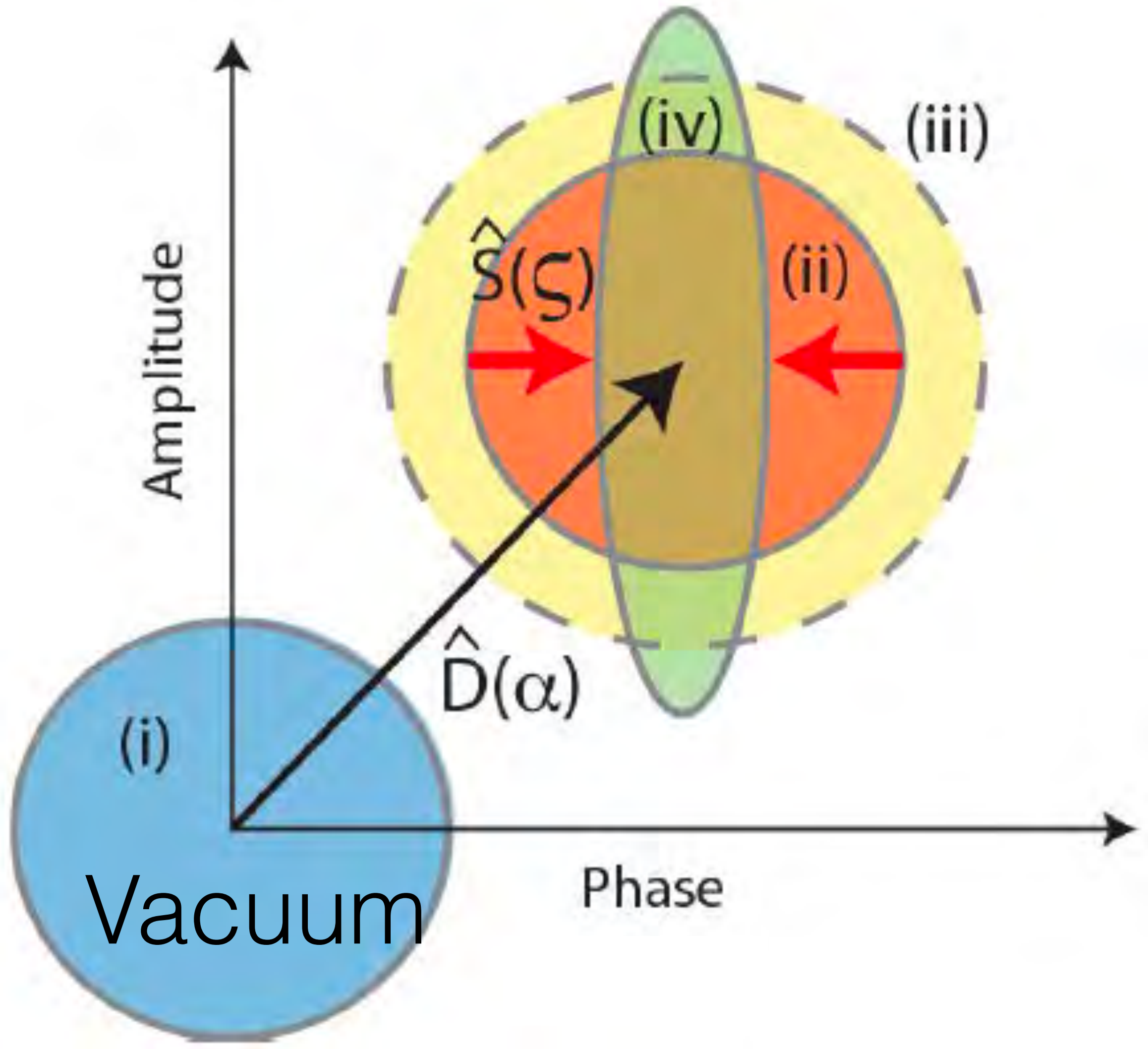
$$\hat{q} = (\hat{a}+\hat{a}^\dagger)/(\sqrt{2}) \qquad [\hat{q},\hat{p}] = i \qquad \hat{q}|s\rangle_q = s|s\rangle_q$$
$$\hat{p} = (\hat{a}-\hat{a}^\dagger)/(i\sqrt{2}) \qquad\qquad \hat{p}|s\rangle_p = s|s\rangle_p$$

e.g. : quadratures $\hat{q}$ and $\hat{p}$ of the em field



Coherent state

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



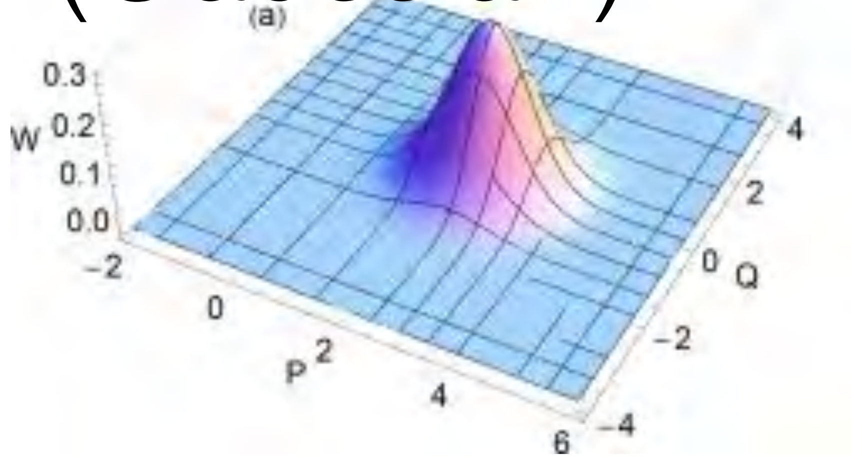
Quadrature eigenstates are infinitely squeezed states, e.g. $|0\rangle_p$

Representing CV systems in phase space

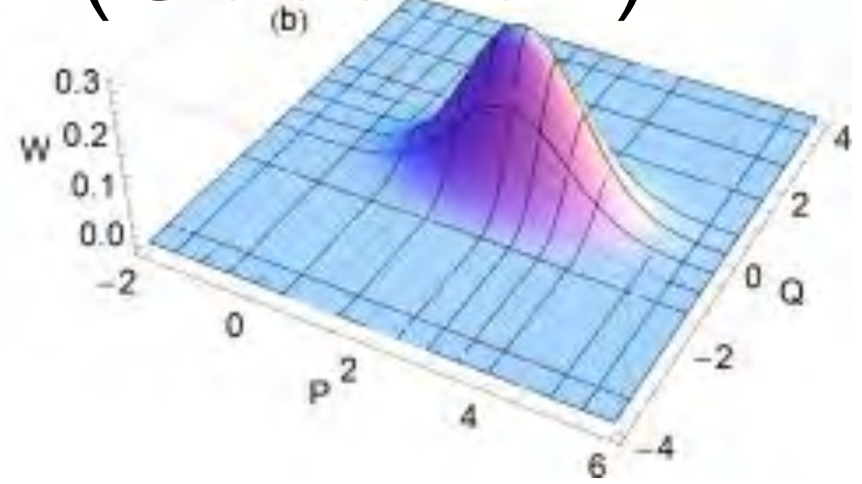
Wigner function: Representation of quantum systems in the classical phase space

$$W(q,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} dy \langle q+y|\hat{\rho}|q-y\rangle e^{-2ipy}$$

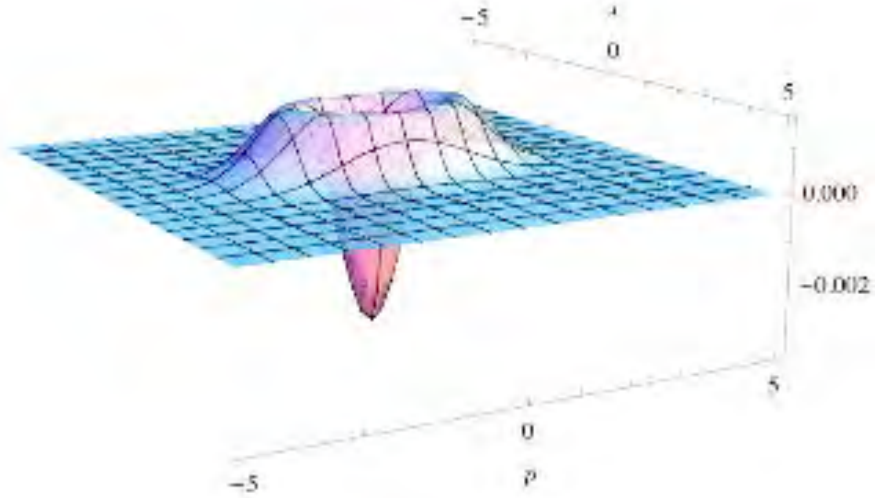
Coherent state $|\alpha\rangle$ (Gaussian)



Squeezed state $|\xi\rangle$ (Gaussian)



Single photon state $|1\rangle$ (non-Gaussian)

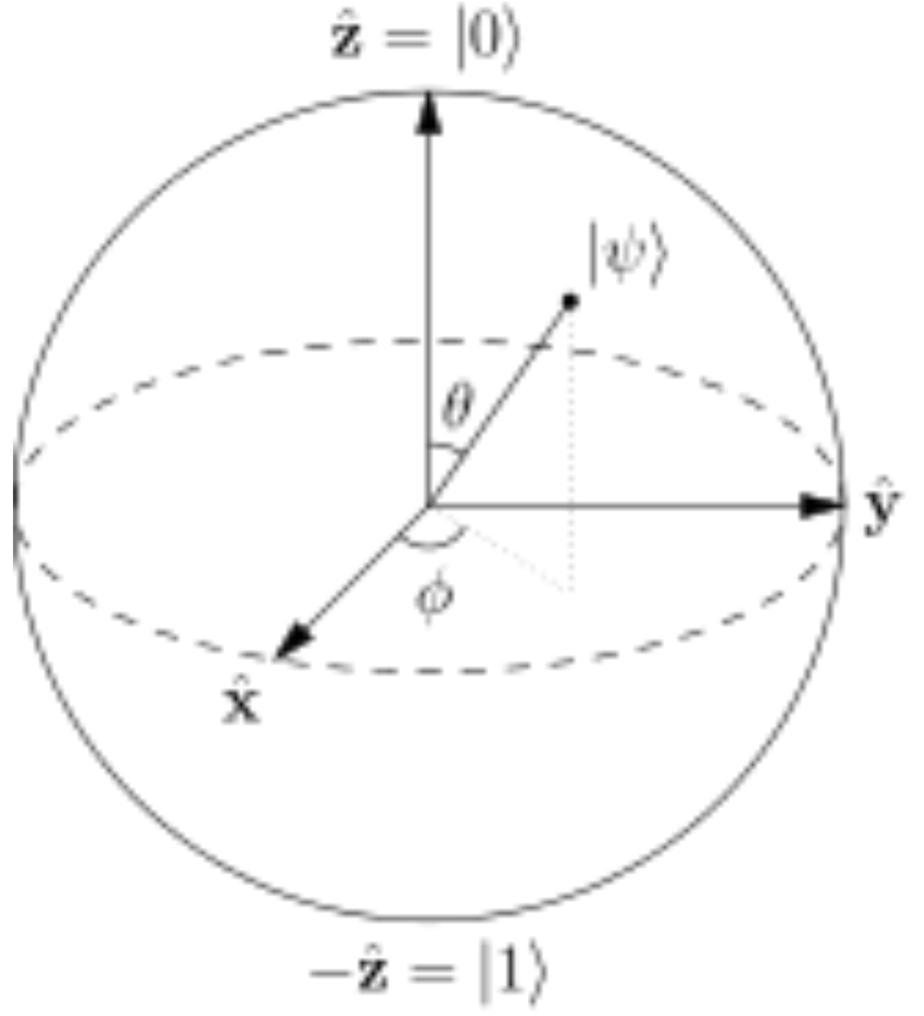


Also suitable for measurements: $\hat{\rho} \rightarrow \hat{\Pi}$

Examples: $\hat{n}, \hat{p}, \hat{q} \ldots .$

DV vs CV encoding of quantum information

DV: Information encoded in qubits



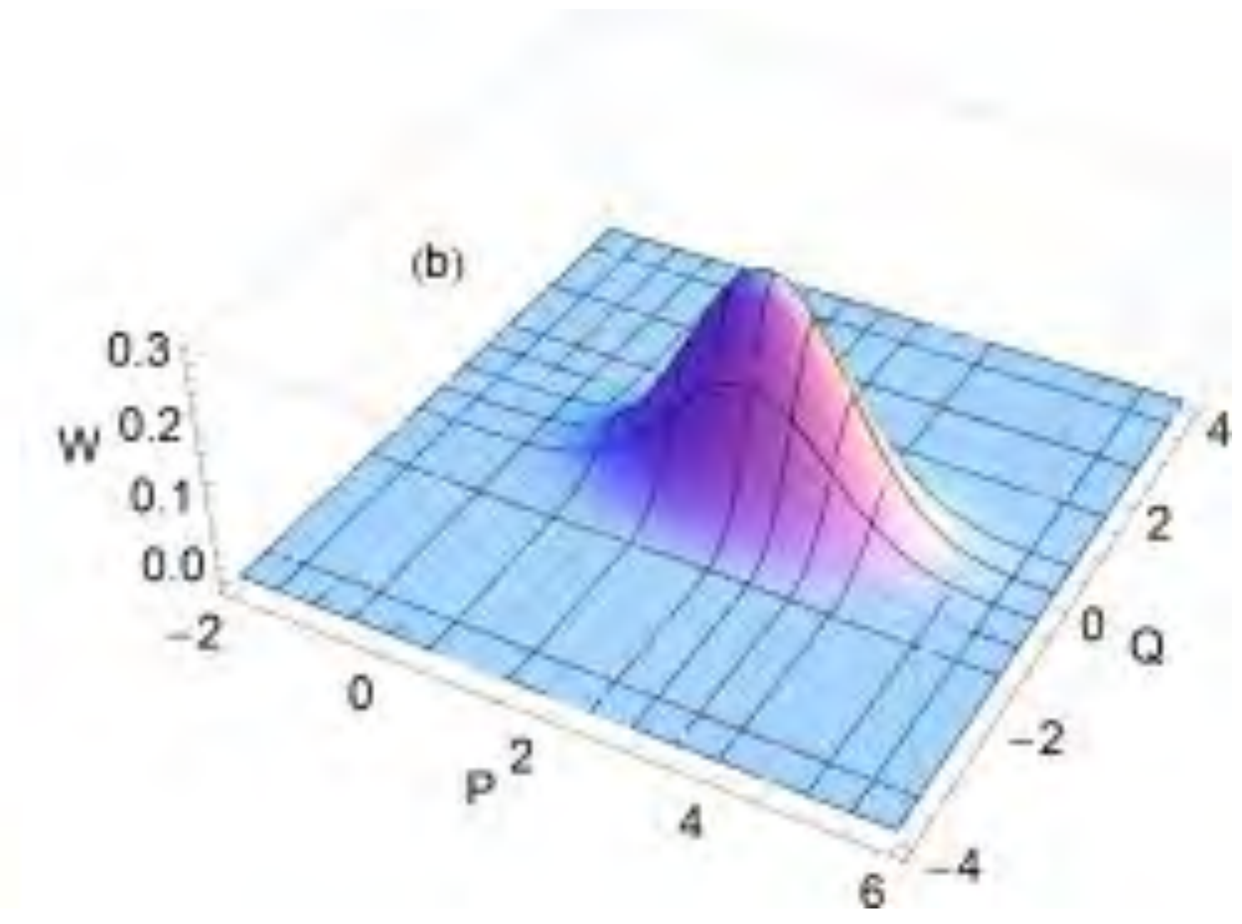
Phase space = Bloch Sphere

Finite-dimensional Hilbert space

Example of operation: $X|0\rangle = |1\rangle$

$Z|1\rangle = -|1\rangle$

CV: Information encoded in qumodes



Phase space = Complex Plane

Infinite-dimensional Hilbert space

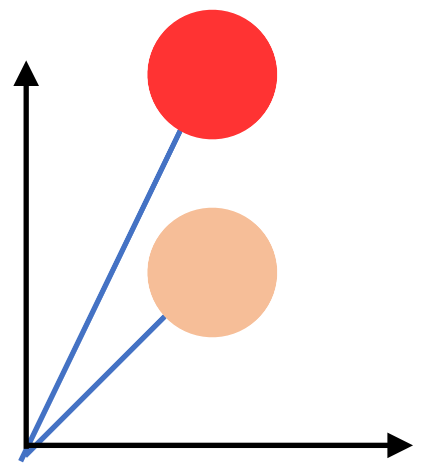
Example of operation: $X(s) = e^{-is\hat{p}}$

$Z(s) = e^{is\hat{q}}$

Operations in continuous variables: Gaussian operations

- Displacements

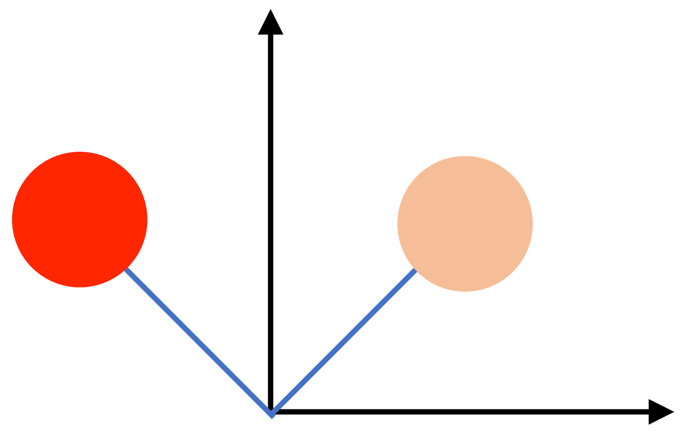
$$X(s) = e^{-is\hat{p}}, \quad \text{such that} \quad X(s)|r\rangle_q = |r+s\rangle_q$$
$$Z(s) = e^{is\hat{q}}, \quad \text{such that} \quad Z(s)|r\rangle_p = |r+s\rangle_p$$



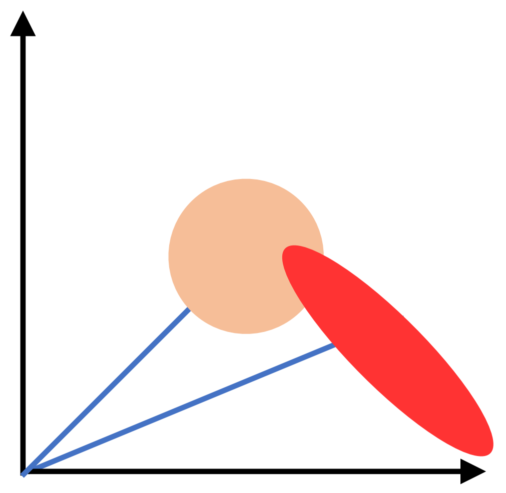
- Fourier transform

$$F = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}$$

$$F|s\rangle_q = \frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty} dr e^{irs}|r\rangle_q = |s\rangle_p$$
$$F^\dagger|s\rangle_p = \frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty} dr e^{-irs}|r\rangle_p = |s\rangle_q$$



- (Squeezing $S(s) = e^{-\frac{is(\hat{q}\hat{p}+\hat{p}\hat{q})}{2}}$)



- Shear $D_{2,q} = e^{\frac{is\hat{q}^2}{2}}$
- Cz gate $e^{i\hat{q}_1\otimes\hat{q}_2}$

$$e^{i\hat{q}_1\hat{q}_1}|s\rangle_q|r\rangle_p = |s\rangle_q|r+s\rangle_p$$

with ${}_q\langle r|s\rangle_p = \frac{e^{irs}}{\sqrt{2\pi}}$ and $|s\rangle_p, |s\rangle_q$

New results by ABC

CHALMERS UNIVERSITY OF TECHNOLOGY

- Only Gaussian (quadratic) Hamilonians are not enough to achieve arbitrary operations!

$$e^{-iAt}e^{-iBt} = e^{-i(A+B)t-[A,B]t^2/2} + O(t^3, A, B)$$

- Commutators of quadratic gates leave us in the space of quadratic gates

S. Lloyd and S. Braunstein PRL 82, 1784 (1999)

Universality in continuous variables

CHALMERS UNIVERSITY OF TECHNOLOGY

- A higher-order gate such as the Kerr gate $e^{i\hat{n}^2 s}$ is sufficient to promote Gaussian operations to universal quantum computation

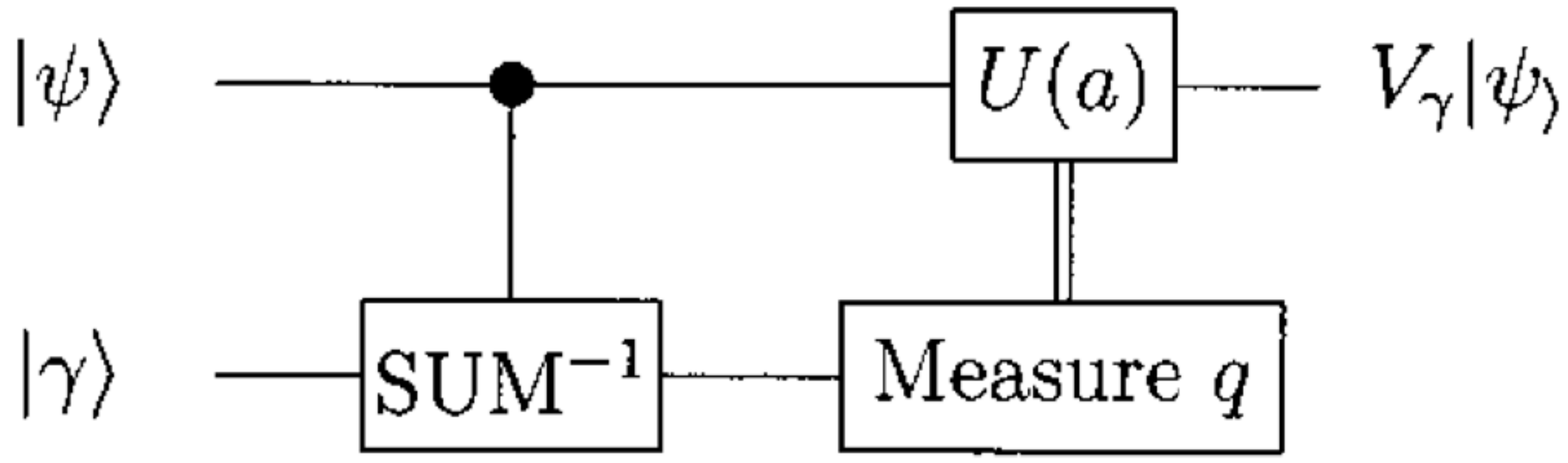
- Universal CV quantum computation if we can approximate evolution from polynomial Hamiltonians in the quadratures, $e^{i\mathcal{H}(q_i,p_i)}$, to arbitrary precision.

- CV universal gate set $\{e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(q^2+p^2)}, e^{i\hat{q}_1\hat{q}_2}, e^{i\hat{n}^2 s}\}$

S. Lloyd and S. Braunstein PRL 82, 1784 (1999)

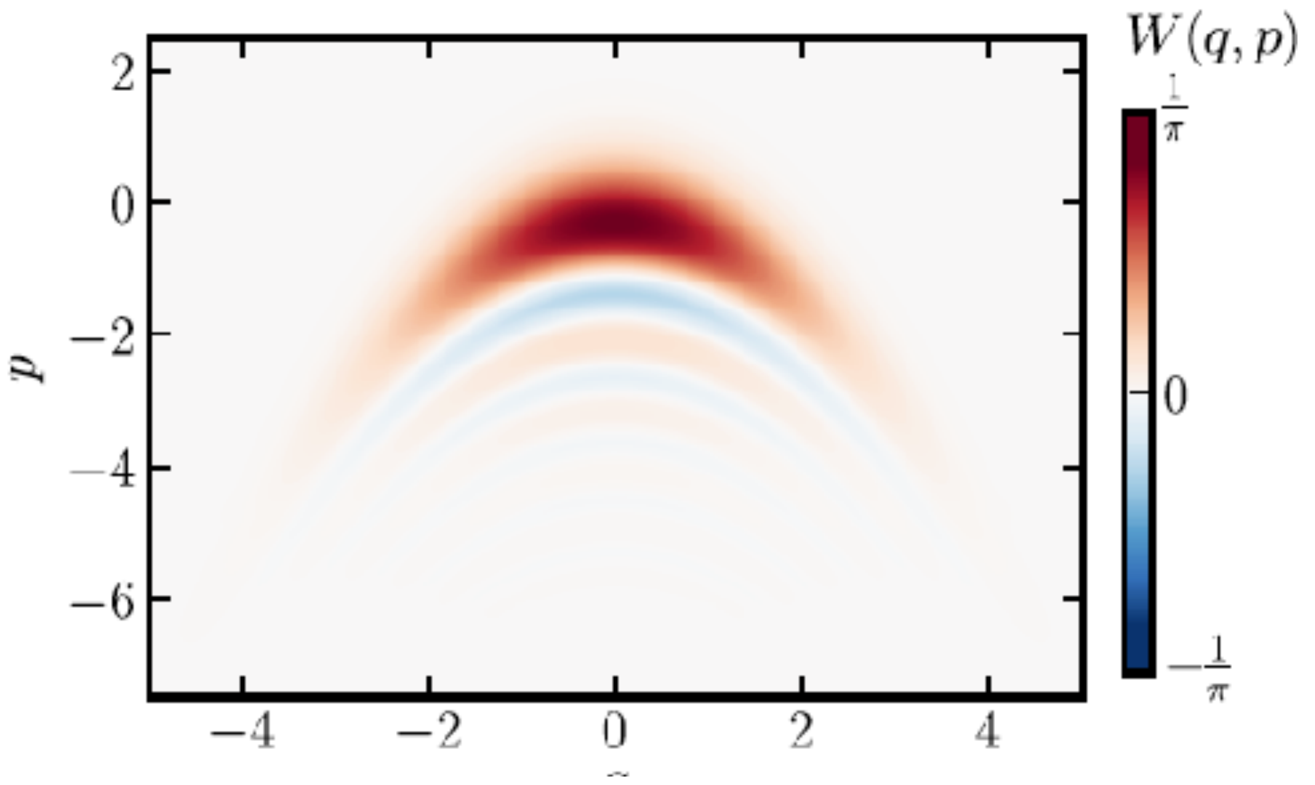
F. Arzani, R. Booth and U. Chabaud, Nat. Comm. 16, 9744 (2025)

Implementing cubic gates with the cubic phase state



$$V_\gamma = e^{i\gamma\hat{q}^3}$$

$$|\gamma\rangle = e^{i\gamma\hat{q}^3}|\xi\rangle$$



picture from Timo Hillmann, PRL 2020

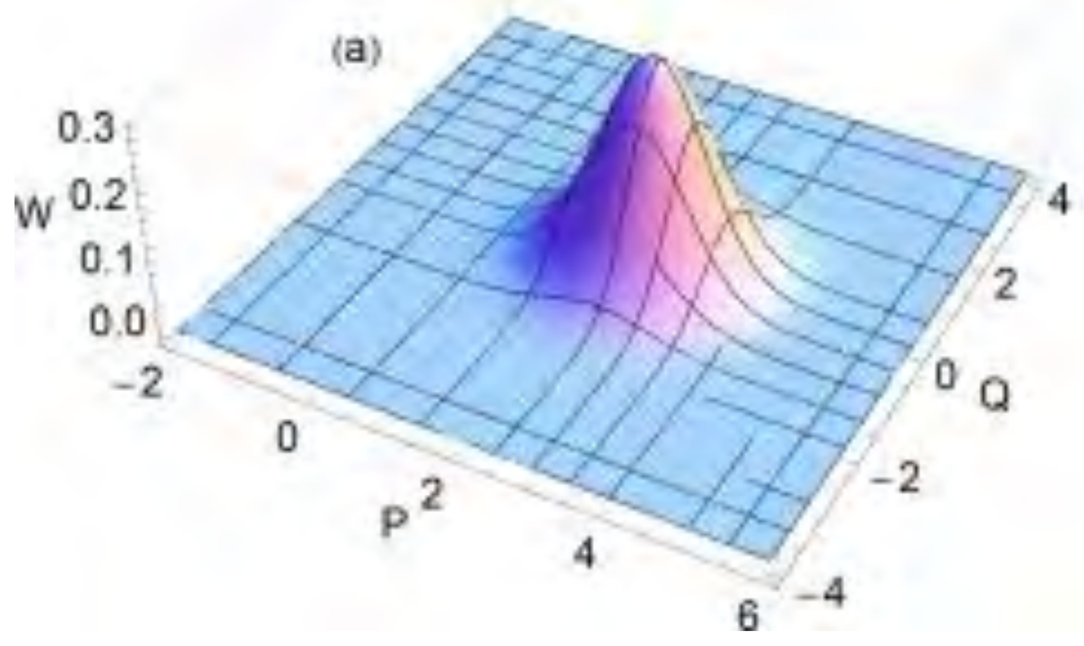
Cubic phase state + Gaussian operations
= cubic phase gate (CV universality?)

D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001)
F. Arzani, R. Booth and U. Chabaud, Nat. Comm. 16, 9744 (2025)

CV QC simulatable architectures

For continuous variables, a QC based on:

(i) “easy” input states (positive Wigner function)
(ii) “easy” operations (mapping positive W to positive W)
(iii) “easy” measurements (positive Wigner function)



can be simulated efficiently with a classical computer

- No exponential quantum advantage with Wigner positive (Gaussian) quantum computation!

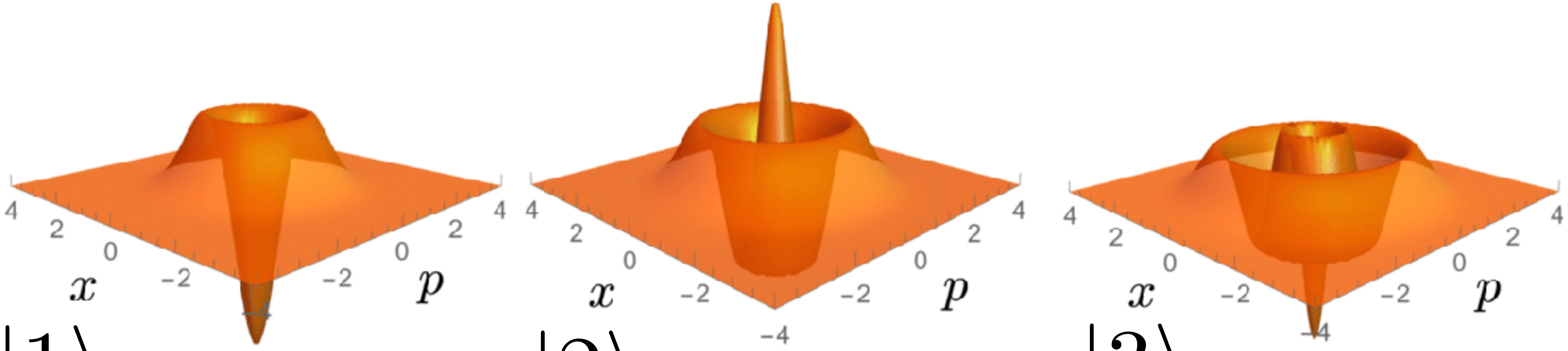
$$P(\hat{\Pi}|\hat{\rho}_{\text{out}}) = \int_{-\infty}^{\infty} W_{\hat{\rho}_{\text{out}}}(q,p) W_{\hat{\Pi}}(q,p)\, dq dp.$$

S. D. Bartlett et al, PRL 88, 097904 (2002);
A. Mari, J. Eisert, PRL 109, 230503 (2012);
V. Veitch et al, New J. Phys. 14, 113011 (2012)
S. Rahimi-Keshari et al, PRX 6, 021039 (2016)

Wigner negativity is a necessary resource for exponential quantum advantage

Examples of Wigner-negative (non-Gaussian) states

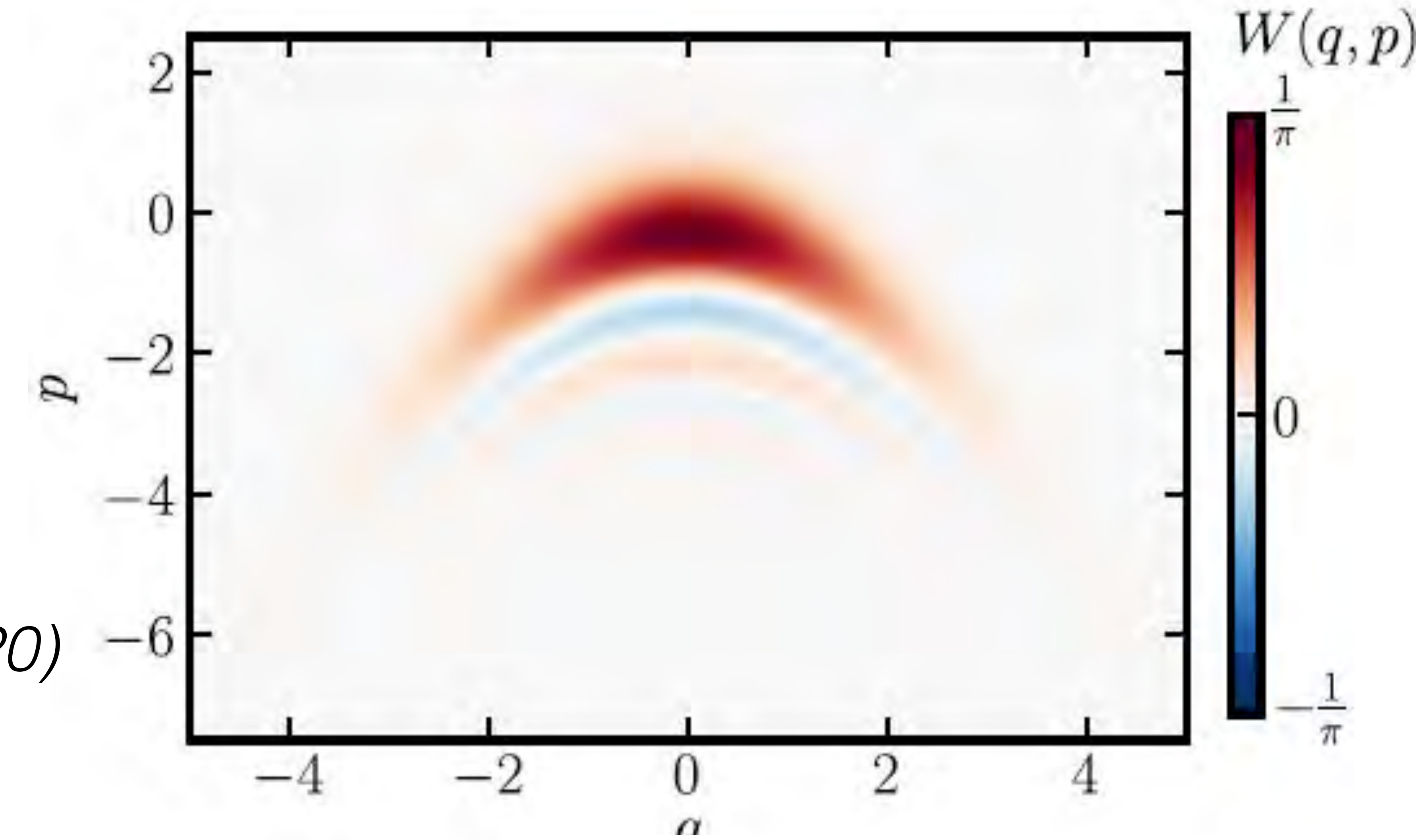
- Fock states



$|1\rangle$ $|2\rangle$ $|3\rangle$

pictures from A. Ketterer PhD thesis

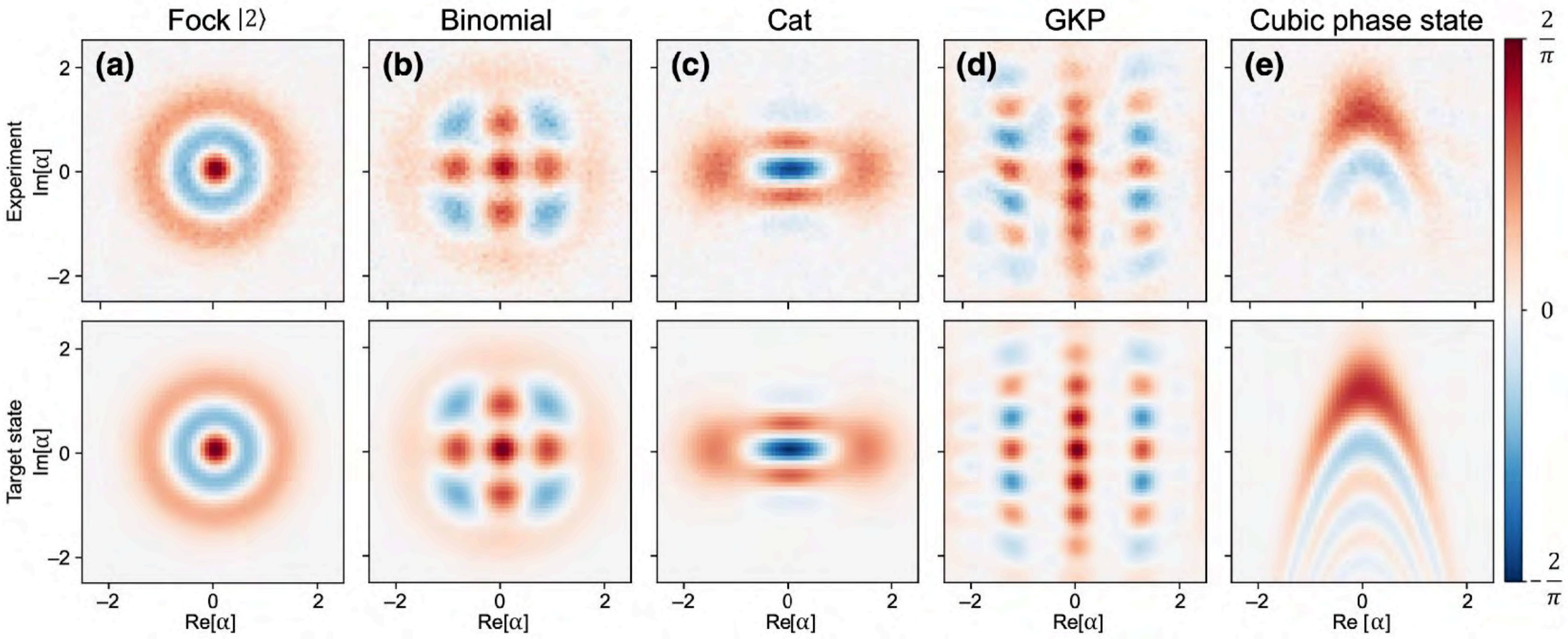
- Cubic phase state $|\gamma\rangle = e^{i\gamma\hat{q}^3}|\xi\rangle$



picture from Timo Hillmann, PRL 2020

- *Production and applications of non-Gaussian quantum states of light,*
A.I. Lvovsky, … Valentina Parigi, … Rosa Tualle-Brouri, arXiv:2006.16985 (2020)
- *Non-Gaussian quantum states and where to find them,*
M Walschaers, PRX quantum 2 (3), 030204 (2021)

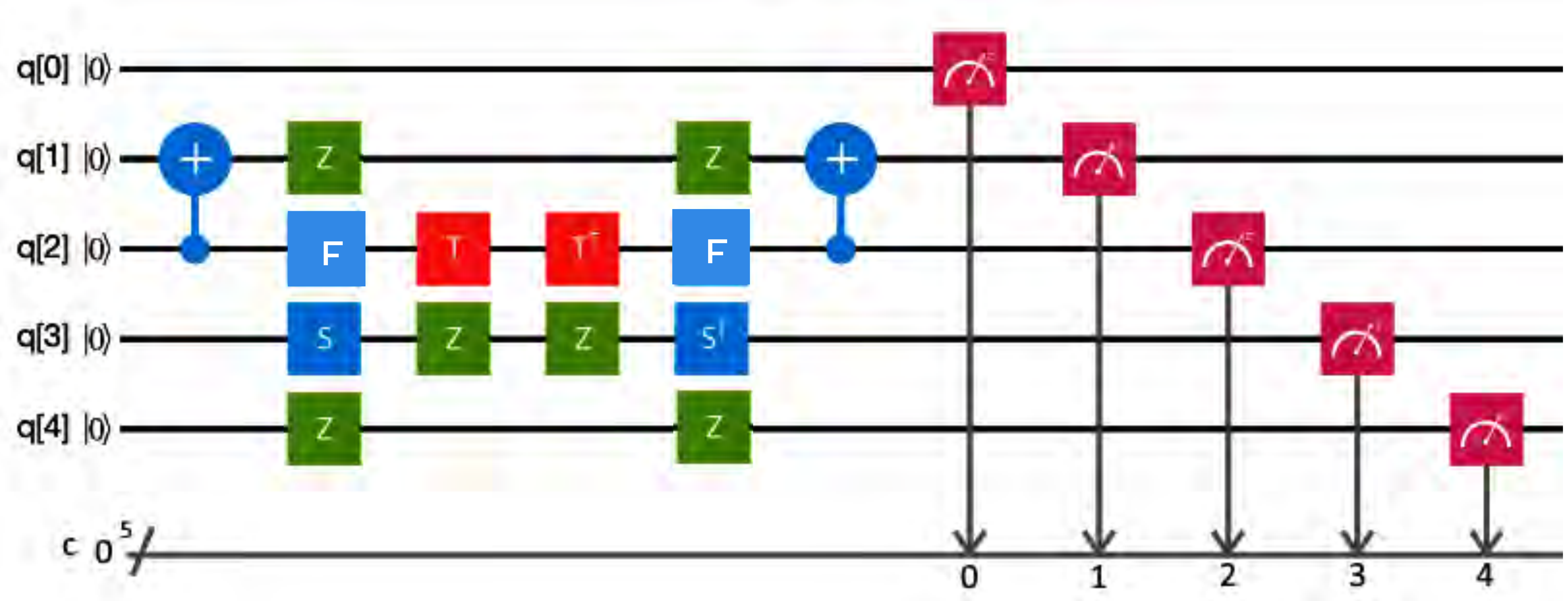
Experimental measurement of bosonic state Wigner function



Kudra et al, PRX Quantum 3, 030301 (2022)

$$W(\alpha) = Tr[\hat{D}(\alpha)\hat{\rho}\hat{D}^{\dagger}(\alpha)\hat{\Pi}]$$

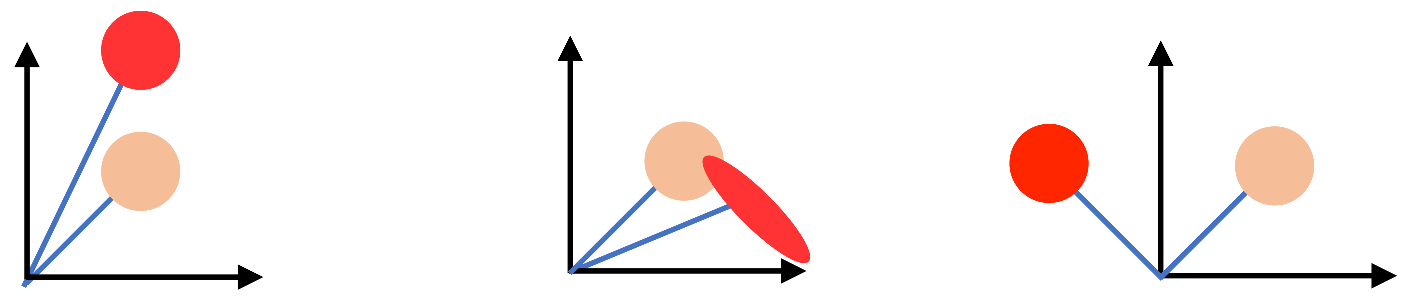
CV "circuit model"

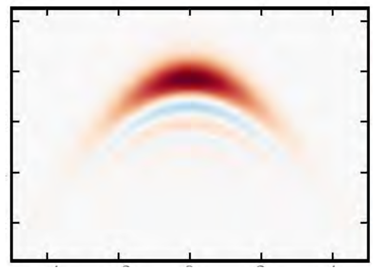


$$\{\hat{Z}(s) = e^{i\hat{q}s}, e^{i\hat{q}^2 s}, \hat{F} = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, \hat{C}_Z = e^{i\hat{q}_1\hat{q}_2}, e^{i\hat{q}^3 s}\}$$

CV universal gate set

Lloyd and Braunstein, PRL





non-Gaussian gate

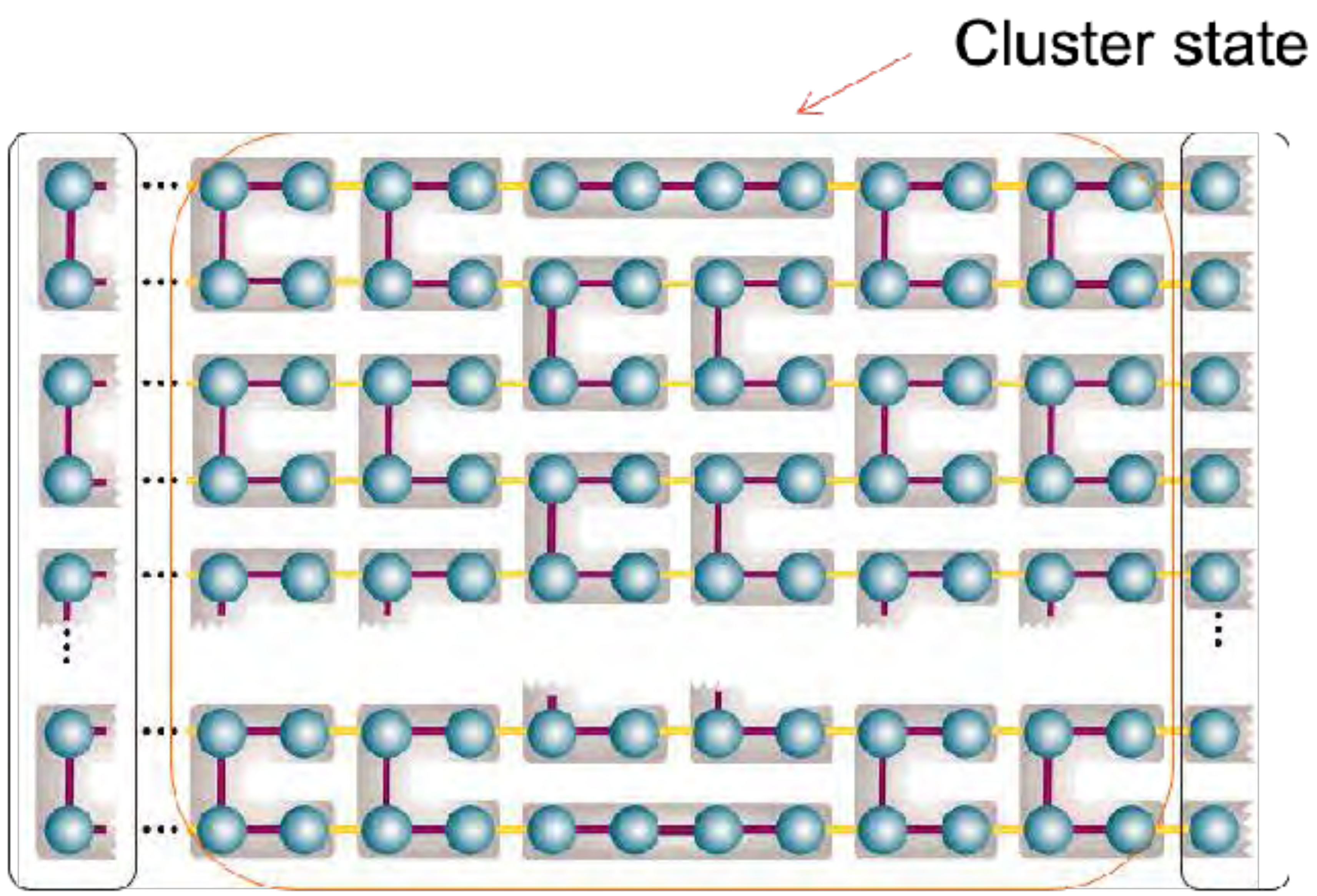
Microwave: *T. Hillmann, F. Quijandría, G. Johansson, A. Ferraro, S. Gasparinetti, G. Ferrini, PRL 125,160501 (2020)*
A. M: Eriksson et al, Nat. Comm. 15, 2512 (2024)

Optics: *A. Sakaguci et al, Nat. Comm. 14, 3817 (2023)*

Measurement-based quantum computing with CV

CV Measurement-based model: CV cluster states

$|\psi_{in}\rangle$



Manipulation of the input state achieved by entangling it with a cluster state and by performing suitable local measurements on its nodes

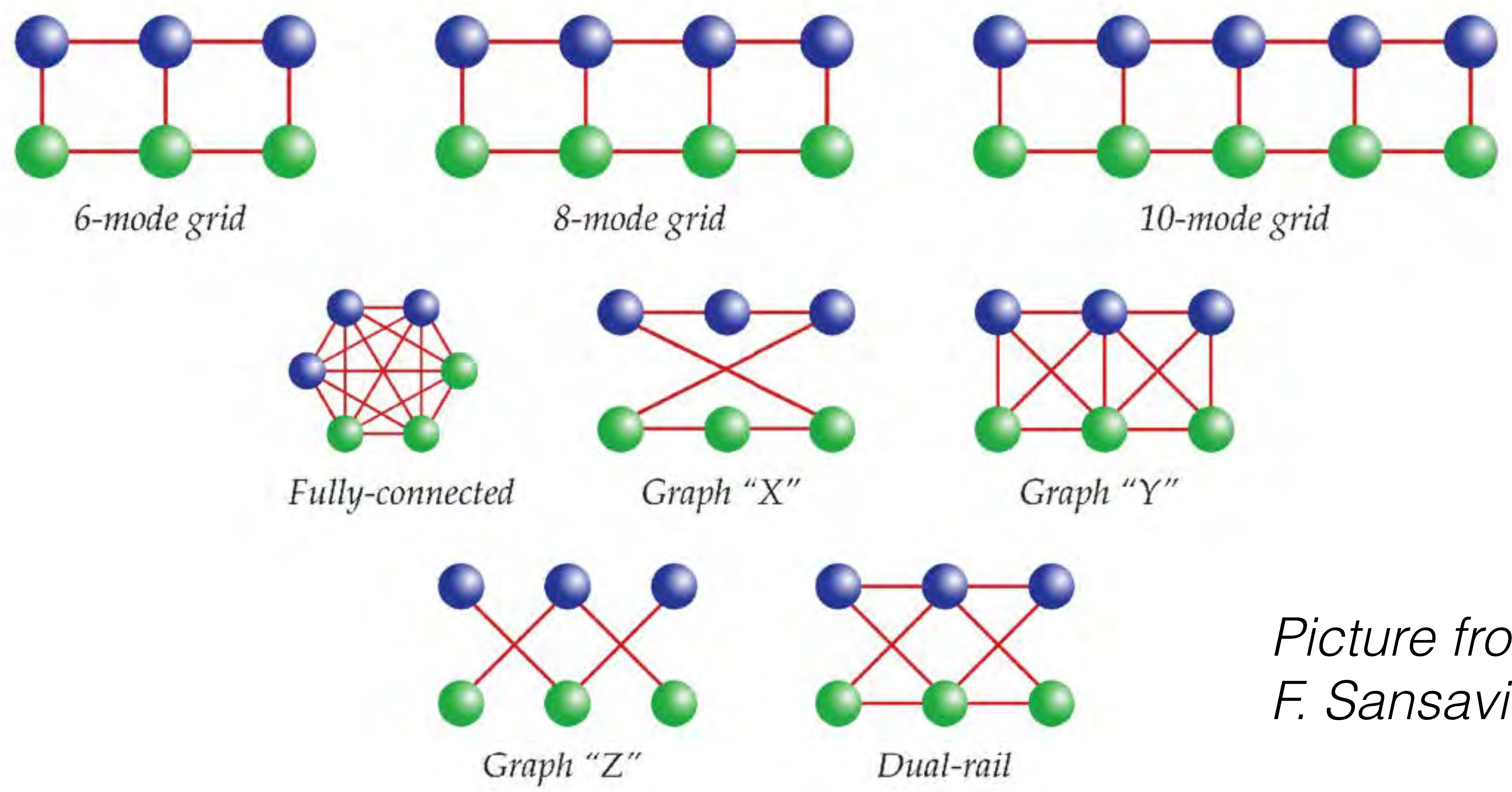
⟶ Unmeasured nodes projected on $|\psi_{out}\rangle = e^{iH(\hat{q}_i,\hat{p}_i)}|\psi_{in}\rangle$

R. Raussendorf, H. Briegel et al, PRL 86, 5188 (2001)
N. Menicucci, P. Van Loock et al, PRL 97, 110501 (2006)

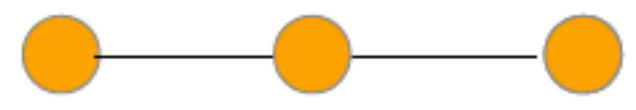
Cluster states

- Cluster state: state associated to a graph, operationally defined as:

  - start with as many $|0\rangle_p$ states as the nodes of the graph
  - apply CZ gate if two nodes are related by an edge



- Example: linear cluster state



$$|\psi_V\rangle = C_Z^{1,2} C_Z^{2,3} |0\rangle_p |0\rangle_p |0\rangle_p$$

$$C_Z^{1,2} = e^{i\hat{q}_1\hat{q}_2}$$

$$V = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Picture from:
F. Sansavini and V. Parigi, Entropy 22, 26 (2019)

- General expression:

$$|\psi_V\rangle = \hat{C}_Z[V]\,|0\rangle_p^{\otimes N} = \prod_{j,k}^{N} e^{\frac{i}{2}V_{jk}\hat{q}_j\hat{q}_k}\,|0\rangle_p^{\otimes N}$$

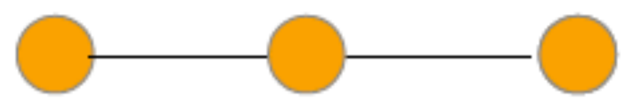
- Cluster states admit stabilizers:

$$K_i\,|\psi_V\rangle = |\psi_V\rangle \qquad K_i = e^{-is\hat{p}_i}\prod_k V_{i,k}e^{is\hat{q}_k} = X_i(s)\prod_k V_{i,k}Z_k(s)$$

- From which follows they also admits nullifiers:

$$\Delta^2\left(\hat{p}_i - \sum_k V_{i,k}\hat{q}_k\right) = 0$$

- Example: linear cluster state



$$|\psi_V\rangle = C_Z^{1,2}C_Z^{2,3}|0\rangle_p|0\rangle_p|0\rangle_p$$

$$V = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Delta^2\left(\hat{p}_1 - \hat{q}_2\right) = 0$$

$$\Delta^2\left(\hat{p}_2 - \hat{q}_1 - \hat{q}_3\right) = 0$$

$$\Delta^2\left(\hat{p}_3 - \hat{q}_2\right) = 0$$

Demonstrations: see A. Frisk Kockum et al, Lecture notes on quantum computing, arXiv:2311.08445 (2023)

How to generate CV cluster states in practice?

Symplectic representation for gaussian states : $\hat{\rho} \leftrightarrow S$ such that $\begin{pmatrix} \vec{q} \\ \vec{p} \end{pmatrix} = S \begin{pmatrix} \vec{q}(0) \\ \vec{p}(0) \end{pmatrix}$

$$\begin{pmatrix} \vec{q} \\ \vec{p} \end{pmatrix} = S \begin{pmatrix} \vec{q}(0) \\ \vec{p}(0) \end{pmatrix} = \begin{pmatrix} X & -Y \\ Y & X \end{pmatrix} \begin{pmatrix} \Lambda^{-\frac{1}{2}} & 0 \\ 0 & \Lambda^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} X' & -Y' \\ Y' & X' \end{pmatrix} \begin{pmatrix} \vec{q}(0) \\ \vec{p}(0) \end{pmatrix}$$

Bloch-messiah decomposition

real matrices

Multiport splitter (linear optics)

diagonal matrix

Off-line squeezing

Uneffective

$$U_{lin} = X + iY$$

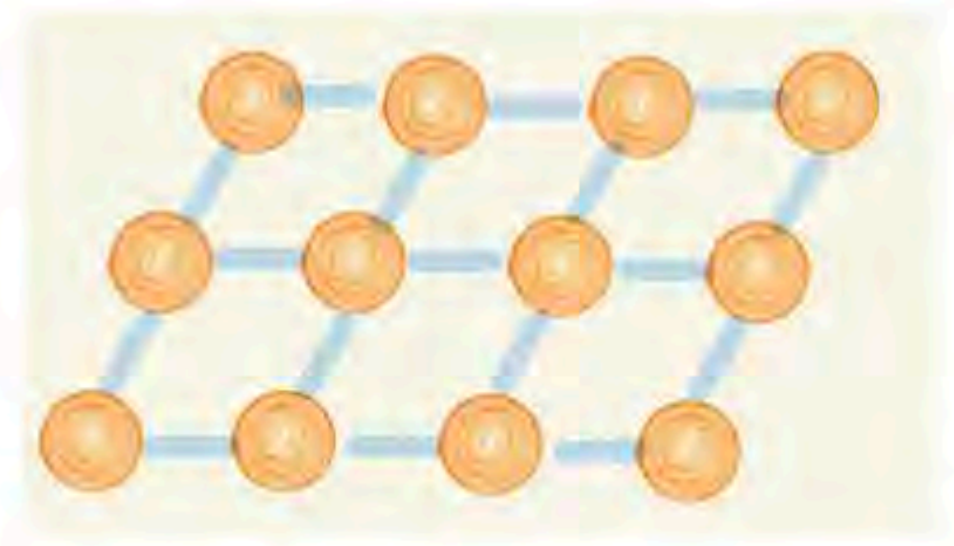
$$\begin{pmatrix} \vec{a}_{cluster} \\ \vec{a}^{\dagger}_{cluster} \end{pmatrix} = \begin{pmatrix} U_{lin} & 0 \\ 0 & U^{*}_{lin} \end{pmatrix} \begin{pmatrix} \vec{a}_{squeezed} \\ \vec{a}^{\dagger}_{squeezed} \end{pmatrix}$$

$$\vec{a}_{cluster} = U_{lin} \vec{a}_{squeezed}$$

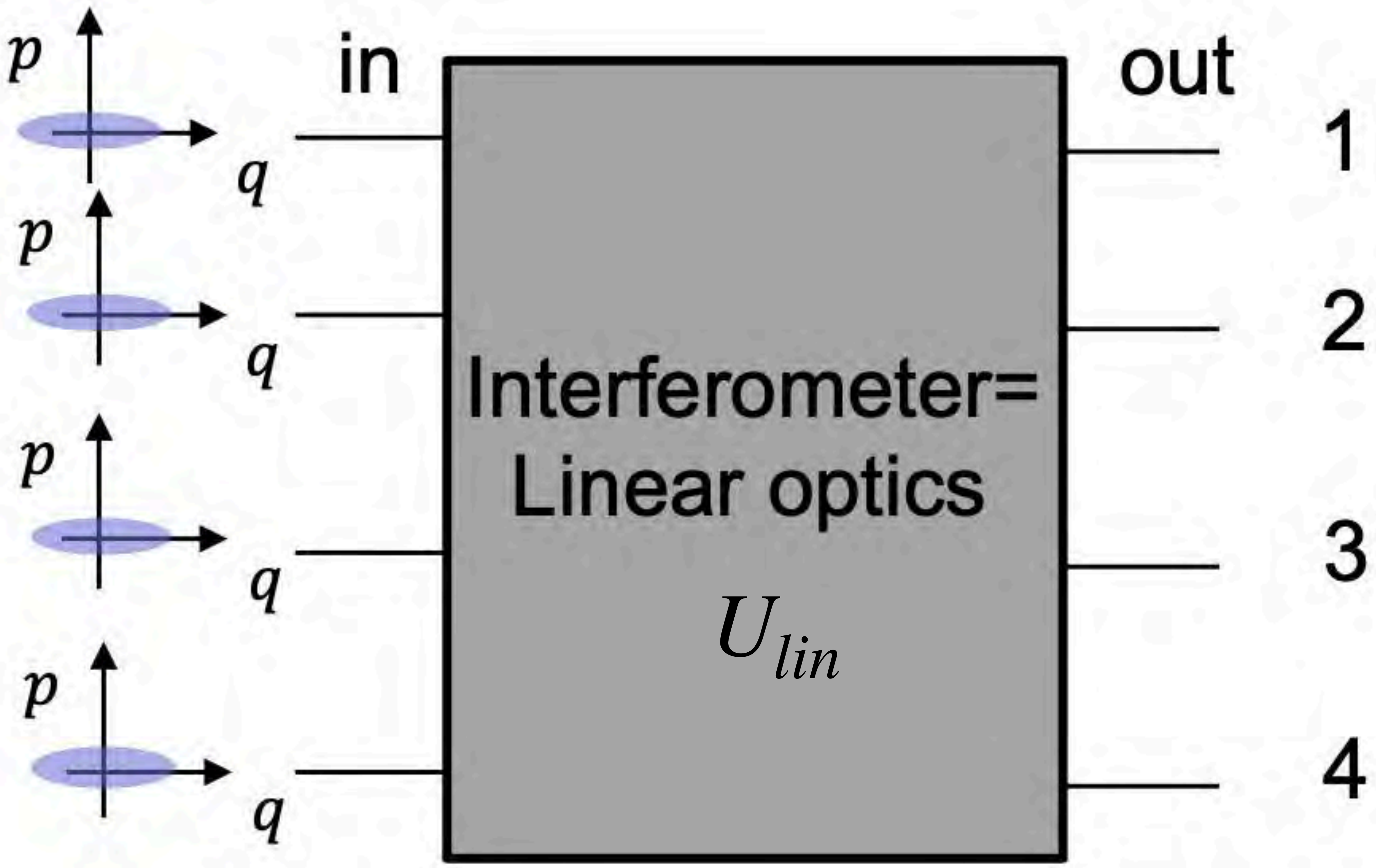
P. Van Loock et al, PRA 76, 032321 (2007)
G. Ferrini et al, PRA 91, 032314 (2015)

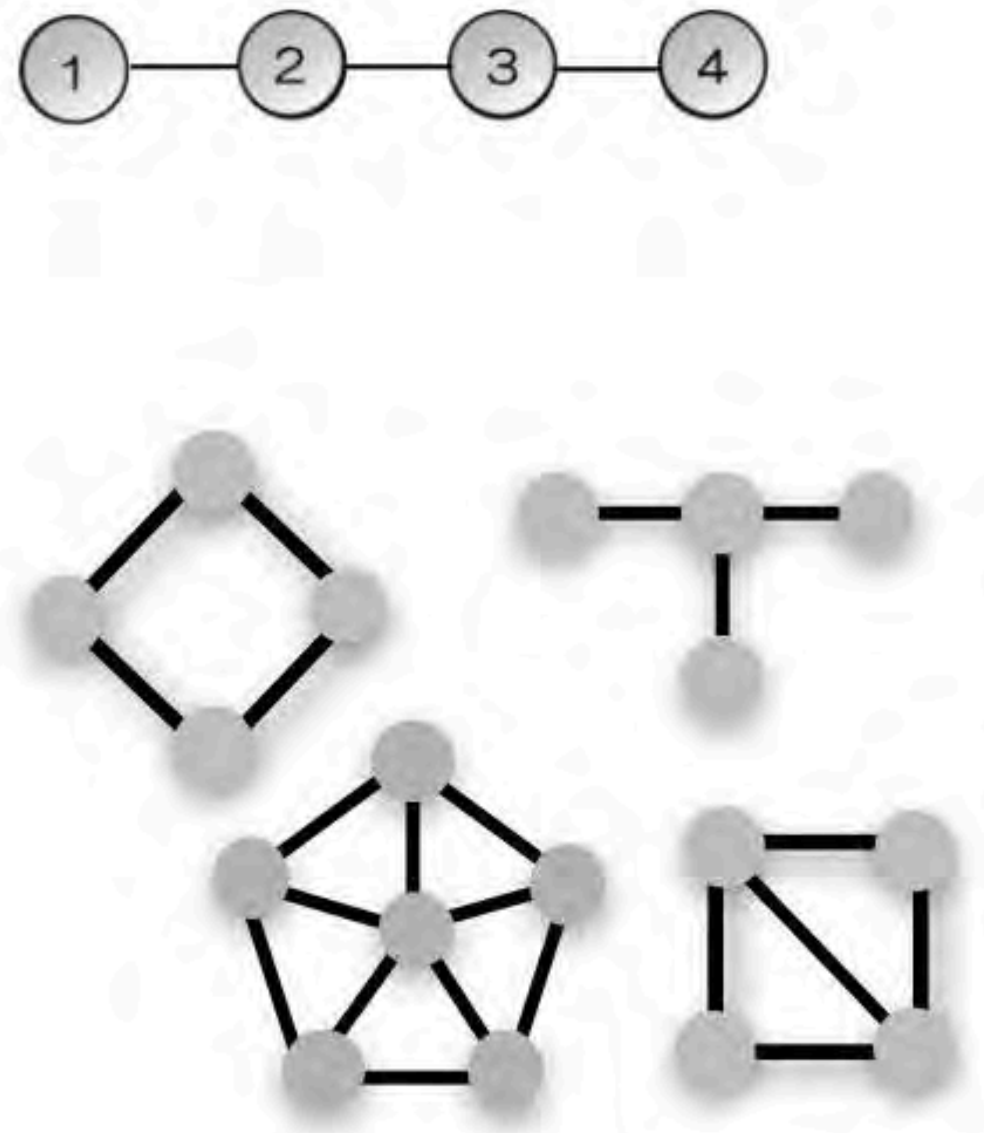
How to generate CV cluster states in practice?

Input squeezed states + linear interferometer



≡





$$\vec{a}_{cluster} = U_{lin}\vec{a}_{squeezed}$$

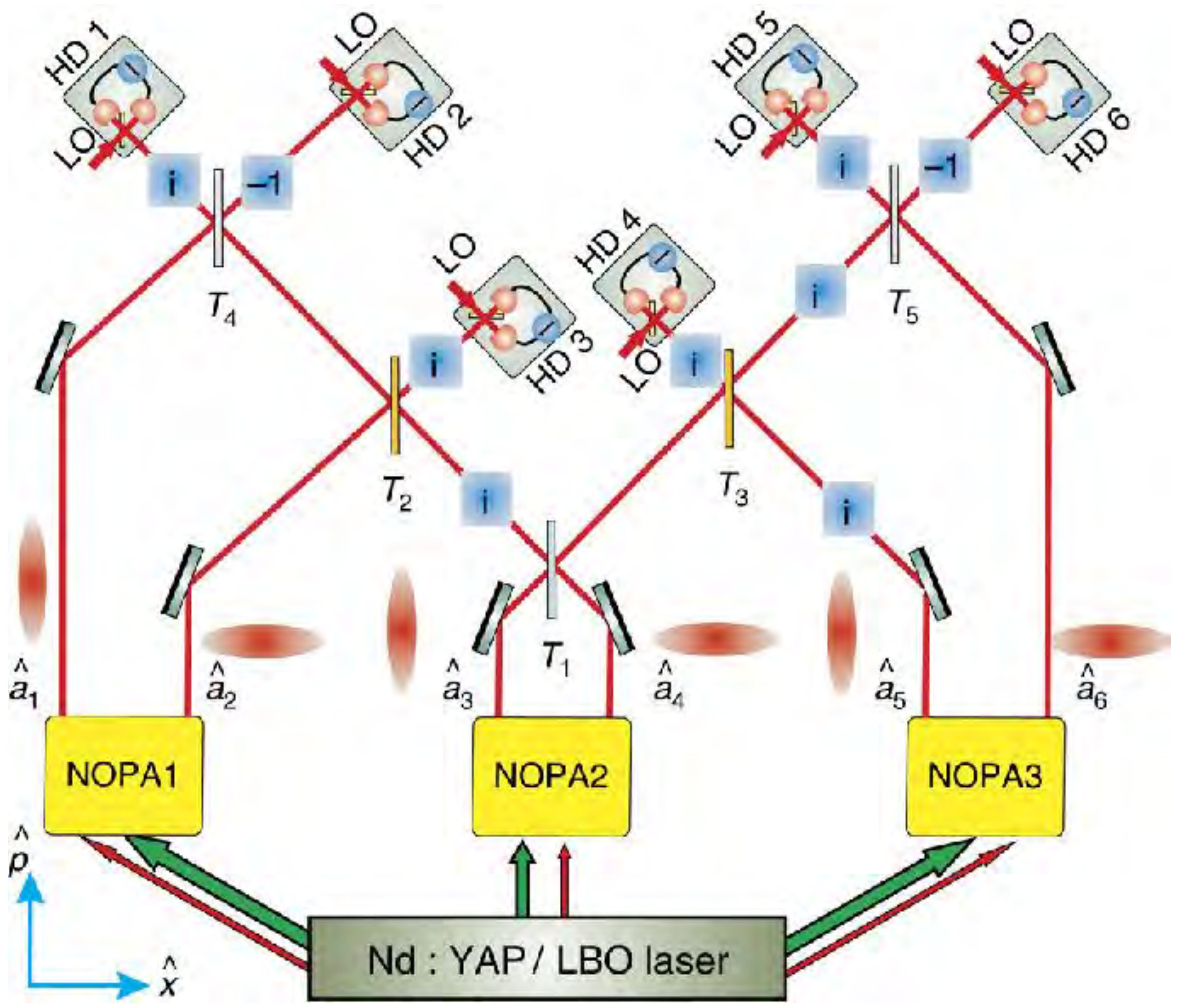
P. Van Loock et al, PRA 76, 032321 (2007)
G. Ferrini et al, PRA 91, 032314 (2015)

Slide: courtesy of V. Parigi

Generation of CV cluster states with spatial modes

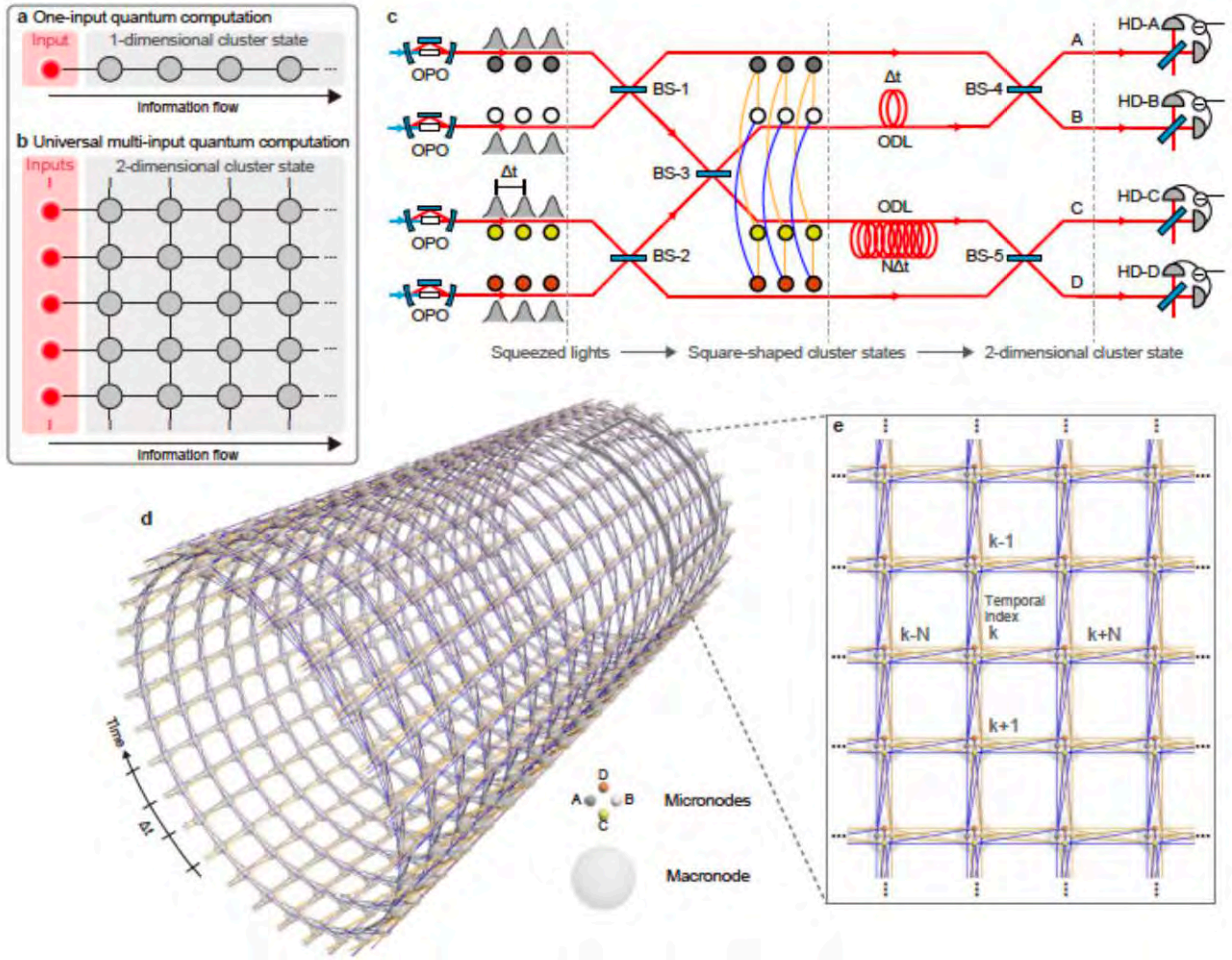
If we want simultaneously available modes, in general:

- large number of optical elements are needed (scales with N)
- Need to reconfigure entirely the hardware if you want to change the target cluster



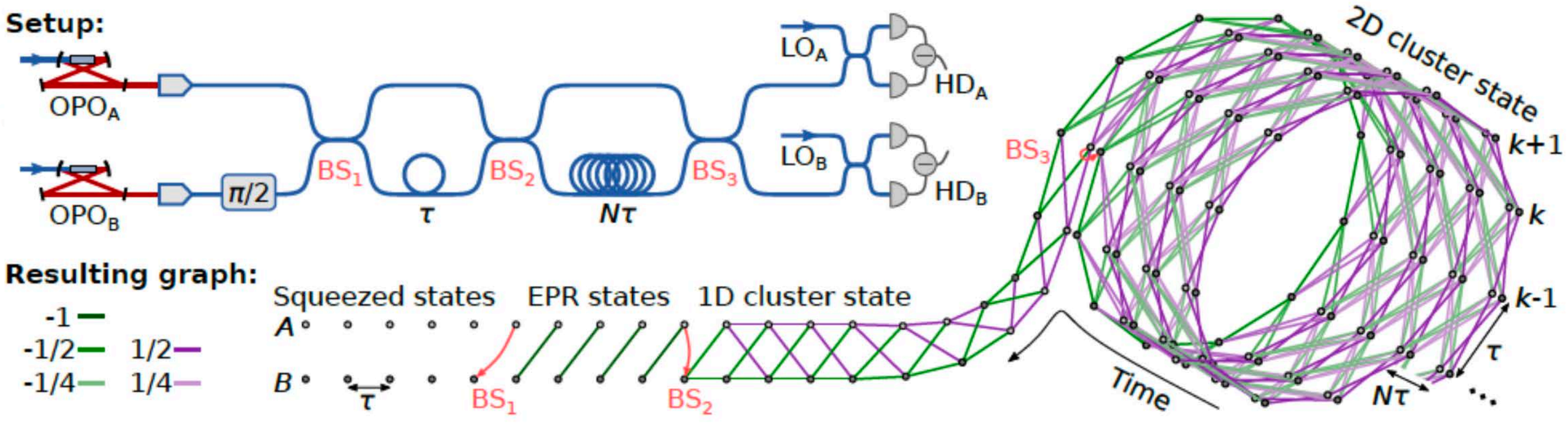
X. Su et al, Nat. Comm. 4, Article number: 2828 (2013)

Generation of CV cluster states with time-bin modes



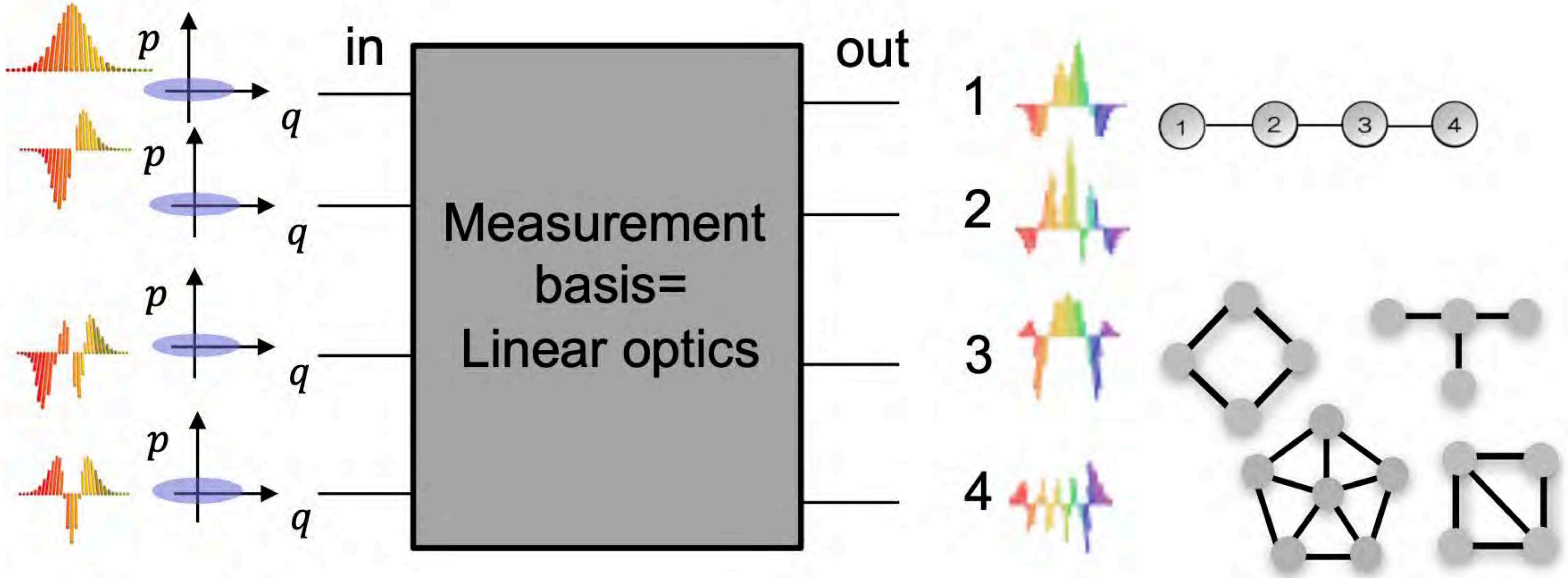
W. Asavanant, *et al.* Science 366, 373-376 (2019)

Theoretical proposal:
N. Menicucci, PRA 83, 062314 (2011)



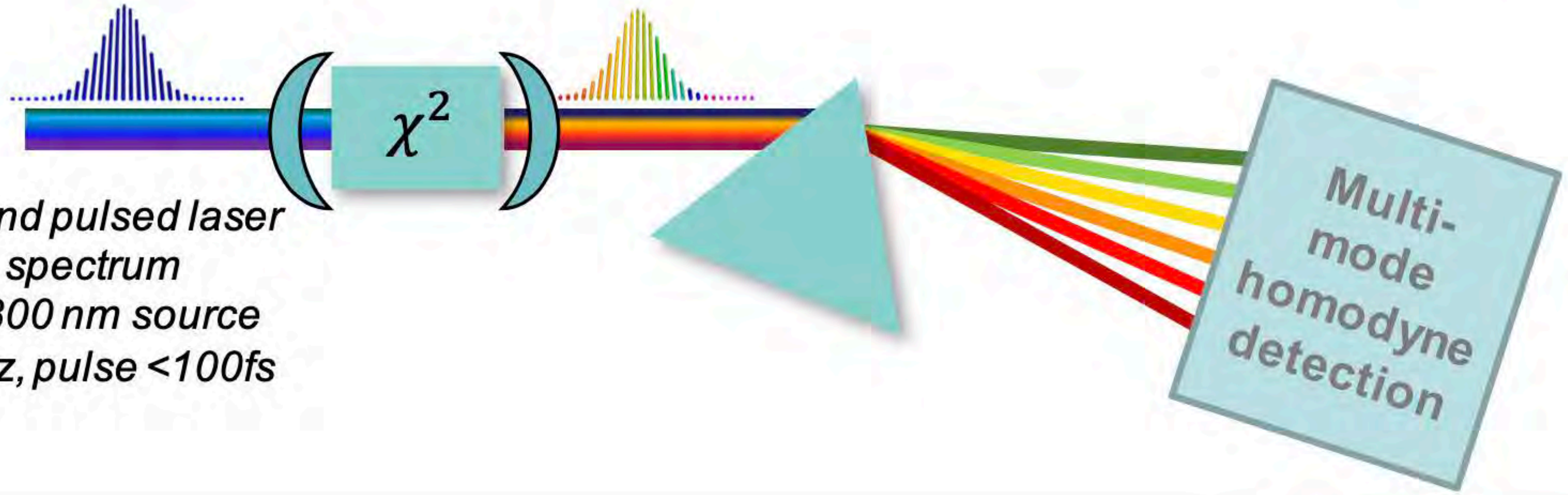
Mikkel V Larsen, *et al.* Science 366, 369-372 (2019)

Generation of CV cluster states with frequency modes



Multi-color(mode) parametric process

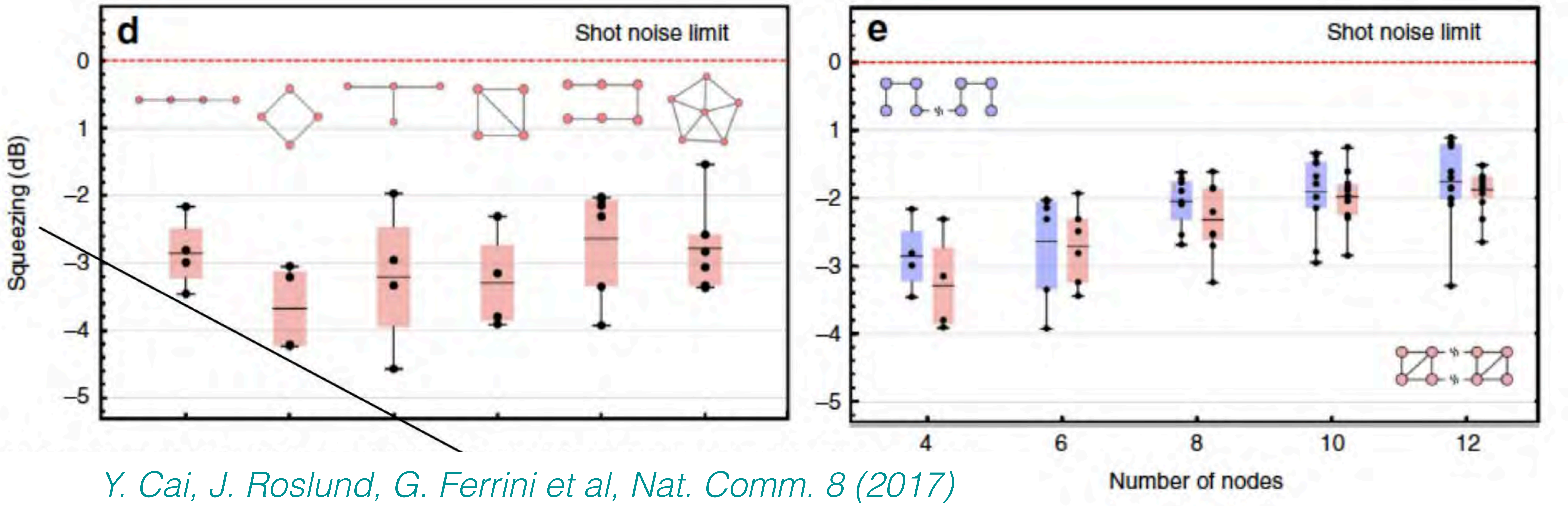
LKB strategy



femtosecond pulsed laser
large spectrum
SHG of 800 nm source
RR 80MHz, pulse <100fs

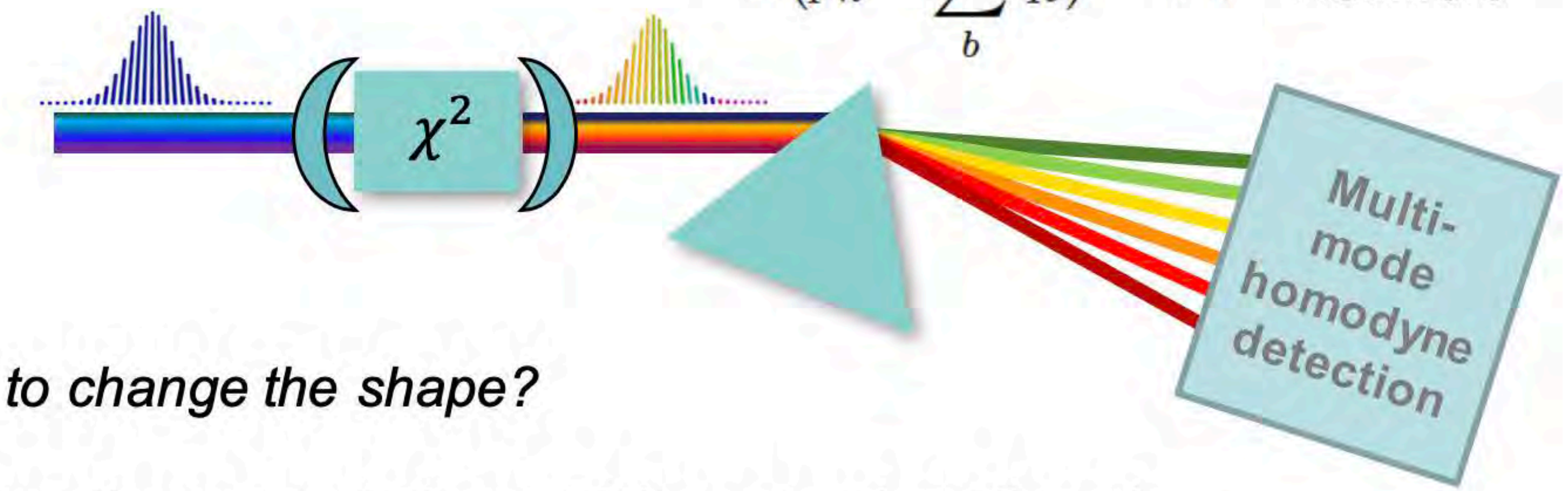
Slide: courtesy of V. Parigi

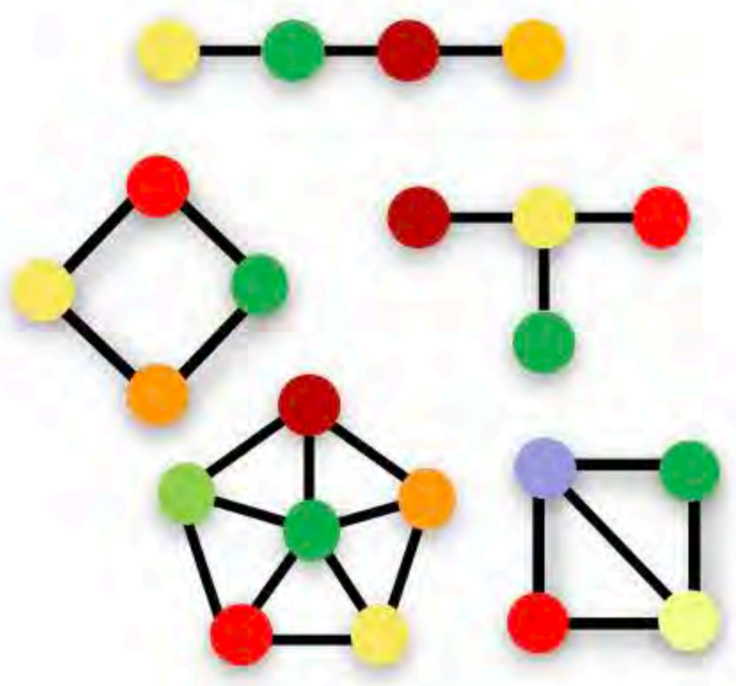
Generation of CV cluster states with frequency modes



Y. Cai, J. Roslund, G. Ferrini et al, Nat. Comm. 8 (2017)

$$\Delta\left(\hat{p}_n - \sum_b \hat{q}_b\right) \longrightarrow 0$$ **Nullifiers**





See also works by Pfister's group: Chen et al, PRL112, 120505 (2014)

and at telecom frequencies: Roman-Rodriguez et al, PRR 043113 (2023) (Parigi's group)

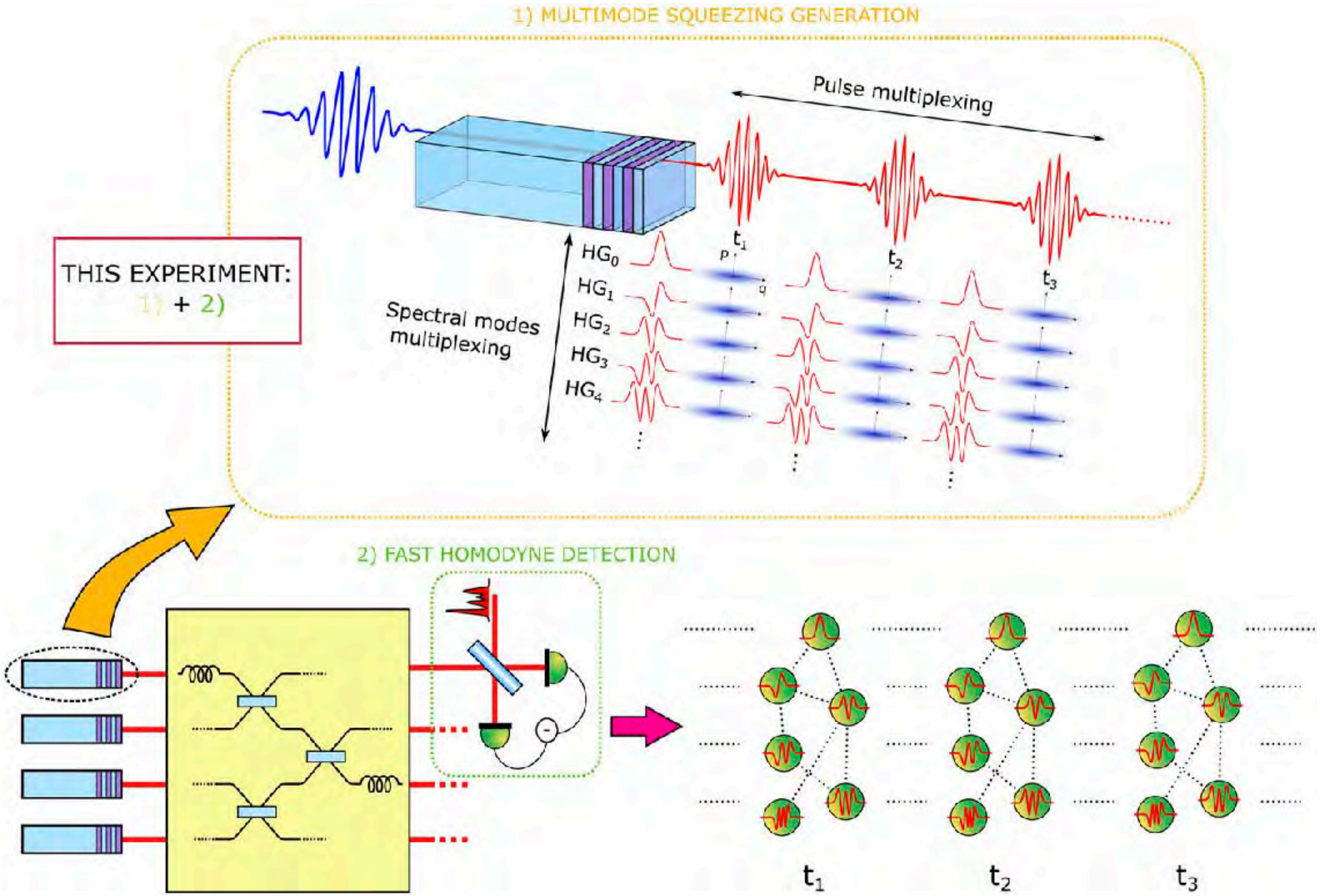
How to change the shape?

Change the spectral shape of the pump (non-linear)
or
Change the measurement basis in homodyne detection (linear)

Slide: courtesy of V. Parigi

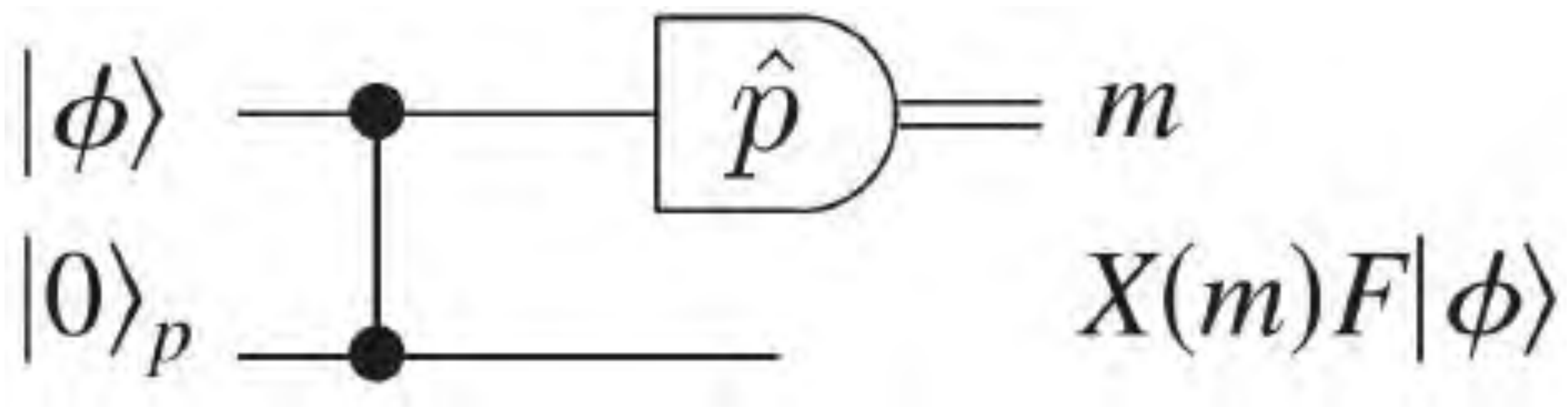
F. Arzani et al, PRA 97, 033808 (2018); G. Ferrini et al, New Journ. Phys. 15, 093015 (2013)

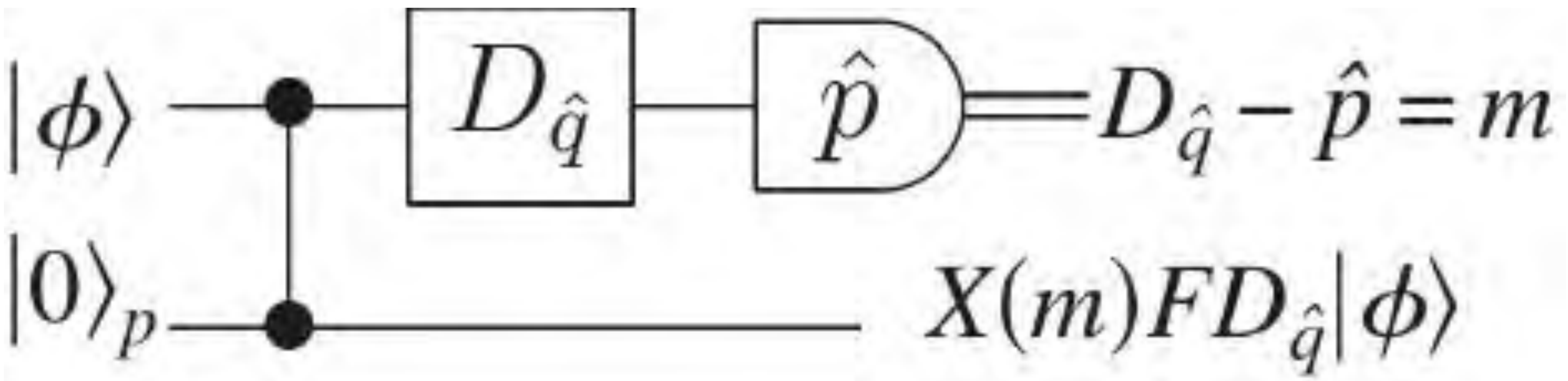
Combining frequency and time modes



T. Kouadou, F. Sansavini, M. Ansquer; J. Henaff; N. Treps; V. Parigi, APL Photonics 8, 086113 (2023)

CV Measurement-based quantum computation





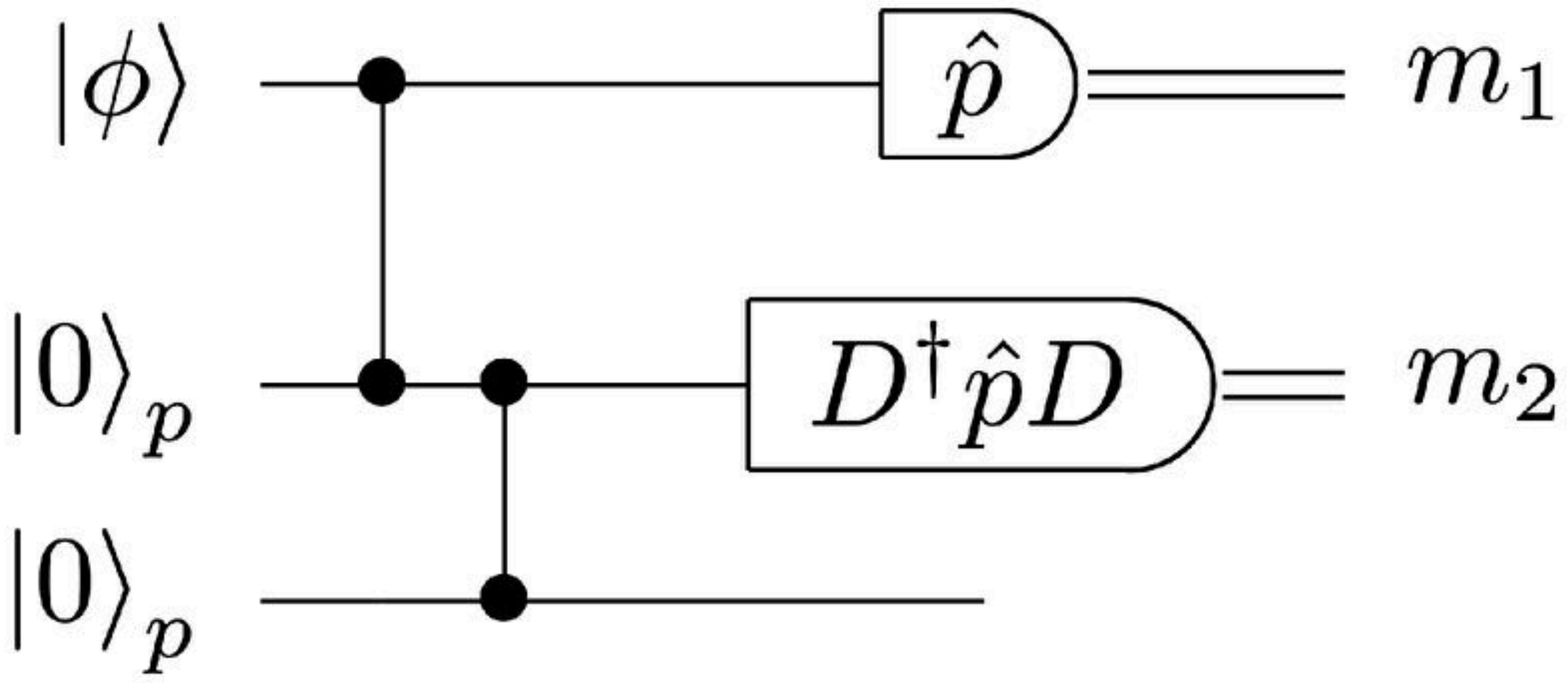
$$D_{\hat{q}} = e^{if(\hat{q})}$$

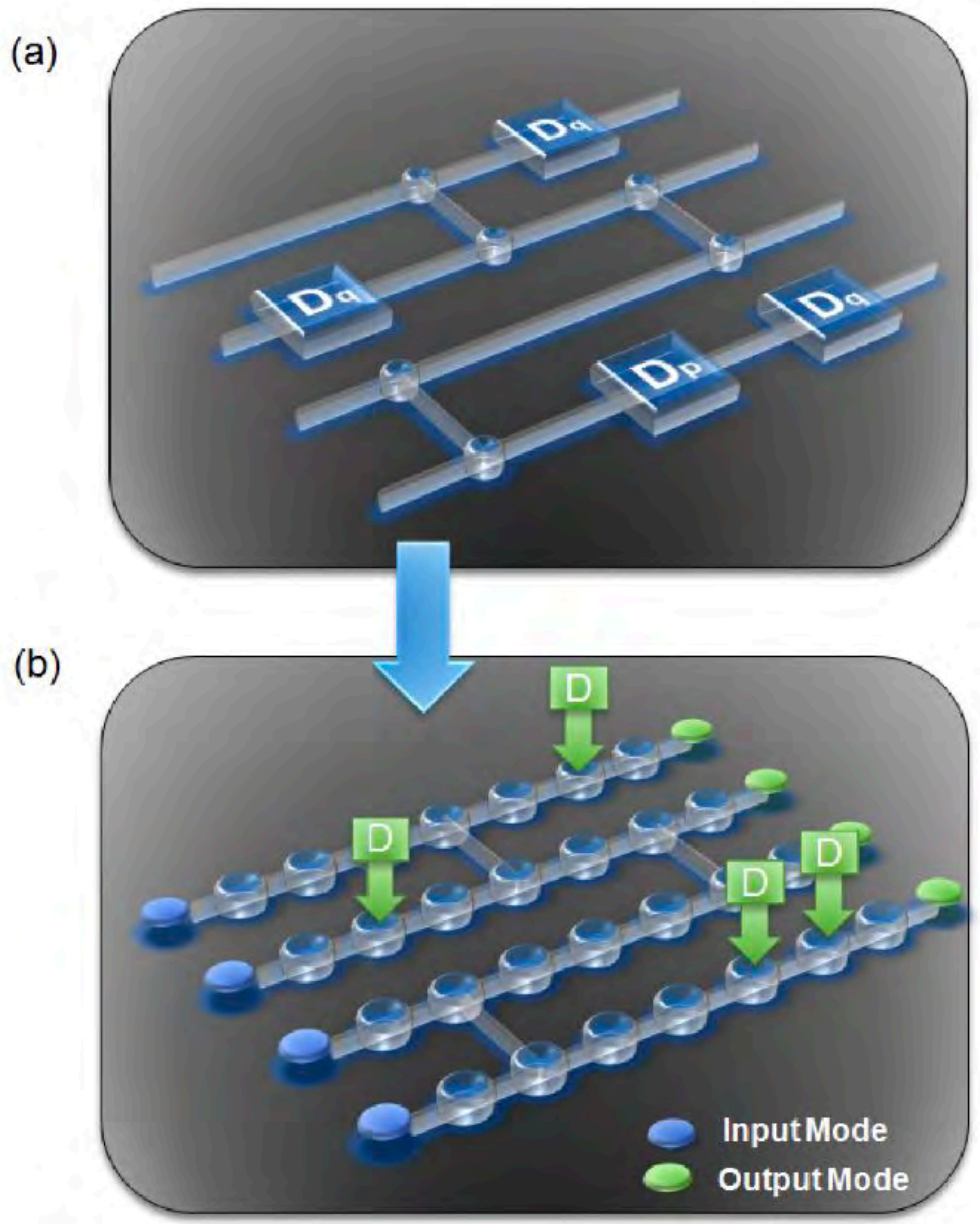
Equivalent to measuring $D_{\hat{q}}^{\dagger}\hat{p}D_{\hat{q}} \equiv \hat{p}_{f(\hat{q})}$

N. Menicucci, P. Van Loock et al, PRL 97, 110501 (2006)

see also calculations details in A. Frisk Kockum et al, Lecture notes on quantum computing, arXiv:2311.08445 (2023)

CV Measurement-based quantum computation



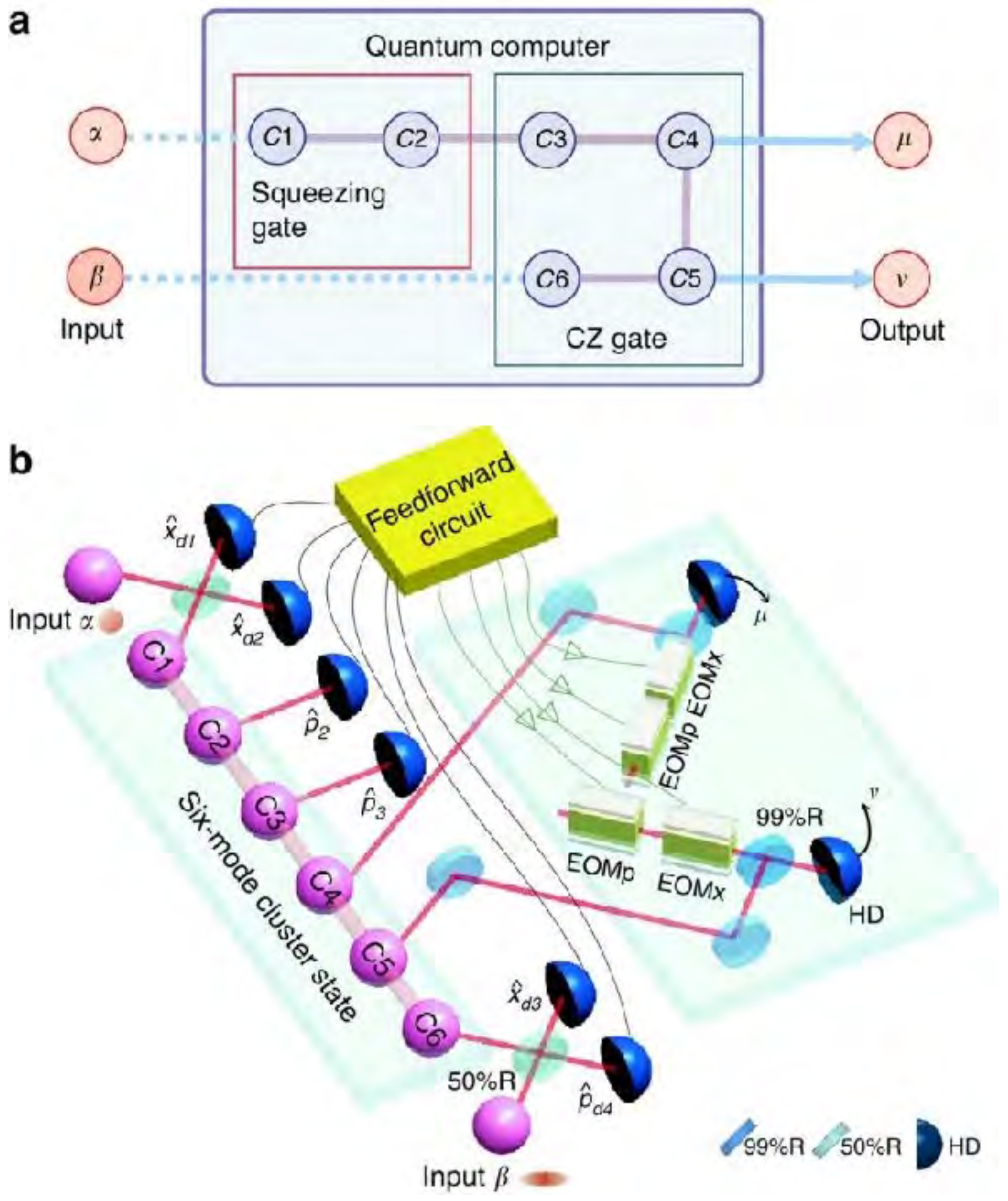


M. Gu et al, Phys. Rev. A 79, 062318 (2009)

see also calculations details in A. Frisk Kockum et al, Lecture notes on quantum computing, arXiv:2311.08445 (2023)

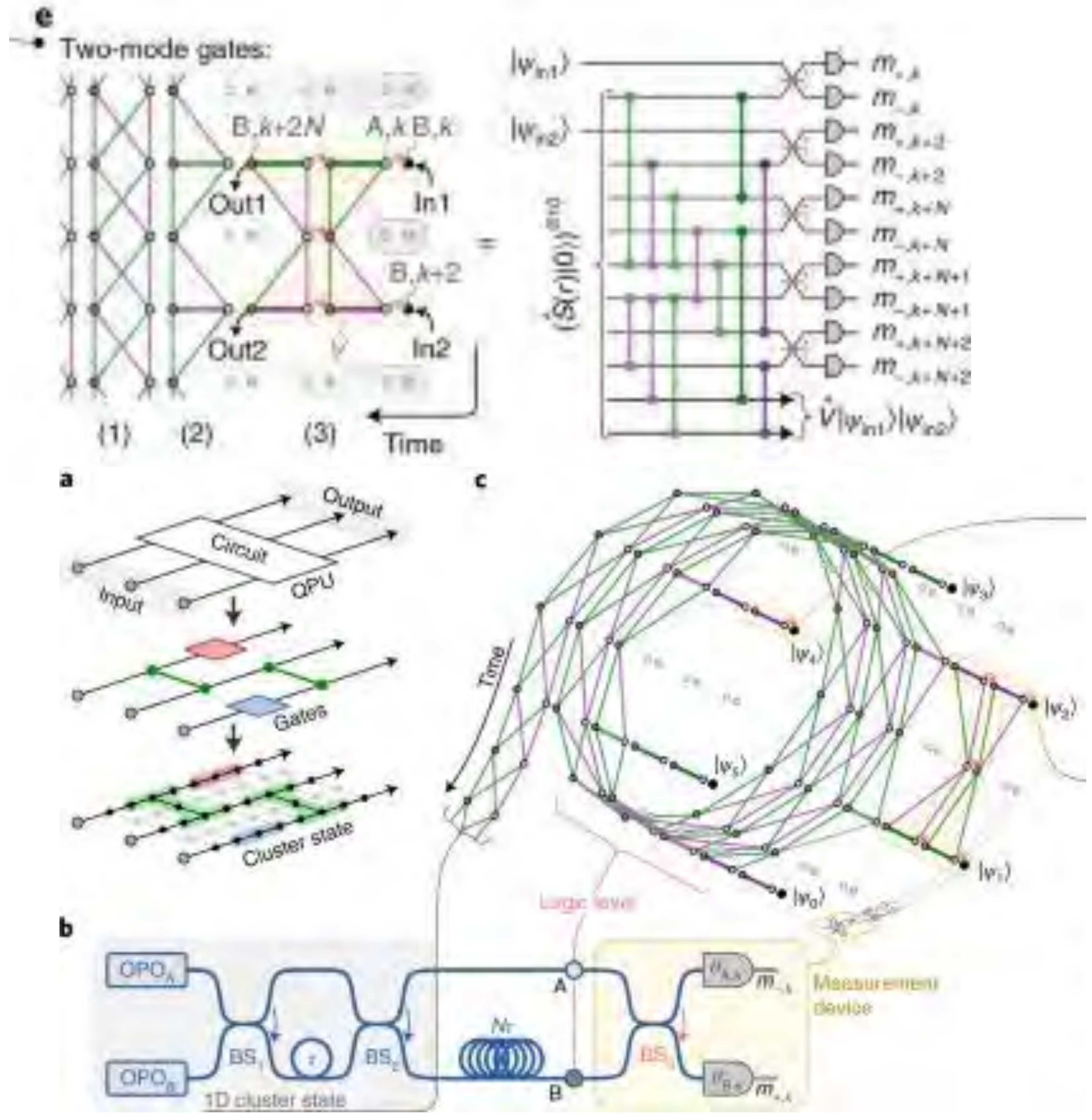
Experimental implementations of CV MBQC

With spatial modes



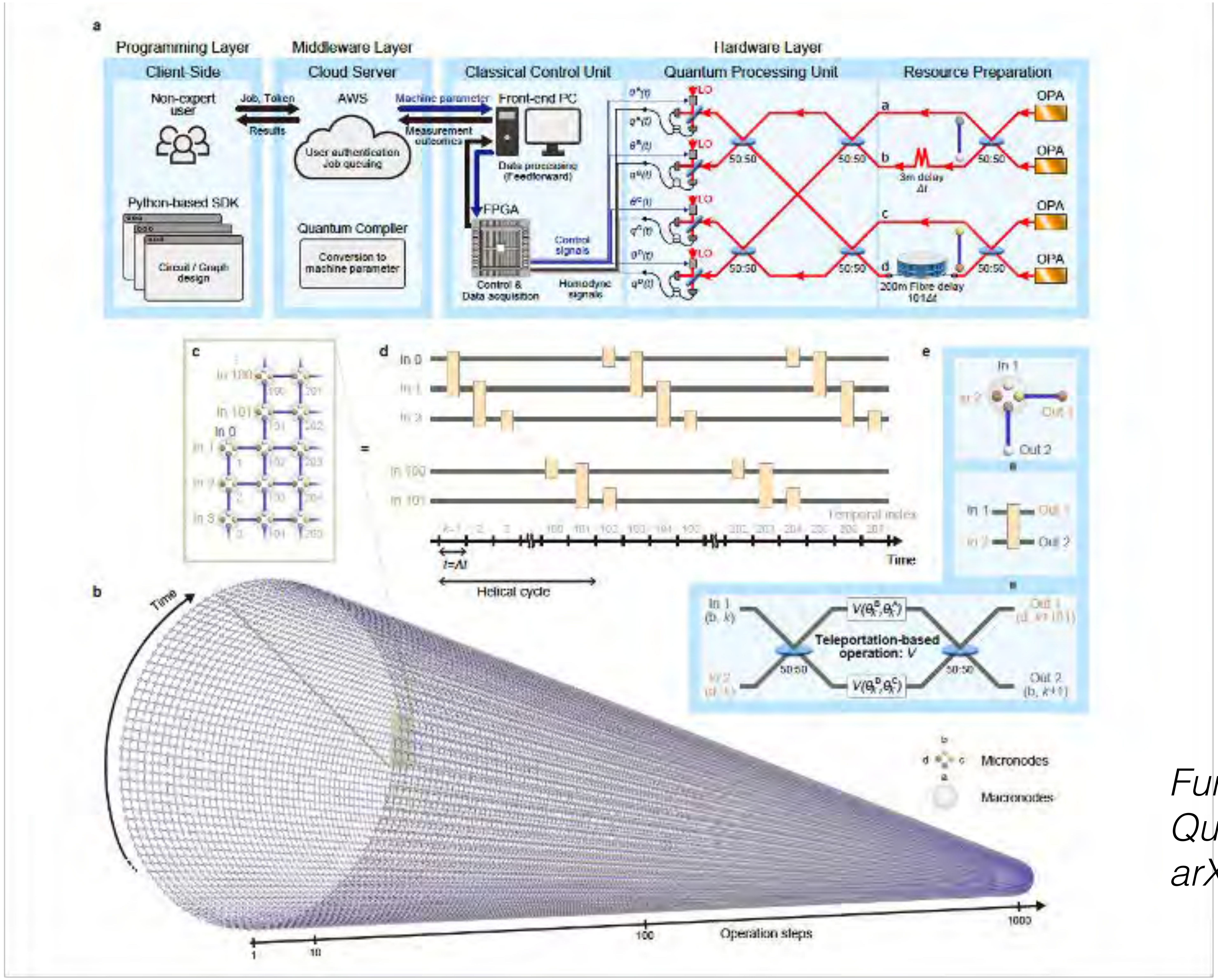
X. Su et al, Nat. Comm. 4, Article number: 2828 (2013)

With time-bin modes



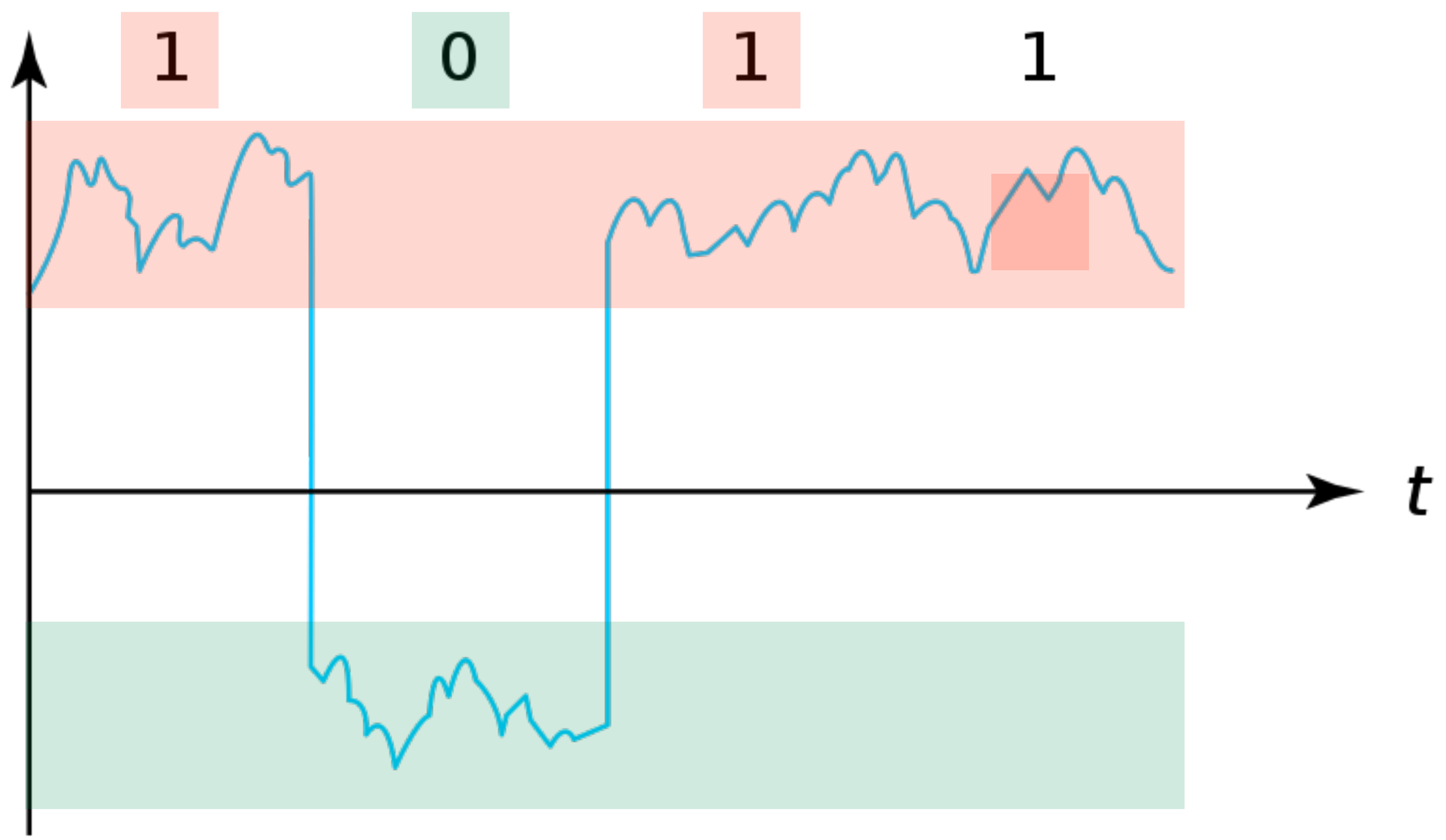
M. V. Larsen et al, Nat. Phys. 17, 1018 (2021)

Generation of CV cluster states with time-bin modes



Furusawa's group: Full-stack Analog Optical Quantum Computer with A Hundred Inputs arXiv:2506.16147v1

Analogously as normal computers, to correct for errors is easier to resort to discrete levels encoding the information



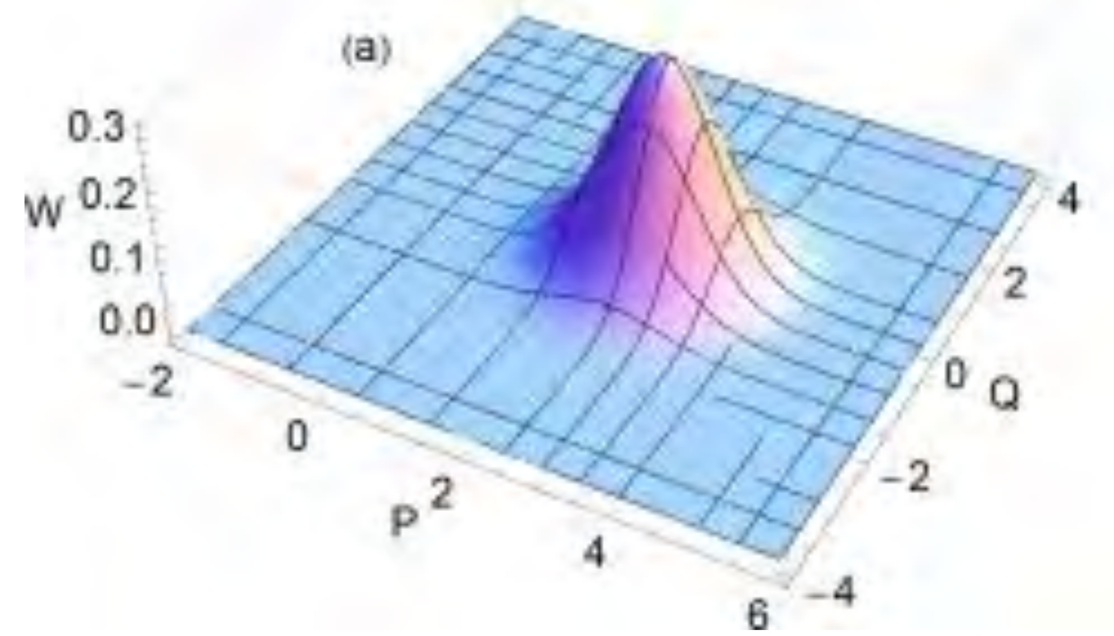
-> Bosonic codes (tomorrow morning)

Summary and conclusion on parts 1a

Continuous-Variable quantum computing

- Infinite dimensional systems modelled by the quantised harmonic oscillator; can be used for quantum information processing by defining transformations on the bosonic field

- MBQC particularly useful model for quantum computation with continuous-variables due to possibility of preparing deterministically large cluster states



Any question?

Part 1b: Sampling models

What is exponential quantum advantage?

- Theoretical prediction : quantum computers allow for solving some computational task efficiently, while hard for normal computers !
  - Efficient = takes a polynomial time t(L) in the input size L
  - Hard = takes an exponential time t(L) in the input size L

E.g: Factoring:
15 = 5x3

- Efficient for a quantum computer (Shor)
- Hard for normal computers

10433 x 16453 = ? (easy)

? x ? = 171654149 (hard)

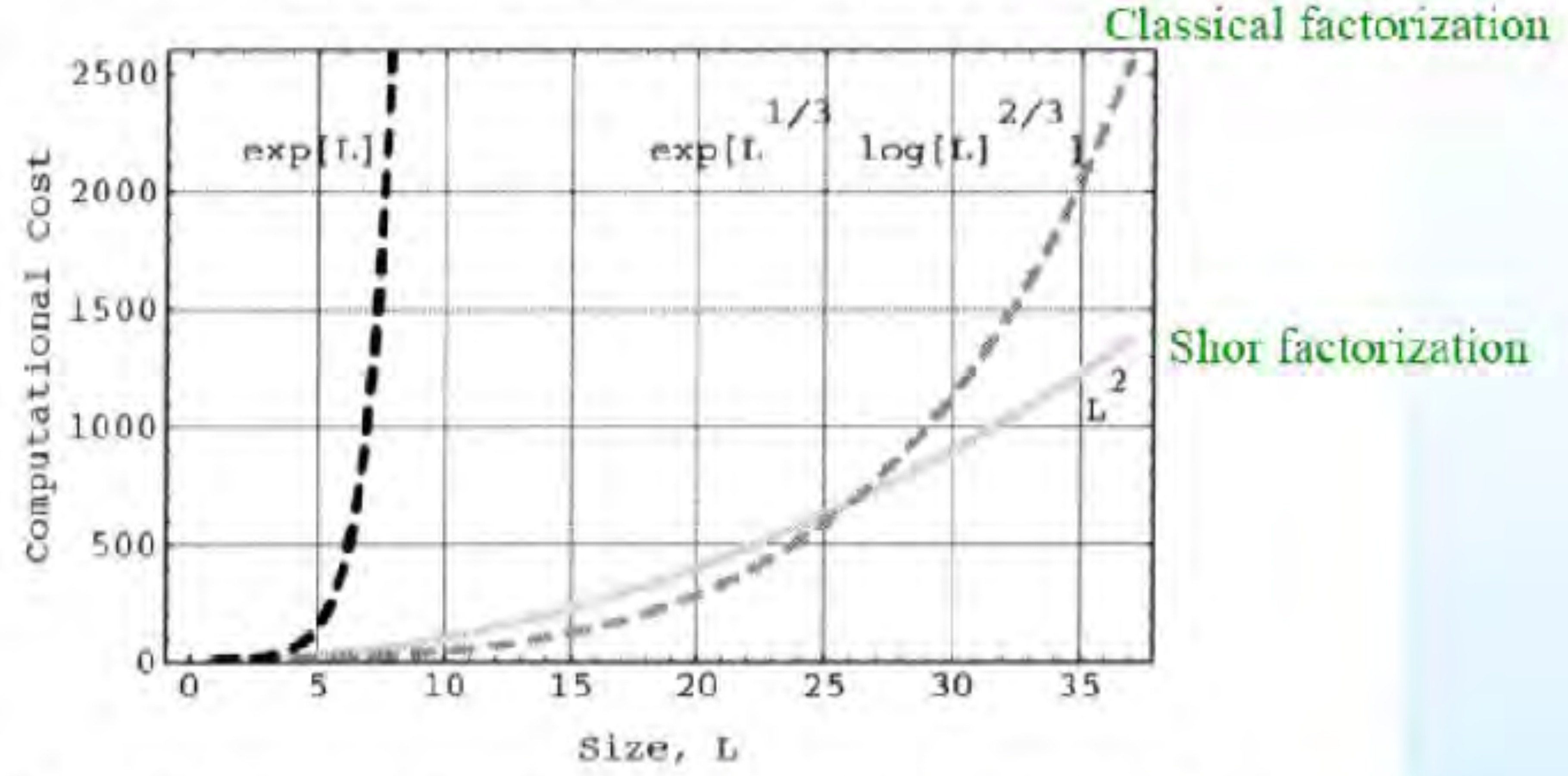


Fig. 2.5 The best factoring algorithms grow subexponentially (but super-polynomially) in *L*, the number of bits needed to specify the number being factored.

Extended Church-Turing thesis

WACQT Wallenberg Centre for Quantum Technology

“Any “reasonable” model of computation can be simulated by a probabilistic Turing machine with at most polynomial overhead” (‘70s/‘80s)

Because of Shor’s algorithm, at least one of the following options must hold (Shor’s trilemma);

- It exists a classical efficient algorithm for factoring;
- It is not possible to build quantum computers;
- The Extended Church-Thesis is false

Extended Church-Turing thesis

Shor's algorithm might be good to disproof, but:

- we don't have guarantees that an efficient classical algorith doesn't exist
- so far we don't have large enough quantum computers

We want a provably hard problem, easier to implement than Shor's algo

Digression: when can we run Shor's algorithm?

Quantum Physics

[Submitted on 23 May 2019 (v1), last revised 13 Apr 2021 (this version, v3)]

How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney, Martin Ekerå

Quantum Physics

[Submitted on 21 May 2025]

How to factor 2048 bit RSA integers with less than a million noisy qubits

Craig Gidney

Quantum Physics

[Submitted on 12 Feb 2026]

The Pinnacle Architecture: Reducing the cost of breaking RSA-2048 to 100 000 physical qubits using quantum LDPC code

Paul Webster, Lucas Berent, Omprakash Chandra, Evan T. Hockings, Nouédyn Baspin, Felix Thomsen, Samuel C. Smith, Lawrence Z. Cohen

Quantum Physics

[Submitted on 30 Mar 2026]

Shor's algorithm is possible with as few as 10,000 reconfigurable atomic qubits

Madelyn Cain, Qian Xu, Robbie King, Lewis R. B. Picard, Harry Levine, Manuel Endres, John Preskill, Hsin-Yuan Huang, Dolev Bluvstein

- So far there is no "proper" experimental implementation of Shor's algorithm (See: Craig Gidney blog "Why haven't quantum computers factored 21 yet?")

Sampling models of quantum computation

WACQT Wallenberg Centre for Quantum Technology

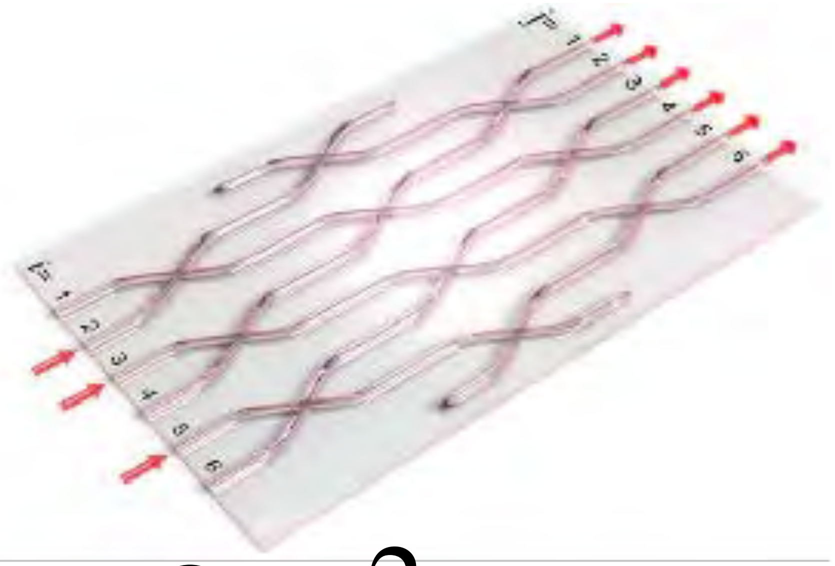
Goal : prove quantum advantage by solving simpler problems (e.g. than factoring), yet faster than with classical power (quantum "primacy").

Classical Computer

Universal Quantum Computer

Calculator

Sampling model



Task: sample from

$$P(\vec{x}) = |\langle \vec{x} | U_C | 0 \rangle^{\otimes n}|^2$$

B. Terhal, D. DiVicenzo, Quant. Inf. Comp. 4, 134 (2004); S. Aaronson, A.Arkhipov, Theory Comput. 9, 143 (2013)

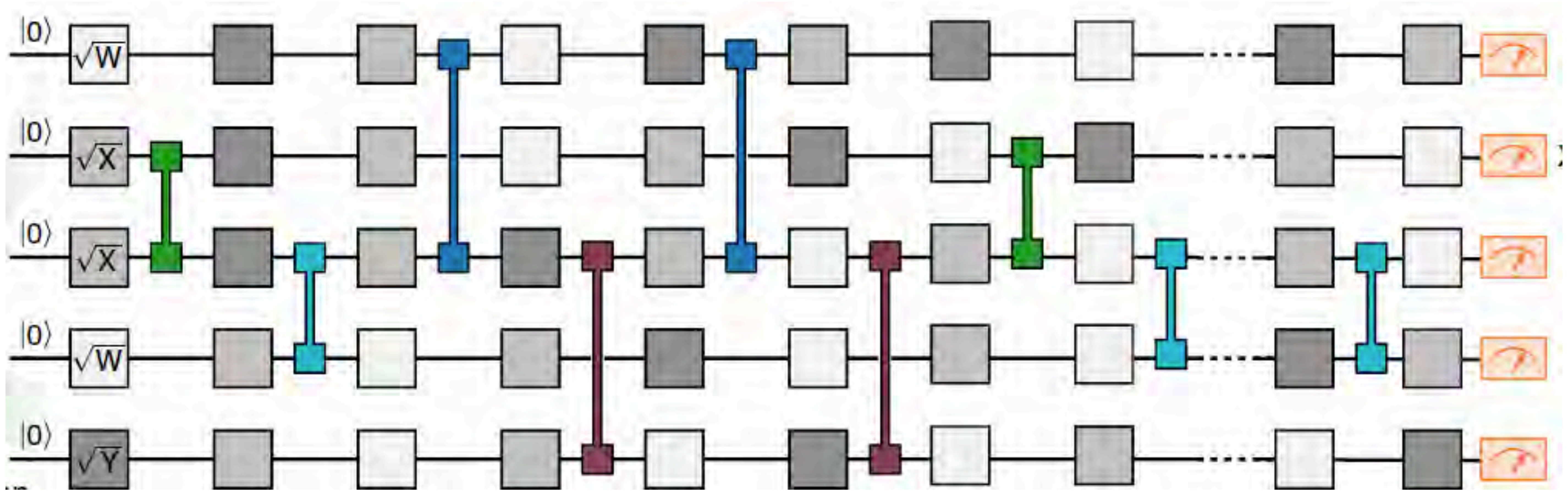
What does “faster” mean? Two notions:

- (1) Theoretic-computer science:
  - it takes an exponential time for the classical algorithm, T = exp(n)
  - polynomial time for quantum algorithm, T = poly(n)

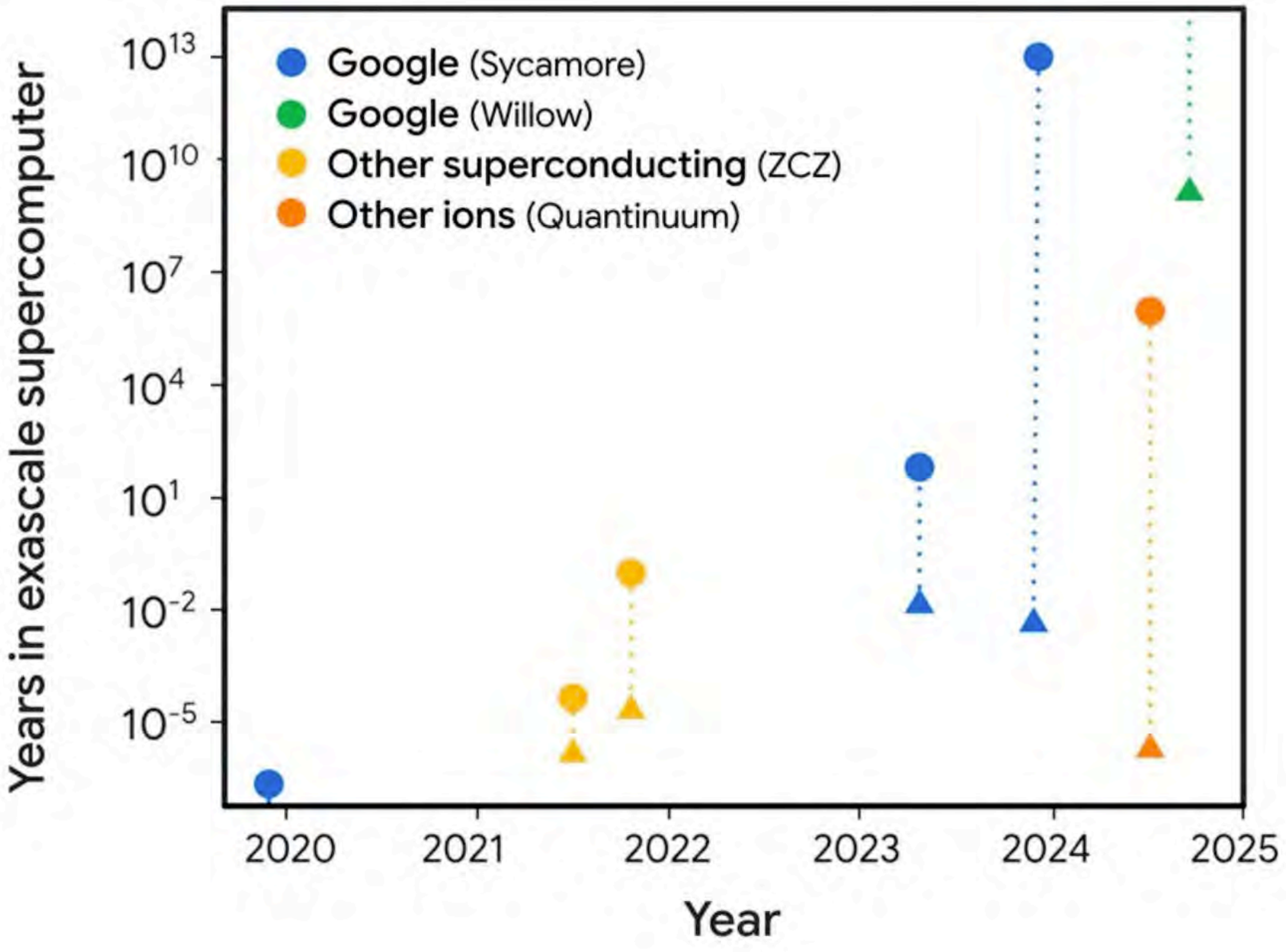
- (2) Practical:
  - Todays supercomputer cannot simulate the outcome

Task: sample from

$$P(\vec{x}) = |\langle \vec{x} | U_C | 0 \rangle^{\otimes n}|^2$$



$\vec{x}$



Computational costs are heavily influenced by available memory. Our estimates therefore consider a range of scenarios, from an ideal situation with unlimited memory (▲) to a more practical, embarrassingly parallelizable implementation on GPUs (●).

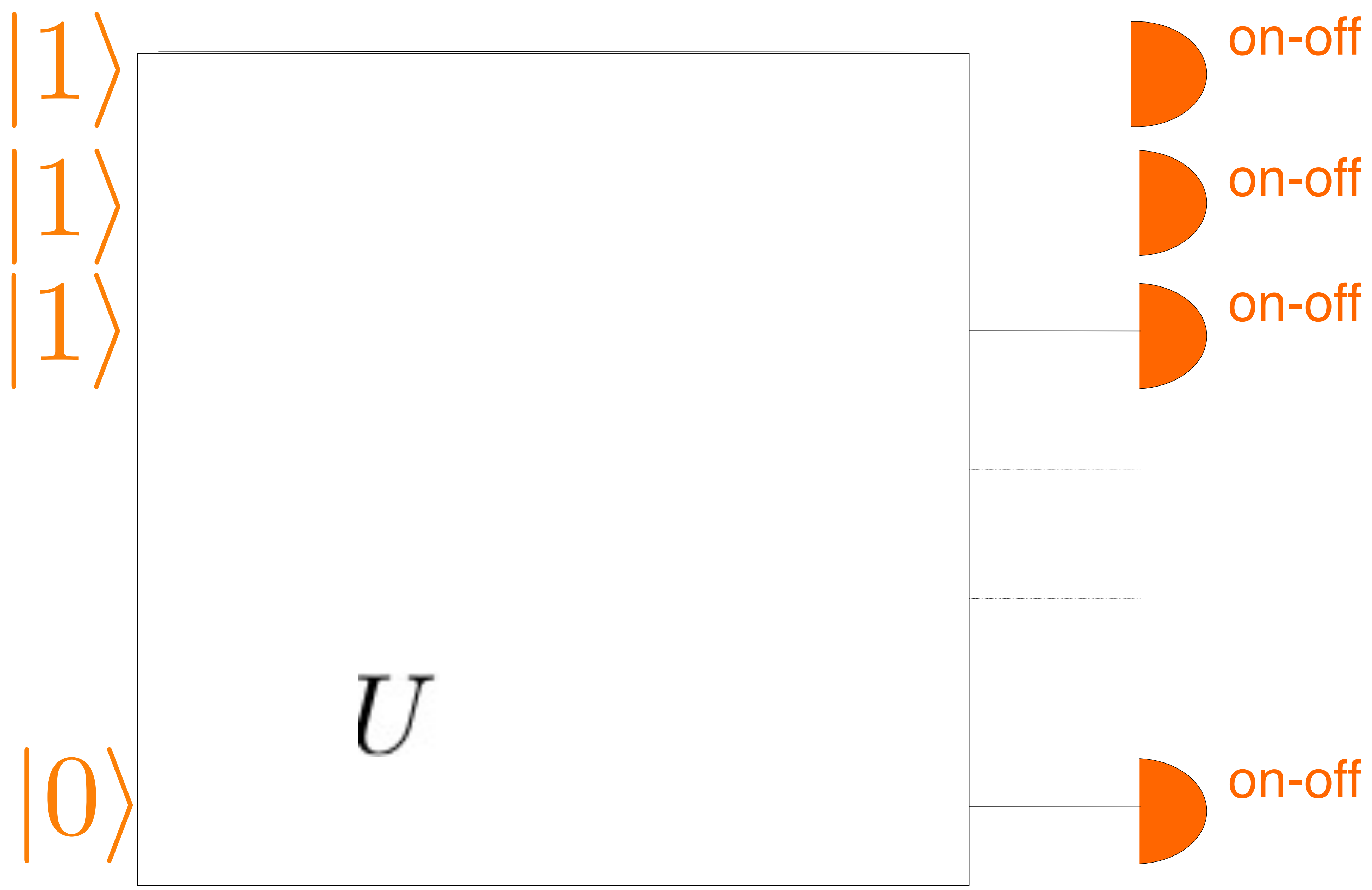
- Took Willow chip 5 minutes, would require 10^{25} years for a normal computer

Spoofing quantum advantage

- (practic): There are better and better classical algorithms! Sycamore 53-qubits experiment has been simulated! 15 hours, 512 GPUs *[Pan et al., 2022]*
  Energy cost estimated 100 to 1000 times larger than quantum chip

- (theoretic): Random Circuit Sampling become simulatable with noise! Simulation time is exponential in $1/\gamma$ (very inefficient for low noise)
  "A polynomial-time classical algorithm for noisy random circuit sampling"
  Dorit Aharonov et al, STOC 2023

  - Need to find a sweet spot noise-size
  - Need for more architectures for showing quantum Primacy!



single photon detection

$$S = \{s_1, ..., s_m\}$$

$$\Pr[S] = \frac{|\mathrm{Per}(U_S)|^2}{s_1!...s_m!}$$

Approximating multiplicatively the permanent is #P-hard!

BosonSampling is classically hard, unless some complexity theory conjecture is broken

S. Aaronson, A.Arkhipov, Theory Comput. 9, 143 (2013)

Relevant classical complexity classes

- P = set of decision problems solvable in polynomial (poly) time by a Turing machine;
- BPP = set of decision problems solvable in poly time by a probabilistic Turing machine (err <1/3)
- NP = set of decision problems which solution can be verified in polynomial time;

$$\mathrm{P} \subseteq \mathrm{NP}$$

- $\sharp$P = set of problems that count the number of solutions of NP problems
- $\sharp$P-hard = set of problems which solution allows solving all problems in $\sharp$P
- Polinomial Hierarchy PH: $\Delta_0^{\mathsf{P}} := \Sigma_0^{\mathsf{P}} := \mathsf{P}$ $\quad \Sigma_{i+1}^{\mathsf{P}} := \mathsf{NP}^{\Sigma_i^{\mathsf{P}}}$ $\quad \Delta_{i+1}^{\mathsf{P}} := \mathsf{P}^{\Sigma_i^{\mathsf{P}}}$

$$\Sigma_i^{\mathrm{P}} \subseteq \Sigma_{i+1}^{\mathrm{P}}$$

- Toda's theorem: implies that PH is contained in $\mathsf{P}^{\#\mathsf{P}}$

Boson Sampling: proof of hardness

- Approximating the probability of a specific measurement outcome at the output of a linear interferometer is a #P-hard problem

- If a polynomial-time classical algorithm for exact boson sampling existed, then the above probability could have been approximated in BPP^{NP}, i.e. within the second level of the polynomial hierarchy (using Stockmeyer counting algorithm)

- This would imply $\#P \subseteq BPP^{NP} \Rightarrow \mathrm{PH} \underset{\text{Toda}}{\subseteq} \mathrm{P}^{\#\mathrm{P}} \subseteq \mathrm{P}^{\mathrm{BPP}^{\mathrm{NP}}} \subseteq \Sigma_3^P$.

collapse to the third level!

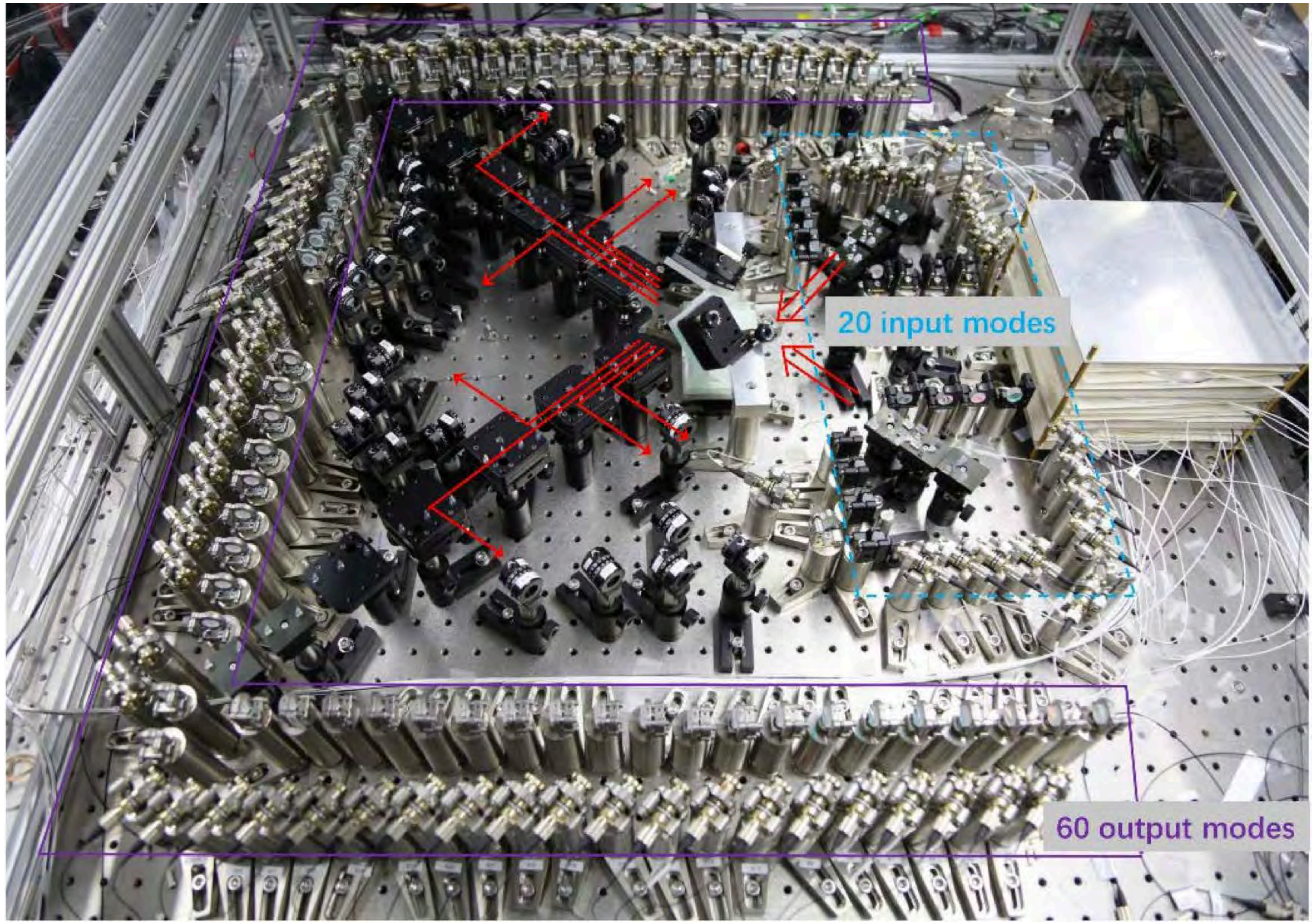
$(\mathrm{BPP}^{\mathrm{NP}} \subseteq \Sigma_2^P)$

➡ BosonSampling is classically hard, unless the polynomial hierarchy collapses

S. Aaronson, A.Arkhipov, Theory Comput. 9, 143 (2013)

First experiments with 3 photons in a handful of modes (2013):
Sciarrino (Rome), Wamsley (Oxford), White (Brisbane), Walter (Vienna)

Most recent*:* 20 photons in 60-mode interferometer, sample Hilbert space of dimensionality 10^14
H. Wang et al, Phys. Rev. Lett. 123, 250503 (2019) Pann group



Also simulatable with noise (for specific noise models)
Oh et al, arXiv:2301.11532

Better and better classical algorithms
Neville et al., Nat. Phys. 2017;
Oh et al, PRL 2022

Rhetoric question

Do we need all those non-Gaussian circuit elements, or can we do with less?

Reminder, Mari-Veitch theorem:

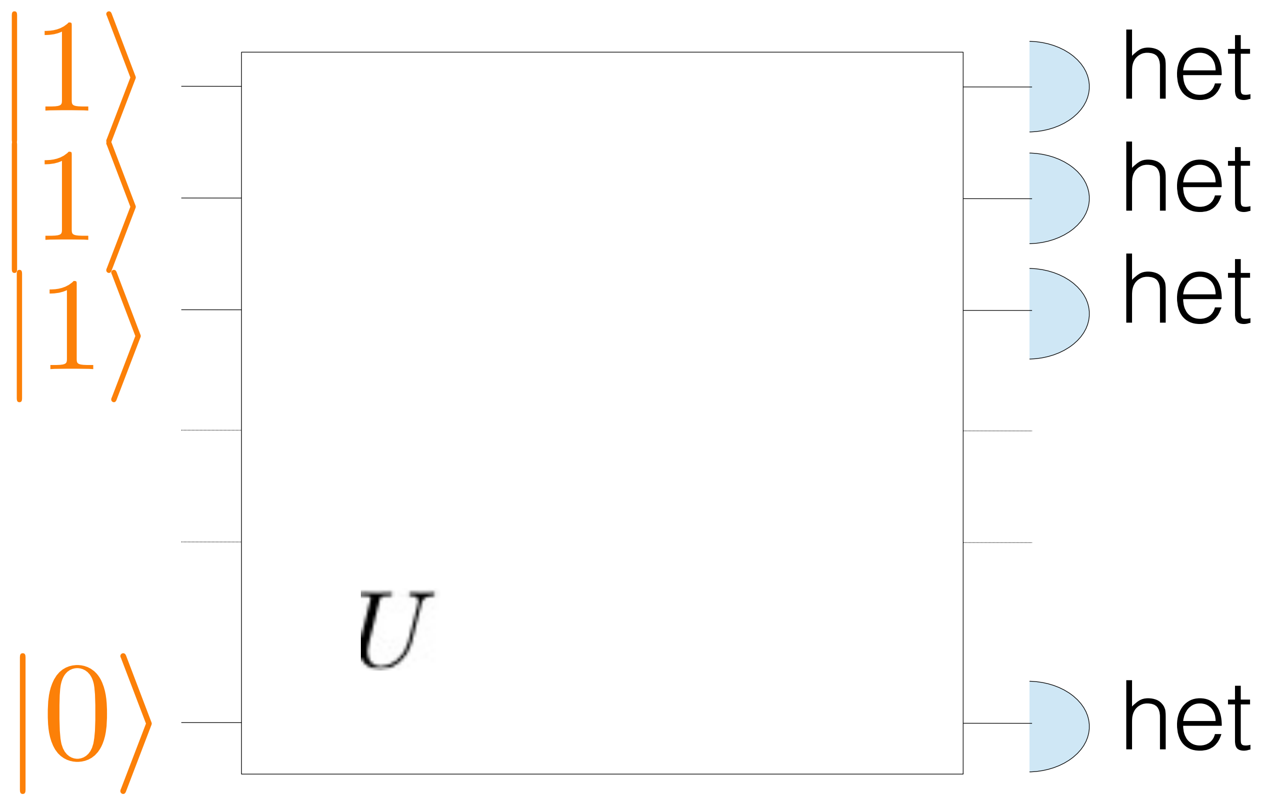
if all Wigner functions (input state, “gates”, measurements) are positive, there is no exponential quantum advantage!

S. D. Bartlett et al, PRL 88, 097904 (2002);
Mari, J. Eisert, PRL 109, 230503 (2014);
V. Veitch et al, NJP 16, 013009 (2014)
S. Rahimi-Keshari et al, PRX 6, 021039 (2016)

WACQT Wallenberg Centre for Quantum Technology

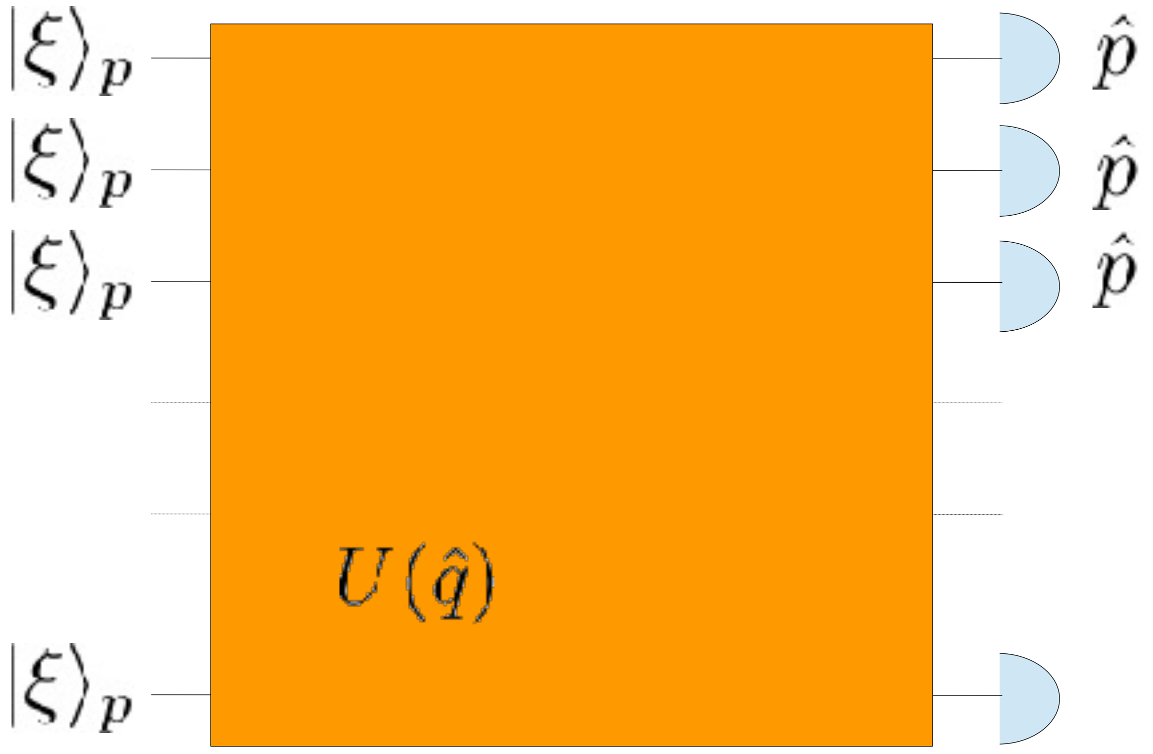
Exiting minimally the hypothesis of Mari-Veitch theorem

1)…the input state



CV Non-Gaussian input circuit

2)…the unitary evolution



CV Instantaneous Quantum Polytime

3)…the detection

Gaussian Boson Sampling

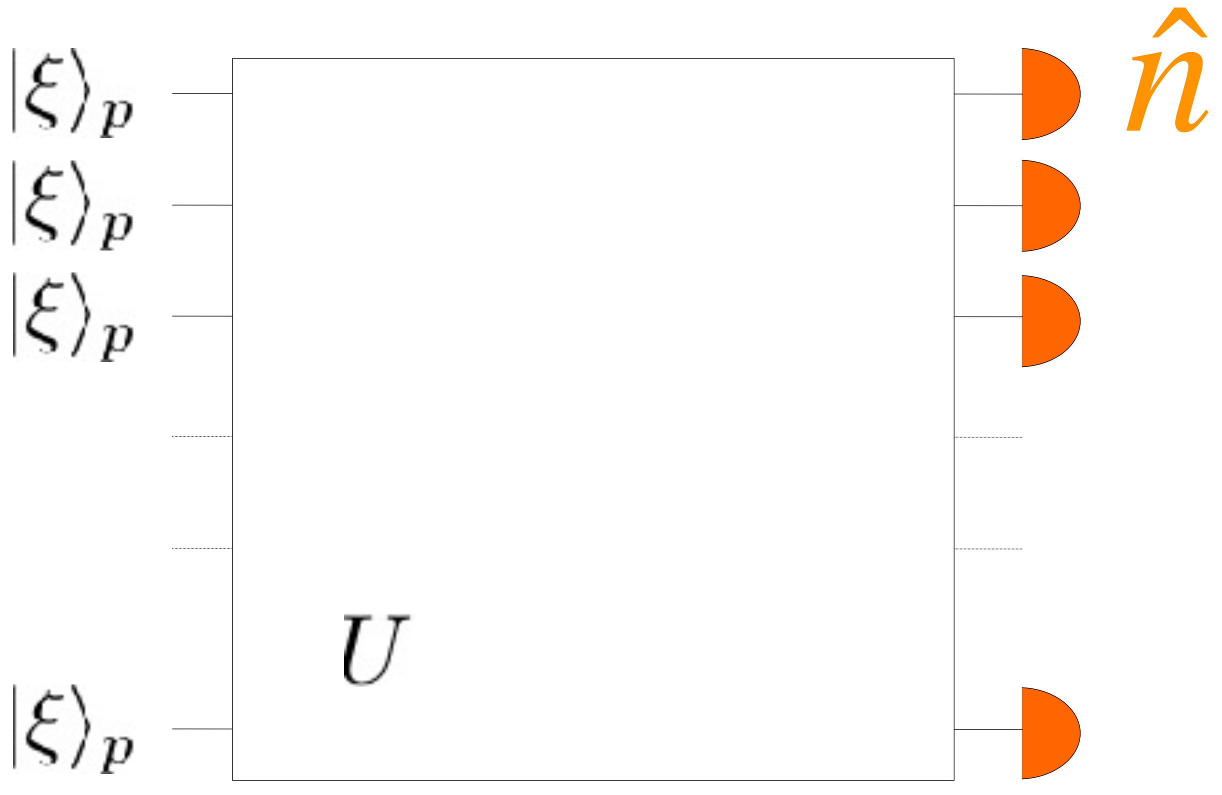
Chabaud et al, PRA 062307 (2017)
Chakhmakhchyan, PRA 032326 (2017)
Lund et al, PRA 022301 (2017)

Douce et al, PRL 118 070503 (2017)
Douce et al, PRA 99, 012344 (2019)

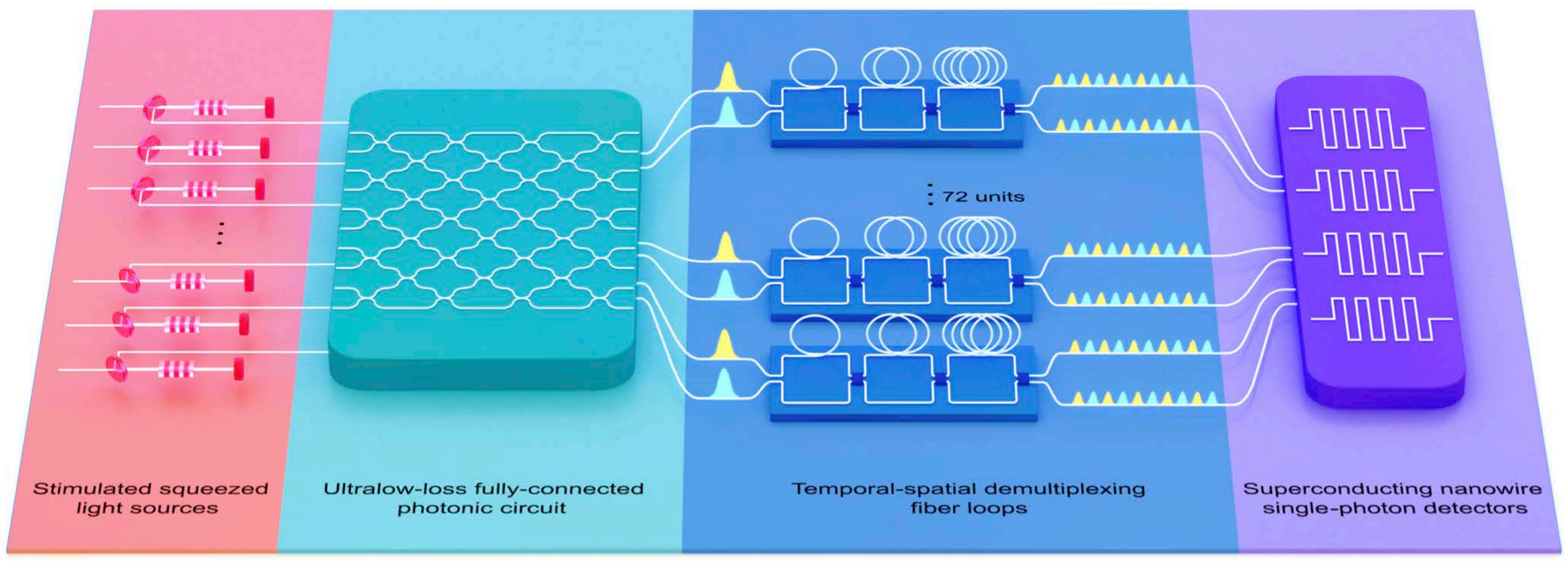
Hamilton et al, PRL 119, 170501 (2017)

Non-Gaussian measurement: Gaussian Boson Sampling

WACQT Wallenberg Centre for Quantum Technology



Gaussian Boson Sampling



Deng et al., Phys. Rev. Lett. 2023

- *Quantum computational advantage using photons: 50 squeezed states, 100 optical modes [Zhong et al., Science 2020];*
- *Quantum circuits with many photons on a programmable nanophotonic chip [Arrazola et al, Nature, 591, 54-60 (2021)];*
- *Quantum computational advantage with a programmable photonic processor, 216 squeezed states [Madsen et al., Nature 2022] (Xanadu)*
- *Gaussian Boson Sampling with Pseudo-Photon-Number-Resolving Detectors and Quantum Computational Advantage, 25 two-mode squeezed sources in 75 optical modes, 255 photon clicks [Deng et al., Phys. Rev. Lett. 2023] (Pann group)*

Gaussian Boson Sampling

Also simulatable with noise (with tensor network simulations)
M. Liu et al, Phys. Rev. A 108, 052604 (2023)
M.V. Umanskii et al, arXiv:2404.01004

Better and better classical algorithms

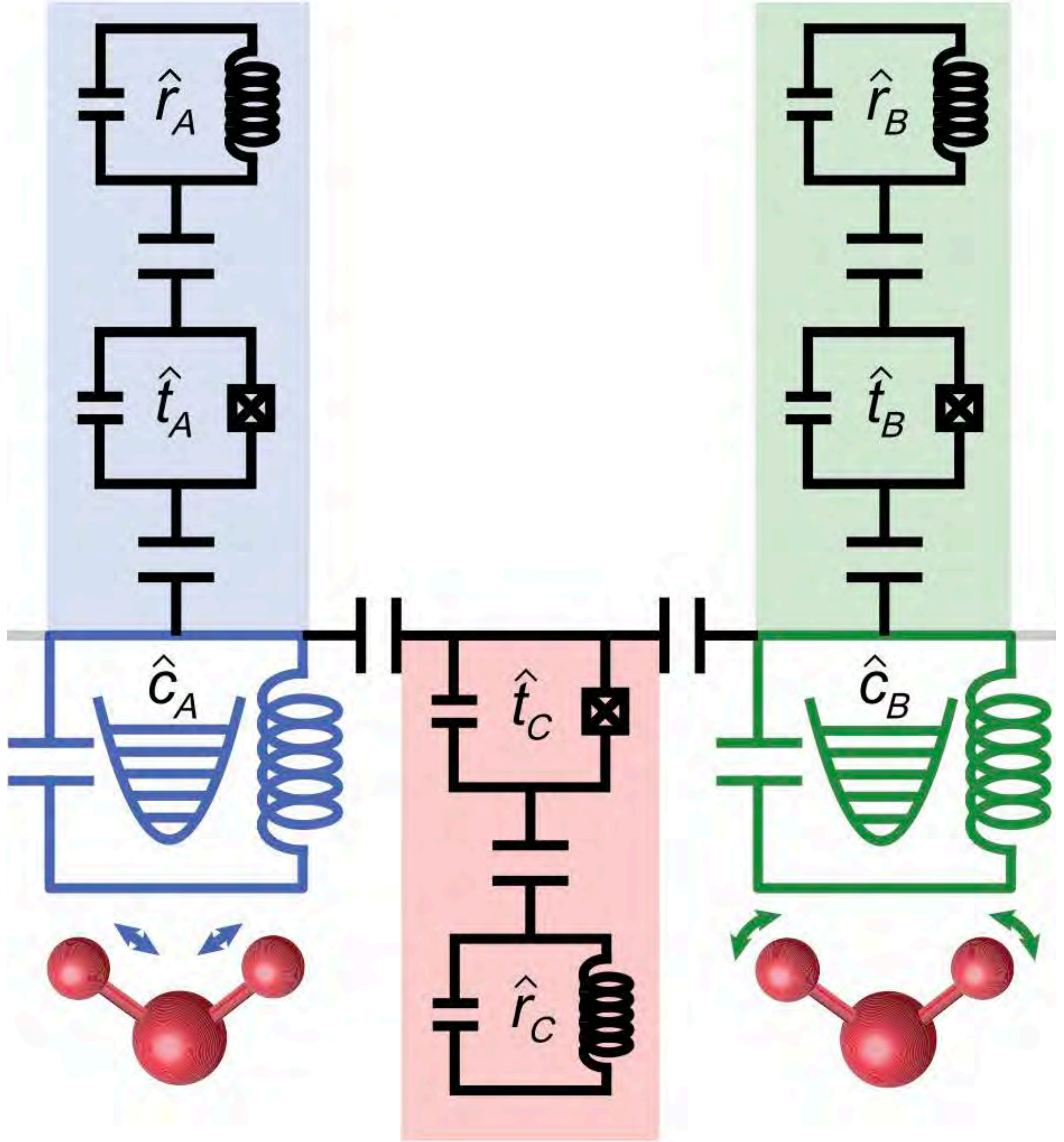
Y. Li et al, "Benchmarking 50-Photon Gaussian Boson Sampling on the Sunway TaihuLight," IEEE Transactions on Parallel and Distributed Systems 33, 1357 (2022),

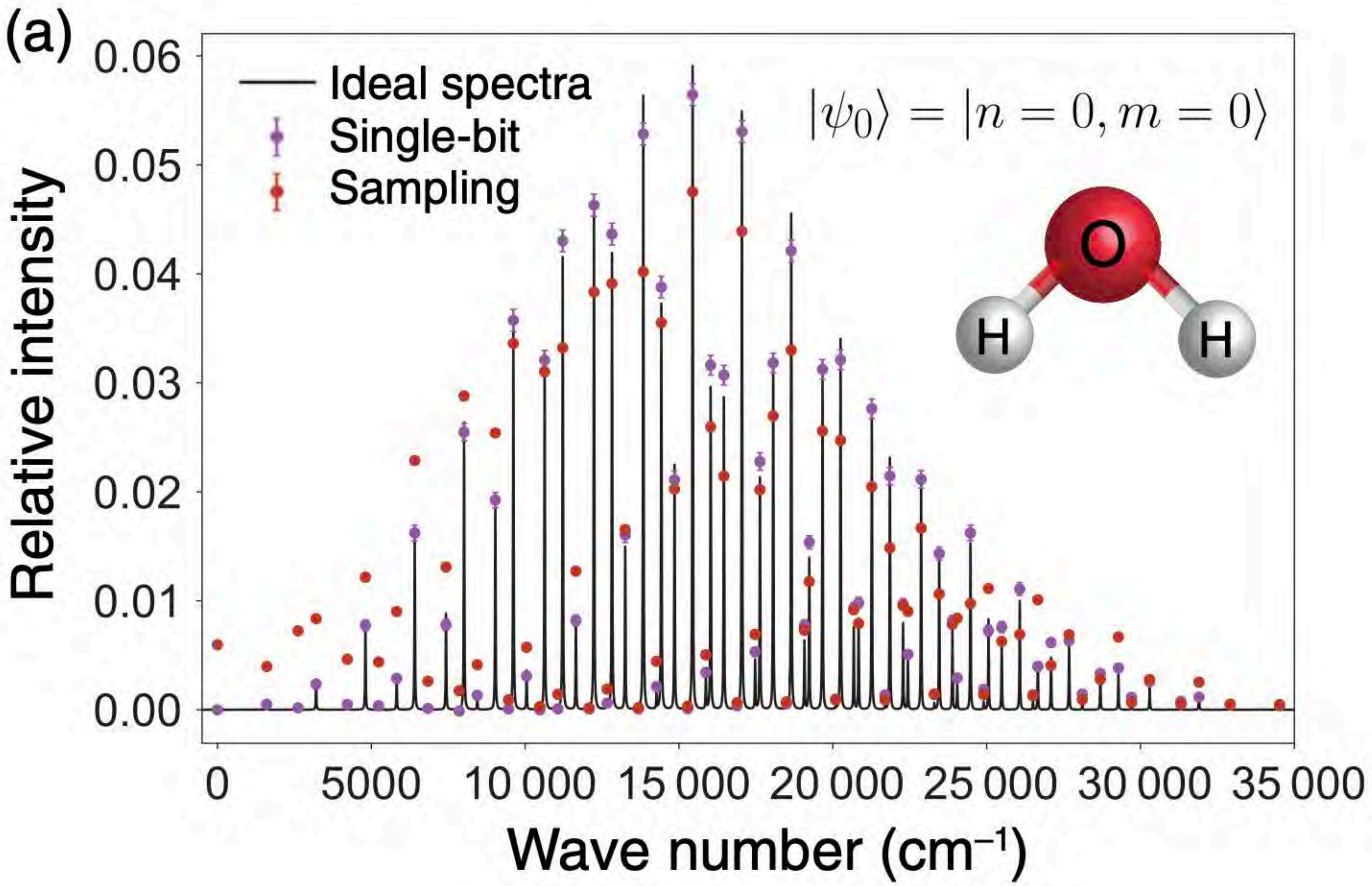
C. Oh et al,"Classical Simulation of Boson Sampling Based on Graph Structure," Physical Review Letters 128, 190501 (2022) +
C. Oh et al Nature Physics (2024) spoofed quantum advantage!

Gaussian Boson Sampling

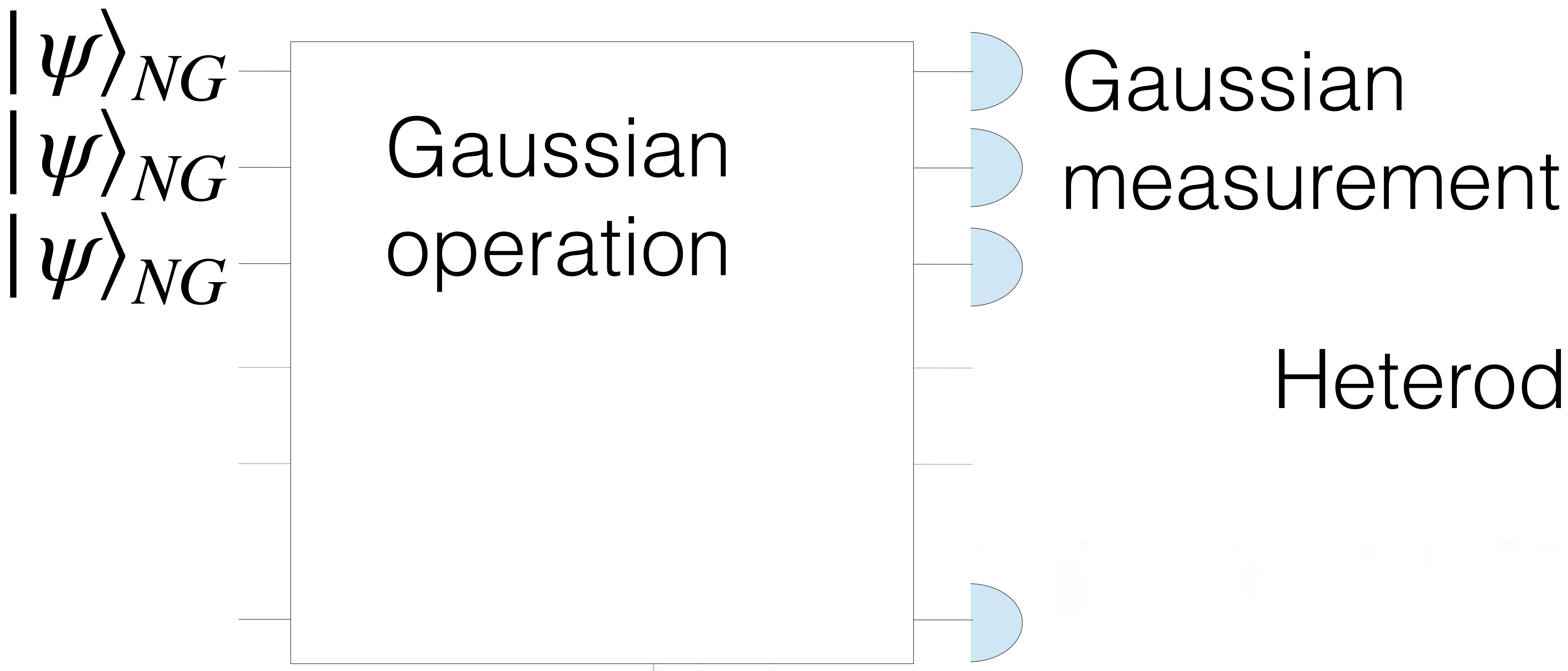
Maps into computing the vibrionic spectra *J. Huh et al, Nature Physics 9, 615 (2015)*

Experiment with 2 superconducting cavities *C. S. Wang et al, Phys. Rev. X 10, 021060 (2020)*





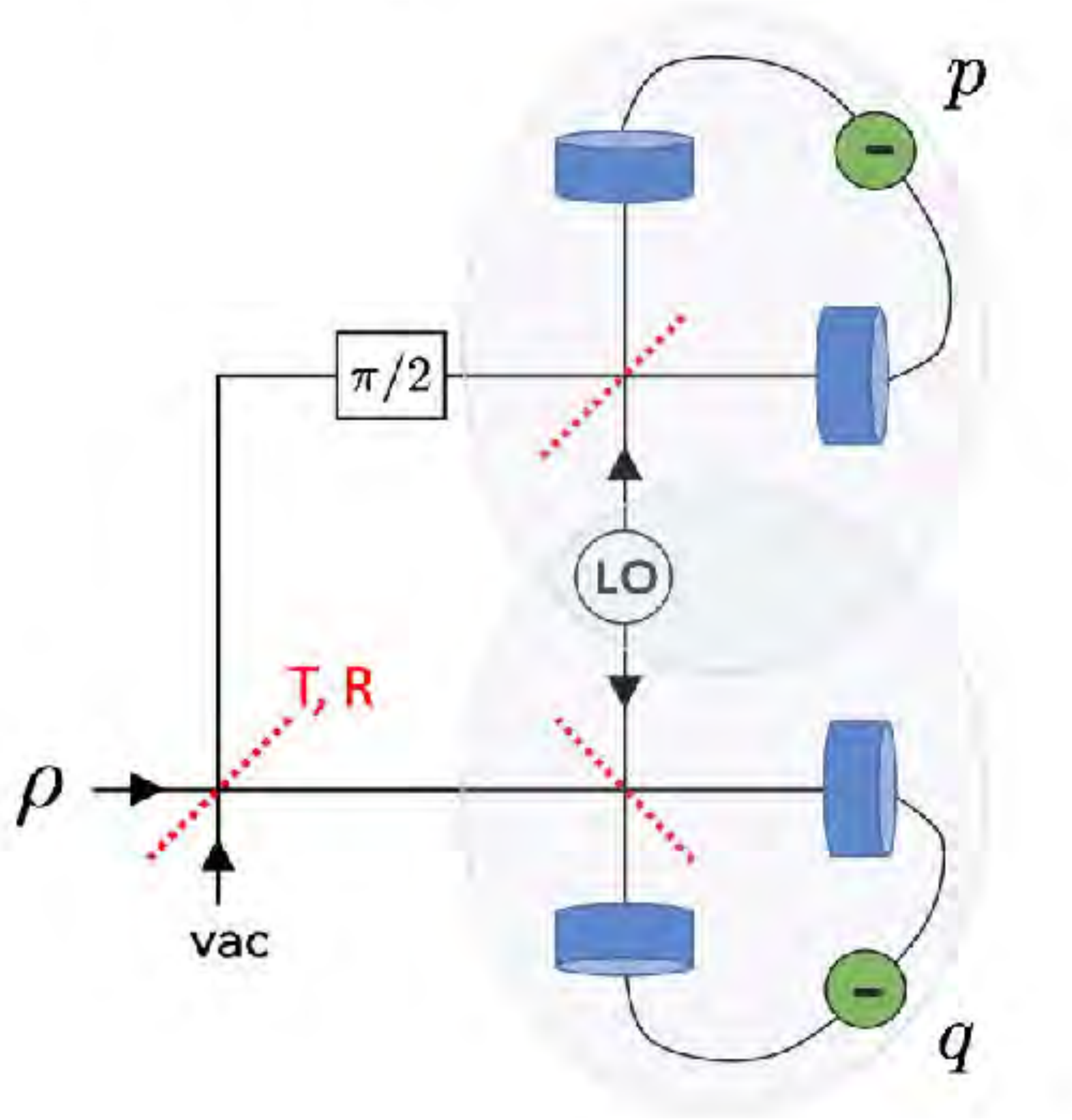
Non-Gaussian input: Continuous-variable sampling



Heterodyne detection = projection onto

$$|\alpha_i, r\rangle = \hat{D}(\alpha_i)\hat{S}(r)\,|0\rangle$$

$$\alpha_i = \sqrt{(1+r^2)/2}(q_i + i p_i/r)$$



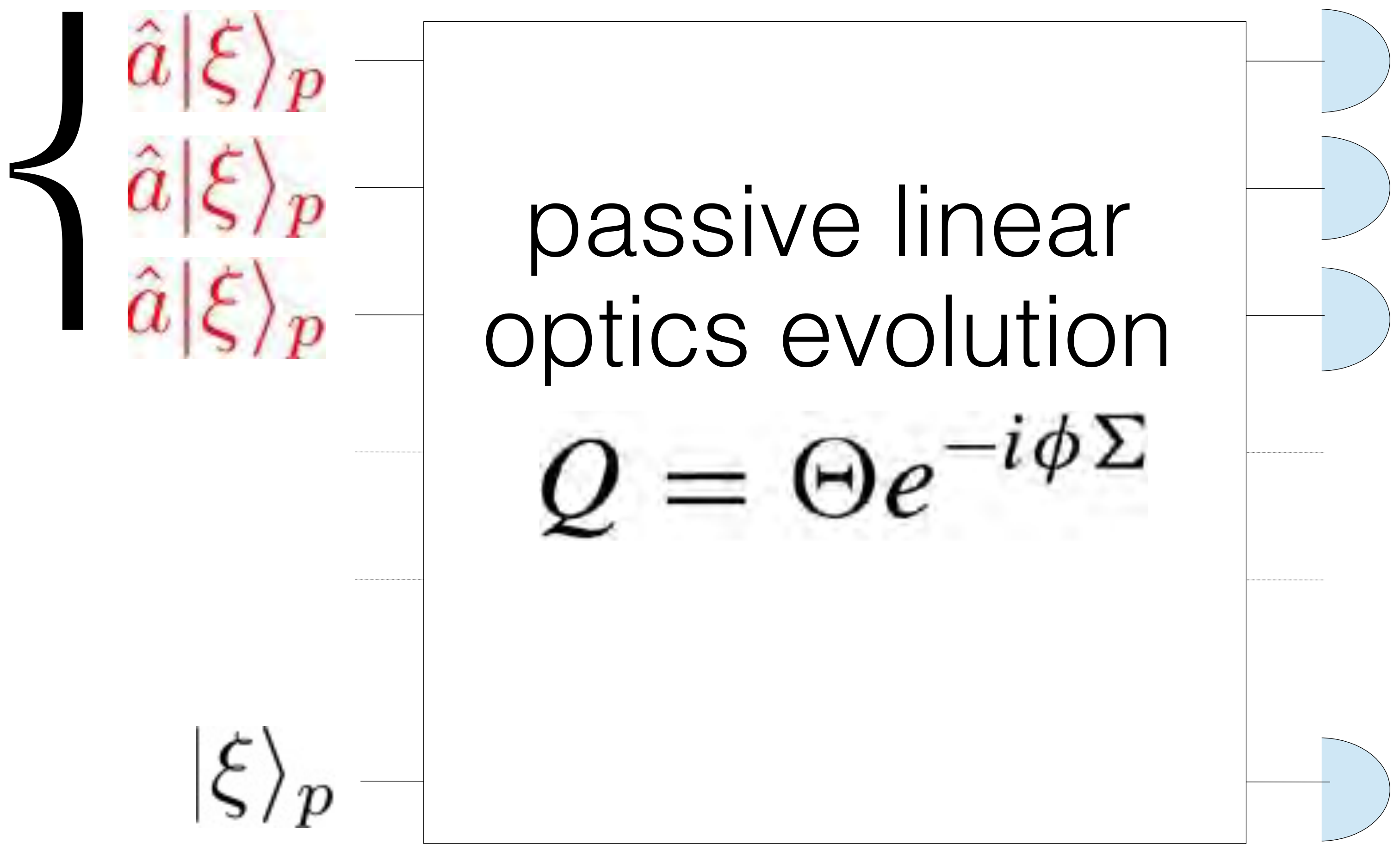
$$\alpha = q + ip$$

Non-Gaussian input: Continuous-variable sampling

WACQT Wallenberg Centre for Quantum Technology

Continuous-Variable Sampling (CVS) circuits: M total number of modes

m photon subtracted squeezed states



heterodyne detection

$$\Sigma_m = \nu \begin{pmatrix} 0 & X \\ X^T & 0 \end{pmatrix}$$

Θ real orthogonal; Σ symmetric real orthogonal; m even, $M \geqslant 2m$

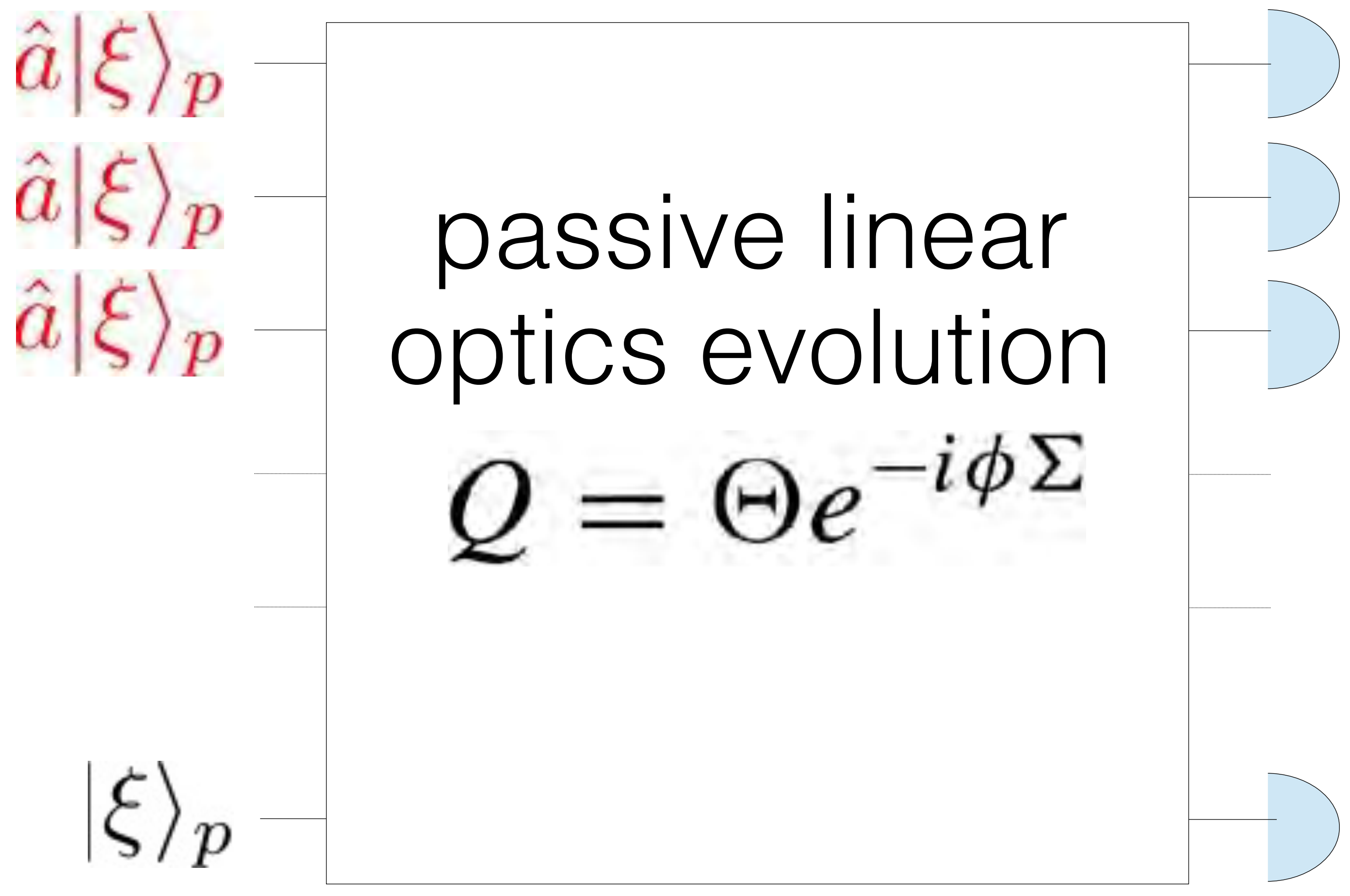
$$\Pr_{\mathrm{CVS}}(0,\ldots,0|\bar{m}) = \frac{f(k,l,\phi)^m \nu^m}{\sqrt{\det(\sigma_{\mathrm{out}} + \mathbb{1}_{2M}/2)}} \mathrm{Perm}(X)^2$$

U. Chabaud, T. Douce, D. Markham, P. van Loock, E. Kashefi and G. Ferrini, PRA 96 062307 (2017)

Non-Gaussian input: Continuous-variable sampling

Continuous-Variable Sampling (CVS) circuits: M total number of modes

m photon subtracted squeezed states



heterodyne detection

$$\Sigma_m = \nu \begin{pmatrix} 0 & X \\ X^T & 0 \end{pmatrix}$$

Θ real orthogonal; Σ symmetric real orthogonal; m even, $M \geqslant 2m$

Sampling from the output exact probability distribution is classically hard, or the polynomial hierarchy collapses

U. Chabaud, T. Douce, D. Markham, P. van Loock, E. Kashefi and G. Ferrini, PRA 96 062307 (2017)

Non-Gaussian input: Continuous-variable sampling

WACQT Wallenberg Centre for Quantum Technology

The input state is $\hat{a}\,|s\rangle$

$|s\rangle = \hat{S}\,|0\rangle$ squeezed state

$\hat{S} = e^{-i\frac{\ln(s)}{2}\hat{q}\hat{p}+\hat{p}\hat{q}}$ squeezing operator

Using: $\hat{S}^{\dagger}(s)\hat{a}\hat{S}(s) = c_s\hat{a} - s_s\hat{a}^{\dagger}$ with $c_s = \cosh(\ln s)$ $s_s = \sinh(\ln s)$

we obtain $\hat{a}\,|s\rangle = c_s\hat{S}(s)\hat{a}\,|0\rangle - s_s\hat{S}(s)\hat{a}^{\dagger}\,|0\rangle = -s_s\hat{S}(s)\,|1\rangle$

In the zero squeezing limit, equivalent to input single-photon state

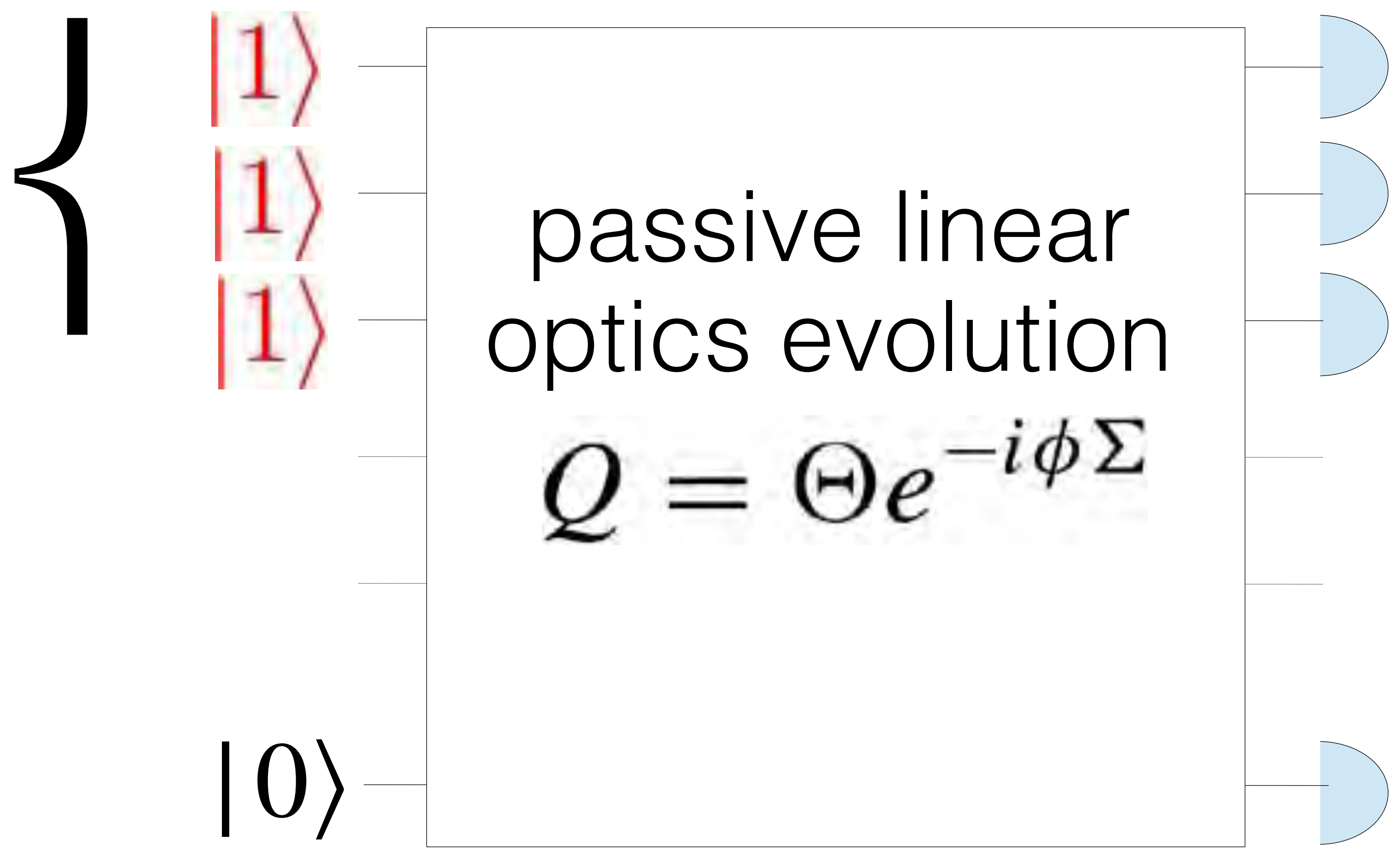
U. Chabaud, T. Douce, D. Markham, P. van Loock, E. Kashefi and G. Ferrini, PRA 96 062307 (2017)

Non-Gaussian input: Boson sampling with heterodyne

Zero-squeezing limit

m single photons

poly (M)



heterodyne detection

Θ real orthogonal; Σ symmetric real orthogonal; m even, $M \geqslant 2m$

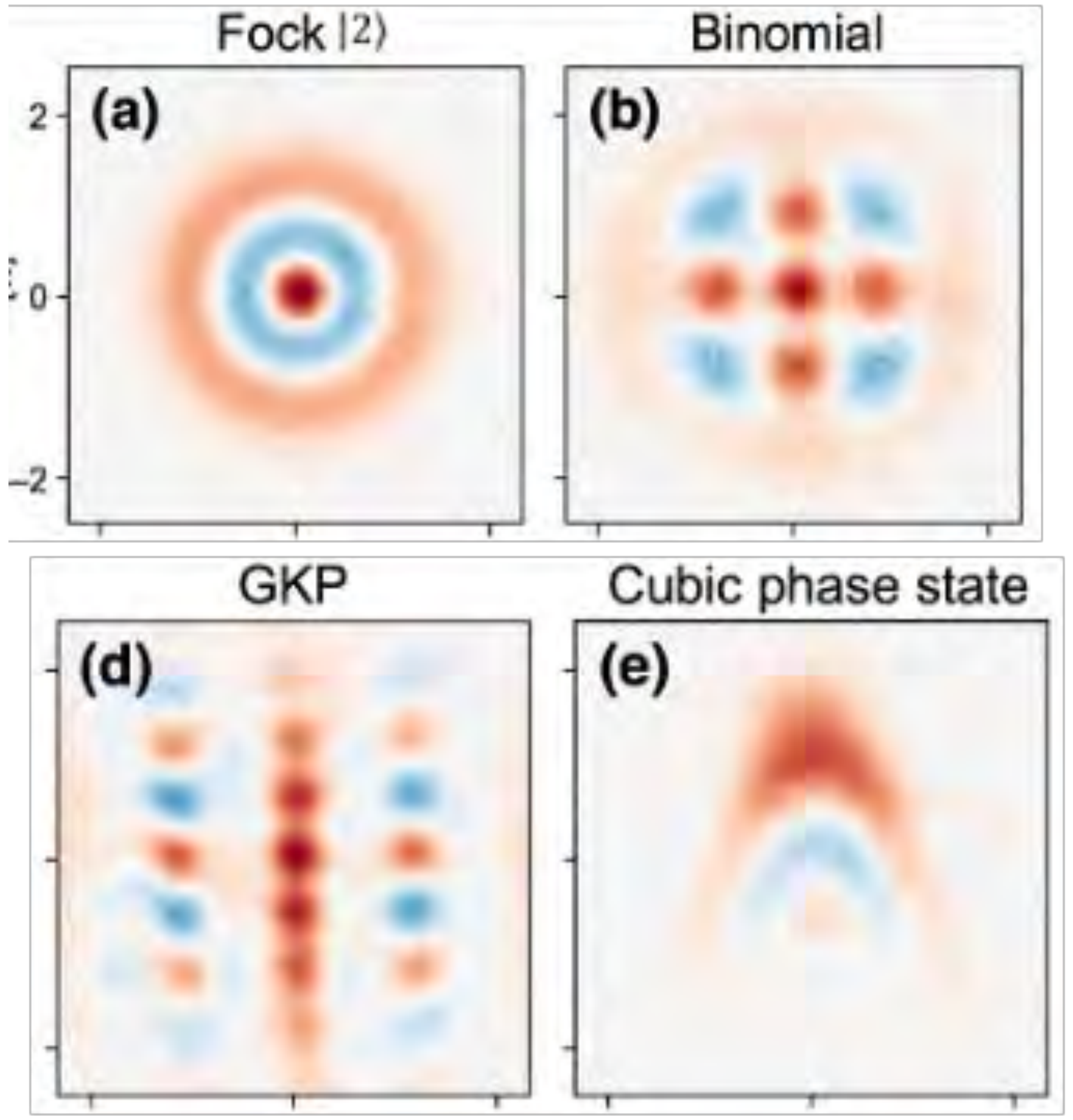
Boson sampling with heterodyne detection is classically hard, or the polynomial hierarchy collapses (to the third level)

U. Chabaud, T. Douce, D. Markham, P. van Loock, E. Kashefi and G. Ferrini, PRA 96 062307 (2017)

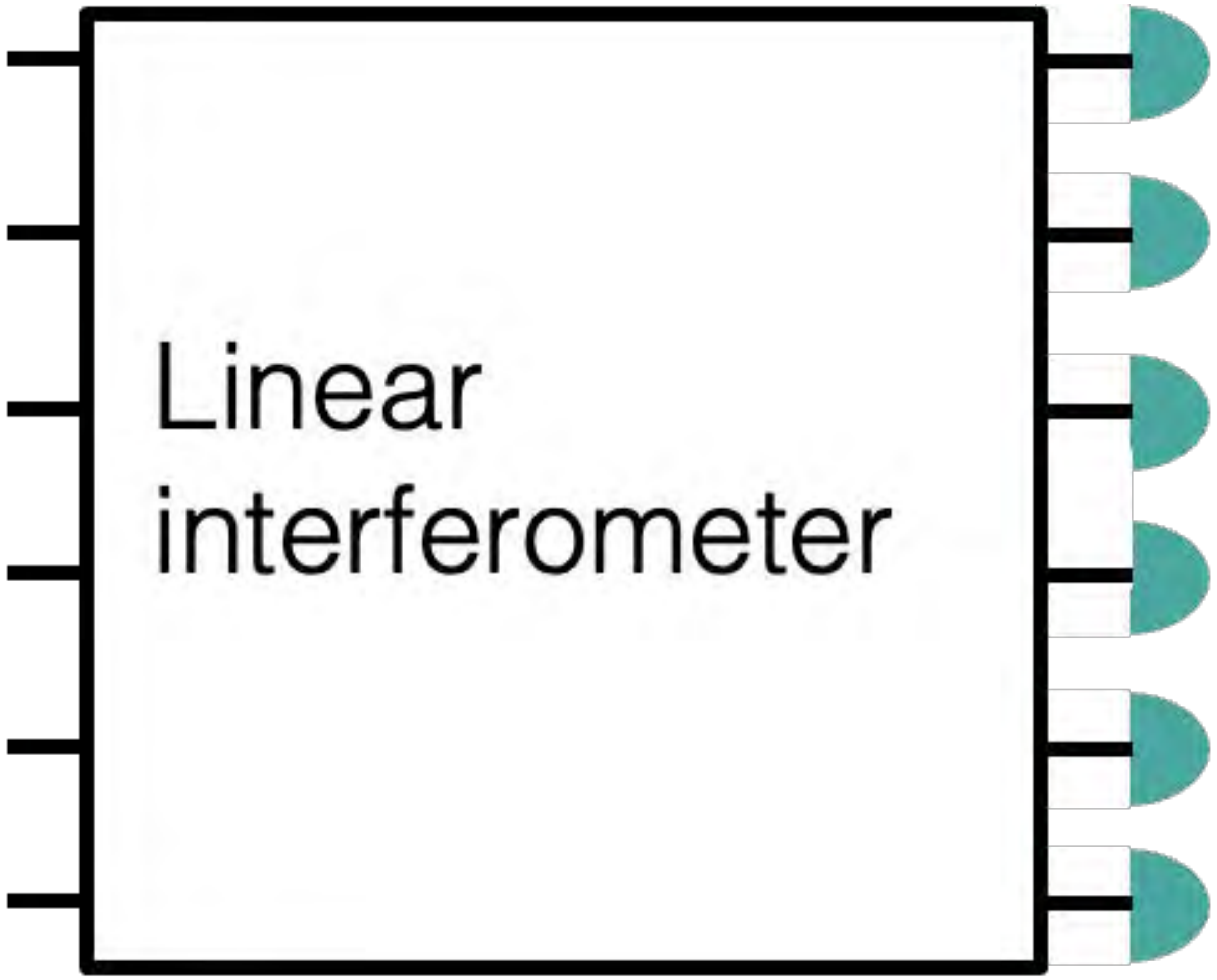
If *m~ log(M)*, we can efficiently simulate *U. Chabaud, G. Ferrini, F. Grosshans, D. Markham, PRR 3 033018 (2021)*

Proof of hardness beyond boson sampling?

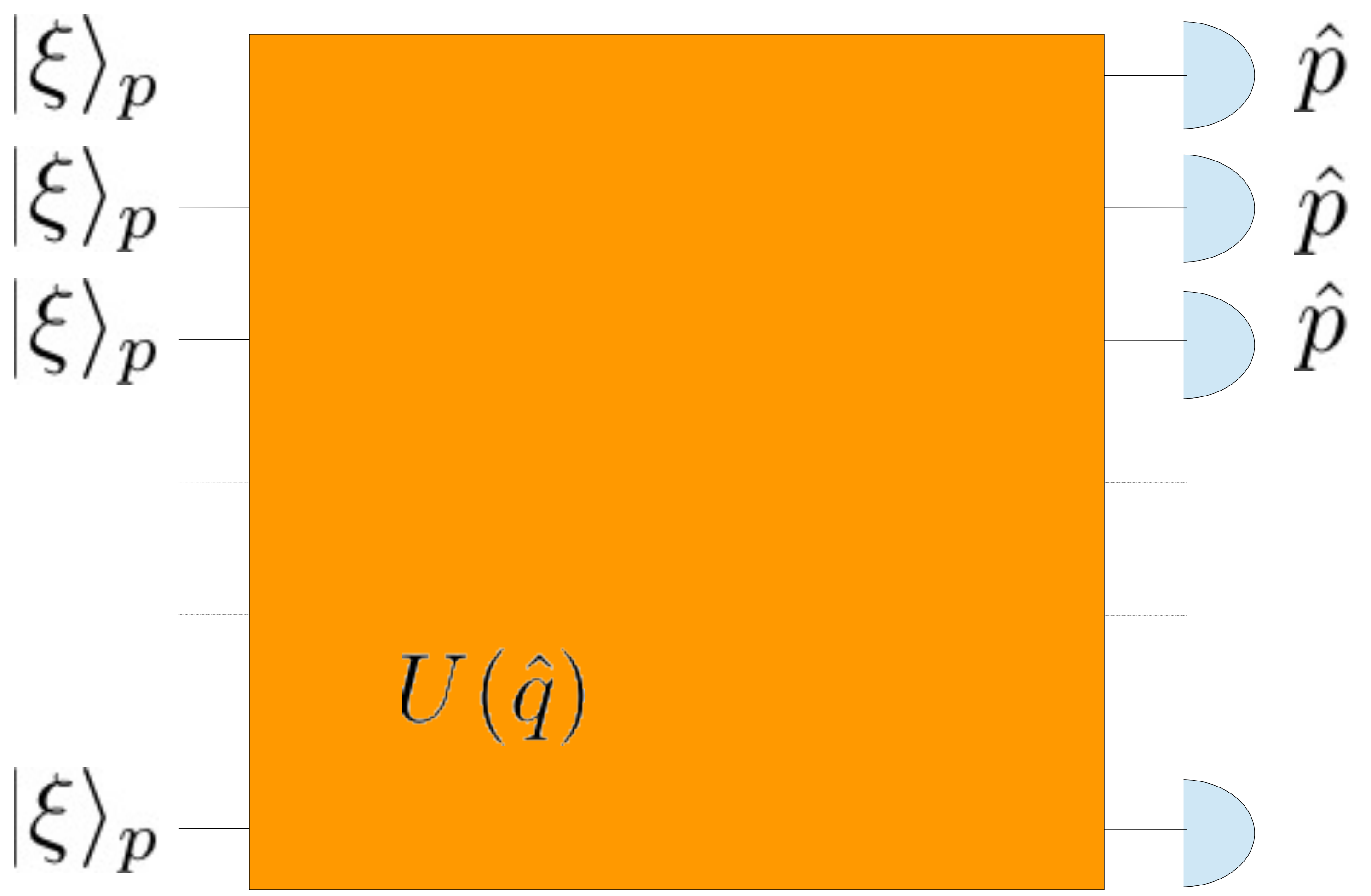
- Unknown if general non-Gaussian states yield models that are hard to sample ("Veriqub")



Kudra et al, PRX-Q, 2022



Non-Gaussian evolution

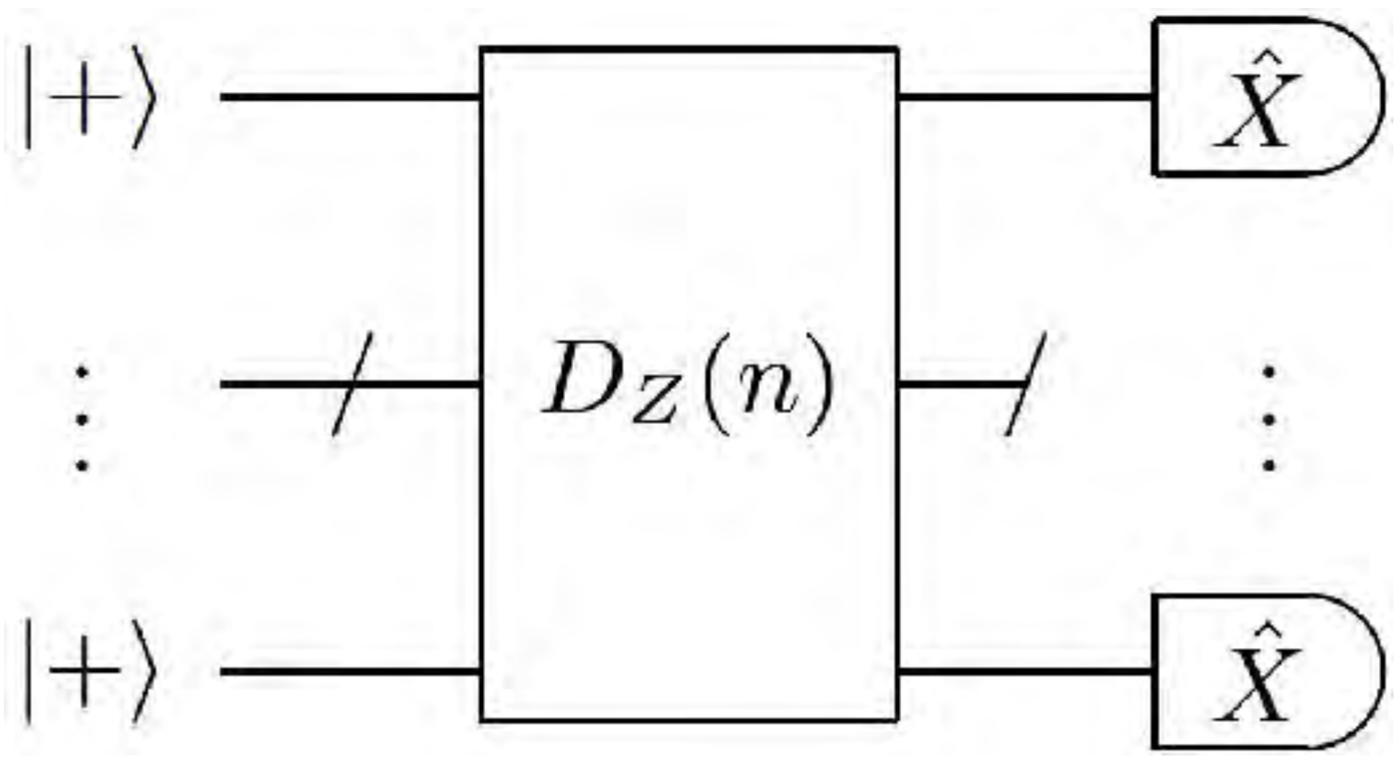


CV Instantaneous Quantum Polytime

Douce et al, PRL 118 070503 (2017)

Douce et al, PRA 99, 012344 (2019)

Instantaneous Quantum Polytime



- Input : X eigenstates $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
- Evolution : Diagonal in Z
- Measurement: X $\begin{aligned} \hat{X}|+\rangle &= |+\rangle \\ -\hat{X}|-\rangle &= |-\rangle \end{aligned}$

- Gates commute, hence they can be performed simultaneously (« Instantaneous »)

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \quad C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

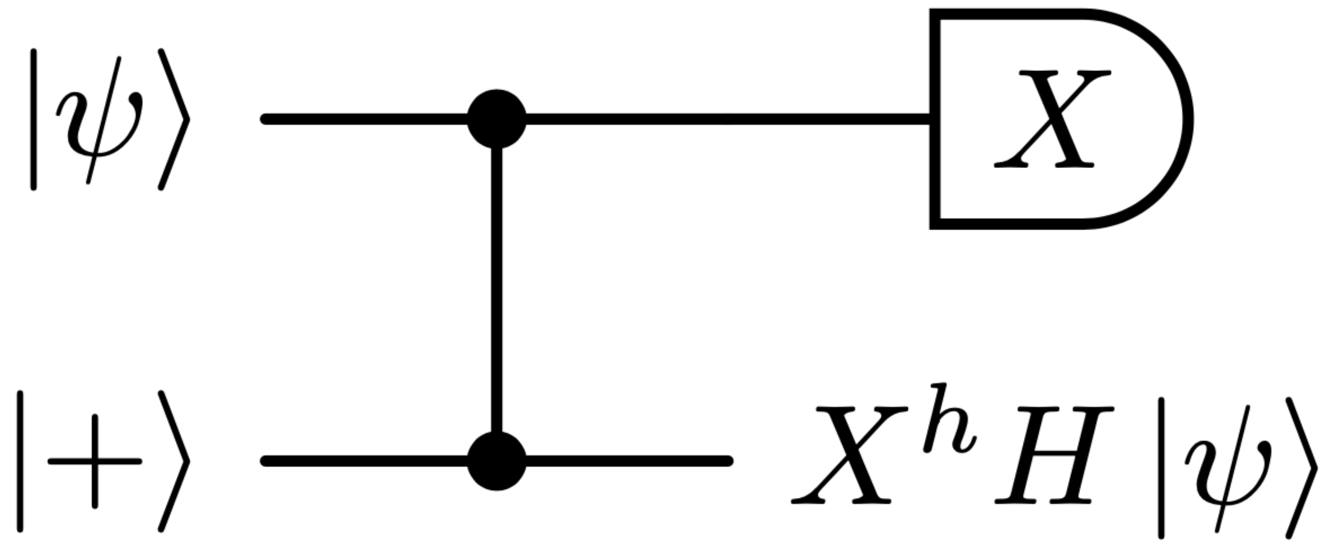
- The probability distribution of the measurement outcomes is hard to sample, or the polynomial hierarchy collapses

M. J. Bremner, R. Josza, and D. Shepherd, Proc. R. Soc. A 459, 459 (2010).
M. J. Bremner, A. Montanaro, and D. J. Shepherd, Phys. Rev. Lett. 117, 080501 (2016)

Proof of hardness

- Adding post-selection to the model makes it universal

Hadamard gadget:



$h = 0$ if outcome +1

$h = 1$ if outcome -1

Post-selection allows for recovering the Hadamard gate → Universal set of gates

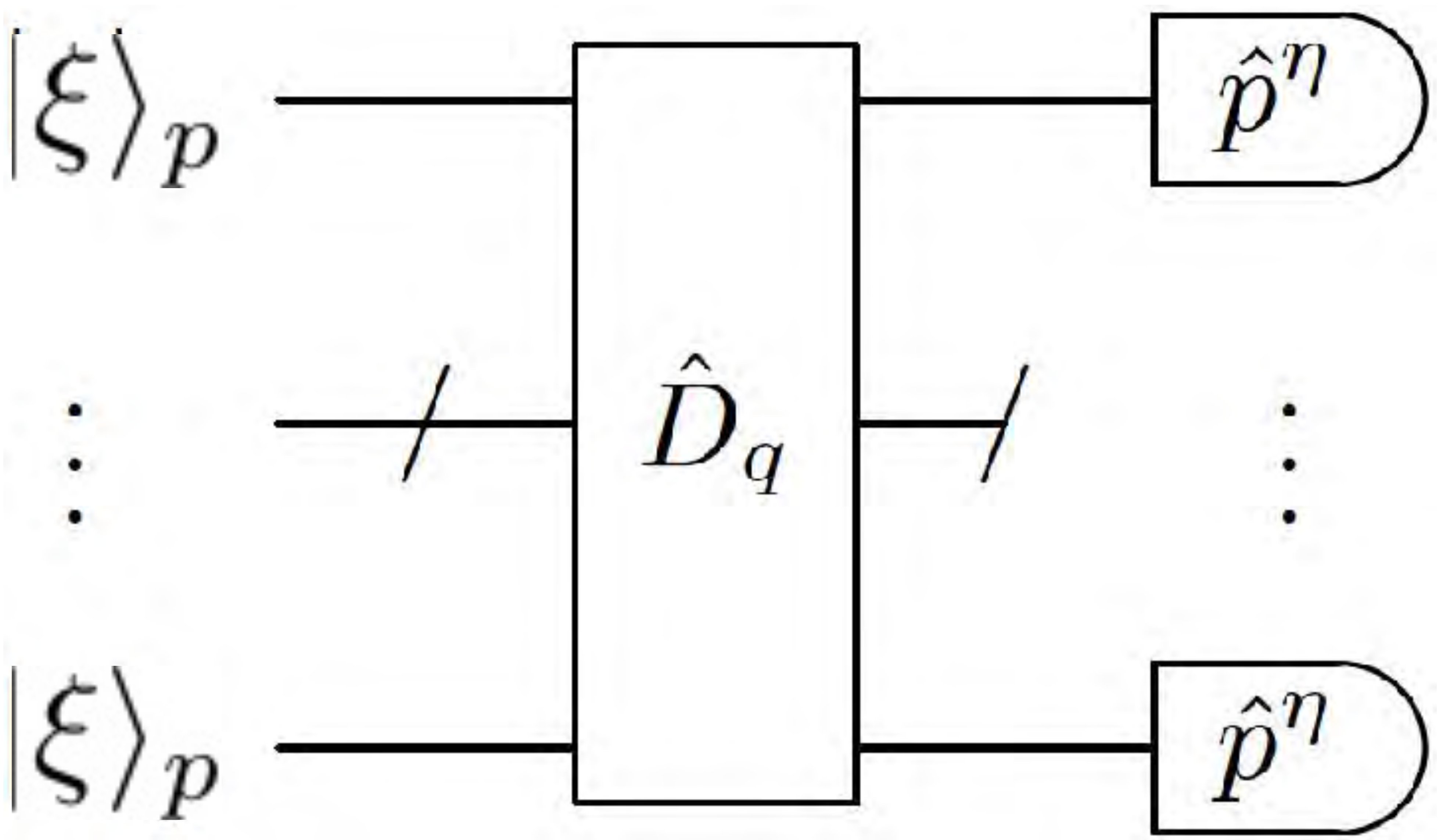
- If it was possible to efficiently simulate IQP on a classical computer, then a post-selected classical computer would be at least as powerful as post selected quantum computer

This yields a collapse of the polynomial hierarchy!

- Hence it must not be possible to efficiently classically simulate IQP circuits ∎

M. J. Bremner, R. Josza, and D. Shepherd, Proc. R. Soc. A 459, 459 (2010).

Non-Gaussian gates: Instantaneous Quantum Polytime



- Input : p-squeezed states

$$|\xi\rangle_p = \frac{1}{\sqrt{\xi}\pi^{1/4}} \int \mathrm{d}t\, e^{-\frac{t^2}{2\xi^2}} |t\rangle_p$$

- Evolution : Diagonal in q

- Measurement: p homodyne detection (finite resolution)

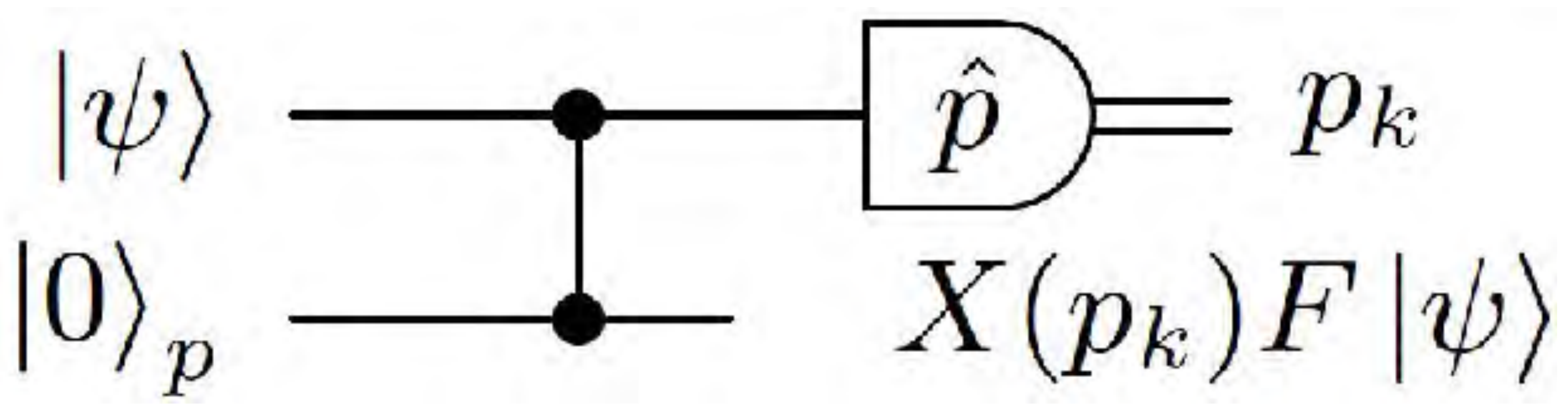
$$\{\hat{Z}(s) = e^{i\hat{q}s}, e^{i\hat{q}^2 s}, \hat{F} = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, \hat{C}_Z = e^{i\hat{q}_1\hat{q}_2}, e^{i\hat{q}^3 s}\}$$

Douce et al, PRL 118 070503 (2017), Douce et al, PRA 99, 012344 (2019)

Proof of hardness

- Adding post-selection to the model makes it universal

Fourier gadget:



$\hat{p}|0\rangle_p = 0$ infinitely p-squeezed state

$\hat{X}(p_k) = e^{-i\hat{p}p_k}$

Post-selection allows for recover the Fourier transform → Universal set of CV gates

- If it was possible to efficiently simulate CV-IQP on a classical computer, then a post-selected classical computer would be at least as powerful as post selected quantum computer

This yields a collapse of the polynomial hierarchy!

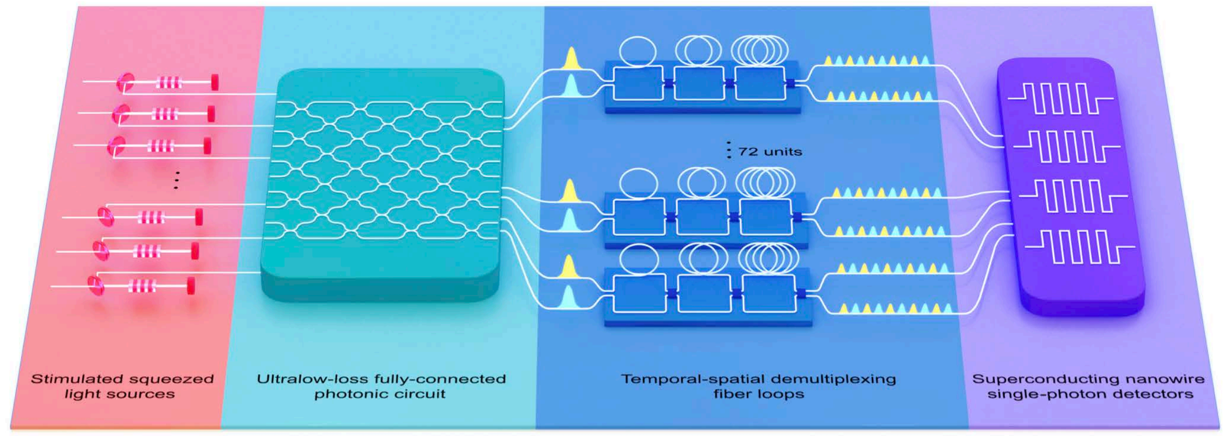
- Hence it must not be possible to efficiently classically simulate IQP circuits ■

Douce et al, PRL 118 070503 (2017), Douce et al, PRA 99, 012344 (2019)

Summary and conclusion on parts 1b

Summary and conclusion on parts 1b

- Sampling models aim at disproving the extended Church-Turing thesis and proving exponential quantum computational advantage



- Beyond random circuit sampling, Boson Sampling-type are good candidates

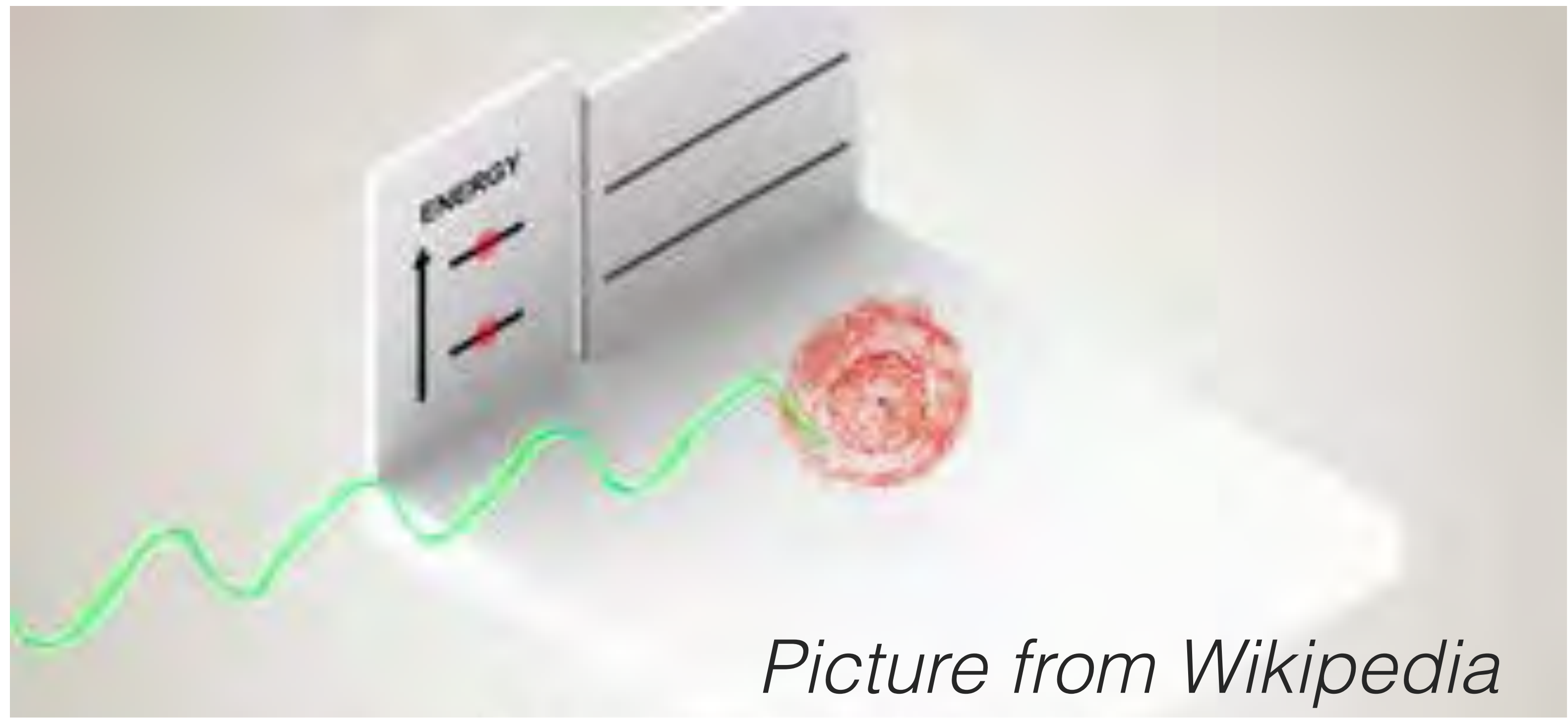
- We can have less non-Gaussian elements than in Boson Sampling: either input, evolution or measurement can be enough (required new proofs of computational hardness)

- Showing practical quantum advantage is a race with classical algorithms and requires low noise!

Part 2a: Bosonic codes

One challenge: build a reliable QC

- Environment affects quantum computers by inducing decoherence



Picture from Wikipedia

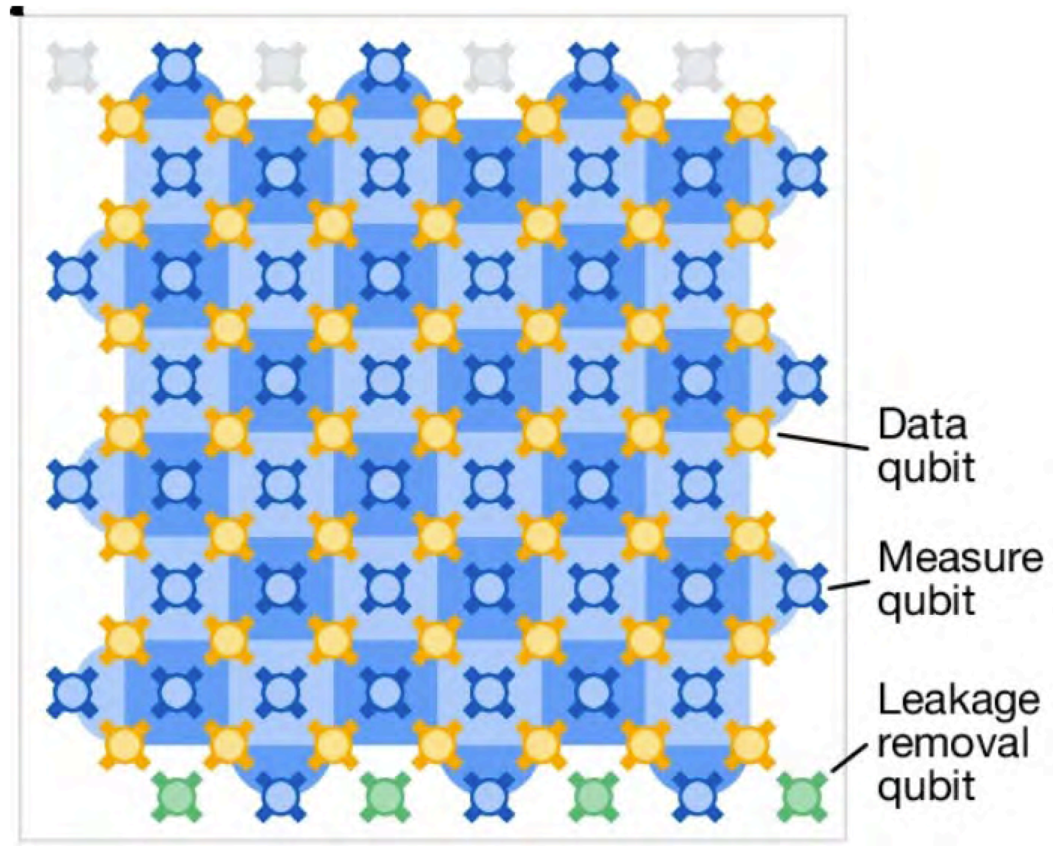
- Redundancy is needed in order to restore quantum information via Quantum Error correction

Repetition code

$$|\Psi\rangle = \alpha|00000\rangle + \beta|11111\rangle$$

Surface code
(d = 7, 105 qubits)

Google AI, Nature 2025



How many qubits for Shor's algorithm?

Quantum Physics

[Submitted on 23 May 2019 (v1), last revised 13 Apr 2021 (this version, v3)]

How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney, Martin Ekerå

Quantum Physics

[Submitted on 21 May 2025]

How to factor 2048 bit RSA integers with less than a million noisy qubits

Craig Gidney

Quantum Physics

[Submitted on 12 Feb 2026]

The Pinnacle Architecture: Reducing the cost of breaking RSA-2048 to 100 000 physical qubits using quantum LDPC code

Paul Webster, Lucas Berent, Omprakash Chandra, Evan T. Hockings, Nouédyn Baspin, Felix Thomsen, Samuel C. Smith, Lawrence Z. Cohen

Quantum Physics

[Submitted on 30 Mar 2026]

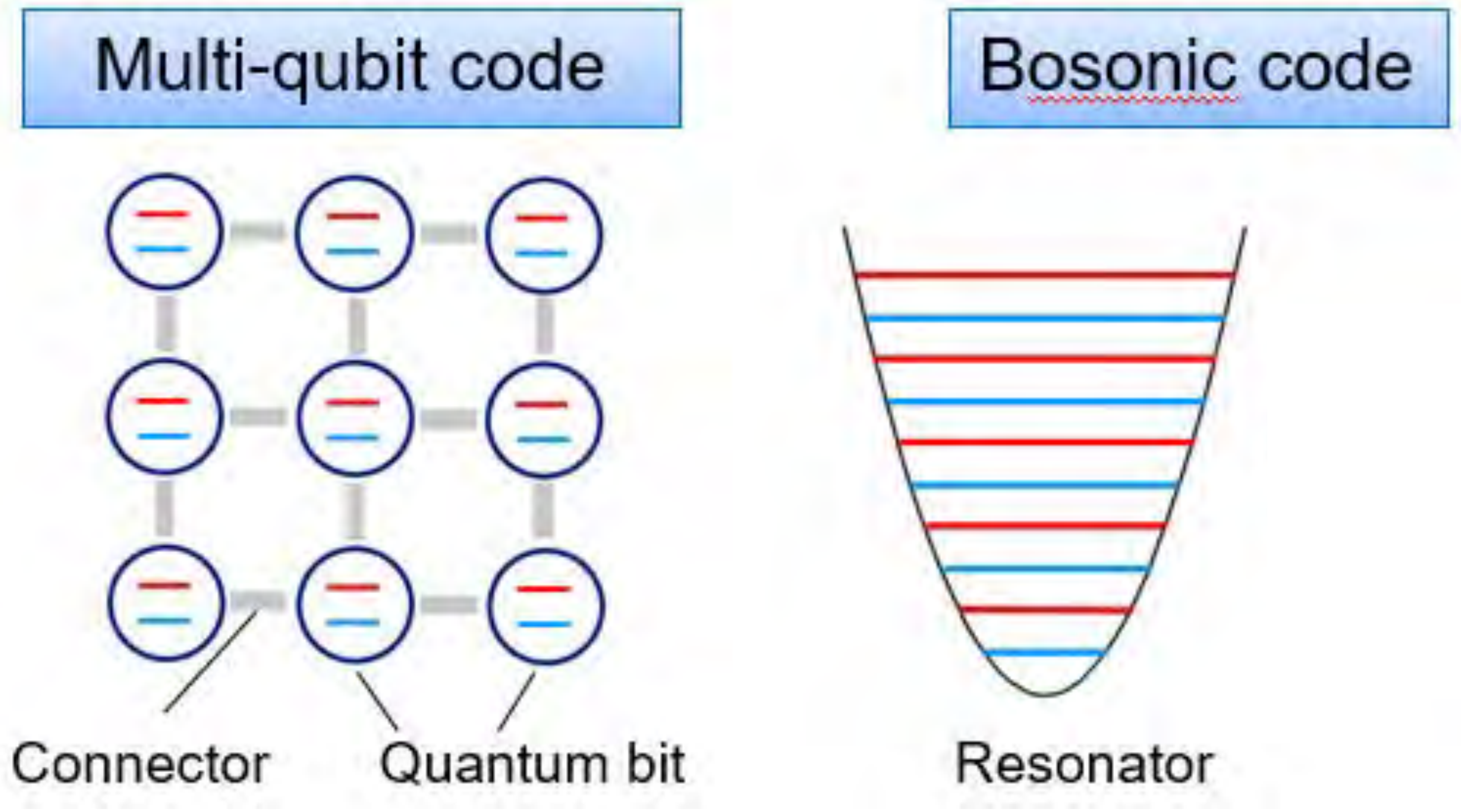
Shor's algorithm is possible with as few as 10,000 reconfigurable atomic qubits

Madelyn Cain, Qian Xu, Robbie King, Lewis R. B. Picard, Harry Levine, Manuel Endres, John Preskill, Hsin-Yuan Huang, Dolev Bluvstein

- So far there is no "proper" experimental implementation of Shor's algorithm (See: Craig Gidney blog "Why haven't quantum computers factored 21 yet?")

Encoding a qubit in a oscillator gives a compact solution to QEC

$$|0\rangle_L = \sum_n c_n^0 |n\rangle \qquad |1\rangle_L = \sum_n c_n^1 |n\rangle$$



Courtesy of Japan Science and Technology Agency

Factor 2048 RSA integer in 4 days using 349133 cat qubits ($d = 15$)

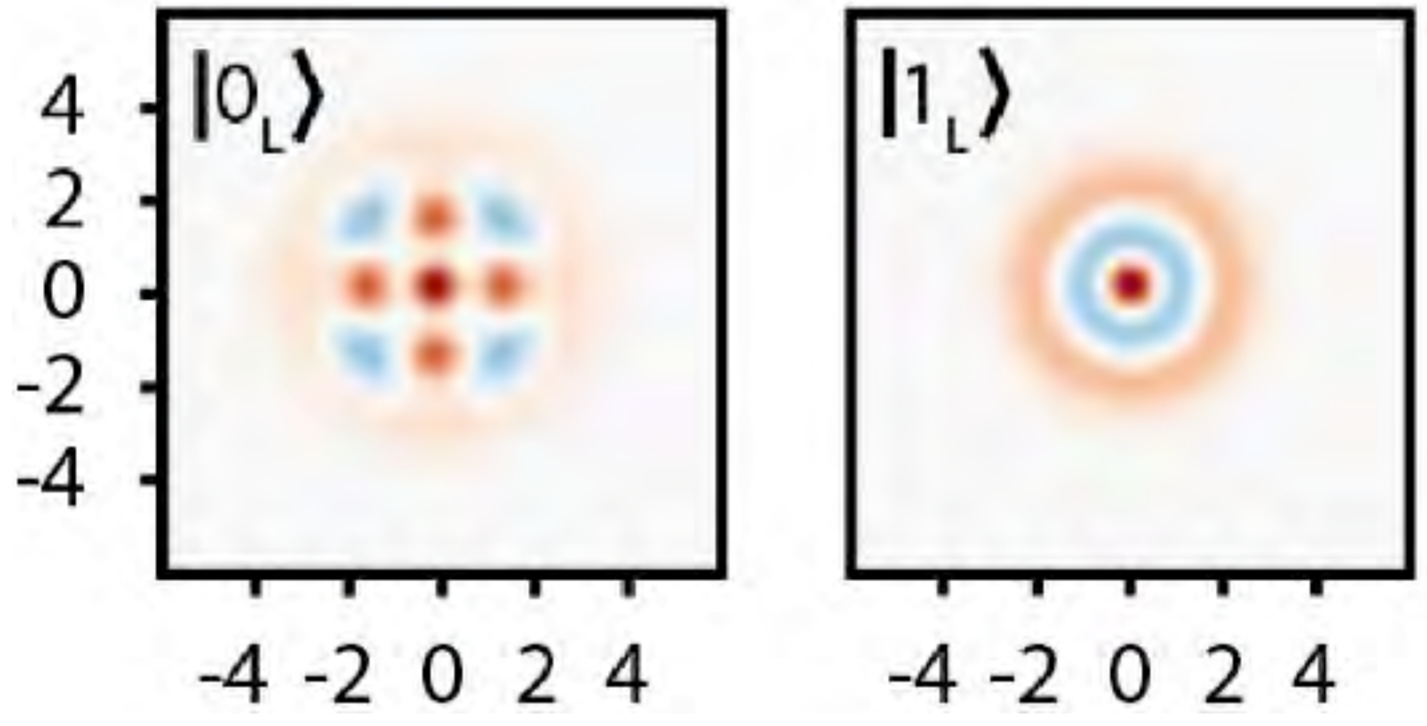
“Performance Analysis of a Repetition Cat Code Architecture: Computing 256-bit Elliptic Curve Logarithm in 9 Hours with 126133 Cat Qubits”, E. Gouzien et al, Phys. Rev. Lett. 131, 040602 (2023)

Rotationally invariant bosonic codes

Bosonic error-correcting codes

- Binomial code

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |4\rangle), \quad |1_L\rangle = |2\rangle \qquad |\psi_L\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$$



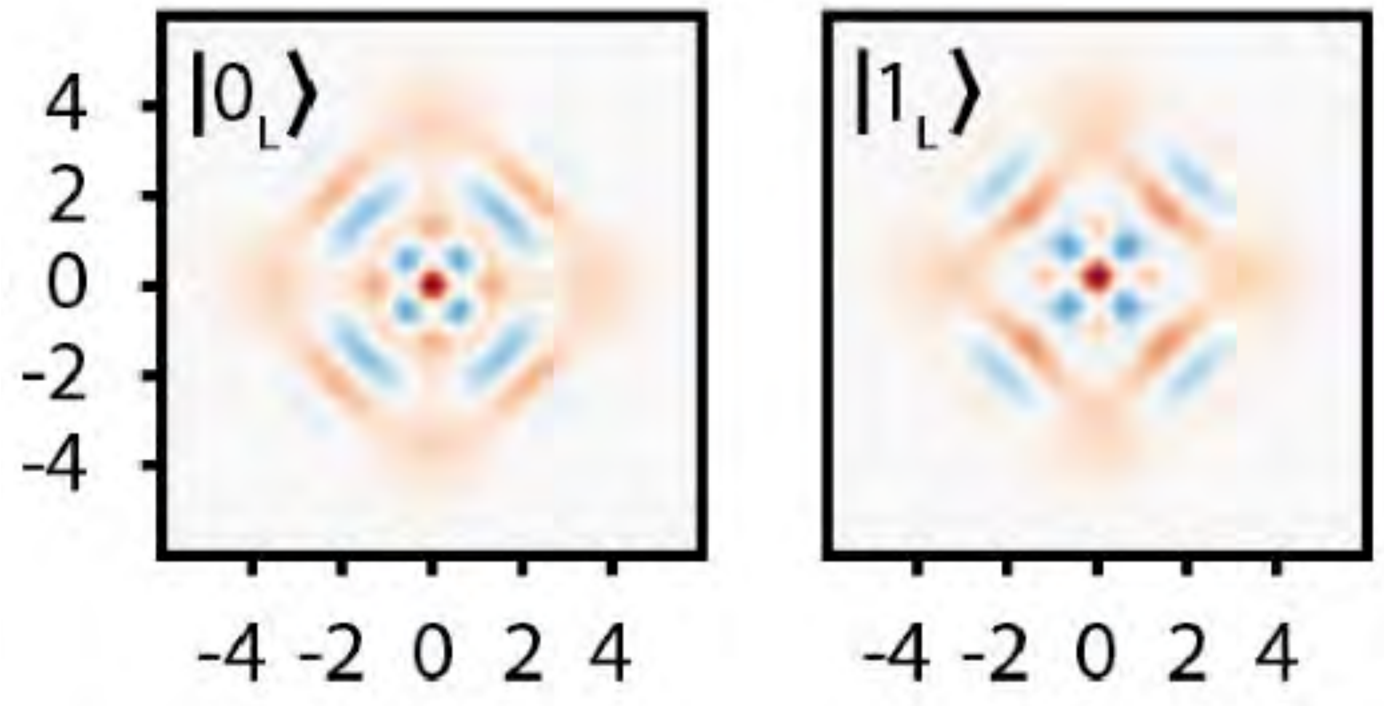
- Designed to correct against photon losses $\hat{a}$, yielding $\hat{a}|\psi_L\rangle = \sqrt{2}(\alpha|3\rangle + \beta|1\rangle)$
- Errors can be detected by measuring parity $\hat{\Pi}_2 = (-1)^{\hat{n}}$
- We can recover the data qubit by mapping back to the code space

$$\hat{a}|\psi_L\rangle = \sqrt{2}(\alpha|3\rangle + \beta|1\rangle) \xrightarrow{\text{recovery}} \sqrt{2}(\alpha|0_L\rangle + \beta|1_L\rangle) \propto |\psi_L\rangle$$

Joshi et al, Qu. Sci. Tech. 6, 2021

- Four-component cat code:

$$|0_L\rangle \propto |\alpha\rangle + |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle$$
$$|1_L\rangle \propto |\alpha\rangle - |i\alpha\rangle + |-\alpha\rangle - |-i\alpha\rangle$$



Invariant under 180 degrees rotations; they correct for $\{\hat{I}, \hat{a}, \hat{a}^\dagger \hat{a}\}$

…they are part of the same family! $\hat{R}_N = e^{i\frac{2\pi}{N}\hat{n}}$ (on this slide $N = 2$)

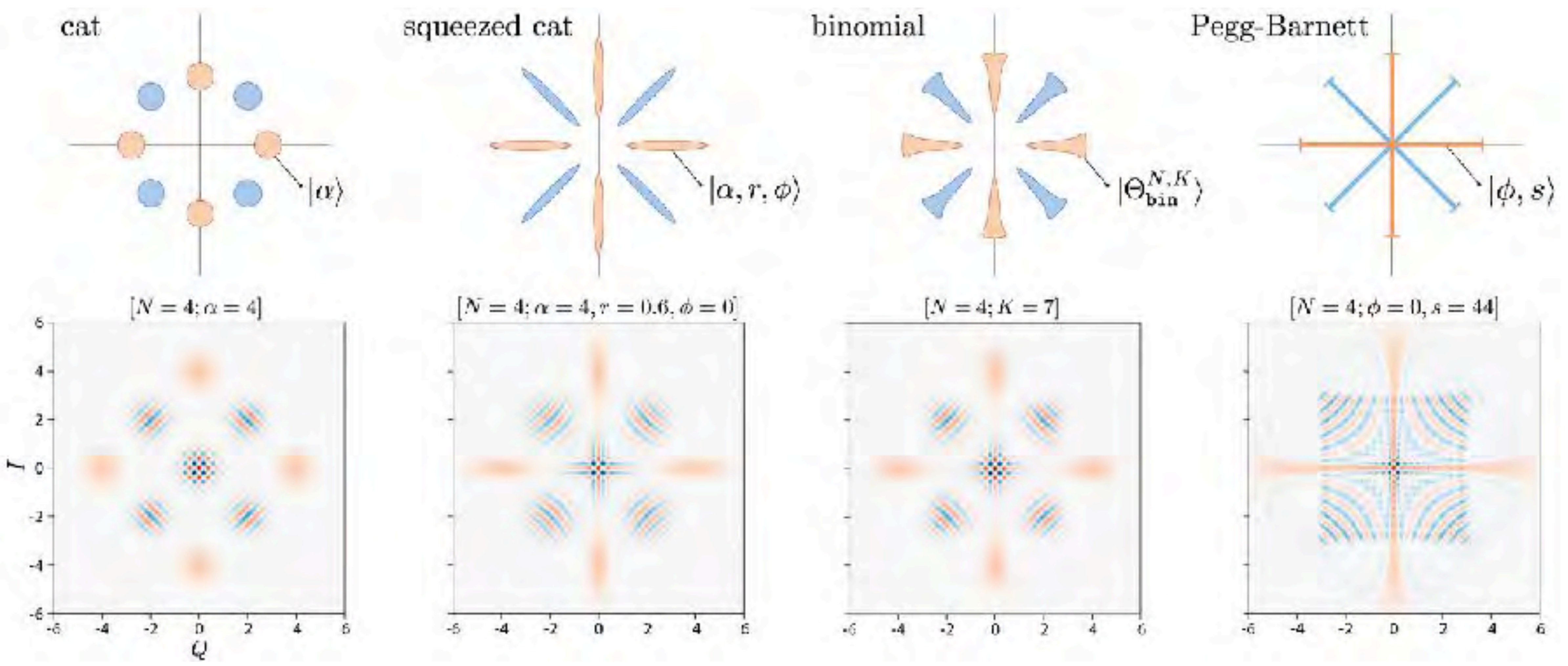
A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

Rotationally symmetric bosonic codes (RSB)

$$|0_{N,\Theta}\rangle := \frac{1}{\sqrt{\mathcal{N}_0}} \sum_{m=0}^{2N-1} e^{i(m\pi/N)\hat{n}} |\Theta\rangle,$$

$$|1_{N,\Theta}\rangle := \frac{1}{\sqrt{\mathcal{N}_1}} \sum_{m=0}^{2N-1} (-1)^m e^{i(m\pi/N)\hat{n}} |\Theta\rangle$$

Invariant under rotations $\hat{R}_N = e^{i\frac{2\pi}{N}\hat{n}}$



Consequence of rotational invariance:

$$|0_N\rangle = \sum_n c_n^0 |n\rangle \qquad e^{i\frac{2\pi\hat{n}}{N}}|0_N\rangle = |0_N\rangle \Rightarrow \sum_n c_n^0 e^{i\frac{2\pi n}{N}}|n\rangle = \sum_n c_n^0|n\rangle \qquad e^{i\frac{2\pi n}{N}} = 1 \Rightarrow n = kN$$

k integer

$$\Rightarrow |0_N\rangle = \sum_k c_{kN}^0 |kN\rangle \qquad |1_N\rangle = \sum_k c_{kN}^1 |kN\rangle$$

only multiples of the rotational order are allowed in the Fock expansion

A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

Rotationally symmetric bosonic codes (RSB)

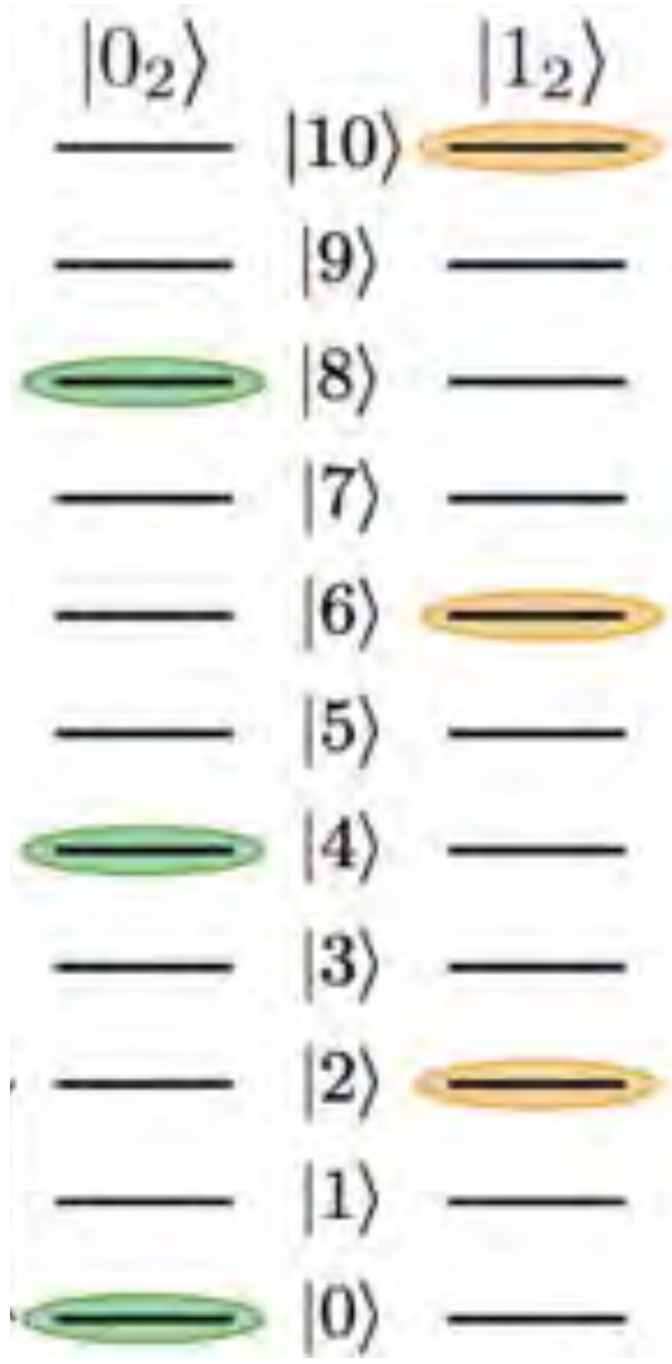
$$\hat{Z}\,|0_N\rangle = |0_N\rangle \Rightarrow c_n = 0 \text{ a part from those for which } e^{i\frac{\pi\hat{n}}{N}} = 1 \Rightarrow n = 2kN$$

$$\hat{Z}\,|1_N\rangle = -|1_N\rangle \Rightarrow c_n = 0 \text{ a part from those for which } e^{i\frac{\pi\hat{n}}{N}} = -1 \Rightarrow n = (2k+1)N.$$

$$\hat{Z} = e^{i\frac{\pi}{N}\hat{n}}$$

$$|0_N\rangle = \sum_k c^0_{2kN}\,|2kN\rangle$$

$$|1_N\rangle = \sum_k c^1_{(2k+1)N}\,|(2k+1)N\rangle$$



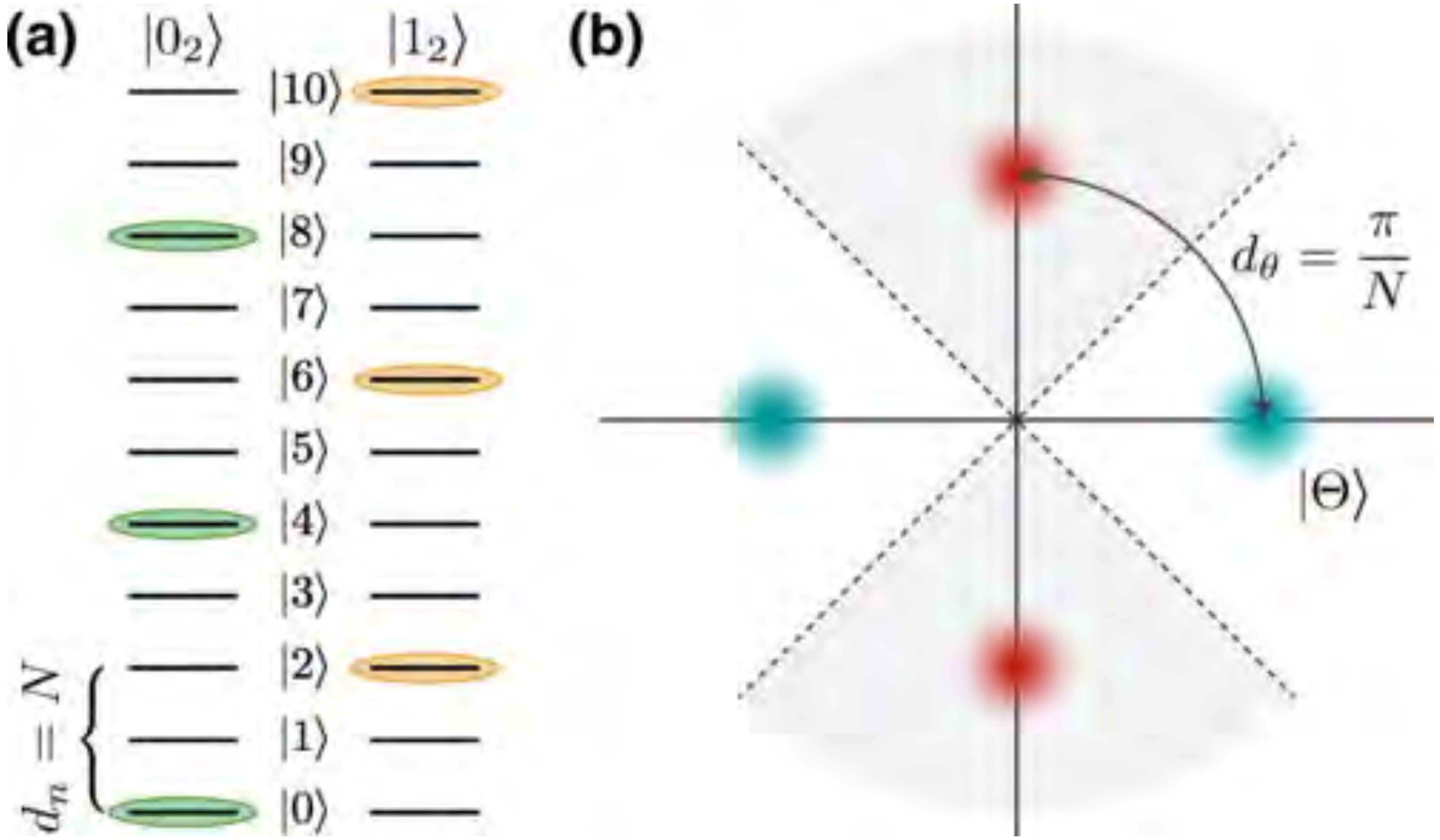
A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

Rotationally symmetric bosonic codes (RSB)

Tradeoff in the code distance: can detect

- Loss or gain up to $N-1$ photons ($d_N = N$)
- Phase errors up to $d_\theta = \pi/N$

$$\hat{R} = e^{i\frac{\pi}{2N}\hat{n}}$$



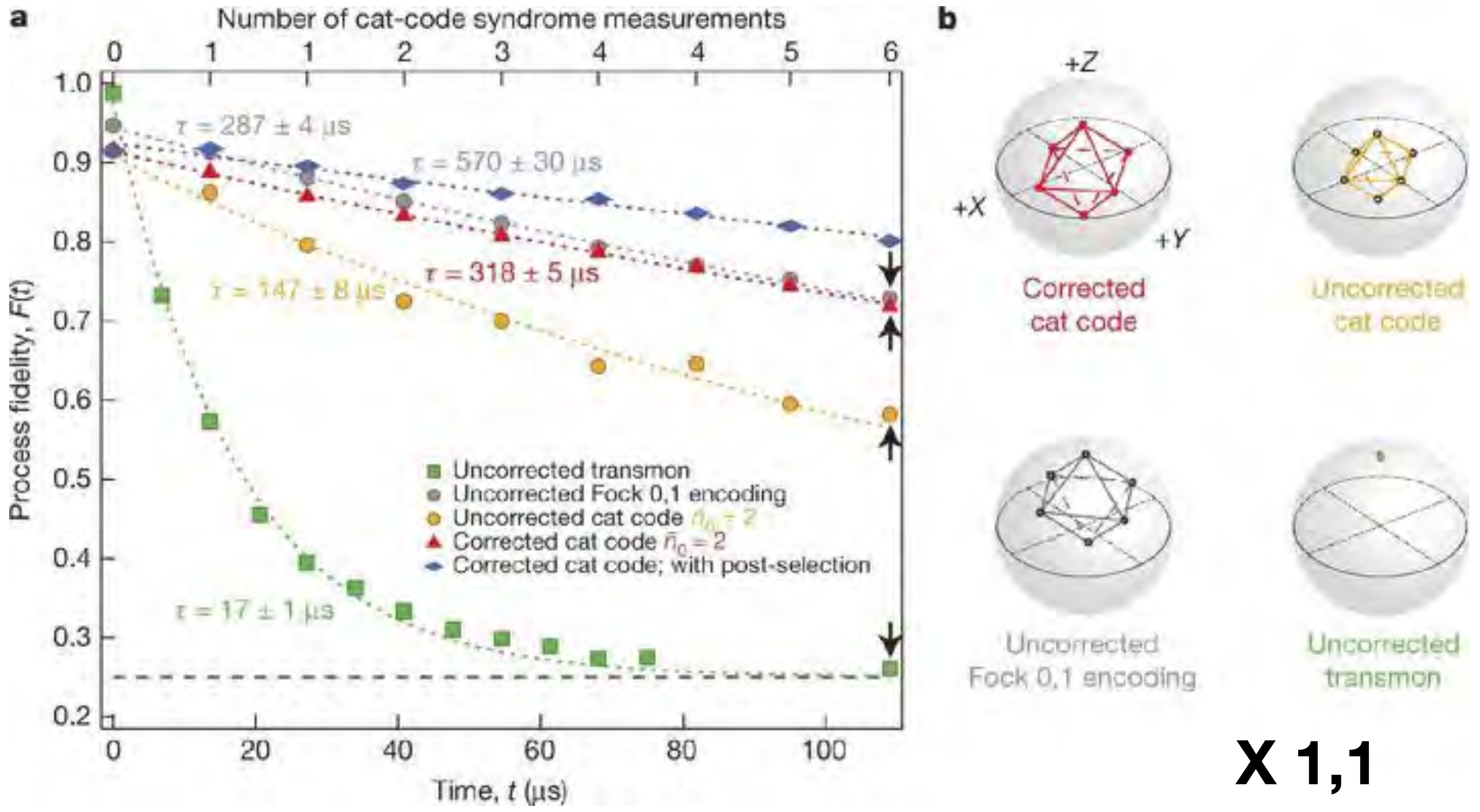
Courtesy of Timo Hillmann

A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

“Break even” with RSB in microwave systems

WACQT Wallenberg Centre for Quantum Technology

Break-even: the quantum coherence using the bosonic code is significantly longer than that of all the imperfect quantum components

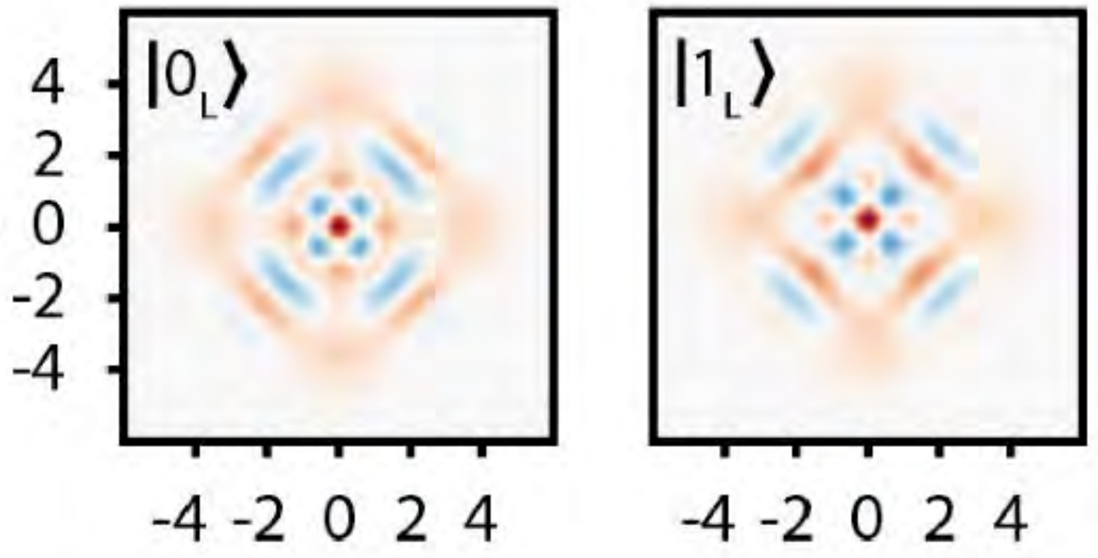


X 1,1

- 4-component cat code

$$|0_L\rangle \propto |\alpha\rangle + |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle$$

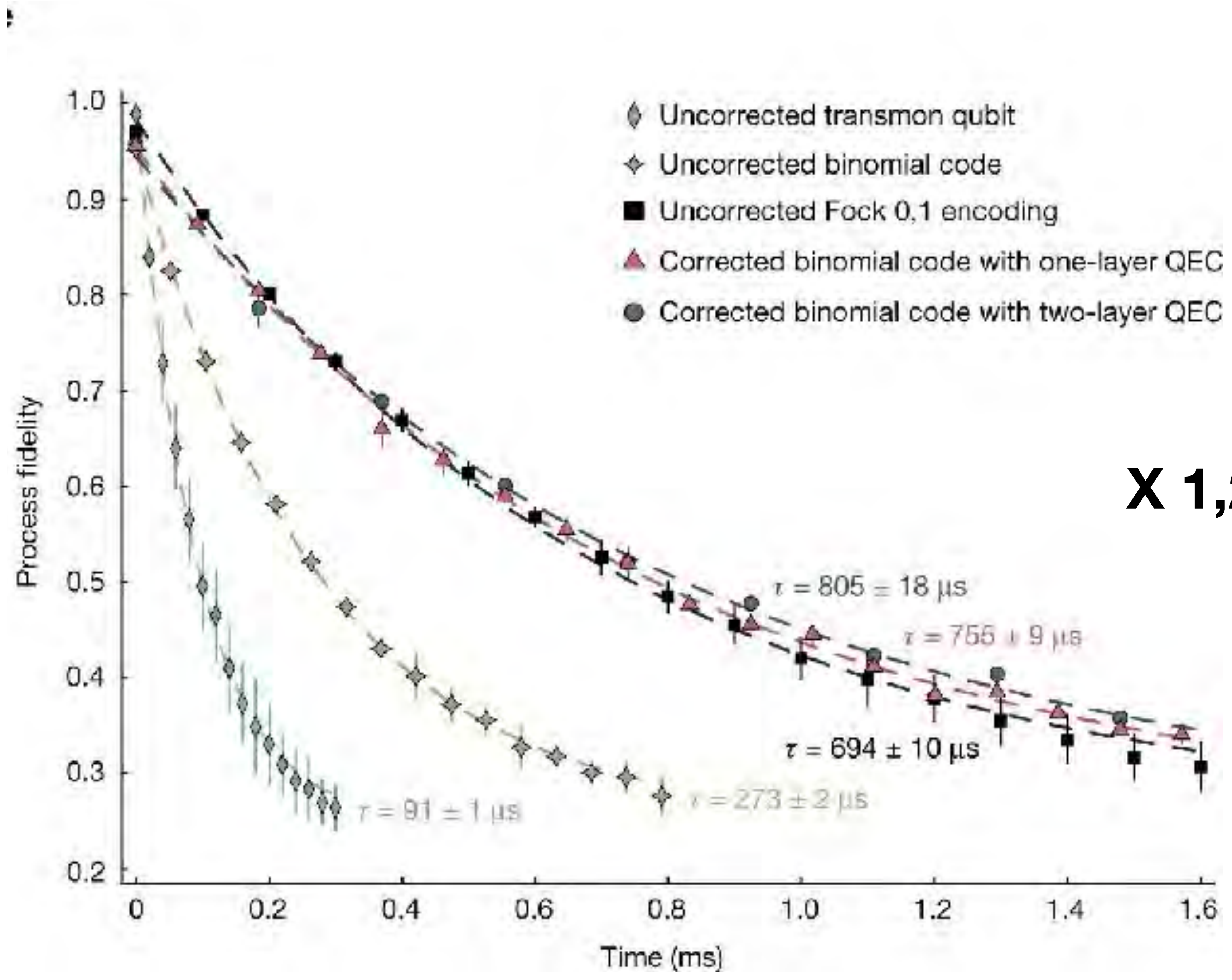
$$|1_L\rangle \propto |\alpha\rangle - |i\alpha\rangle + |-\alpha\rangle - |-i\alpha\rangle$$



“Extending the lifetime of a quantum bit with error correction in superconducting circuits”, Ofek et al, Nature 536, 416 (2016)

"Break even" with RSB in microwave systems

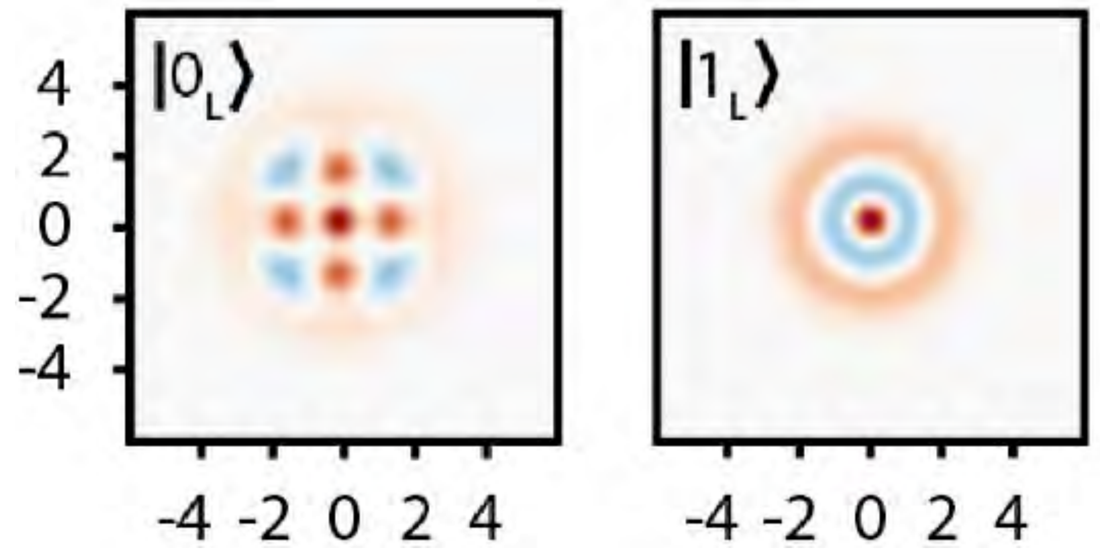
Break-even: the quantum coherence using the bosonic code is significantly longer than that of all the imperfect quantum components



X 1,2

- Binomial code

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |4\rangle), \quad |1_L\rangle = |2\rangle$$



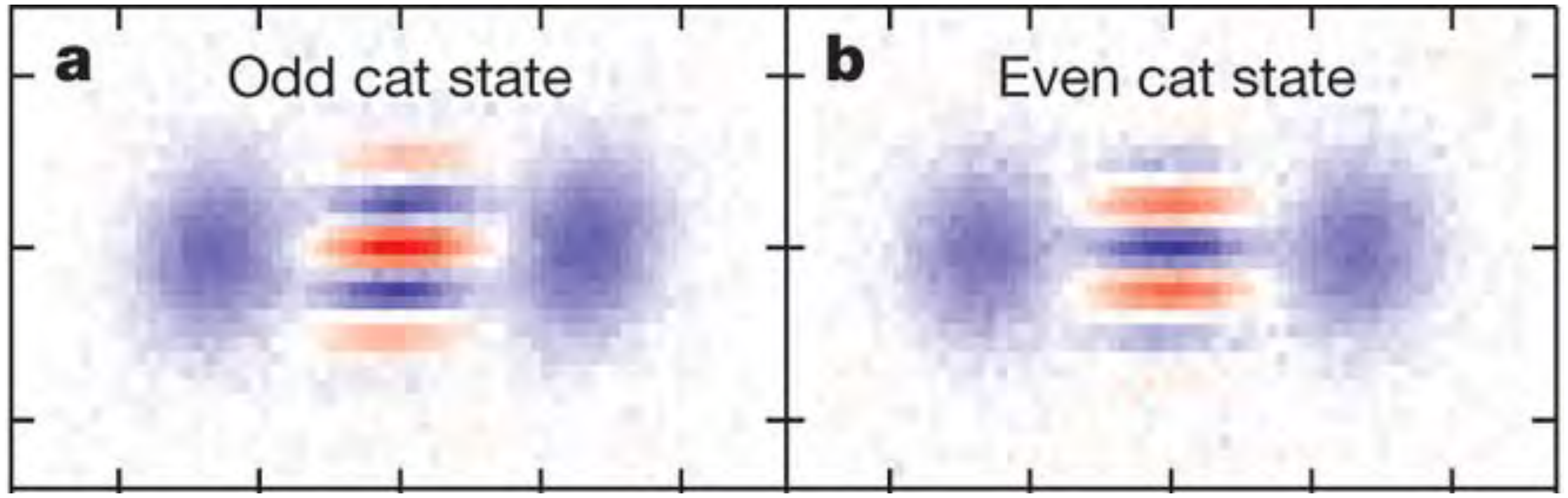
"Beating the break-even point with a discrete-variable-encoded logical qubit", Ni et al, Nature 616, 56 (2023)

Cat qubit (N = 1)

Consider the case $N = 1, |\Theta\rangle = |\alpha\rangle$, can detect but not correct errors

$$|0\rangle_L = \frac{1}{N_0}(|\alpha\rangle + |-\alpha\rangle) \quad \text{(even n)}$$

$$|1\rangle_L = \frac{1}{N_1}(|\alpha\rangle - |-\alpha\rangle) \quad \text{(odd n)}$$



Sun et al, Nature 511, 7510 (2013)

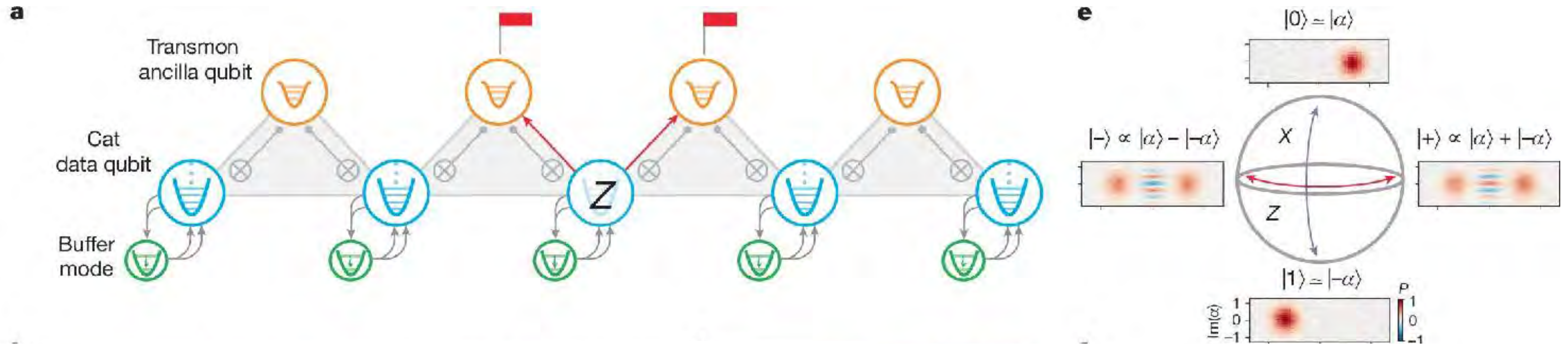
$$\hat{a}|0\rangle_L \propto (|\alpha\rangle - |-\alpha\rangle) = |1\rangle_L$$

The dominant error (photon loss) results in bit flips!
-> *Noise bias*

S. Puri et al, Science Advances 6, (2020) and several related paper by Shruti Puri's group

Repetition code with cat qubits ($d = 5$)

Hardware-efficient quantum error correction via concatenated bosonic qubits, Putterman et al, Nature 2025 (Amazon group)

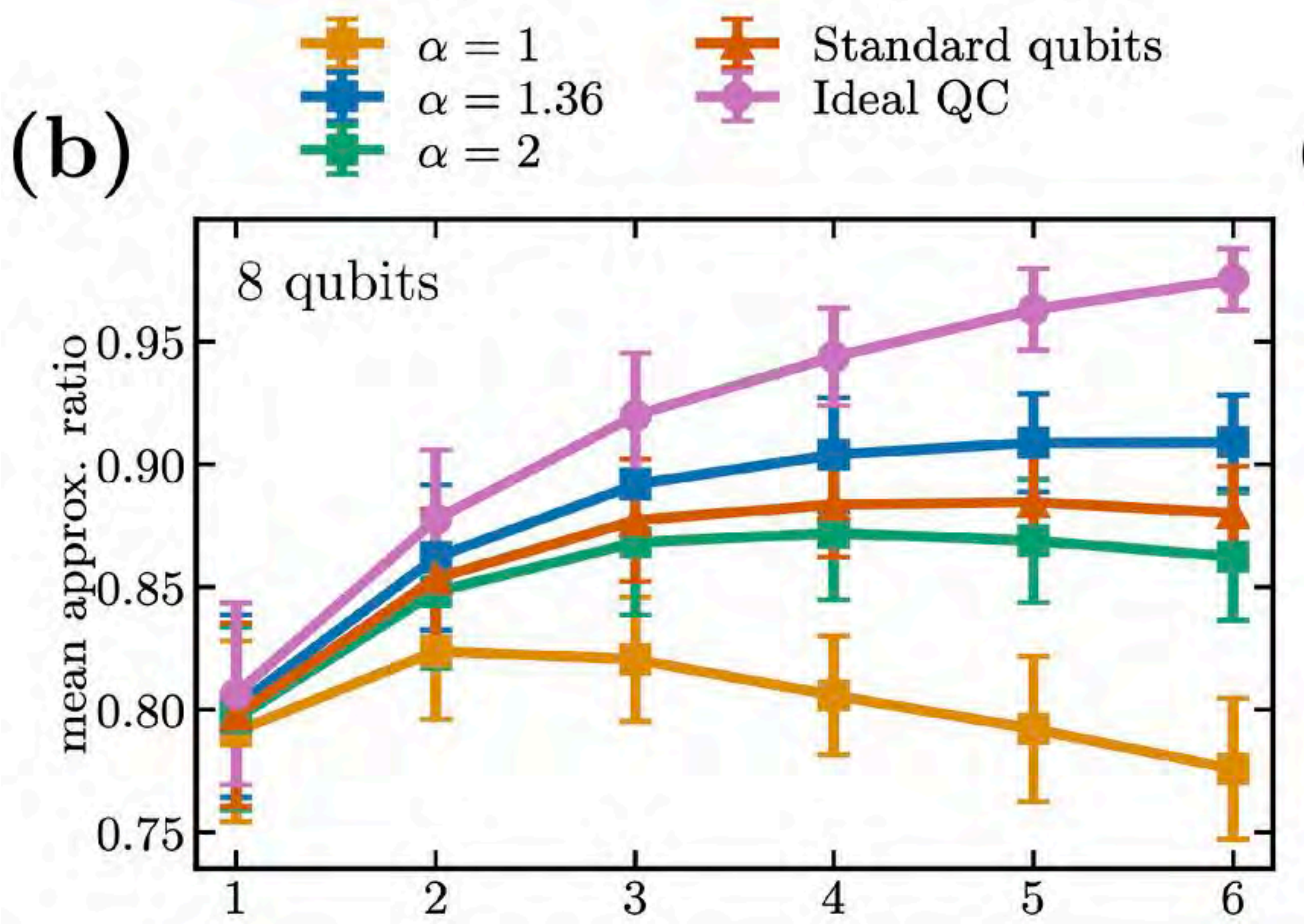


$$\Gamma_X \propto \left|\langle \bar{0}|\hat{a}|\bar{1}\rangle\right|^2 = |\alpha|^2 e^{-2|\alpha|^2}$$

$$\Gamma_Z \propto |\langle \bar{\mp}|\hat{a}|\bar{-}\rangle|^2 \sim |\alpha|^2,$$

S. Puri et al, Science Advances 6, (2020) and several related paper by Shruti Puri's group

Enhanced algorithmic performance in algorithms with dominant Z-gates



"Quantum Approximate Optimization Algorithm with Cat Qubits" P. Vikstål, L. García-Álvarez, S. Puri, G. Ferrini, arXiv:2305.05556 (2023)

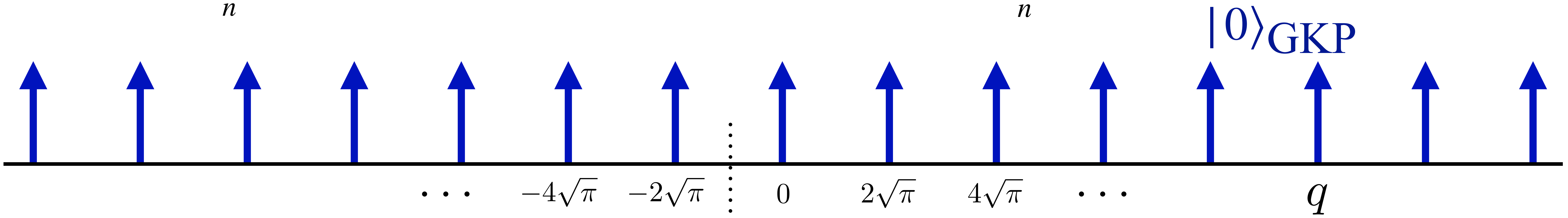
Translationally invariant bosonic codes: the GKP code

Gottesman-Kitaev-Preskill (GKP) code

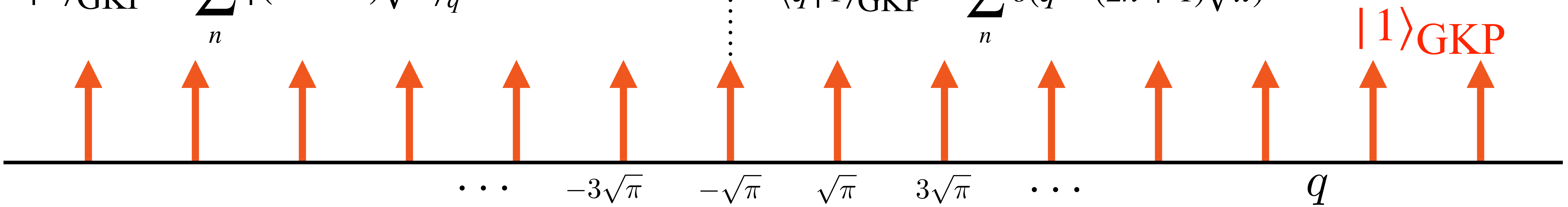
CHALMERS
UNIVERSITY OF TECHNOLOGY

Code-words invariant under translations of $2\sqrt{\pi}$, stabilisers: $e^{i2\sqrt{\pi}\hat{p}}$, $e^{-i2\sqrt{\pi}\hat{q}}$

$$|0\rangle_{\mathrm{GKP}} = \sum_n |2n\sqrt{\pi}\rangle_q \qquad \langle q|0\rangle_{\mathrm{GKP}} = \sum_n \delta(q - 2n\sqrt{\pi})$$



$$|1\rangle_{\mathrm{GKP}} = \sum_n |(2n+1)\sqrt{\pi}\rangle_q \qquad \langle q|1\rangle_{\mathrm{GKP}} = \sum_n \delta(q - (2n+1)\sqrt{\pi})$$



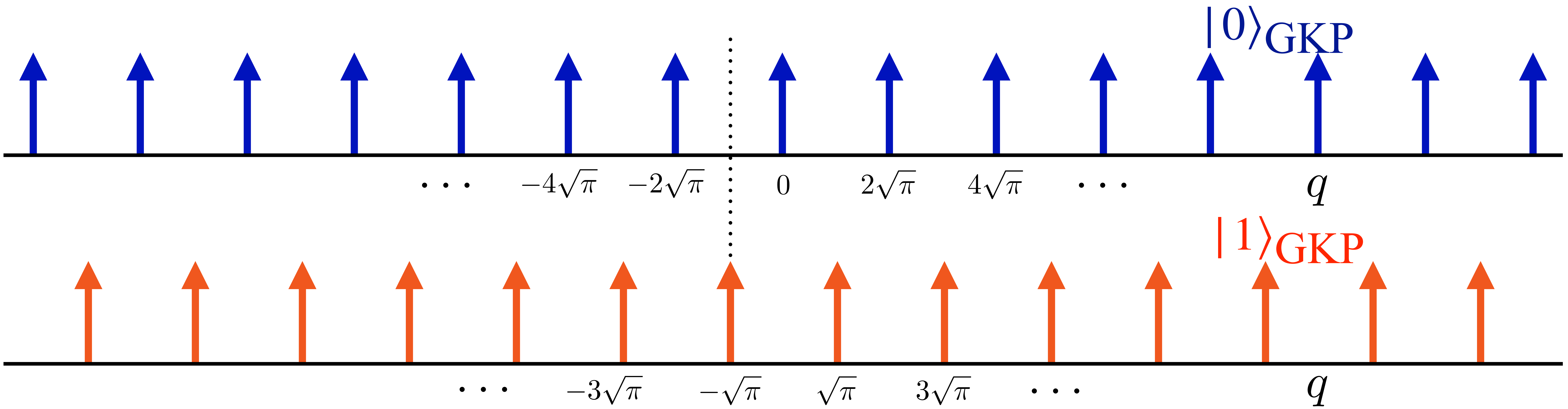
Th: Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001)

$$d_q = d_p = \sqrt{\pi}$$

GKP code: measurement

CHALMERS UNIVERSITY OF TECHNOLOGY

Measurement of $\hat{q}$ (homodyne measurement) = measurement in the computational basis



GKP code: Clifford operations via Gaussian operations

Logical operations on GKP states

$$X = \exp\left(-i\sqrt{\pi}p\right)$$

$$Z = \exp\left(i\sqrt{\pi}q\right)$$

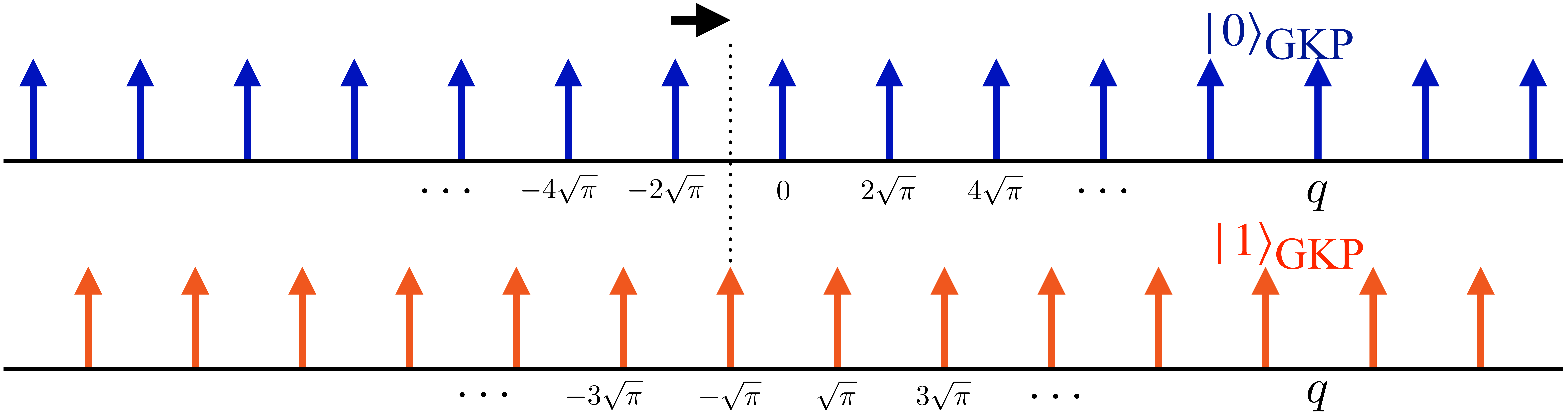
Discrete set of displacements

$$X|0\rangle_{\text{GKP}} = |1\rangle_{\text{GKP}}$$

$$H = \exp\left(i\frac{\pi}{2}(q^2 + p^2)\right)$$

$$S = \exp\left(iq^2/2\right)$$

$$C_Z = \exp\left(iq_1q_2\right)$$



T-gate and notion of universal quantum computation in CV (2)

- One possible universal qubit gate set: $\{T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}, H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\}$

When given a bosonic encoding, one can perform universal quantum computation if one can perform universal encoded qubit quantum computation

- Example with the GKP code:

$$\hat{T} = \mathrm{e}^{\frac{i\pi}{4}\left[2\hat{q}^3/\sqrt{\pi}^3 + \hat{q}^2/\pi - 2\hat{q}/\sqrt{\pi}\right]}$$

$$\overline{H} = \exp(i\frac{\pi}{2}(q^2 + p^2))$$

$$\overline{C}_Z = \exp\left(iq_1 q_2\right)$$

Wigner function of GKP states

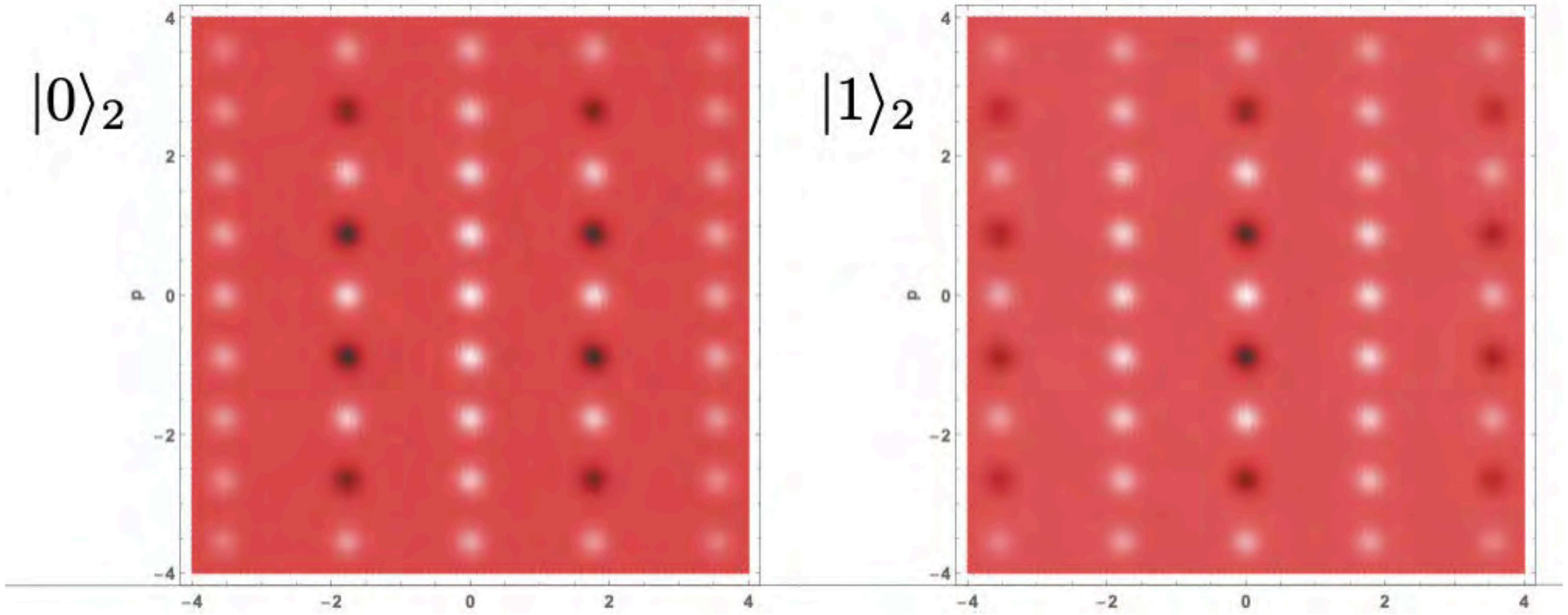
$$W^{(j)}(q,p) = \frac{1}{2n\alpha}\sum_{st}(-1)^{st}\delta\left(p-\frac{\pi}{n\alpha}s\right)\delta\left(q-\alpha j-\alpha\frac{nt}{2}\right)$$

For |0> and |1> states: 1/4 of the summands are negative and 3/4 are positive

$n=2$

$q:\sqrt{\pi}$

$p:\sqrt{\pi}/2$

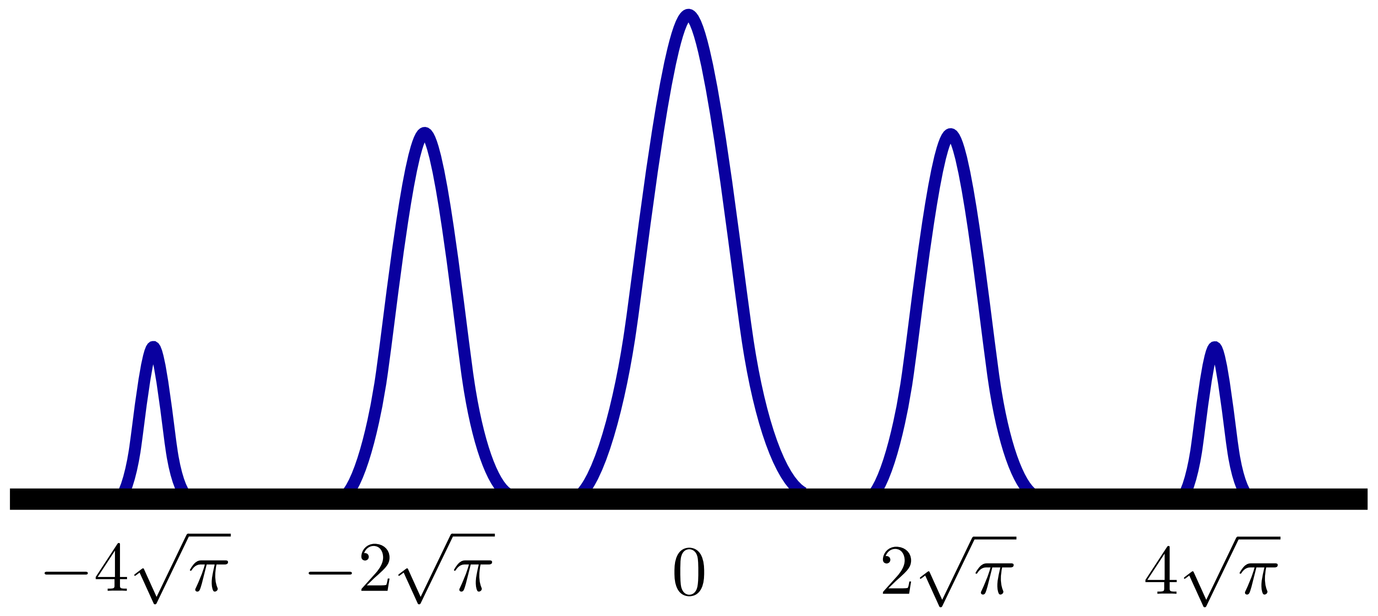


Expression for general states in: *L. García-Álvarez, A. Ferraro, G. Ferrini, International Symposium on Mathematics, Quantum Theory, and Cryptography p.79, Springer (2021)*

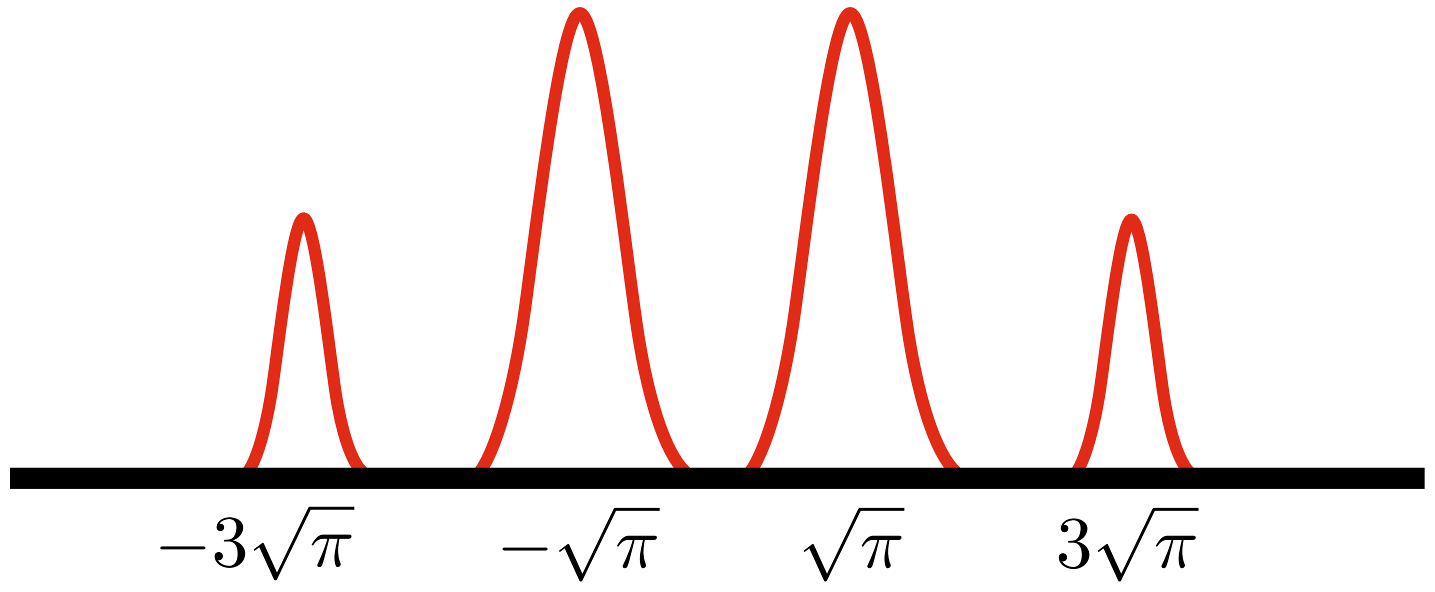
CHALMERS
UNIVERSITY OF TECHNOLOGY

Ideal code states must have a finite energy and must be regularized

$$|0^{\Delta}\rangle_{\mathrm{GKP}} = e^{-\Delta\hat{n}}|0\rangle_{\mathrm{GKP}}$$

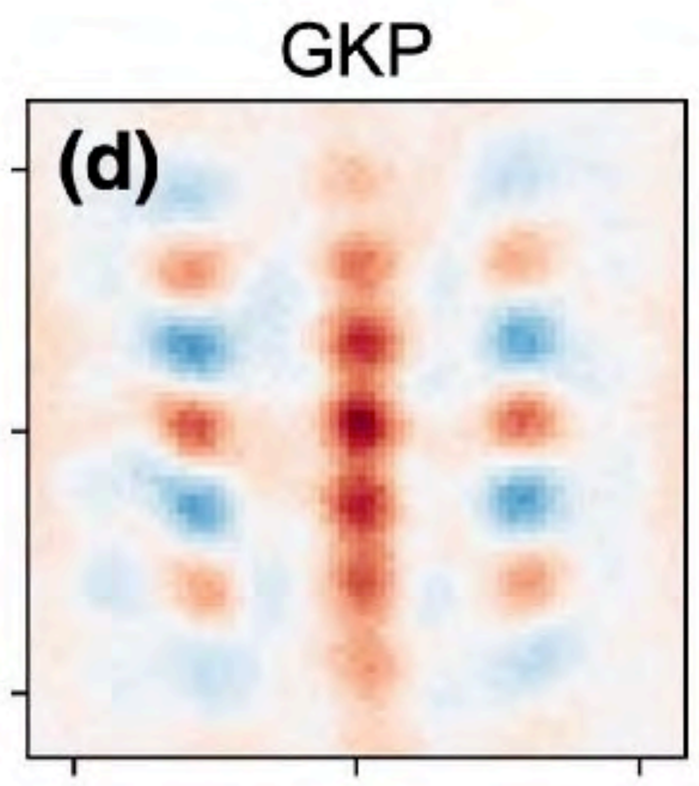


$$|1^{\Delta}\rangle_{\mathrm{GKP}} = e^{-\Delta\hat{n}}|1\rangle_{\mathrm{GKP}}$$



D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001)

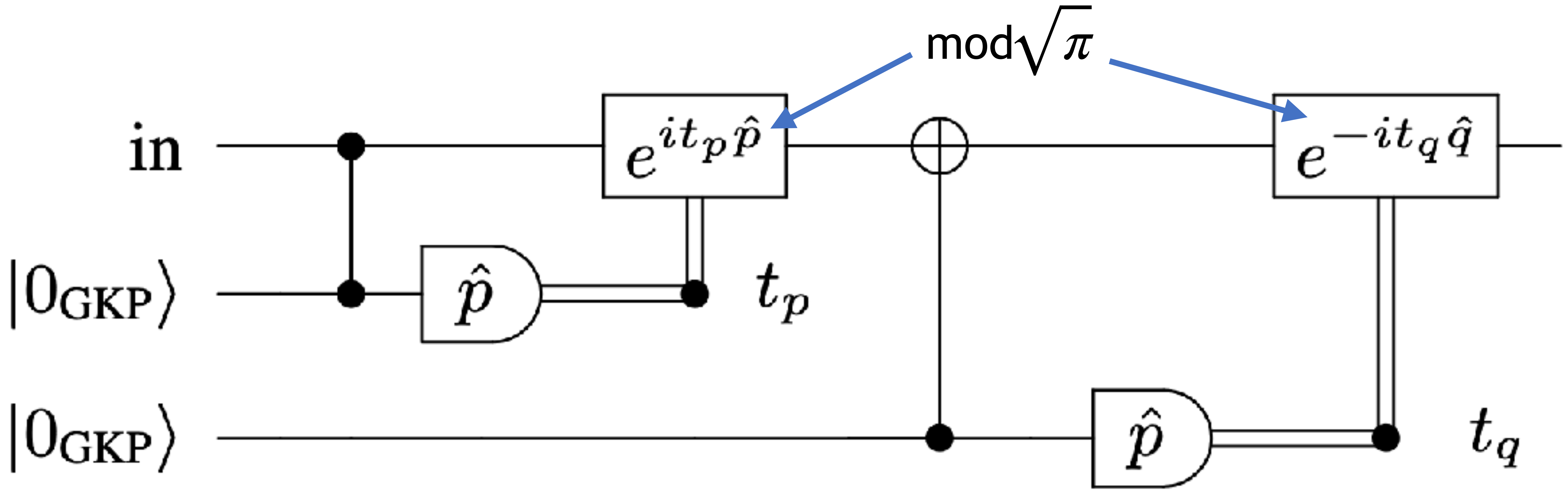
Ions: C. Flühmann et al, Nature 566, 513 (2019);
Microwaves: P. Campagne-Ibarque, Nature 584, 368 (2020);
V. V. Sivak et al, Nature 616, 50 (2023)
Optics: S. Konno et al. Science 383. 6680 (2024)



Kudra et al, PRX Quantum 3, 030301 (2022)

Error correction with GKP states

CHALMERS UNIVERSITY OF TECHNOLOGY



$e^{i2\sqrt{\pi}\hat{p}}, e^{-i2\sqrt{\pi}\hat{q}}$

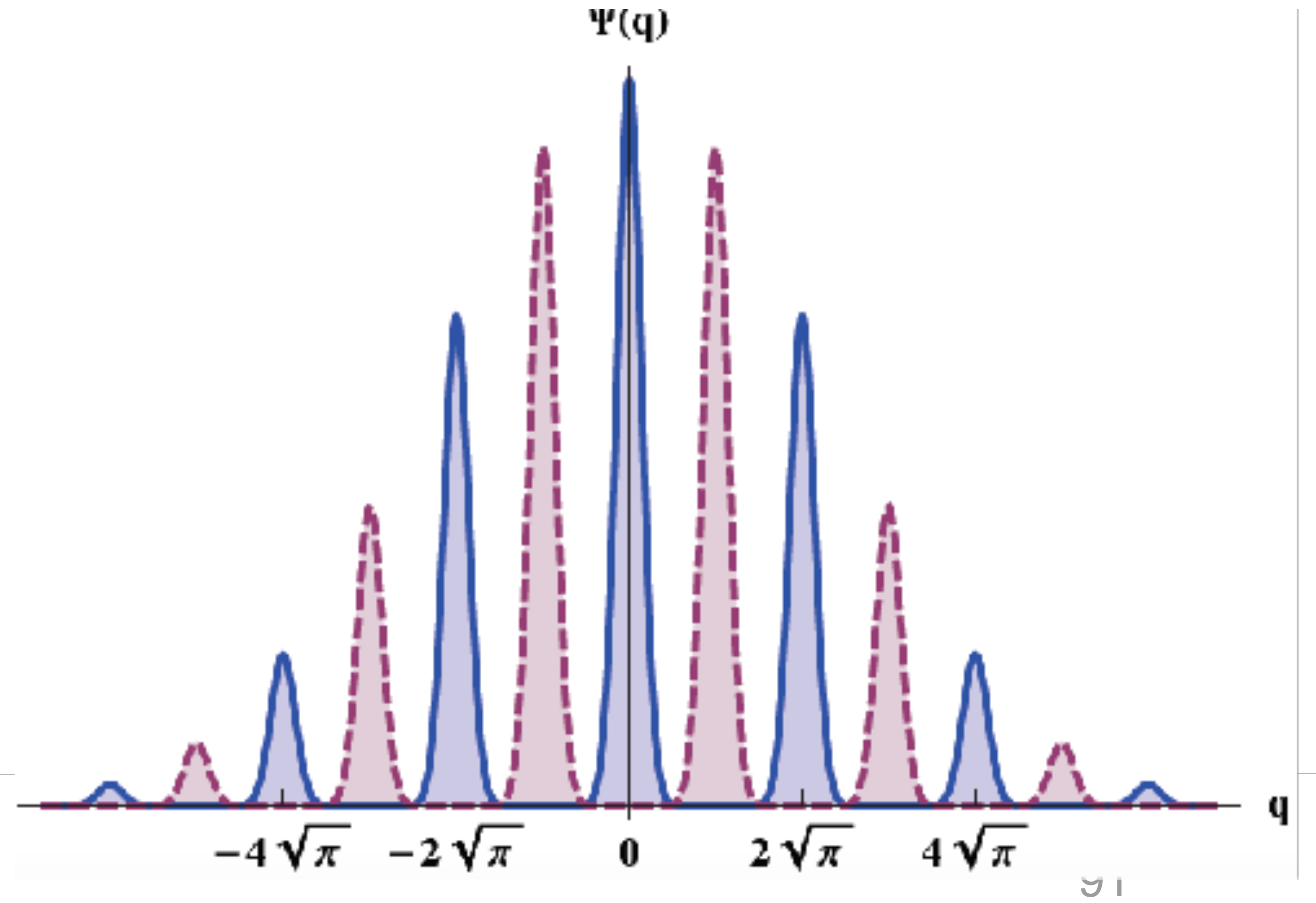
- Syndrome measurement: non-destructive measure of $\hat{q}$ and $\hat{p}$ on the data qubit
- Correction: displacement back of the measured amount (modulo $\sqrt{\pi}$)

D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001); N. Menicucci, PRL 112, 120504 (2014)

For a pedagogical presentation

S. Glancy and E. Knill, Phys. Rev. A 73, 012325 (2006) + our course notes

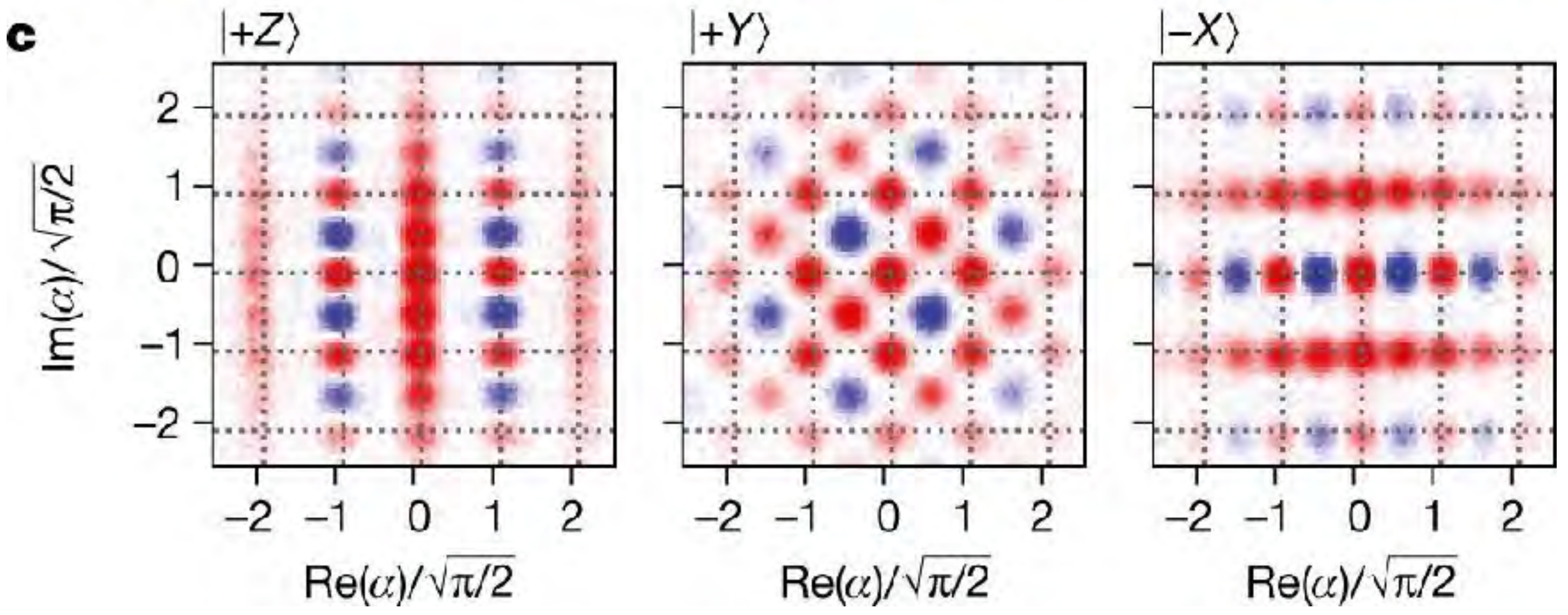
Due to finite squeezing concatenation with qubit QEC is required

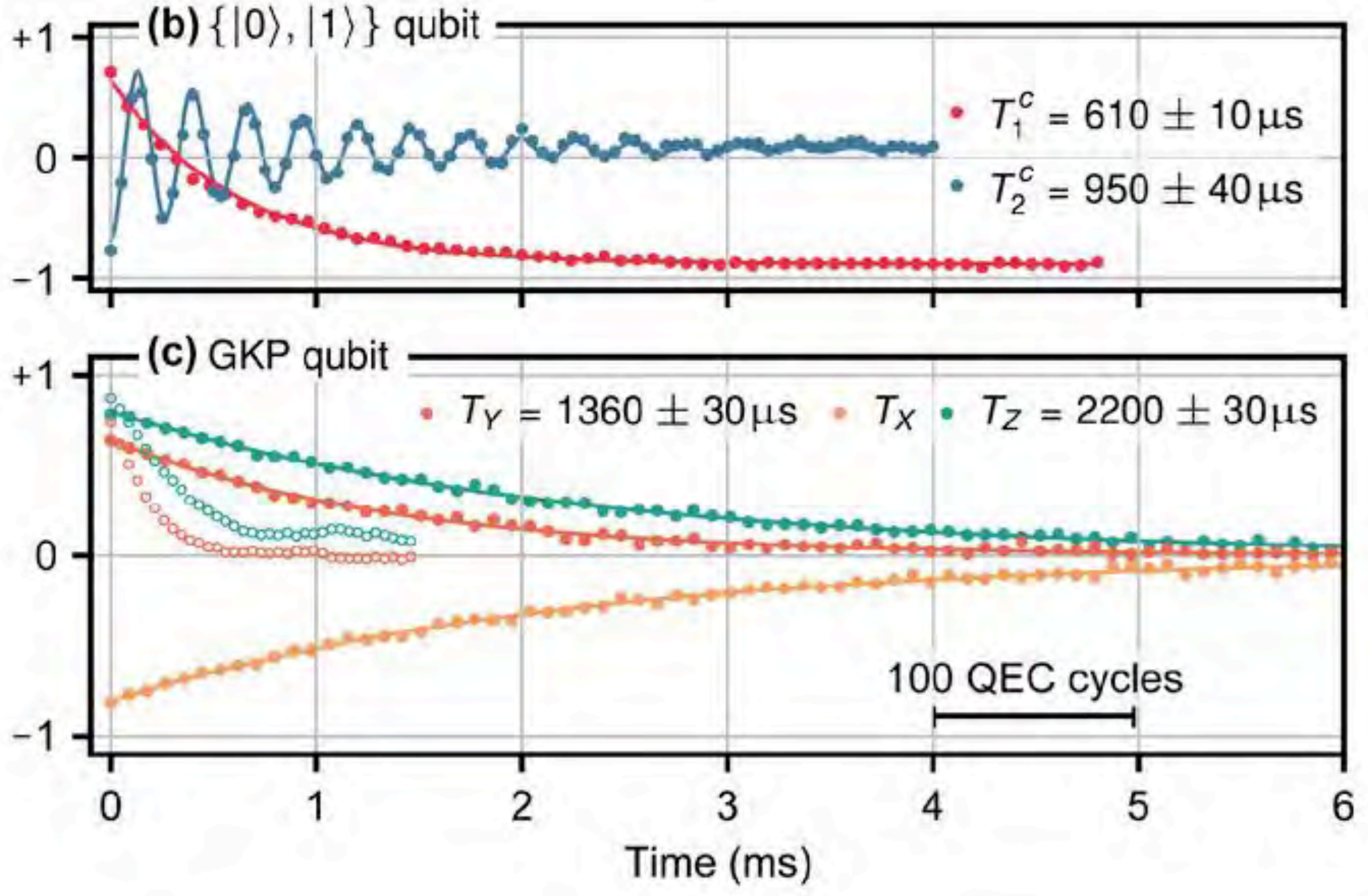


Break even with the GKP code in microwave systems

WACQT Wallenberg Centre for Quantum Technology

Break-even: the quantum coherence using the bosonic code is significantly longer than that of all the imperfect quantum components

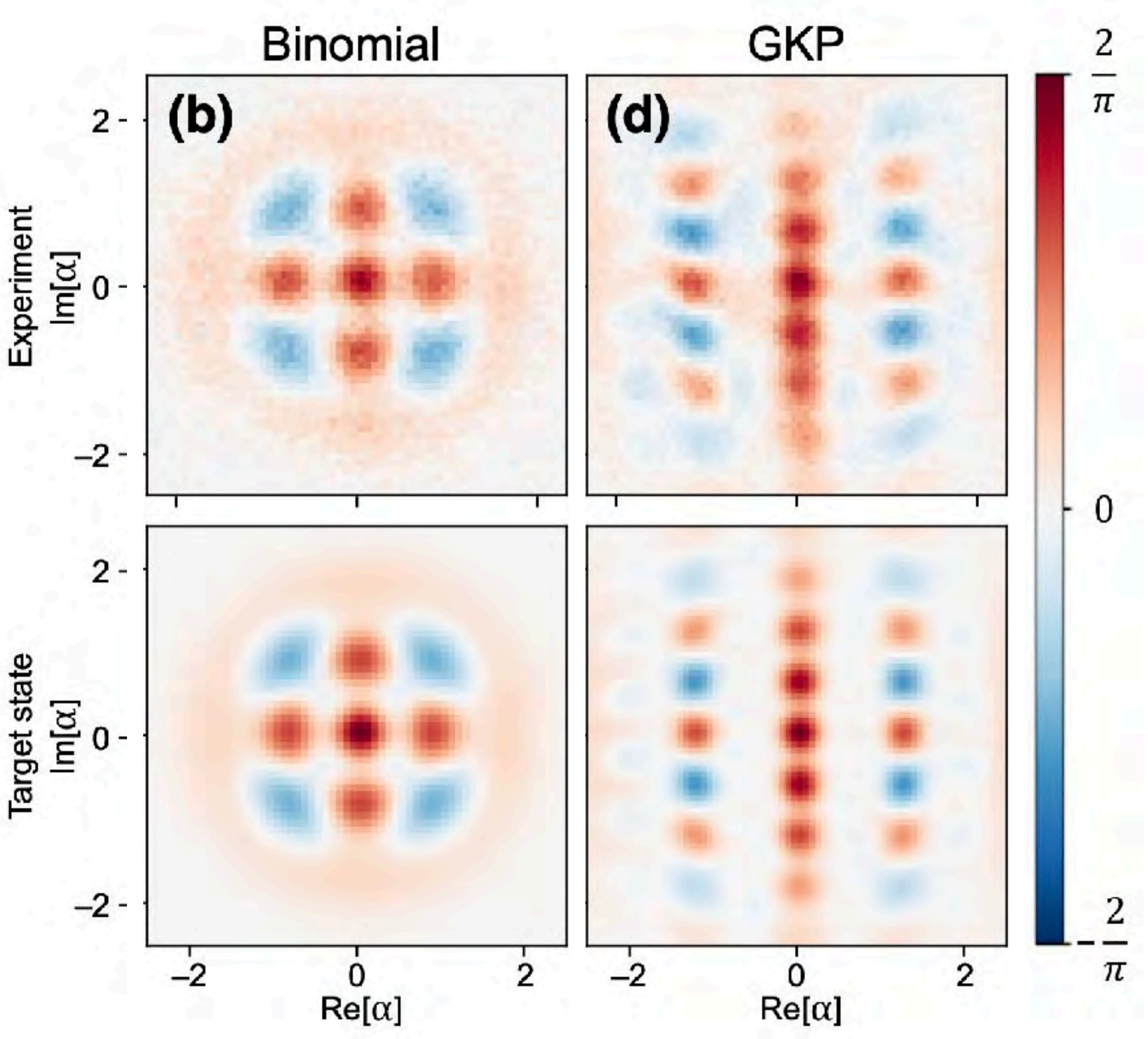




X 2,27

"Real-time quantum error correction beyond break-even", V. V. Sivak et al, Nature 616, 50 (2023)

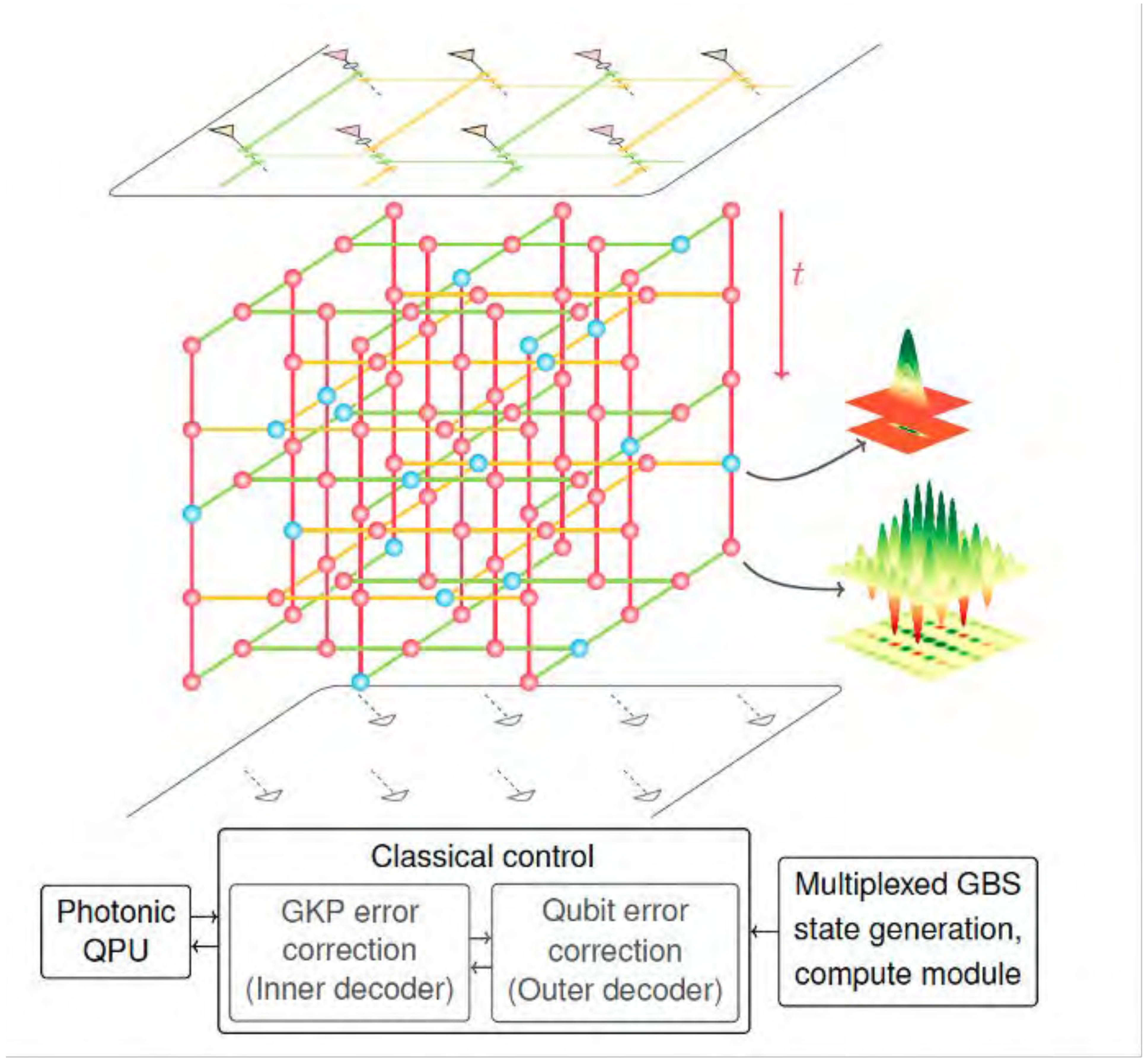
Translationally and rotational symmetric bosonic codes



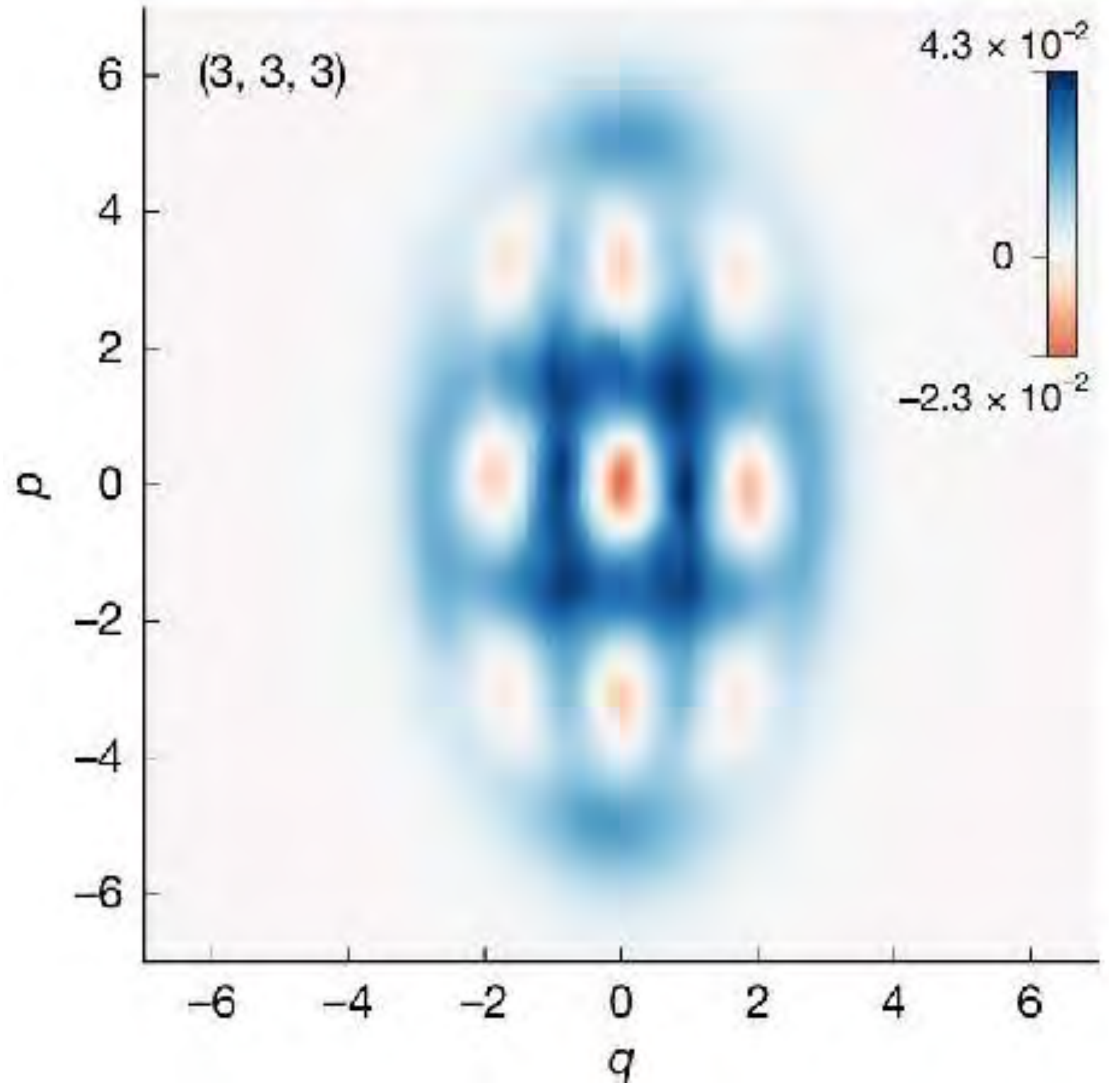
Implemented at Chalmers in microwave cavities

Kudra et al, PRX Quantum 3, 030301 (2022)

Photonic implementations: state generation

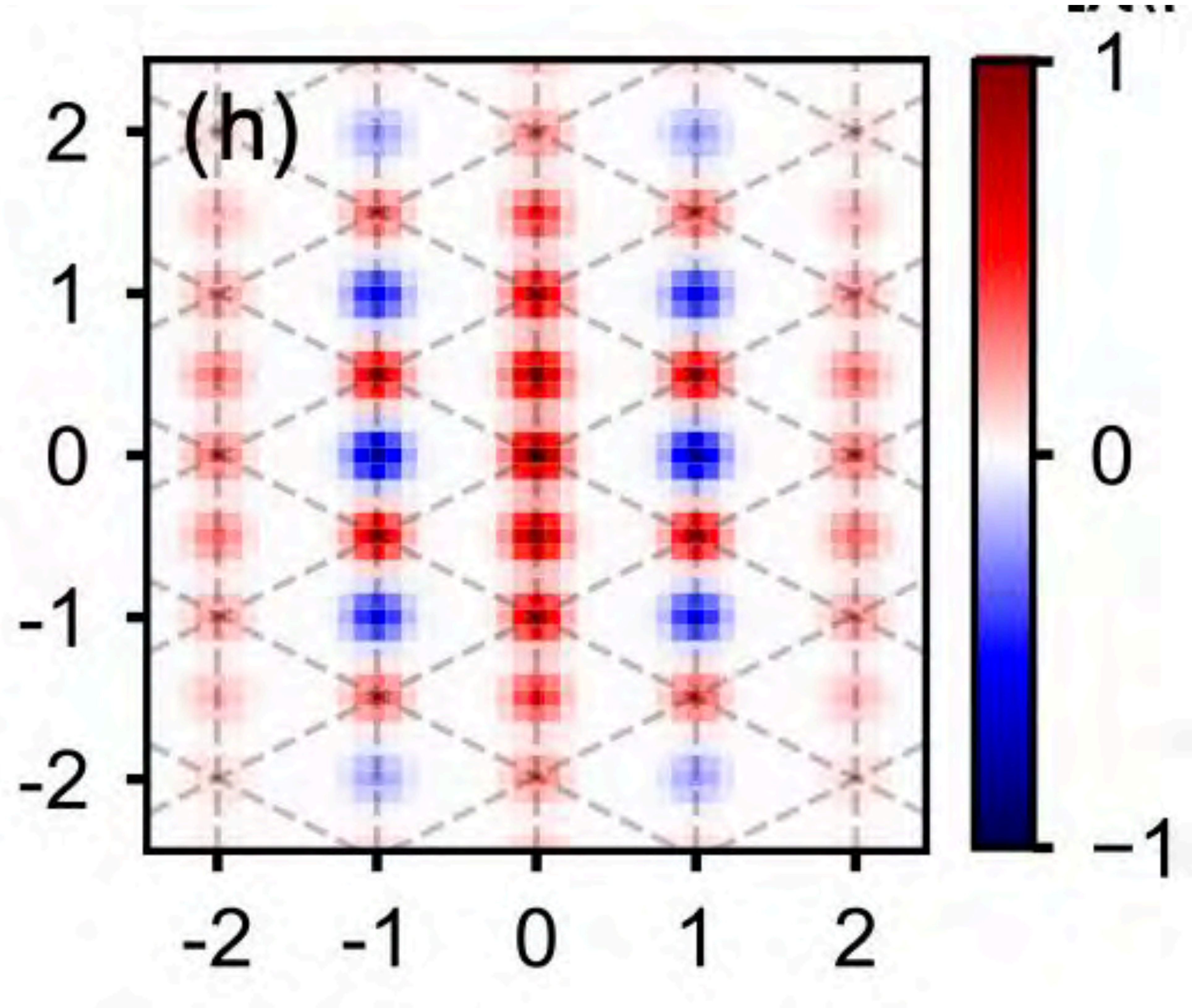


Bourassa et al. Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer, Quantum 5, 392 (2021) Xanadu



Larsen et al, Nature 642, 587 (2025);
See also implementation by Furusawa's group

Trapped ion implementations: state generation



Matsos et al, Phys. Rev. Lett. 133, 050602 (2024)
See also implementation by Home's group

Concatenation of GKP code with surface code

CHALMERS
UNIVERSITY OF TECHNOLOGY

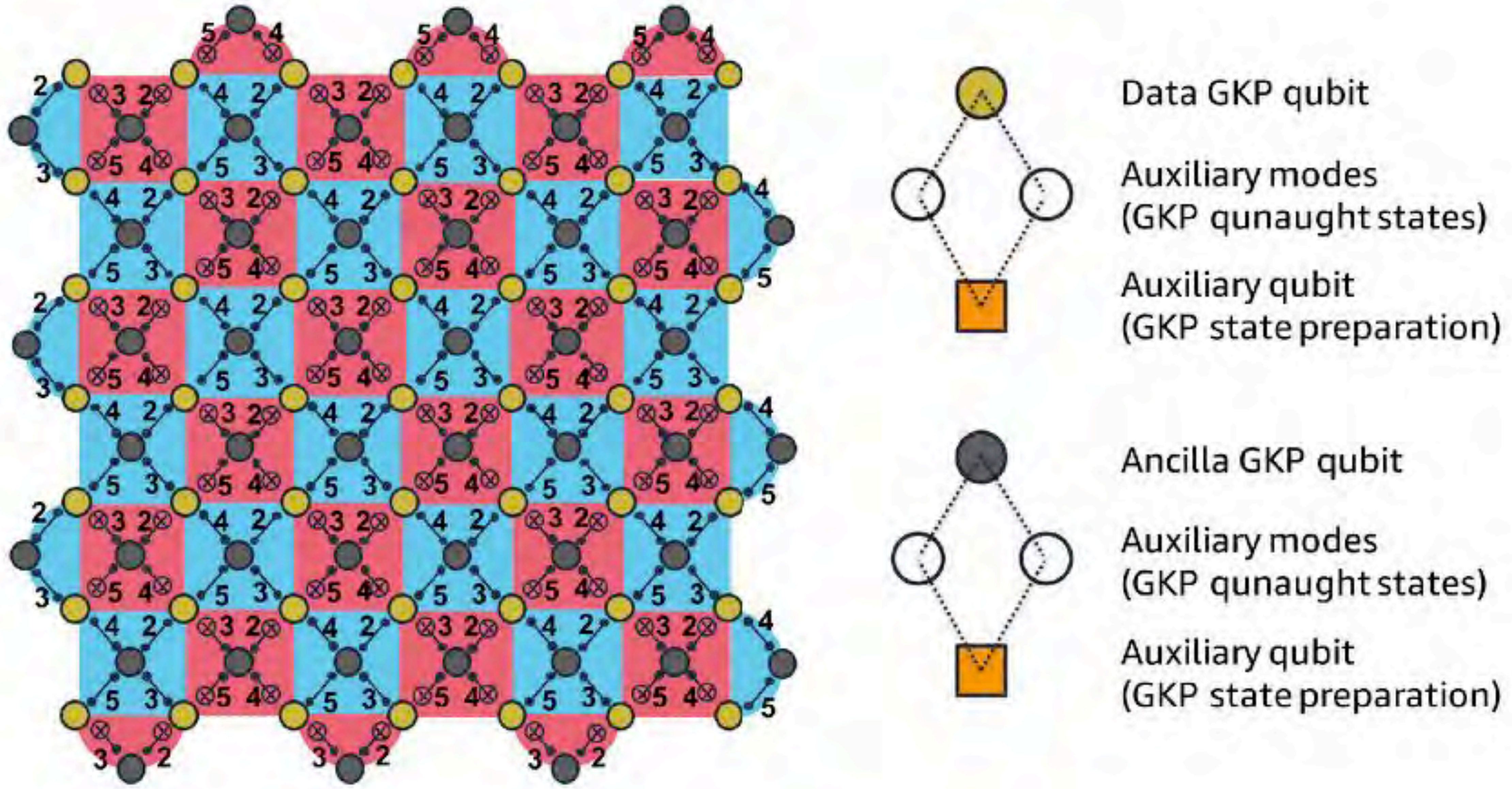
PRX QUANTUM **3**, 010315 (2022)

Low-Overhead Fault-Tolerant Quantum Error Correction with the Surface-GKP Code

Kyungjoo Noh,[1,2,*] Christopher Chamberland,[1,2,†] and Fernando G.S.L. Brandão[1,2,‡]
[1] *AWS Center for Quantum Computing, Pasadena, California 91125, USA*
[2] *IQIM, California Institute of Technology, Pasadena, California 91125, USA*

concatenating the GKP code with the surface code, we find that the threshold GKP squeezing is given by 9.9 dB under the the assumption that finite squeezing of the GKP states is the dominant noise source. More importantly, we show that a low logical failure rate $p_L < 10^{-7}$ can be achieved with moderate hardware requirements, e.g., 291 modes and 97 qubits at a GKP squeezing of 12 dB as opposed to 1457 bare qubits for the standard rotated surface code at an equivalent noise level (i.e., $p = 0.36\%$).

Makes us of clever decoding exploiting continuous information of the syndrome p-measurements

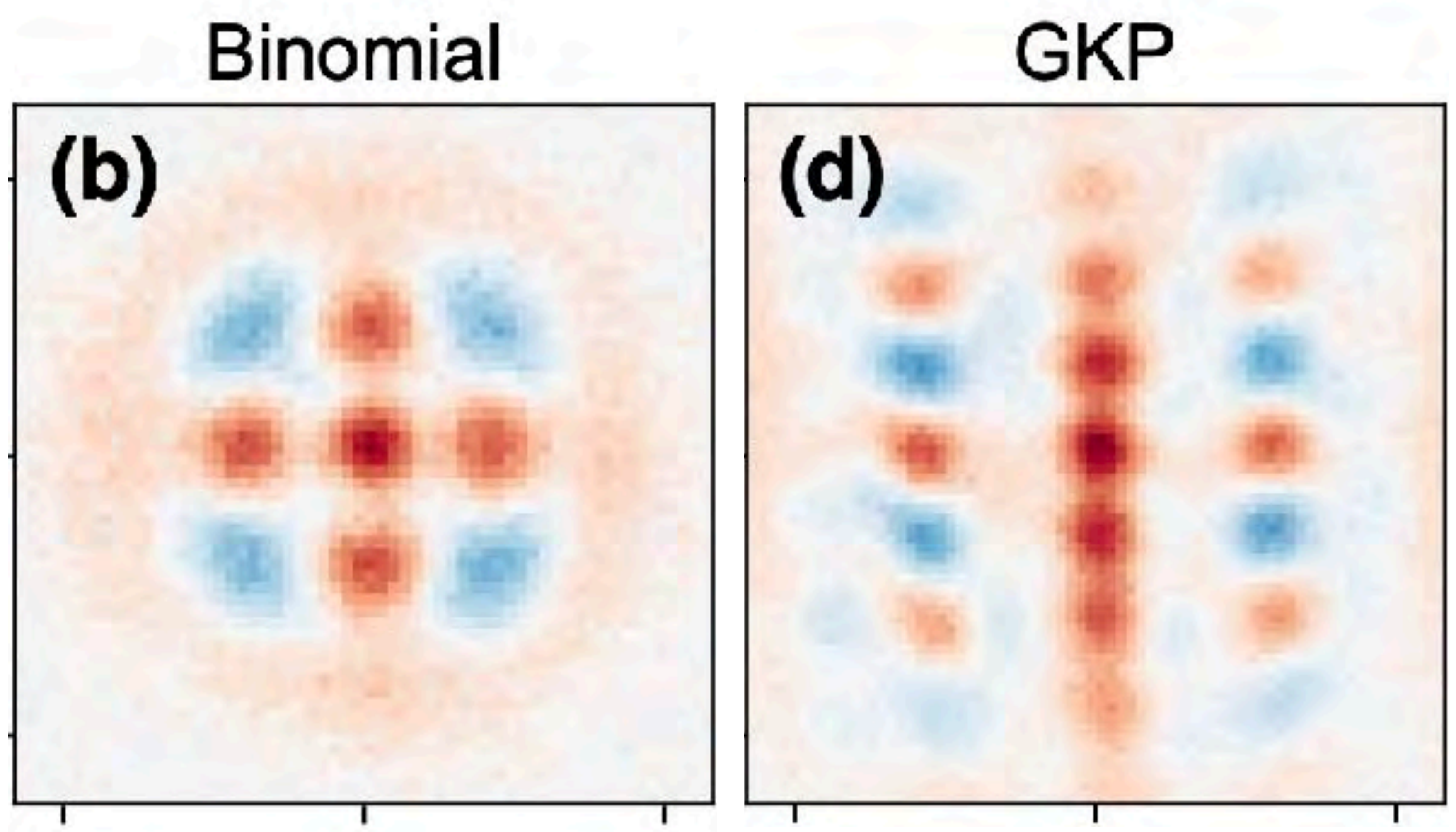


See also later proposals for photonic implementation:
- J. Eli Bourassa et al, Quantum 5, 392 (2021)
- I. Tzitrin et al, Phys. Rev. X Quantum 2, 040353 (2021)

Summary and conclusions on part 2a

Summary and conclusions part 2a

- Bosonic codes
  - Allow for encoding a qubit into a oscillator
  - Serve for quantum error correction with possibly easier decoding
  - Yield reduced code distance when concatenated to the surface code
  - Codewords have either discrete or rotational symmetry



Any question?

Part 2b: Bosonic codes: new research results

Comparing the performance of RSB and GKP codes for realistic detection efficiency and under dephasing

- Translationally symmetric: GKP codes

  Invariant under translation $2\sqrt{\pi}$;

  Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001)

  Extends lifetime of quantum information (x 2,27)

  Sivak et al, Nature 615, 50 (2023)

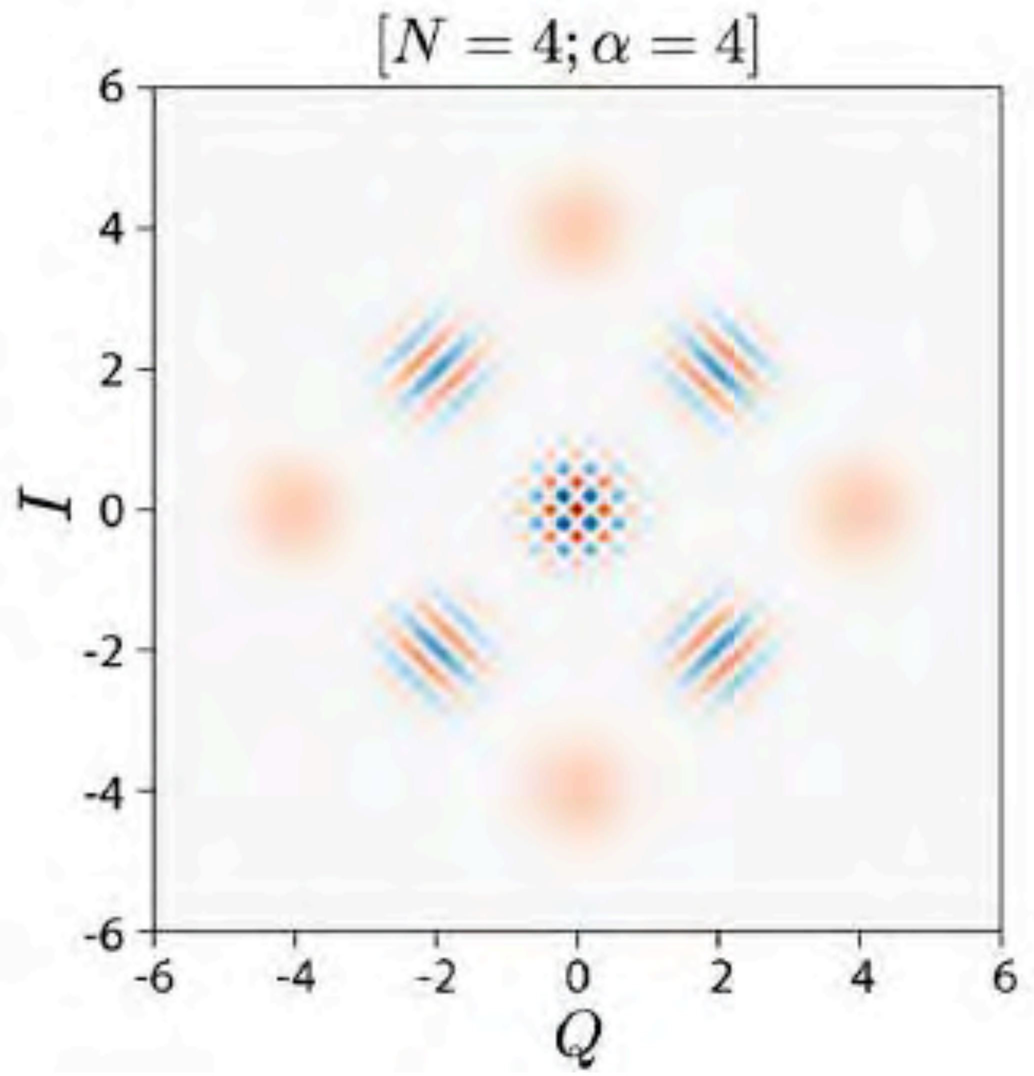
- Rotationally symmetric: RSB codes

  Invariant under rotations of $2\pi/N$;

  A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

  Extends lifetime of quantum information (x 1,2)

  Ni et al, Nature 615, 56 (2023)



Comparison with Rotationally Symmetric Bosonic (RSB) codes

Previous results by Albert *et al.* showed GKP perform better than RSB in the case of

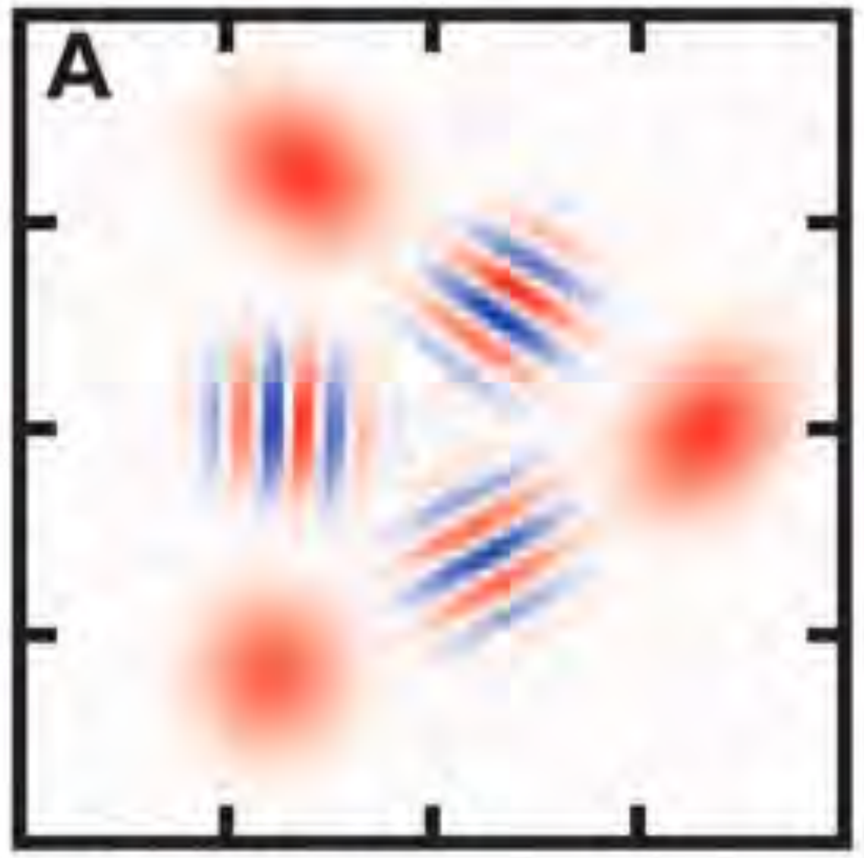
- pure loss & optimal recovery
- similar photon number $\overline{n}_{\mathrm{code}}$

Albert *et al.*, PRA **97**, 032346 (2018)

What if:

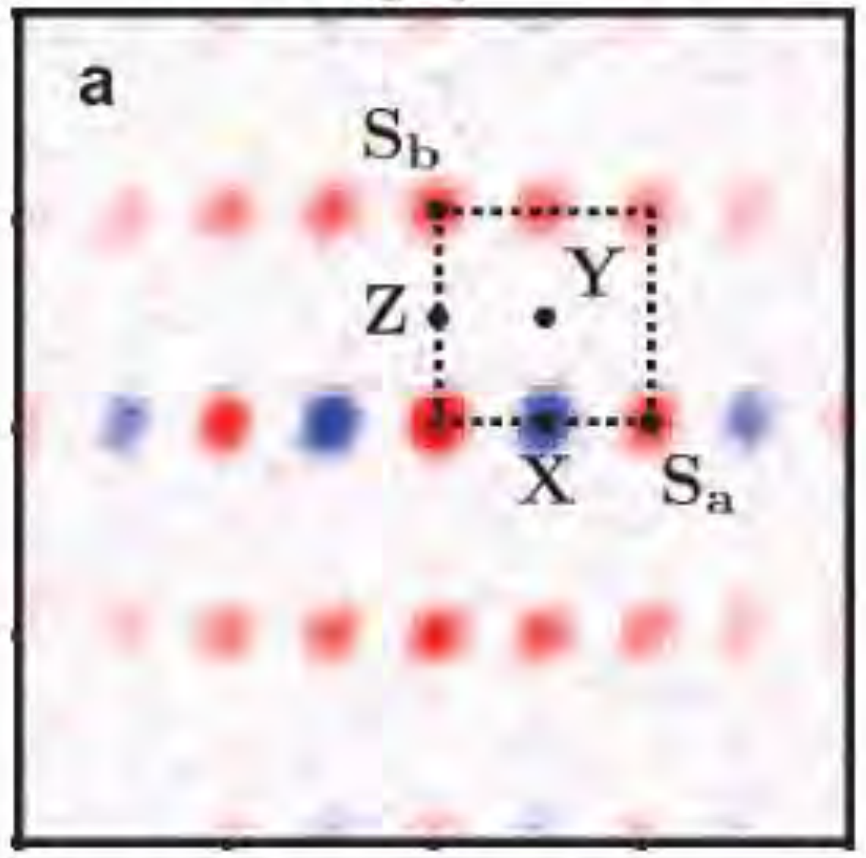
- dephasing & Knill-EC
- *arbitrary* $\overline{n}_{\mathrm{code}}$

$\max \overline{n}_{\mathrm{cat}} > 100$



Vlastakis *et al.*, Science **342**, 607 (2013)

$\max \overline{n}_{\mathrm{gkp}} < 10$



Campagne-Ibarcq *et al.*, Nature **584**, 368 (2020)

Noise channels: losses and dephasing

CHALMERS UNIVERSITY OF TECHNOLOGY

Any quantum channel $\mathcal{N}_{A\to B}$ can be written as a sum of *Kraus operators*, which are linear maps $K_i \colon \mathcal{H}_A \to \mathcal{H}_B$ such that

$$\mathcal{N}_{A\to B} = \sum_i K_i \bullet K_i^\dagger,$$

Loss channel:

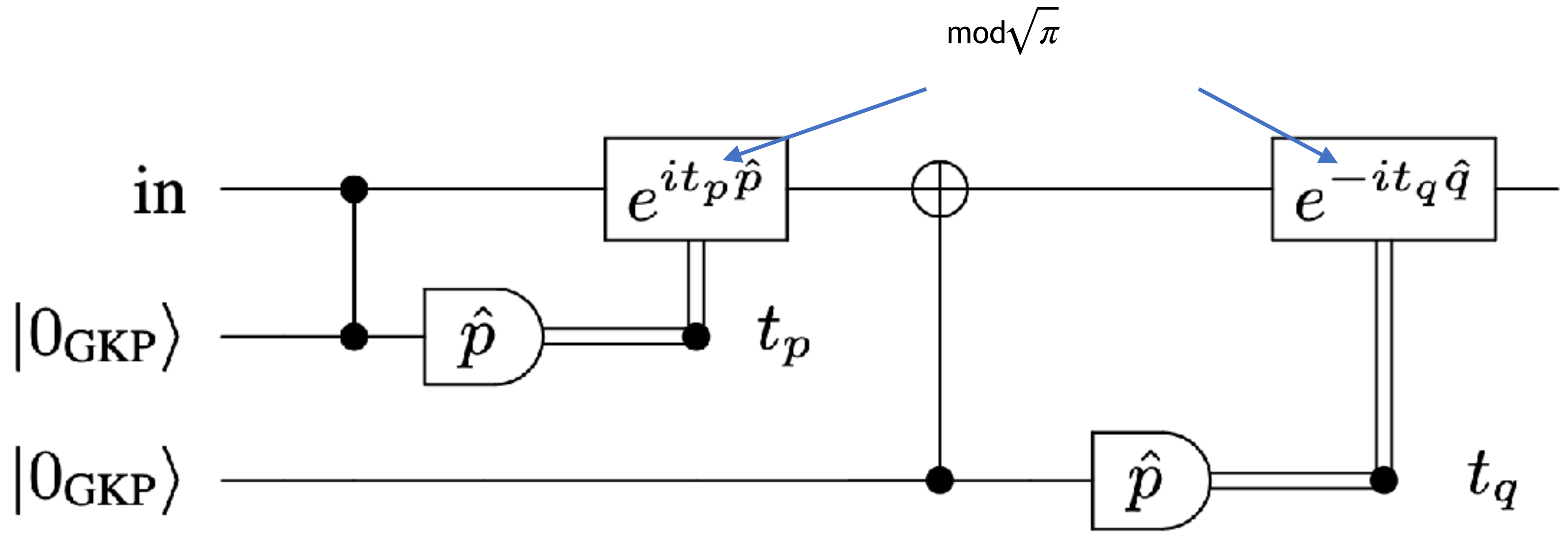
$$\mathcal{N}_L[\gamma] = \sum_{n=0}^{\infty} \hat{L}_k \bullet \hat{L}_k^\dagger,\ \hat{L}_k = \sqrt{\frac{\gamma^k}{k!}}(1-\gamma)^{\frac{\hat{n}}{2}}\hat{a}^k$$

Dephasing channel:

$$\mathcal{N}_D[\gamma_\phi] = \sum_{k=0}^{\infty} \hat{D}_k \bullet \hat{D}_k^\dagger,\ \hat{D}_k = \sqrt{\frac{\gamma_\phi^k}{k!}e^{-\frac{\gamma_\phi}{2}\hat{n}^2}}\hat{n}^k$$

CHALMERS
UNIVERSITY OF TECHNOLOGY

Steane QEC

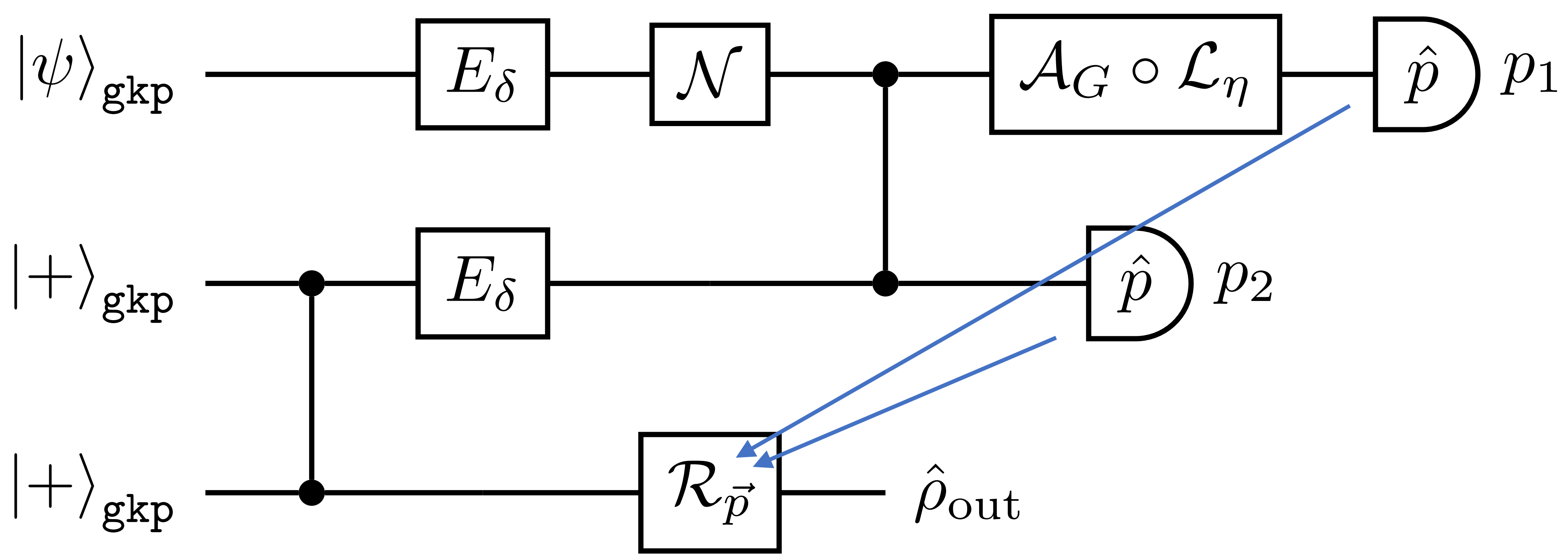


- stabiliser measurement with auxiliary system

Steane, PRL 77, 793 (1996)

Gottesman et al, PRA 64, 012310 (2001)

Knill (teleportation) QEC



Used to calculate F, tells how similar is the recovered state to the initial one (good if =1)

- logical information is teleported to last mode

Knill, Nature 434, 39 (2005)

T. Hillmann, F. Quijandría, A. L. Grimsmo, G. Ferrini, PRX Quantum 3, 020334 (2022)

Take-away messages:

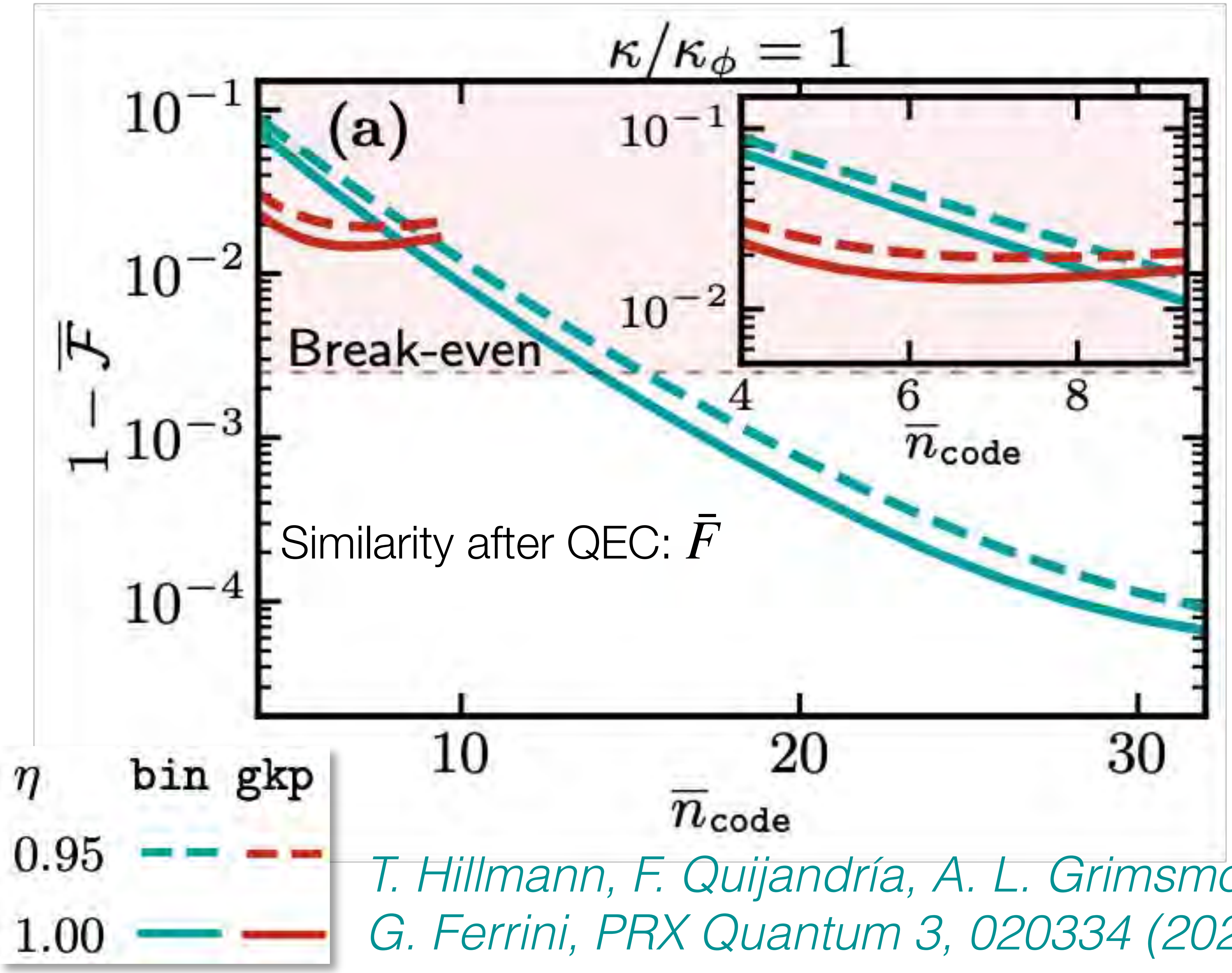
1. Bin codes can outperform GKP codes in the presence of dephasing

$$|0\rangle_{\text{bin}} = \frac{1}{\sqrt{2}}(|0\rangle + |2N\rangle)$$

$$|1\rangle_{\text{bin}} = |N\rangle$$

$N = 4$

- binomial order-$N = 4$ code
- adaptive homodyne measurement
- photon loss rate: $\kappa\tau = 5 \cdot 10^{-3}$



T. Hillmann, F. Quijandría, A. L. Grimsmo, G. Ferrini, PRX Quantum 3, 020334 (2022)

Take-away messages:

1. Bin codes can outperform GKP codes in the presence of dephasing
2. If dephasing is negligible, finite measurement efficiencies quickly diminish GKP advantages
3. Possibly greater near-term potential of Bin codes over GKP codes

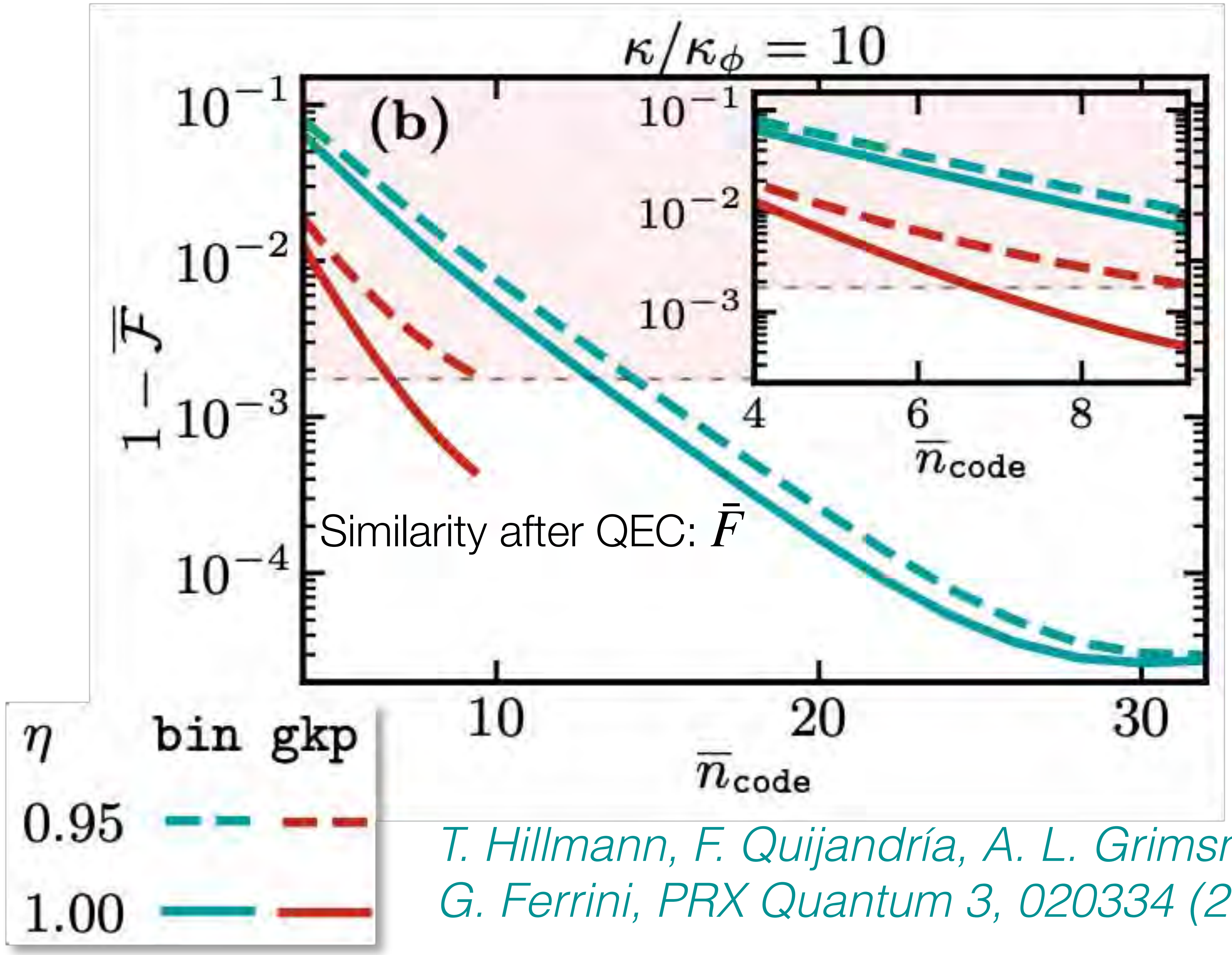
(0.95-efficiency currently unreached)

$$|0\rangle_{\text{bin}} = \frac{1}{\sqrt{2}}(|0\rangle + |2N\rangle)$$

$$|1\rangle_{\text{bin}} = |N\rangle$$

$N = 4$

- binomial order-$N = 4$ code
- adaptive homodyne measurement
- photon loss rate: $\kappa\tau = 5 \cdot 10^{-3}$



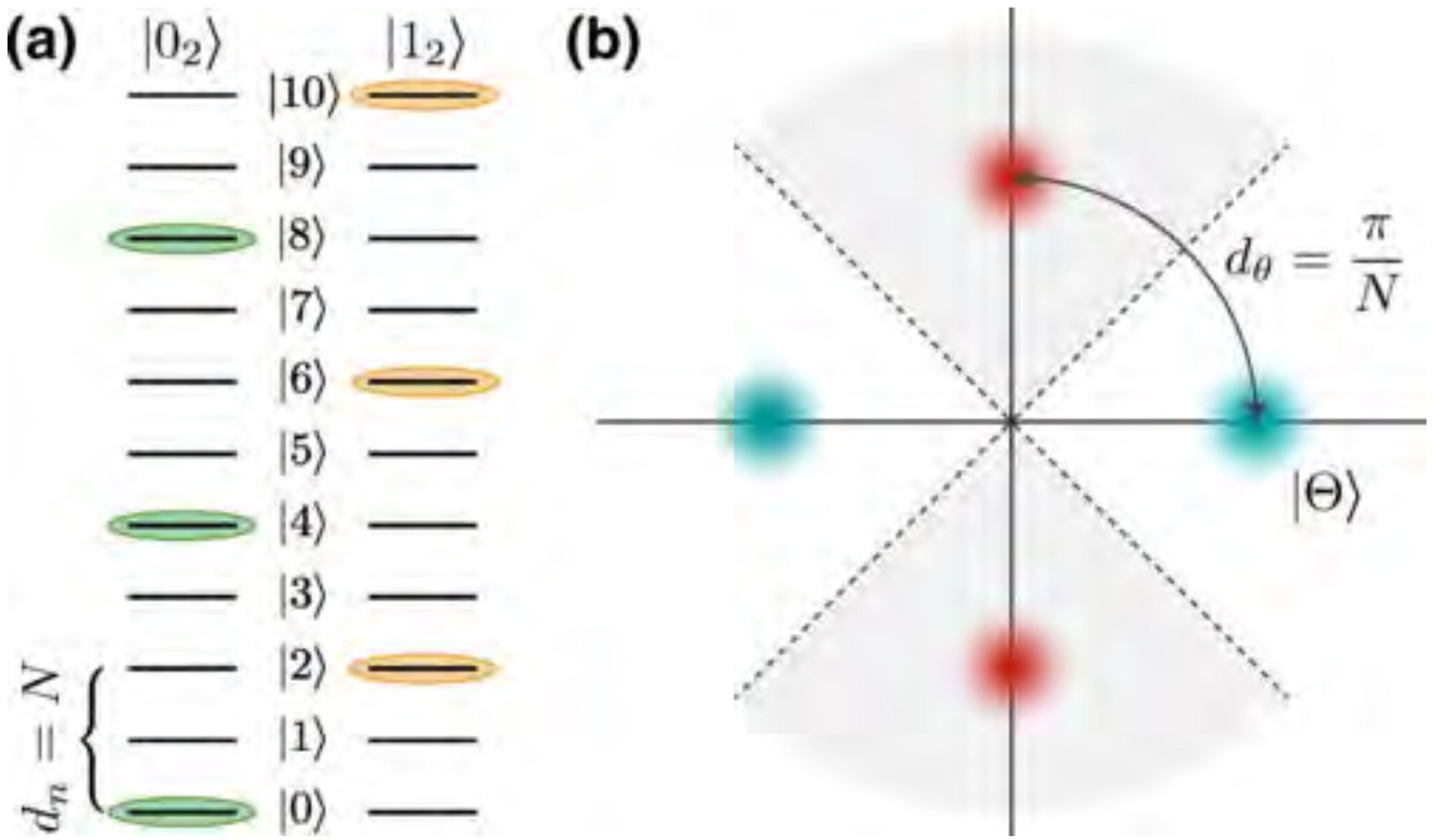
T. Hillmann, F. Quijandría, A. L. Grimsmo, G. Ferrini, PRX Quantum 3, 020334 (2022)

Single-mode RSB have limitations

Tradeoff in the code distance: can detect

- Loss or gain up to $N-1$ photons ($d_N = N$)
- Phase errors up to $d_\theta = \pi/N$

Can we go around this tradeoff?



Courtesy of Timo Hillmann

A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

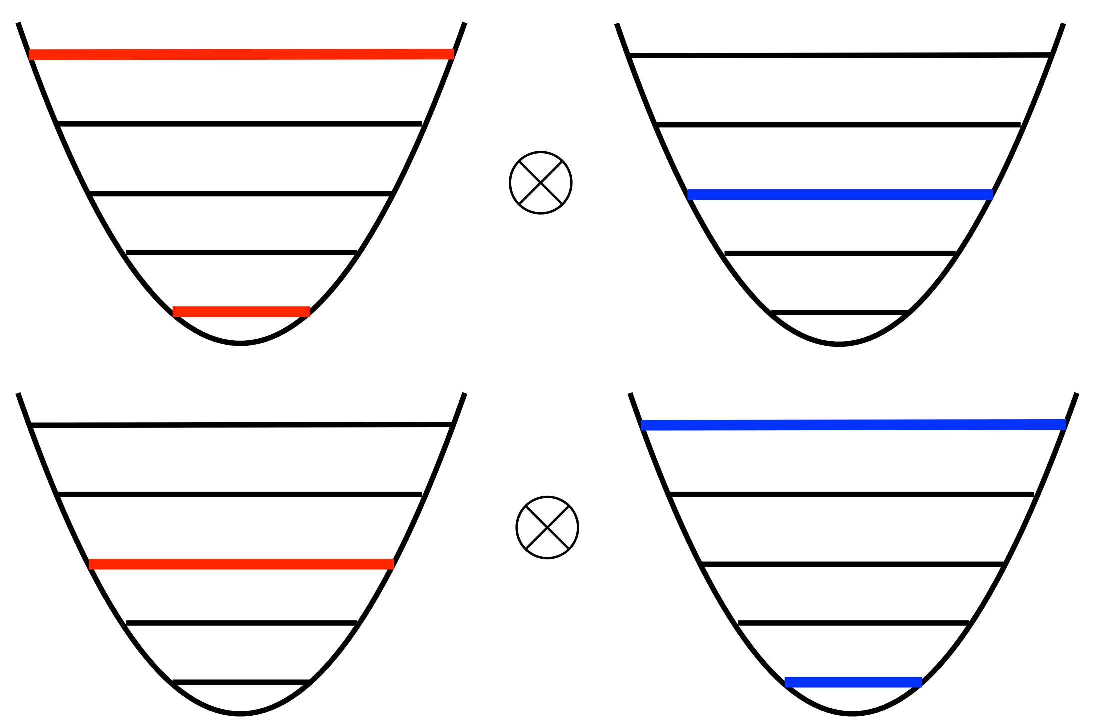
Solution: two-mode RSB codes

WACQT | Wallenberg Centre for Quantum Technology

- Example: Dual-rail binomial code

$$|0\rangle_L = \hat{U}_{BS}(\delta,\phi)\frac{1}{\sqrt{2}}(|0\rangle + |2N\rangle)\otimes|N\rangle$$

$$|1\rangle_L = \hat{U}_{BS}(\delta,\phi)|N\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |2N\rangle)$$



- Based on group-theoretic construction *A. Denys and A. Leverrier. Phys. Rev. Lett. 133 (2024)*
- Parameterized by two angles δ and ϕ

$|0_{\text{dual rail}}\rangle = |01>$
$|1_{\text{dual rail}}\rangle = |10>$

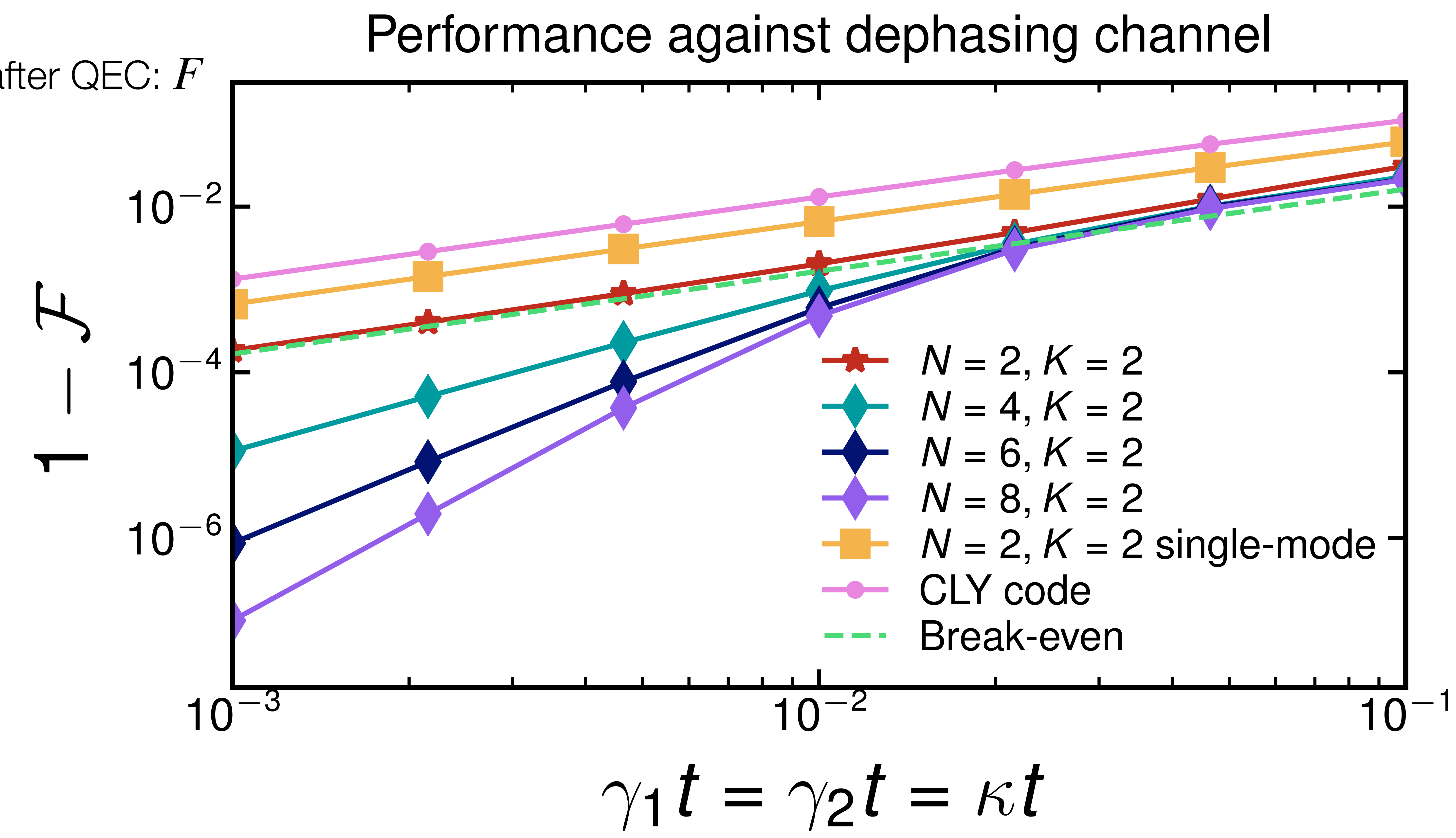
$$U_{\text{BS}} = \exp(-i\delta(G^{+}_{12}\cos\phi + G^{-}_{12}\sin\phi)$$

$$G^{+}_{12} = a_1 a_2^{\dagger} + a_1^{\dagger} a_2 \quad G^{-}_{12} = i(a_1 a_2^{\dagger} - a_1^{\dagger} a_2)$$

R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)

Performance against Gaussian dephasing

Similarity after QEC: F

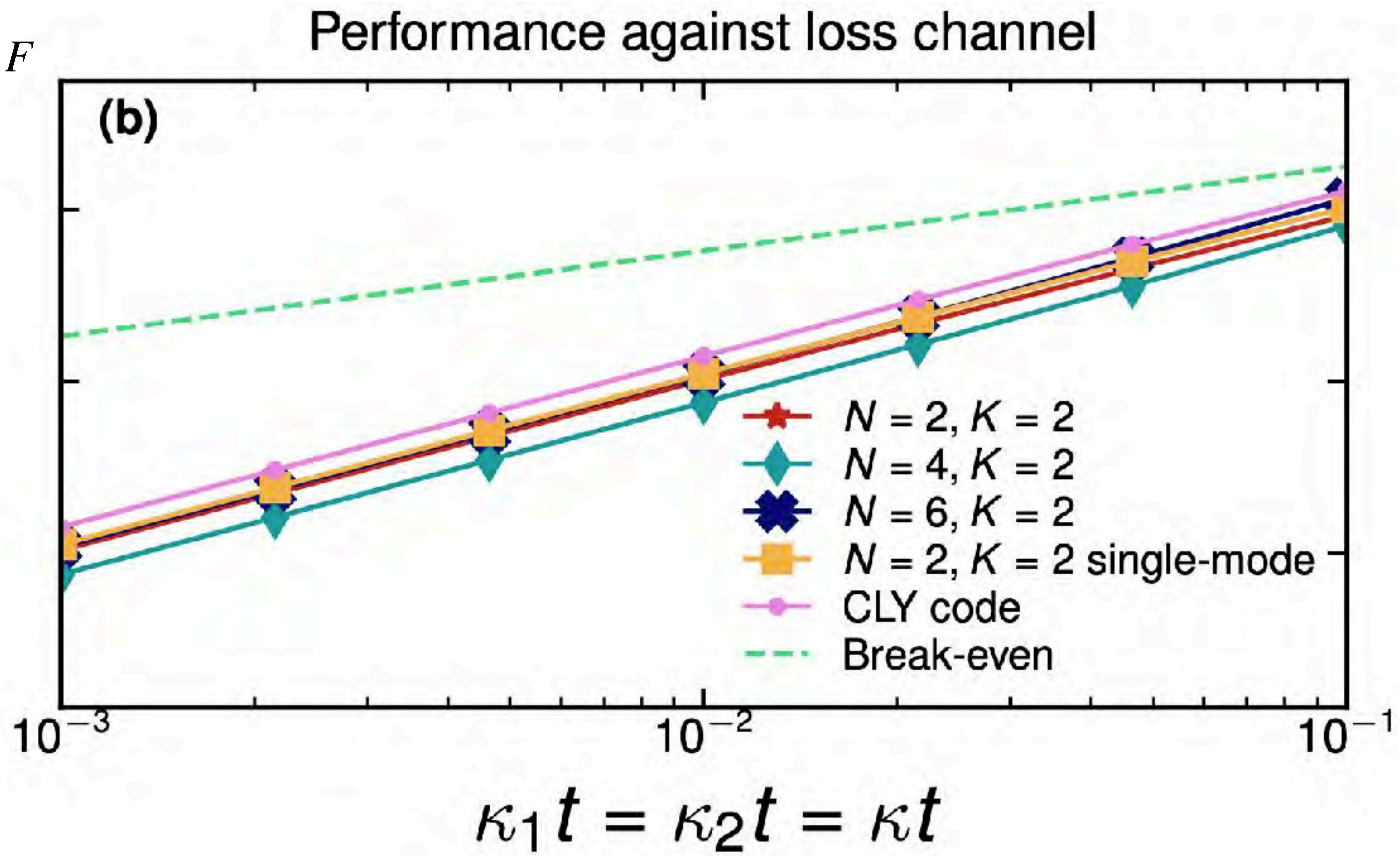


- For $\delta = \pi/4, \phi = \pi/(2N)$ performance against dephasing increases with increasing N!!!

R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)

Performance against losses

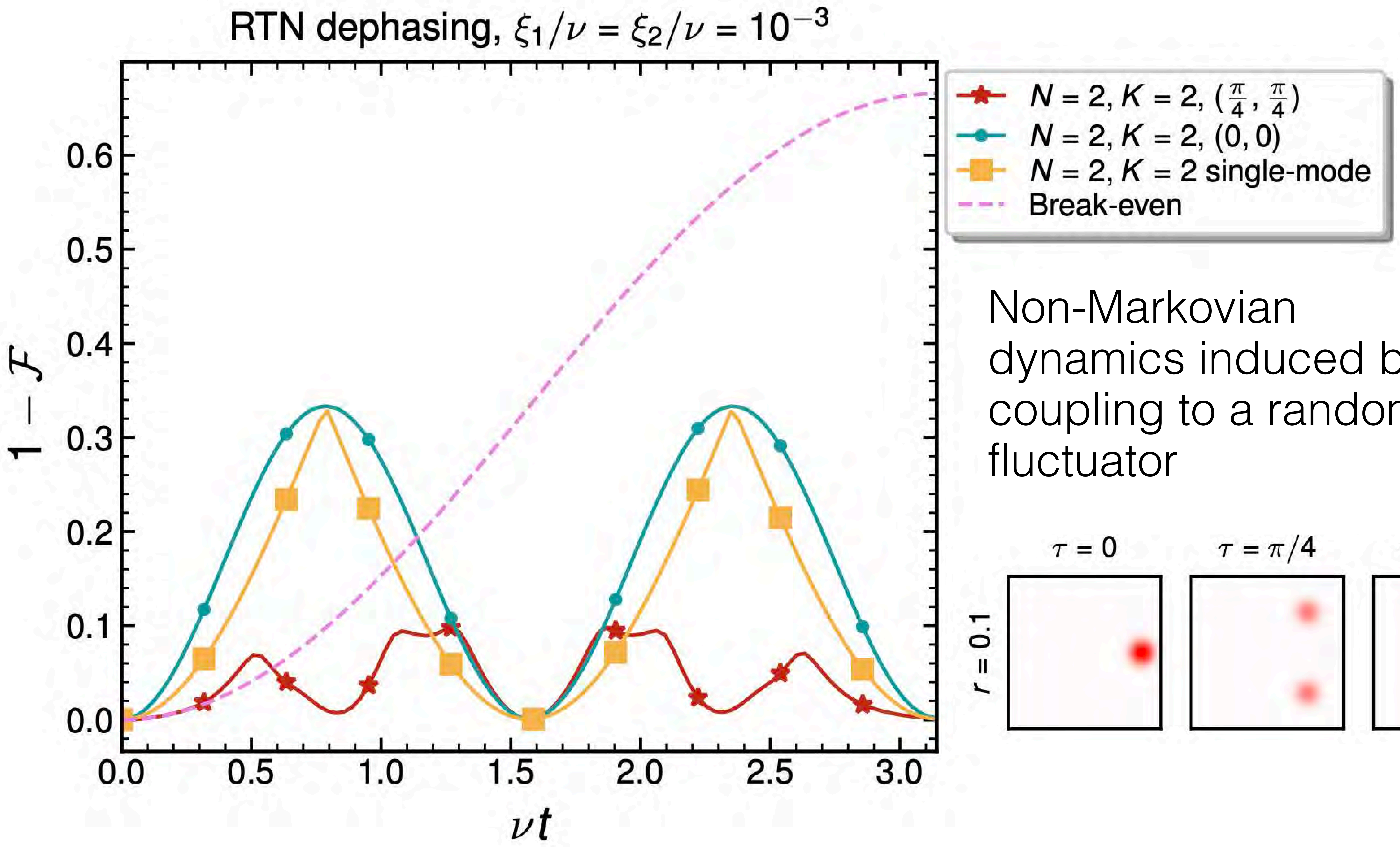
Similarity after QEC: F



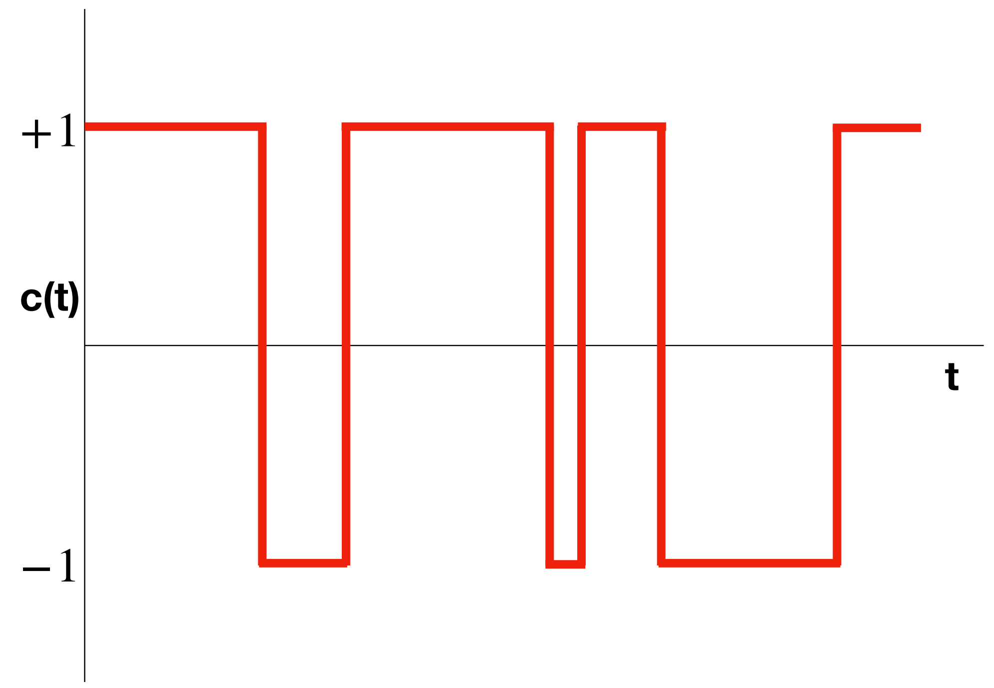
- For $\delta = \pi/4, \phi = \pi/(2N)$ the tradeoff in phase and loss sensitivity is mitigated!

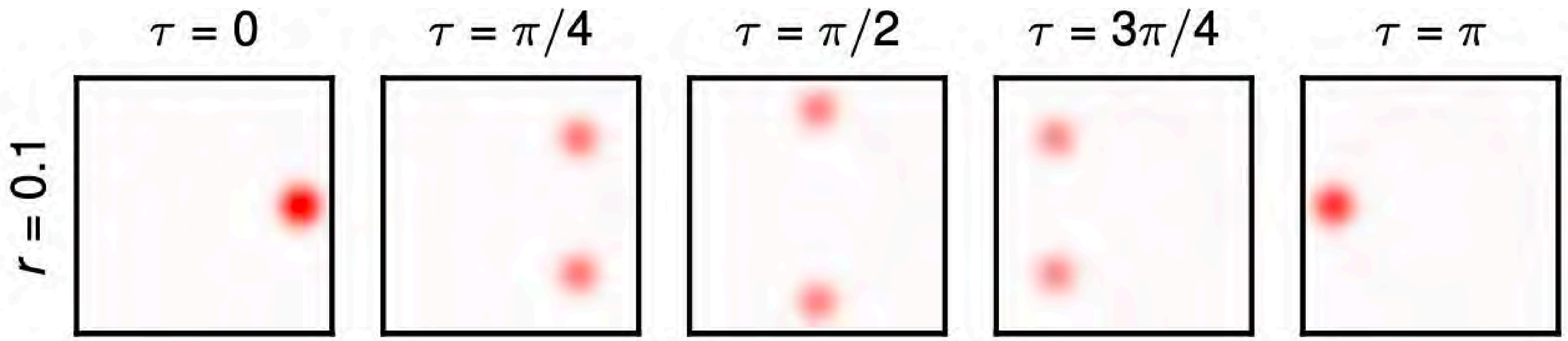
R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)

Performance against random telegraph noise



Non-Markovian dynamics induced by coupling to a random fluctuator





A. Udupa, T. Hillmann, R. G. Ahmed, A. Smirne and G. Ferrini, Phys. Rev. Research 8, 023007 (2026)

Part of a larger family

Most general codewords:

$$|0_L\rangle = U_{\mathrm{BS}} \sum_{mn} f_{mn} \, |2mN\rangle |(2n+1)N\rangle$$

$$|1_L\rangle = U_{\mathrm{BS}} \sum f_{mn} \, |(2n+1)N\rangle |2mN\rangle$$

- Have order *N* rotational symmetry in modes $U_{BS}^{\dagger} a_i U_{BS}$

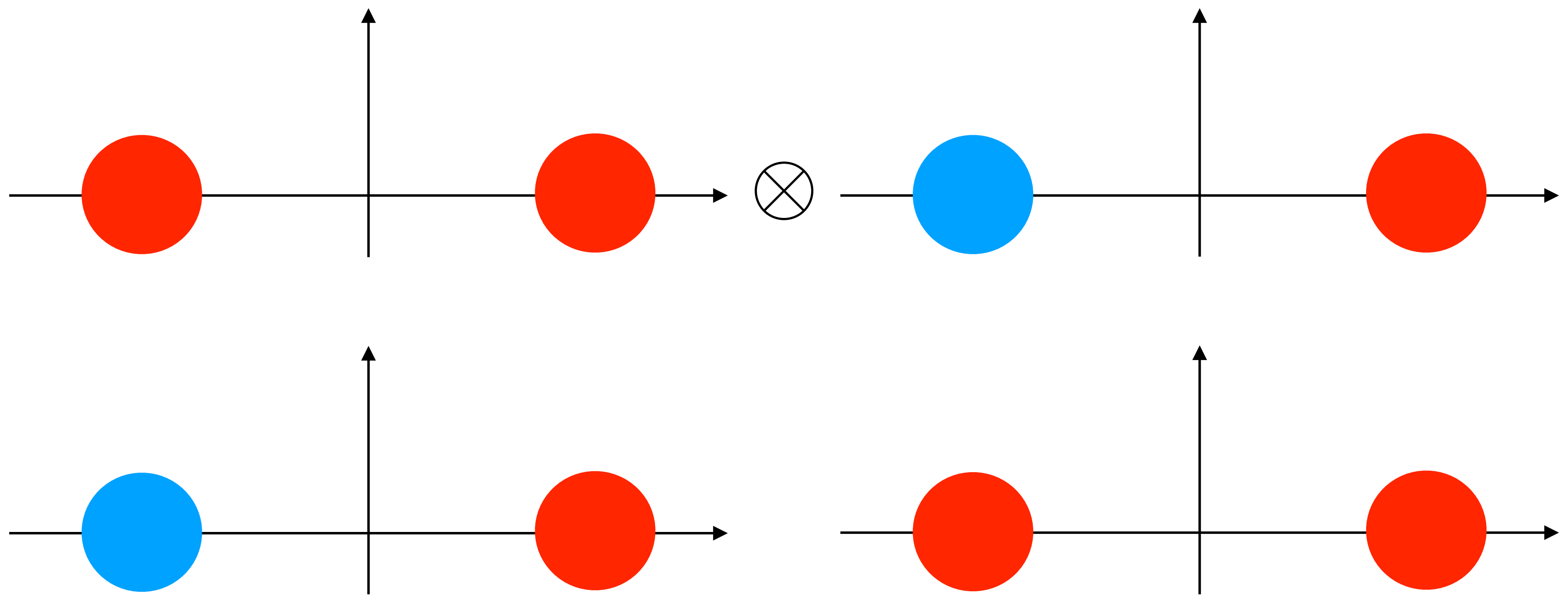
Generalization of RSB codes to two modes!

- For arbitrary even *N*, the Pauli group gates are implemented by linear optics

R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)

Dual rail cat code instance *N=1* (DRCC-1)

$$|0_L\rangle = U_{\mathrm{BS}}|C^+\rangle|C^-\rangle$$
$$|1_L\rangle = U_{\mathrm{BS}}|C^-\rangle|C^+\rangle$$



$$|C^{\pm}\rangle = \frac{1}{N_{\pm}}(|\alpha\rangle \pm |-\alpha\rangle)$$

$$N_{\pm} = \sqrt{2(1 \pm \exp(-2|\alpha|^2))}$$

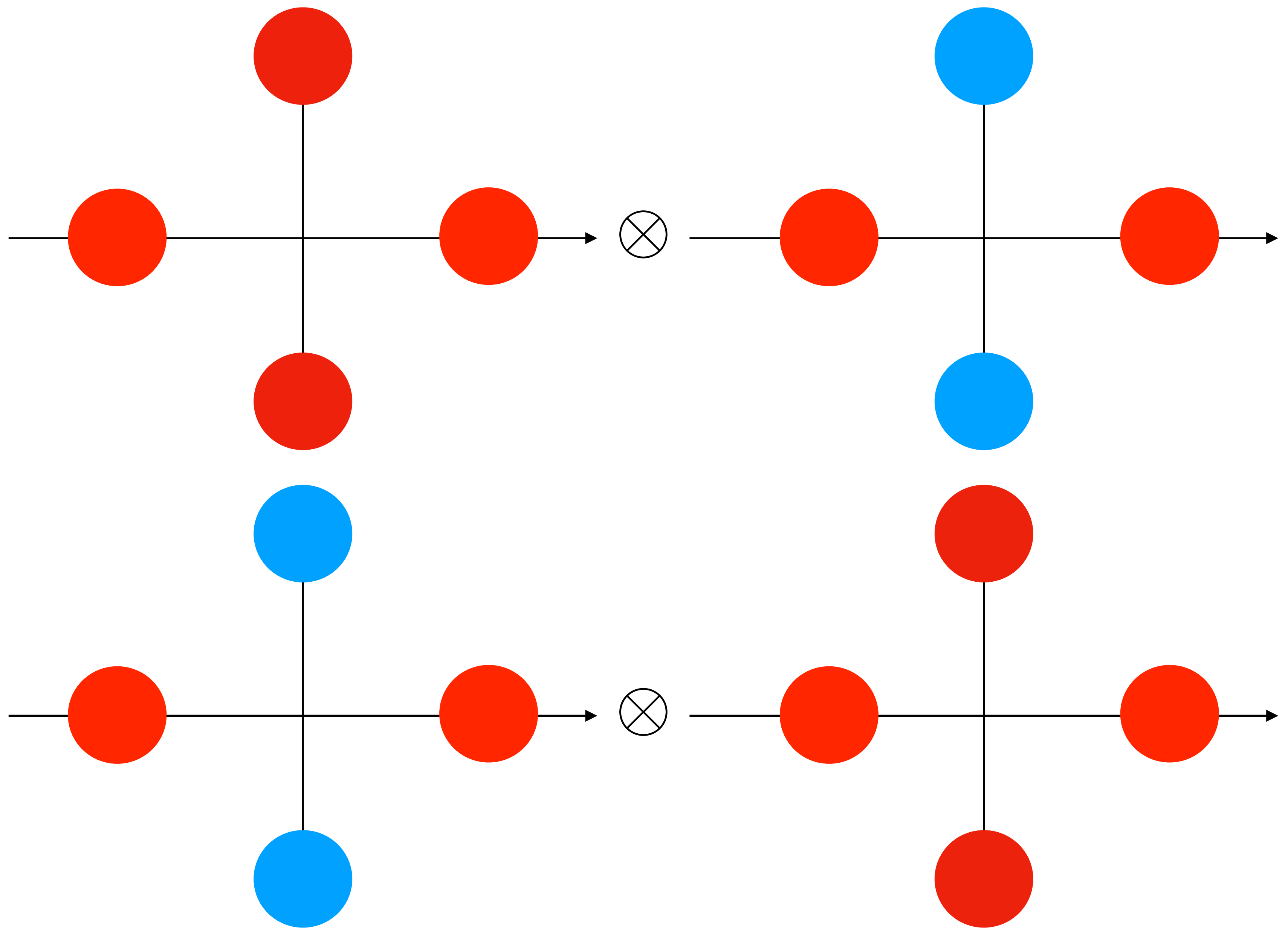
$$|0_{\text{dual rail}}\rangle = |01>$$
$$|1_{\text{dual rail}}\rangle = |10>$$

$$f_{mn} = e^{-2|\alpha|^2}\frac{\alpha^{2mN}\alpha^{(2n+1)N}}{\sqrt{((2n+1)N)!}\sqrt{(2mN)!}}$$

D. Biswas et al, in preparation

Dual rail cat code instance *N=2* (DRCC-2)

$$|0_L\rangle = U_{\mathrm{BS}}|C_4^+\rangle|C_4^-\rangle$$
$$|1_L\rangle = U_{\mathrm{BS}}|C_4^-\rangle|C_4^+\rangle$$



$$|C_4^{\pm}\rangle = \frac{1}{N_{4\pm}}(|\alpha\rangle + |-\alpha\rangle \pm (|i\alpha\rangle + |-i\alpha\rangle))$$

$$N_{4\pm} = \frac{1}{\sqrt{4+4e^{-2|\alpha|^2} \pm 8e^{-|\alpha|^2}\cos(|\alpha|^2)}}$$

$$f_{mn} = e^{-2|\alpha|^2}\frac{\alpha^{2mN}\alpha^{(2n+1)N}}{\sqrt{((2n+1)N)!}\sqrt{(2mN)!}}$$

D. Biswas et al, in preparation

Can encode d-level systems in d modes

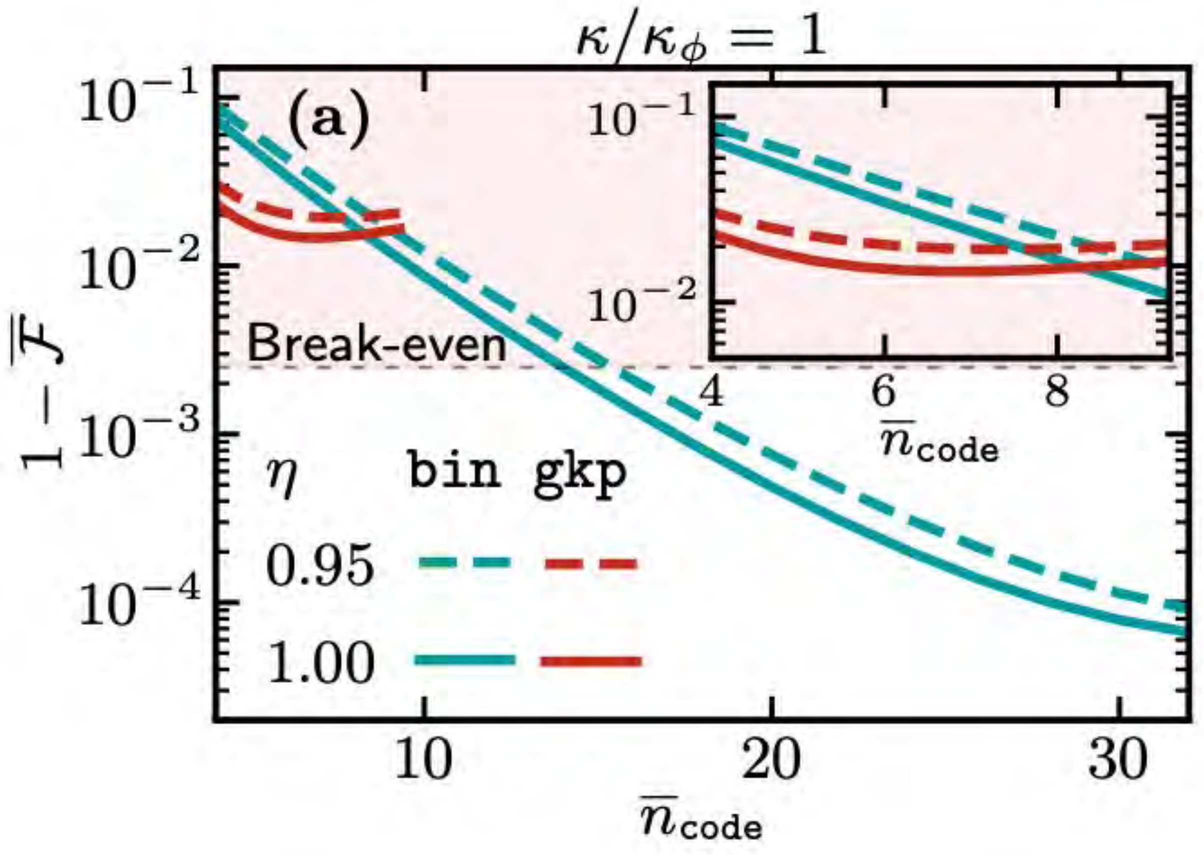
$$|k\rangle_L = \hat{U}_{BS} \sum_{\{n_i\}=0}^{\infty} f_{n_0 \dots n_{d-1}} \bigotimes_{j=k}^{k \oplus (d-1)} \big|(dn_j + j)N\big\rangle$$

- From same group-theoretic construction, Pauli gates still implemented by linear optics
- Still have *N*-fold discrete rotational symmetry
- Performance still to be analysed!

R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)

Summary and conclusions on part 2b

Conclusions

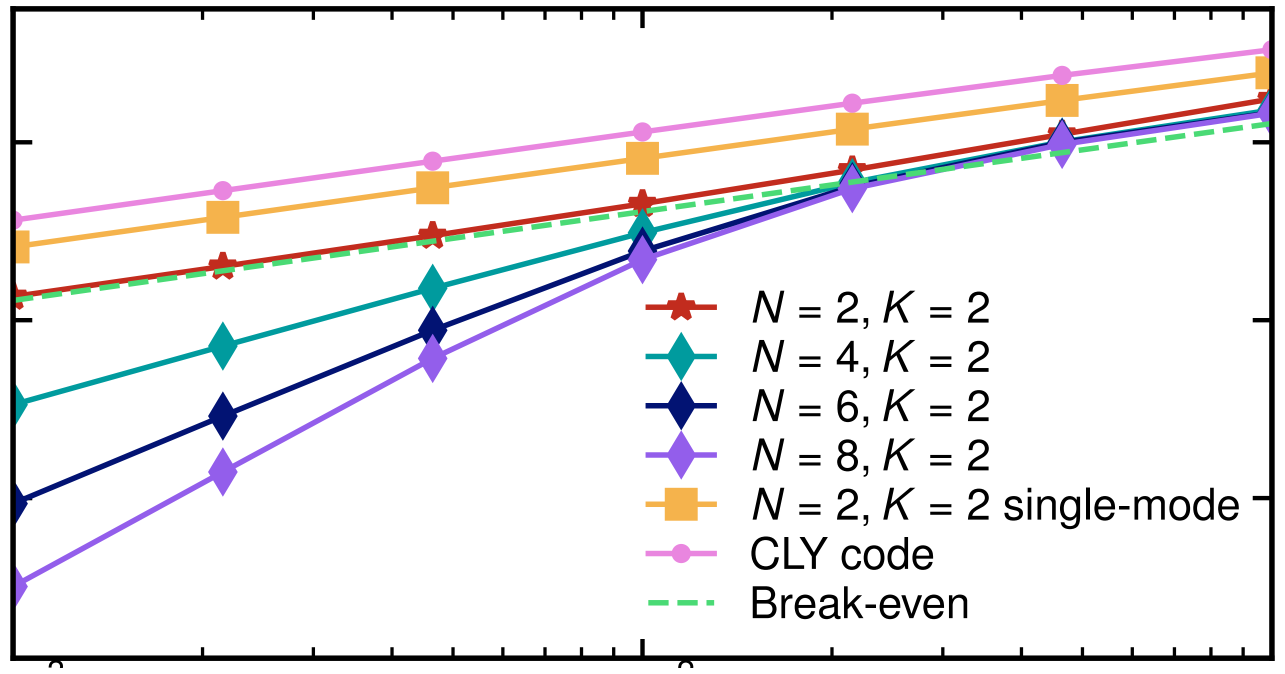


- Binomial code outperforms GKP codes under error model with
1. finite measurement efficiencies
2. number dephasing noise

T. Hillmann, F. Quijandría, A. L. Grimsmo, G. Ferrini, PRX Quantum 3, 020334 (2022)

- Beam-splitted two-mode RSB codes resolve the performance tradeoff between performance against losses and dephasing!

R. Galib Ahmed, A. Udupa, G. Ferrini, arXiv:2508.20647 (2025)
A. Udupa et al, Phys. Rev. Research 8, 023007 (2026)



- We generalized RSB codes to multi-mode based on group theory construction; many qubit and qudit codes to be explored!

Think about:

One concept that you have understood;

One concept that you have not understood;

2 minutes by yourself;

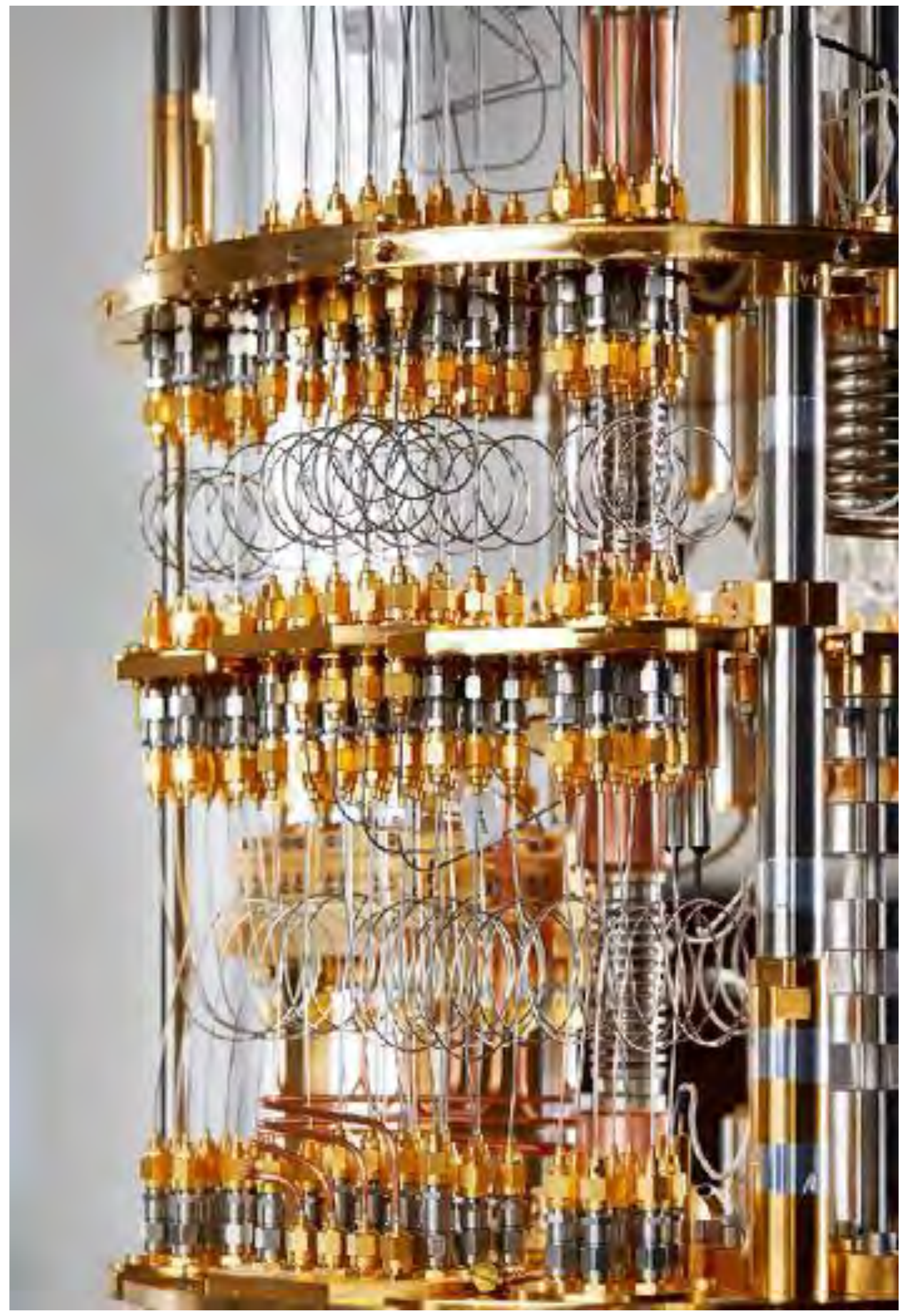
2 minutes with your neighbour;

5 minutes ask me questions!

Part 3a: Vacuum provides quantum advantage

Why classical simulation of quantum computers?

“Simulating”: computing or approximating the outcomes probability distribution

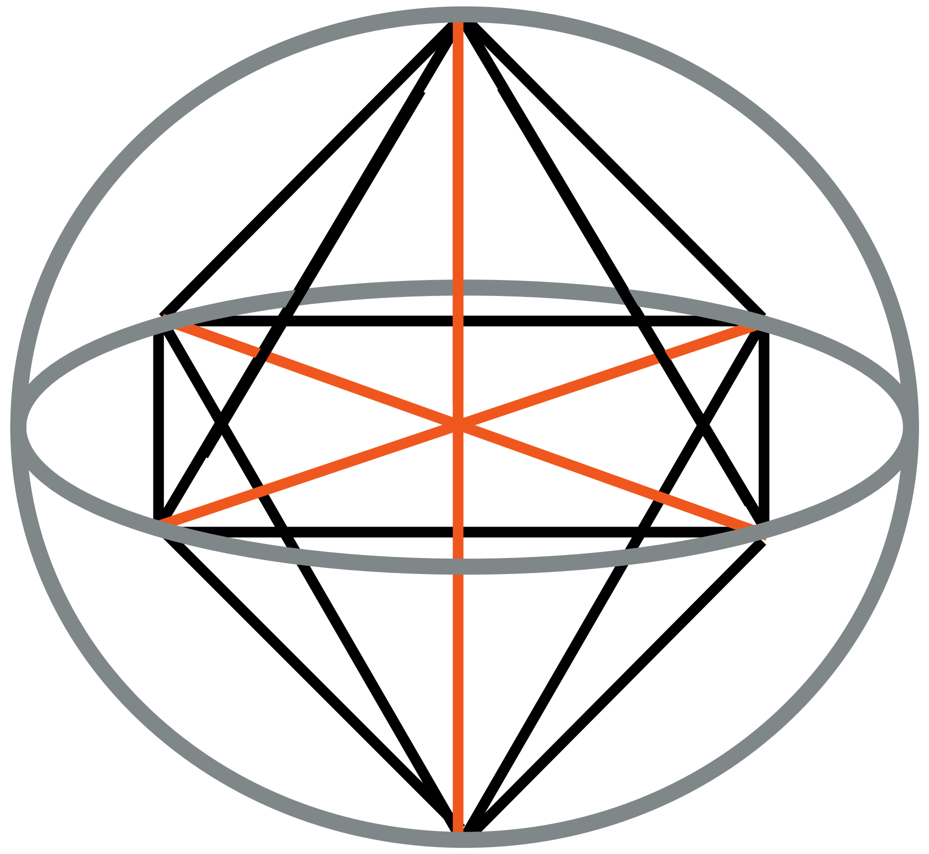


- Benchmark small-size quantum computers *Arute et al, Nature 574, 505 (2019)*
- Understanding which architectures are capable of providing quantum advantage

A restricted simulatable qubit architecture

For discrete variables, a QC based on:

(i) qubits initialised in “easy” states (“Pauli eigenstates”)
(ii) “easy” operations (“Clifford group”)
(iii) “easy” measurements (computational basis)



can be simulated efficiently with a classical computer

Gottesman-Knill theorem

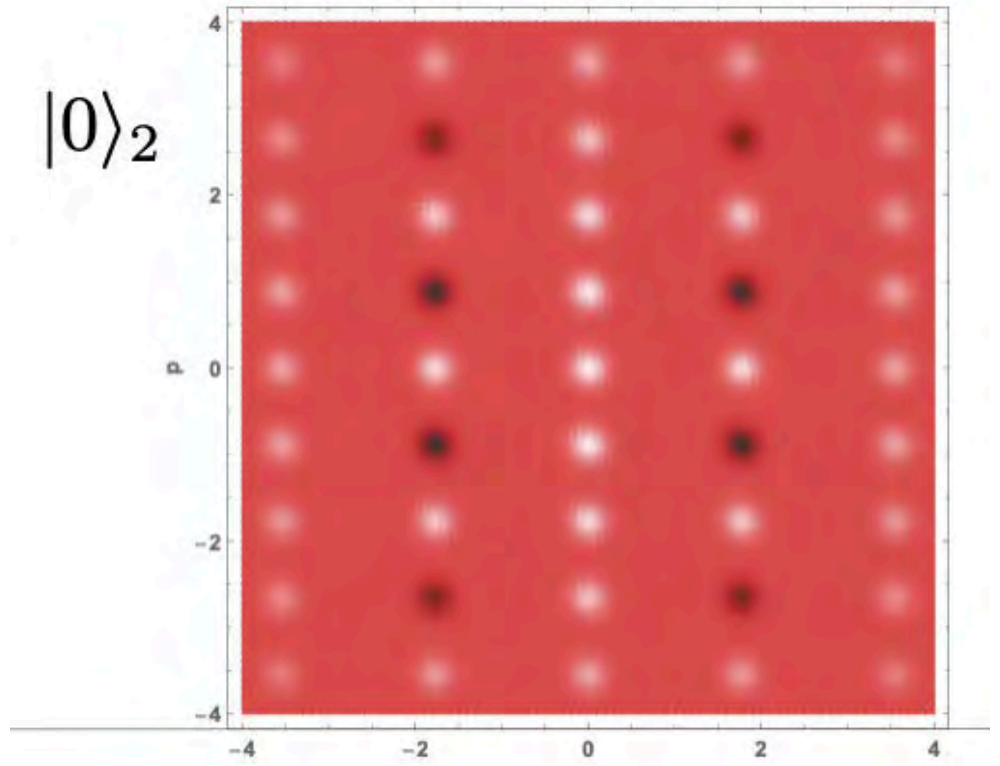
- No exponential quantum advantage with Clifford quantum computation!

What about the simulation of CV system?

We have seen that:

- Wigner negativity is a necessary condition in a bosonic quantum circuit for quantum advantage

- GKP states are highly (infinitely) Wigner-negative

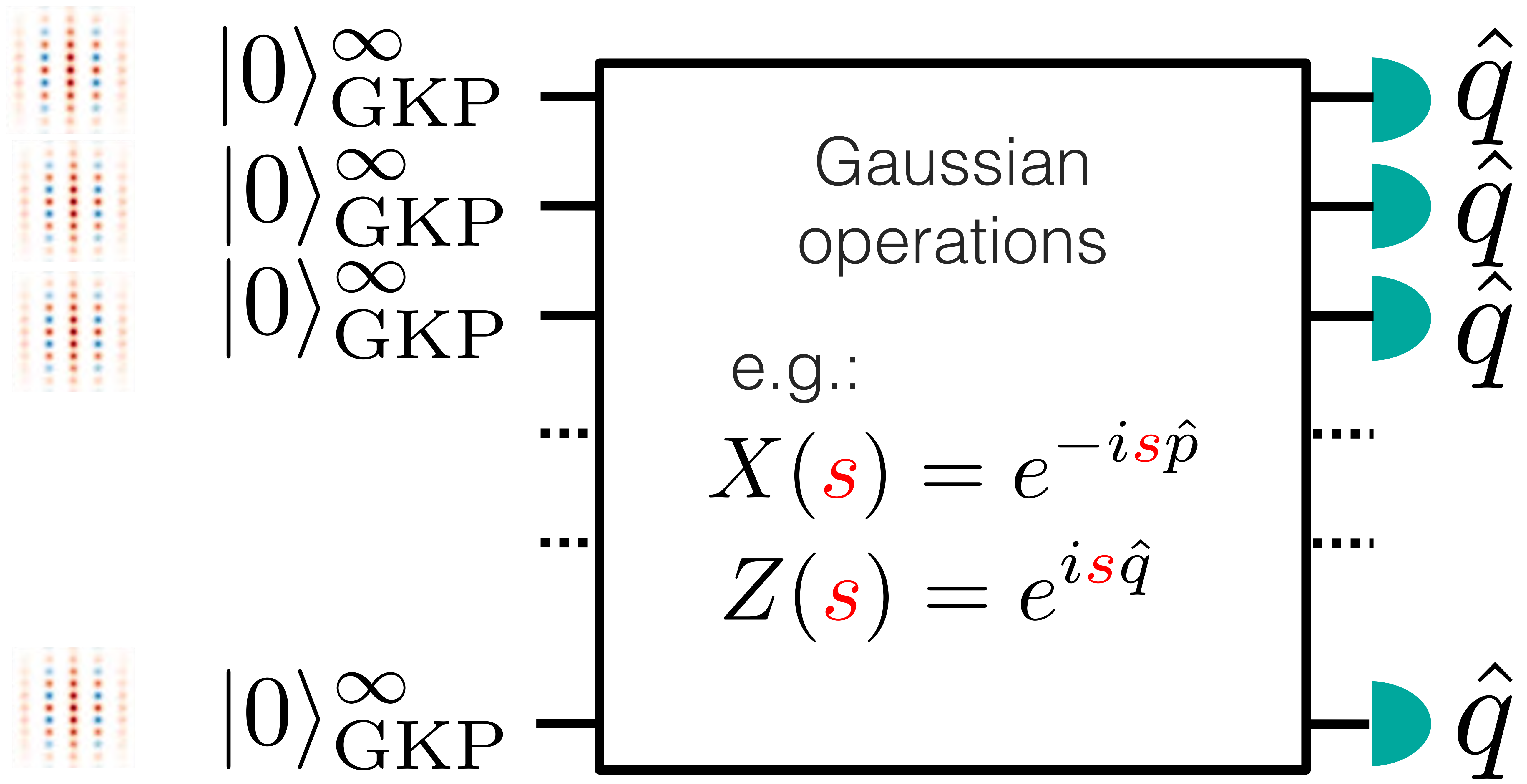


Are Gaussian circuits with input GKP states classically efficiently simulatable, or do they provide quantum advantage?

Simulatability of GKP circuits / vacuum provides quantum advantage

GKP states with Gaussian circuits

First we look at infinite squeezing case: $|0\rangle_{\mathrm{GKP}}^{\infty}$



Are these circuits universal or classically simulatable?

Simulatable = we can compute in polytime the probability density function (PDF)

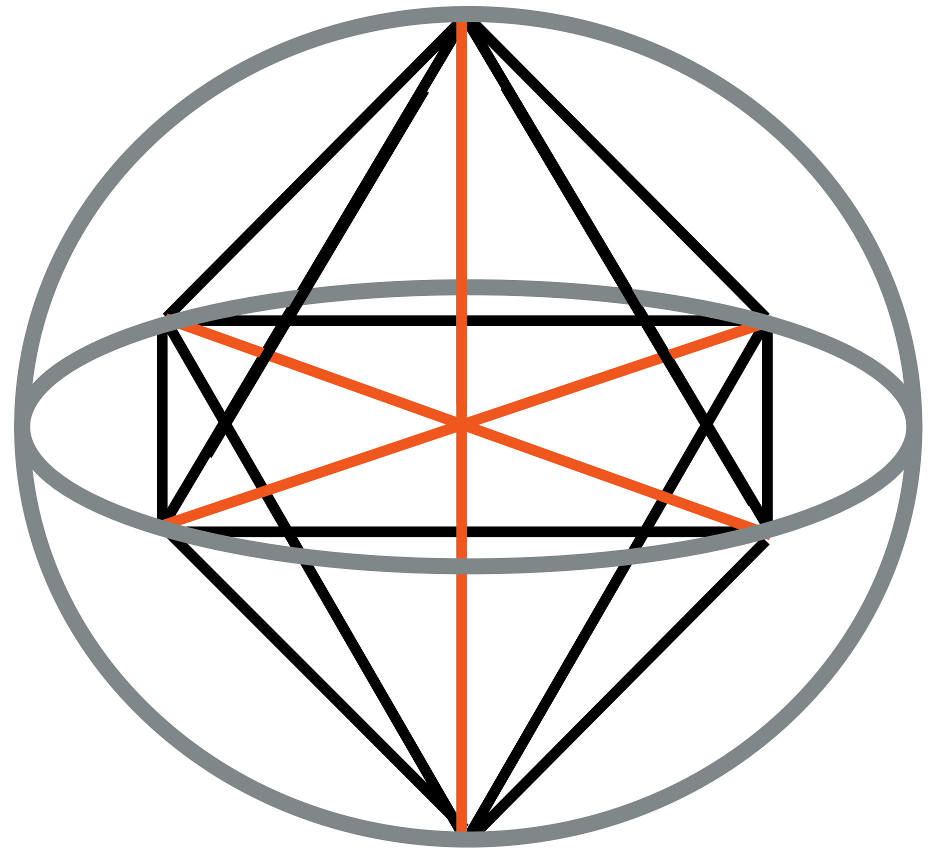
$$\mathrm{PDF} = \left| \prod_j \langle \hat{q}_j = x_j | \hat{U} | 0 \rangle_{\mathrm{GKP}}^{n} \right|^2$$

Gottesman-Knill theorem (DV-analog of Mari-Veitch)

A QC based only on:

(i) qubits initialised in a Pauli eigenstate
(ii) Clifford group operations
(iii) Pauli measurements

can be simulated efficiently with a classical computer



$$\mathcal{C}_2^n = \langle H, S, \mathrm{CNOT} \rangle \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

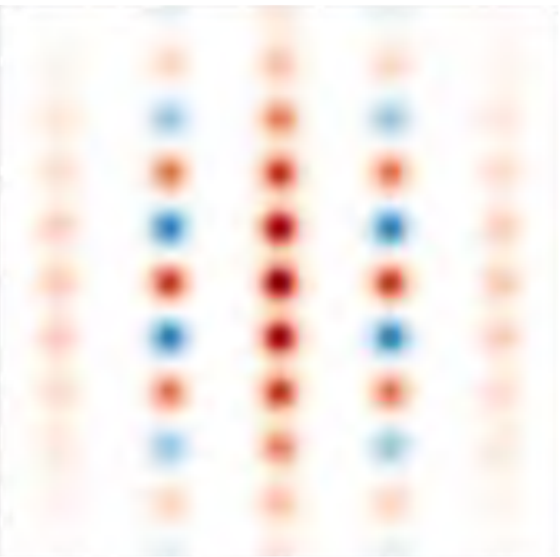
Includes Pauli matrices X, Y, Z

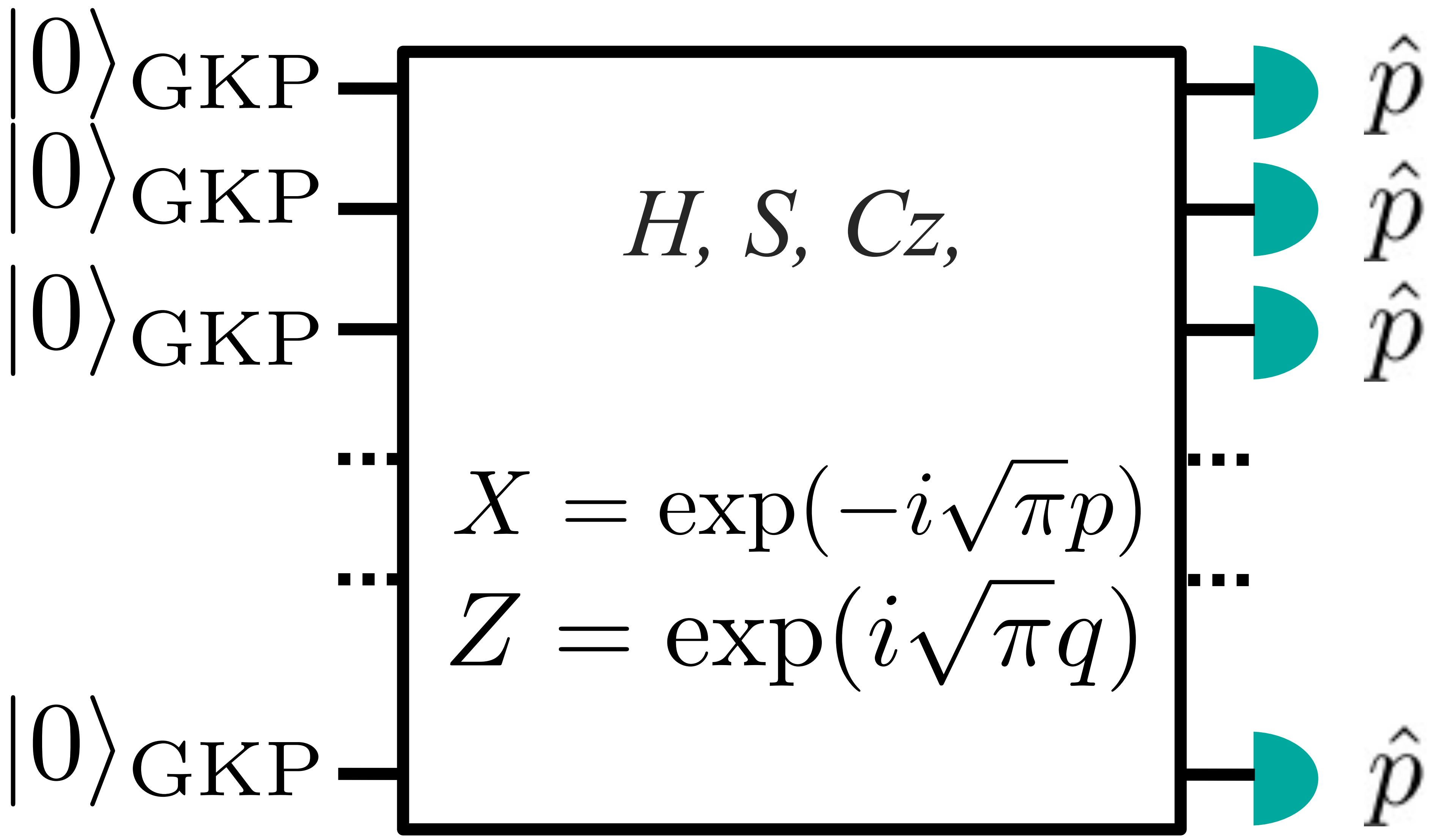
- A trivial example of a circuit with high Wigner negativity and classically efficiently simulatable is a GKP-encoded Clifford circuit!

Wigner function is not sufficient for quantum advantage

Construct a trivial example: encode Clifford operations with GKP states

Pauli-eigenstate GKP states





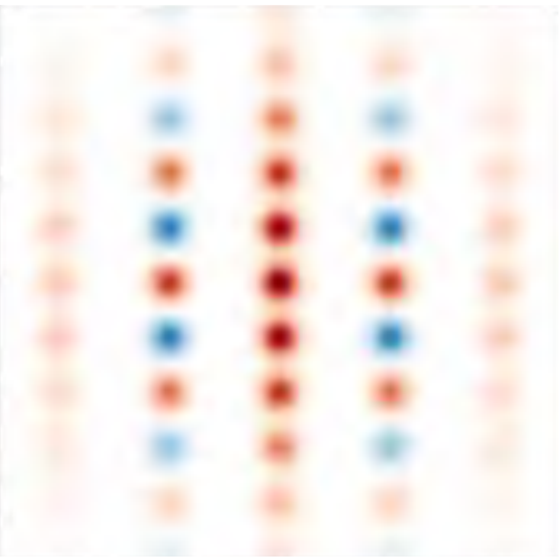
Pauli measurement

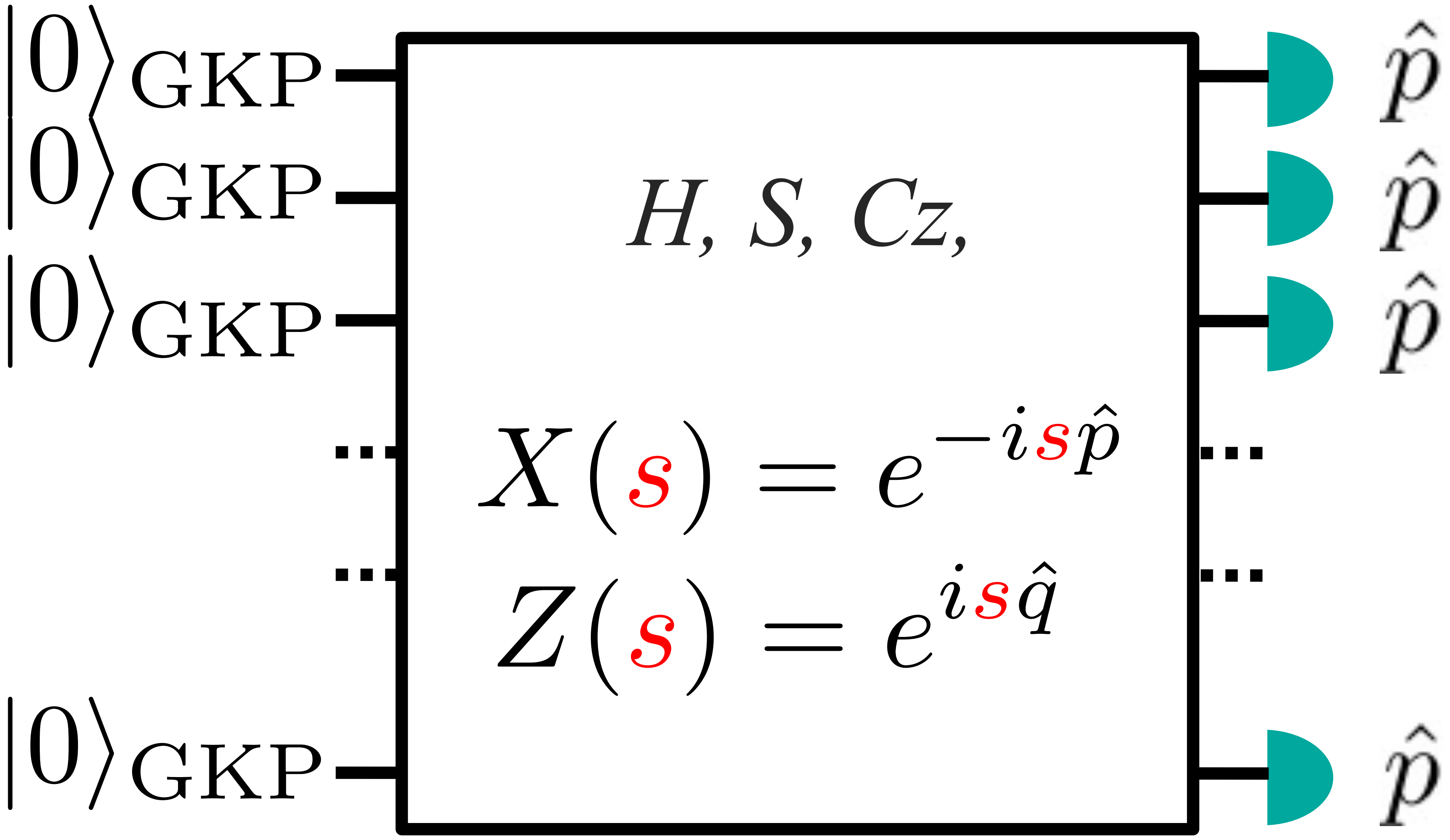
Encoded Clifford operations

This circuit is simulatable due to the Gottesman-Knill theorem for qubits

Is this circuit simulatable or hard to sample?

Pauli-eigenstate GKP states





$\hat{F} = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}$, $\hat{C}_Z = e^{i\hat{q}_1\hat{q}_2}$

$S = e^{i\hat{q}^2/2}$, $\hat{Z}(s) = e^{i\hat{q}s}$

Pauli measurement

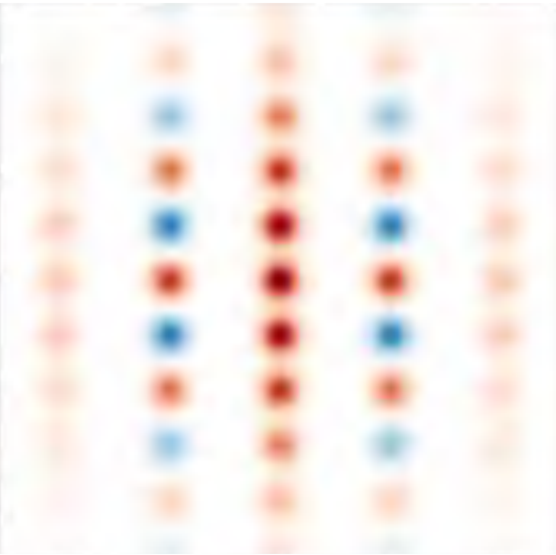
Displacements do not correspond to encoded Clifford operations

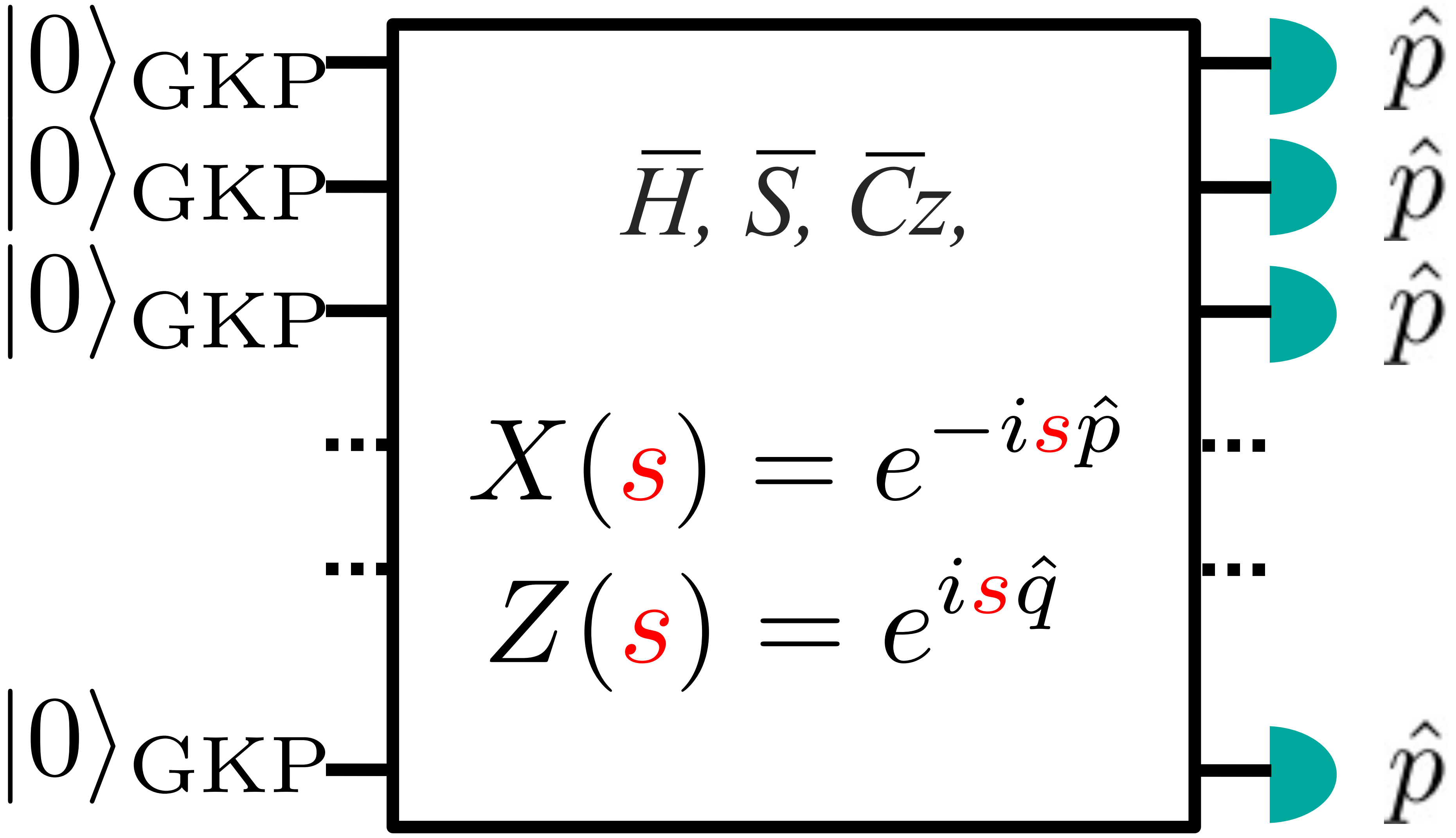
- Gottesman-Knill th. not applicable because operations are outside qubit space
- Mari-Veitch theorem not applicable because Wigner function is negative

A non-trivial class of circuits

We show that these circuits are classically efficiently simulatable for displacements that are an arbitrary fraction of the ones that represent encoded qubit operations

Pauli-eigenstate GKP states





$\bar{F} = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}$, $\bar{C}_Z = e^{i\hat{q}_1\hat{q}_2}$

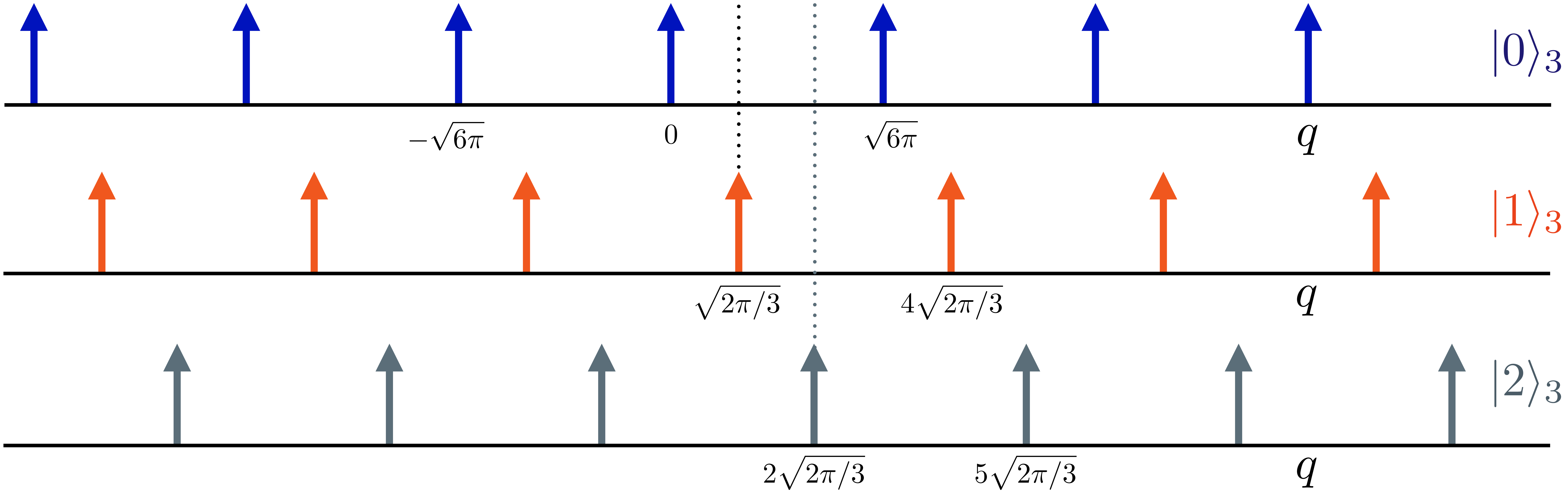
$\bar{S} = e^{i\hat{q}^2/2}$

$\bar{Z}(s) = e^{i\hat{q}s}$

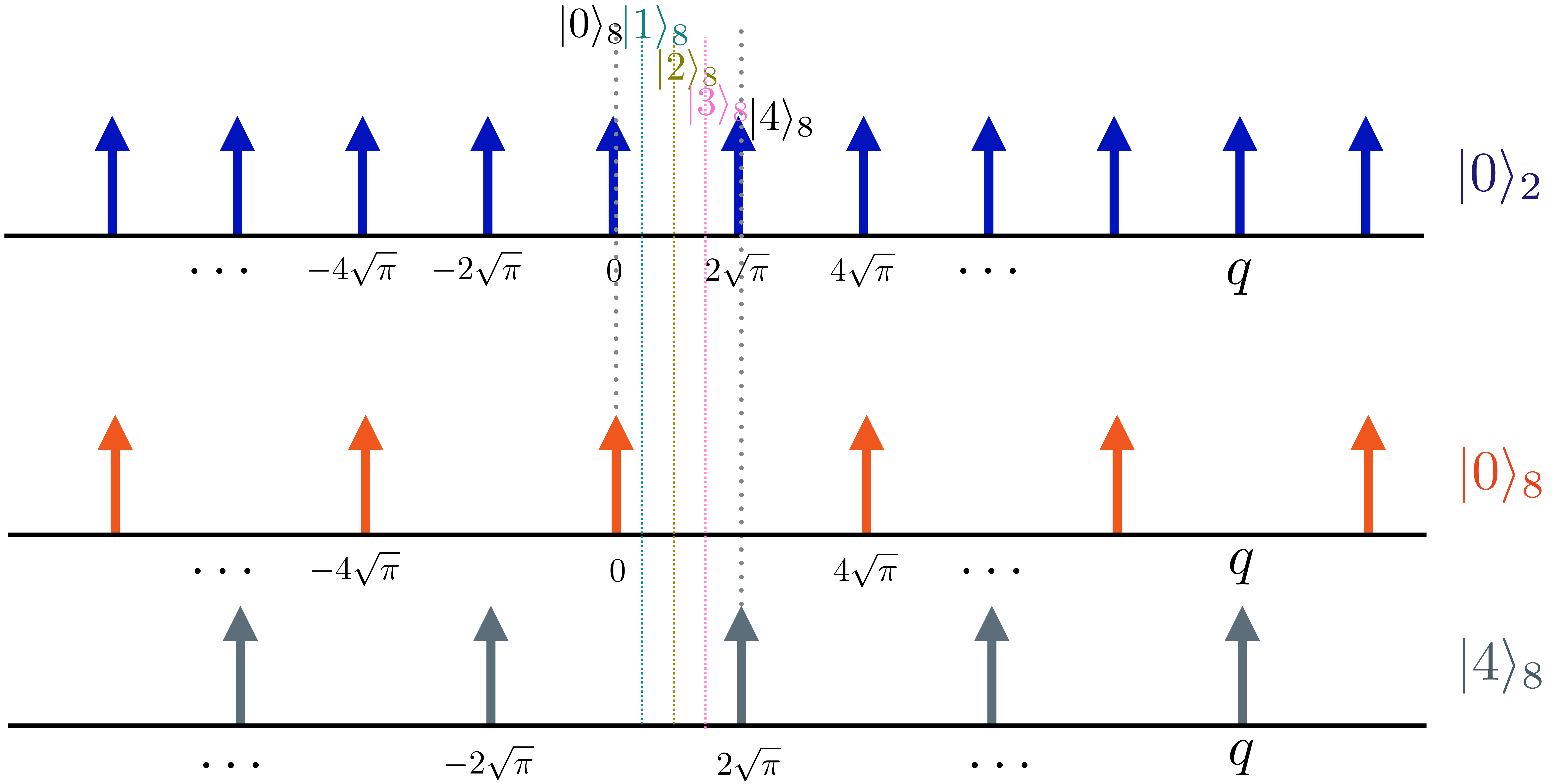
$$s = \sqrt{\pi}\frac{k}{a}$$

L. García-Álvarez, C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. Research 2, 043322 (2020)

Our idea: interpret displacements outside the qubit subspace as Clifford displacements onto a higher-dimensional GKP code space



Identifying a GKP qubit with a GKP qudit



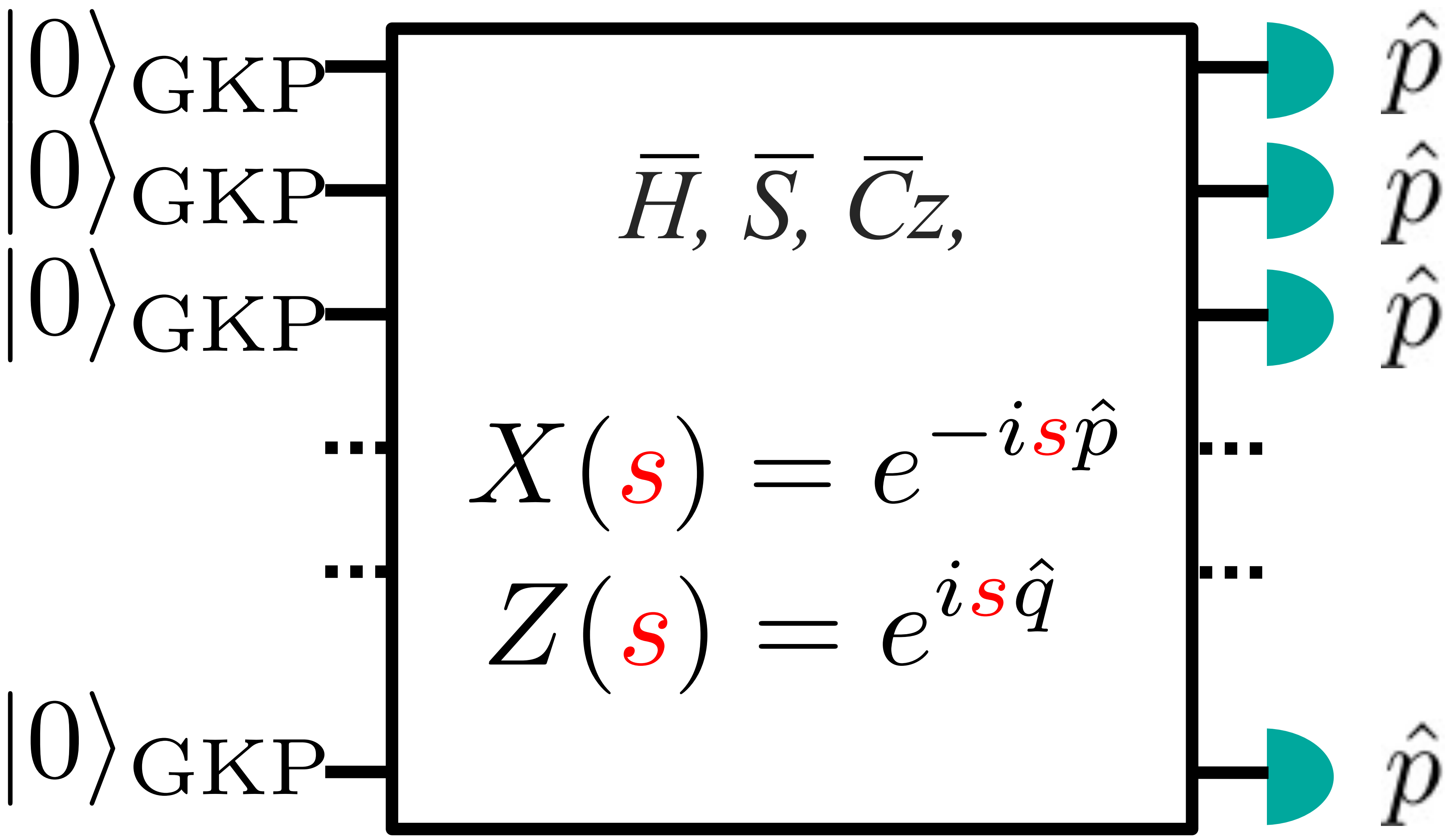
$$|0\rangle_2 = |0\rangle_8 + |4\rangle_8$$

Works for any qudit dimension $d = 2a^2 \quad a \in \mathcal{N}$

Arbitrarily small displacements can be interpreted as Clifford operations on qudits

Result

Input GKP states



$$H = \exp(i\frac{\pi}{2}(q^2 + p^2))$$

$$S = \exp\left(iq^2/2\right)$$

$$C_Z = \exp\left(iq_1 q_2\right)$$

$$X(s) = e^{-is\hat{p}} \qquad Z(s) = e^{is\hat{q}}$$

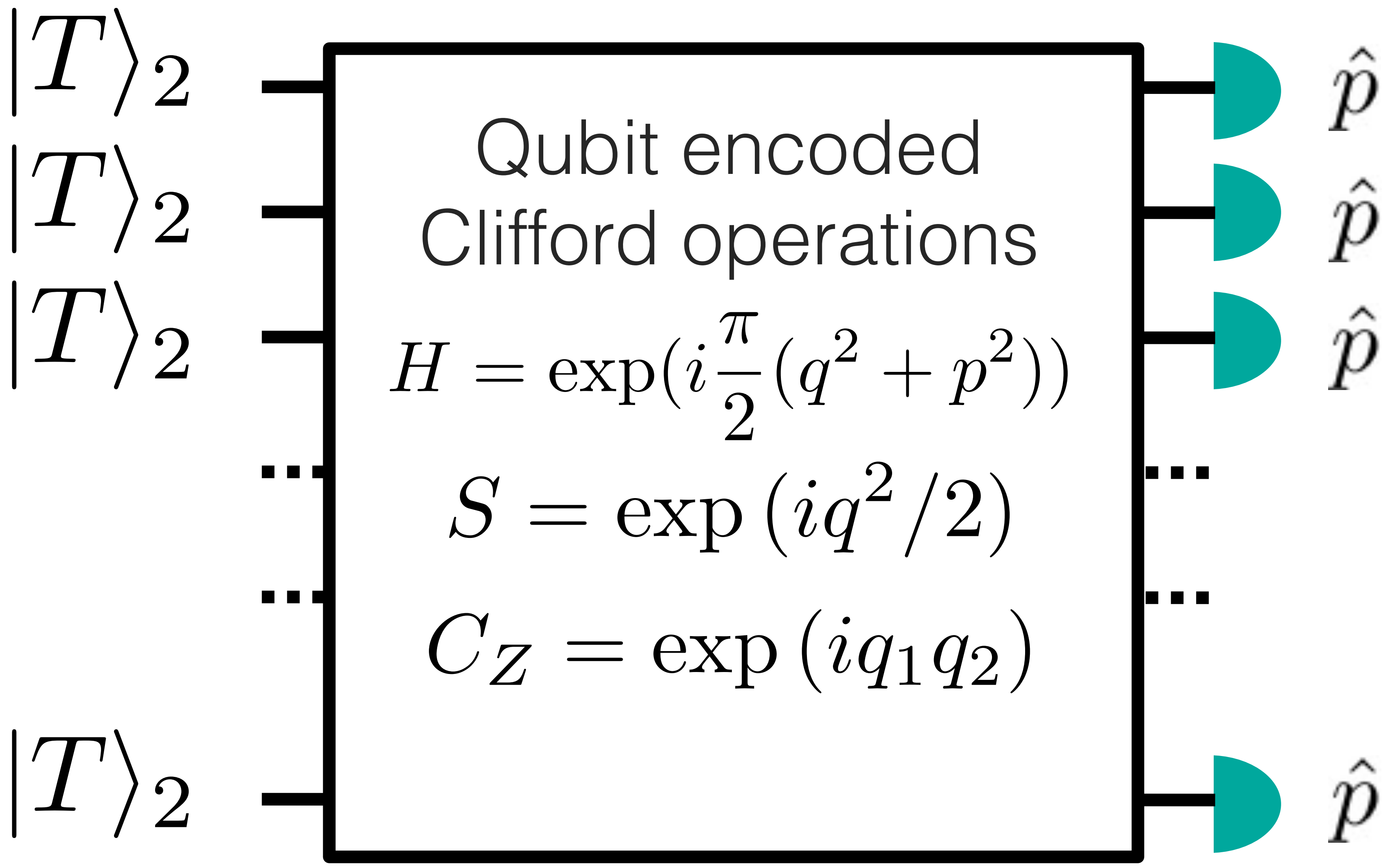
$$s = \sqrt{\pi}\frac{k}{n}$$

These circuits are classically efficiently simulatable due to the extensions of the Gottesman-Knill theorem to qudits *N. de Beaudrap, Quantum Information & Computation 13, 73 (2013)*

L. García-Álvarez, C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. Research 2, 043322 (2020)

Consequences

- It doesn't mean that fault-tolerant QC with GKP states is simulatable: the data qubit is most probably not in a stabiliser state
- You can still do quantum-primacy type of experiments by injecting GKP magic states



"Quantum advantage of unitary Clifford circuits with magic state inputs"

M. Yoganathan, R. Jozsa, and S. Strelchuk, Proc. R. Soc. A 475 (2019)

$$|T\rangle_2 = |0\rangle_2 + e^{i\frac{\pi}{4}}|1\rangle_2$$

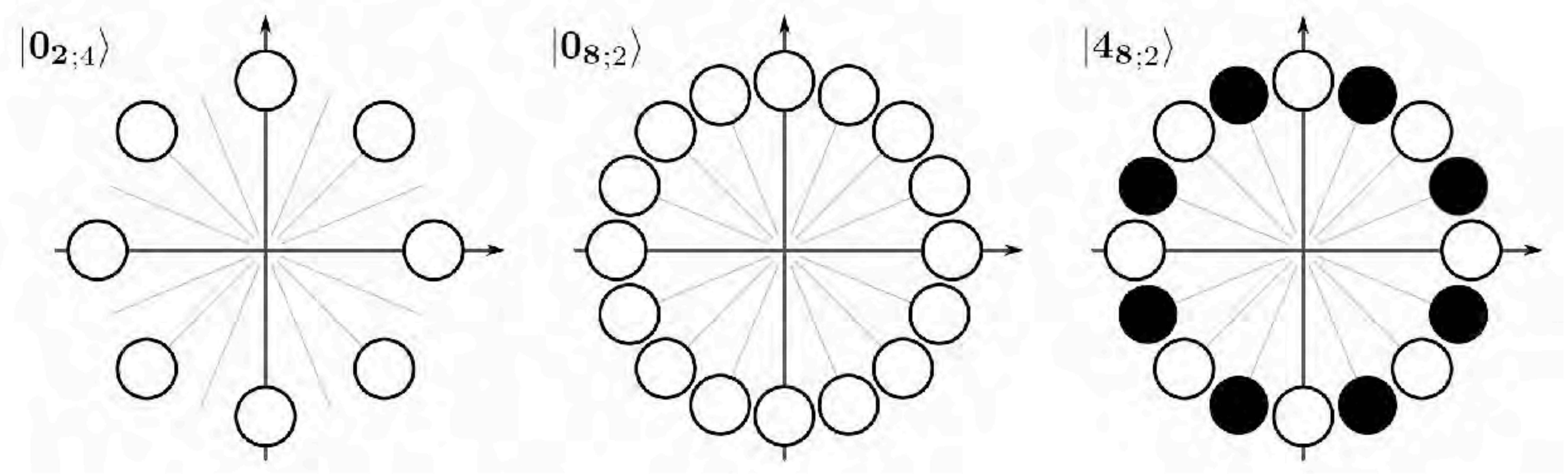
L. García-Álvarez, C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. Research 2, 043322 (2020)

RSB codes

A. Grimsmo, J. Combes, B. Baragiola, Phys. Rev. X 10, 011058 (2020)

$$|j_{2;N,\varphi}\rangle = \frac{1}{\sqrt{N_j}} \sum_{m=0}^{2N-1} (-1)^{jm} e^{i\frac{m\pi}{N}\hat{n}} |\varphi\rangle$$

$$\hat{R}_N = e^{i\frac{2\pi}{N}\hat{n}}$$

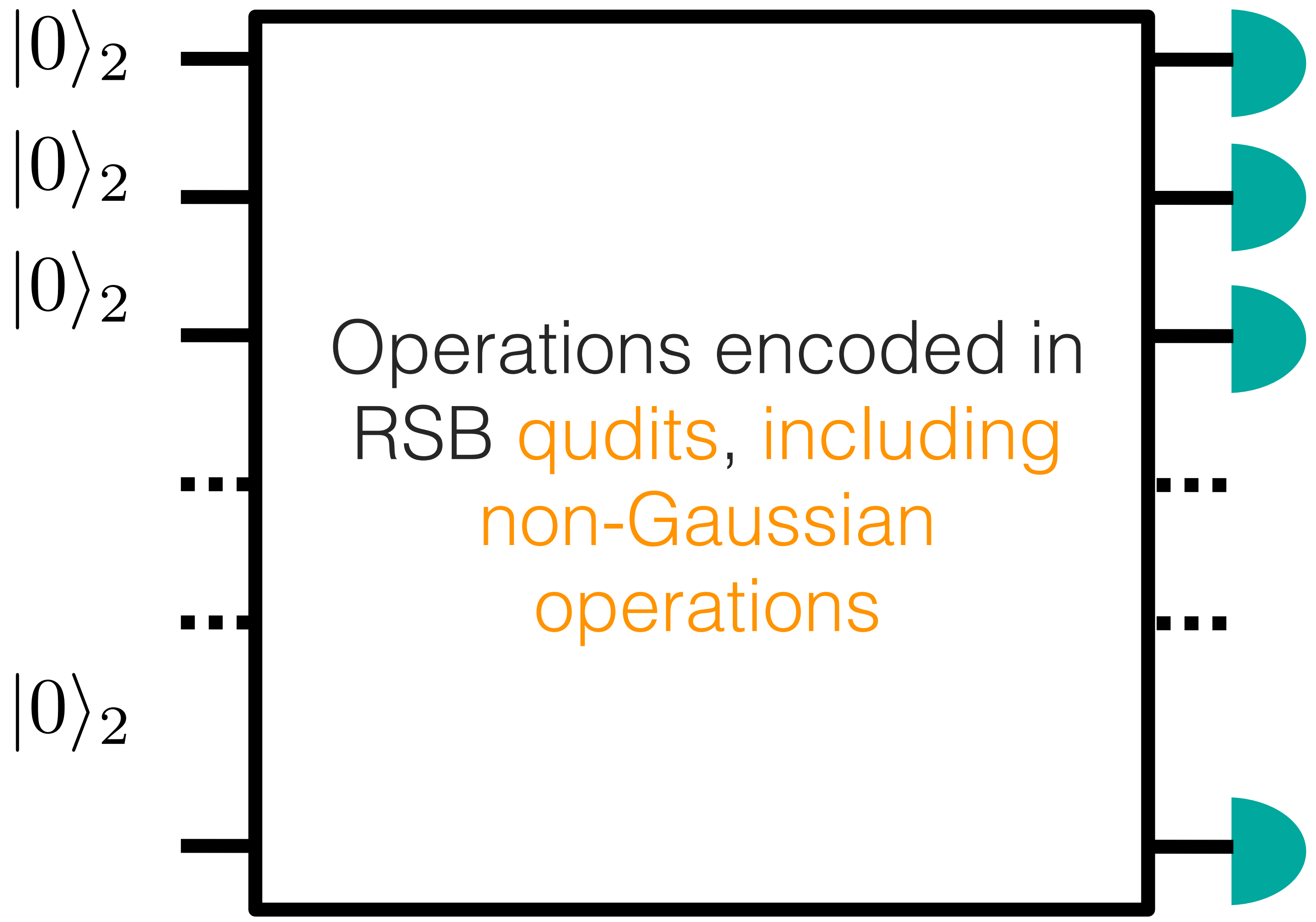


We can identify a RSB qubit with a RSB qudit with different rotational symmetry

L. García-Álvarez, C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. Research 2, 043322 (2020)

Input RSB states

$$|j_{2;N,\varphi}\rangle = \frac{1}{\sqrt{\mathcal{N}_j}} \sum_{m=0}^{2N-1} (-1)^{jm} e^{i\frac{m\pi}{N}\hat{n}} |\varphi\rangle$$



Phase measurement

$$\{e^{i\frac{2\pi}{ad_1 N}\hat{n}}, e^{i\frac{\pi}{d_1}\left(\frac{\hat{n}^2}{N^2} - \beta\frac{\hat{n}}{aN}\right)}, e^{i\frac{2\pi}{d_1 N^2}\hat{n}_k \hat{n}_l}, \mathbf{PM}\}$$

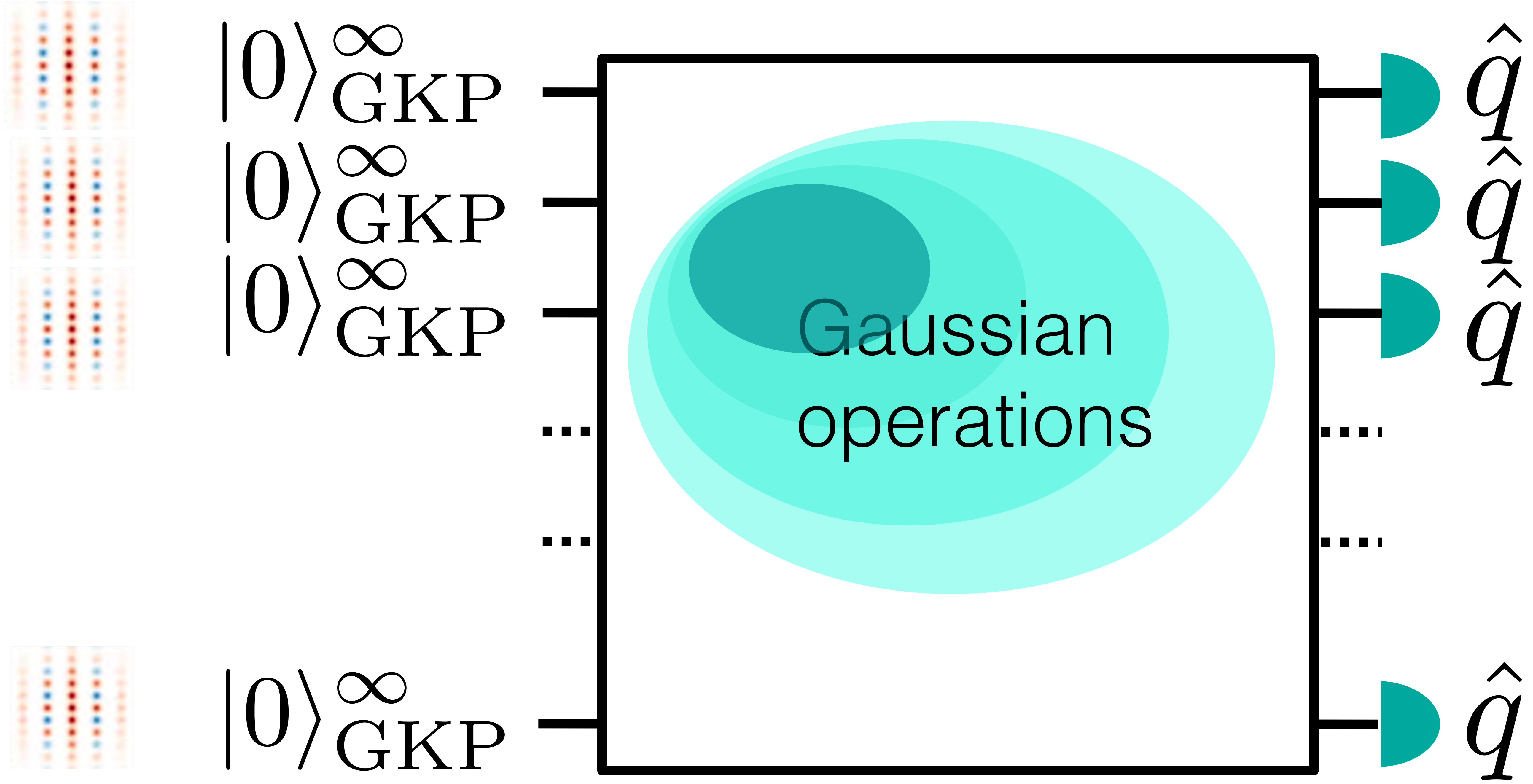
These circuits are classically efficiently simulatable due to the extensions of the Gottesman-Knill theorem to qudits *N. de Beaudrap, Quantum Information & Computation 13, 73 (2013)*

L. García-Álvarez, C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. Research 2, 043322 (2020)

Generalizing the simulatability proof to arbitrary Gaussian operation

History of simulatability proofs

First we look at infinite squeezing case: $|0\rangle_{\mathrm{GKP}}^{\infty}$



Simulatable architecture: SGKP circuits

1. Encoded Clifford in dimension *d* *L. García-Álvarez, C. Calcluth, A. Ferraro and G. Ferrini, PRR 2, 043322 (2020)*
2. Larger class defined by cumbersome condition *C. Calcluth, A. Ferraro, G. Ferrini, Quantum 6, 867 (2022)*
3. "Almost all" Gaussian operations *C. Calcluth, A. Ferraro, G. Ferrini, Phys. Rev. A 107, 062414 (2023)*
4. All Gaussian operations (weak simulation) *O. Hahn, G. Ferrini, R. Takagi, Phys. Rev. X Quantum 6, 010330 (2025)*

What are the resource states for this model?

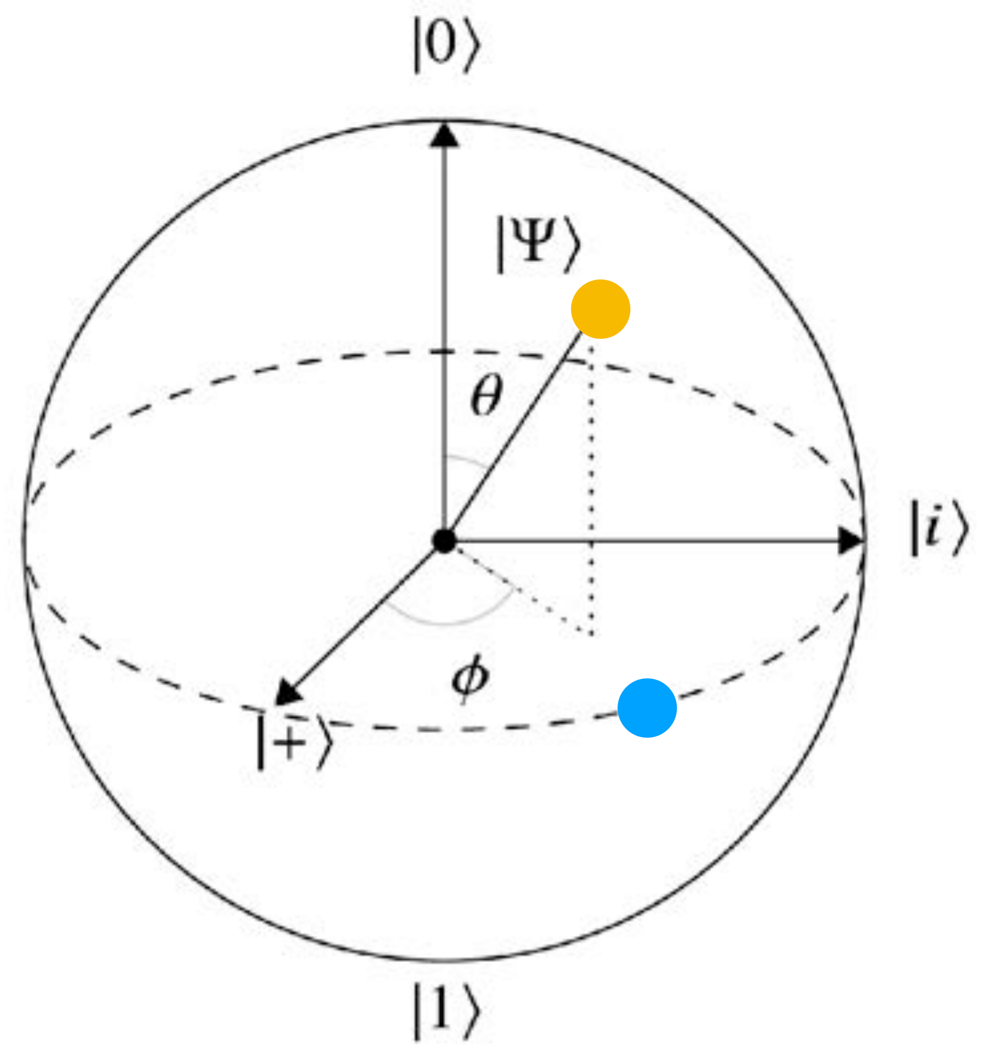
Resource states

- States that when added to a simulatable architecture make the QC universal are called resource states

Simulatable architecture + Resource state $|\psi\rangle$ = Universal QC

Example: Magic states unlock universality in Clifford QC

Clifford quantum computation + magic state $|H\rangle$ = Universal QC

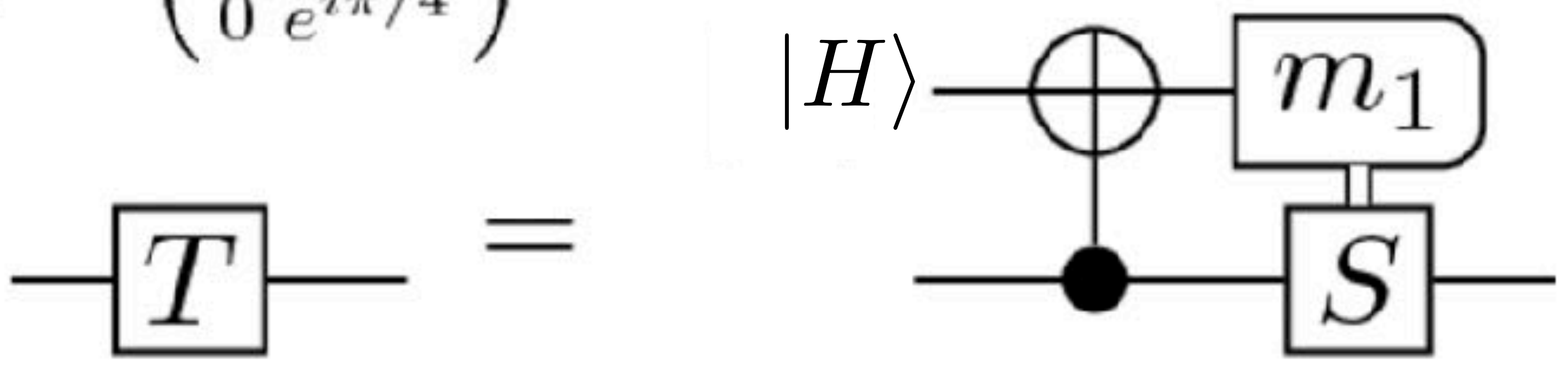


$$|T\rangle = \cos\tfrac{\theta}{2}|0\rangle + \sin\tfrac{\theta}{2}e^{i\frac{\pi}{4}}|1\rangle \quad \text{with} \quad \theta = \arccos\left(\tfrac{1}{\sqrt{3}}\right)$$

$$|H\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle\right),$$

- From magic states we obtain the missing gate ("T-gate")

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$|H\rangle$ states (+Cliffords) enable T gates

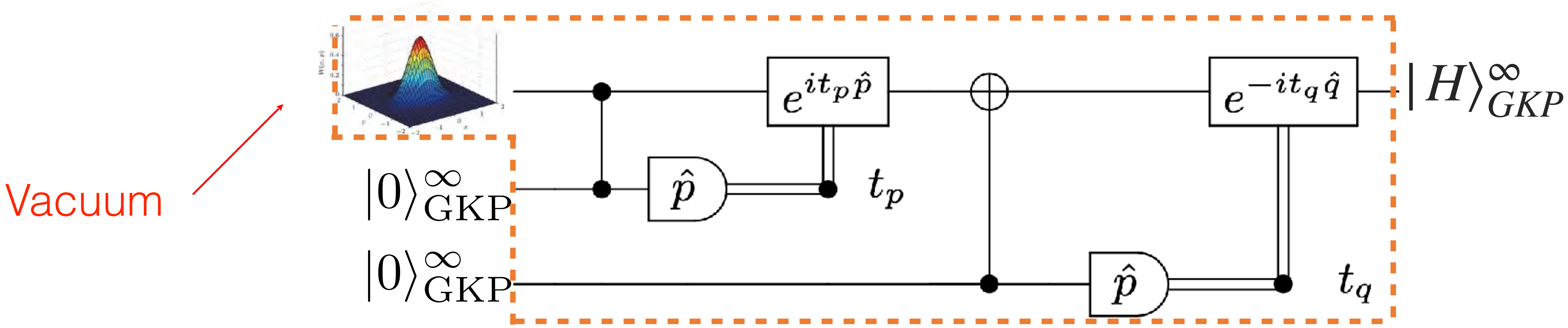
Sergey Bravyi and Alexei Kitaev, PRA 71 022316 (2005)

A new class of simulatable circuits

We have identified a non-trivial class of simulatable GKP (SGKP) circuits

SGKP circuits + Mistery state = Universal QC

What are the resource states for the class of SGKP circuits?

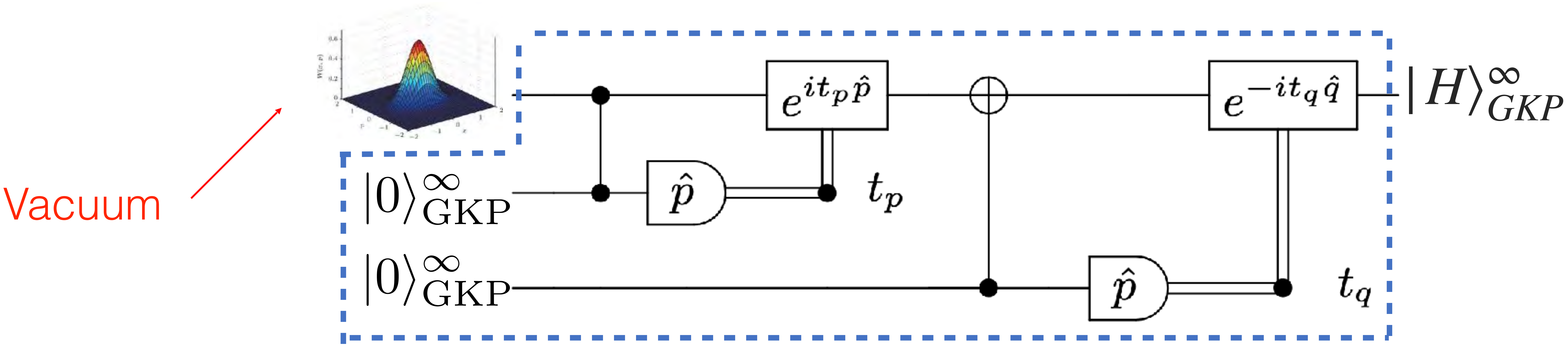


For almost all outcomes t_q, t_p the output state is approximatively a magic state

Baragiola et al, Physical Review Letters 123, 200502 (2019)

“All Gaussian universality with GKP states”

WACQT | Wallenberg Centre for Quantum Technology

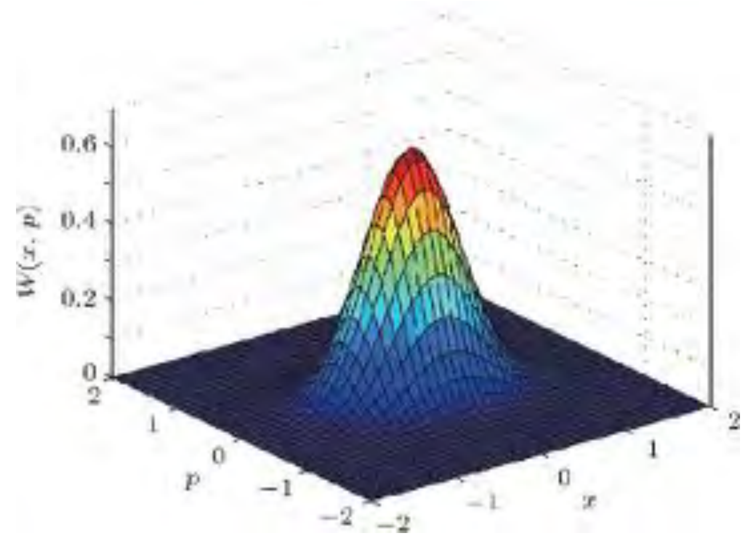


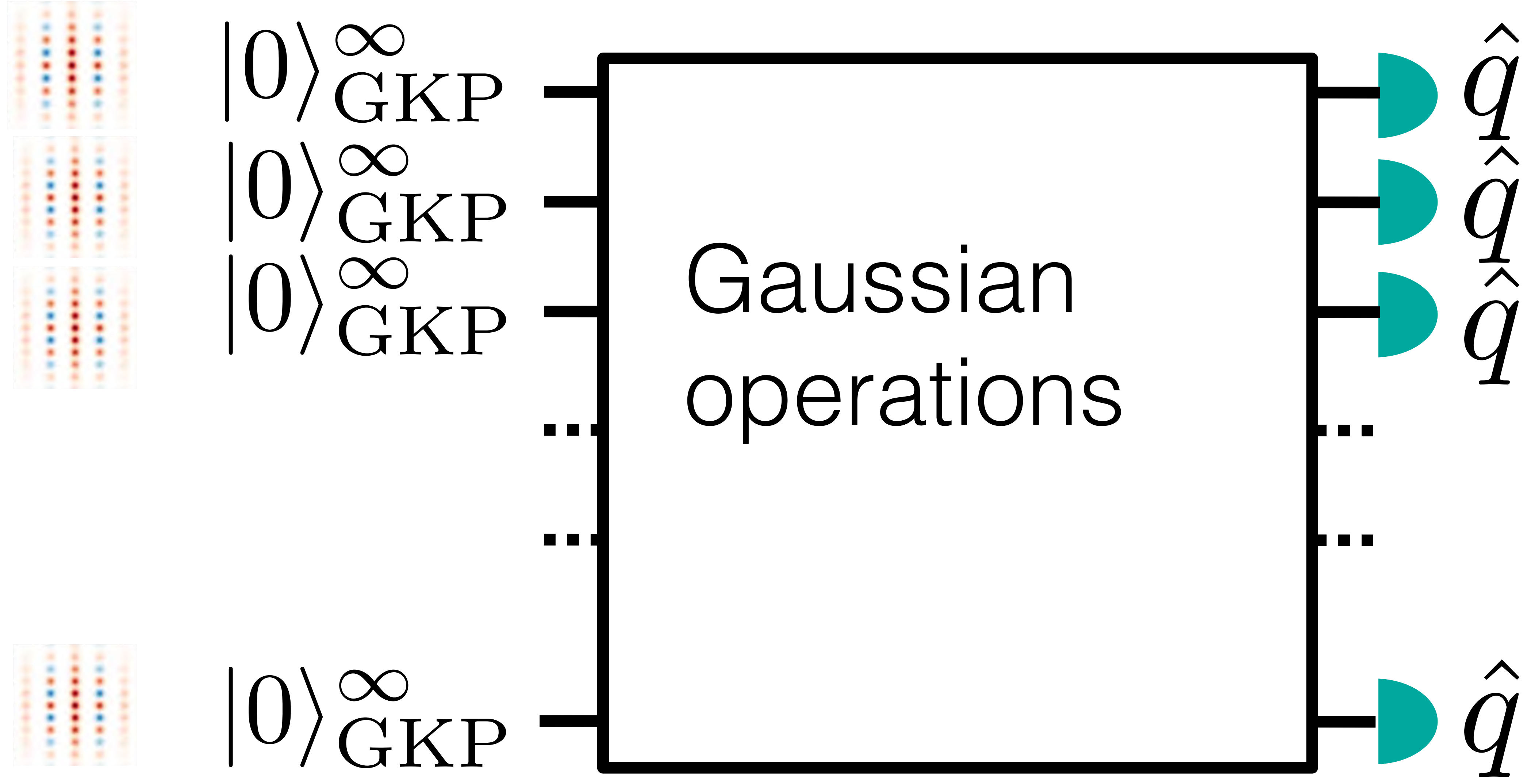
For almost all outcomes t_q, t_p the output state is approximatively a magic state

Baragiola et al, Physical Review Letters 123, 200502 (2019)

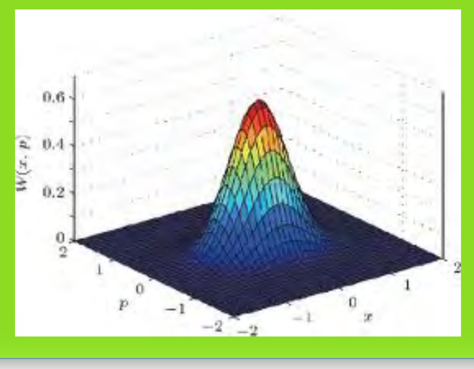
…but everything inside the dashed box is simulatable by our result!

Vacuum unlocks universality in SGKP circuits





SGKP circuits + Vacuum



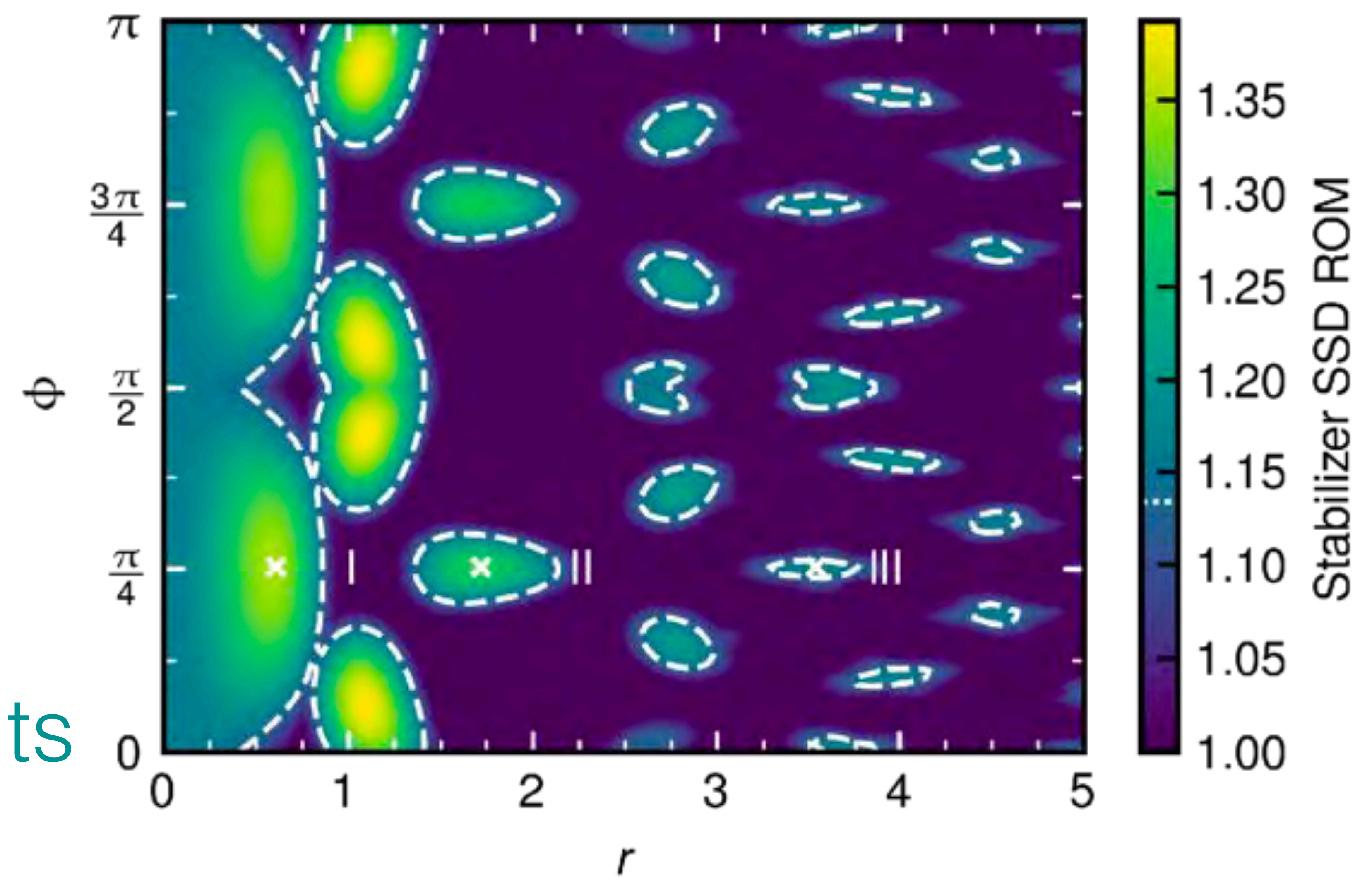
= Universal QC

"Nothing provides quantum advantage" (Cameron Calcluth)

C. Calcluth, A. Ferraro, G. Ferrini, PRA 107, 062414 (2023)

A sufficient condition for universality in SGKP circuits

SGKP circuits + Precisely characterized CV states = Universal QC



$\alpha = re^{i\phi}$

$|\alpha\rangle + |-\alpha\rangle$

Sufficient condition for universality with SGKP circuits

C. Calcluth, A. Ferraro, G. Ferrini, PRX Quantum 5, 020337 (2024)

All Gaussian + GKP and few other known states = Universal QC

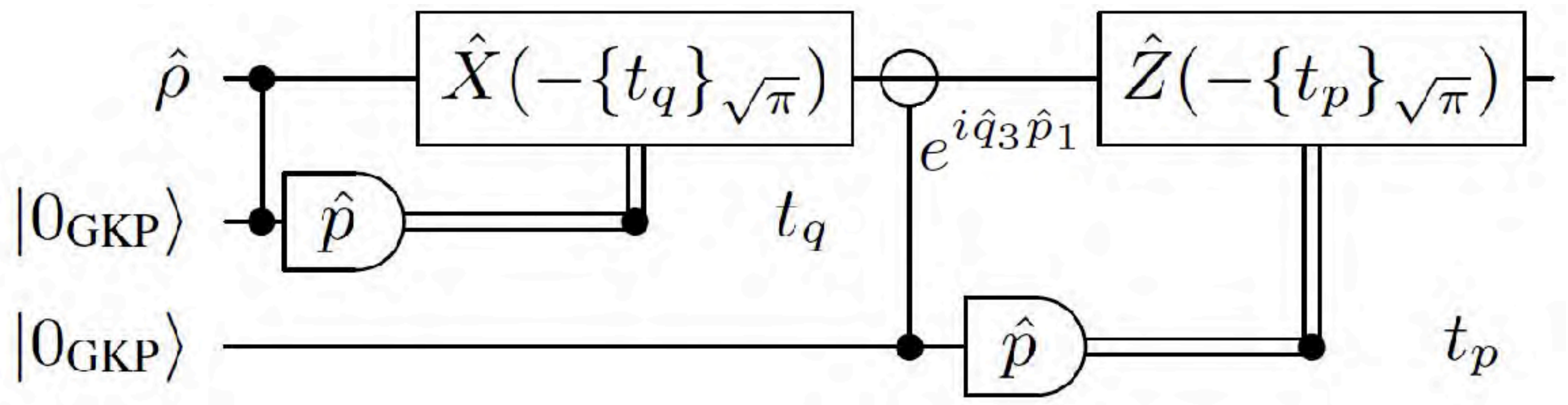
No sufficient criterion for universality with Gaussian circuits!

Sufficient condition for quantum advantage in CV

Map CV state to a qubit state with SGKP circuits

Example: stabiliser subsystem decomposition

$$\hat{\rho} \to \hat{\rho}_\Pi$$



$$\hat{\rho}_\Pi(\mathbf{t}) = \hat{\Pi}\hat{V}(-\mathbf{t})\hat{\rho}\hat{V}^\dagger(-\mathbf{t})\hat{\Pi}$$

$$\hat{V}(-\mathbf{t}) = e^{-i\hat{q}t_p}e^{i\hat{p}t_q}$$

$$\hat{\Pi} = |0_{\rm GKP}\rangle\langle 0_{\rm GKP}| + |1_{\rm GKP}\rangle\langle 1_{\rm GKP}|$$

(Infinitely-squeezed GKP states)

$$\hat{\rho}_\Pi = \int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} dt_q \int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} dt_p \hat{\rho}_\Pi(\mathbf{t})$$

Mackenzie H. Shaw, Andrew C. Doherty, Arne L. Grimsmo, PRX Quantum 5, 010331 (2024)

Sufficient condition for universality with SGKP circuits

Given a CV state $\hat{\rho}$:

- Compute $\hat{\rho}_{\Pi}$ using an allowed SGKP map (e.g. stabilizer subsystem dec.)
- Computing the fidelity of $\hat{\rho}_{\Pi}$ to $|T\rangle$ or its “cousins” (Clifford-rotated $|T\rangle$s)
- If the fidelity is larger than $F^* = (1+\sqrt{3/7})/2$, the CV state is universal with SGKP circuits

F^* threshold for magic state distillation *Sergey Bravyi and Alexei Kitaev, PRA 71 022316 (2005)*

C. Calcluth, A. Ferraro, G. Ferrini, PRX Quantum 5, 020337 (2024)

Fidelity to $|T\rangle$ is equivalent to robustness of magic

- Robustness of magic (ROM) for one single qubit:

$$\mathcal{R}^{(1)}(\hat{\rho}) = \left|\mathrm{Tr}\left(\hat{\rho}\hat{X}\right)\right| + \left|\mathrm{Tr}\left(\hat{\rho}\hat{Y}\right)\right| + \left|\mathrm{Tr}\left(\hat{\rho}\hat{Z}\right)\right|$$

M. Howard and E. Campbell, PRL 118, 090501 (2017)

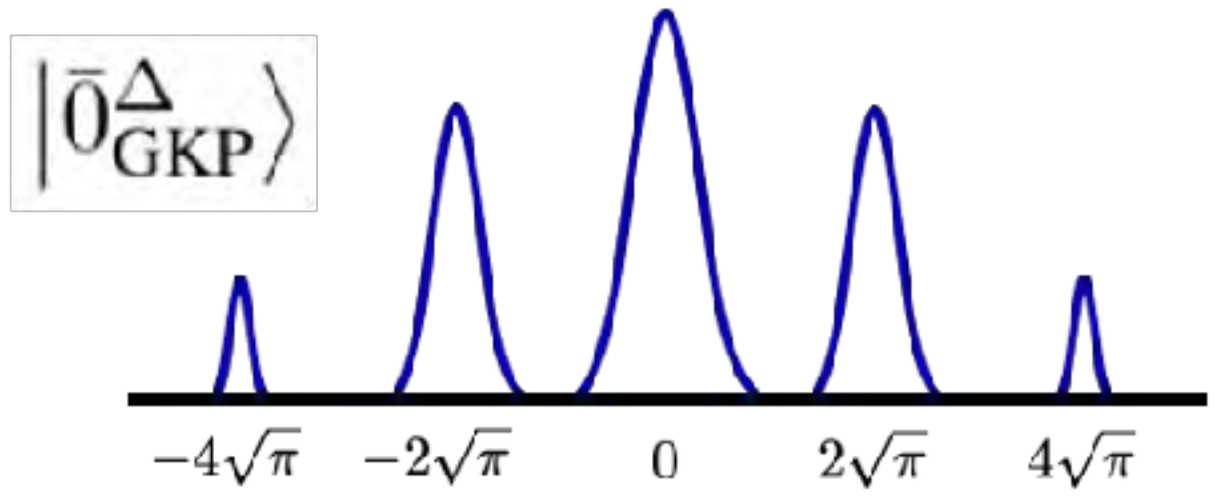
- Fidelity to $|T\rangle$ and robustness are equivalent for a single qubit

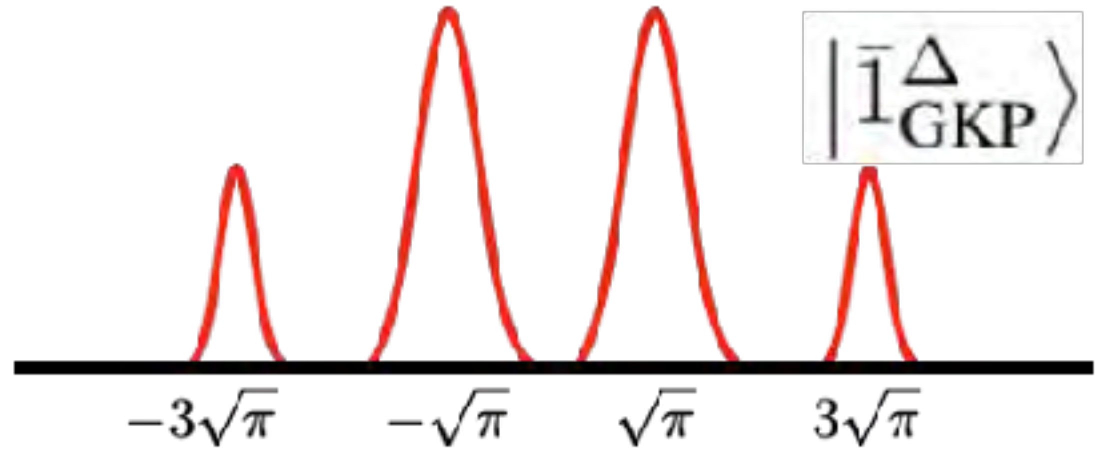
$$F_T^{\max}(\hat{\rho}) = \frac{1}{2\sqrt{3}}\mathcal{R}^{(1)}(\hat{\rho}) + \frac{1}{2}$$

- Threshold for magic state distillation:

$$F_T^{\max}(\hat{\rho}) > F^* = \tfrac{1}{2}(1 + \sqrt{3/7}) \Longrightarrow \mathcal{R}^{(1)}(\hat{\rho}) > \mathcal{R}^* = \frac{3}{\sqrt{7}} \approx 1.134$$

ROM of mapped finitely-squeezed GKP states





$$|\psi^{\Delta}_{\rm GKP}\rangle = \frac{1}{\sqrt{\mathcal{N}_{\rm GKP}}}\left(\cos(\theta/2)\,|\bar{0}^{\Delta}_{\rm GKP}\rangle + \sin(\theta/2)e^{i\pi/4}\,|\bar{1}^{\Delta}_{\rm GKP}\rangle\right)$$

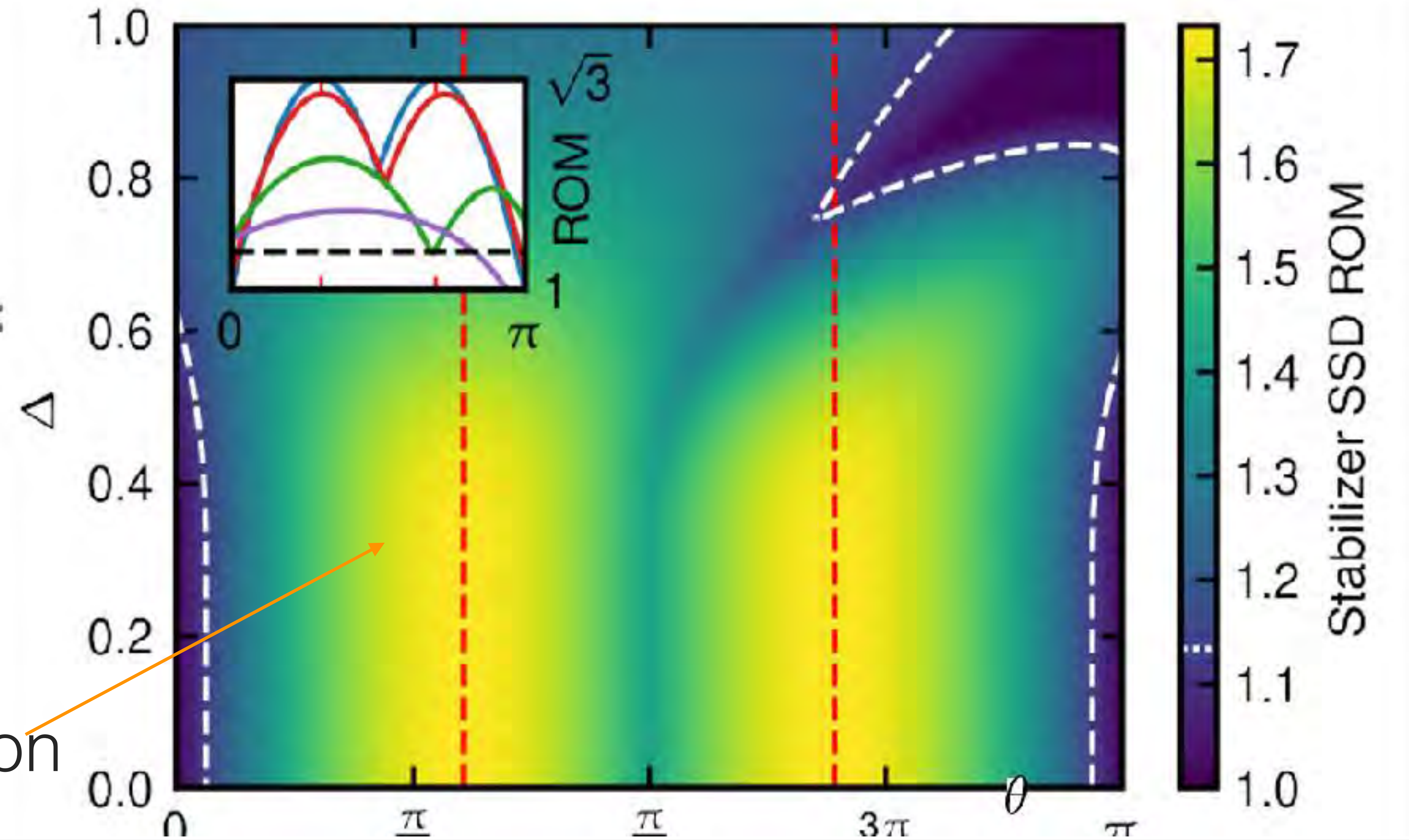
Maximal achievable ROM is obtained for a T-like state:

$$\theta = \arccos(\pm 1/\sqrt{3})$$

$$\Delta = 0$$

$$\mathcal{R} = \sqrt{3}$$

Distillability region

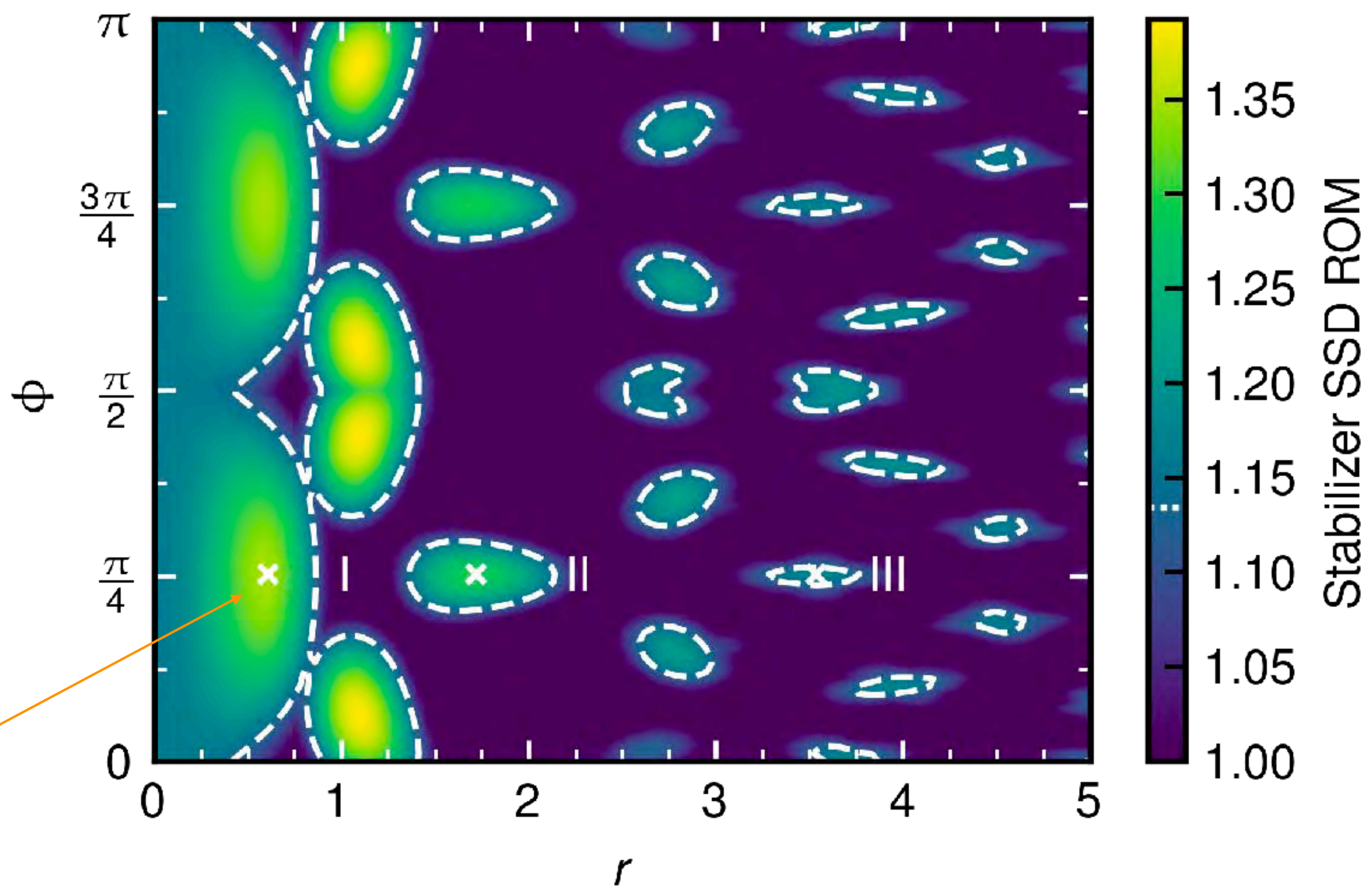


ROM of mapped cat states

$$|\bar{0}^{\alpha}_{\text{cat}}\rangle = (|\alpha\rangle + |-\alpha\rangle)$$

$$\alpha = re^{i\phi}$$

max ROM value = 1.36

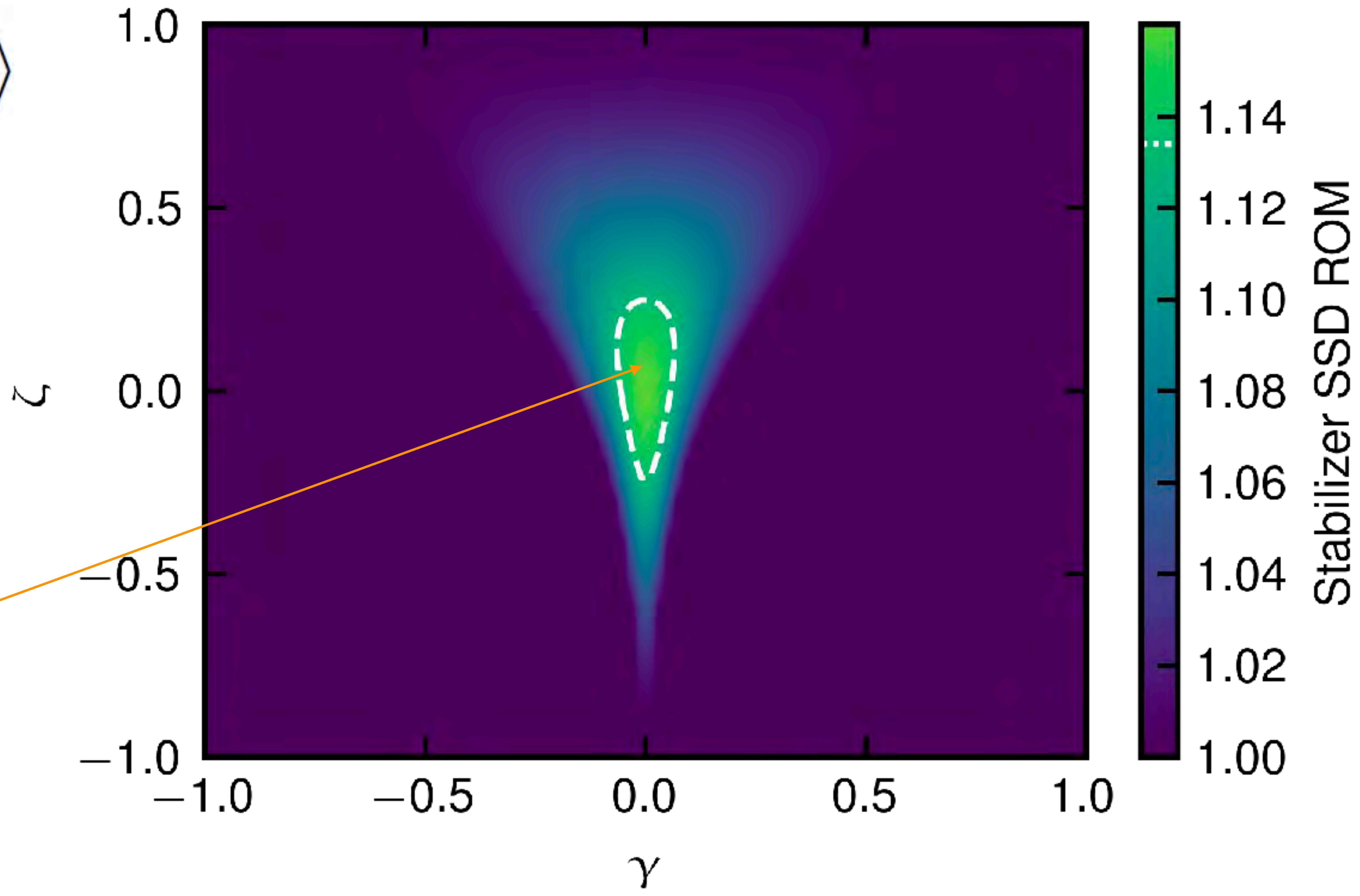


ROM of cubic phase state

$$|\gamma,\zeta\rangle = e^{i\gamma\hat{q}^3}\,\hat{S}(\zeta)\,|0\rangle$$

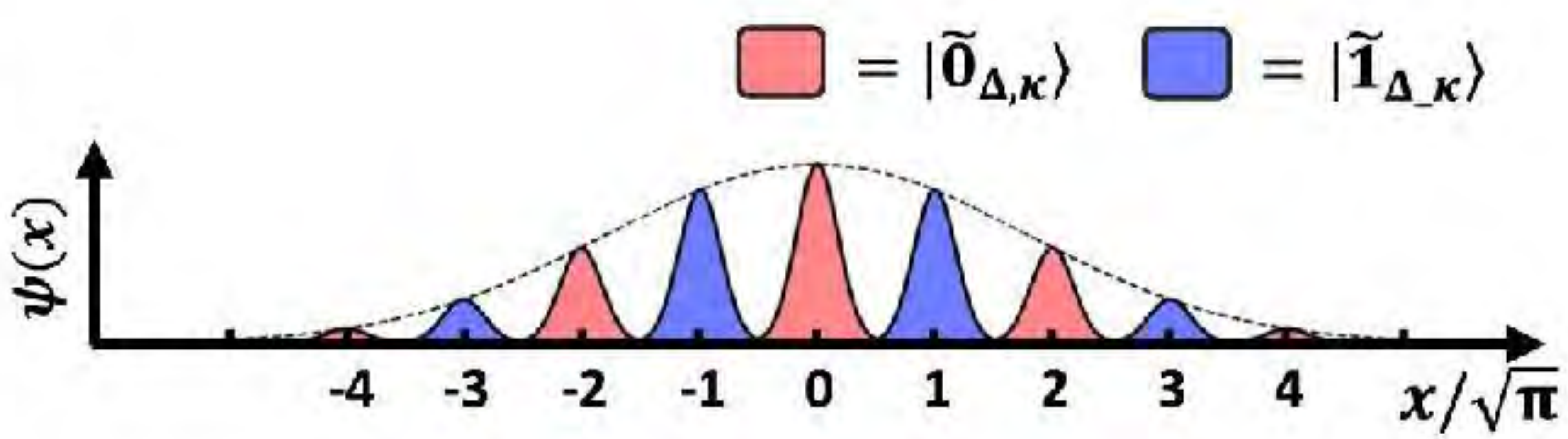
$$\hat{S}(\zeta) = e^{-\frac{i}{2}\zeta(\hat{q}\hat{p}+\hat{p}\hat{q})}$$

Higher resourcefulness for lower cubicity and squeezing

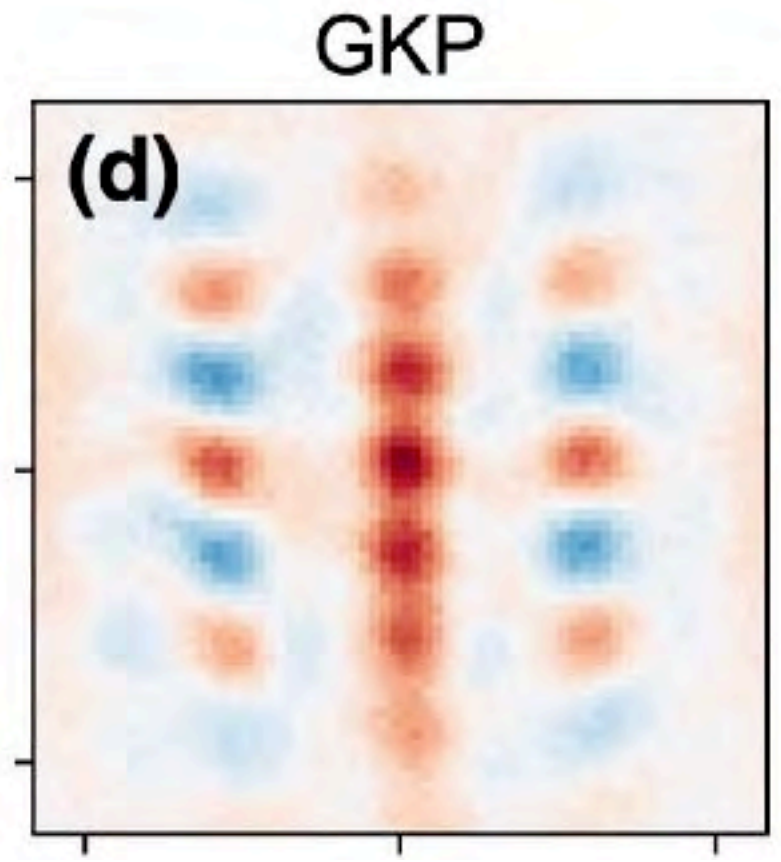


But how do we simulate realistic GKP circuits?

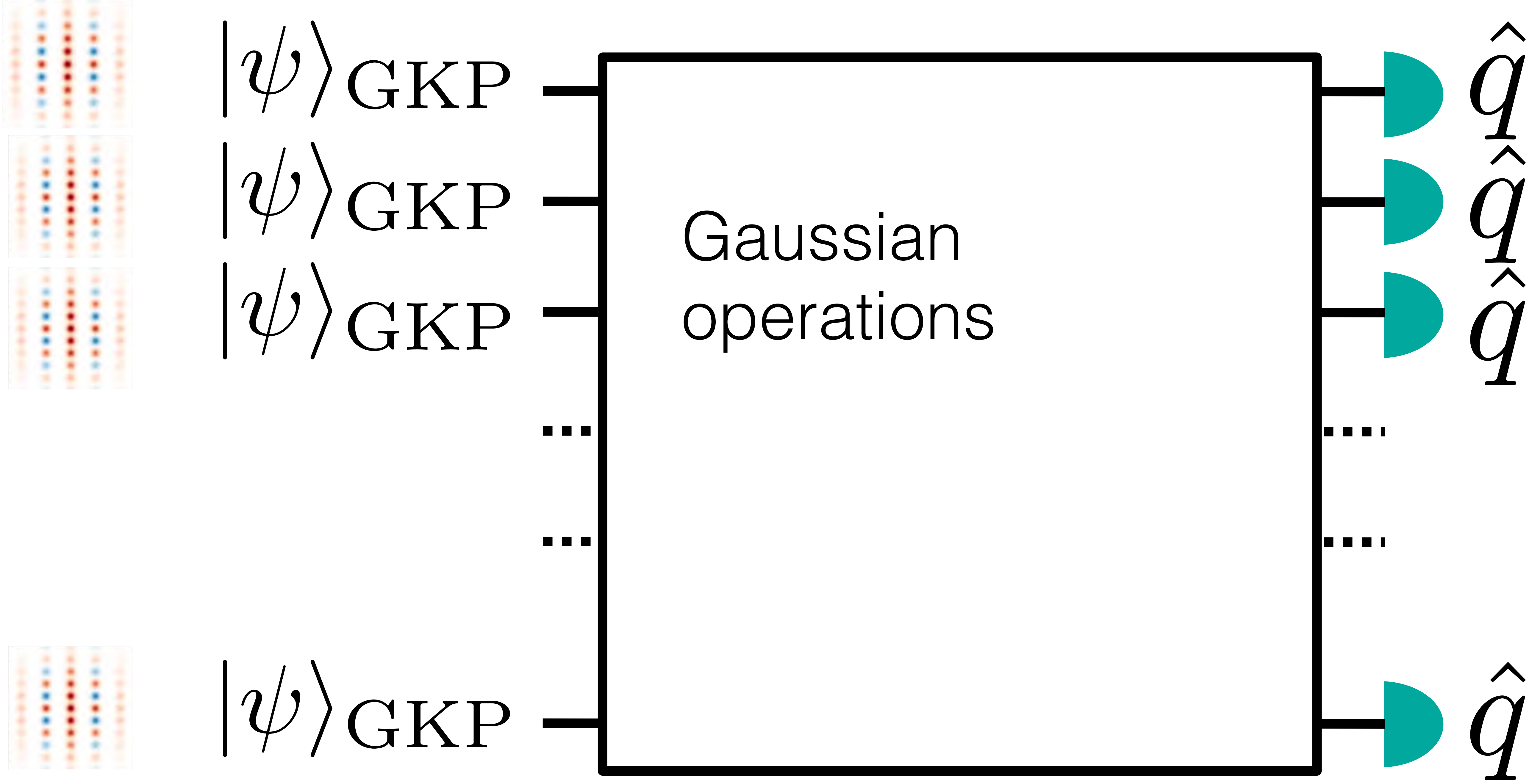
WACQT Wallenberg Centre for Quantum Technology



Courtesy of Takase et al, npj Q. Info 2023



Kudra et al, PRX Quantum 3, 030301 (2022)



Two years of attempts….

shutterstock.com · 2416571501

Simulating realistic GKP circuits with Zak-Gross Wigner function

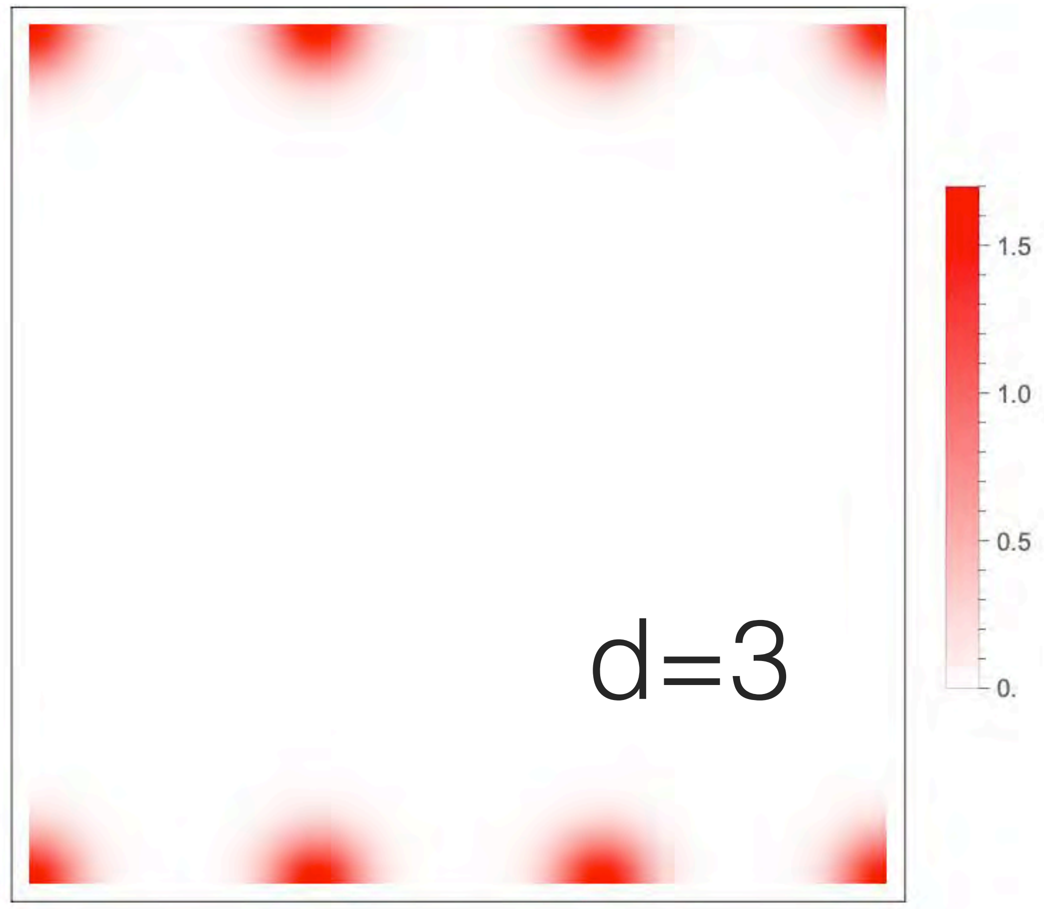
Zak-Gross Wigner (ZGW) function

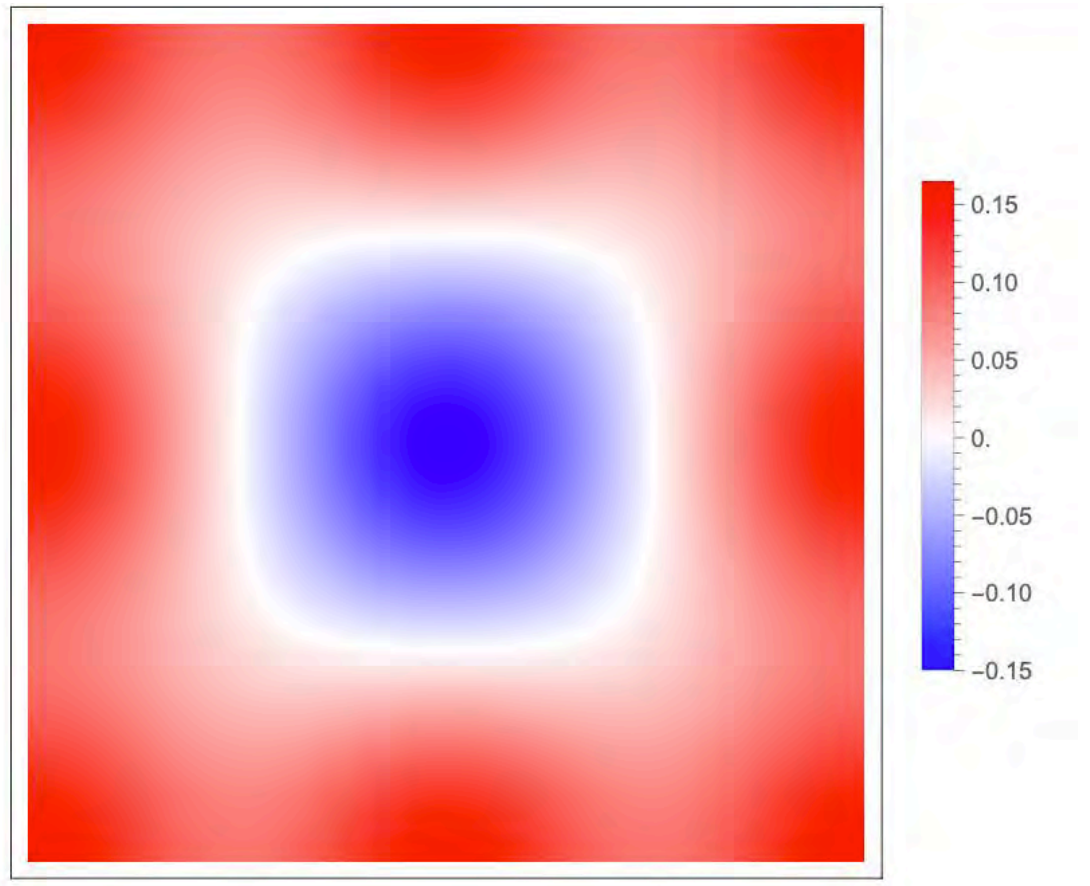
$$W_{\hat{A}}^{\mathcal{C}_{\text{GKP}}}(u,v) = \frac{1}{2\pi}\sum_{m,n\in\mathbb{Z}}(-1)^{mn}e^{i\ell(nv-mu)}\,\text{Tr}[\hat{D}(n\ell,m\ell)\hat{A}]$$

$$\hat{D}(x,p) = e^{i\frac{xp}{2}}e^{ip\hat{x}}e^{-ix\hat{p}}$$

J. Davis, N. Fabre and U. Chabaud arXiv:2407.18394 (2024)

Infinitely squeezed GKP states $|0\rangle_{\text{GKP}}^{\infty}$ have positive ZGW (d odd), vacuum negative





- We generalize the ZGW function to multimode
- We provide a simulation algorithm which time cost scales with negativity of ZGW function: the higher the negativity, the slower the simulation

$$p_{\mathbf{z}} = \int \mathrm{d}\boldsymbol{\eta}\, W_{\hat{\rho}}(\boldsymbol{\eta})W_{\hat{M}_{\mathbf{z}}}(\boldsymbol{\eta})$$

see also Pashayan et al, PRL 115, 070501 (2015)

C. Calcluth, O. Hahn, J. Bermejo-Vega, A. Ferraro, G. Ferrini, Phys. Rev. Lett. 135, 010601 (2025)

Simulating GKP circuits with Zak-Gross Wigner function

WACQT | Wallenberg Centre for Quantum Technology

- If ZG-Wigner function is positive everywhere we can simulate efficiently by sampling from it

$$p_{\mathbf{z}} = \int \mathrm{d}\boldsymbol{\eta}\, W_{\hat{\rho}}(\boldsymbol{\eta}) W_{\hat{M}_{\mathbf{z}}}(\boldsymbol{\eta})$$

- If it is negative, we can simulate with sample complexity scaling quadratically with negativity of Zak-Gross Wigner function (error ϵ, success probability $1-\delta$)

$$N = \frac{2\,\mathcal{M}^2_{\hat{\rho}_0}}{\epsilon^2} \log\left(\frac{2}{\delta}\right) \qquad \mathcal{M}_{\hat{\rho}} = \int d\boldsymbol{\eta}\, |W_{\hat{\rho}}(\boldsymbol{\eta})|$$

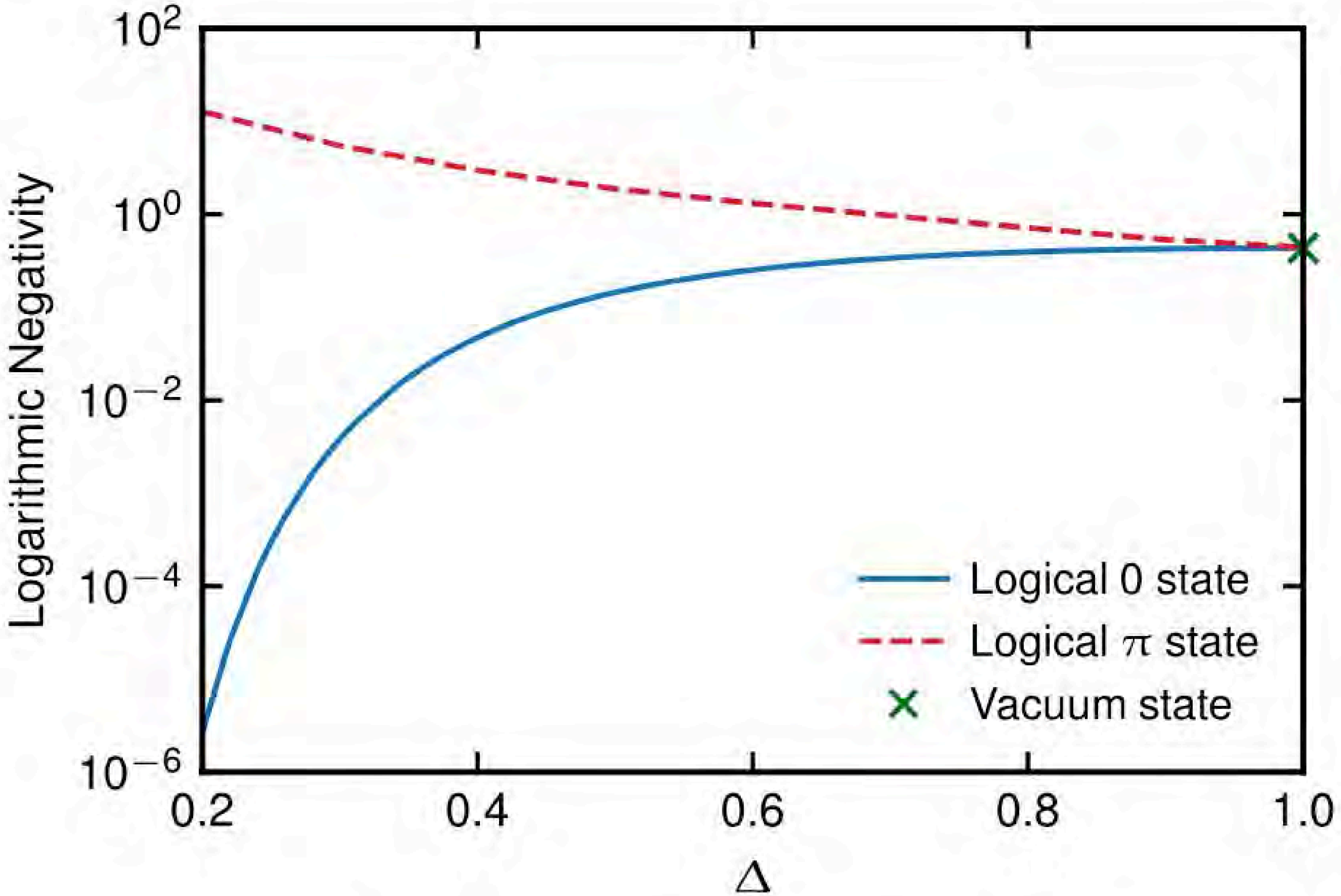
Same technique as for standard Wigner function in Pashayan et al, PRL 115, 070501 (2015)

ZG-Wigner negativity is a necessary resource for quantum computational advantage

C. Calcluth, O. Hahn, J. Bermejo-Vega, A. Ferraro, G. Ferrini, Phys. Rev. Lett. 135, 010601 (2025)

Plot Insight:

- For a realistic 0-logical GKP state, the Zak–Gross logarithmic negativity decreases as squeezing improves (smaller Δ).
- Non-stabilizer (magic) states, such as the π-logical state, retain significant negativity.

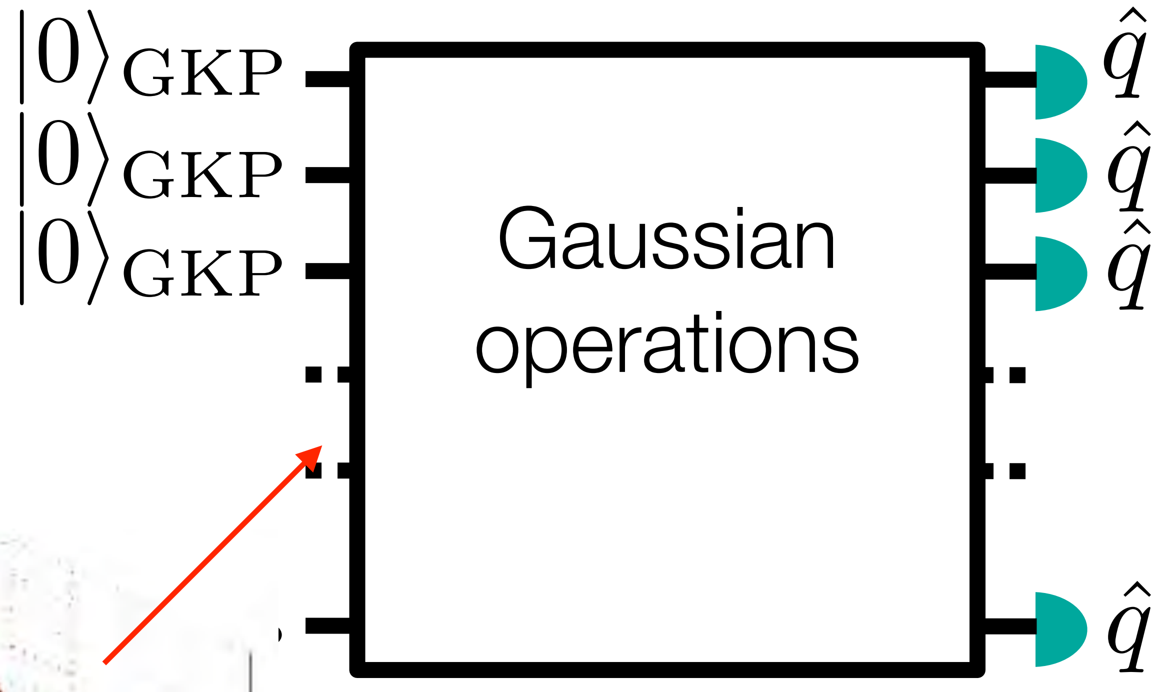


- E.g., for stabilizer GKP states with 12 dB squeezing we can simulate 1,000 modes with less than double the number of samples required for a single input mode

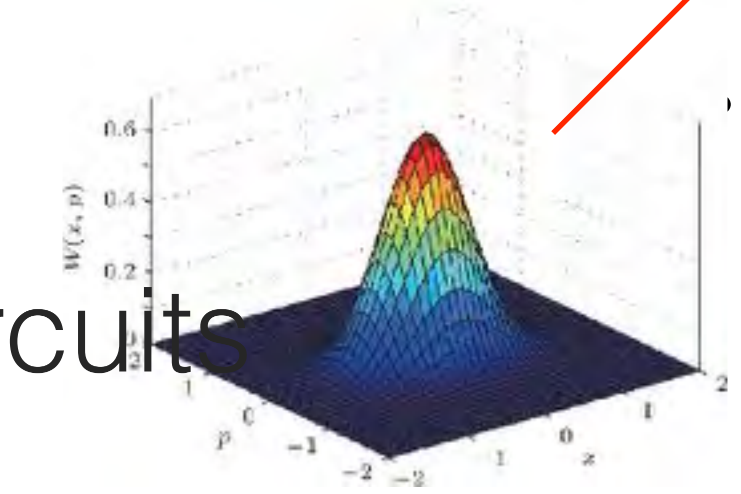
C. Calcluth, O. Hahn, J. Bermejo-Vega, A. Ferraro, G. Ferrini, Phys. Rev. Lett. 135, 010601 (2025)

Summary and conclusions on part 3a

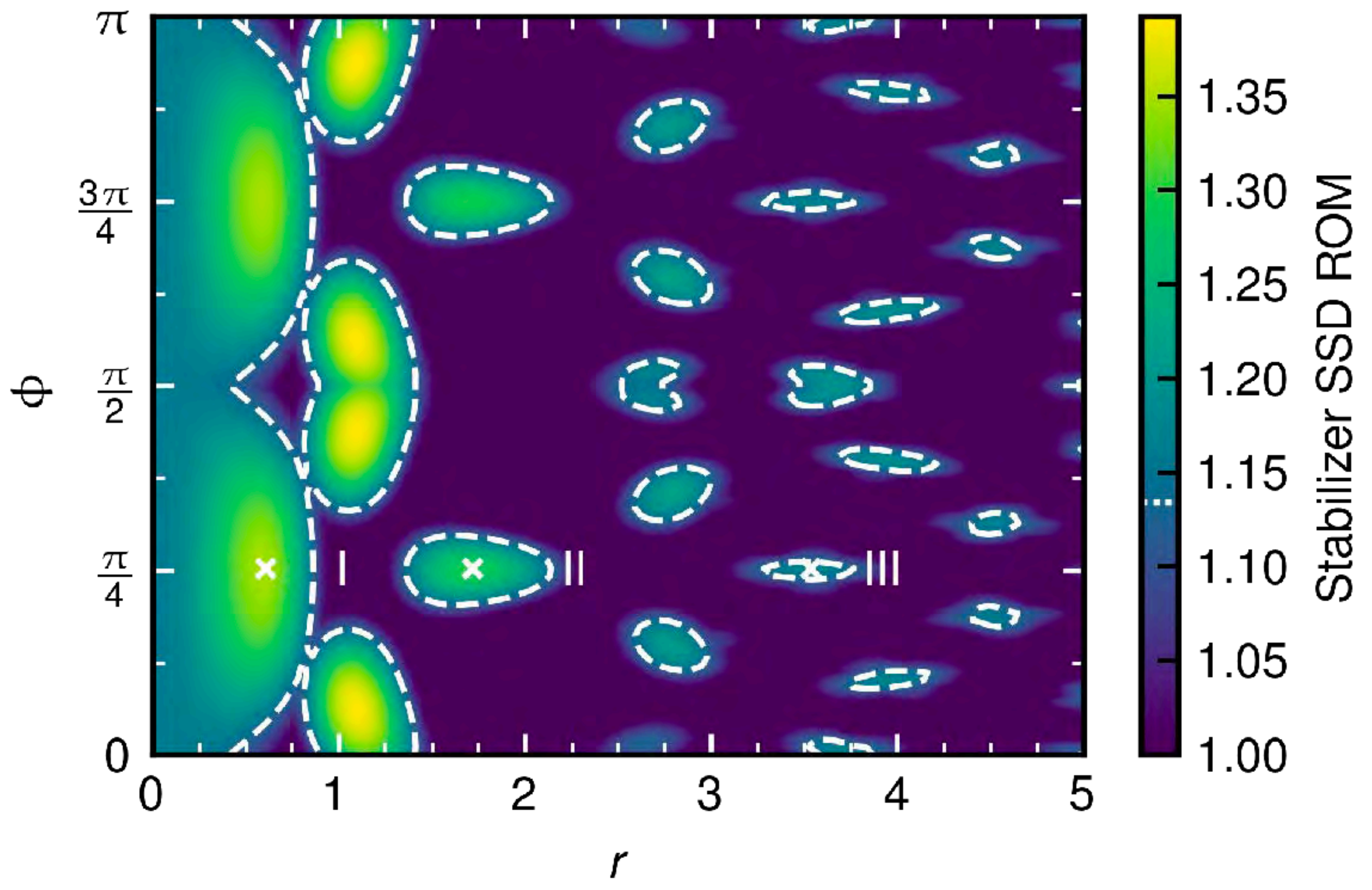
Conclusions of part 3a



- Stabiliser GKP states with Gaussian operations are classically efficiently simulatable (SGKP circuits)
- Vacuum provides quantum advantage to SGKP circuits



- New classical simulation algorithms for realistic GKP circuits
- Negativity of the Zak-Gross Wigner function is a necessary condition to quantum advantage
- Sufficient condition for quantum advantage in CV



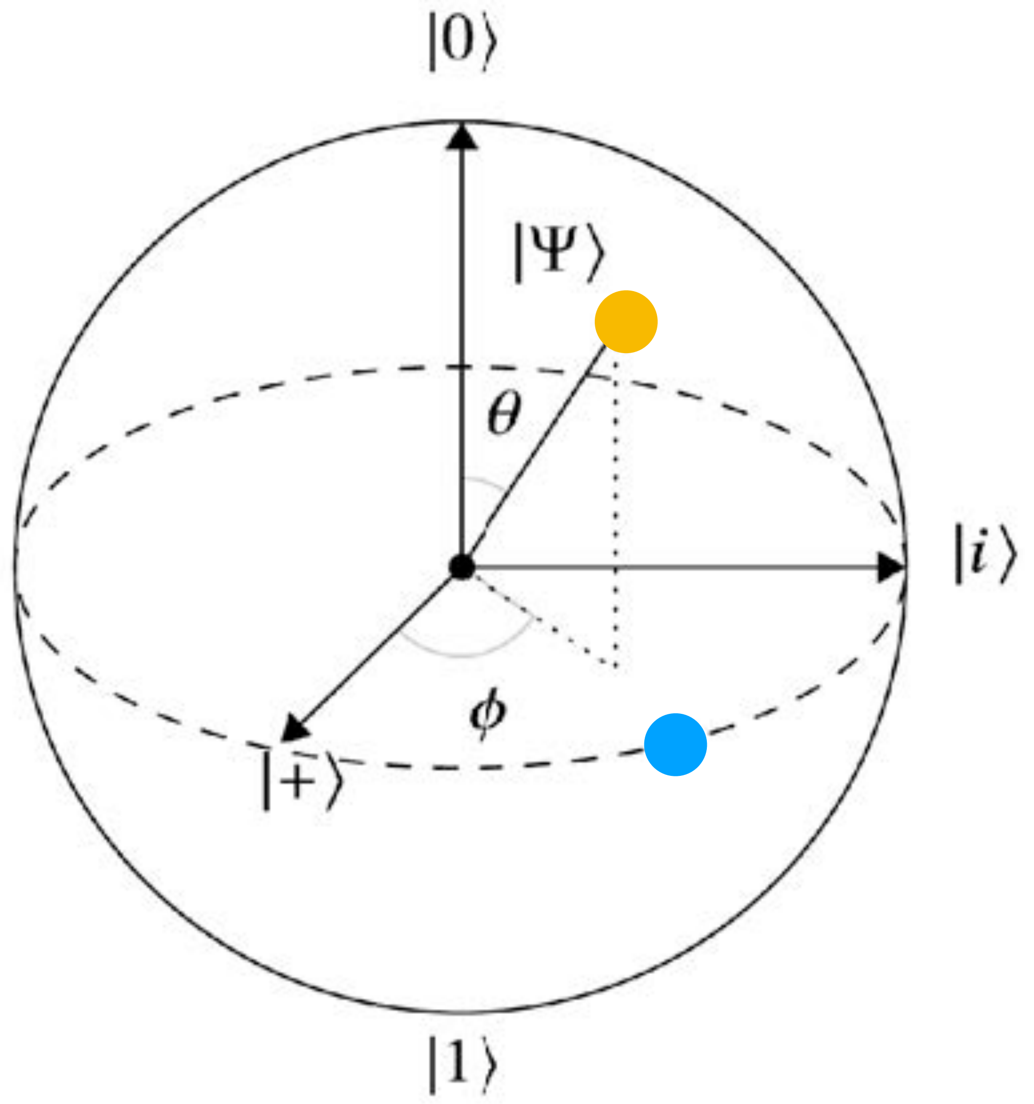
L. García-Álvarez et al, PRR 2, 043322 (2020)
C. Calcluth et al, Quantum 6, 867 (2022)
C. Calcluth, A. Ferraro, G. Ferrini, PRA 107, 062414 (2023)
C. Calcluth, A. Ferraro, G. Ferrini, PRX Quantum 5, 020337 (2024)
C. Calcluth et al, PRL 135, 010601 (2025)

Part 3b: A magic measure based on GKP encoding

A new magic measure for qubits using bosonic codes

Example of magic states

- Magic states are non-stabiliser states that allow to promote Clifford operations and Pauli measurements to universal QC



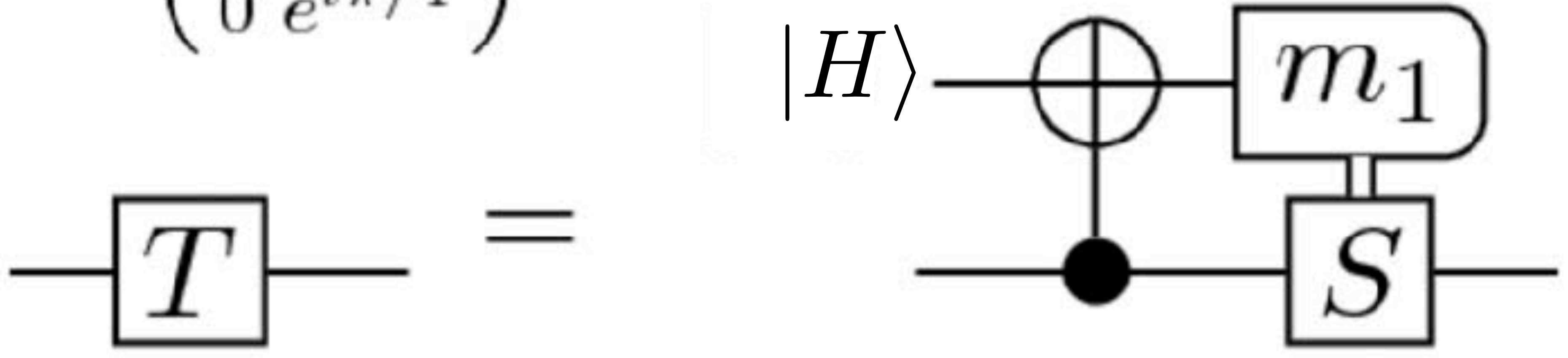
- T-state and H-state:

$$|T\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\frac{\pi}{4}}|1\rangle \quad \text{with} \quad \theta = \arccos\left(\frac{1}{\sqrt{3}}\right)$$

$$|H\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle\right),$$

- From magic states to the T-gate:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$|H\rangle$ states (+Cliffords) enable T gates

Sergey Bravyi and Alexei Kitaev, PRA 71 022316 (2005)

Why is it important to quantify magic?

WACQT | Wallenberg Centre for Quantum Technology

- Magic states and magic gates are costly to implement in leading paradigms for fault-tolerant quantum computation such as surface-code quantum computing

- In a given algorithm, we want to implement non-Clifford gates using as few magic states as possible

- Magic measures allow for identifying bounds on the "T-count"!

How magic is a state?

Magic monotones: satisfy properties and quantify magic

We want to define a new one!

How magic is a state? Magic monotones

- Relative entropy of magic, *V. Veitch et al, New Journal of Physics 16, 013009 (2014)*

$$r_{\mathcal{M}}(\rho) \equiv \min_{\sigma \in \mathrm{STAB}(\mathcal{H}_d)} S(\rho \| \sigma) \quad \text{with} \quad S(\rho\|\sigma) = \mathrm{Tr}(\rho \log \rho) - \mathrm{Tr}(\rho \log \sigma)$$

- Robustness of magic, *Howard et al, Phys. Rev. Lett. 118, 090501 (2017)*
- Stabilizer rank, *Bravyi et al, Quantum 3, 181 (2019)* $\mathcal{R}(\rho) = \min_x \left\{ \sum_i |x_i| ; \rho = \sum_i x_i \sigma_i \right\}$ with $\sum_i x_i = 1$
- Stabilizer nullity and the dyadic monotone, *Beverland et al, Quantum Science and Technology 5, 035009 (2020)*
- Dyadic negativity, mixed state extend, generalised robustness *Seddon et al, PRX Quantum 2, 010345 (2021)*

- Stabilizer Rény Entropy *Leone et al, PRL 128, 050402 (2022)*
- Bell magic *Haug et al, PRX Quantum 4, 010301 (2023)*
- Quantum Jensen-Shannon divergence *Tian and Sun, J. Phys. A: Mat. and Th. (2024)*
- Pauli instability *Garcia et al, Phys. Rev. Research 7, 033271 (2025)*
- Quantum Ruzsa divergence, quantum-doubling constant *IEEE Trans. Inf. Th. (2025)*

Proving that GKPmagic is a magic measure

(i) Monotonicity: $\mathcal{G}(\mathcal{E}(|\psi\rangle)) \leq \mathcal{G}(|\psi\rangle)$ for $\mathcal{E}(|\psi\rangle)$ a Clifford unitary.

(ii) Faithfulness: $\mathcal{G}(|\psi_S\rangle) = 0$ iff $|\psi_S\rangle$ is a stabilizer state

(iii) Additivity: $\mathcal{G}(|\psi\rangle_A \otimes |\phi\rangle_B) = \mathcal{G}(|\psi\rangle) + \mathcal{G}(|\phi\rangle)$

(iv) Invariance under composition with stabilizer states: $\mathcal{G}(|\psi\rangle \otimes |\phi_S\rangle) = \mathcal{G}(|\psi\rangle)$

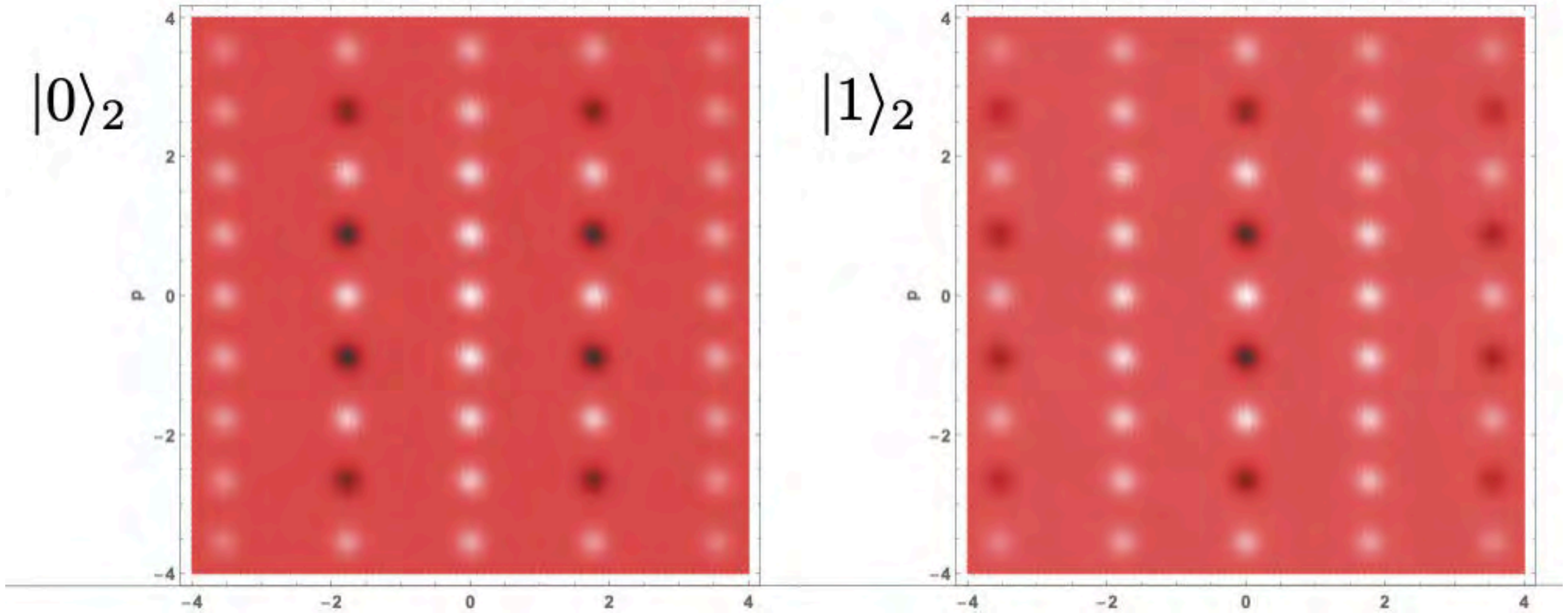
Wigner function of GKP states is highly negative

For |0> and |1> states: 1/4 of the summands are negative and 3/4 are positive

$n = 2$

$q : \sqrt{\pi}$

$p : \sqrt{\pi}/2$

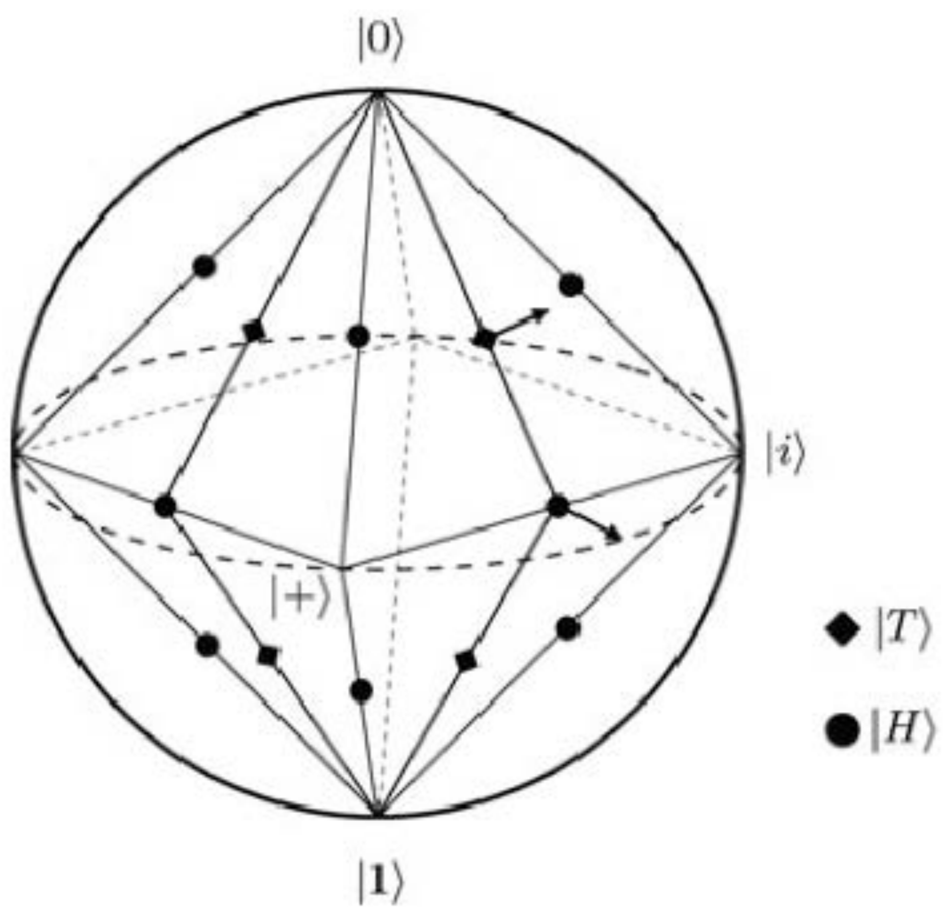


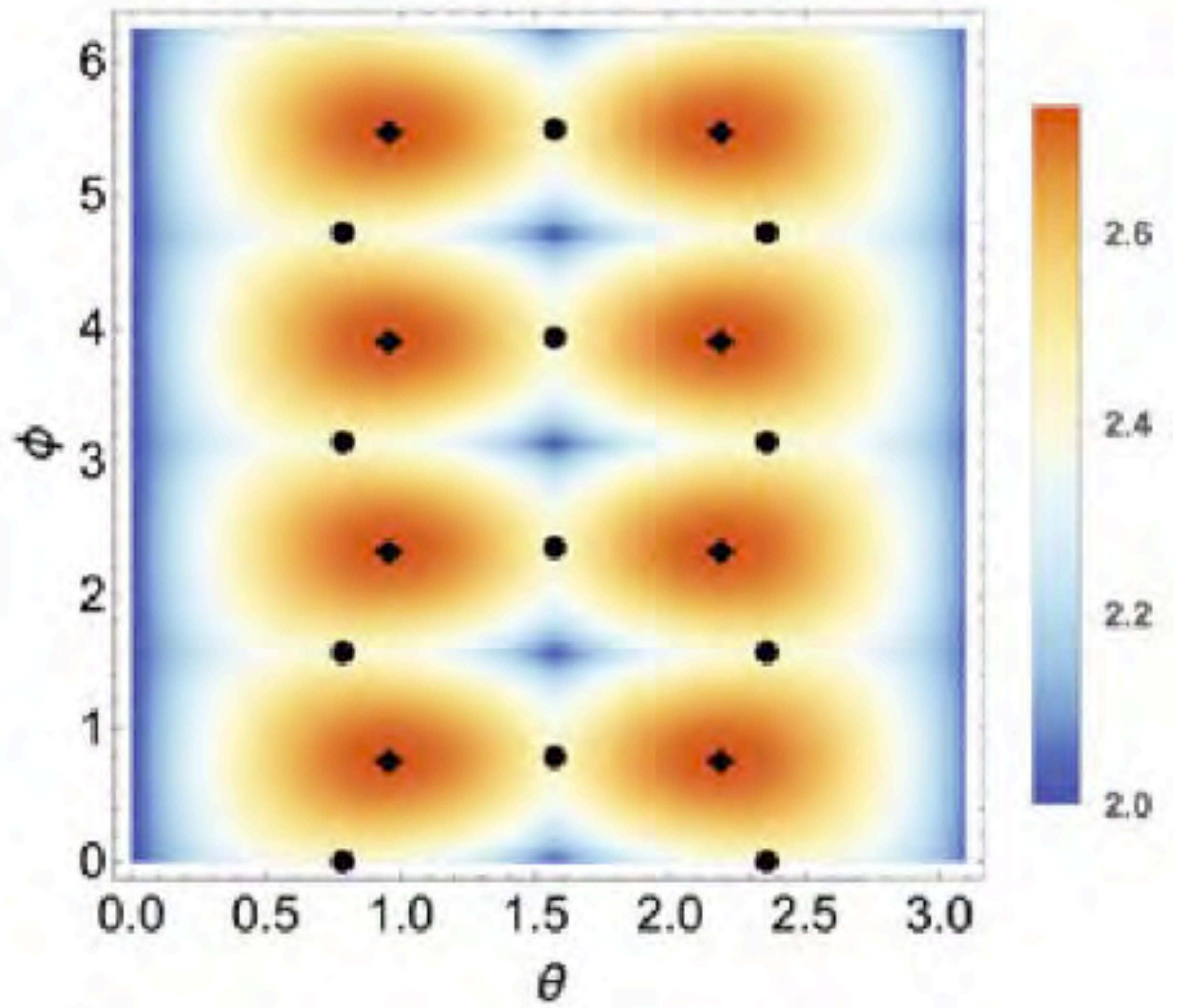
Expression for general states in: *L. García-Álvarez, A. Ferraro, G. Ferrini, International Symposium on Mathematics, Quantum Theory, and Cryptography p.79, Springer (2021)*

- The Wigner function of encoded GKP states depends on the encoded state
- It is periodic, we restrict to a cell

- Volume of the negative region of the Wigner function depends on the encoded GKP state!

	$\sqrt{\pi}\int\lvert W_{\mathrm{cell}}\rvert$
$\lvert 0\rangle$	2
$\lvert +\rangle$	2
$\lvert i\rangle$	2
$\lvert H\rangle$	$1+\sqrt{2}\approx 2.41$
$\lvert T\rangle$	$1+\sqrt{3}\approx 2.73$





- Constant value for Pauli eigenstates
- High for *magic* states

O. Hahn et al, Phys. Rev. Lett 128, 210502 (2022)
L. García-Álvarez, A. Ferraro, G. Ferrini, International Symposium on Mathematics, Quantum Theory, and Cryptography p.79, Springer (2021)

We can use $\mathcal{W}_{\mathrm{cell}}(\theta,\phi)$ to define a magic measure! *GKPmagic*

GKP magic

WACQT Wallenberg Centre for Quantum Technology

- Wigner function for N encoded GKP qubits: $|\psi\rangle = \sum_{i\in\mathbb{F}_2^n} c_i\,|i\rangle$ $\quad |i\rangle = |i_1 i_2 \ldots i_n\rangle \quad i_l \in \{0,1\}$
- Compute negative volume in the cell $\quad \mathcal{W}_C(\hat\rho) = \log_2\left(\int_0^{2\sqrt{\pi}-\epsilon} \mathrm{d}^n\boldsymbol{q}\,\mathrm{d}^n\boldsymbol{p}\,|W_{\hat\rho}(\boldsymbol{q},\boldsymbol{p})|\right)$
- Final expression:

$$\mathcal{G}(|\psi\rangle) \equiv \log_2\left(\sum_{i,j\in\mathbb{F}_2^n}\left|\sum_{k\in\mathbb{F}_2^n}\frac{(-1)^{i\cdot k}}{2^n}c_k\, c^*_{k+j}\right|\right)$$

GKPmagic

= st-norm, Howard et al, PRL 118, 090501 (2017)
= stab. Rényi entropy α= 1/2, Leone et al, PRL 128, 050402 (2022)

- We prove the measure properties using Wigner function properties
- Easily computable for 12 qubits on laptop (962 seconds)

O. Hahn, A. Ferraro, L. Hultquist, G. Ferrini, L. García-Álvarez, PRL 128, 210502 (2022)

Proving that GKPmagic is a magic measure

WACQT | Wallenberg Centre for Quantum Technology

(i) Monotonicity: $\mathcal{G}(\mathcal{E}(|\psi\rangle)) \leq \mathcal{G}(|\psi\rangle)$ for $\mathcal{E}(|\psi\rangle)$ a Clifford unitary.

(ii) Faithfulness: $\mathcal{G}(|\psi_S\rangle) = 0$ iff $|\psi_S\rangle$ is a stabilizer state

(iii) Additivity: $\mathcal{G}(|\psi\rangle_A \otimes |\phi\rangle_B) = \mathcal{G}(|\psi\rangle) + \mathcal{G}(|\phi\rangle)$

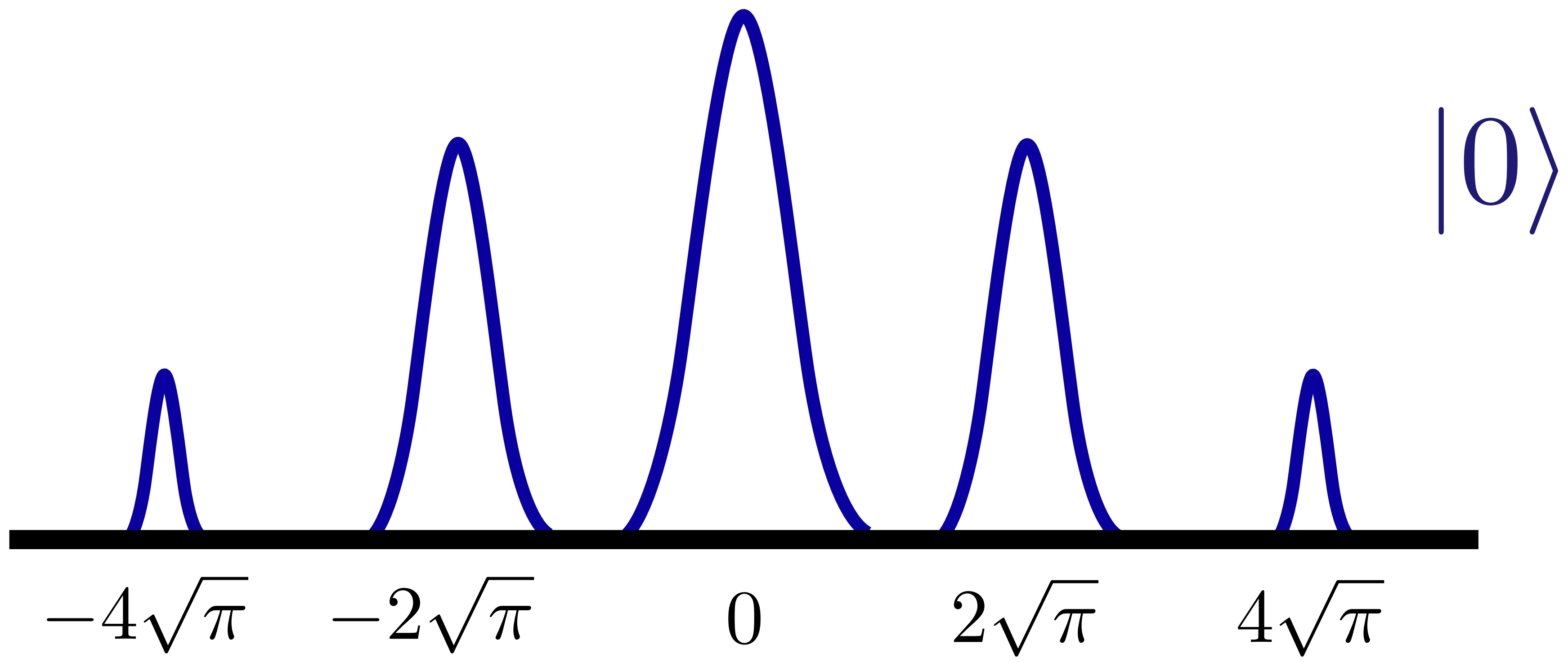
(iv) Invariance under composition with stabilizer states: $\mathcal{G}(|\psi\rangle \otimes |\phi_S\rangle) = \mathcal{G}(|\psi\rangle)$

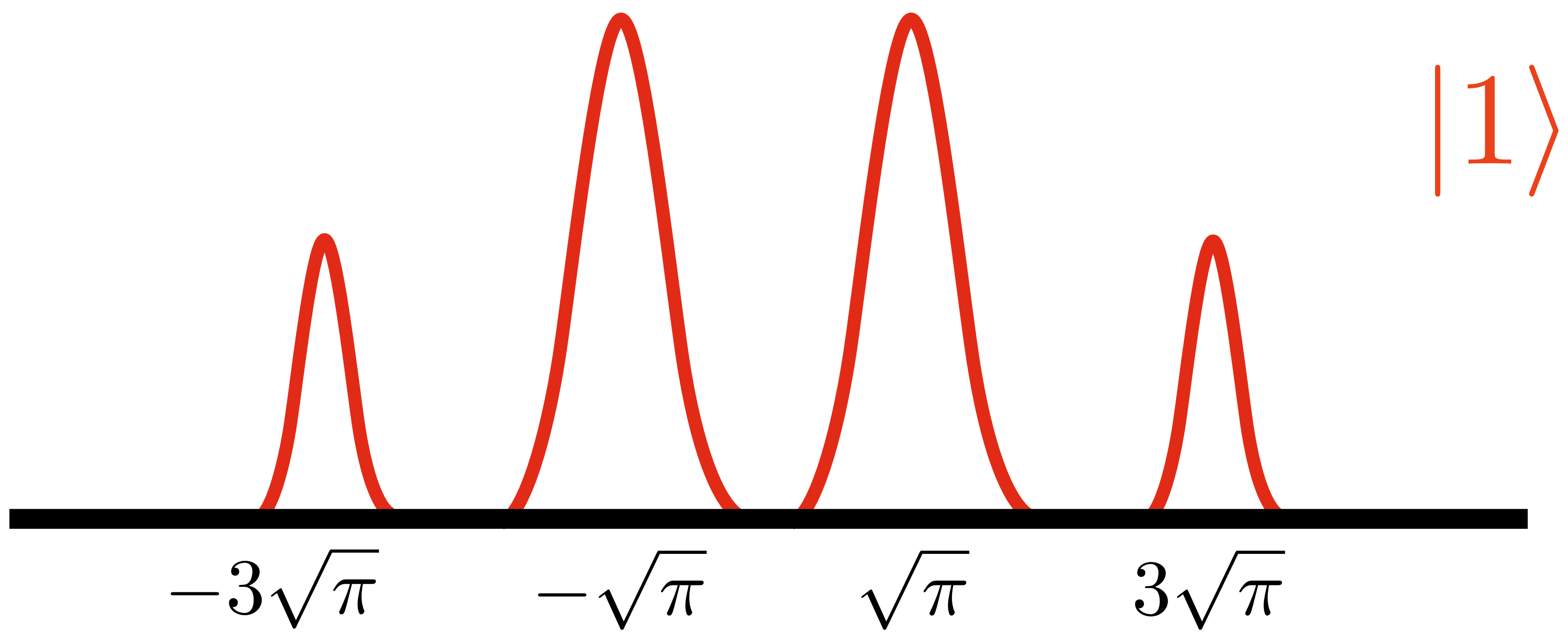
- Only possible to define for pure states, because for mixed states faithfulness does not hold
- Can increase with stabilizer measurements

Finitely-squeezed GKP states

Finite energy case $\delta(0) \longrightarrow |\psi_0\rangle = \int dq \frac{1}{(\pi\Delta^2)^{1/4}} e^{-\frac{q^2}{\Delta^2}} |q\rangle$

$$|\tilde{k}\rangle = N_k \sum_s e^{-\frac{1}{2}\kappa^2[(2s+k)\alpha]^2} T[(2s+k)\alpha]|\psi_0\rangle$$





Theory: D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A 64, 012310 (2001)
Ions: C. Flühmann et al, Nature 566, 513 (2019);
Microwaves: P. Campagne-Ibarque, Nature 584, 368 (2020)

Wigner logarithmic negativity for finitely-squeezed GKP states computed in:

Yamasaki et al, Phys. Rev. Research 2, 023270 (2020)

Used to bound conversion from $|H\rangle_{GKP}$ to $|0\rangle_{GKP}$

Extension to qudits and link to simulation cost

Generalization to arbitrary dimension

WACQT Wallenberg Centre for Quantum Technology

We want to quantify magic for a qudit state

$$|\psi\rangle = \sum_{i=0}^{d-1} \alpha_i |i\rangle$$

The negativity of the discrete Wigner function in odd dimensions is a magic measure ("mana") *V Veitch et al New J. Phys. 14, 113011 (2012)*

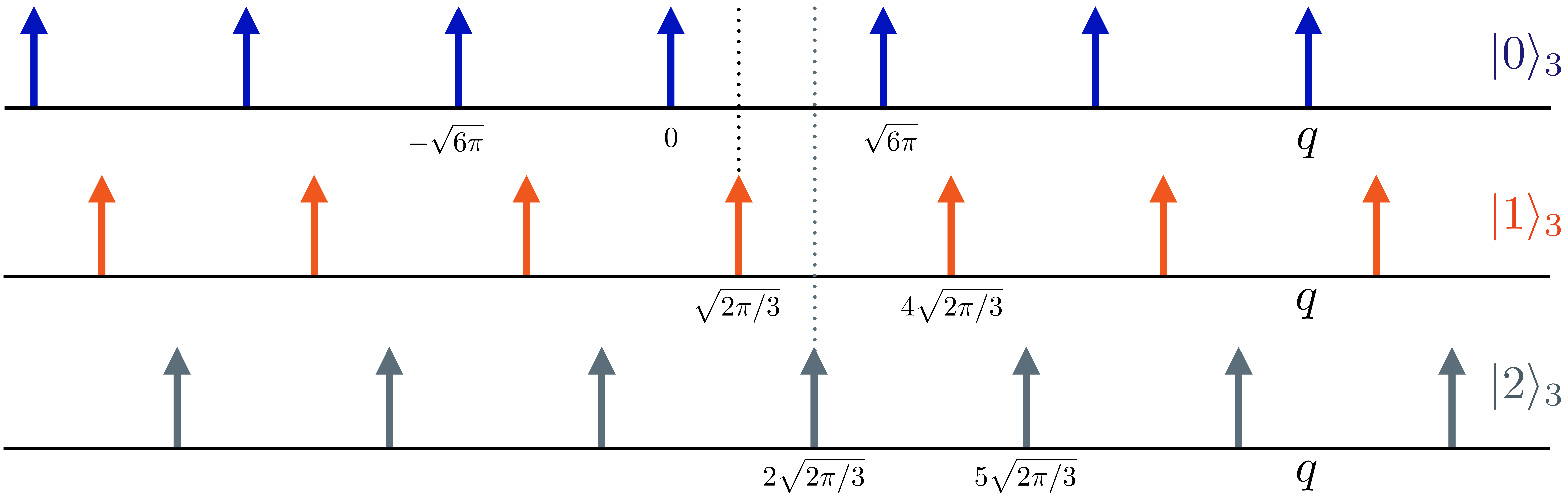
$$\|W_\rho^{\mathrm{DV}}\|_1 = \sum_{\boldsymbol{u}} |W_\rho^{\mathrm{DV}}(\boldsymbol{u})|$$

But we can also consider the negativity of the CV Wigner function for GKP-encoded qudit states, for arbitrary dimension!

We can also encode qudits with the GKP code

$$|\psi\rangle = \sum_{i=0}^{d-1} \alpha_i |i\rangle$$

E.g. qutrit (d = 3)



Rescaled negativity of the CV Wigner function

WACQT | Wallenberg Centre for Quantum Technology

- For all dimensions:

$$\frac{\|W^{\mathrm{CV}}_{\rho_{\mathrm{GKP}}}\|_{p,\mathrm{cell}}}{\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}}},$$

with
$$\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}} := \|W^{\mathrm{CV}}_{\phi_{\mathrm{GKP}}}\|_{p,\mathrm{cell}} = (4d)^{n/p}/(8\pi d)^{n/2}$$
for p=1 is a magic monotone

- For odd d
$$d^{n(1-1/p)}\|W^{\mathrm{DV}}_{\rho}\|_p = \frac{\|W^{\mathrm{CV}}_{\rho_{\mathrm{GKP}}}\|_{p,\mathrm{cell}}}{\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}}}$$
for all p is a magic monotone

O. Hahn et al, PRX Quantum 6, 010330 (2025)

Definition independent from GKP states

WACQT Wallenberg Centre for Quantum Technology

$$x_\rho(\boldsymbol{l},\boldsymbol{m}) := d^{-n}\,\mathrm{Tr}(O_{\boldsymbol{l},\boldsymbol{m}}\rho) \qquad l,m \in \mathbb{Z}_{2d}$$

with $O_{l,m} = e^{-i\pi ml/d} M_l Z_d^m$ $\qquad M_l = \sum_{\substack{u,v\in\mathbb{Z}_d \\ u+v \mod d=l}} |u\rangle\langle v|$

- Reduce to Pauli operators for d = 2
- Relate to phase space point operators for d odd $\quad O_{l,m} = (-1)^{ml} A\left(\frac{l}{2}, -\frac{m}{2}\right)$
- They are a basis, we can express any operator

O. Hahn et al, PRX Quantum 6, 010330 (2025)

See also C. Miquel et al, PRA 65, 062309 (2002)

Rescaled negativity of the CV Wigner function

- For all dimensions:

$$d^{n(1-1/p)}\|x_\rho\|_p = \frac{\|W^{\mathrm{CV}}_{\rho_{\mathrm{GKP}}}\|_{p,\mathrm{cell}}}{\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}}},$$

where

$$\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}} := \|W^{\mathrm{CV}}_{\phi_{\mathrm{GKP}}}\|_{p,\mathrm{cell}} = (4d)^{n/p}/(8\pi d)^{n/2}$$

- For odd d $\quad d^{n(1-1/p)}\|x_\rho\|_p = d^{n(1-1/p)}\|W^{\mathrm{DV}}_\rho\|_p = \frac{\|W^{\mathrm{CV}}_{\rho_{\mathrm{GKP}}}\|_{p,\mathrm{cell}}}{\|W^{\mathrm{CV}}_{\mathrm{STAB}_n,\mathrm{GKP}}\|_{p,\mathrm{cell}}}$

where $\|x_\rho\|_p = \left(\sum_{\boldsymbol{l},\boldsymbol{m}\in\mathbb{Z}_d^n} |x_\rho(\boldsymbol{l},\boldsymbol{m})|^p\right)^{1/p}$

O. Hahn et al, PRX Quantum 6, 010330 (2025)

The l1 norm of x is a measure of magic

WACQT Wallenberg Centre for Quantum Technology

1. Invariance under Clifford unitaries U_C:
$\left\|x_{U_C \rho U_C^\dagger}\right\|_1 = \|x_\rho\|_1$

2. Multiplicativity: $\|x_{\rho\otimes\sigma}\|_1 = \|x_\rho\|_1 \|x_\sigma\|_1$

3. Stabilizer states achieve the minimum value:
$\|x_\phi\|_1 = 1$ for every pure stabilizer state ϕ, and $\|x_\psi\|_1 \geq 1$ for every pure state ψ.

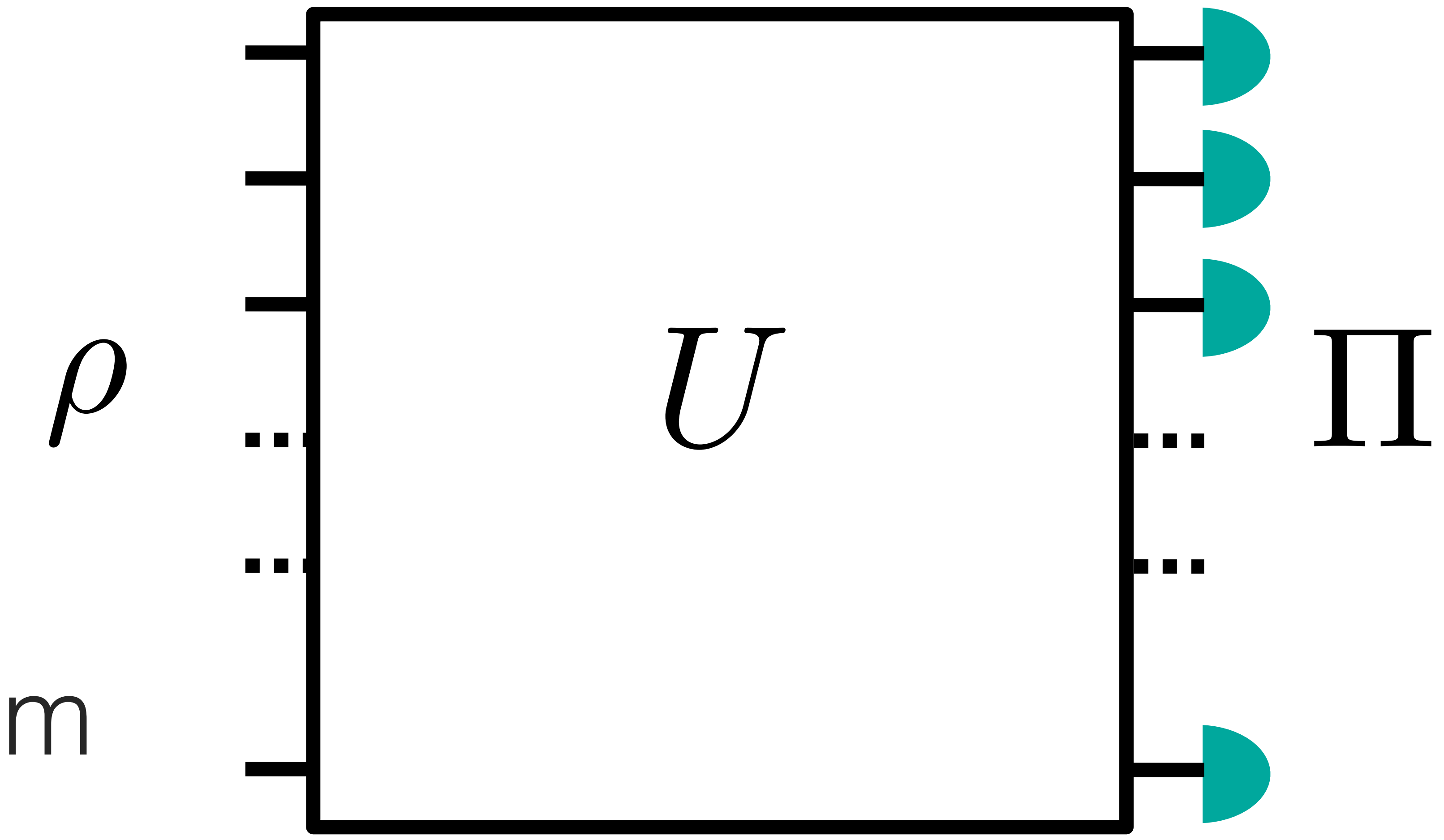
Odd dimensions: it reduces to negativity of discrete Wigner function (OK for mixtures):

4. Faithfulness: For a pure state ϕ, $\|x_\phi\|_1 = 1$ if and only if ϕ is a stabilizer state.

(This property is defined in terms of pure states)

5. Non increasing under stabilizer protocols (composition with stab, Pauli measurements, partial trace, the above operations conditioned on outcomes of Pauli measurements)

$$P(\Pi|U\rho U^\dagger) = \sum_{\lambda,\lambda'} x_\Pi(\lambda') x_U(\lambda',\lambda) x_\rho(\lambda)$$



The simulation cost scales with the aggregated l1 norm

$$\mathcal{M}_\rightarrow = \max_\lambda \|x_\rho(\lambda)\|_1 \max_{\lambda,\lambda'} \|x_U(\lambda,\lambda')\|_1 \max_{\lambda'} |x_\Pi(\lambda')|$$

Need $K \geq 2\mathcal{M}^2_\rightarrow \frac{1}{\epsilon^2} \ln\left(\frac{2}{p_f}\right)$ samples for precision ϵ with probability $(1-p_f)$

O. Hahn et al, PRX Quantum 6, 010330 (2025)

Same technique as in Pashayan et al, Phys. Rev. Lett. 115, 070501 (2015)

Bonus: non-Gaussian operations are needed to implement non-Clifford operations on the GKP code space

Do we need non-Gaussian operations, when GKP states are non-Gaussian themselves?

Suppose that the non-stabilizer state $\hat{\rho}$ is obtained from the stabilizer state $\hat{\sigma}$ with a Gaussian protocol, then

$$||W^{\mathrm{CV}}_{\rho_{\mathrm{GKP}}}||_{1,\mathrm{cell}} = ||W^{\mathrm{CV}}_{\mathcal{G}(\sigma_{\mathrm{GKP}})}||_{1,\mathrm{cell}} \leq ||W^{\mathrm{CV}}_{\sigma_{\mathrm{GKP}}}||_{1,\mathrm{cell}}$$

> Wstab

= Wstab

Contradiction! Non-Gaussian operations are needed for non-Clifford gates

O. Hahn et al, PRX Quantum 6, 010330 (2025)

Summary and conclusions on part 3b

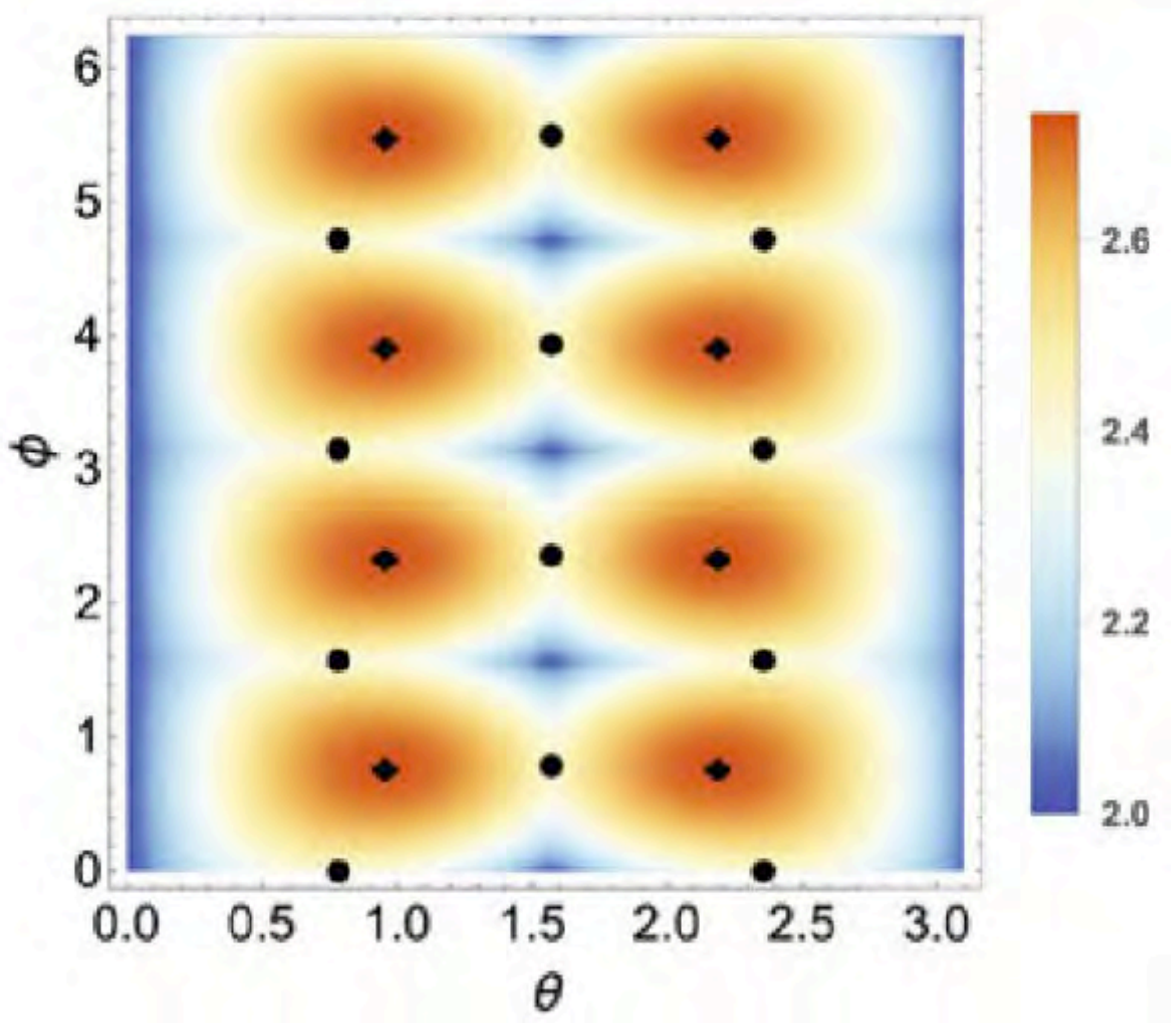
Conclusions and open questions

- We have defined a new magic measure, easier to compute, by mapping onto GKP states; equivalent to st-norm, as well as to Renyi-entropy with $\alpha = 1/2$

- Generalization to arbitrary dimensions yields negativity of discrete (Gross) Wigner function in odd-dimensions

- It quantifies the cost of simulating the circuit

- Non-Gaussian gates needed to implement non-Clifford gates



L. García-Álvarez et al, Springer (2021)
O. Hahn et al, PRL 128, 210502 (2022)
O. Hahn et al, PRX Quantum 6, 010330 (2025)

Continuous-variable quantum computing theory group /WACQT

Wallenberg Centre for Quantum Technology

Thanks to:

- My amazing group and collaborators!!
- The sources of my financial support

Thank you for coming and listening!!!

WACQT | Swedish Research Council | Area of Advance NANO | veriqub EIC Pathfinder CHALLENGES | OLLE ENGKVISTS STIFTELSE