Atomic Structure and Radiation

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Few notes

- Not a complete course on A&M data for plasmas
- Don’t feel discouraged if you see something familiar (you certainly will)
- Ask questions
- Talk with the other students
- Talk with the lecturers

What is this about?...and not

- Most general overview of atomic parameters that are relevant to plasma spectroscopy
- Qualitative estimates
- Selection rules
- Scalings
- Fascinating things
- …

- Radiation transport
- Line broadening
- Continuum

A few terrific textbooks on Atomic Physics/Plasma Spectroscopy

R.D. Cowan
- Theory of Atomic Structure and Spectra (1981)

I.I Sobelman
- Atomic Spectra and Radiative Transitions (1979)

A. Thorne et al
- Spectrophysics (1999)

L.A. Vainshtein and V.P. Shevelko
- Atomic Physics for Hot Plasmas (1993)

H.R. Griem
- Plasma Spectroscopy (1964)
- Principles of Plasma Spectroscopy (1997)

W. Lochte-Holtgreven (ed.)
- Plasma Diagnostics (1968)

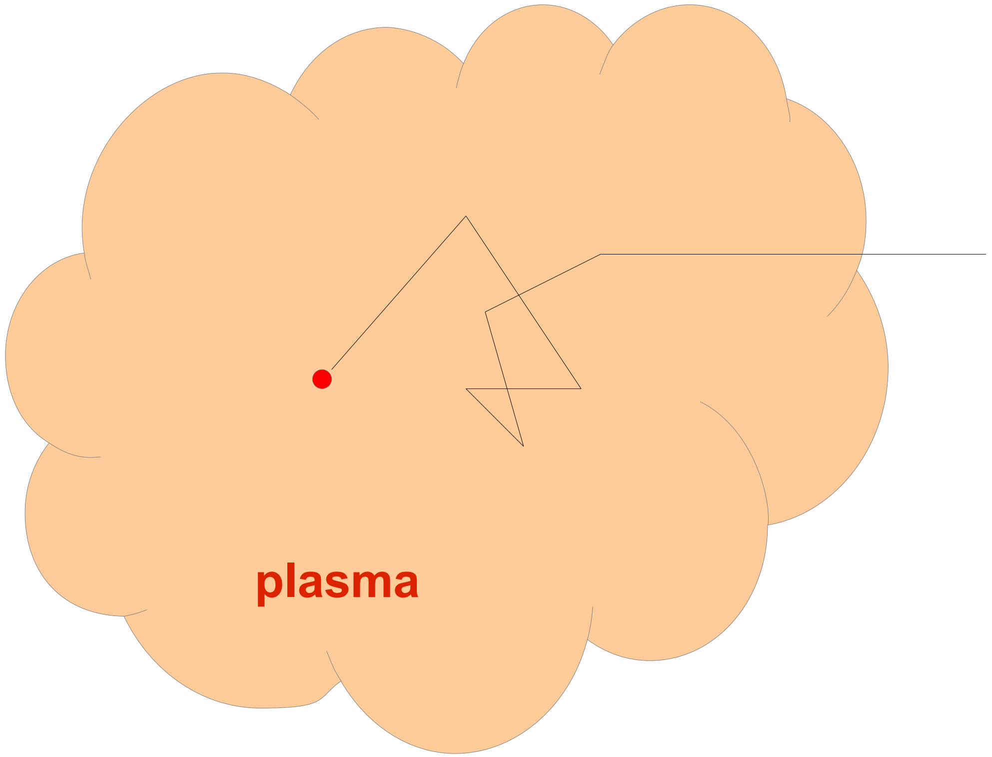
D. Salzmann
- Atomic Physics in Hot Plasmas (1998)

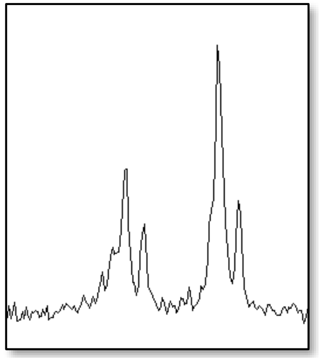
T. Fujimoto
- Plasma Spectroscopy (2004)

H.-J. Kunze
- Introduction to Plasma Spectroscopy (2009)

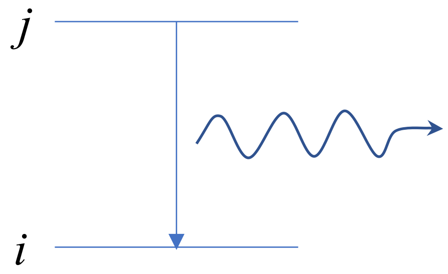
J. Bauche, C. Bauche-Arnoult, O. Peyrusse
- Atomic Properties in Hot Plasmas (2015)

Life of a photon





Example: Spectral Line Intensity



Upper state density (cm^{-3})

Einstein coefficient or transition probability (s^{-1})

strongly depends on plasma conditions

$$I_{ij} = N_j \cdot A_{ij} \cdot E_{ij}$$

(*almost*) purely atomic parameters

Energy emitted due to a specific transition from a unit volume per unit time

Photon energy (eV)

http://physics.nist.gov/cuu/Units/

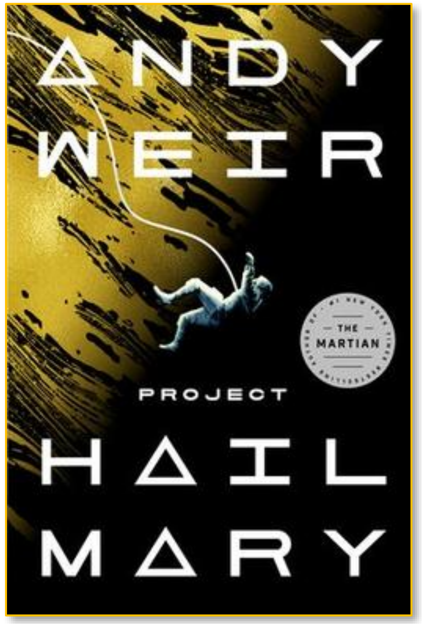
Major atomic units/constants

- Energy
  - **1 Ry** ≈ 13.6057 eV ≈ 109 737 cm^{-1} = ½ Hartree (a.u.)
    - (*ionization energy of H*)
  - **1 eV** ≈ 8065.5439 cm^{-1}
- Length
  - $\mathbf{a_0} \approx 5.29 \cdot 10^{-9}$ cm = 0.529 Å (radius of H atom) [dimension]
- Area (cross section)
  - $\pi \mathbf{a_0}^2 \approx 8.8 \cdot 10^{-17}$ cm^2 (area of H atom)
- Velocity
  - $\mathbf{v_0} \approx 2.2 \cdot 10^8$ cm/s = $\alpha c \approx c/137$

A glimpse into the future…

20th Century Fox

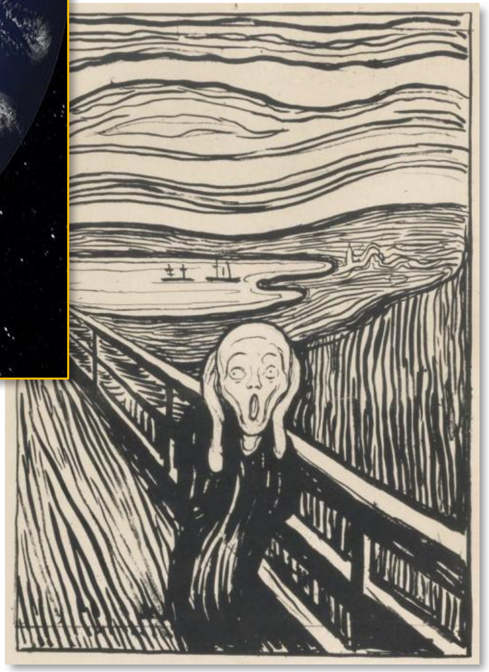
Photo by Jonathan Olley / Amazon MGM Studios



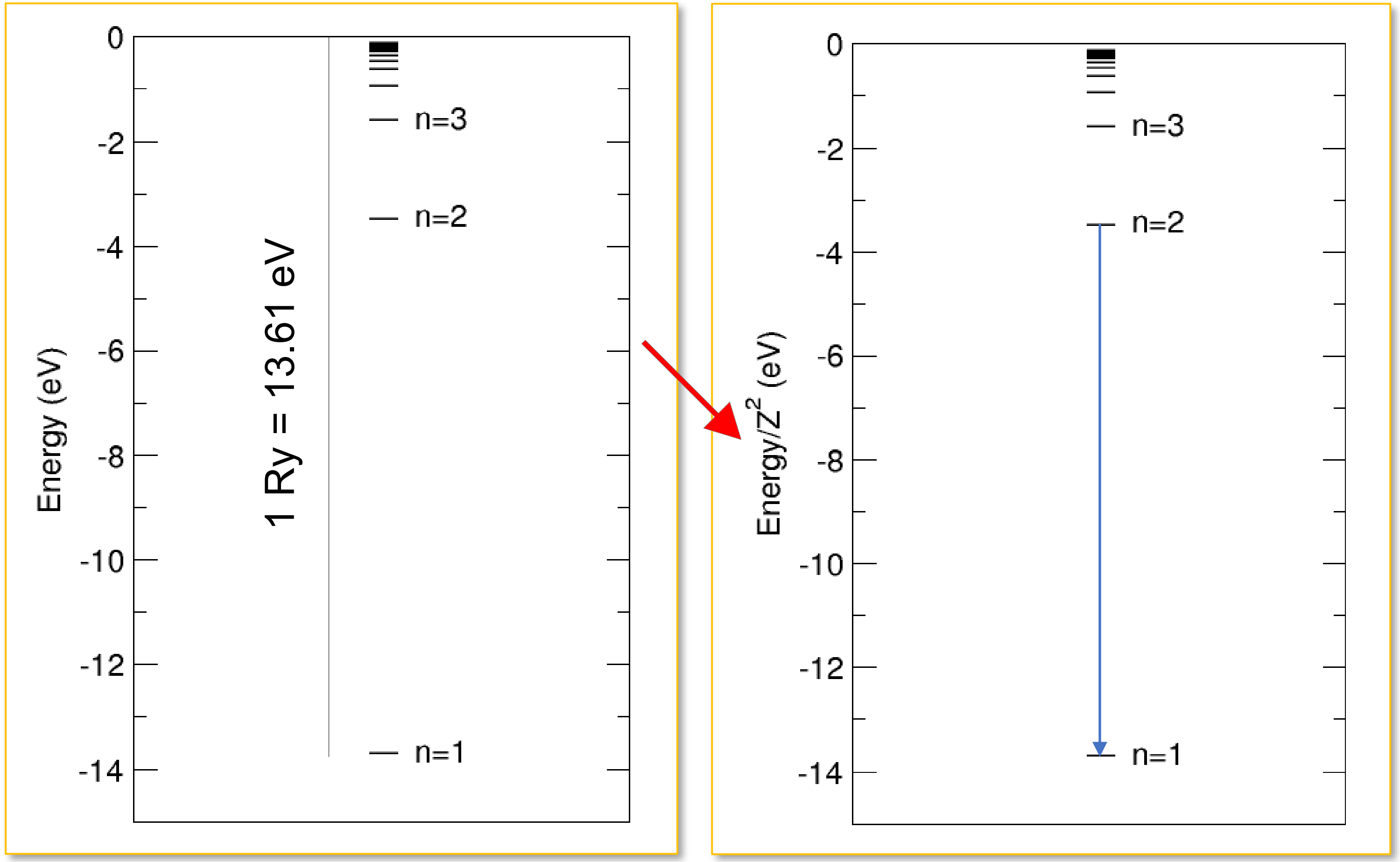
Photos: Amazon MGM Studios

NASA Goddard Space Flight Center / Image by Reto Stöckli

Rocky: to save the Earth, you've got 20 seconds to calculate the Lyα energy (no tools, within 3%) for H-like **XX**!!!



Edvard Munch / *The Scream*



Hydrogen atom

H-like ion

Radius: $a_n \sim \dfrac{n^2}{Z_N}$

Energy: $E_n = -\dfrac{Z_N^2 Ry}{n^2}$

$$E_{21} = Z_N^2 Ry\left(1 - \frac{1}{2^2}\right) = Z_N^2 Ry \cdot \frac{3}{4}$$

Homework: study Schrödinger equation with *ρ=rZ*

General mathematical scale-invariance

$$f(\lambda x) = \lambda^{\alpha} f(x)$$

Examples: $f(x) = Ax^n$

But NOT: $f(x) = A + Bx$

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\nabla^2 \boldsymbol{B} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{B}}{\partial t^2}$$

$$x \to \lambda x, t \to \lambda t$$

$$a_n \sim \frac{n^2}{Z_N}$$

Complex atoms (non-relativistic)

We know all important interactions:

$$H = H_{kin} + H_{elec-nucl} + H_{elec-elec} + H_{s-o} + \ldots$$

$$= -\sum_i \frac{1}{2}\nabla_i^2 - \sum_i \frac{Z_n}{r_i} + \sum_{i>j} \frac{1}{r_{ij}} + \sum_i \frac{1}{2}\xi_i(r_i)(\boldsymbol{l}_i \cdot \boldsymbol{s}_i) + \ldots$$

$$H\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots) = E\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots)$$

The Schrödinger equation for multi-electron atoms cannot be solved exactly…

Each atomic state (wavefunction) is characterized by a set of quantum numbers

- Generally speaking, **only two are exact**:
  - Total angular momentum (*rotation invariance*)
  - Parity = $(-1)^{\sum_i l_i}$ (*spatial invariance*)

- **Everything else (L,S,*n*,…) is not exact!**

“Standard” procedure

- Use central-field approximation to approximate the effects of the Coulomb repulsion among the electrons:
  - $H \approx H_0 = \sum_i^N \left(-\frac{1}{2}\nabla_i^2 \; - \frac{Z}{r_i} + V(r_i) \right)$
- Properly choose the potential V(r) (e.g., self-consistent field)
- Find configuration state functions $\Phi(\gamma_j LS)$ (accounting also for antisymmetry): ***n*,*l***
- Assume that the atomic state function is a linear combination of CSFs: $\Psi(\gamma LS) = \sum_j^M c_j \Phi(\gamma_j LS)$
- Solve Schrodinger equation for mixing coefficients:
  - $(\hat{H} - E\hat{I})\hat{c} = 0, H_{ij} = \langle \Phi(\gamma_i LS) | H | \Phi(\gamma_j LS) \rangle$
- Include other effects (perturbation theory)

Relativistic atomic structure: heavy and not so heavy ions

$$H_{DC} = \sum_i (c\,\boldsymbol{\alpha}_i \cdot \boldsymbol{p}_i + V_{nuc}(r_i) + \beta_i c^2) + \sum_{i>j} \frac{1}{r_{ij}}$$

Dirac-Coulomb Hamiltonian

$\boldsymbol{p} \equiv -i\boldsymbol{\nabla}$ electron momentum operator

$\boldsymbol{\alpha}, \beta$ 4x4 Dirac matrices

$V_{nuc}(r)$ extended nuclear charge distribution

Transverse photons (magnetic interactions and retardation effects):

$$H_{TP} = -\sum_{j>i} \left[\frac{\alpha_i \cdot \alpha_j \cos(\omega_{ij} r_{ij}/c)}{r_{ij}} + (\boldsymbol{\alpha}_i \cdot \boldsymbol{\nabla}_i)(\boldsymbol{\alpha}_j \cdot \boldsymbol{\nabla}_j) \frac{\cos(\omega_{ij} r_{ij}/c) - 1}{\omega_{ij}^2 r_{ij}/c^2} \right]$$

QED effects: self energy (SE), vacuum polarization (VP)

$$H_{DCB+QED} = H_{DC} + H_{TP} + H_{SE} + H_{VP} + \ldots$$

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Relativistic notations

	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	$f_{5/2}$	$f_{7/2}$
	s	p_-	p_+	d_-	d_+	f_-	f_+
l	0	1	1	2	2	3	3
j	½	½	3/2	3/2	5/2	5/2	7/2

Atomic Structure Methods and Codes

- Coulomb approximation (Bates-Damgaard)
- Thomas-Fermi (**SUPERSTRUCTURE**, **AUTOSTRUCTURE**)
- Single-configuration Hartree-Fock (self-consistent field)
  - **Cowan's code + modifications**
- Model potential (including relativistic)
  - **HULLAC**, **FAC**
- Multiconfiguration Hartree-Fock (**`http://nlte.nist.gov/MCHF`**)
- Multiconfiguration Dirac-Hartree-Fock (**MCDHF**)
  - **GRASP2K** (**`http://nlte.nist.gov/MCHF`**)
  - **Desclaux's code**
- Various perturbation theory methods (RMBPT…)
- B-splines

Z_c-scaling of one-electron energies

Spectroscopic charge: **Z_c= ion charge + 1** (H I, Ar XV…)

This is the charge that is seen by the outermost (valence) electron

$$E = E_0 \boldsymbol{Z_c^2} + E_1 \boldsymbol{Z_c} + E_2 + E_3 \boldsymbol{Z_c^{-1}} + \ \ldots$$

non-relativistic

$$E_0 = -\frac{1}{n^2}$$ hydrogenic term

Therefore, for high Z_c the energy structure looks more and more H-like!

Of course, relativistic effects slightly modify this dependence
but the general trend remains valid

K I: $1s^2 2s^2 2p^6 3s^2 3p^6 4s$

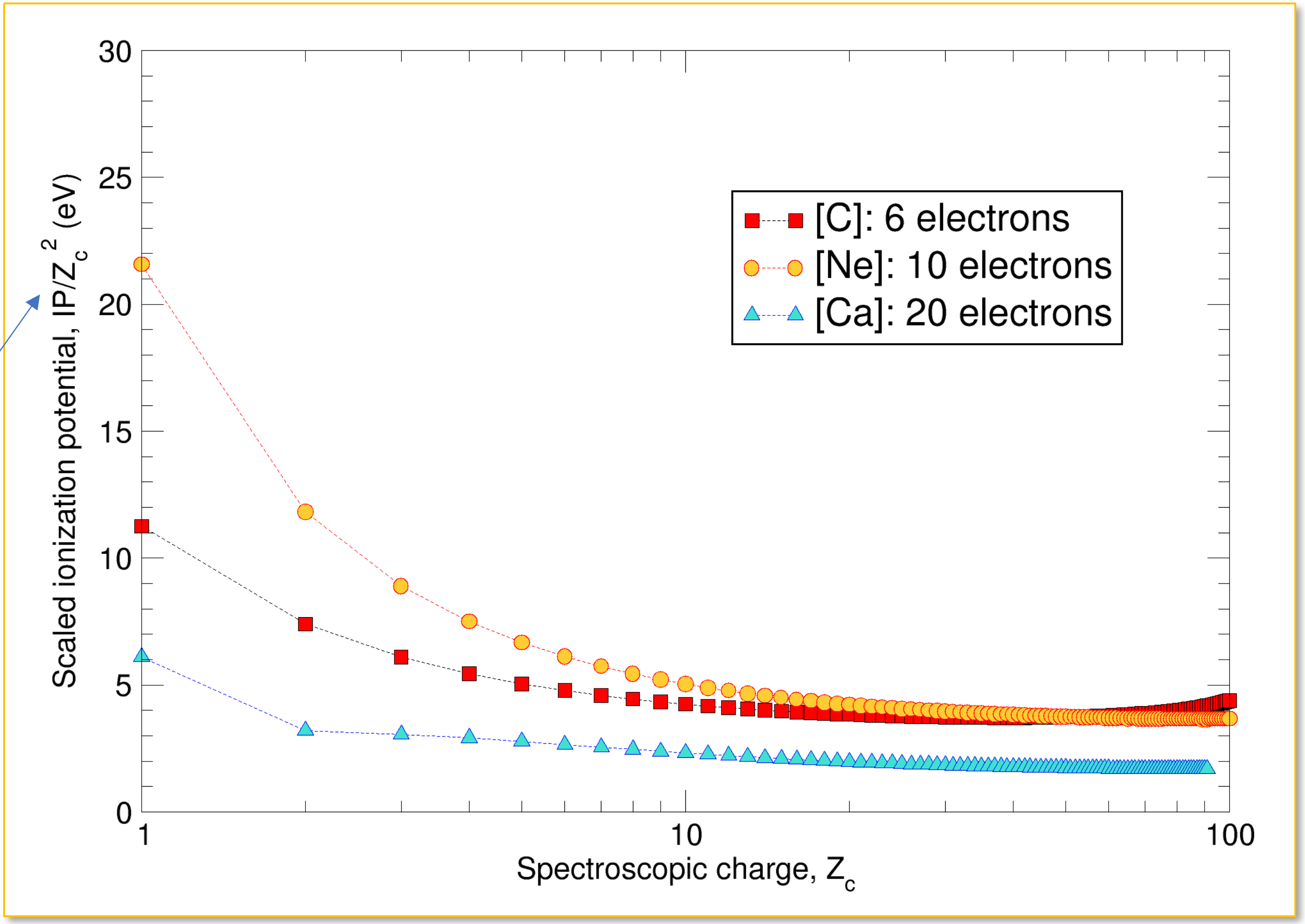
W LVI: $1s^2 2s^2 2p^6 3s^2 3p^6 3d$

Ionization potentials

NIST ASD

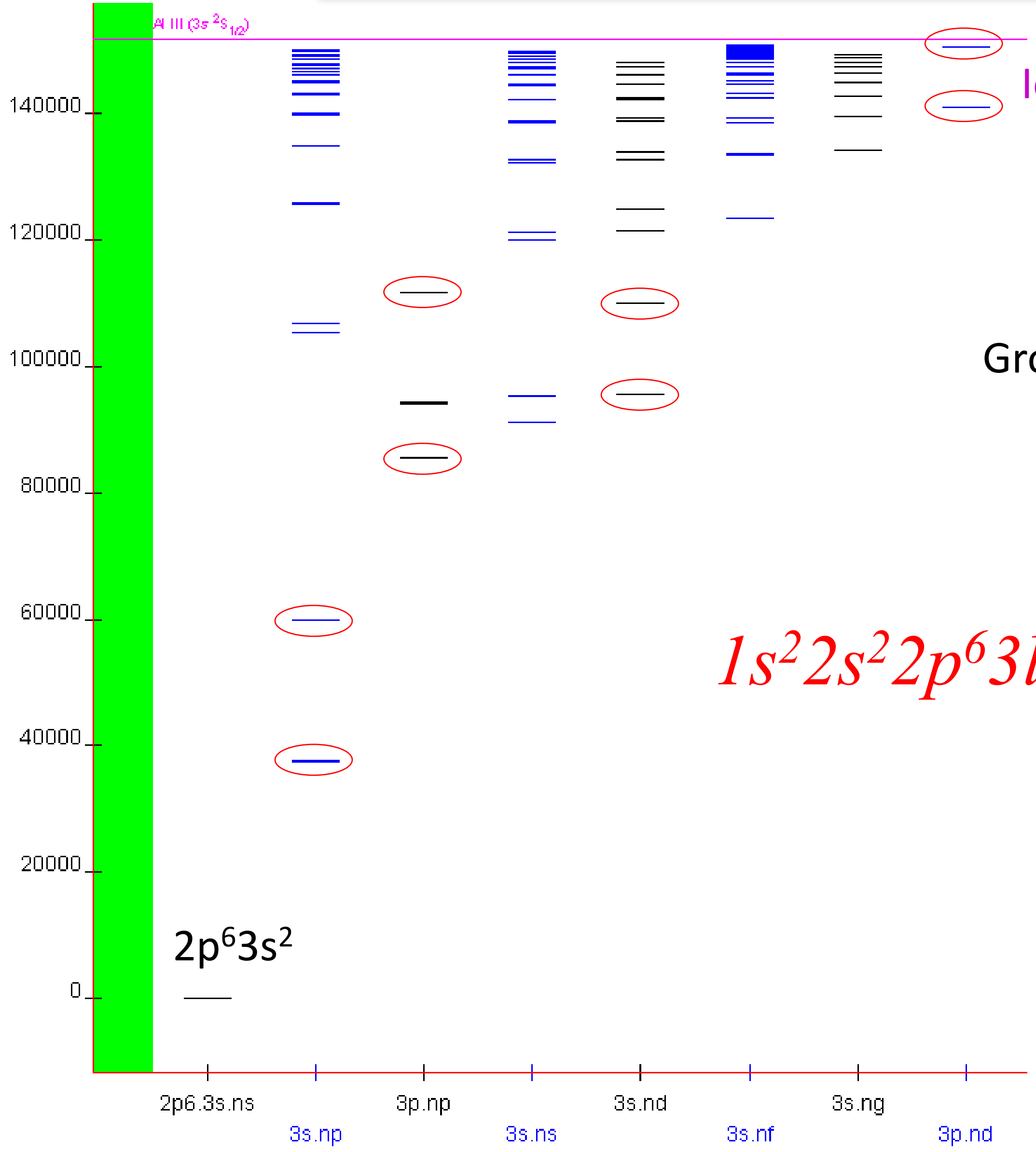
- IPs are directly connected with ionization distributions in plasmas
- Most often are determined from Rydberg series

Scaled!



Mg-like Al II: *3l3l'*

$$E = E_0 Z_c^2 + E_1 Z_c + E_2 + E_3 Z_c^{-1} + \ldots$$



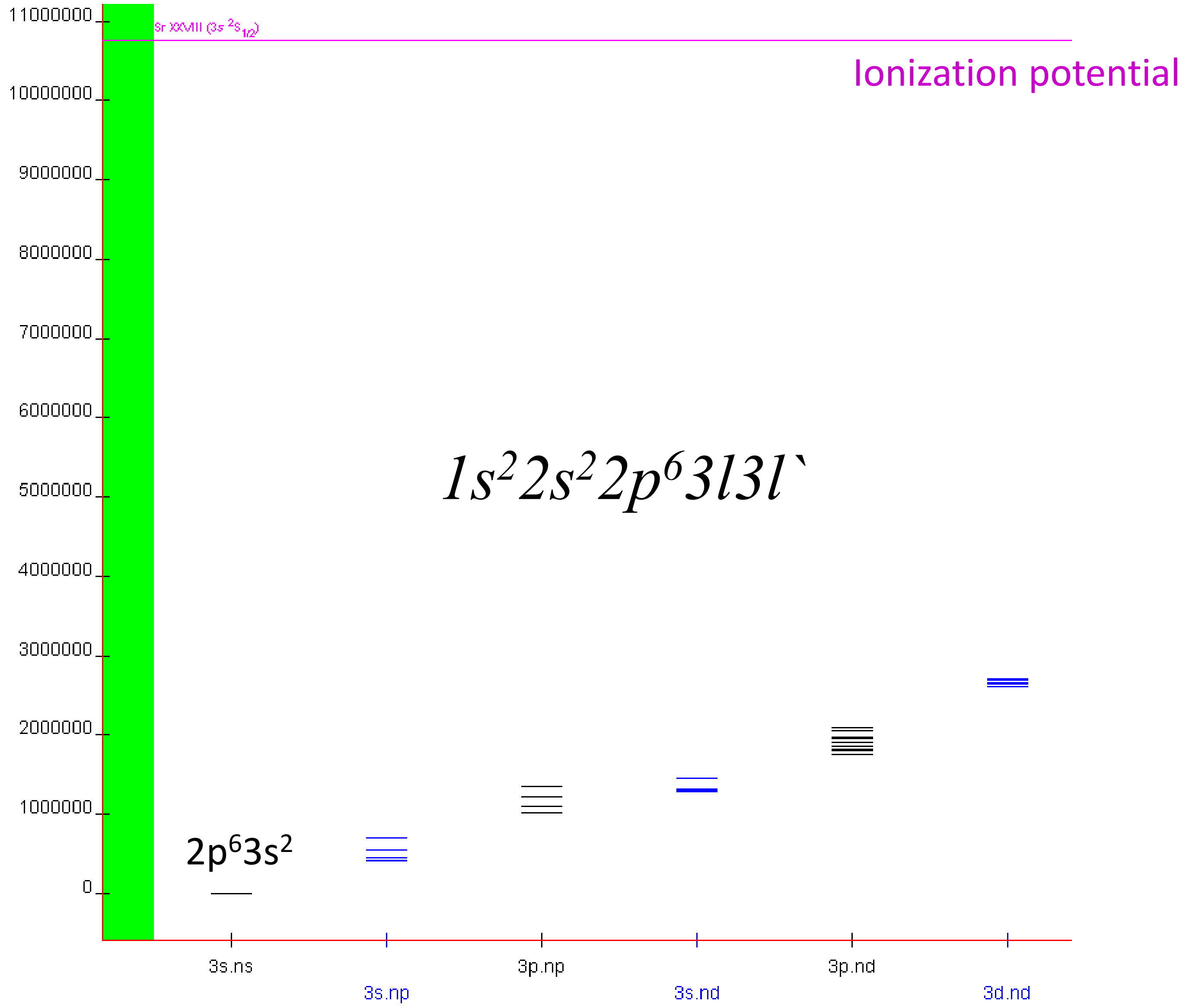
Ionization potential

Grotrian diagram

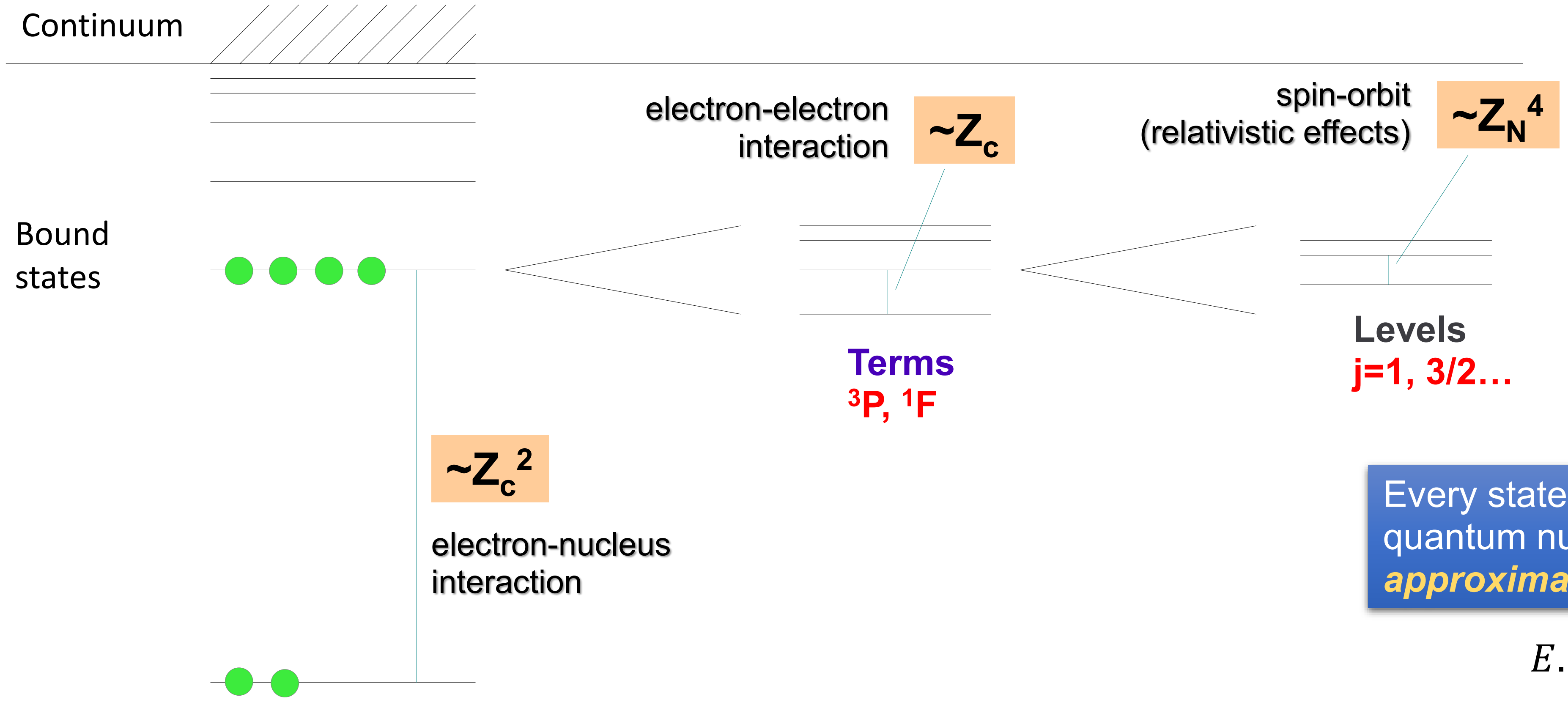
$1s^2 2s^2 2p^6 3l3l`$

Mg-like Sr XXVII: *3l3l′*

$$E = E_0 Z_c^2 + E_1 Z_c + E_2 + E_3 Z_c^{-1} + \ldots$$



Energy structure of a (relatively light) ion



Every state is defined by a set of quantum numbers which are mostly ***approximate***

$E.g.:\ 1s^2 2s^2 2p^5 3d \quad {}^3F_3^o$

Electrons are grouped into shells ***nl***
(K *n*=1, L *n*=2, M *n*=3,...)
producing **configurations**

$$\Delta E(\Delta n \neq 0) \sim Z_c^{\,2}$$
$$\Delta E(\Delta n = 0) \sim Zc$$

Spin-orbit (relativistic!) interaction

Hydrogenic ion: $$\zeta_{nl} = \frac{Ry\,\alpha^2\,Z_N^4}{n^3\,l\,(l+1/2)\,(l+1)}$$

Semi-theoretical **Lande formula**: $$\zeta_{nl} = \frac{Ry\,\alpha^2\,Z_c^2\tilde{Z}^2}{n^{*3}\,l\,(l+1/2)\,(l+1)}$$

n*: effective n $$IP = \frac{Ry\,Z_c^2}{n^{*2}}$$ (ionization potential)

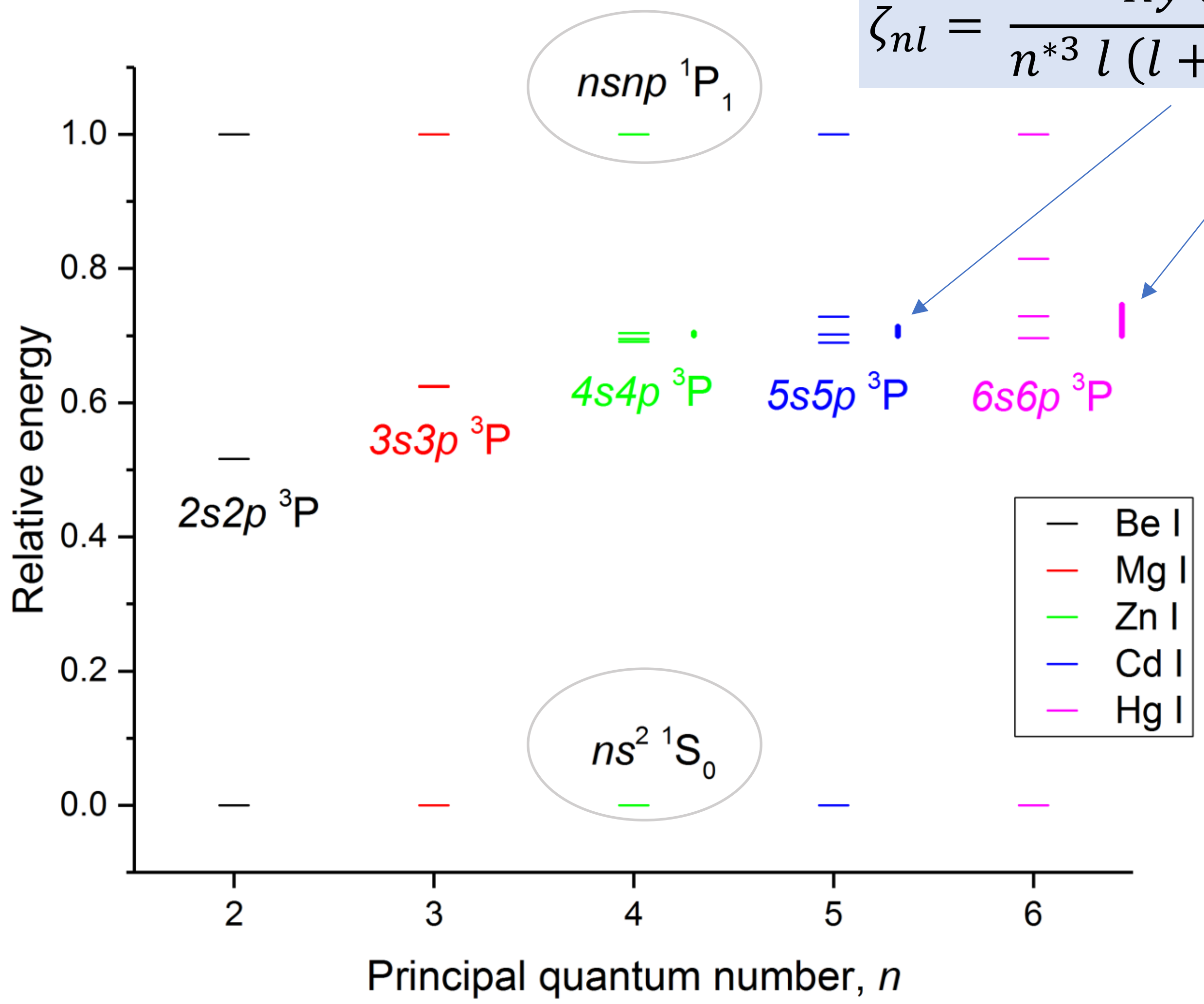
$\tilde{Z}$: effective nuclear charge (for penetrating orbits) = $\boldsymbol{Z_N}$**-*n*** for *np* orbitals

$$H = -\sum_i \frac{1}{2}\nabla_i^2 - \sum_i \frac{Z_N}{r_i} + \sum_{i>j}\frac{1}{r_{ij}} + \sum_i \frac{1}{2}\xi_i(r_i)(\boldsymbol{l_i}\cdot\boldsymbol{s_i}) + \ldots$$

Spin-orbit interaction does depend on nuclear charge!

$$\zeta_{nl} = \frac{Ry\ \alpha^2\ Z_c^2 \tilde{Z}^2}{n^{*3}\ l\ (l+1/2)\ (l+1)}$$

Lande formula



Types of coupling

- **LS-coupling**: electron-electron » spin-orbit
  - light elements

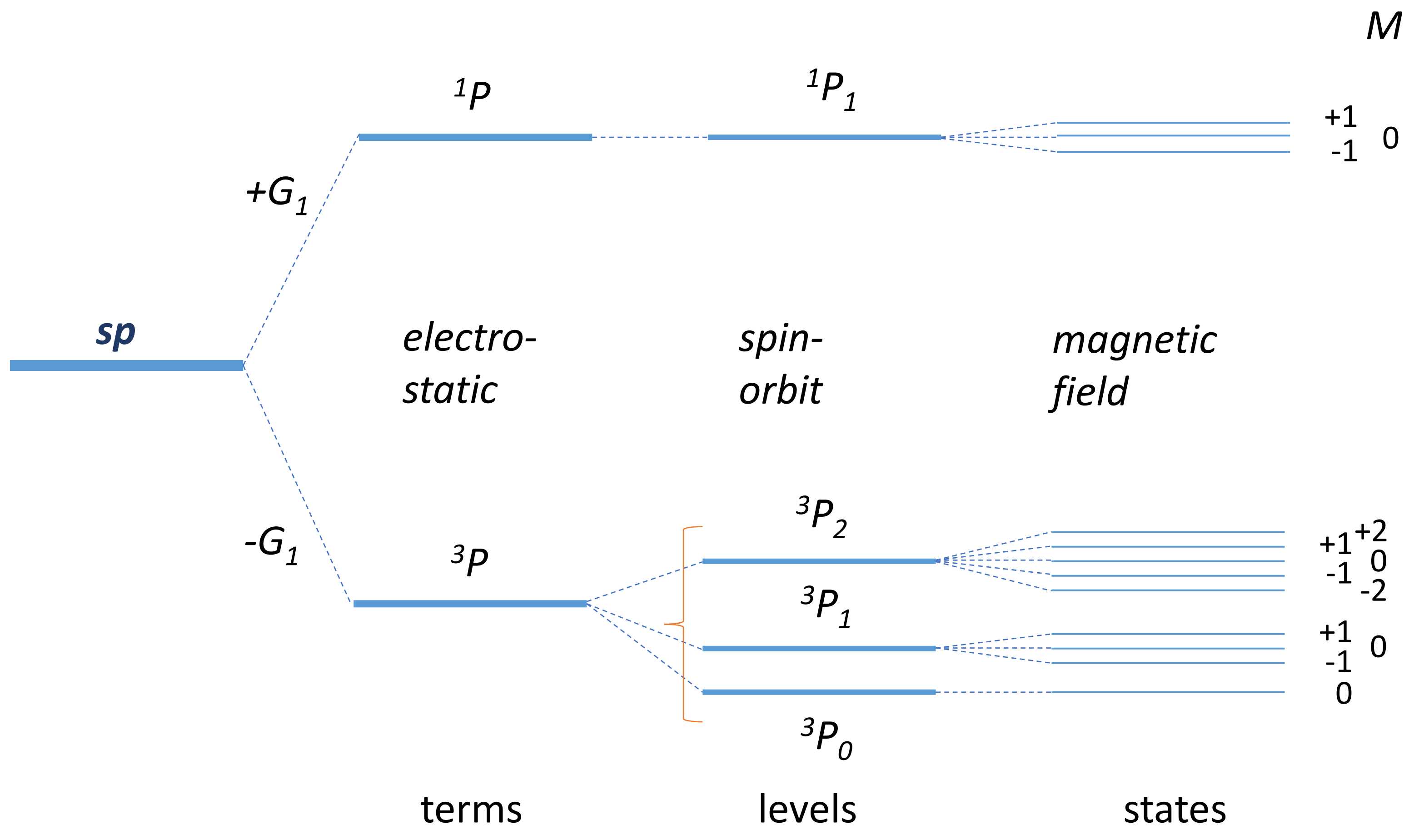
$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \ldots, \quad \vec{S} = \vec{s}_1 + \vec{s}_2 + \ldots, \quad \vec{J} = \vec{L} + \vec{S}$$

- **jj-coupling**: spin-orbit » electron-electron
  - heavy elements
  - 2s2p: $(2s_{1/2}, 2p_{1/2})$ or $(2s, 2p_-)$
  - $3d^5$: $((3d_-^3)_{5/2}, (3d_+^2)_2)_{3/2}$

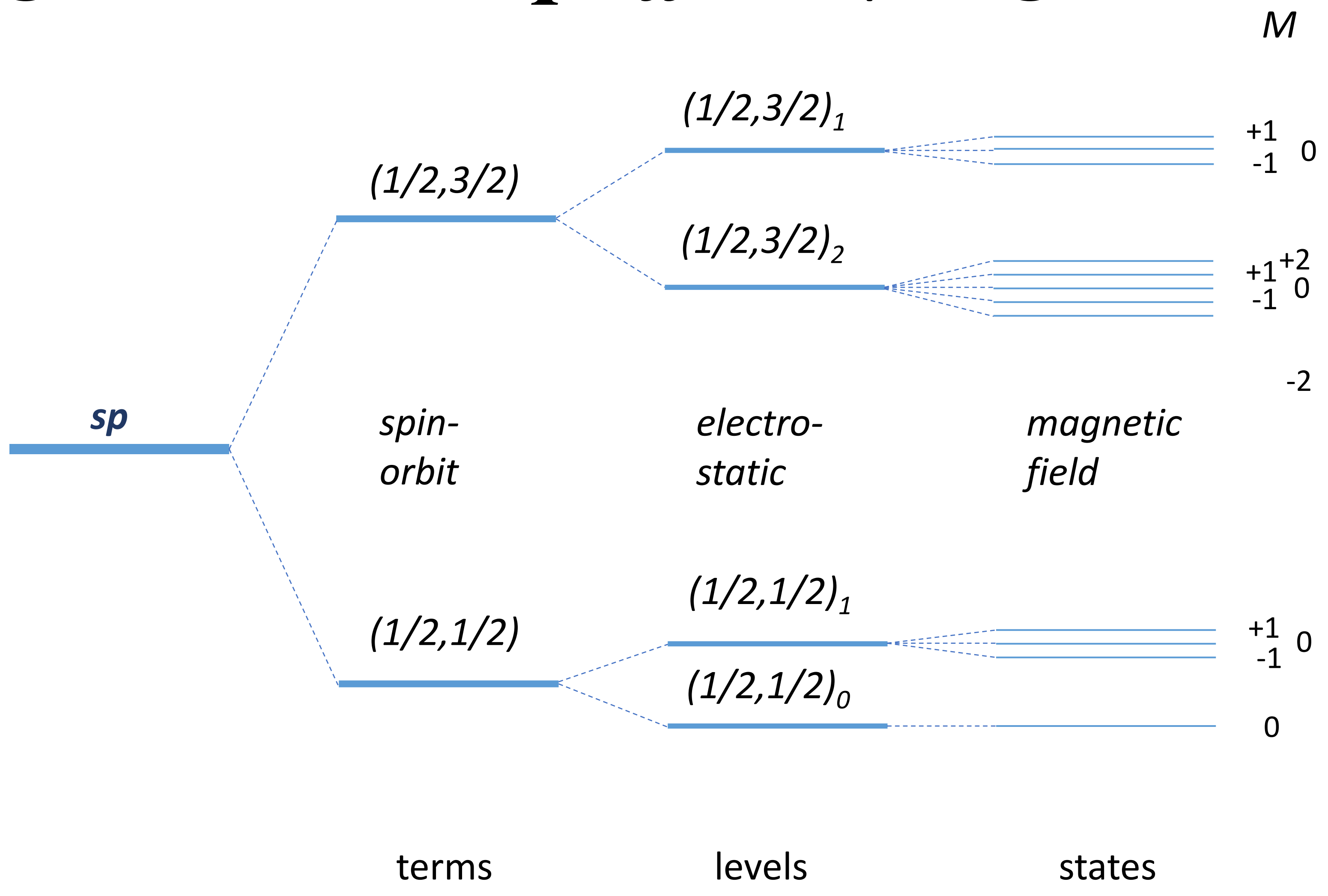
$$\vec{j}_1 = \vec{l}_1 + \vec{s}_1, \; \vec{j}_2 = \vec{l}_2 + \vec{s}_2, \; \ldots \quad \vec{J} = \vec{j}_1 + \vec{j}_2 + \ldots$$

- ***Intermediate coupling***
  - neither LS or jj is overwhelmingly strong
- Other types of couplings exist (jK, LK, J_1J_2,…)

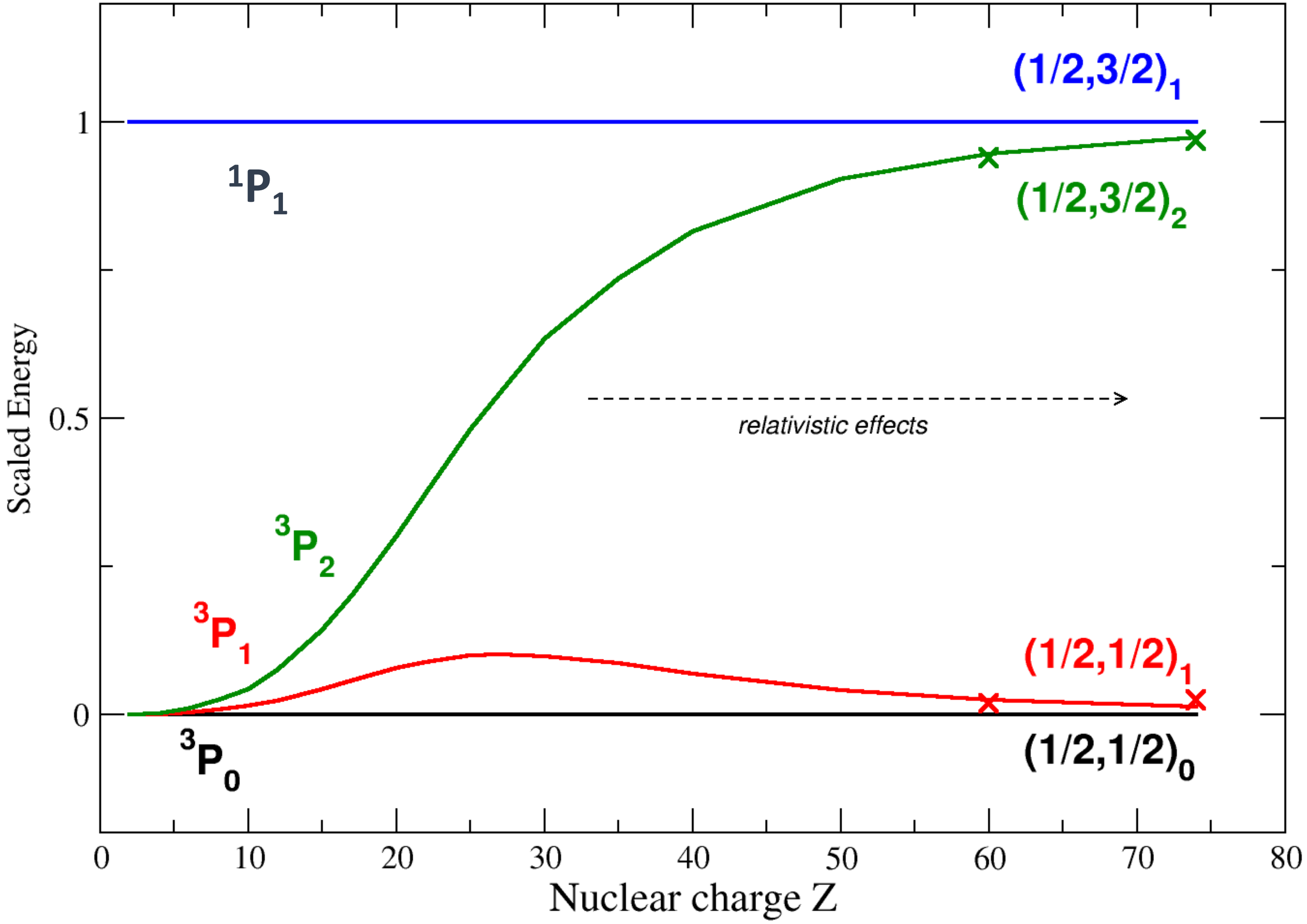
Configuration *nsn'p*: LS coupling (LSJ)



Configuration *nsn'p*: jj coupling



From LS to jj: **1*s*2*p*** in He-like ions



Non-trivial coupling (J_1J_2)

Np-like ion: 93 electrons

Closed shells: unimportant

$1s^22s^22p^63s^23p^63d^{10}4s^24p^64d^{10}4f^{14}5s^25p^65d^{10}5f^46s^26p^66d7s7p$

J=9/2

$5f^4(^5I_4)6d_{3/2}\ (4,{}^3/_2)_{11/2}7s7p(^1P^\circ{}_1)\ \ ({}^{11}/_2,\ 1)^\circ{}_{\mathbf{9/2}}$

State mixing: intermediate coupling, configuration interaction

$$|\Psi(a,b,c,\ldots)\rangle = \sum_i \boldsymbol{\alpha_i} \Psi_i(a',b',c',\ldots)$$

expansion coefficients

He-like Na^{9+}: $1s2p\ {}^3P_1 = \mathbf{0.999}\ {}^3P + \mathbf{0.032}\ {}^1P$

He-like Fe^{24+}: $1s2p\ {}^3P_1 = \mathbf{0.960}\ {}^3P + \mathbf{0.281}\ {}^1P$

He-like Mo^{40+}: $1s2p\ {}^3P_1 = \mathbf{0.874}\ {}^3P + \mathbf{0.486}\ {}^1P$

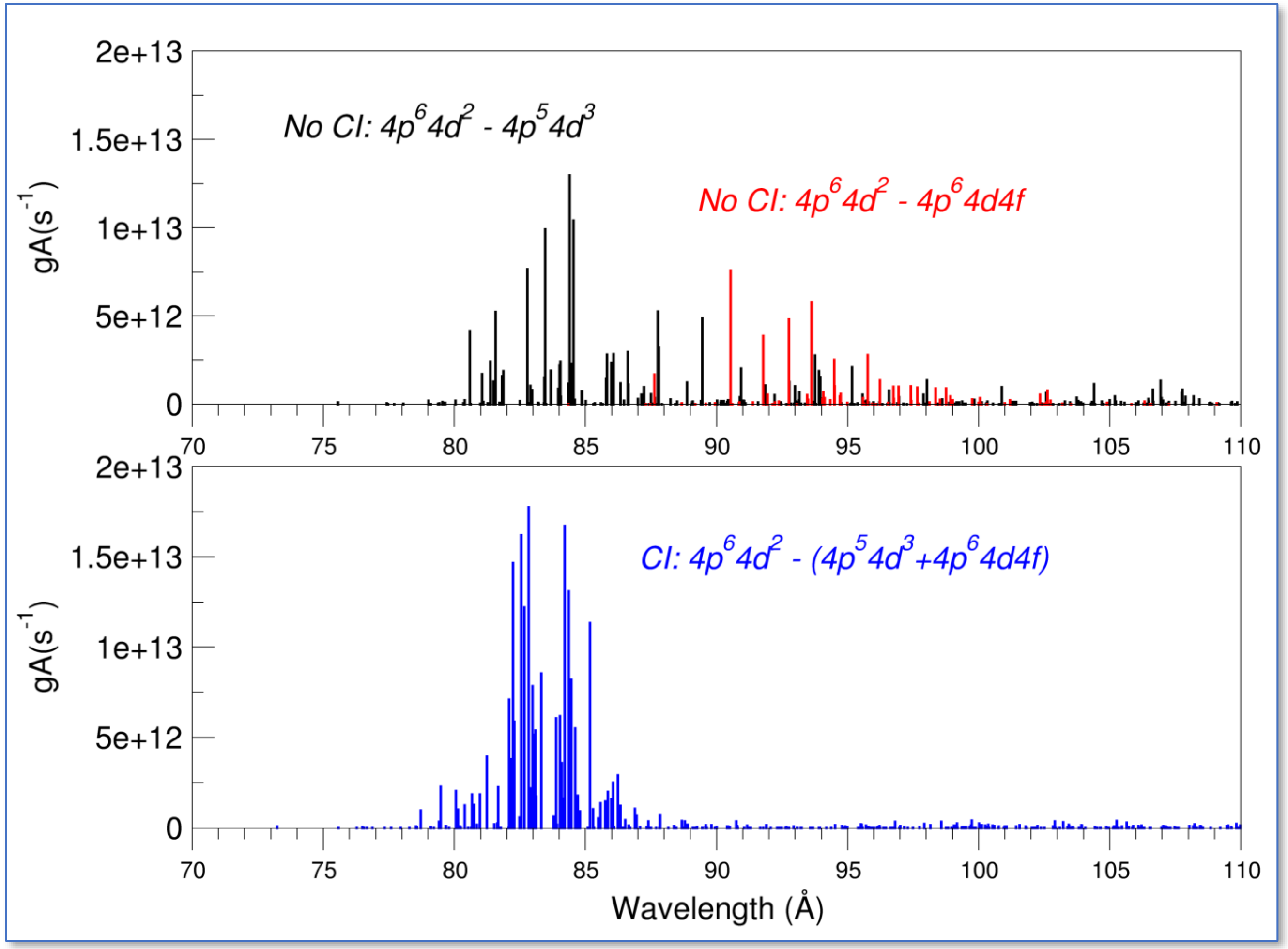
s-o coupling increases with Z $\Rightarrow$ change of coupling scheme

Very, VERY important for radiative transitions!!!

Configuration interaction example

From J. Bauche et al

Pr XXII: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^2$



Hund's rules (equivalent electrons, LS)

- Largest *S* has the lowest energy
- Largest *L* with the same *S* has the lowest energy
- For atoms with less-than half-filled shells, ***lowest J has lowest energy***

- Lande interval rule:
  - $E(J)-E(J-1) = \beta J$

C I

Configuration	Term	J	Level (cm^{-1})	Reference
$2s^22p^2$	^3P	0	0.00	L7288
		1	16.40	
		2	43.40	
$2s^22p^2$	^1D	2	10 192.63	
$2s^22p^2$	^1S	0	21 648.01	
$2s2p^3$	$^5S°$	2	*33 735.20*	
$2s^22p3s$	$^3P°$	0	*60 333.43*	
		1	*60 352.63*	
		2	*60 393.14*	
$2s^22p3s$	$^1P°$	1	*61 981.82*	
$2s2p^3$	$^3D°$	3	*64 086.92*	
		1	*64 089.85*	
		2	*64 090.95*	

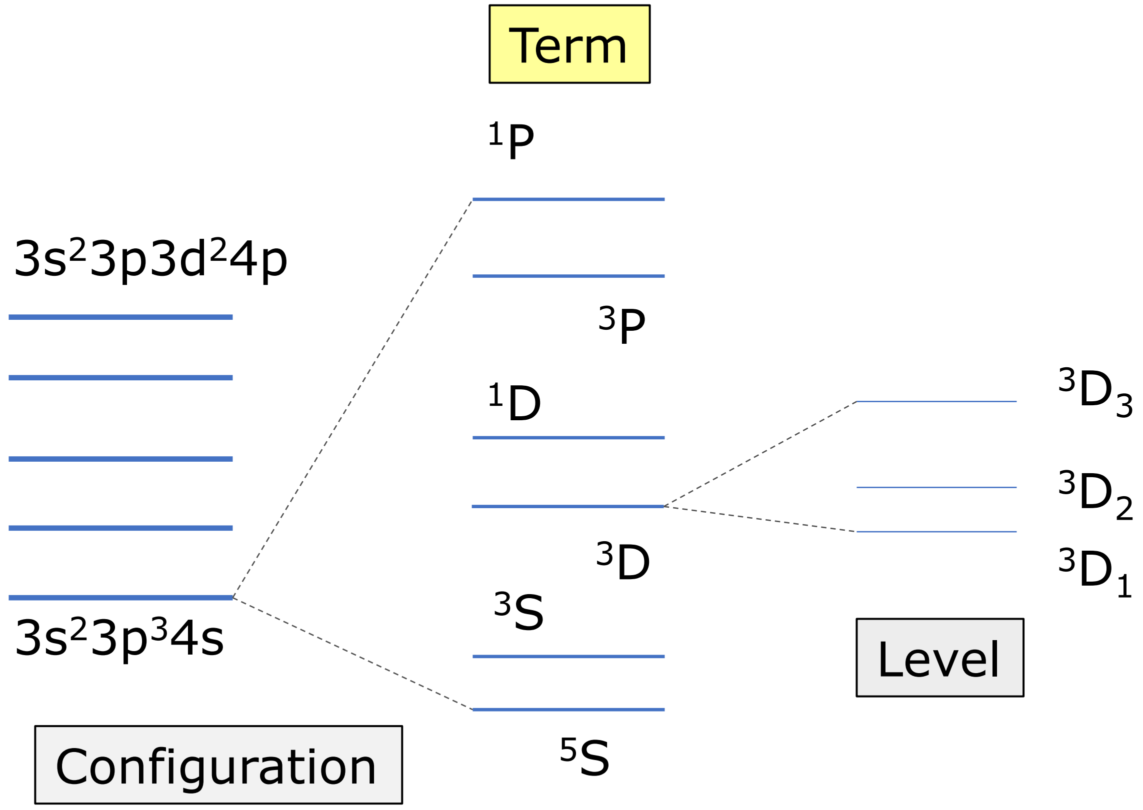
S I

- For atoms with more-than half-filled shells, largest J has lowest energy

- Lande interval rule:
  - $E(J)\text{-}E(J\text{-}1) = \beta J$

Configuration	Term	J	Level (cm^{-1})
$3s^2 3p^4$	^3P	2	0.000
		1	396.055
		0	573.640
$3s^2 3p^4$	^1D	2	9 238.609
$3s^2 3p^4$	^1S	0	22 179.954
$3s^2 3p^3(^4S°)4s$	$^5S°$	2	52 623.640
$3s^2 3p^3(^4S°)4s$	$^3S°$	1	55 330.811
$3s^2 3p^3(^4S°)4p$	^5P	1	63 446.065
		2	63 457.142
		3	63 475.051

16-electron ion (S-like)



Even parabolic states for motional Stark effect!

Superconfigurations

Motivation: for very complex atoms (ions) not only the **number of levels** is overwhelmingly large, but also the **number of configurations**

Example:

$1s^2 2s^2 2p^5 3s$
$1s^2 2s^2 2p^5 3p$
$1s^2 2s^2 2p^5 3d$
$1s^2 2s 2p^6 3s$
$1s^2 2s 2p^6 3p$
$1s^2 2s 2p^6 3d$

→

different *n*'s

$(1s)^2(2s2p)^7(3s3p3d)^1 \equiv (1)^2(2)^7(3)^1$

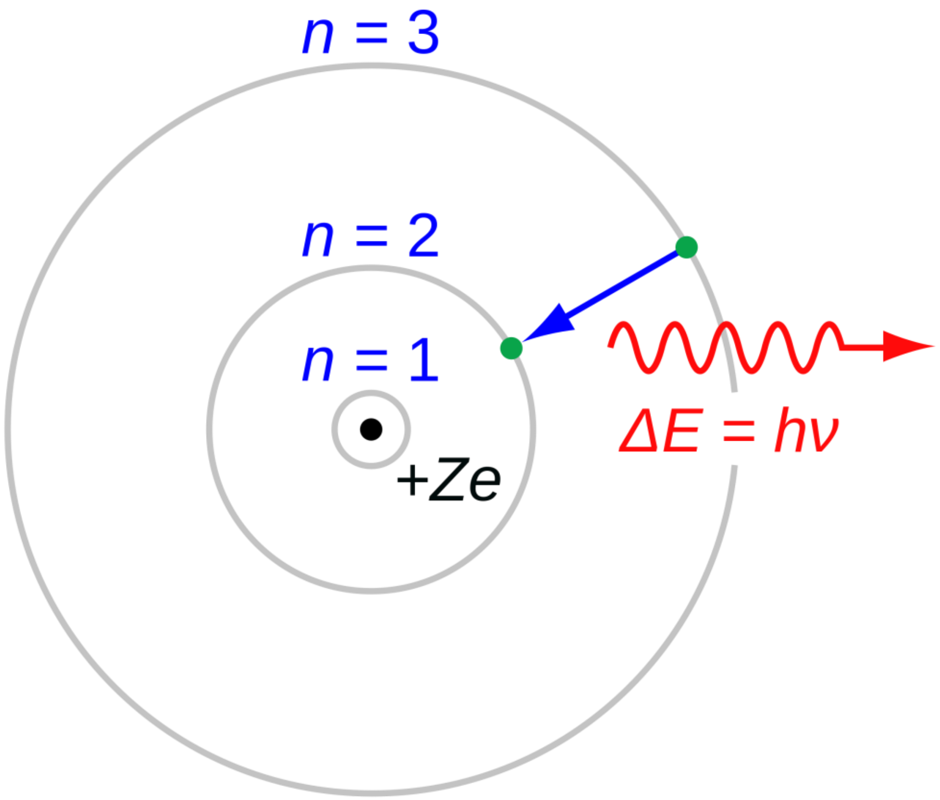
BUT: $(1s)^2(2s2p)^7(3s3p3d4s4p4d4f)^1$

Instead of producing millions or billions of lines, SCs are used to calculate Super Transition Arrays → Statistical methods

FLYCHK, CRETIN, DEDALE…

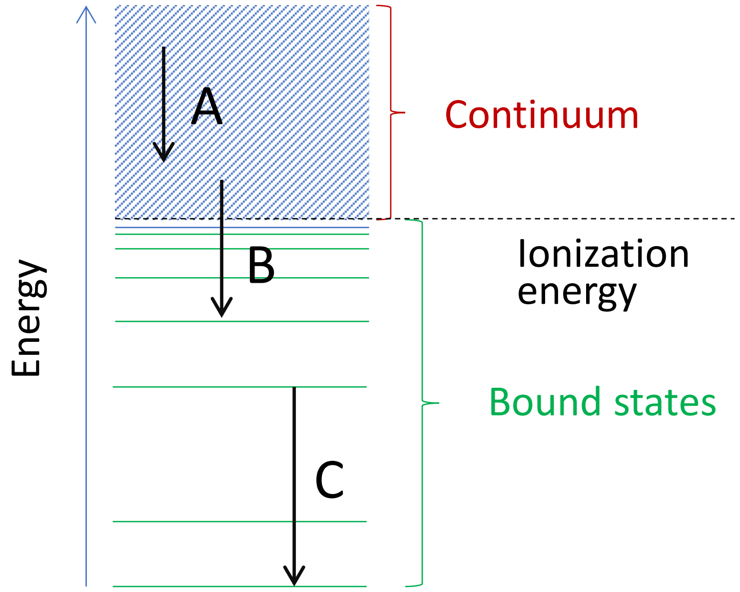
See J. Bauche et al's book (2015)

Now to radiative processes…



Three major sources of photons

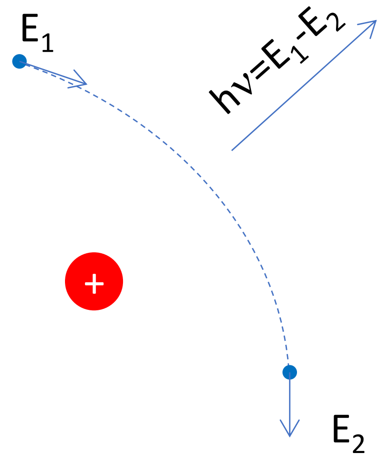
- Free-free transitions (bremsstrahlung)
  - $A^{z+} + e \rightarrow A^{z+} + e + h\nu$
- Free-bound transitions (radiative recombination)
  - $A^{z+} + e \rightarrow A^{(z-1)+} + h\nu$
- Bound-bound transitions
  - $A_j^{z+} \rightarrow A_i^{z+} + h\nu$



Bremsstrahlung (free-free)

- Calculation is straightforward for Maxwellian electrons off bare nuclei of Z:

$$\varepsilon_\lambda^{ff}(\lambda) = \frac{32\sqrt{\pi}c(\alpha a_0)^3 Ry}{3\sqrt{3}} N_Z n_e Z^2 \left(\frac{Ry}{T_e}\right)^{1/2} \frac{1}{\lambda^2} e^{-\frac{hc}{\lambda T_e}} G^{ff}(T_e, \lambda)$$



- Total power loss $\varepsilon^{ff} = 4.51 \times 10^{-45} Z^2 \left(\frac{T_e}{Ry}\right)^{1/2} N_Z n_e \left[\frac{W}{sr \cdot cm^3}\right]$

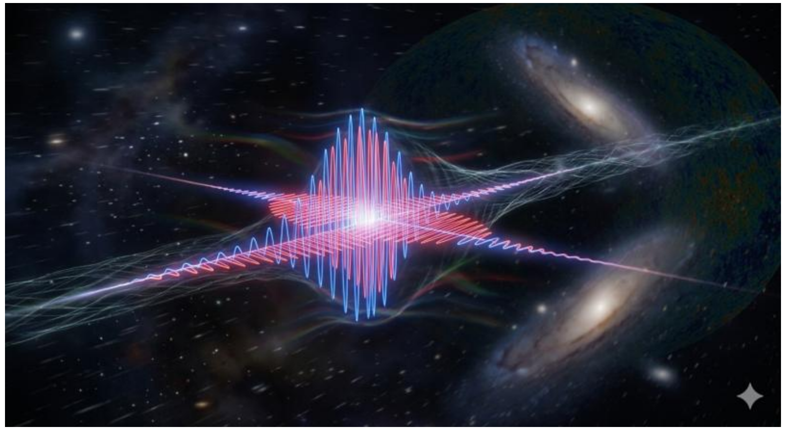
- Multicomponent plasma:

$$\varepsilon_\lambda^{ff}(\lambda) = z_{eff}\varepsilon_\lambda^{ff}(\lambda)[H]; \quad z_{eff} = \frac{1}{N_e}\sum_{i,z} z_i^2 N_z^i = \frac{\sum_{i,z} z_i^2 N_z^i}{\sum_{i,z} z_i N_z^i}$$

Dominant at longer wavelengths

Maximum emission at $\lambda_{\max} = \frac{620\ nm}{T_e[eV]}$

Photons



Quantum electrodynamics: there are two types of photons

Electric 2^J-pole:
Total angular momentum = **J**
Parity = $(-1)^J$

Electric-dipole E1
Electric-quadrupole E2
Electric-octupole E3
…

Magnetic 2^J-pole:
Total angular momentum = **J**
Parity = $(-1)^{J+1}$

Magnetic-dipole M1
Magnetic-quadrupole M2
Magnetic-octupole M3
…

Positive parity
Negative parity

Atomic Processes

Almost all relevant physics is inside this matrix element

$$\langle \Psi_f(a', b', c', \ldots) | \hat{O} | \Psi_i(a, b, c, \ldots) \rangle$$

final state

interaction operator

initial state

- Wavelengths
- Energies
- Transition probabilities (radiative and non-radiative)
- Collisional cross sections
- …

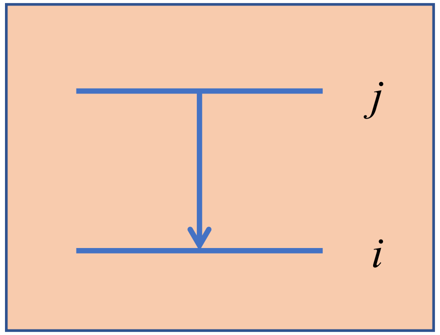
Radiative transitions: decay rate

- Classical rate of loss of energy: dE/dt ~ $|\mathbf{a}|^2$, and decay rate ~ $|\mathbf{r}|^2$ for harmonic oscillator

- Quantum treatment: $$\left\langle \Psi_f \middle| \vec{\varepsilon} \cdot \vec{p}\ e^{i\vec{k}\vec{r}} \middle| \Psi_i \right\rangle$$
  - $e^{ikr} = 1 + ikr + \ldots \approx 1$ (electric dipole or **E1** = allowed)
  - Length form: $\langle \Psi_f | r | \Psi_i \rangle$
  - Velocity form: $\langle \Psi_f | \nabla | \Psi_i \rangle$

must be equal for an exact wavefunction (*good test!*)

S, *f*, and A



- **Line strength**
  - Symmetric w/r to initial-final

$$S_{ji} = |\langle i \| r \| j \rangle|^2 = S_{ij}$$

- **Oscillator strength** (*absorption*)
  - $g_j f_{ij} = g_i f_{ji}$ $(g_j = 2J_j+1)$; dimensionless
  - Typical values for strong lines: ~ 0.1-1

$$f_{ji} = \frac{1}{3g_i}\frac{\Delta E}{Ry}S$$

- **Transition probability** (or Einstein coefficient)

$$A_{ij}[s^{-1}] = 4.34 \cdot 10^7 \frac{g_i}{g_j}(\Delta E[eV])^2 f_{ji}$$

- Typical values for neutrals: ~10^8 s^{-1}

Selection rules and Z-scaling

Fundamental law:
parity and J do not change

Before: P_j $\vec{J}_j$

After: $P_i \cdot P_{ph}$ $\vec{J}_i + \vec{J}_{ph}$

$P_{ph} = -1$
$J_{ph}(E1) = 1$

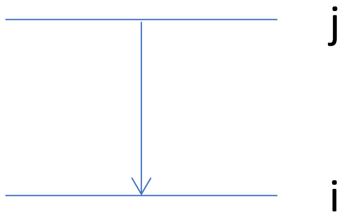
Exact selection rules:
$P_j = - P_i$
$|\Delta J| \leq 1$, $0 \not\to 0$ $(J_j + J_i \geq 1)$

Approximate selection rules (for LS coupling):
$\Delta S = 0$, $|\Delta L| \leq 1$, $0 \not\to 0$

Intercombination transitions: $\Delta S \neq 0$
$1s^2\ ^1S_0 - 1s2p\ ^3P_1$

E1: Z-scaling

$$a_n \sim \frac{n^2}{Z}$$



1. Basic matrix element $\langle \Psi_i | r | \Psi_j \rangle$ Z^{-1}
2. Line strength $S_{ji} = \left| \langle \Psi_i \| r \| \Psi_j \rangle \right|^2 = S_{ij}$ Z^{-2}
3. Oscillator strength $f_{ji} = \frac{1}{3g_i} \frac{\Delta E}{Ry} S_{ji}$

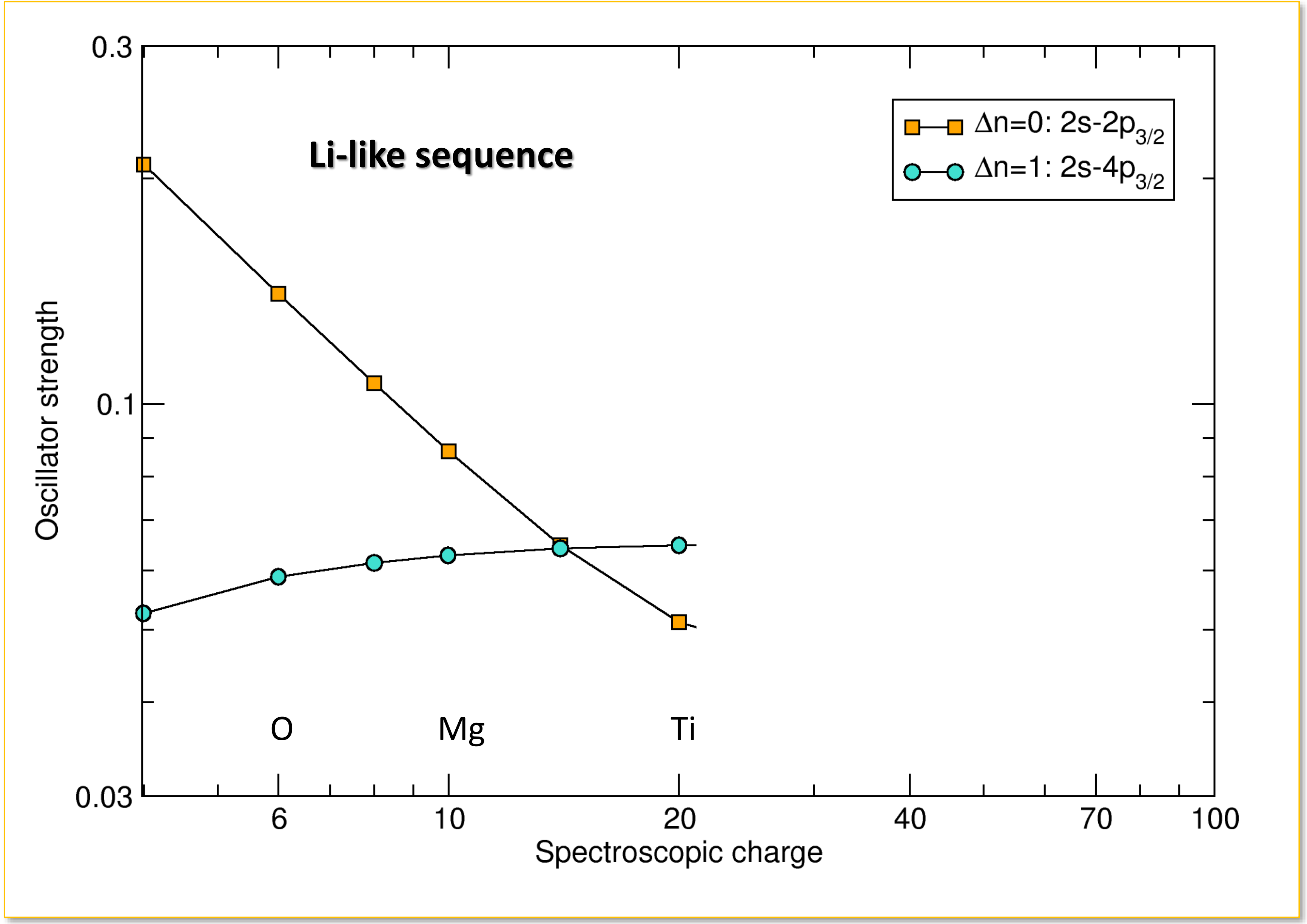
Δn=0	Δn≠0
Z^{-1}	Z^{0}

4. Transition probability $A_{ij} = 4.34 \cdot 10^7 \frac{g_i}{g_j} (\Delta E[eV])^2 f_{ji}$

Δn=0	Δn≠0
Z^{1}	Z^{4}

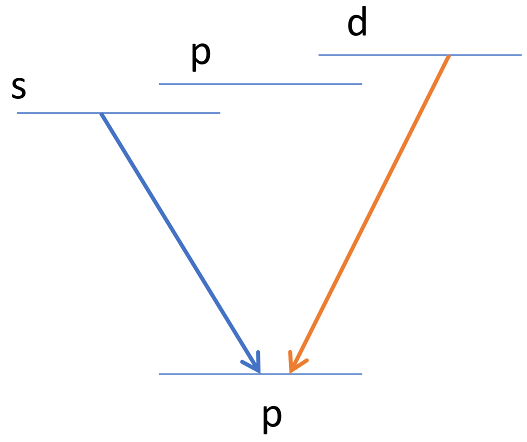
f(Δn=0) ~ 1/Z

f(Δn≠0) ~ Z^0

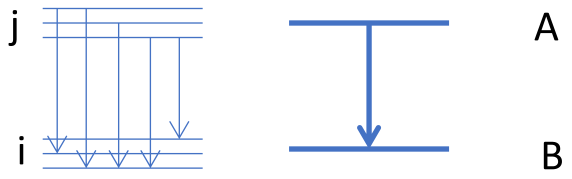


Some useful info

- "Left" is stronger than "right"
  - $f(\Delta l = -1) > f(\Delta l = +1)$
  - He I
    - $f(1s2p\ ^1P_1 - 1s3s\ ^1S_0)$ = **0.049**
    - $f(1s2p\ ^1P_1 - 1s3d\ ^1D_2)$ = **0.71**



- Level grouping
  - Average over initial states
  - Sum over final states
  - Example: from levels to terms
  - Any physical parameter



$$\alpha_{BA} = \sum_i \frac{\sum_j g_j \alpha_{ij}}{\sum_j g_j}$$

Level => term => configuration => …

Principal quantum number *n*

- *n*-dependence for *f*:

$$f(n_1 \rightarrow n_2) \approx \frac{32}{3\pi\sqrt{3}}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^{-3} \frac{1}{n_1^5}\frac{1}{n_2^3}$$

$$f(\Delta n = 1) \approx \frac{4}{3\pi\sqrt{3}} n \approx 0.245\boldsymbol{n}$$

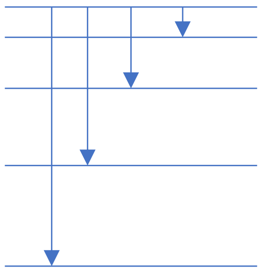
$$f(n_2 >> n_1) \propto \frac{1}{\boldsymbol{n_2^3}}$$

- *n*-dependence for *A*

$$A(n_2 >> n_1) \propto \frac{1}{n_2^3}$$

- Total radiative rate from a specific *n*

$$A_Z(n) \approx 1.6 \times 10^{10} \frac{\boldsymbol{Z^4}}{n^{9/2}} (s^{-1})$$



Forbidden transitions (high multipoles)

- QED: **En**, **Mn** (n=1, 2,…)
- E1/M1 *dipole*, E2/M2 *quadrupole*, E3/M3 *octupole*,…
- Selection rules
  - $P_j \cdot P_i$
    - +1 for M1, E2, M3,…
    - -1 for E1, M2, E3,…
  - J_{ph}(En/Mn) = n
- M3 and E3 were measured!

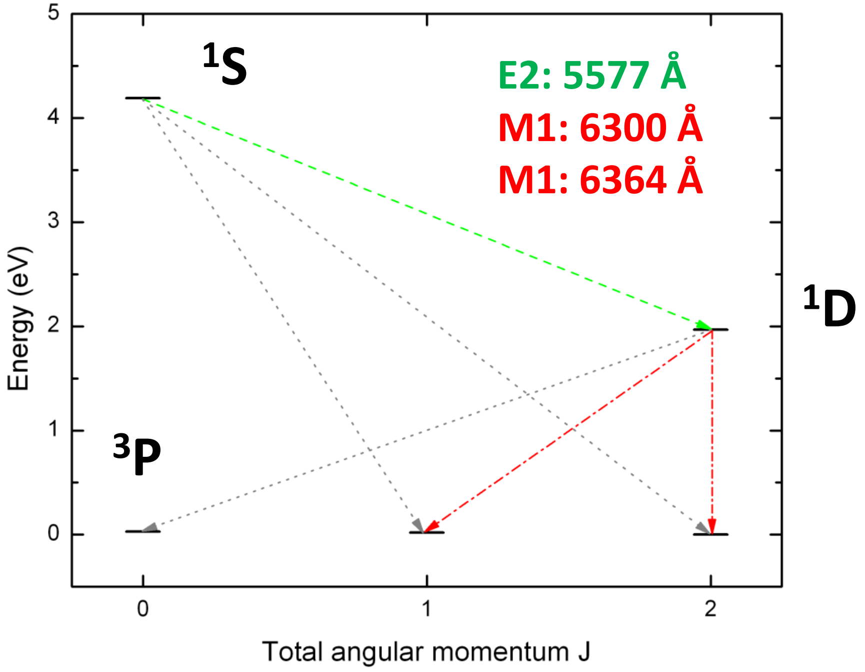
- Magnetic dipole (M1)
  - Stronger within the same configuration/term
  - $A \propto Z^6$ or stronger
  - Same parity, $|\Delta J| \leq 1$, $J_j + J_i \geq 1$
- Electric quadrupole (E2)
  - Stronger between configurations/terms
  - $A \propto Z^6$ or stronger
  - Same parity, $|\Delta J| \leq 2$, $J_j + J_i \geq 2$

Generally weak…

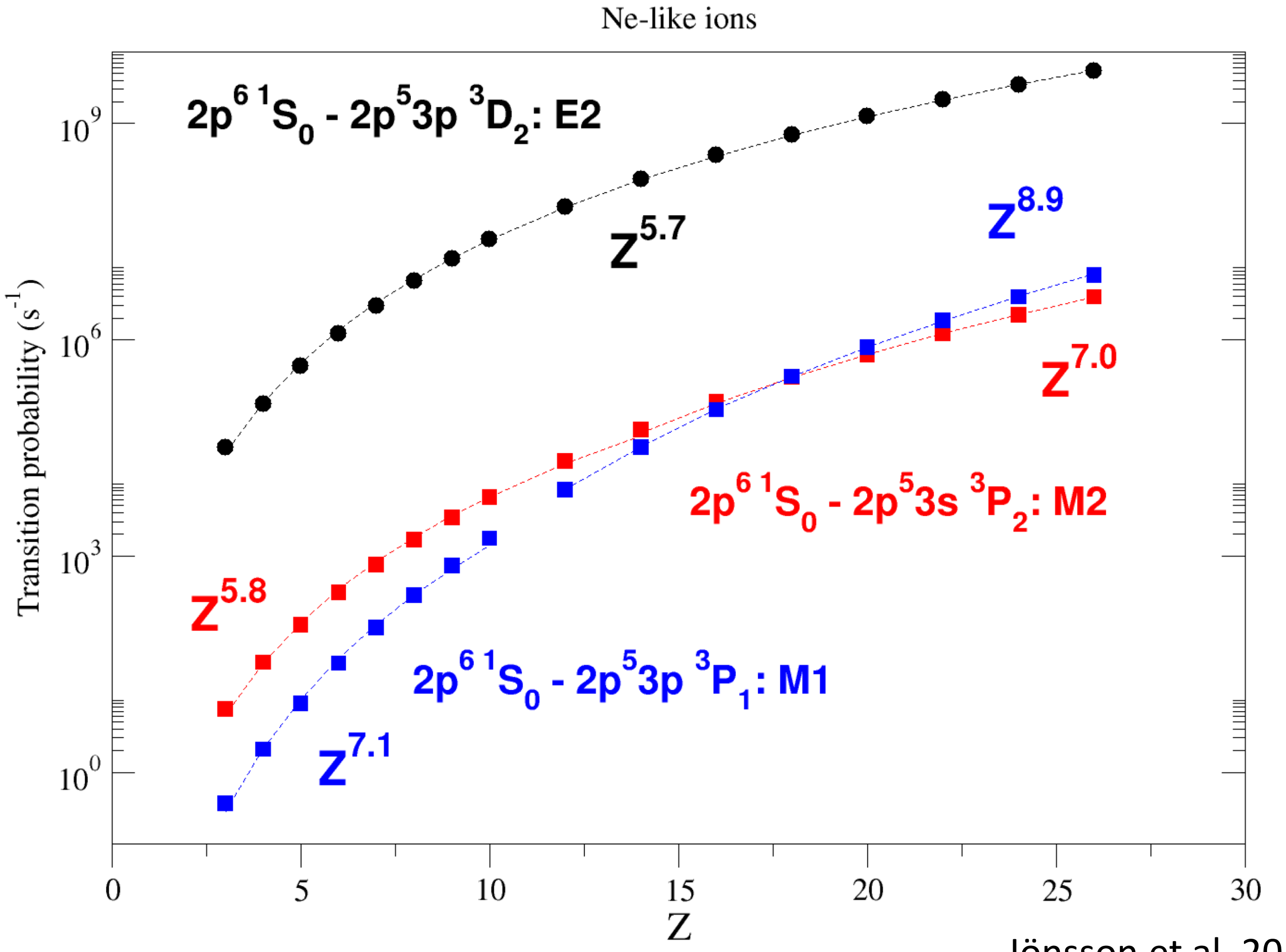
Forbidden transitions: auroras

Wavelength	Transition	A(s^{-1})
2958	$^1S_0$-$^3P_2$	E2: 2.42(-4)
2972	$^1S_0$-$^3P_1$	M1: 7.54(-2)
5577	**$^1S_0$-$^1D_2$**	**E2: 1.26(+0)**
6300	**$^1D_2$-$^3P_2$**	**M1: 5.63(-3)**
6300	$^1D_2$-$^3P_2$	E2: 2.11(-5)
6364	**$^1D_2$-$^3P_1$**	**M1: 1.82(-3)**
6364	$^1D_2$-$^3P_1$	E2: 3.39(-6)
6392	$^1D_2$-$^3P_0$	E2: 8.60(-7)

O I $2p^4$



Scaling in Ne-like ions

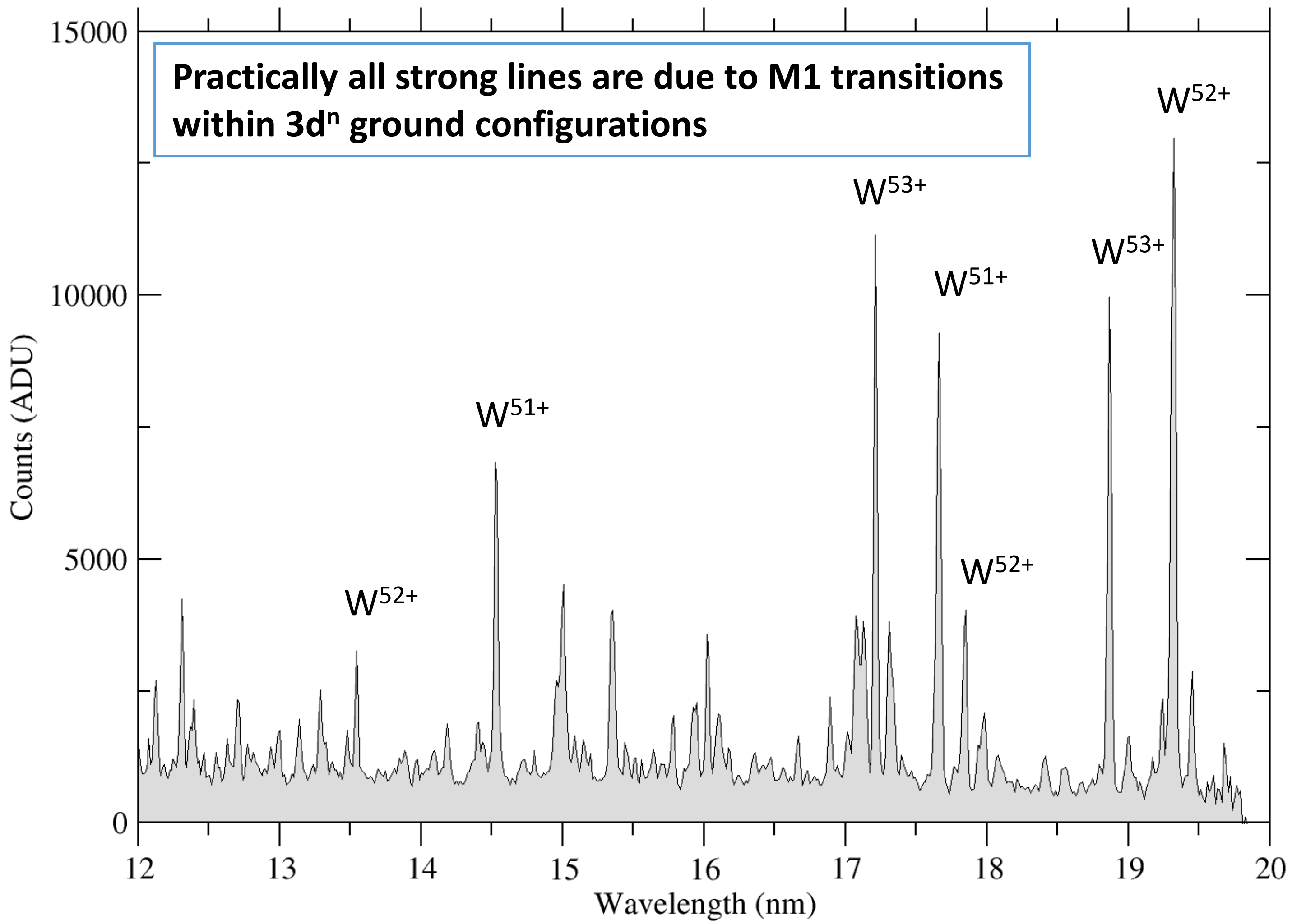


Jönsson et al, 2011

Forbidden transitions: highly-charged W

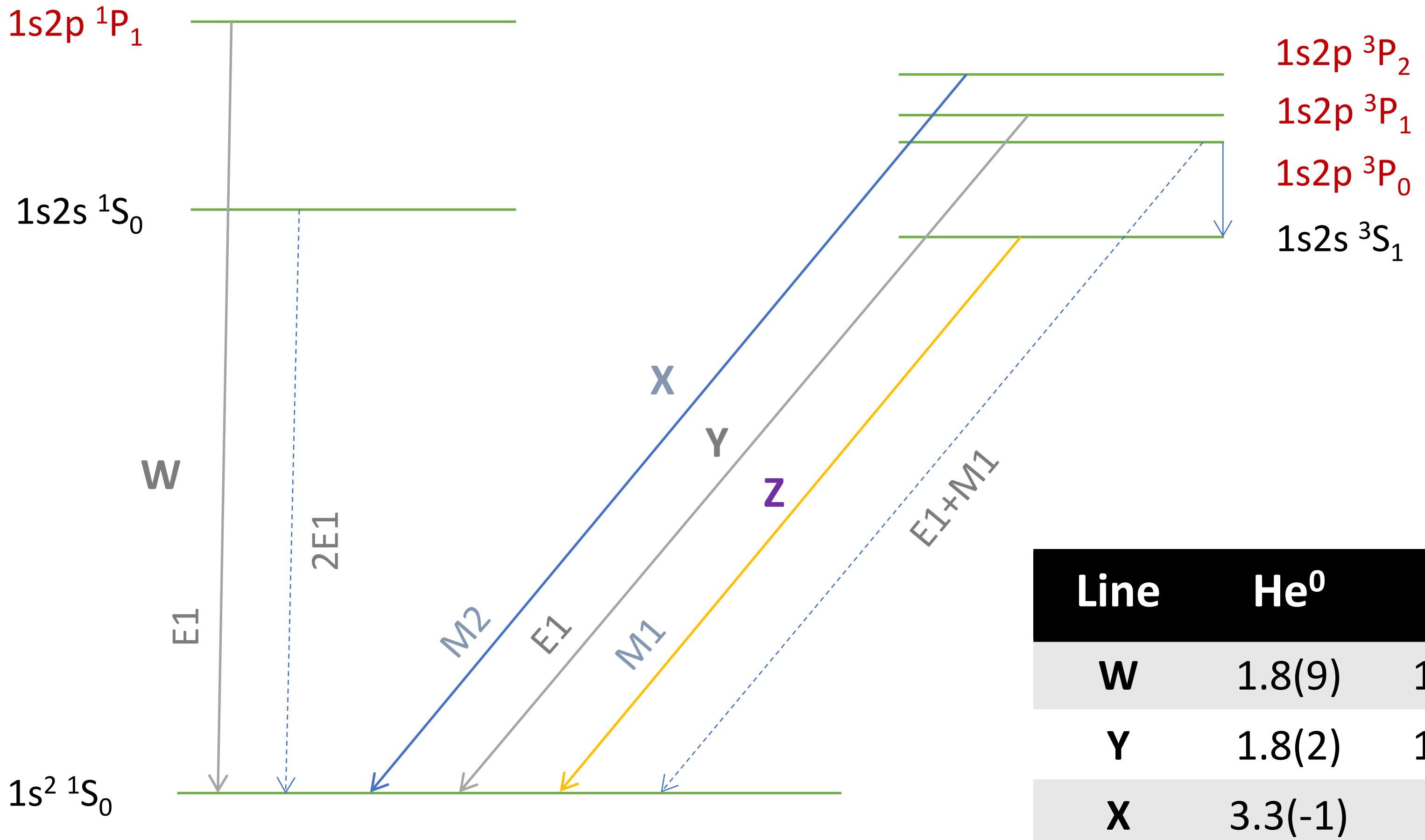
$A \sim 10^4$-10^6 s^{-1}

EBIT spectrum:
E_B = 5500 eV



NIST EBIT

He-like n=2 levels and lines



Line	He^0	Ar^{16+}	Fe^{25+}	Kr^{34+}	Scaling
W	1.8(9)	1.1(14)	4.6(14)	1.5(15)	**Z^4**
Y	1.8(2)	1.8(12)	4.4(13)	3.9(14)	**Z^{10} to Z^4**
X	3.3(-1)	3.1(8)	6.5(9)	9.3(10)	**Z^8**
Z	1.3(-4)	4.8(6)	2.1(8)	5.8(9)	**Z^{10}**